FEEDBACK COMMUNICATION SYSTEMS
FOR TIME-VARYING CHANNELS

THESIS

for

DOCTOR OF PHILOSOPHY

at the

UNIVERSITY OF ASTON

IN BIRMINGHAM

by

RAJAN SRINIVASAN

AUGUST, 1975

DEPARTMENT OF ELECTRICAL ENGINEERING

ACKNOWLEDGEMENT

The author is grateful to Dr. Ronald L. Brewster for his invaluable guidance and constant encouragement throughout the course of this work, and to Professor J.E. Flood and the University of Aston for the award of a research studentship.

He is also grateful to his fellow research students for their kind assistance and to Miss N. Freeman for her patient typing of the manuscript.

SUMMARY

The growing use of centralized data processing systems has led to a demand for data transmission facilities from remote locations, mobile stations, and even from space. To meet this demand the possibilities of data transmission over radio channels have been widely investigated. A major problem encountered on such channels is that of fading. This thesis is concerned with the possibility of using feedback to combat the effects of fading on digital communication systems.

A number of interesting techniques have appeared in the literature but these have usually been based on the assumption that the receiver obtains perfect knowledge of channel conditions. In the system described in the first part of this work, this assumption is removed by the simple expedient of devoting some transmitter energy to a channel estimation pilot-tone signal. Channel information thus derived by the receiver is fed back to the transmitter to control transmitter power and/or transmission rate. The resulting improvement in receiver error probabilities is calculated. It is shown that this system is superior in performance to dual and fourth-order diversity systems.

In the second part, a feedback system is proposed which uses two-level transmitted power, controlled by an intermediate decision fed back from the receiver. This is essentially a nonsequential decision feedback system based on Schalkwijk's generalised centre of gravity feedback technique. The system has been assessed analytically and by

analog computer simulation to determine its superiority over corresponding feedback - less systems. Though not matching the variable rate system in performance, it has the advantage of simplicity and is therefore easily implementable.

Both techniques described are especially applicable to low-data-rate multi-tone modems for short distance HF channels where propagation delays are small in comparison with baud lengths. Performance comparisons with existing data transmission systems such as Janet and Kathryn are given.

CONTENTS

LIST OF SYMBOLS

a_1, a_2	Independent identically distributed zero-mean Gaussian channel processes
V	envelope of the channel processes (Rayleigh distributed)
v	dummy variable for V
β	V^2, the squared envelope
b	dummy variable for β
θ	uniformly distributed channel phase
$\sigma_a^{\ 2}$	variance of the processes a_1 or a_2
p	probability density function
$p_V(v)$	probability density function of the channel envelope as a function of v
$p_\beta(b)$	probability density function of the squared envelope as a function of b
$P_V^e(v)$	probability of error given that the channel envelope $V = v$
$P_\beta^e(b)$	probability of error given that the squared envelope $\beta = b$
P_e	probability of error
$R(.)$	instantaneous rate
R_{av}	average transmission rate
R_{max}	maximum transmission rate
$r(.)$	instantaneous normalized rate
r_o	maximum normalized rate
λ, μ	Lagrange multipliers
$\overline{x}$	expectation or average value of the random variable x
$\hat{x}$	estimate of the random variable x
ε_x	estimation error for the random variable x
T	signalling period, i.e. duration of one bit

T_m	duration of channel measurement signal
T_s	duration of information signal
Δ	round-trip propagation delay
$n(t)$	white Gaussian noise
N_o	single sided noise power density
E	transmitted energy/bit in information signal
E_m	transmitted energy/bit in channel measurement signal
E_T	total transmitted energy
P_s	power content of information signal
P_m	power content of measurement signal
α_e	ratio of information and measurement energies
α_p	ratio of information and measurement powers
γ	total transmitted energy to noise ratio.

CHAPTER 1

INTRODUCTION

1.1 Introduction

The introduction of centralized control systems in the early 1950's brought with it the need to communicate to the control centre from remote locations. Where these locations are especially inaccessible, it is convenient to establish contact via radio rather than the use of the more conventional telephone link. A number of radio bands are, however, subject to fading. The last decade has, therefore, seen an increasing interest in the use of feedback links to combat the effects of fading on digital transmissions. A number of interesting and potentially successful systems have arisen out of this interest. These systems have shown definite improvement over conventional diversity techniques.

The basic principle regarding the use of feedback links in fading channels has centred around controlling one or more of the transmitter parameters, through knowledge of channel conditions fed back from the receiver. The techniques are applicable in that region of the radio spectrum where fading is frequently encountered, i.e. at hf, vhf and uhf. This chapter contains a brief review of fading channels and the work to date on feedback systems. This provides the necessary basis for the formulation of the problems studied in this thesis.

1.2 An Outline of Fading Channels

In many important radio applications communication channel characteristics are often strongly nonstationary, as evidenced by fading of the received signal. The two major channels in which fading is encountered in practice are the ionospheric high frequency 'skywave' and tropospheric scatter at vhf and above. These channels usually exhibit both fading and dispersive characteristics.

Fading dispersive channels are usually best described as random linear time-varying filters. They are characterized mathematically by the statement: conditioned on the transmitted waveform the received waveform is a sample function of a Gaussian stochastic process. An equivalent statement is that the impulse response of the filter is a Gaussian random process. Hence the statistical characterization of a fading dispersive channel is complete by specifying the mean and correlation function either of the channel's random impulse response or of the received process conditioned upon the transmitted waveform. Moreover, many channels are adequately modelled by taking the mean to be zero [31]. Hence, in this thesis, attention is restricted to such channels.

A vast body of literature is available on fading channels, their characterizations and the analyses of performances of communication links in fading environments [1 - 35]. For simplicity and convenience a brief consideration of the major aspects of fading channels is given here.

Given the channel input, i.e. the transmitted signal

$$s(t) = \mathrm{Re}\left[u(t)\ \exp(j2\pi f_o t)\right]$$

where Re denotes the real part of $[\cdot]$, $u(t)$ the complex envelope of $s(t)$ and f_o the nominal carrier frequency, the complex envelope of the received channel output signal is

$$w(t) = \int U(f)\ H(f,t)\ \exp(j2\pi ft)\ df$$

$$= \int u(t - \xi) g(t, \xi) d\xi$$

where $U(f)$ is the Fourier transform of $u(t)$, $H(f,t)$ is the time varying transfer function of the channel and $g(t,\xi)$ the time varying channel impulse response, both being complex zero-mean gaussian random processes. Under this characterization the magnitude and phase of the channel transfer function or impulse response have respective Rayleigh and uniform probability density functions. This same statement applies to the channel output process since the channel acts as a linear filter.

The time and frequency selective behaviour of the fading dispersive channel can be described with the aid of the channel functions $H(f,t)$ and $g(t,\xi)$. Various other methods of representing random channels are available in the references cited earlier. Of interest here, are three special cases of fading channels that are most frequently encountered in practice. These are:

(i) the nonselective nondispersive fading channel

(ii) the frequency selective (time dispersive channel) and

(iii) the time selective (frequency dispersive) channel.

The nonselective fading channel, also called the flat-flat channel, is characterized by an output signal containing a constant complex gaussian channel multiplier, implying thereby that the received signal differs from the transmitted signal only in an amplitude change and a phase shift, so that if the transmitted signal is

$$s(t) = \mathrm{Re}\left[u(t)\exp(j2\pi f_o t)\right], \quad 0 < t < T$$

the received signal is

$$y(t) = v\mathrm{Re}\left[u(t)\exp(j2\pi f_o t - \theta)\right], \quad 0 < t < T$$

where v and θ are statistically independent random variables that are constant over the observation period T, the former being Rayleigh and the latter being uniformly distributed in the interval 0 to 2π. This simple model of a fading channel has received considerable attention in the past and the analysis of most digital links has been carried out using this model.

The frequency selective fading channel, also called the time flat fading channel, is characterized by an input output relationship of the form

$$W(f) = U(f)\ H(f)$$

where U(f) is the Fourier transform of the transmitted complex envelope, W(f) the transform of the received envelope and H(f) the random transfer function of the channel which is a zero mean complex gaussian process as is W(f) when U(f) is specified. Thus this type of channel is modelled as a random but time-invariant filter. A manifestation of frequency selectivity, or equivalently, time dispersion, is the presence of time spreading or distortion, or both, in the received waveform.

The time selective fading channel or the frequency flat channel, is characterized by the input output relationship

$$w(t) = g(t)\ u(t)$$

where g(t) is the time varying complex zero mean gaussian channel gain. That is, the channel may be thought of as producing the received waveform by amplitude and phase modulating the transmitted waveform with the magnitude and argument of g(t), respectively. This type of fading is also termed fast fading, since the channel selectivity alters certain time segments of the transmitted waveform.

In conclusion, channels that are dispersive in both time and frequency are called doubly dispersive and such channels generally exhibit both time selective and frequency selective fading. Because of this, they are difficult to analyse and have not received as much attention as the simpler cases of fading.

1.3 A Brief Survey of Feedback Communication Systems

A number of papers on feedback communication systems have appeared since 1966[36 - 57], describing a variety of feedback schemes. Schalkwijk's[53] center of gravity feedback technique represents a generalisation of a number of these schemes. The technique depends on a simple property of additive noise channels: it is possible to subtract a common signal at the transmitter and add it back in at the receiver without affecting the probability of error. Thus the noiseless feedback channel is used to shift the centre of gravity of the signal progressively, both at the transmitter and receiver, in such a way as to minimize transmitted power. One of the feedback systems proposed in this thesis is a straightforward application of this principle to fading channels.

Since the modulation in both forward and feedback links can be varied, and the channel models are subject to variation, every author has his own variation of feedback strategy. The two basic modes of operation, however, involve sequential and nonsequential feedback. Sequential feedback systems are those in which the decision times are not fixed a priori, but depend upon the noise in that a sequential test is performed at the receiver. Typically the receiver calculates the likelihood ratio and terminates the test when it crosses a given threshold. The transmitter is then informed of the time of decision or continuously informed of the received signal (or some function of it) so as to continuously optimize the transmitted waveform.

Nonsequential systems use fixed-length transmission blocks in which there are typically N forward transmissions and N-1 feedback transmissions. At the end of each forward transmission the receiver tells the transmitter something about what has been received and the next forward transmission is suitably modified. The aim is to use the feedback information in such a way as to minimize the average transmitter power for a given error probability. When the receiver uses analog feedback to communicate say, the a posteriori probabilities back to the transmitter, the system is said to employ information feedback. If the feedback is digital and consists of tentative decisions, such as which symbol is most likely at a given time, the system is one of decision feedback. The various schemes have been shown to afford considerable performance improvement over feedback-less systems. It must be mentioned, however, that extension of the simple noiseless feedback model to allow for more realistic situations, such as noisy feedback channels and peak power constraints, reveals a certain sensitivity in the ideal feedback results.

Turning our attention to feedback communication systems for fading channels, Glave [58] has proposed an intermittent on-off noiseless feedback scheme for slow and fast Rayleigh fading. The scheme uses nonsequential decision feedback in that the signalling interval is divided into a number of subintervals with the receiver computing the log-likelihood ratio at the end of each subinterval and feeding this back to the transmitter. The transmitter then adjusts the amplitude of the signal according to the value of the log-likelihood ratio in a pre-determined manner. Performance bounds are derived and improvements over the equivalent one-way communication system are calculated. The system becomes rather complicated, however, when more than two sub-intervals are used. A typical power saving of 7 dB is achieved for slow fading and two subintervals.

Hayes[59] has used the concepts of receiver adaptivity and feedback in an attempt to combat the multiplicative noise effects of fading channels. The feedback channel is assumed noise free and delayless, and it has also been assumed that the receiver learning of the channel state is perfect. A slow nonselective Rayleigh fading model of the channel is used and the fading is assumed slow enough to ensure that no distortion of the digitial signalling pulse occurs. The receiver tracks the channel gain perfectly and this is communicated to the transmitter via the feedback link, whereupon the transmitter adjusts its transmitted power according to an optimum rule designed to minimize the probability of error in the receiver. The scheme has been shown to be reasonably effective in combating fading. A more detailed discussion is presented in Chapter 2.

Cavers[60] suggested the technique of modulating the rate of transmission of the information bits in accordance with variation of channel gain. Again a similar set of assumptions concerning the feedback link and fading channel have been made and it has been shown that, under idealized conditions, feedback rate control can completely eliminate the effects of fading. The effects of propagation delay have been taken into account and performance degradation due to delay has been evaluated. Again, Chapter 2 contains a more complete evaluation of Caver's work.

An adaptive technique was suggested by Montgomery[76] in relation to meteor-burst communication (JANET[77]) systems. Here, the transmission rate or bandwidth are made functions of the received signal-to-noise ratio and varied in an effort to reduce receiver error probability. An examination of the theoretical analysis reveals that this system is identical to feedback rate control as proposed by Cavers.

Van Duuren[78] suggested a feedback system for Rayleigh fading channels wherein detected errors at the receiver cause repetition of transmitted signals at the transmitter. This is a fairly succesful technique and is described in more detail in Chapter 3.

Skinner and Cavers[61] applied the techniques of feedback power control to a diversity system operating through a Rayleigh fading channel. The power levels in the diversity subchannels are allowed to vary independently and the effect of round trip delay on this 'selective diversity' system is evaluated. It has been shown that the scheme is a potentially successful one for low distance ionospheric hops.

Of other adaptive systems worth mentioning are the AN/GSC-10 (KATHRYN[79]) and the system proposed by Betts[80]. Both these systems use pilot tone sounding of the HF channel to control respectively, transmission rate and frequency of operation. Again, more detailed descriptions and comparisons with the systems suggested in this thesis can be found in Chapter 3.

In 1974, Hentinen's paper[62] on adaptive transmission for fading channels was more in the nature of a brief review and devotes some attention to the problem of controlling alphabet size using feedback. Essentially no new results have appeared here.

This completes a brief summary of previous work on feedback systems for fading channels.

1.4 Scope of the Present Study

The communication system has been assumed to operate in a flat-flat fading channel with the fading slow enough to ensure that the channel gain is constant at least over one signalling interval but varies from baud to baud. Two feedback systems have been proposed.

The first solves the problem of optimum control of transmitter parameters by a noise-fee feedback link when receiver learning of channel conditions is imperfect. In fact, at the transmitter, a certain fraction of the total transmitted energy is allocated to a pilot tone signal which is used by the receiver to generate estimates of the channel processes. These estimates are fedback to the transmitter and are used to optimally modulate the power content or duration (and hence overall transmission rate) of the information-carrying binary signal. The technique removes the need to assume perfect channel learning at the receiver and leads towards a more realizable system with respect to round trip propagation delays owing to the fact that the information is transmitted only upon transmitter reception of channel estimates.

Pilot tone or transmitted reference systems[63-65,79,80] have received some attention in the past, but none have reported great success. In the present context however, it is shown that transmitted reference systems used with a feedback link provide potentially successful schemes for combating multiplicative noise (i.e. fading) with certain advantages.

The second system proposed is applicable mainly to short distance high frequency hops and is a straightforward application of the centre of gravity feedback technique to binary transmission over fading channels. During a first part of the signalling interval the receiver makes an inferior estimate of the transmitted binary signal and communicates this estimate to the transmitter via a noiseless feedback link. Upon reception of this estimate (the effect of the round trip delay is taken into account) the transmitter forms an error signal with the remaining portion of the signal that awaits transmission. This error signal is then subjected to a fixed though optimum gain and transmitted. In effect the transmitted power is then either zero or an increased value such that the average power over a large number of bits is constrained at a specified level. In the receiver, a locally generated signal, made up of the channel gain and the initial inferior message estimate, is added to the received signal. An energy advantage approximately proportional to the square of the transmitter gain is thereby realized for the purpose of detection. This advantage is due to the fact that the average transmitted energy is constrained to equal that of a feedbackless signalling system operating over an identical channel. In conclusion, this technique predicts reasonable performance improvement with the advantage of simplified implementability.

1.5 Summary of Results

It has been shown that feedback control of transmission power and/or rate via receiver derived channel estimates are promising techniques with performances that exceed those of dual and fourth-order diversity systems. In a comparison made with other adaptive systems using pilot tone signalling, it has been shown that feedback power control performs better than Bett's system for error rates of less than about 10^{-3} and signal energy to noise ratios of about 20 dB. Van Duuren's system of automatic repetition of signals, however, has a 1 dB advantage over power control at an error rate of about 10^{-3} but falls behind power control for error rates of less than about 10^{-4}.

On the other hand, feedback rate control, and especailly hybrid control of power and rate, are, in comparison with all other adaptive systems, closest in performance to that of an equivalent signalling system operating over a steady non-fading channel. The price paid for this enhanced performance is an increased system bandwidth and a transmission rate that fluctuates about a specified average. It is expected that this latter will lead to increased complexity in implementation.

The nonsequential decision feedback system suggested in the latter part of the thesis is, however, quite easily implementable. The performance of this system exceeds that of feedback power control up to error rates of about $10^{-2.5}$ but is about 1 dB poorer than Van Duuren's system for error rates of less than 10^{-3}. However, it exceeds Bett's system in performance for error rates of less than 10^{-3}. An analog computer simulation of this system results in performances that generally agree with the theoretical analysis.

In conclusion, it must be mentioned that, as a result of assumptions made regarding signalling conditions and round-trip propagation delays, the feedback techniques proposed here are applicable mainly to multi-tone modems for short range HF ionospheric links.

CHAPTER 2

FEEDBACK FORMULATION

2.1 Introduction

This chapter begins with a review of the work on feedback techniques carried out by Hayes and Cavers. A discussion of their results ensues and this forms the basis of the work described in the rest of the thesis. The problems to be considered are formulated mathematically and certain error probability relationships are derived which are necessary for their solution. A signalling scheme is envisaged consisting of a channel estimation pilot-tone signal together with a binary information signal set. Finally, the advantages of the scheme are evaluated.

2.2 Formulation of the Basic Feedback Problem

Throughout the thesis a slow nonselective Rayleigh fading model of the channel is used. For convenience it is specified here in a more complete manner.

Consider the input signal to the channel as

$$\cos \omega_o t, \quad 0<t<T$$

where T may be taken as an arbitrary signalling interval. The channel output under the assumption of slow nonselective Rayleigh fading is

$$V \cos(\omega_o t - \theta) \ , \qquad 0<t<T$$

where V is the Rayleigh distributed channel gain and θ is the statistically independent random phase shift introduced by the channel and is uniformly distributed over the range 0 to 2π. It is well known[66] that the channel output is in fact a zero mean gaussian process and can be rewritten as

$$a_1 \cos\omega_o t + a_2 \sin \omega_o t \ , \qquad 0<t<T$$

where a_1 and a_2 can be considered as channel processes that operate on quadrature components of the transmitted signal. In fact a_1 and a_2 are uncorrelated zero mean gaussian processes (variables in this case of slow fading) with variances denoted by $\sigma_a{}^2$. It can also be shown that[66] the channel gain and phase are expressible as

$$V = (a_1{}^2 + a_2{}^2)^{1/2}$$

and

$$\theta = \tan^{-1}\left(\frac{a_2}{a_1}\right)$$

with associated probability density functions

$$p_V(v) = \frac{v}{\sigma_a{}^2} \exp(-v^2/2\sigma_a{}^2) \ , \quad v>0$$

$$p_\theta(\theta) = \begin{cases} 1/2\pi \ , & -\pi<\theta<\pi \\ 0 \ , & \text{elsewhere} \end{cases}$$

and

$$p_\beta(b) = \frac{1}{2\sigma_a{}^2} \exp(-b/2\sigma_a{}^2) \ , \ b>0$$

where $\beta = V^2$ the squared channel gain. With this channel model, the feedback problem is described next.

The communication system uses a binary orthogonal signal set at the transmitter. The transmitted signal on hypothesis H_i, $i = 0, 1$, is

$$g(t) = E^{1/2} s_i(t) \cos\omega_o t \ , \ 0<t<T$$
$$i = 0, 1$$

where it is assumed that both hypotheses are equally likely and where E is the energy content in the signal, ω_o is the carrier frequency and T the signalling interval with the information bearing binary signal satisfying the orthogonality condition

$$\int_o^T s_1(t)\ s_o(t)\ dt = 0$$

The transmitted signal is subject to fading and the received signal is

$$f(t) = a_1 E^{1/2} s_i(t) \cos\omega_o t + a_2 E^{1/2} s_i(t) \sin\omega_o t + n(t).\ 0<t<T$$

where n(t) is independent additive noise which is assumed to be zero mean white gaussian with single sided power density N_o.

In the receiver, it is assumed that a_1 and a_2 are known exactly from bit to bit and these are fed back to the transmitter via the feedback link which is noise free. (The assumption of a noise free feedback link is one which has been used by most workers in the field. Actually, as will be seen later, the Rayleigh distributed channel envelope V rather than the channel processes a_1 and a_2 is fed back to the transmitter. It is assumed that sufficient protection against feedback noise can be provided by quantizing and coding the envelope V at the receiver before transmission over the feedback channel. Moreover, the slowly varying Rayleigh envelope is expected to have a lower bandwidth requirement (and consequently, a better signal to noise ratio) than the forward channel, especially where there are no rapid channel fluctuations from bit to bit). The feedback problem then is one of determining the optimum method of controlling the energy E or the transmission rate 1/T using the feedback channel processes, so as to achieve minimum probability of error communication, given that the receiver processes the received signal in an optimum manner.

It is well known[67] that the optimum receiver for Rayleigh faded orthogonal signals is the matched filter-envelop detector or, equivalently, the correlation-squarer receiver. In the case of varying energy/rate, the receiver is only required to be adaptive to energy/rate with the basic configuration remaining unchanged. The probability of error for correlation-squarer or incoherent detection, as it is sometimes called, conditioned on knowledge of the channel processes, is well known[68] and is given by

$$P_V^e(v) = \frac{1}{2} \exp(-Ev^2/2N_o)$$

where V is the Rayleigh distributed channel gain and the notation for probability indicating the probability of error given the value of the channel gain V. From this expression it can be deduced that only the channel gain (and not the phase) need be fed back to the transmitter. The feedback power or energy control problem is then solved by making the energy E a function of V and determining the optimum function that minimizes the average error probability

$$P_e = \frac{1}{2} \int_o^{\infty} \exp(-E(b)\ b/2N_o)\ p_\beta(b)\ db$$

where $\beta=V^2$, subject to the constraint

$$\overline{E} = \int_o^{\infty} E(b)\ p_\beta(b)\ db$$

which is the constraint on the average transmitted energy, the bar denoting the expectation operation. The minimization of the error probability is then the solution of

$$\frac{\partial P_e}{\partial E(b)} + \lambda \frac{\partial \overline{E}}{E(b)} = 0$$

where λ is a Lagrange multiplier determinable from the constraint equation.

The feedback rate problem is solved by recognizing that the energy E can be expressed in terms of the transmitted power and rate as

$$E = T \cdot \frac{1}{T} \int_{0}^{T} E\, s_i^2(t) \cos^2\omega_o t \; dt$$

where the term

$$\frac{1}{T} \int_{0}^{T} E\, s_i^2(t) \cos^2\omega_o t \; dt \quad \triangleq \; P$$

denotes the transmitter power and the inverse rate is defined by

$$\frac{1}{R} \triangleq T \quad .$$

Without loss of generality it has been assumed that the binary signalling waveform satisfies

$$\int_{0}^{T} s_i^2(t) \; dt = 2 \quad .$$

Hence the transmitted energy per bit is

$$E = P/R$$

and the instantaneous probability of error conditioned on the channel gain is

$$P^e_{\beta}(b) = \frac{1}{2} \; \exp(- Pb/2N_oR) \quad .$$

While the rate R is made a function of b and optimized, the power P is held constant, with the result that rate modulation becomes an indirect method of energy control. The average probability of error for a variable rate scheme of this type has been shown by Cavers to be

$$P_e = \frac{1}{2} \int_0^\infty r(b) \exp(-Pb/2N_o R_{av}\, r(b))\, p_\beta(b)\, db$$

where r(b) is an instantaneous normalized rate defined by

$$r(b) = R(b)/R_{av}$$

and R_{av} is the average transmission rate given by

$$R_{av} = \int_0^\infty R(b)\ p_\beta(b)\ db \quad .$$

This leads to the constraint on average rate as

$$\bar{r} = 1 = \int_0^\infty r(b) p_\beta(b)\ db \quad .$$

As before, the optimum rate function is found from

$$\frac{\partial P_e}{\partial r(b)} + \lambda \frac{\partial \bar{r}}{\partial r(b)} = 0$$

where λ is a Lagrange multiplier satisfying the average rate constraint.

2.3 Solution for Ideal Feedback Control

The problem formulation of the last section is based on the work done by Hayes and Cavers. In this section the solutions are given and the results discussed briefly.

The optimum energy (or power) function is determined by first using, for convenience, an average constraint on the transmitted energy to noise ratio rather than on the transmitted energy. Hence with

$$\gamma(b) \triangleq E(b)/N_o$$

the average error probability is

$$P_e = \frac{1}{2} \int_0^\infty \exp(-\gamma(b)b/2)p_\beta(b)\ db$$

with the constraint

$$\overline{\gamma} = \int_0^\infty \gamma(b)p_\beta(b)\ db \quad .$$

The minimization of the error probability then proceeds as

$$\frac{\partial P_e}{\partial\gamma(b)} + \lambda\frac{\partial\overline{\gamma}}{\partial\gamma(b)} = 0$$

which yields

$$\gamma(b) = 2\,\frac{\ln(b/\lambda)}{b} \quad .$$

The obvious constraint

$$\gamma(b) \geq 0$$

leads to the complete solution

$$\gamma(b) = \begin{cases} \dfrac{2\ln(b/\lambda)}{b}, & b \geq \lambda \\ 0\ , & b < \lambda \end{cases} ,$$

which then specifies the optimum method of modulating the transmitted energy in order to achieve minimum probability of error given by

$$P_e = \frac{1}{2}\int_0^\lambda p_\beta(b)\ db + \frac{\lambda}{2}\int_\lambda^\infty \frac{1}{b}\ p_\beta(b)\ db \quad .$$

The multiplier λ is chosen to satisfy the constraint

$$\bar{\gamma} = 2 \int_{\lambda}^{\infty} \frac{\ln(b/\lambda)}{b} \, p_{\beta}(b) \, db \quad .$$

From the solution it is observed that the transmitted energy is maximum in a middle range of channel gain. When the gain is low, little energy is wasted in attempting to compensate. In fact, below a certain threshold of channel gain, no energy is transmitted. The performance of the system is shown in Figure 2.1.

The optimum rate function is determined in a similar manner to that of energy above by solving

$$\frac{\partial P_e}{\partial r(b)} + \mu \frac{\partial \bar{r}}{\partial r(b)} = 0,$$

which yields

$$(1 + \frac{Pb}{2N_o R_{av} r(b)}) \exp(Pb/2N_o R_{av} r(b)) = \text{constant}$$

and this has the solution

$$r(b) = kb$$

where k is a constant determinable from the average rate constraint

$$\bar{r} = \int_{0}^{\infty} kb \, p_{\beta}(b) \, db = 1 \quad .$$

This yields

$$k = 1/2\sigma_a^{\,2} \quad .$$

The optimum rate function, therefore, is

$$r(b) = b/2\sigma_a^{\,2},$$

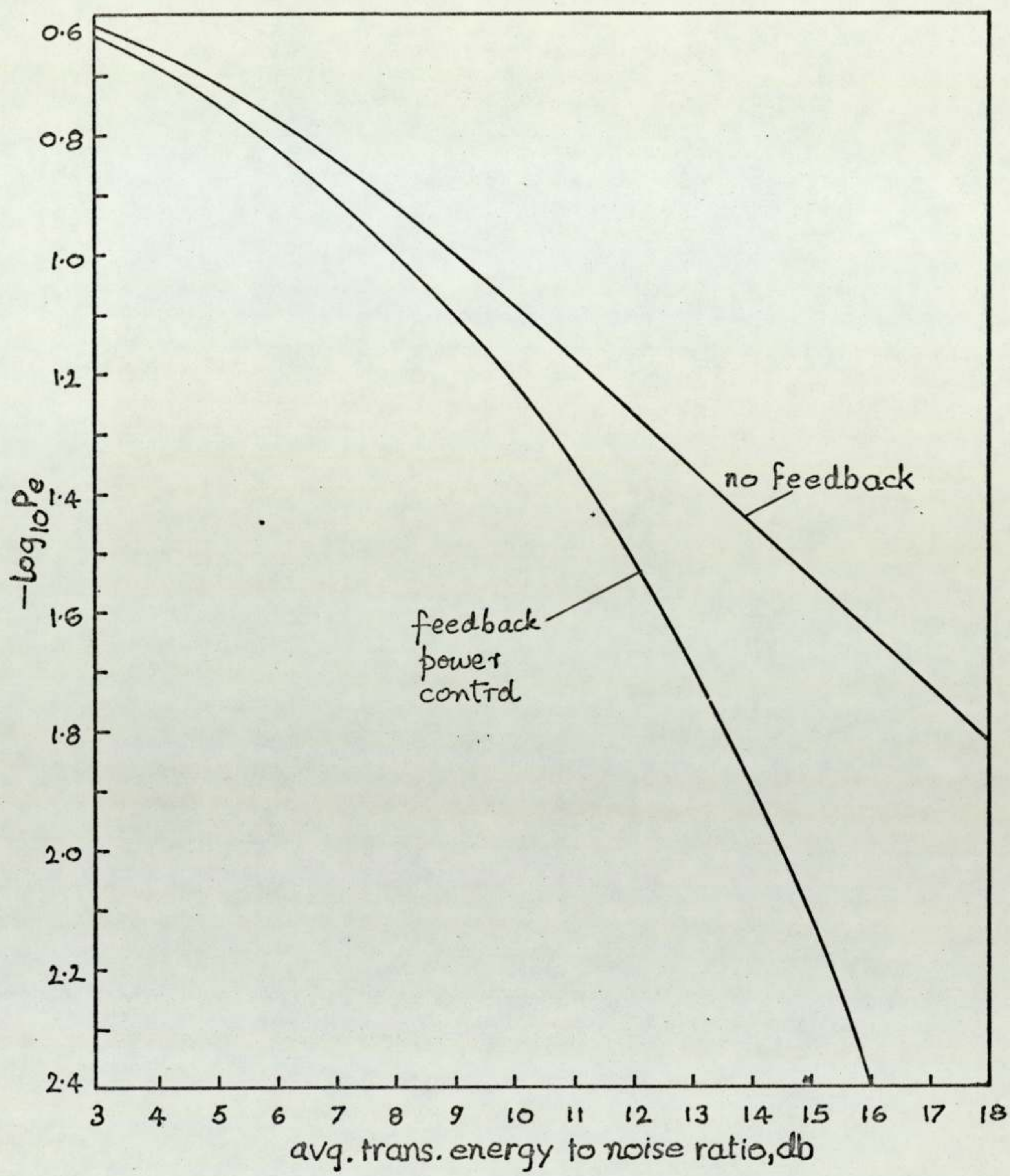


FIG. 2.1 Performance with feedback power control.

with the associated minimum error probability given by

$$P_e = \frac{1}{2} \int_0^\infty \frac{b}{2\sigma_a^2} \exp(- P\sigma_a^2/N_o R_{av})\ p_\beta(b)\ db$$

$$= \frac{1}{2} \exp(-P\sigma_a^2/N_o R_{av}) \quad .$$

Defining the average normalized transmitted energy to noise ratio as

$$\gamma = 2\sigma_a^2 P/N_o R_{av},$$

the error probability becomes

$$P_e = \frac{1}{2} \exp(- \gamma/2),$$

which is seen to be exactly the error probability of a feedbackless system using orthogonal signals and operating in a non-fading (steady) channel.

Feedback rate control according to the optimum rule derived above can, therefore, ideally, eliminate the effects of fading. The penalty paid for this improvement in performance over a feedbackless system is an unconstrained bandwidth (rate) which can become very large when the channel gain becomes very large. Cavers has therefore calculated the degradation in performance that results from imposing an upper bound i.e. a maximum rate constraint on the rate function. A rate function of the type

$$r(b) = \begin{cases} kb \quad , & b \leq r_o/k \\ r_o \quad , & b > r_o/k \end{cases}$$

has been used, where r_o denotes the maximum allowed expansion of the rate around the specified average rate R_{av} and k is again determined from the average rate constraint equation. The performances are shown in Fig.2.2. The degradation due to the maximum rate constraint is seen to be not very large and it is observed that feedback rate control is a potentially successful scheme that affords considerable improvement over diversity systems.

2.4 An Initial Formulation of the Feedback Problem

The results described in the previous sections (Hayes, Cavers) and most other work on feedback systems for fading channels are based on the assumption of perfect receiver learning of the channel state. This assumption, although furnishing performance bounds and reasonably simple insight into the action of feedback systems, is justified only in situations where the fading is essentially constant over a large number of bits. Moreover, although Cavers has calculated the performance degradation due to the effects of round trip propagation delay, no attempts have been made to compensate for it. Delays affect performance of feedback systems by preventing the transmitter from responding instantaneously to control signals from the receiver.

With this in view, a signalling scheme has been suggested which obviates the need to assume perfect channel measurement and compensates for small round trip delays. The overall feedback system, using this signalling, is thus realizable and provides an accurate prediction of system operation and performance.

In the last section it was seen that imposing a maximum rate constraint at the transmitter results in some performance degradation. To counteract this degradation, a hybrid form of feedback control has been suggested. The hybrid system effects transmitter power control whenever the rate saturate Performance bounds have been calculated assuming ideal channel learning

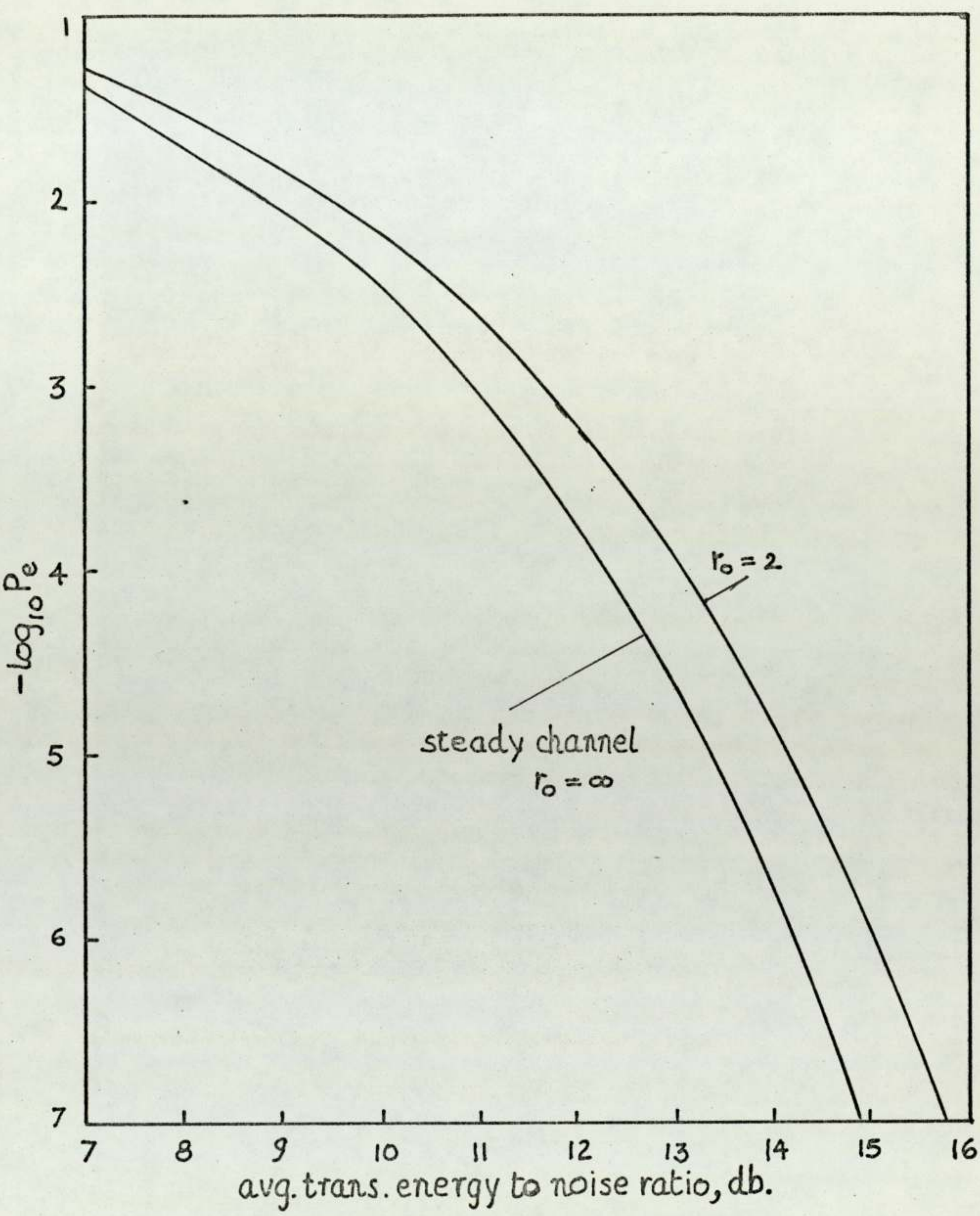


FIG. 2.2 Feedback rate control with and without a rate constraint.

conditions and the predicted performance is close to that of a system operating under a non-fading conditions.

Certain situations with respect to fading channels arise, however, where the presence of multipath necessitates low data rate transmission. In such cases, feedback rate control may not be very effective owing to rate limitations imposed by the channel. A simple decision feedback technique has therefore been suggested. The feedback link is used to convey to the transmitter the receiver's estimate of the transmitted message. Depending upon the receiver decision the transmitter then transmits an increased power or zero power. The transmitter is therefore not required to be explicitly adaptive to channel state, i.e. no continuous control of a transmitter parameter from bit to bit is needed. This implies that only the receiver is obliged to learn the channel conditions. The presence of round trip delay is taken into account while designing the transmitted signal and optimum receiver. It must be mentioned that the feedback technique is applicable, owing to the assumption of small magnitudes of propagation delay, to short distance ionospheric links.

2.5 Mathematical Preliminaries and Discussion of the Transmitted Reference Signalling Scheme

The signalling scheme used for the first feedback technique studied in this thesis envisages a channel estimation signal occupying a certain portion of the signalling time interval. This is followed by the information bearing binary antipodal set for the remaining portion of the interval. The channel estimation signal acts as a transmitted reference and is used to provide information about channel conditions to the receiver and hence, to the transmitter. Since the reference and the information signals occupy different time slots, they are orthogonal. The transmitted signal, before modulation onto a carrier, is

$$g(t) = \begin{cases} E_m^{\frac{1}{2}}\ s_m(t) & , 0 < t < T_m \\ \pm E^{\frac{1}{2}}\ s(t) & , T_m < t < T \end{cases}$$

where $s_m(t)$ denotes the channel estimation signal of duration T_m and energy content E_m, $s(t)$ is the baseband signal of duration $T-T_m$ and energy content E with the information contained in the polarity and T is the overall signalling interval. Hence

$$\int_0^T s_m(t)\ s(t)\ dt = 0$$

and it has been assumed, for convenience that

$$\int_0^T s_m^2(t)\ dt = \int_0^T s^2(t)\ dt = 2 \ .$$

This signal is modulated onto a carrier of angular frequency ω_o using double sideband suppressed carrier amplitude modulation. On transmission, it is subjected to slow non-selective Rayleigh fading. The received signal is perturbed by additive noise and is given by

$$f(t) = a_1 g(t)\ \cos\omega_o t + a_2 g(t)\ \sin\omega_o t + n(t), \quad 0 < t < T$$

where, as before, a_1 and a_2 are the gaussian channel processes and $n(t)$ is zero mean white gaussian noise with single sided power density N_o.

It is observed that, for $E = E_m$, i.e. equal energies in the estimation and information signals, the composite transmitted signal forms an orthogonal set. As a result of this, and the lack of exact channel information, the performance of the optimum receiver for the above signalling scheme must be identical to that of a correlation-squarer or incoherent (optimum) receiver for Rayleigh faded orthogonal signals. This is shown later in the section. It is also proved that optimum performance, without feedback, is achieved only when $E = E_m$. With feedback the same equality is needed to be maintained, but in an average sense. This is because the information energy is a random variable, being dependent on estimates of the channel gain.

In the receiver, the received signal is processed in the interval $(0, T_m)$ for estimates of the channel processes. These estimates are then available at the end of this interval. The estimator, shown in Figure 2.3 generates the maximum a posteriori estimates $\hat{a}_1$ and $\hat{a}_2$ of the processes a_1 and a_2. This is the simple case of linear estimation[67] of a gaussian parameter in independent white gaussian noise and the estimator consists of a correlator followed by an amplifier. The estimates are therefore given by

$$\hat{a}_1 = G(a_1 E_m^{\frac{1}{2}} + \int_0^{T_m} n(t)\, s_m(t) \cos\omega_o t\, dt) \tag{2.1}$$

and

$$\hat{a}_2 = G(a_2 E_m^{\frac{1}{2}} + \int_0^{T_m} n(t)\, s_m(t) \sin\omega_o t\, dt) \tag{2.2}$$

where the estimator gain is

$$G = \sigma_a^{\,2} E_m^{\frac{1}{2}} \Big/ (\sigma_a^{\,2} E_m + N_o/2) .$$

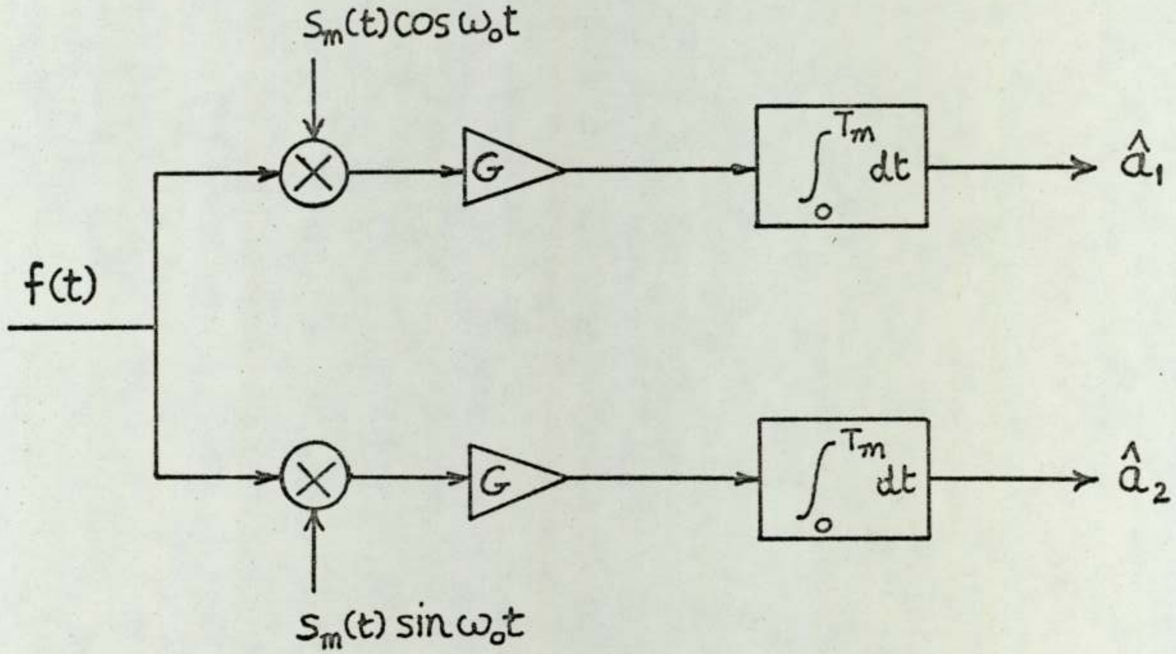


FIG. 2.3 Linear estimator for gaussian channel.

Since the estimation operation, i.e. correlation-gain, is linear, the outputs retain a gaussian distribution. A measure of the quality of these channel estimates is the estimation error, defined by

$$\varepsilon_a \quad \underline{\Delta} \quad \langle (a_1 - \hat{a}_1)^2 \rangle \quad / \quad \langle a_1^2 \rangle$$

$$\underline{\Delta} \quad \langle (a_2 - \hat{a}_2) \rangle \quad / \quad \langle a_2^2 \rangle \qquad (2.3)$$

where $\langle \, . \, \rangle$ indicates the expectation operation. This is, in fact the normalized mean squared error of the estimates and it can be shown using (2.1) and (2.2) in (2.3) that

$$\varepsilon_a = 1/(1 + 2\sigma_a^{\,2}E_m/N_o).$$

The estimation error decreases inversely for large transmitted estimation energy to noise ratios.

Because the estimates are gaussian, the estimated channel gain, denoted by

$$\hat{V} \;\underline{\Delta}\; (\hat{a}_1^{\,2} + \hat{a}_2^{\,2})^{\frac{1}{2}} \quad ,$$

is Rayleigh distributed with probability density function

$$p_{\hat{V}}(\hat{v}) = \frac{\hat{v}}{\sigma_a^{\,2}e_m} \exp(- \hat{v} / 2\sigma_a^{\,2}e_m), \quad \hat{v} > 0$$

where $e_m = \dfrac{2\sigma_a^{\,2}E_m}{N_o} \,\Big|\, (1 + \dfrac{2\sigma_a^{\,2}E_m}{N_o})$.

As a matter of interest it is noted that in the case of large energy to noise ratios i.e.

$$\frac{2\sigma_a{}^2 E_m}{N_o} \quad \gg \quad 1,$$

the factor e_m approaches unity and the statistics of the estimated gain can be approximated by those of the actual gain.

At the end of the time interval $(0,\ T_m)$, the decision portion of the receiver, shown in Figure 2.4 , begins processing the rest of the received signal , i.e. the Rayleigh faded antipodal information set. A decision regarding the transmitted signal is generated by the end of the total signalling interval. This is done by correlating, in two quadrature channels, estimates of the received signal with the actual received signal and comparing with a zero threshold. The probability of error, under the usual assumption of equally likely transmitted hypotheses, is

$$P_e = \text{Prob. } (M_c + M_s < 0) \quad ,$$

with the quadrature decision statistics given by

$$M_c = \hat{a}_1 (a_1 E^{\frac{1}{2}} + \int_{T_m}^{T} n(t)s(t)\cos\omega_o t \; dt)$$

and

$$M_s = \hat{a}_2 (a_2 E^{\frac{1}{2}} + \int_{T_m}^{T} n(t)s(t)\sin\omega_o t \; dt) \; .$$

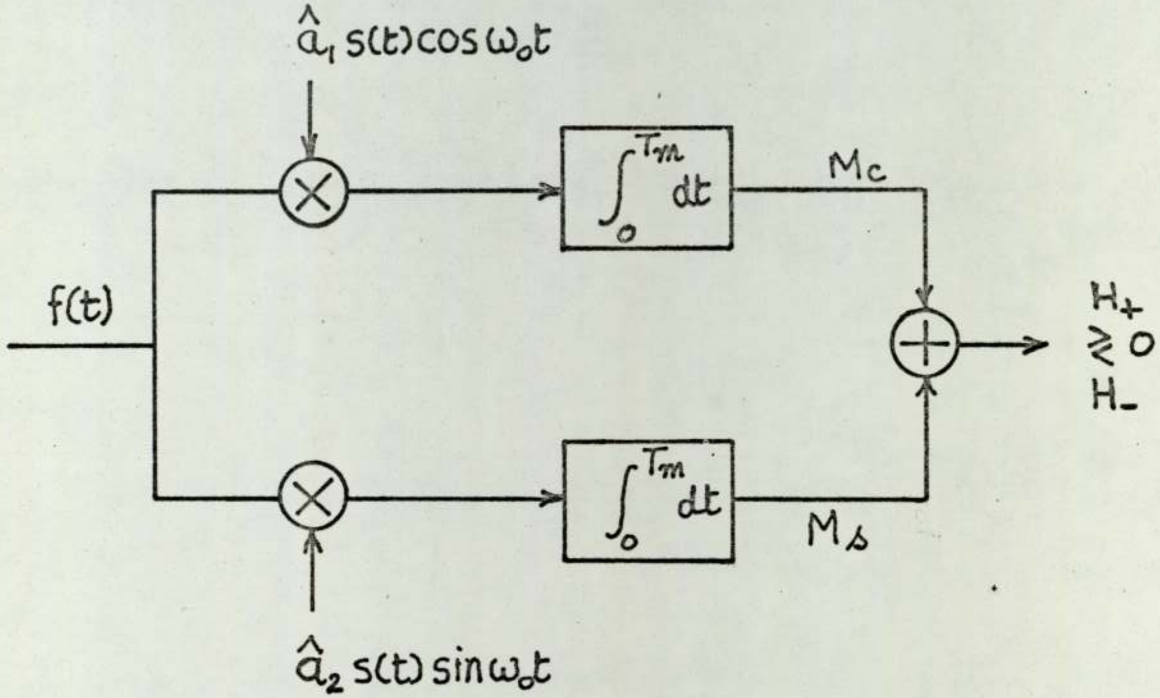


FIG.2.4 Receiver decision.

In the case of feedback, the estimates $\hat{a}_1$ and $\hat{a}_2$ are communicated to the transmitter (as will be seen, only $\hat{V}$ need be fed back) in order to enable control of power or rate. Thus the received signal contains parameters (energy, rate) that are actually random variables and functions of the estimates $\hat{a}_1$ and $\hat{a}_2$. Since these estimates are known at the receiver and transmitter, the optimum dependence of the signal parameters on them can be determined through an instantaneous error probability. In a statistical sense, this implies a probability of error given the values of the estimates $\hat{a}_1$ and $\hat{a}_2$. This conditional error probability is

$$P^{e}_{\hat{a}_1,\, \hat{a}_2} \;\underline{\Delta}\; \text{Prob.}\ (M_c + M_s < 0 \mid \hat{a}_1,\, \hat{a}_2).$$

Using a result on probability derived in the appendix, the conditional probability has been shown to be

$$P^{e}_{\hat{a}_1,\, \hat{a}_2} = \operatorname{erfc}\{\xi^{\frac{1}{2}}(\hat{v})\}$$

$$\underline{\Delta}\; P^{e}_{\hat{V}}\ (\hat{v})$$

where the above notation indicates probability of error given that the channel gain $\hat{V} = \hat{v}$,

$$\operatorname{erfc}(x) = (2\pi)^{-\frac{1}{2}} \int_{x}^{\infty} \exp(-\, t^2/2)\, dt,$$

and

$$\xi(\hat{v}) = \left(\frac{2\sigma_a^2 E/N_o}{1+2\sigma_a^{\,2}E_m/N_o} + 1 \right)^{-1} \frac{2\sigma_a^{\,2}E}{N_o} \; \frac{\hat{v}^2}{\sigma_a^{\,2}} \quad .$$

Since the conditional error probability is a function of $\hat{V}$, only the estimated channel gain need be fed back to the transmitter.

As yet no feedback has been introduced, and therefore, the average error probability of the straight system is evaluated by averaging the conditional error probability over the statistics of the estimated channel gain. That is

$$P_e = \int_o^\infty P^e_{\hat{V}}\,(\hat{v})\; p_{\hat{V}}\,(\hat{v})\; d\hat{v} \;,$$

which can be shown to be

$$P_e = \tfrac{1}{2}(1 - \mu)$$

where

$$\mu^2 = \frac{\sigma_a^{\,4}E\,E_m}{(\sigma_a^{\,2}E_m + \frac{N_o}{2})\,(\sigma_a^{\,2}E + \frac{N_o}{2})}$$

The above expression represents, in fact, the error probability of a communication system employing channel estimation detection of Rayleigh faded binary antipodal information signals with unequal energies in the estimation and information parts. The minimization of this error probability with respect to the energy division between the estimation and information signals is achieved by first defining the total transmitted energy as

$$E_T \triangleq E + E_m$$

and the ratio of information to estimation energies as

$$\alpha \triangleq E/E_m,$$

whereby

$$E = E_T\alpha/(1 + \alpha)$$

and

$$E_m = E_T/(1 + \alpha) .$$

These are introduced into the experession for μ, which is then maximized with respect to α by setting

$$\frac{\partial\mu}{\partial\alpha} = 0$$

and solving for the optimum α . This is equivalent to the minimization of P_e and it yields

$$\alpha_{optimum} = 1,$$

implying thereby that performance is best for equal energies in the estimation and information signals. Under this condition (i.e. $E = E_m$), the composite transmitted signal set is orthogonal with performance

$$P_e = (2 + 2\sigma_a{}^2 E_T/N_o)^{-1} \quad .$$

This is the well known result that, as far as a feedbackless system operating in a Rayleigh channel is concerned, orthogonal signals are the best.

The major advantage, however, of devoting part of the transmitted energy to channel estimation and using binary antipodal signals is that channel estimates are explicitly available in the receiver for the purposes of adaptivity and feedback at the end of the interval $(0, T_m)$. An examination of the optimum (incoherent) correlation-squarer receiver for orthogonal signals [67] reveals that MAP (maximum a posteriori) channel estimates are generated during the detection process. The estimation error for these is given by

$$1/(1 + 2\sigma_a{}^2 E_T/N_o) \; ,$$

since the total transmitted energy E_T of the orthogonal signals is used for estimation (as well as detection). This means that the estimates are nearly twice as good as the ones generated by channel estimation signalling.

There is, however, a disadvantage. Firstly, the estimates exist only in that branch of the receiver in which the hypothesis is true. In such a situation, there are two ways of obtaining waveforms representing channel estimates on either transmitted hypothesis. These are the techniques of decision-directed and non-decision-directed channel measurement.[69] Very simply, the first technique involves selection of, as channel estimates, the outputs of that branch which has been decided by the receiver to contain the true hypothesis. Obviously, this selection suffers from the same probability of error as the detection process itself; with the result that, on an average, such estimates will be inferior to MAP estimates. The second technique, of non-decision-directed channel measurements, involves summing up of the two branch outputs to generate channel estimates. Again, these estimates will not be as good as MAP estimates. This is simply due to the fact that the noise output of that branch of the receiver not containing the true hypothesis is an additively degrading quantity to the MAP estimate from the branch which does contain the hypothesis. The associated estimation error is therefore equal to that of channel estimation signalling. This is because the effect of the additional noise is equivalent to the use of only half the signalling energy for generating estimates. Secondly, although decision-directed techniques yield better channel estimates than non-decision-directed methods, the estimates are available only at the end of the signalling interval (O, T).

The availability of estimates at the end of the interval (O, T_m) (using channel estimation signalling) adds to the realizability of the feedback system. This is due to the fact that the information is transmitted only at the end of the portion of the bit interval devoted to the channel measurement signal. Whereas for orthogonal signalling even the assumption of zero propagation delay would require the channel estimate (which would only be available at the end of (O,T)) from the receiver to reach the transmitter at the start of the bit interval to be able to effectively control the transmitted power or rate of the signal, in the case of channel estimation signalling the transmitter has the channel estimates at the start of ((T_m, T)) transmission of the antipodal information set. Therefore the power or rate of the antipodal set can be modulated. Moreover, small round trip delays can easily be combated by allowing a compensatory duration gap between the channel estimation and information portions of the transmitted signal. Hence, for the purposes of receiver adaptivity and feedback, the above arguments and the requirement of bit by bit detection (with considerations of propagation delays) preclude the use of orthogonal signals (see Chapter 6), in favour of a channel estimation signalling scheme. It must be mentioned, however, that the requirement of small round trip delay is a result of the possibility of channel drift from bit to bit. The minimum range of the HF channel to ensure Rayleigh fading is around 200 miles, with a loop delay on the order of 2 milliseconds. This would then require baudlengths which are typically tens of milliseconds. Therefore, with typical fading rates of 1 to 10 hz, the main application of the signalling scheme is to long baud multi-tone modems for short range HF channels.

CHAPTER 3

FEEDBACK CONTROL

3.1 Introduction

The solutions to the problems of feedback control of transmitter power and/or rate are presented in this chapter, with a discussion of the results.

3.2 Problem formulation

Given the channel estimates that were studied in the last chapter the optimum methods of controlling the power content or inverse duration (and hence, overall rate) of the antipodal information set are determined. This is done with a view to minimizing the receiver error probability.

Under the assumption of perfect channel knowledge, bounds on performance of a hybrid system utilizing simultaneous power and rate modulation are derived. This is intended for situations where a maximum rate constraint on the transmitter has been imposed.

3.3 Feedback power control

The feedback communication system is shown in Figure 3.1. Control of transmitter power is effected by modulating the energy content in the antipodal set while keeping the energy content in the measurement signal constant. It is noted that thus far the terms power and energy have been used interchangeably. In the case of transmitter power control, this does not present a problem since the transmission rate is a fixed quantity; which implies that power and energy ratios are identical for the nominal condition $T_m = T/2$. For the case of varying rate (section 3.4), however, a distinction is necessary and has been

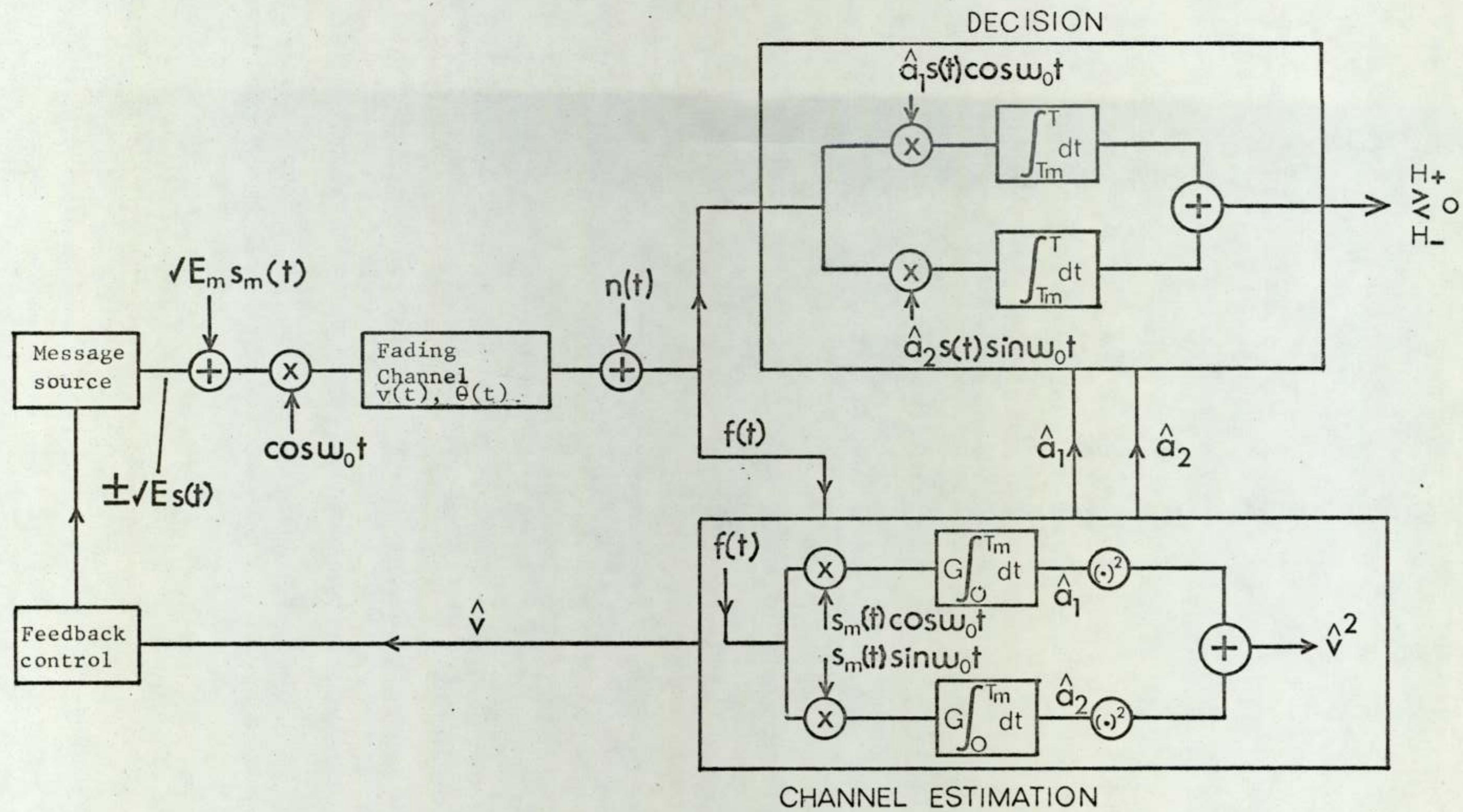


FIG.3.1 Communication system with feedback of estimated channel gain.

made.

Hence, a ratio between the energies of the information and estimation signals is defined as

$$\alpha_e \triangleq E/E_m$$

and this is made a function of the fedback estimated channel gain. The total transmitted signal, before modulation onto the carrier, can then be written as

$$g(t) = \begin{cases} E_m^{\frac{1}{2}} \, s_m(t) \,, & 0 < t < T_m \\ \pm\, \alpha_e^{\frac{1}{2}}(\hat{v})\, E_m^{\frac{1}{2}}\, s(t), & T_m < t < T \end{cases}$$

with the usual notations. The average total transmitted energy therefore is

$$E_T = \int_0^\infty (E_m + \alpha_e(\hat{v})\, E_m)\, p_{\hat{V}}(\hat{v})\, d\hat{v}$$

$$= E_m(1 + \overline{\alpha}_e)$$

where the bar denotes the expectation operation. The average normalized total transmitted energy to noise ratio is defined as

$$\gamma \triangleq 2\sigma_a^{\,2} E_T/N_o$$

$$= \frac{2\sigma_a^{\,2}E_m}{N_o}\,(1 + \overline{\alpha}_e)\,.$$

From the results of the previous chapter, the conditional (instantaneous) error probability without feedback is

$$P^e_{\hat{V}}(\hat{v}) = \mathrm{erfc}\,\{\,\xi^{\frac{1}{2}}(\hat{v})\,\}$$

where

$$\xi(\hat{v}) = \left(\frac{2\sigma_a{}^2E/N_o}{1+2\sigma_a{}^2E_m/N_o} + 1\right)^{-1} \frac{2\sigma_a{}^2E}{N_o} \; \frac{\hat{v}^2}{\sigma_a{}^2} \; .$$

With feedback, the energy E, or equivalently α_e, is a random variable that varies from bit to bit, its value at each bit, as a function of $\hat{V}$, being known. Therefore the average bit error probability is the conditional error probability averaged over the statistics of $\hat{V}$. That is

$$P_e = \int_o^\infty P^e_{\hat{V}}(\hat{v})\; p_{\hat{V}}(\hat{v})\; d\hat{v} \; .$$

Introducing the ratios α_e and γ into $\xi(\hat{v})$ and denoting this new function by $\xi_F(\hat{v})$ yields

$$\xi_F(\hat{v}) = \gamma \; \frac{\alpha_e(\hat{v})}{\alpha_e(\hat{v}) + 1 + \dfrac{1+\overline{\alpha}_e}{\gamma}} \left(\frac{1}{\gamma} + \frac{1}{1+\overline{\alpha}_e}\right) \frac{\hat{v}^2}{\sigma_a{}^2} \; .$$

The value of $\overline{\alpha}_e$ can be determined by considering the situation without feedback and solving

$$\xi(\hat{v}) = \xi_F(\hat{v}) \Big|_{\text{no feedback}} \; ,$$

the no feedback condition implying

$$E = E_m .$$

This results in

$$\bar{\alpha}_e = 1,$$

so that

$$\xi_F(\hat{v}) = \gamma \frac{\alpha_e(\hat{v})}{\alpha_e(\hat{v}) + 1 + 2/\gamma} \left(\frac{1}{\gamma} + \frac{1}{2} \right) \frac{\hat{v}^2}{\sigma_a} .$$

The error probability is then

$$P_e = \int_o^\infty \mathrm{erfc}\{\xi_F^{\frac{1}{2}}(\hat{v})\} \; p_{\hat{V}}(\hat{v}) \, d\hat{v} .$$

Noting that

$$p_{\hat{V}}(\hat{v}) = \frac{\hat{v}}{\sigma_a{}^2 e_m} \exp(-\hat{v}^2/2\sigma_a{}^2 e_m) , \quad \hat{v} > 0$$

where

$$e_m = \frac{2\sigma_a{}^2 E_m}{N_o} \bigg/ \left(1 + \frac{2\sigma_a{}^2 E_m}{N_o}\right) ,$$

the substitution

$$\hat{v}^2 = 2\sigma_a{}^2 e_m y$$

in the P_e intregal yields

$$P_e = \int_o^\infty \mathrm{erfc}\left\{ \left(\gamma \frac{\alpha_e(y)}{\alpha_e(y) + 1 + \frac{2}{\gamma}} \, y \right)^{\frac{1}{2}} \right\} \exp(-y) \, dy .$$

The constraint on α_e is

$$\overline{\alpha}_e = \int_0^{\infty} \alpha_e(y)\ \exp(-y)\ dy = 1$$

and the feedback problem is to find the function $\alpha_e(y)$ that minimizes the error probability subject to the above constraint. This is obtained from

$$\frac{\partial P_e}{\partial \alpha_e(y)} + \lambda \frac{\partial \overline{\alpha}_e}{\partial \alpha_e(y)} = 0$$

where λ is a Lagrange multiplier determinable from the average constraint. A numerical evaluation has been carried out since an analytic solution of the above equation is quite intractable. Figure 3.2 shows the variation of the optimum α_e with the square of the estimated channel gain. Transmitted energy-to-noise ratio is a parameter and it has been assumed for convenience that $2\sigma_a^2 = 1$. These optimum energy division rules are then used in the equation for average error probability to yield the performance shown in Figure 3.3. Also shown are, for the purposes of comparison, the error probability lower bound obtained by Hayes for perfect channel learning and the performance of incoherently detected orthogonal signals.

As expected, allocation of some transmitted energy to a channel measurement signal degrades the performance of the feedback system in comparison with a perfect channel learning system. However, definite improvement in performance over a system without feedback is still maintained, the advantage being of the order of 4 dB at an error rate of 10^{-2} and increasing with decreasing probability of error.

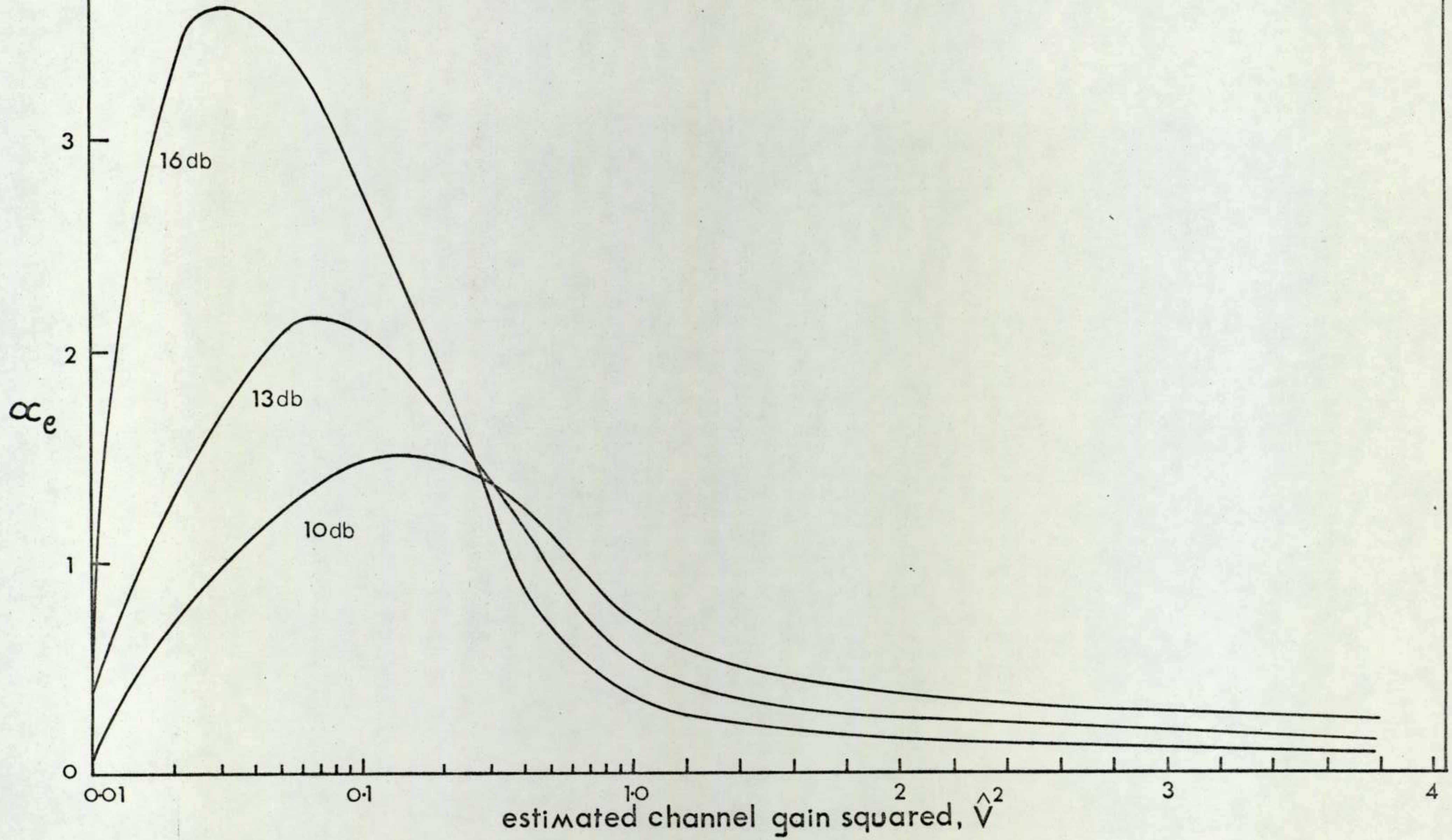


FIG.3.2 Transmitter power division rules.

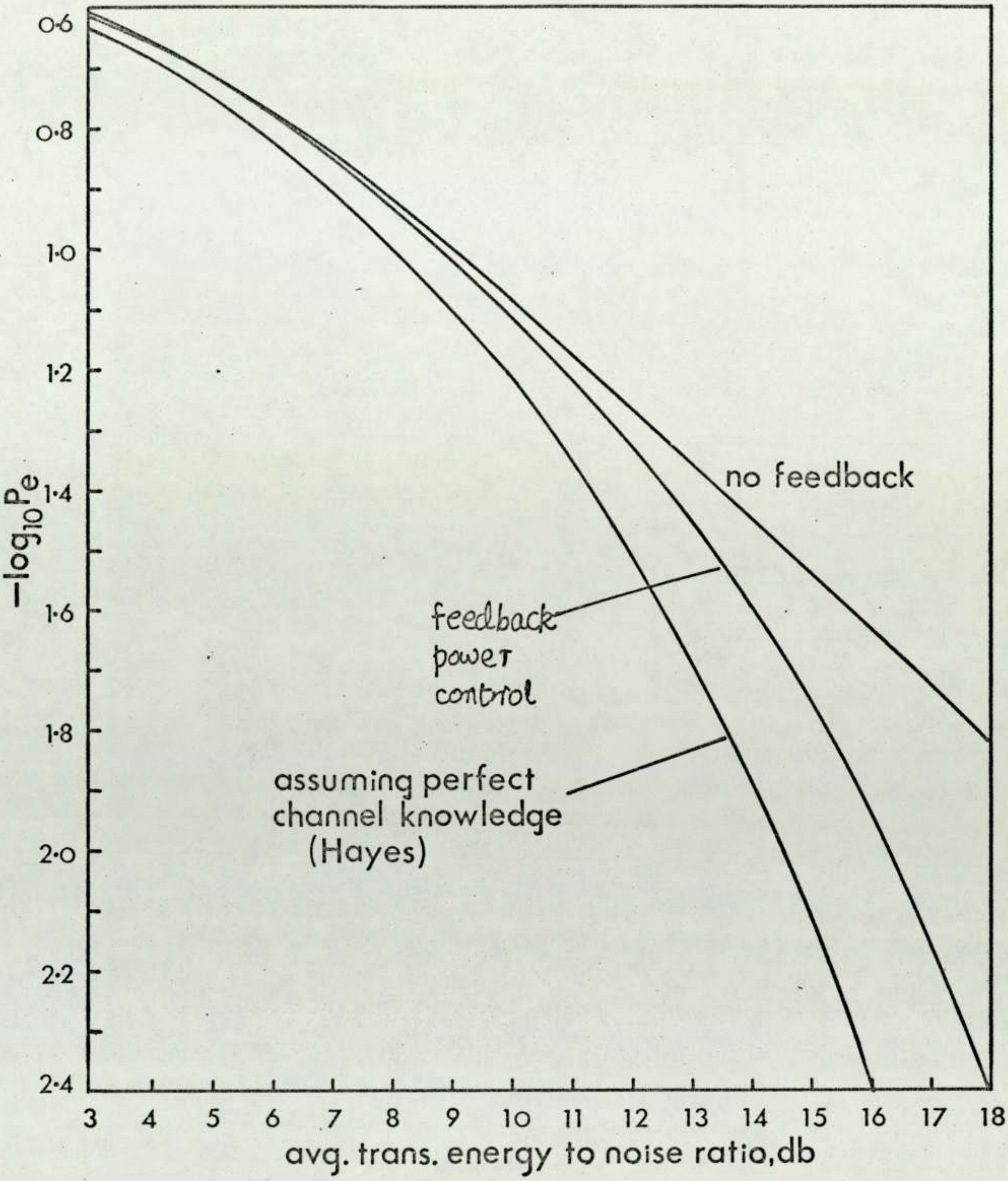


FIG3.3 Performance with feedback power control.

3.4 Feedback rate control

Control of transmission rate is effected by varying the inverse duration of the antipodal information signal. The duration of the channel estimation signal and the powers in both signals are held constant. It is noted that control of estimation energy is not possible simply because the controlling estimate is available at the transmitter only after transmission of the estimation signal.

Solution of the rate problem necessitates definition of the following quantities:

$$T_s \triangleq T - T_m$$

duration of the antipodal signal,

$$P_s \triangleq \frac{1}{T_s} \int_{T_m}^{T} E\, s^2(t) \cos^2\omega_o t \, dt = \frac{E}{T_s}$$

power in the antipodal signal,

$$P_m \triangleq \frac{1}{T_m} \int_{o}^{T_m} E_m\, s_m{}^2(t) \cos^2\omega_o t \, dt = \frac{E_m}{T_m}$$

power in the channel estimation signal,

$$\alpha_p \triangleq \frac{P_s}{P_m}$$

the corresponding power ratio, R_{av} the average specified transmission rate,

$$R_{max} \triangleq \frac{1}{T_m}$$

maximum allowed transmission rate by virtue of the fact that T_m is the irreducible duration of the transmitted signal,

$$R \triangleq \frac{1}{T}$$

the instantaneous rate,

$$R_s \triangleq \frac{1}{T_s}$$

an instantaneous pseudo-rate for the information signal,

$$r \triangleq \frac{R}{R_{av}}$$

the normalized instantaneous rate,

$$r_s \triangleq \frac{R_s}{R_{av}}$$

the normalized instantaneous pseudo-rate and

$$r_o \triangleq \frac{R_{max}}{R_{av}}$$

the rate expansion factor.

Rate control is achieved by making the normalized pseudo-rate an optimum function of the estimated channel gain, subject to a constraint on the average rate. Assuming ergodicity of the estimated channel processes and using an argument identical to that given by Cavers, it can be shown that the average error probability of a system employing rate modulation is given by

$$P_e = \int_0^{\infty} r(\hat{v})\ P^e_{\hat{V}}(\hat{v})\ p_V(\hat{v})\ d\hat{v}$$

where the instantaneous error probability $P^e_{\hat{V}}(\hat{v})$ is a function of the rate $r(\hat{v})$, whose relation to $r_s(\hat{v})$ is, using the quantities defined above,

$$r(\hat{v}) = \frac{r_o r_s(\hat{v})}{r_o + r_s(\hat{v})} \ .$$

Also, from the previous section the conditional error probability is

$$P^e_{\hat{V}}(\hat{v}) = \operatorname{erfc}\ \{\ \xi^{\frac{1}{2}}(\hat{v})\ \}$$

where

$$\xi(\hat{v}) = \left(\frac{2\sigma_a{}^2 E/N_o}{1 + 2\sigma_a{}^2 E_m/N_o} + 1\right)^{-1} \frac{2\sigma_a{}^2 E}{N_o} \cdot \frac{\hat{v}^2}{\sigma_a{}^2} \ .$$

In the absence of any feedback the system is to behave as a straight orthogonal system requiring

$$E \doteq E_m$$

and

$$R_{av} = \frac{1}{T} = \frac{1}{T_m + T_s} \ .$$

This immediately leads to

$$r_o = \frac{R_{max}}{R_{av}} = \frac{\alpha_p + 1}{\alpha_p}$$

since the equal energies condition implies

$$\alpha_p = \frac{P_s}{P_m} = \frac{T_m}{T_s} \quad .$$

The above relation determines, for a specified rate expansion factor, the necessary power division between the information and measurement signals. And in keeping with previous definitions, the average total transmitted energy is

$$\begin{aligned} E_T &= E_m + \overline{E} \\ &= P_m T_m + \frac{P_s}{\overline{R}_s} \\ &= P_m T_m \left(1 + \frac{1 + \alpha_p}{\overline{r}_s}\right) \quad , \end{aligned}$$

so that the normalized average total transmitted energy to noise ratio is

$$\gamma = \frac{2\sigma_a{}^2 E_T}{N_o} = \frac{2\sigma_a{}^2 P_m T_m}{N_o} \left(1 + \frac{1 + \alpha_p}{\overline{r}_s}\right) \quad .$$

Introducing the quantities γ, α_p and $r_s(\hat{v})$ into the expression for $\xi(\hat{v})$ results in the corresponding feedback function $\xi_F(\hat{v})$ given by

$$\xi_F(\hat{v}) = (1 + \alpha_p) \frac{\left\{1 + \dfrac{\gamma}{1 + (1 + \alpha_p/\overline{r}_s)}\right\}}{(1 + \alpha_p) + A_1 r_s(\hat{v})} \quad \frac{\hat{v}^2}{\sigma_a{}^2}$$

where

$$A_1 = \frac{\gamma + 1 + (1 + \alpha_p)/\overline{r}_s}{\gamma} \quad .$$

The average error probability then becomes

$$P_e = \int_o^\infty r(\hat{v}) \text{ erfc } \{ \xi^{\frac{1}{2}}{}_F(\hat{v}) \} \; p_{\hat{V}}(\hat{v}) \; d\hat{v} \; .$$

Invoking the relationship between $r(\hat{v})$ and $r_s(\hat{v})$ and using the substitution

$$\hat{v}^2 = 2\sigma_a{}^2 e_m y$$

in the above integral yields

$$P_e = \int_o^\infty \frac{r_o r_s(y)}{r_o + r_s(y)} \text{ erfc } \{ (\frac{2\gamma A_2}{1 + \alpha_p + A_1 r_s(y)} \, y)^{\frac{1}{2}} \} \exp(-y) \; dy$$

where

$$A_2 = \frac{1 + \alpha_p}{1 + (1 + \alpha_p)/\overline{r}_s} \quad .$$

This error probability is to be minimized subject to the average rate constraint

$$\overline{r} = \int_o^\infty r(y) \exp(-y) \; dy = 1$$

or equivalently

$$\overline{r} = \int_o^\infty \frac{r_o r_s(y)}{r_o + r_s(y)} \exp(-y) \; dy = 1$$

by determining an optimum $r_s(y)$. For ease of computation, a sub-optimal solution in the form

$$r_s(y) = ky^t,$$

with k and t parameters to be determined, has been assumed. Although this choice has not been made on any rigorous considerations, it nevertheless predicts system performances reasonably close to lower bounds on error probability.

With this form of solution the overall rate function is

$$r(y) = r_o \frac{y^t}{y^t + r_o/k} \quad .$$

The computation then proceeds to jointly solve for the minimum error probability and the optimum values of the parameters k and t required to achieve this minimum.

The results of the computation are shown in Figures 3.4 - 3.6 with Fig. 3.4 showing the variation of the optimum k and t with energy to noise ratio γ. The behaviour of the overall rate function with the square of the estimated channel gain is shown in Fig. 3.5. Use of these rate control rules results in the system performances shown in Fig. 3.6 with the rate expansion factor as parameter. Also shown for the purposes of comparison are the performances of maximal ratio predetection combining diversity and an incoherently detected straight orthogonal signalling system under non-fading conditions, the latter representing in fact the lower bound on error probability achieved by a feedback system assuming perfect learning of a fading channel (Cavers)

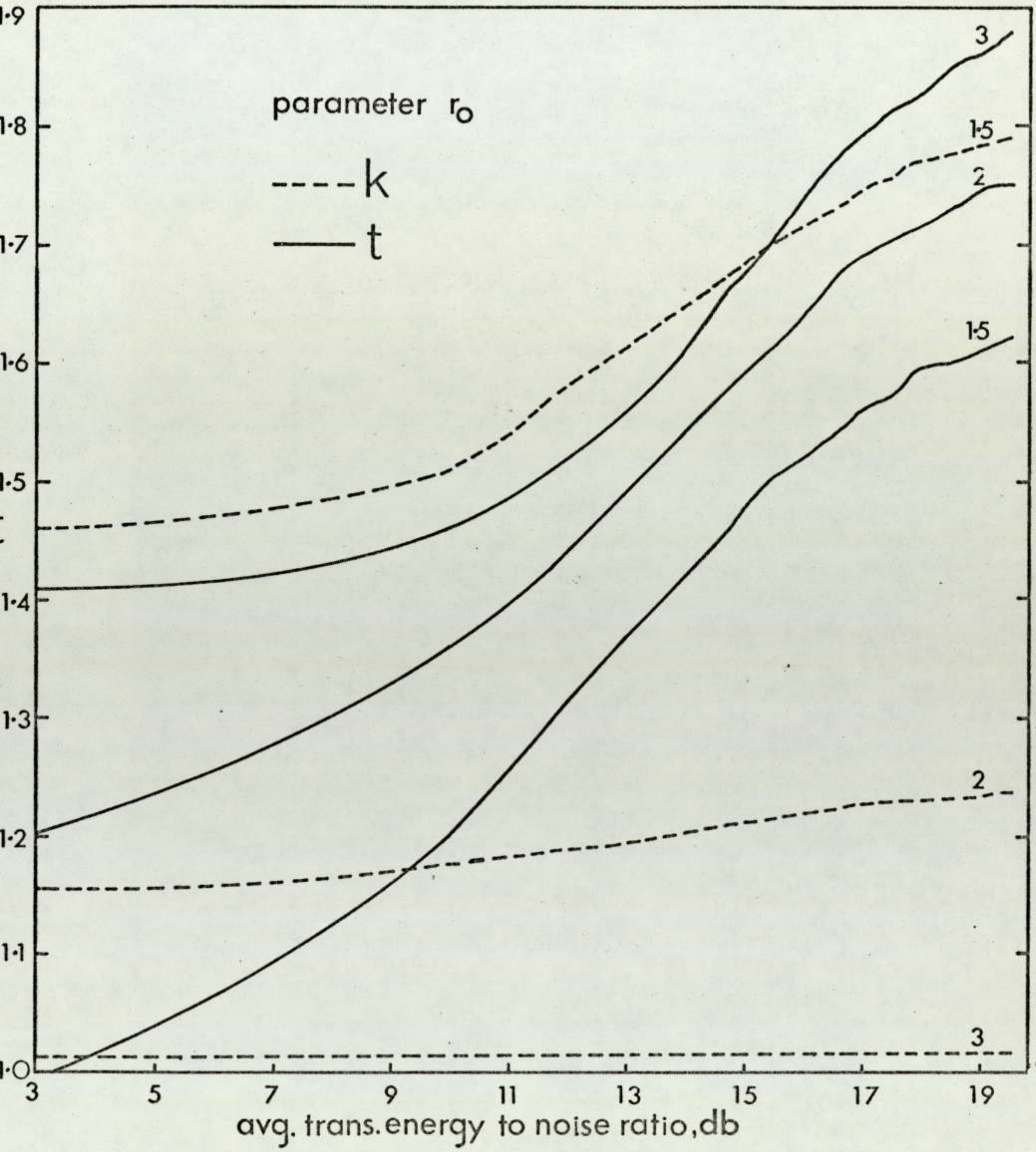


FIG.3.4 Variation of k and t with transmitted energy.

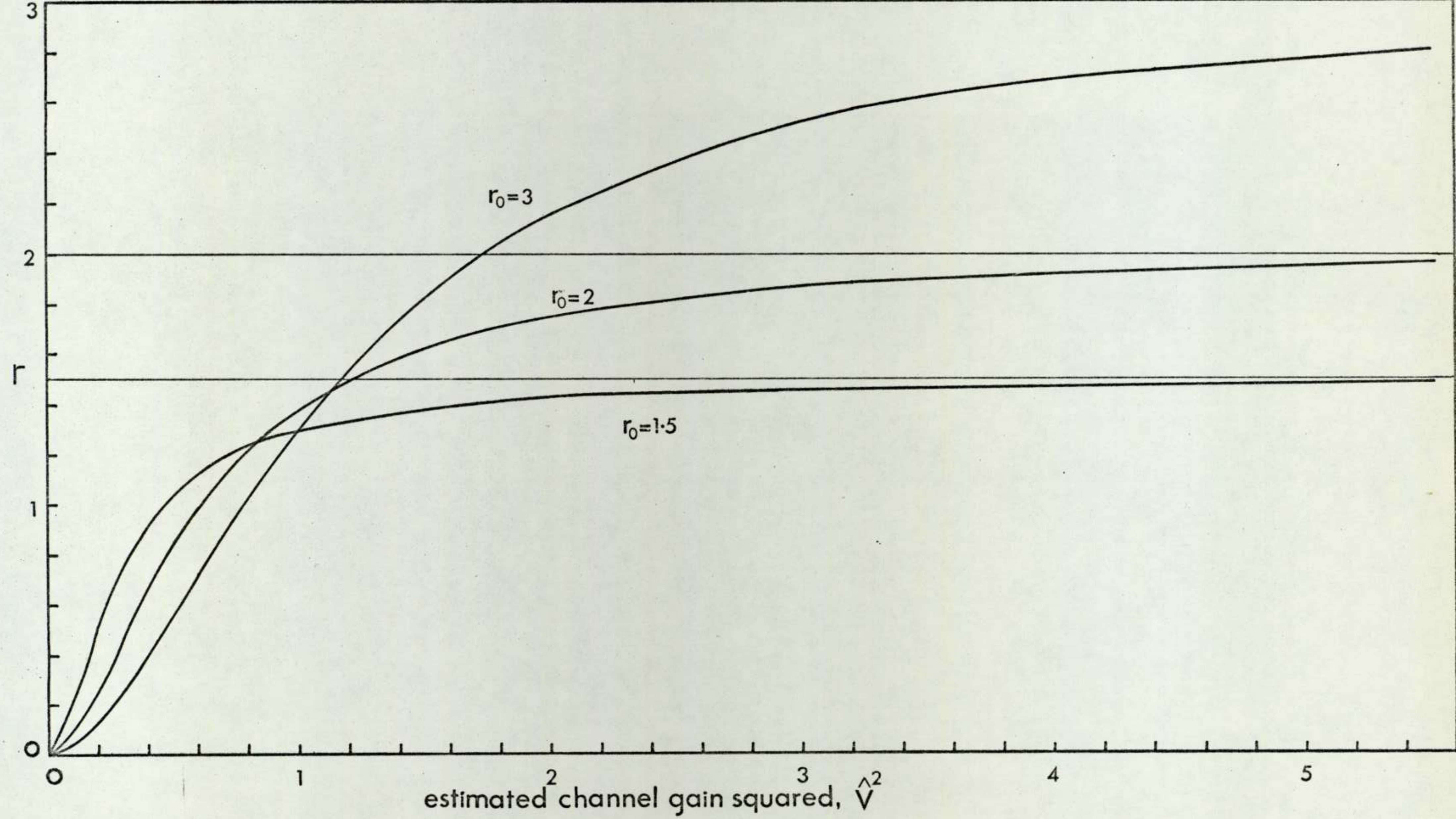


FIG.3.5 Transmitter rate control rules for Rayleigh faded signals with channel estimation reception.

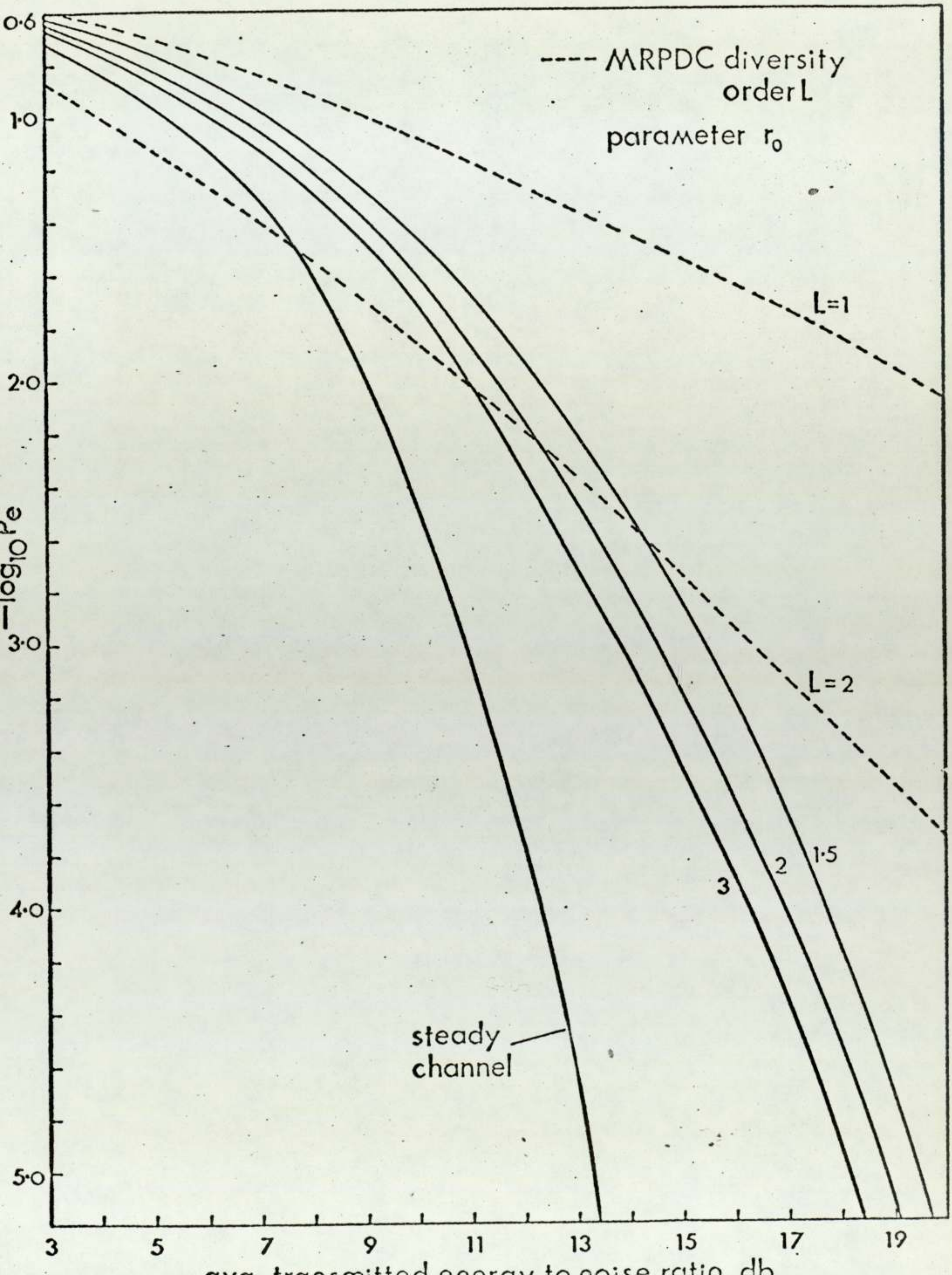


FIG.3.6 Performance with feedback rate control.

and allowing uncontrolled expansion of rate.

It is observed that the channel estimation scheme provides considerable improvement in performance over second order diversity for values of transmitted energy to noise ratios greater than about 15 dB. At higher energy to noise ratios improvement over fourth order diversity is expected. A comparison with error probability bounds corresponding to maximum rate constrained perfect channel learning systems is given in the last section of this chapter.

3.5 Feedback power and rate control

Bounds on performance and optimum control rules are now derived for a hybrid system with simultaneous power and rate control.

It was seen that imposing a maximum rate constraint (note that the channel estimation signalling scheme discussed in the last section incorporates, inherently, this constraint) caused performance degradation. This degradation can be reduced by modulating transmitter power whenever the system rate saturates at the imposed maximum.

For mathematical convenience exact channel knowledge is assumed and the signalling is binary orthogonal. The transmitted signal on hypothesis H_i, $i = 0, 1$ is

$$g\ (t) = E^{\frac{1}{2}}\, s_i(t) \cos\omega_o t \quad , 0 < t < T$$
$$i = 0,\ 1.$$

with the signal power defined by

$$P = \frac{1}{T} \int_0^T E\, s_i^{\,2}(t)\, \cos^2\omega_o t\ dt = ER$$

where R denotes, as usual, the transmission rate. The fed-back channel gain is used to control either the transmission rate or power, depending on whether the gain is below or above a certain threshold. This threshold depends upon the specified maximum rate expansion. It is noted that at any given instant only one of the transmitter parameters, i.e. power or rate, is being controlled.

The instantaneous error probability conditioned on knowledge of the channel gain (page 16) is

$$P^{e}_{\beta}(b) = \frac{1}{2}\exp(-Pb/2N_oR_{av}r)$$

$$\triangleq P_e(b,r,P)$$

where $\beta = V^2$ the squared channel gain and the remaining symbols have their usual meaning. With either P or the normalized rate r as functions of β the average probability of error is

$$P_e = \int_0^{\beta_o} r(b)P_e(b,r(b),P_o)p_\beta(b)db$$

$$+ \int_{\beta_o}^{\infty} r_oP_e(b,r_o,P(b))p_\beta(b)db$$

where

$$P(b) = P_o \quad , \quad 0 < b \leq \beta_o$$

and

$$r(b) = r_o = \frac{R_{max}}{R_{av}} \quad , \quad b \geq \beta_o \; .$$

Thus the system is made to switch over from rate control to controlling the power P(b) whenever the squared gain exceeds the threshold β_o. It is noted that the receiver employs the usual matched filter-envelope detector or correlation-squarer detection.

The problem now consists of determining the optimum r(b) and P(b) that minimize P_e subject to constraints on the average rate and average transmitted power. These constraints are

$$\bar{r} = 1 = \int_o^{\beta_o} r(b) p_\beta(b)\, db + \int_{\beta_o}^{\infty} r_o p_\beta(b)\, db$$

and

$$P_{av} \triangleq \int_o^{\beta_o} P_o p_\beta(b)\, db + \int_{\beta_o}^{\infty} P(b)\, p_\beta(b)\, db$$

with the boundary condition

$$P(b)\Big|_{b = \beta_o} = P_o \, .$$

The minimization is then the solution of

$$\frac{\partial P_e}{\partial r(b)} + \lambda \frac{\partial \bar{r}}{\partial r(b)} = 0$$

and

$$\frac{\partial P_e}{\partial P(b)} + \mu \frac{\partial P_{av}}{\partial P(b)} = 0$$

where λ and μ are Lagrange multipliers that can be determined from the constraint equations. The solutions of the above equations are easily shown to be

$$r(b) = \begin{cases} b/c \quad , & b < \beta_o \\ r_o \quad , & b \geq \beta_o \end{cases}$$

and

$$P(b) = \begin{cases} P_o \quad , & b < \beta_o \\ \dfrac{2N_oR_{av}r_o}{b} \ln \left(\dfrac{b}{cr_o}\right) + \dfrac{cP_or_o}{b} , & b \geq \beta_o \end{cases}$$

where $\beta_o = cr_o$ and c is determined from the average rate constraint equation as

$$1 - \exp(-cr_o/2\sigma_a^2) = c/2\sigma_a^2 \quad .$$

Use of these optimum rules yields for the error probability

$$P_e = \frac{1}{2} \exp(-cP_o/2N_oR_{av}) \left\{ \frac{cr_o}{2\sigma_a^2} + 1 - r_o + \frac{cr_o^2}{2\sigma_a^2} E_1(cr_o/2\sigma_a^2) \right\}$$

where

$$E_1(x) = \int_x^\infty \frac{1}{t} \exp(-t)\, dt \, .$$

The average transmitted energy to noise ratio is defined as

$$\gamma \triangleq 2\sigma_a^2 \; P_{av}/N_oR_{av}$$

and using the average power constraint equation this becomes

$$\gamma = \frac{cP_o}{N_o R_{av}} \{ 1 + E_1(\frac{cr_o}{2\sigma_a^2}) \} + \frac{r_o}{2\sigma_a^2} \int_{cr_o}^{\infty} \frac{\ln(b/cr_o)}{b} e^{-b/2\sigma_a^2} db$$

The above relations have been evaluated numerically and the system performances are shown in Figure 3.7 for rate expansions of 1.5 and 2. Also shown is the error probability of a system using only rate control (Cavers) with an expansion of 2. It is seen that the performance of the hybrid system with an expansion of 1.5 is nearly equal to that of only rate control (expansion 2), thus allowing the hybrid system to closely approach that of one operating in a non-fading channel.

3.6 Conclusion

The results described in the various sections of this chapter are presented together in this section. Of the three modes of control described, hybrid control performs the best, and rate control is superior to power control. The allocation of part of the transmitter's energy to channel estimation in the receiver results in a scheme that obviates the need to assume perfect tracking of channel conditions. Although the hybrid system has been analyzed under this assumption, the bounds derived indicate the limit of performance improvement that can be achieved by using this mode of feedback control. Two other advantages of channel estimation signalling are the realizabiltity of the system with respect to round trip propagation delays and the inherent upper bound on rate (and hence bandwidth) expansion as a result of the fixed duration of the estimation signal.

In the section on rate control (3.4), a sub-optimal solution for rate has been used. Additional performance improvement, with corresponding bounds, can be achieved by a more suitable (optimum) choice for the rate function. The performances of the various techniques with

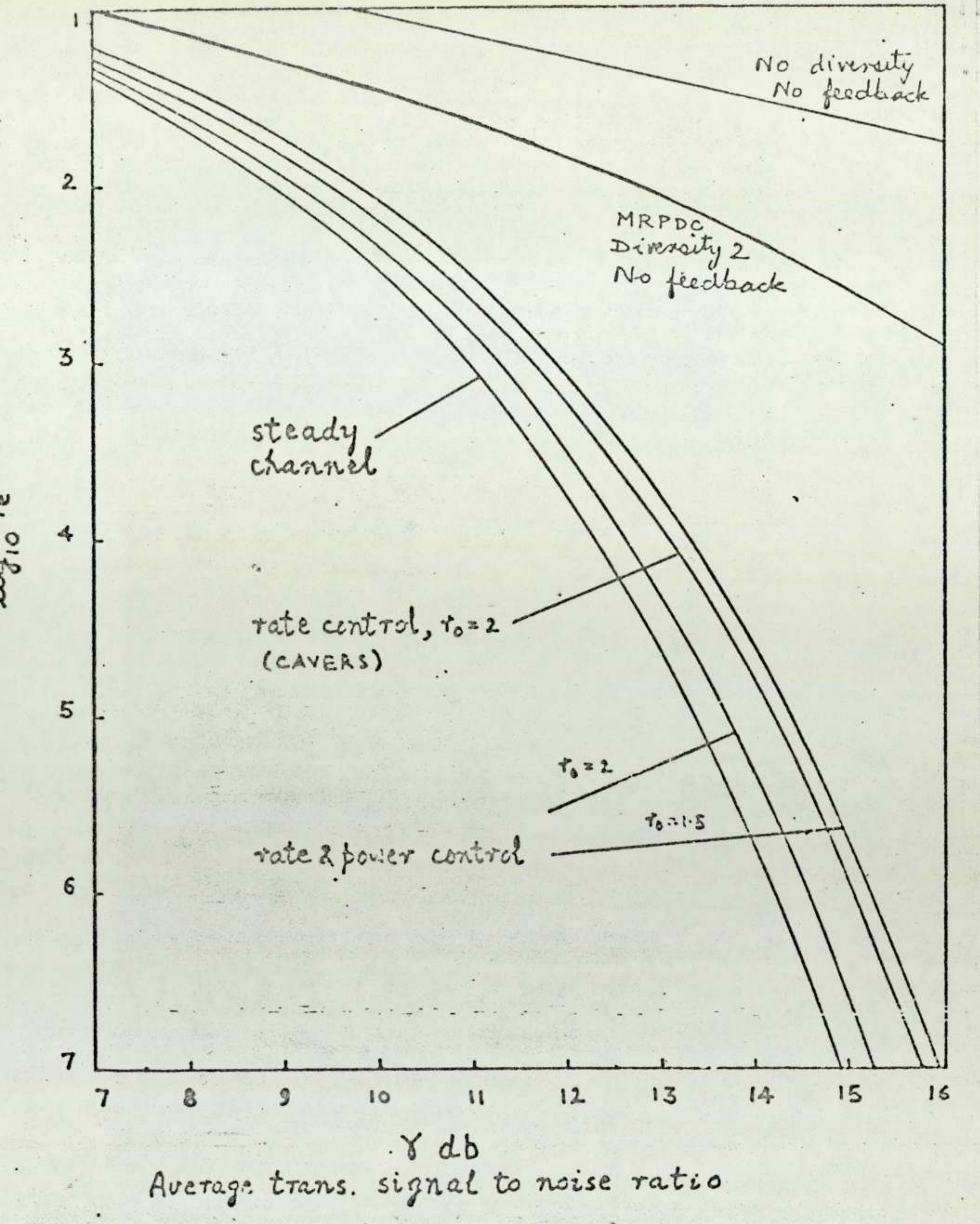
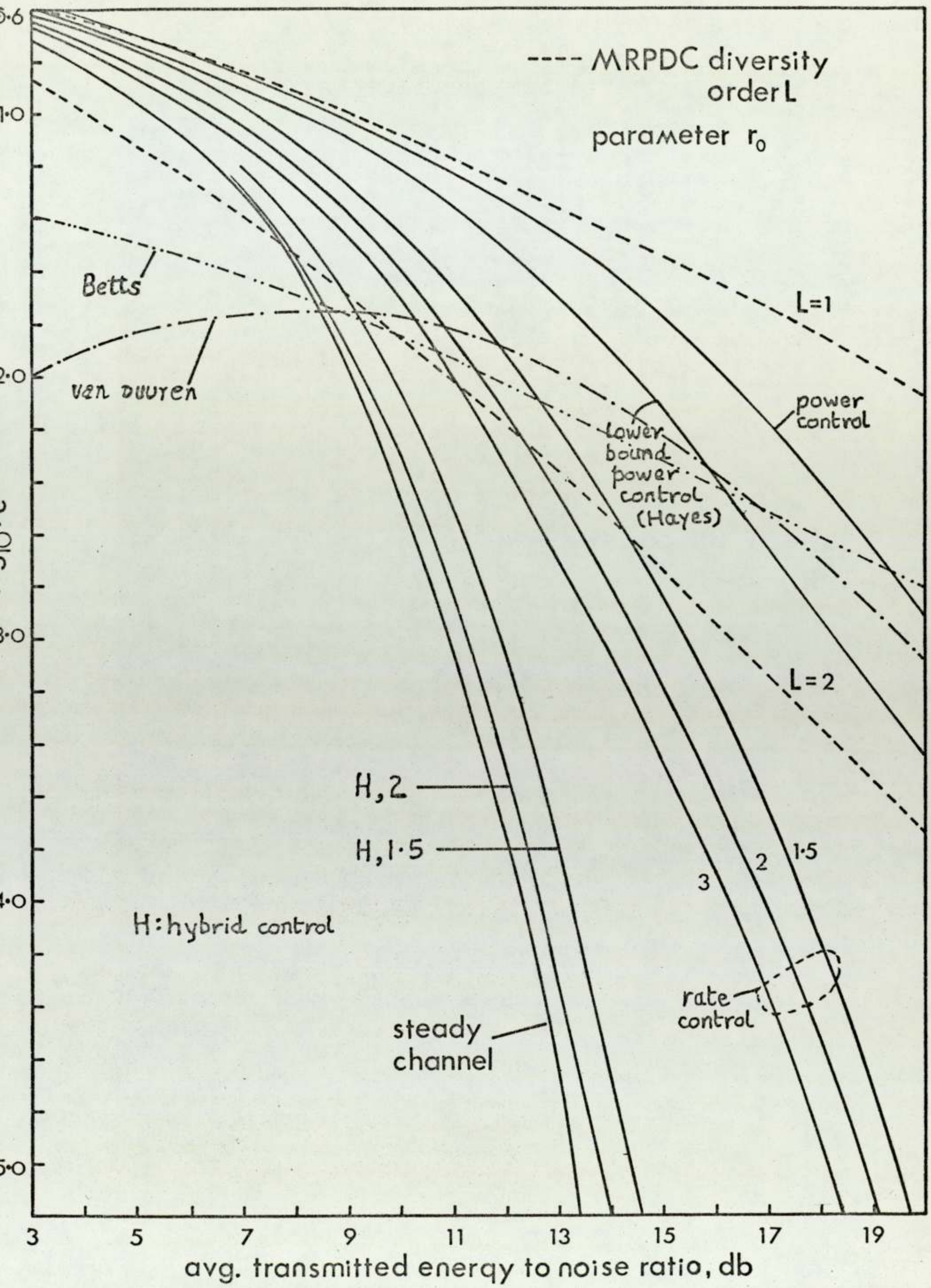


FIG.3.7. Performance of hybrid rate and power control.

the lower bounds on error probabilities have been shown in Figure 3.8 for the purpose of an overall comparison.

Of existing data transmission systems utilizing pilot tone channel measurement the AN/GSC-10 KATHRYN system is a variable rate data modem for use on HF channels. Very simply, a channel measuring signal is transmitted together with the information. This is used by the receiver to extract phase and amplitude information about the channel which is then used to phase correct the information component and implement maximal-ratio combining. Also at the receiver, the channel amplitude measurements are displayed in real time on a RAKE display. This display allows the operator to view the characteristics of the ionosphere channel and select an appropriate data rate. The KATHRYN is not a self-adaptive system in that the variation of data rate depends upon human decisions. A rigorous theoretical analysis of the system performance is not available. However, field tests have been carried out to evaluate performance. Experimental results obtained indicated that there is no clear dependence of error probability on signal to noise ratio implying that additive noise is not the only contributor to bit errors.

A similar pilot-tone adaptive system for HF channels has been described by Betts. Here a limited number of frequencies are allocated for a particular service. A pilot tone accompanying the data and a further pilot tone on each standby frequency provide a monitoring of the data channel performance and also advance information of the state of another channel before a change of frequency is carried out. This frequency management is carried out by the human operator and could lead to a significant improvement in spectrum utilization. The prime aim of the work on this system however, has been to confirm the validity of the system for the estimation of data channel performance from phase measurements of an associated pilot tone. Phase perturbations of the pilot tone are interpreted into a 'phase-error' by means of a time-



IG.3.8 Comparison of the three modes of feedback control.

differential phase comparison with a locally generated reference. The resulting phase-error rate is then interpreted directly as an error rate associated with the main information channel. The result of a theoretical analysis for phase-error probability is shown in Fig.3.8 as a graph against average transmitted energy to noise ratio. A power advantage of about 8 dB is obtained over a straight FSK system operating over a Rayleigh fading channel. However, there is no significant increase in the roll-off of error probability with energy to noise ratio, with the result that the performance of feedback power control overtakes that of this system at an energy to noise ratio of about 20 dB and an error probabilty of nearly 10^{-3}.

Lastly, a brief comparative study of Van Duuren's system is made. This is a system using error detection and automatic repetition of signals. Here, when a character is received mutilated and is detected by the error detecting device, a repetition cycle of four characters is started. The transmission on the return circuit is interrupted for the duration of the cycle and during that time four special signals (signals I) are transmitted. Reception of the first signal I causes the transmitter to emit a cycle of four characters starting with one signal I followed by three characteristics from storage. These characters are a repetition of the three preceeding ones. Error detection in this system is accomplished by means of a constant ratio code. The performance of the system has been analyzed for a number of channel conditions. Of interest here is the model of fading adopted in this thesis. For this, the error probability performance is shown in Fig.3.8. It is observed that there is a 1 dB advantage over feedback power control at an error rate of about 10^{-3}. The roll-off of error probability is, however, somewhat slower than that of power control with the result that power control is expected to perform better for error rates of less than about 10^{-4}.

CHAPTER 4

A NONSEQUENTIAL DECISION FEEDBACK SYSTEM

4.1 Introduction

A simple feedback technique applicable to short distance links is proposed and analyzed in relation to Rayleigh fading channels. The scheme, which matches feedback power control in performance, is characterized by ease of implementability and simplicity of operation. The transmitter uses two level transmitted power controlled by receiver decisions and is not required to be adaptive to channel conditions.

In section 4.5 is studied a feedback system which is, in effect, a fusion of the above technique and feedback power control. Here the transmitter operation is adaptive, with the feedback link carrying the message estimate as well as the channel estimate from the receiver.

4.2 Problem formulation

The basic technique involves subtracting from the transmitter waveform a signal which is the fedback message estimate from the receiver. The difference is subjected to a gain before transmission so that the transmitted energy is equal to that of a feedback-less system. To the received signal is added the message estimate times the transmitter gain thereby realizing an energy advantage. The scheme is shown in Fig. 4.1 for an additive noise channel. Also shown is a corresponding feedback-less system that utilizes the same transmitted energy.

The received signal in the feedback-less system is

$$f(t) = E^{\frac{1}{2}}\, s(t) + n(t), \quad 0 < t < T,$$

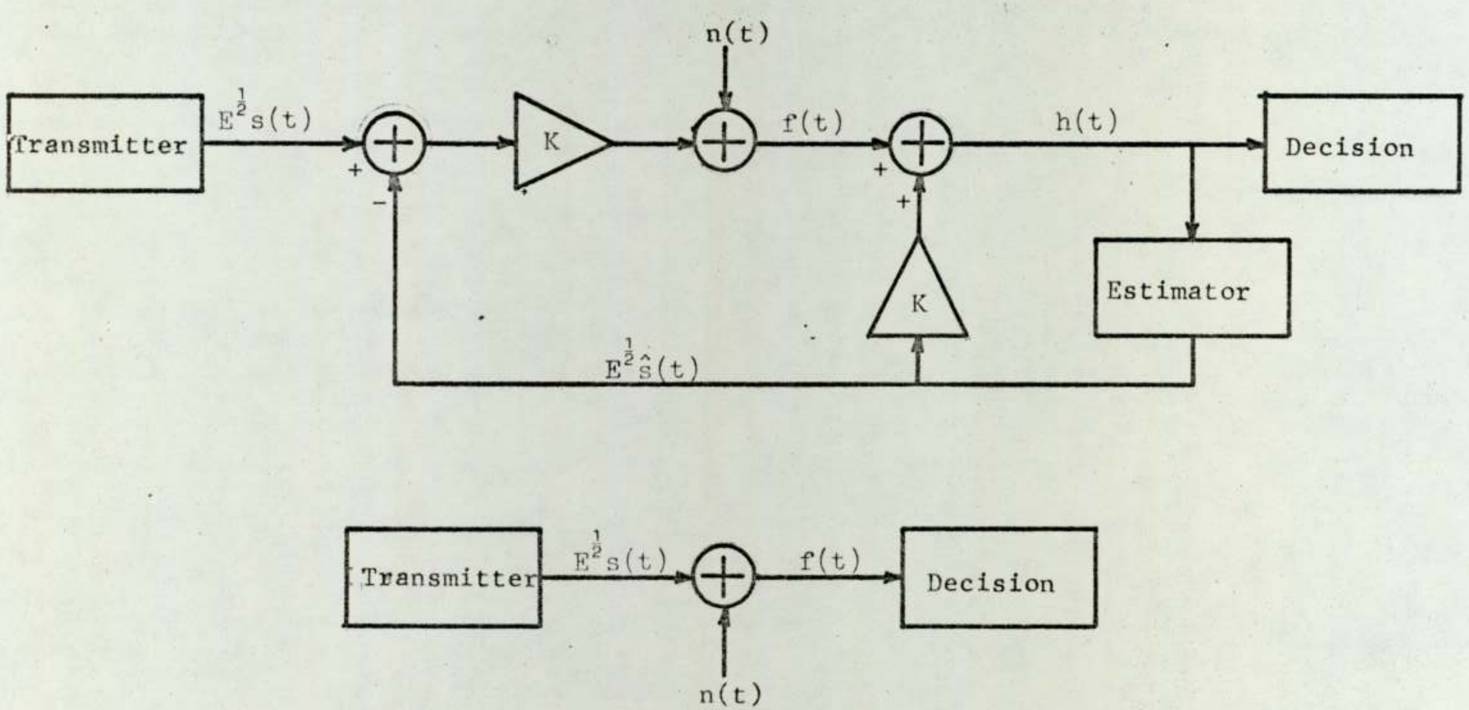


Fig. 4.1 System with feedback of message estimate and the corresponding feedback-less system.

so that the transmitted energy to noise ratio is E/N_o where it is assumed that

$$\int_o^T s^2(t)\,dt = 1 .$$

The signal transmitted in the feedback system is

$$KE^{\frac{1}{2}}\{s(t) - \hat{s}(t)\}, \quad O < t < T$$

where K denotes the transmitter gain and $\hat{s}(t)$ the fedback message estimate from the receiver. Therefore, the transmitted energy, denoted by E_F, is given by

$$E_F = K^2E \quad \overline{\int_o^T \{ s(t) - \hat{s}(t) \}\,dt}$$

where the bar denotes expectation over the error signal statistics. The requirement of equal transmitted energies in the two systems is therefore satisfied if the gain K satisfies

$$K = \left\{ \overline{\int_o^T \{ s(t) - \hat{s}(t) \}^2\,dt} \right\}^{-1/2} .$$

The received signal is

$$f(t) = KE^{\frac{1}{2}}\{s(t) - \hat{s}(t)\} + n(t), \quad O < t < T.$$

To this is added the signal (assuming the receiver knows the value of K)

$$KE^{\frac{1}{2}}\,\hat{s}(t)$$

to yield

$$h(t) = KE^{\frac{1}{2}} s(t) + n(t), \qquad 0 < t < T.$$

This signal is then processed for the message. It is observed that an energy advantage of exactly K^2 over the corresponding feedbackless system is achieved if K satisfies the condition

$$K \geqslant 1.$$

This feedback technique is applied to the problem of digital signalling over Rayleigh fading channels by means of orthogonal waveforms. The system is designed to account for propagation delays that are small in comparison with the bit duration. A situation of this nature might typcially arise in a single hop, low distance (ionospheric E layer), HF link where the absence of specific antimultipath measures necessitates pulse lengths in the 10 to 20 msec. range [70]. The technique can in fact find application in any situation involving low data rate telemetry.

4.3 System description and performance analysis

A binary orthogonal signal set is used for transmission. The receiver processes the received signal for a specified portion of the bit length and generates an estimate of the transmitted hypothesis using incoherent detection. This estimate is communicated to the transmitter via a noiseless feedback link. At the transmitter, it is subtracted from the remaining portion of the transmitter waveform that awaits transmission, to generate an error signal. This error signal is then subjected to a specified gain and transmitted. The total transmitted signal on hypothesis H_i, $i = 0, 1$, before modulation onto a carrier, therefore is

$$g(t) = \begin{cases} E^{\frac{1}{2}} s_i(t) , & 0 < t < T_1 + \Delta \\ KE^{\frac{1}{2}} \{s_i(t) - \hat{s}_i(t)\} , & T_1 + \Delta < t < T \end{cases} \tag{4.1}$$

where the average total transmitted energy over a bit length is constrained to E, $s_i(t)$ is the binary signalling waveform, $\hat{s}_i(t)$ denotes the estimate of $s_i(t)$ that is fed back to the transmitter and K denotes the transmitter gain on the error signal. T_1 is the time taken by the receiver to generate the message estimate, T the overall bit length and Δ represents the sum of the forward and feedback propagation delays with $\Delta < T$. For the purposes of signal design and detection the following orthogonality conditions on the signalling waveform have been imposed:

$$\int_0^{T_1} s_1(t)\, s_0(t)\, dt = 0$$

$$\int_0^{T} s_1(t)\, s_0(t)\, dt = 0$$

and

$$\int_0^{T_1 + \Delta} s_1(t)\, s_0(t)\, dt = 0$$

with

$$\int_{o}^{T} s_i^{2}(t)\cos^2\omega_o t\,dt = 1\ , \qquad i = 0,\ 1$$

$$\alpha \triangleq \int_{o}^{T_1 + \Delta} s_i^{2}(t)\cos^2\omega_o t\,dt\ , \qquad i = 0,\ 1$$

and

$$\alpha_{\wedge} \triangleq \int_{o}^{T_1} s_i^{2}(t)\cos^2\omega_o t\,dt\ , \qquad i = 0,\ 1$$

where ω_o denotes the carrier frequency used for the double sideband suppressed carrier amplitude modulation of the transmitted signal.

The system operation is shown in Fig. 4.2. Under the assumption of slow nonselective Rayleigh fading the received signal is

$$f(t) = a_1 g(t)\cos\omega_o t + a_2 g(t)\sin\omega_o t + n(t),$$

$$0 < t < T \qquad (4.2)$$

with g(t) given by (4.1). The fedback message estimate is the output of the incoherent detection process on the received signal in the interval $(0,\ T_1)$ with the associated probability of error denoted by $P_{\hat{e}}$. The transmitted energy, denoted by E_F is

$$E_F = E\int_{o}^{T_1 + \Delta} s_i^2(t)\cos^2\omega_o t$$

$$+ K^2E\ \overline{\int_{T_1+\Delta}^{T} \{s_i(t) - \hat{s}_i(t)\}^2\ \cos^2\omega_o t\,dt}\ . \qquad (4.3)$$

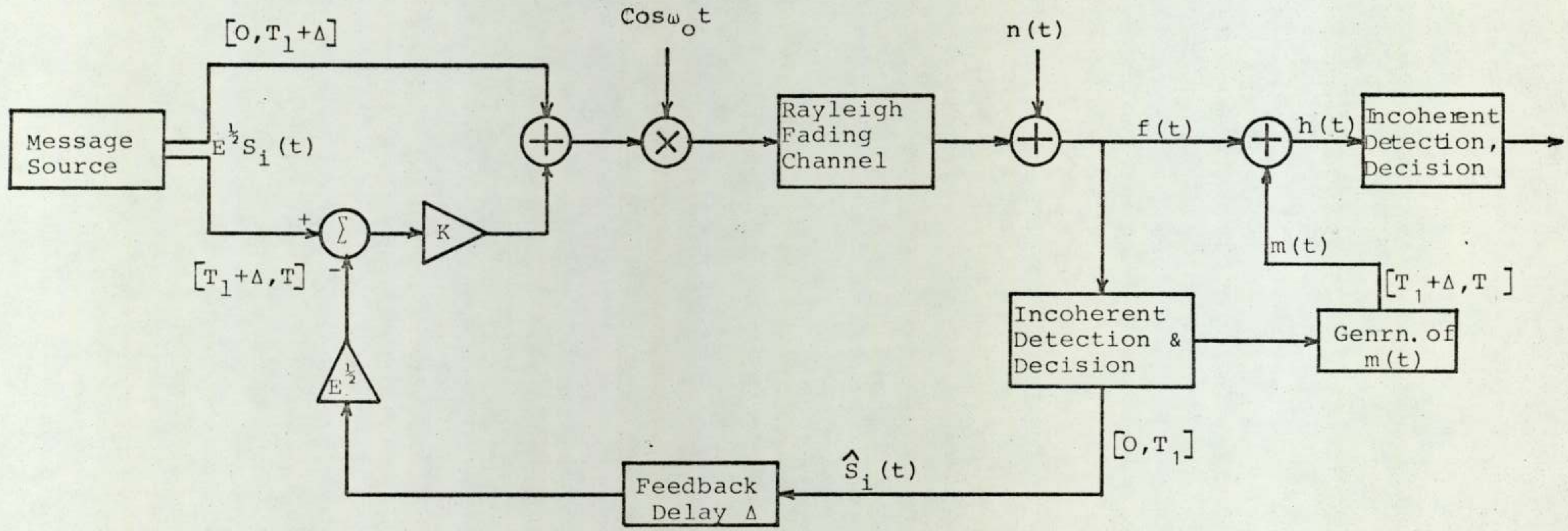


Fig.4.2 Communication system with feedback of estimated transmitted hypothesis.

The second term in (4.3) is evaluated on either one of the transmitted hypothesis since both are equally likely. Thus, on hypothesis H_1, the second term is

$$K^2E \quad \overline{\int_{T_1+\Delta}^{T} \{s_1(t) - \hat{s}_1(t)\}^2 \cos^2\omega_o t \, dt} \quad .$$

Assuming that the receiver has made a correct decision makes this term zero. In the case of an error, occurring with probability $P_{\hat{e}}$, the term is

$$K^2E \int_{T_1+\Delta}^{T} \{ s_1(t) - s_o(t)\}^2 \cos^2\omega_o t \, dt$$

$$= 2K^2E \; (1 - \alpha).$$

Hence the average value of the second term is

$$2K^2P_{\hat{e}}E(1 - \alpha),$$

so that the average transmitted energy is

$$E_F = \alpha E + 2K^2P_{\hat{e}}E(1 - \alpha)$$

which when constrained to equal the energy E of the equivalent feedback-less system, yields

$$K = 1/(2P_{\hat{e}})^{\frac{1}{2}} \tag{4.4}$$

An expression for $P_{\hat{e}}$ can be obtained by noting that α denotes the fraction of the total transmitted energy used by the receiver to generate the message estimate for feedback. And since this is an incoherent detection process the error probability is given by

$$P_{\hat{e}} = (2 + \gamma\alpha_{\wedge})^{-1} \tag{4.5}$$

where the total transmitted energy to noise ratio is

$$\gamma = 2\sigma_a{}^2 E/N_o.$$

Therefore, from (4.4) and (4.5)

$$K = (1 + \frac{\gamma\alpha_{\wedge}}{2})^{\frac{1}{2}} , \tag{4.6}$$

which determines the transmitter gain.

In the main decision portion of the receiver a locally generated signal is added to the received signal Δ seconds after the estimate for feedback has been generated. An assumption is made here that the receiver makes a perfect estimate of the channel processes in the interval $(0, T_1 + \Delta)$. The subsequent main receiver decision, however, is incoherent and does not depend upon this assumption, which is used solely for the generation of the appropriate local signal for addition. This signal is

$$m(t) = a_1 K E^{\frac{1}{2}} \hat{s}_i(t) \cos\omega_o t$$

$$+ a_2 K E^{\frac{1}{2}} \hat{s}_i(t) \sin\omega_o t, \qquad T_1 + \Delta < t < T,$$

which when added to the received signal f(t) (4.1 and 4.2), yields

$$h(t) = \begin{cases} a_1 E^{\frac{1}{2}} s_i(t) \cos\omega_o t + a_2 E^{\frac{1}{2}} s_i(t) \sin\omega_o t \\ \qquad\qquad + n(t),\ 0 < t < T_1 + \Delta \\ a_1 K E^{\frac{1}{2}} s_i(t) \cos\omega_o t + a_2 K E^{\frac{1}{2}} s_i(t) \sin\omega_o t \\ \qquad\qquad + n(t),\ T_1 + \Delta < t < T. \end{cases}$$

Finally, $h(t)$ is incoherently detected for the transmitted hypothesis. It is noted, from $h(t)$, that in the interval $(T_1 + \Delta, T)$ an energy advantage of K^2 over a feedback-less system $(K = 1)$ has been achieved. The overall energy advantage, denoted by F, over the interval $(0,T)$ is easily seen to be

$$F = \{K(1 - \alpha) + \alpha\}^2 \tag{4.7}$$

and using (4.6) in (4.7) yields

$$F = \{(1 + \frac{\gamma\alpha_\wedge}{2})^{\frac{1}{2}} (1 - \alpha) + \alpha\}^2. \tag{4.8}$$

The probability of error of incoherent detection is therefore

$$P_e = (2 + \gamma F)^{-1} \ . \tag{4.9}$$

Minimum error probability is achieved by maximizing F with respect to α. This is done by first expressing $\alpha_\wedge$ in terms of α and some measure of delay. From the definitions of $\alpha_\wedge$ and α is defined the quantity

$$\varepsilon_\Delta \triangleq \alpha - \alpha_\wedge = \int_{T_1}^{T_1+\Delta} s_i^{\,2}(t)\cos^2\omega_o t \, dt \tag{4.10}$$

as the fraction of the total transmitted energy that, owing to non-zero delay, could not be utilized by the receiver in generating the message estimate for feedback. Although this definition does not strictly specify the amount of fractional round trip delay in the link, it avoids the problem of specifying actual signal shapes (section 4.4).

Use of (4.10) in (4.6) and (4.8) yields

$$K = \{1 + \frac{\gamma}{2}\,(\alpha - \varepsilon_\Delta)\}^{\frac{1}{2}} \tag{4.11}$$

and

$$F = (\{1 + \frac{\gamma}{2}\,(\alpha - \varepsilon_\Delta)\}^{\frac{1}{2}}\,(1 - \alpha) + \alpha)^2 \tag{4.12}$$

which latter is maximized by setting

$$\frac{\partial F}{\partial \alpha} = 0,$$

solving for the optimum α and verifying the negative second derivative. This turns out to be quadratic in α and the solution is shown in Fig. 4.3 for parameter values of $\varepsilon_\Delta = 0$ (corresponding to no delay), 0.1 and 0.2. Use of these optimum α in (4.11) yields the optimum gain required at the transmitter and this is also shown in Fig. 4.3. The resulting minimum error probability is evaluated by using (4.12) and the optimum α in (4.9). The performance is shown in Fig. 4.4 along with the performances of feedback power control and orthogonal signalling without feedback. It is

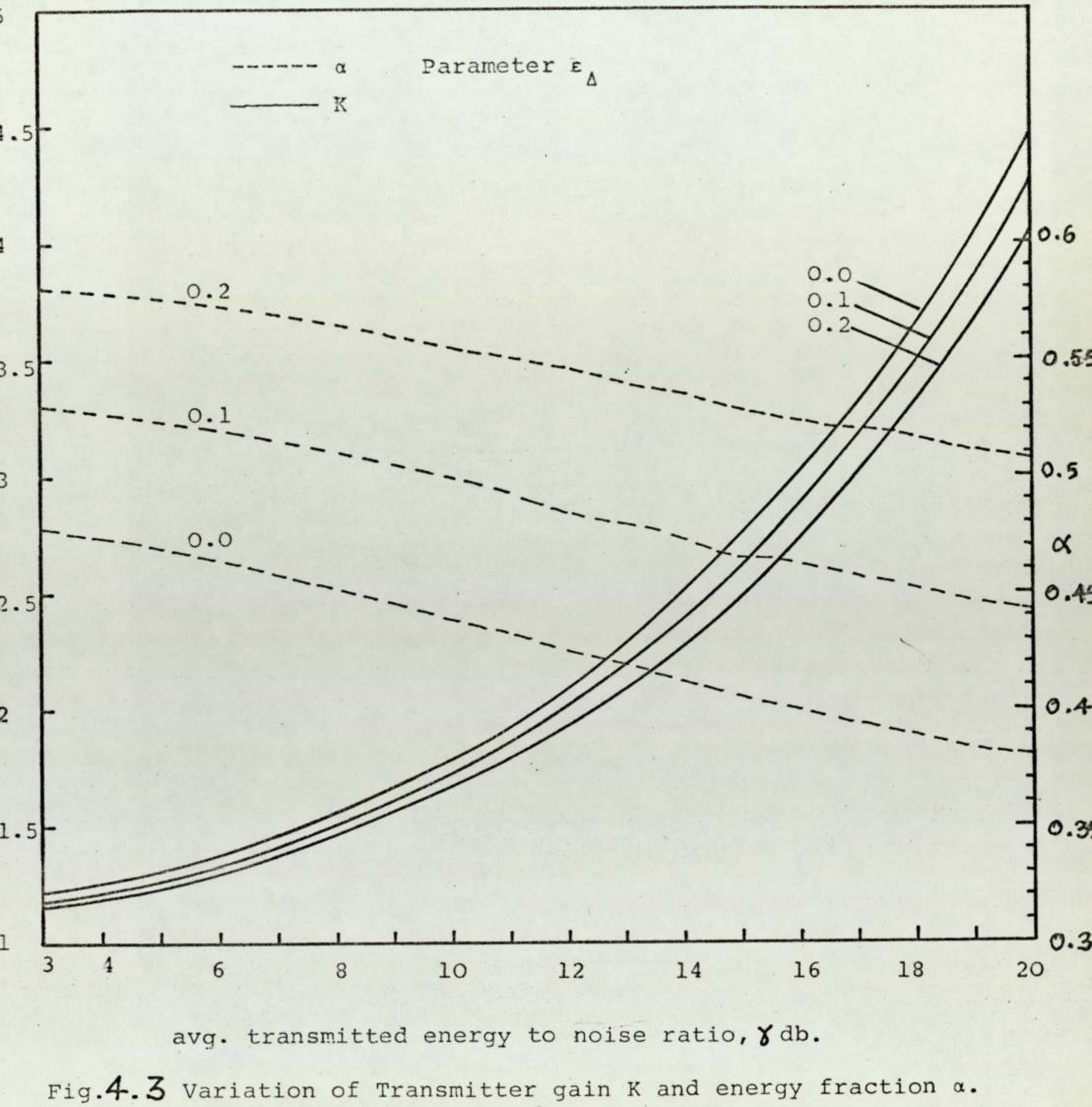


Fig. 4.3 Variation of Transmitter gain K and energy fraction α.

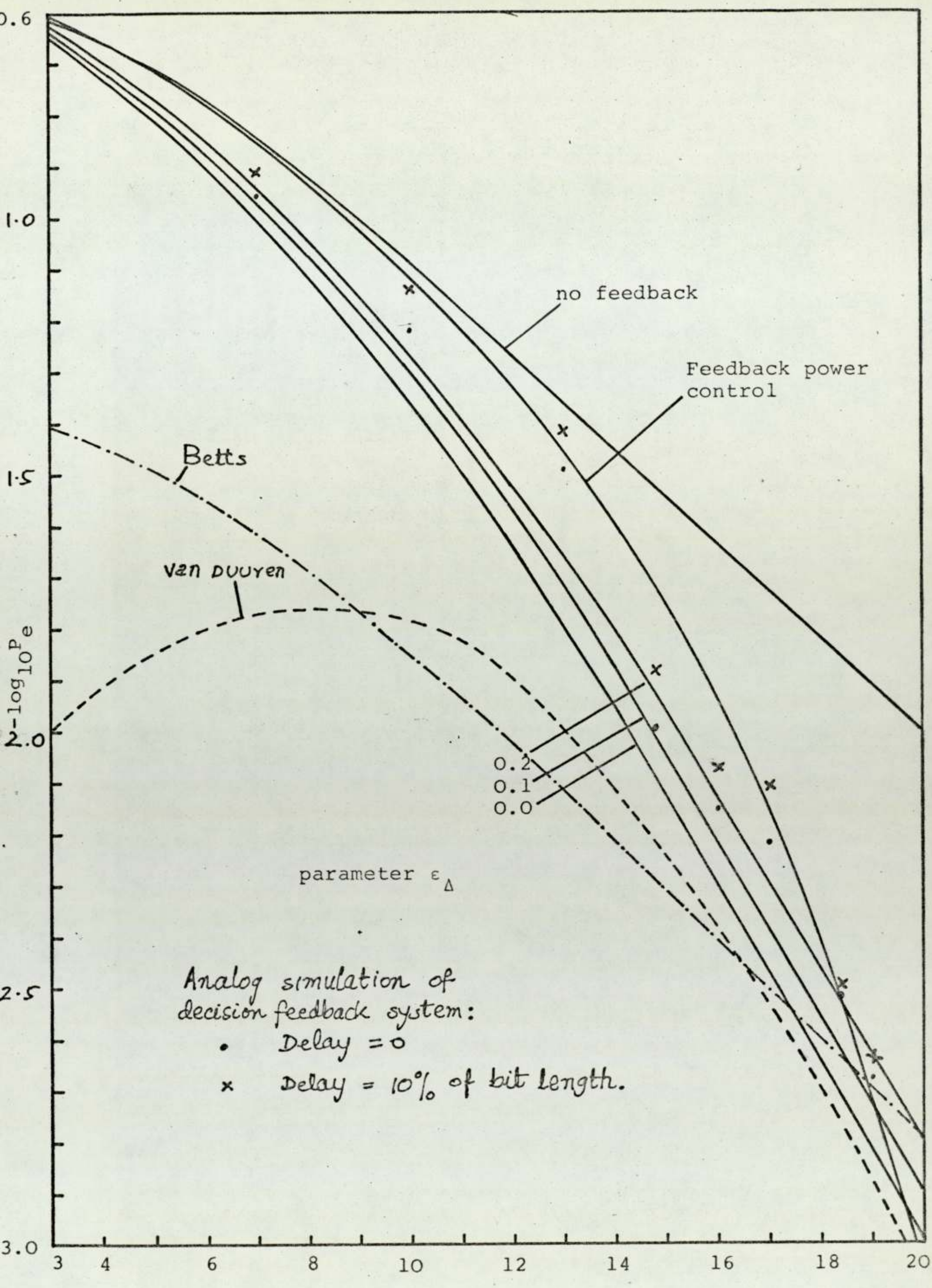


Fig. 4.4 Performance of system with feedback of estimated transmitted hypothesis

observed that the scheme compares well with feedback power control up to energy to noise ratios of about 18 to 20 dB. Also shown in Fig.4.4. are the performances of Van Duuren's and Bett's systems. Van Duuren's system has an advantage of about 1 dB at an error rate of 10^{-3} and an energy to noise ratio of about 20 dB. This advantage is maintained at higher energy to noise ratios.

4.4 Signal Design

To complete the system specification an example signal design is worked out here. From the previous section the conditions imposed on the transmitted signal are

$$\int_{o}^{T} s_1(t)s_o(t)\, dt = 0 \tag{4.13}$$

$$\int_{o}^{T_1} s_1(t)s_o(t)\, dt = 0 \tag{4.14}$$

and

$$\int_{o}^{T_1+\Delta} s_1(t)s_o(t)\, dt = 0 \tag{4.15}$$

with

$$\int_{o}^{T} s_i^{\,2}(t)\cos^2\omega_o t\, dt = 1,\ i = 0,\ 1, \tag{4.16}$$

$$\int_{o}^{T_1+\Delta} s_i^{\,2}(t)\cos^2\omega_o t\, dt \triangleq \alpha,\ i = 0,\ 1, \tag{4.17}$$

$$\int_{o}^{T_1} s_i^{\,2}(t)\cos\omega_o t\, dt \triangleq \alpha_{\wedge},\ \ i = 0,\ 1. \tag{4.18}$$

It is understood that the conditions (4.13) - (4.15) contain carrier frequency terms within the integrals but that these have been neglected since they average to zero under the assumption that there are an integral number of carrier cycles within the respective time slots.

The signal design then proceeds as follows: given the optimum α, delay measure ε_{Δ} and the signalling period T, find a signal set that satisfies the six conditions (4.13) - (4.18). An immediate choice for the signal set is

$$s_1(t) = A \sin \omega t$$

$$, \; 0 < t < T$$

$$s_o(t) = A \cos \omega t$$

where A and ω are respectively the amplitude and angular frequency to be determined. This signal is now tested for the six conditions:

Cond. 4.13

$$\int_o^T s_1(t)\; s_o(t)\; dt = \frac{A^2}{2} \int_o^T \sin 2\omega t \; dt$$

$$= \frac{-A^2}{4\omega} \{\cos 2\omega T - 1\}$$

$$= 0$$

Hence

$$\omega = \frac{n\pi}{T}, \quad n = 0, 1, 2, ---$$

and the frequency

$$f = \frac{\omega}{2\pi} = \frac{n}{2T}, \quad n = 0, 1, 2, ----$$

Cond. 4.14

$$\int_o^{T_1} s_1(t)\; s_o(t)\; dt = \frac{-A^2}{4\omega} \{\cos 2\omega T_1 - 1\}$$

$$= 0,$$

which yields

$$\omega = \frac{m\pi}{T_1} \quad , m = 0, 1, 2, ---$$

and

$$f = \frac{m}{2T_1} \quad , \quad m = 0 , 1, 2, ---$$

Cond. 4.15

In a similar manner

$$f = \frac{j}{2(T_1+\Delta)} \quad , \quad j = 0, 1, 2, ----$$

Cond. 4.16

$$\int_0^T s_i^2(t) \cos^2\omega_o t \, dt = A^2 \int_0^T \sin^2\omega t \cos^2\omega_o t \, dt$$

$$= 1$$

which, using cond. (4.13) and the fact there are integer carrier cycles in (O, T) yields

$$A = \frac{2}{T^{\frac{1}{2}}} \quad .$$

Cond. 4.17

$$\int_0^{T_1+\Delta} s_i^2(t)\cos^2\omega_o t \, dt = \frac{4}{T} \int_0^{T_1+\Delta} \sin^2\omega t \cos^2\omega_o t \, dt$$

$$= \alpha$$

which with integer carrier cycles in $(0, T_1+\Delta)$ yields

$$T_1 + \Delta = \alpha T$$

and

Cond. 4.18

similarly yields

$$T_1 = \alpha_{\wedge} T.$$

From the last section it was seen that

$$\varepsilon_\Delta = \alpha - \alpha_{\wedge}$$

which, from (4.17) and (4.18), is

$$\varepsilon_\Delta = \frac{\Delta}{T}$$

implying thereby that for this choice of signalling waveform the quantity ε_Δ represents the actual fractional round trip propagation delay. The signal frequency, from (4.13) - (4.15), is given by

$$f = \frac{n}{2T} = \frac{m}{2T_1} = \frac{j}{2(T_1+\Delta)}\ ,\ j,\ m,\ n = 0,\ 1,\ 2,\ ---- \qquad (4.19)$$

Use of (4.17) and (4.18) in (4.19) yields

$$m = n\,\alpha_{\wedge}$$

and

$$j = n\,\alpha\,. \qquad (4.20)$$

The integers j, m and n are therefore chosen to satisfy (4.20) and the frequency is determined from (4.19)

To summarize, the signalling set is given by

$$s_1(t) = \frac{2}{T^{\frac{1}{2}}} \cos 2\pi ft\ , \qquad 0 < t < T$$

$$s_o(t) = \frac{2}{T^{\frac{1}{2}}} \sin 2\pi ft, \qquad 0 < t < T$$

where f is determined from (4.19) and (4.20) and the time slots T_1 and $T_1 + \Delta$ from conditions (4.17) and (4.18). The carrier frequency is simply chosen to be an integer times the frequency of the signal set.

In conclusion, therefore, a simple signalling scheme has been suggested for use in a feedback communication system which relies on the transmitter's knowledge of the receiver estimate of the transmitted hypothesis. Transmitter operation is static in the sense that it does not have to modulate any of its parameters from bit to bit. This is an advantage (in terms of implementation) over feedback power or rate control as learning of the channel state is confined to the receiver.

4.5 Conclusion

This section concludes the chapter and the theoretical portion of the thesis by briefly investigating a feedback system which uses a combination of feedback power control and the technique of message estimate feedback studied in this chapter. Although no numerical results are presented, intuitive arguments indicate for this hybrid system, a performance that exceeds that of either one of the two techniques used singly.

The system is adaptive in that the channel gain is fedback to the transmitter along with the initial receiver estimate of the transmitted hypothesis and is used to optimally modulate the transmitter gain K. This is equivalent to modulating the energy content in the transmitted signal over the interval $(T_1 + \Delta, T)$. The signal transmitted, before modulation onto a carrier, is, as usual

$$g(t) = \begin{cases} E^{\frac{1}{2}} s_i(t) & , \quad 0 < t < T_1 + \Delta \\ E^{\frac{1}{2}} K(\beta) \left| s_i(t) - \hat{s}_i(t) \right| & , \quad T_1 + \Delta < t < T \end{cases}$$

where the transmitter gain K has been made a function of the squared channel gain. It has been assumed for convenience that the receiver is able to track channel variations exactly. Following the analysis in section 4.3 the instantaneous transmitted energy is

$$E_T = E\{\alpha + 2K^2(\beta) P_{\hat{e}}(1 - \alpha)\}$$

which when averaged over the statistics of the channel variable yields

$$\overline{E}_T = E = E\{\alpha + 2\, \overline{K^2(\beta)}\; P_{\hat{e}}(1 - \alpha)\}$$

so that

$$\overline{K^2(\beta)} = (2P_{\hat{e}})^{-1}$$

The signal to be finally detected by the receiver is, from page 71

$$h(t) = \begin{cases} a_1 E^{\frac{1}{2}} s_i(t)\cos\omega_o t + a_2 E^{\frac{1}{2}} s_i(t)\sin\omega_o t + n(t) & 0 < t < T_1 + \Delta \\ a_1 E^{\frac{1}{2}} K(\beta) s_i(t)\cos\omega_o t + a_2 E^{\frac{1}{2}} K(\beta) s_i(t)\sin\omega_o t + n(t), & T_1 + \Delta < t < T. \end{cases}$$

This is incoherently detected. From page 15 of chapter 2 the conditional probability of error given the channel state β is easily shown to be

$$P^e_\beta(b) = \tfrac{1}{2}\exp\left(-\frac{E}{2N_o}\{K(b)(1-\alpha)+\alpha\}^2 b\right).$$

The average error probability is therefore

$$P_e = \tfrac{1}{2}\int_0^\infty P^e{}_\beta(b)\ p_\beta(b)\ db.$$

This is to be minimized to determine the optimum $K(b)$ subject to the constraint

$$\overline{K^2(\beta)} = \int_0^\infty K^2(b) p_\beta(b)\ db = (2P_{\hat{e}})^{-1}$$

The above two equations are adjoined with a Lagrange multiplier and differentiated to provide the optimum gain function. This yields the transcendental equation

$$\frac{E}{4N_o}\, b\exp\left(-\frac{E}{2N_o} K_1{}^2(b) b\right) = \frac{\lambda}{(1-\alpha)^2}\ K_1{}^{-1}(b)(K_1(b)-\alpha)$$

where

$$K_1(b) = K(b)(1 - \alpha) + \alpha$$

and λ is the Lagrange multiplier determinable from the constraint equation.

These equations are not evaluated here but the form of the last equation suggests an optimum gain function that varies in a manner similar to that of the variation of optimum energy division derived in section 3.3 of Chapter 3. A rough lower bound on the error probability is observed by noting that only part of the transmitter energy (that contained in the $(T_1 + \Delta, T)$ interval) is modulated by feedback. This implies that the signal to noise advantage of this hybrid system over one without feedback must be less than the sum of the advantage obtained in section 4.3 (Fig. 4.4) and that obtained by using feedback power control assuming perfect channel tracking (page 44, Fig. 3.3).

CHAPTER 5

SYSTEM SIMULATION

5.1 Introduction

An analog computer simulation of a feedback communication link is described in this chapter. The system simulated has been studied in Sections 4.3 and 4.4 of Chapter 4 and a comparison between theoretically predicted and experimentally obtained performance is made.

5.2 Brief System Description

A Hybrid-48 analog computer was used for simulating the system shown in Figure 5.1. This system is identical to the one shown in Figure 4.2 with the receiver and fading channel shown in schematic.

A slow Rayleigh fading channel was simulated by multiplying the transmitted signal and its quadrature component (90° phase shifted at the carrier frequency) each with one of two independent zero mean gaussian noise processes that were sampled and held at the bit rate. The resulting signal processes were then added together to simulate a Rayleigh faded signal process with the fading constant over each bit but varying from bit to bit.

The receiver, being the usual correlation-squarer receiver for orthogonal signals, was built using multipliers and integrators. Owing to a limitation on the number of multipliers available on the computer eight multiplier circuits were built and externally linked with the computer.

Sinusoidal information and carrier signals were provided by two variable phase oscillators. Bit and phase synchronization between the receiver and transmitter were assumed. All timing and logic waveforms

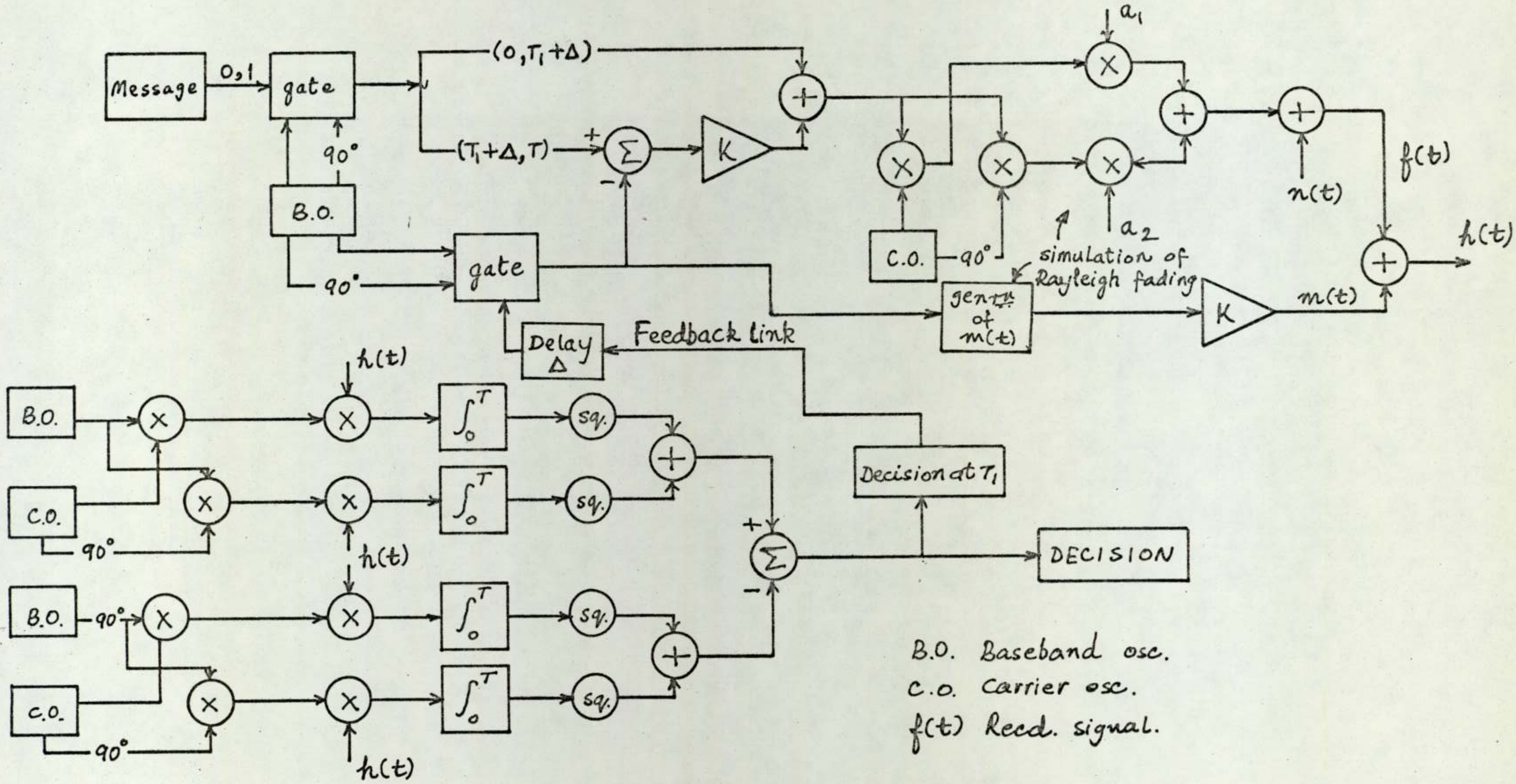


FIG. 5.1 Feedback communication system simulated on an analog computer.

were derived from the computer to control the track-store (sample and hold) amplifiers, reset the integrators, gate the signalling waveforms and to perform the intermediate and final decision operations.

5.3 Choice of Operating Parameters

A typical bit length of 10 msec was chosen. The simulation was carried out for a delayless ($\varepsilon_\Delta = 0$) and a 10% ($\varepsilon_\Delta = 0.1$) delay situation. For each signal energy to noise ratio, the optimum system parameters, i.e. transmitter gain K and energy fraction α were determined from Fig.4.3 for both delay situations. These optimum values were then used to determine the optimum signal parameters, i.e. signal frequency of f and T_1 the time taken by the receiver to generate the feedback decision. For convenience, though, sub-optimum approximations for α, f and T_1 were used. Table 4.1 lists the various parameters and their approximations used in the simulation.

5.4 Results

Some typical signalling waveforms are shown in Figures 5.2 - 5.7. The top waveform in all the pictures indicates the periodic (0,1) binary transmitted information.

Fig.5.2 shows the transmitted message signal, without the carrier, in the interval $(0, T_1)$.

Fig.5.3 shows one gaussian component of a simulated slowly fading Rayleigh channel.

Fig.5.4 shows the total transmitted signal before modulation onto a carrier. The absence of the larger sine wave (K times the smaller one) indicates where the fedback decision at time T_1 from the receiver is correct.

Fig.5.5 shows the received signal and the effect of Rayleigh fading.

Fig.5.6 shows a typical modulated signal used in the receiver for correlation detection.

	$\varepsilon_\Delta = 0.0$					$\varepsilon_\Delta = 0.1$					$P_e = 10^{-x}$			
											$\varepsilon_\Delta = 0.0$		$\varepsilon_\Delta = 0.1$	
γ dB	K	α	α used in simulation	f used in simulation Hz	T_1 used in simulation msec.	K	α	α used in simulation	f used in simulation Hz	T_1 used in simulation	No. of errors	x	No. of errors	x
7	1.46	0.455				1.4	0.515				1090	0.96	1200	0.92
10	1.8	0.44				1.75	0.5				600	1.22	724	1.14
13	2.3	0.42				2.2	0.48				320	1.49	380	1.42
14.77	2.7	0.406	0.4	500	4	2.55	0.465	0.5	1000	4	100	2.0	130	1.89
16	3.0	0.4				2.86	0.46				68	2.15	83	2.08
17	3.3	0.395				3.15	0.456				61	2.22	77	2.11
18.45	3.8	0.385				3.65	0.45				29	2.52	32	2.5
19	4.0	0.385				3.85	0.446				19	2.70	23	2.64

TABLE 4.1

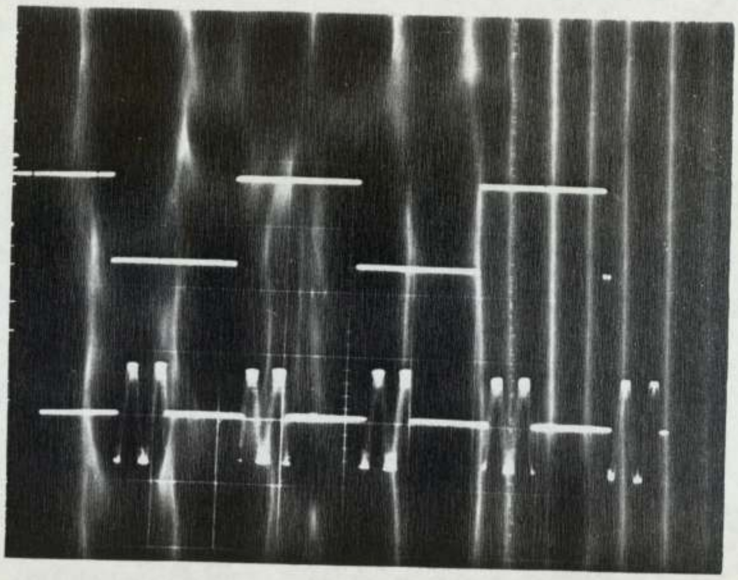


FIG. 5.2

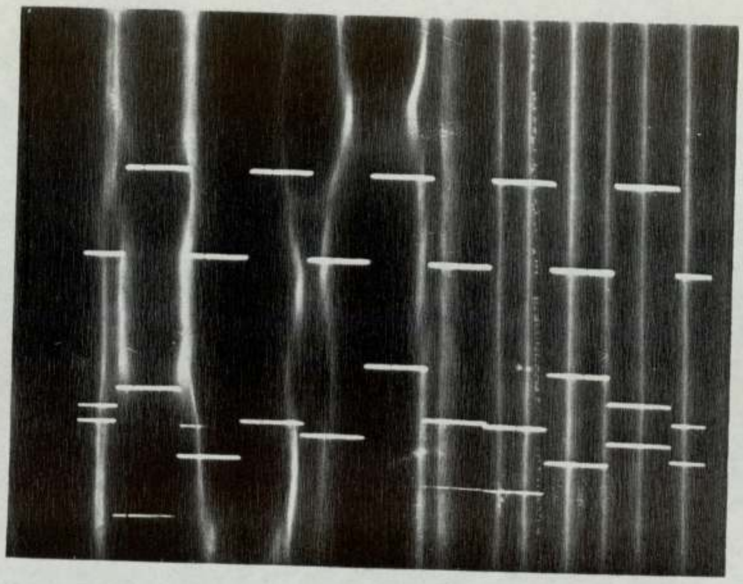


FIG. 5.3

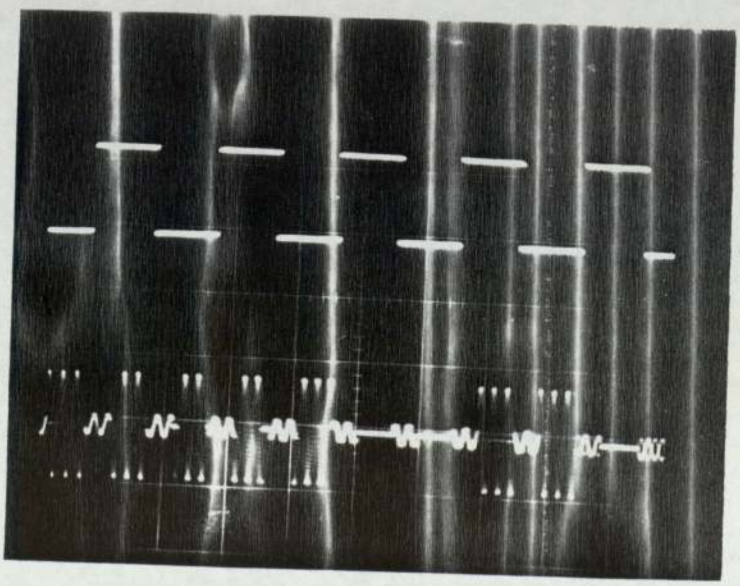


FIG. 5.4

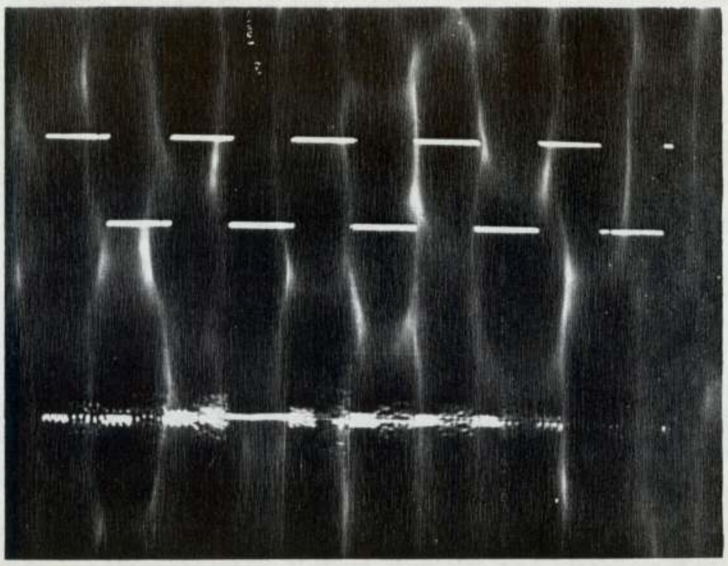


FIG. 5.5

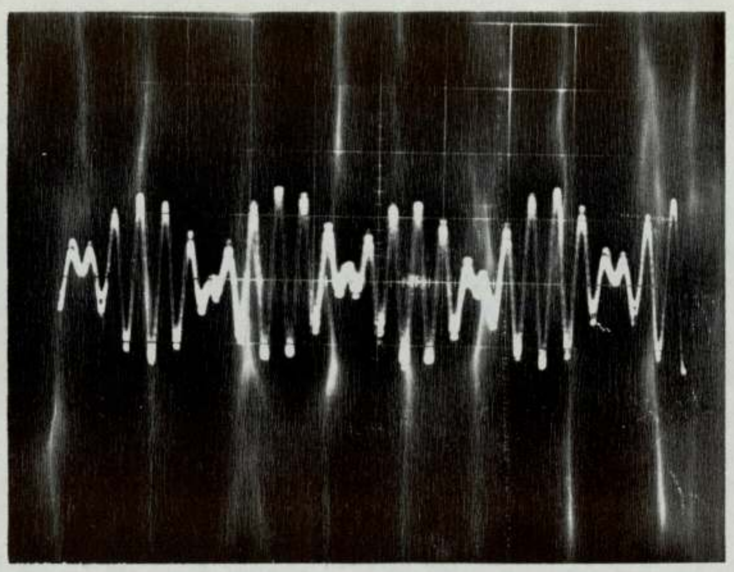


FIG. 5.6

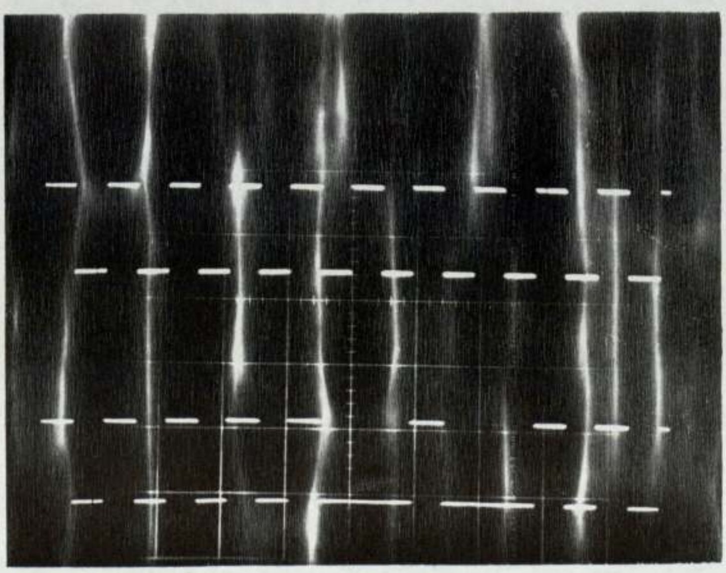


FIG. 5.7

and finally

Fig.5.7 shows the receiver message output.

Error probability measurements were made by counting the number of errors in a stream of 10^4 bits. The results are shown in the last four columns of Table 4.1. It is noted that there is a general agreement with the theoretical results in that a definite advantage over corresponding feedback-less systems has been established. The results given in Table 4.1 are plotted in Fig.4.4. The divergence between the simulation results and the theoretical error rates, however, can be attributed mainly to the fact that sub-optimum values of α and T_1 were used in the simulation.

CHAPTER 6

CONCLUSION

Two different modes of feedback control of digital communication systems operating in slowly fading environments have been studied in this thesis. The first method centres round informing the transmitter of channel conditions as estimated by the receiver. The transmitter then proceeds to modify the transmission power and/or rate accordingly. The second method utilizes the feedback link to communicate to the transmitter an initial message estimate from the receiver generated over a part of the signalling interval. The transmitter then transmits, over the rest of the interval, either no more energy or a signal with increased energy, depending on whether or not the initial message estimate was correct.

The first method, and particularly feedback control of transmission rate, results in systems that are superior to optimum dual diversity and fourth-order diversity systems. An interesting feature of these feedback systems is that some of the transmitter energy is devoted to a reference signal that is used for channel estimation. The second feedback technique, although not as good in performance, is very simple and easily implementable. This is because the system is not required to be adaptive to channel conditions. An analog computer simulation of this system indicates a definite improvement in performance over a feedback-less system. And finally, a system has been briefly described, which is based on the second feedback technique and which uses, at the same time, feedback control of transmitted energy. This is intended for situations where control of rate is not possible and promises enhanced performance approaching that of rate control.

An unavoidable drawback in any feedback communication system is the round trip propagation delay. The presence of delay places restrictions on the use of feedback systems. Consequently, the systems described thus far in this thesis are usable only for short distance ionspheric communication links where the magnitude of propagation delay is smaller than the duration of the signalling bit. For longer distance links, where the delays encountered are larger than the bit durations, a slightly different approach to the feedback problem has to be adopted. Here, the receiver is required to predict the channel state, using observation of the received signal, at a time removed from the final observation instant by an amount equal to the propagation delay. To make a predicted estimate the receiver needs to know the second order statistics of the channel. The predictor would take the form of a Wiener filter under the assumption that the channel processes are wide-sense stationary. This would bring into the picture the autocorrelation of the channel processes.

In the channel estimation signalling system studied in this thesis a filtered estimate of the channel was made on the basis of the observation of the received signal during only one bit. All prior and subsequent bit decisions are independent of this estimate, with the result that knowledge of the channel autocorrelation is not needed. However, the filtered estimate would be better if observations of the received signal are made over a longer period. This applies also to the predicted estimate. And for situations where the delay is larger than a bit duration it is not necessary to transmit a separate pilot tone channel estimation signal. Orthogonal signalling can be used with the channel estimate derived from a decision directed channel measurement process. This is because the quality of the estimate from decision directed channel measurement is better than from channel estimation signalling which is used only for the purpose of generating an estimate before the transmitter transmits the information. In view of the above arguments the assumption of slow Rayleigh fading is no longer necessary and the proposed analysis can be used to study the more realistic situation of time-selective fading wherein distortion of the transmitted signal over one bit occurs.

Having extended the forward channel fading model to a rather more generalized situation, it remains to examine the effect of feedback channel noise on the performance of feedback systems. In carrying out this analysis it should be kept in mind that the most realistic situation is one in which the feedback situation is one in which the feedback channel is, like the forward channel, subject to fading. However, one method of protecting against feedback noise would be to quantize and code the information to be fed back. In a rate control system for example, this would lead to discrete changes in rate at the transmitter which would therefore be more easily implementable than continuous rate control.

APPENDIX

From page 31 the conditional probability to be evaluated is

$$P^e_{\hat{a}_1,\hat{a}_2} = \text{Prob.}\ (M_c + M_s < 0\ |\hat{a}_1, \hat{a}_2)$$

where

$$M_c = \hat{a}_1(a_1E^{\frac{1}{2}} + \int_{T_m}^{T} n(t)\ s(t)\ \cos\omega_o t\ dt),$$

$$M_s = \hat{a}_2(a_2E^{\frac{1}{2}} + \int_{T_m}^{T} n(t)\ s(t)\ \sin\omega_o t\ dt)$$

and the channel estimates are given by

$$\hat{a}_1 = G(a_1E_m^{\frac{1}{2}} + \int_{o}^{T_m} n(t)\ s_m(t)\ \cos\omega_o t\ dt)$$

and

$$\hat{a}_2 = G(a_2E_m^{\frac{1}{2}} + \int_{o}^{T_m} n(t)\ s_m(t)\ \sin\omega_o t\ dt).$$

A new set of variables and their functions are now defined as

$$X_1 = \hat{a}_1$$

$$X_2 = M_c/\hat{a}_1$$

$$X_3 = \hat{a}_2$$

$$X_4 = M_s/\hat{a}_2$$

and

$$Y_1 = X_1$$

$$Y_2 = X_1X_2 + X_3X_4$$

$$Y_3 = X_3$$

$$Y_4 = X_4$$

Using these the required conditional probability can be expressed as

$$P^e_{\hat{a}_1,\hat{a}_2} = \text{Prob.}(Y_2 < 0 \mid Y_1, Y_3).$$

From Baye's theorem on conditional probability

$$\frac{p_{Y_1,\ldots,Y_4}(y_1,\ldots,y_4)}{p_{Y_1,Y_3}(y_1,y_3)} = p_{Y_2,Y_4|Y_1,Y_3}(y_2,y_4|y_1,y_3),$$

where p denotes probability density function.

Also

$$\int_{-\infty}^{\infty} p_{Y_2,Y_4|Y_1,Y_3}(y_2,y_4|y_1,y_3)\,dy_4$$

$$= p_{Y_2|Y_1,Y_3}(y_2|y_1,y_3)$$

and

$$\int_{-\infty}^{0} p_{Y_2|Y_1,Y_3}(y_2|y_1,y_3)\,dy_2$$

$$= \text{Prob.}\,(Y_2 < 0 | Y_1, Y_3)$$

so that

$$P^e_{\hat{a}_1,\hat{a}_2} = \int_{-\infty}^{0} \int_{-\infty}^{\infty} \frac{p_{Y_1,\ldots,Y_4}(y_1,\ldots,y_4)}{p_{Y_1,Y_3}(y_1,y_3)}\, dy_4 dy_2 \qquad \text{(A1)}$$

Invoking a theorem on the joint probability density function of functions of random variables[71] (page 329) leads to

$$p_{Y_1,\ldots,Y_4}(y_1,\ldots,y_4) = p_{X_1,\ldots,X_4}(x_1,\ldots,x_4)\{J(x_1,\ldots,x_4)\}^{-1} \qquad \text{(A2)}$$

where

$$J(x_1,\ldots,x_4) = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_4} \\ \\ \frac{\partial y_4}{\partial x_1} & \cdots & \frac{\partial y_4}{\partial x_4} \end{vmatrix}$$

Since the variables $(X_1, \ldots, X_4)$ are all gaussian with zero-mean their joint density is gaussian with

$$\overline{X_1^2} = \overline{X_3^2} = G^2(\sigma_a^2 E_m + N_o/2) \triangleq \sigma^2 \qquad \text{(A3)}$$

$$\overline{X_2^2} = \overline{X_4^2} = \sigma_a^2 E + N_o/2$$

$$\overline{X_1 X_3} = \overline{X_2 X_4} = \overline{X_1 X_4} = \overline{X_2 X_3} = 0$$

and

$$\mu^2 \triangleq \frac{\overline{X_1 X_2}^2}{\overline{X_1^2}\ \overline{X_2^2}}$$

$$= \frac{\sigma_a^4 E E_m}{(\sigma_a^2 E_m + N_o/2)\ (\sigma_a^2 E + N_o/2)} \qquad \text{(A4)}$$

Setting up the appropriate joint density of $(X_1, \ldots, X_4)$ in (A2) leads to the joint density function of the variables $(Y_1 \ldots, Y_4)$, which is used in (A1), noting that the joint density of (Y_1, Y_3) is gaussian, to yield

$$P^e_{\hat{a}_1, \hat{a}_2} = \text{erfc}\ \{ \frac{\mu/\sigma}{(1-\mu^2)^{\frac{1}{2}}}\ (y_1^2 + y_3^2)^{\frac{1}{2}} \}$$

where μ and σ are specified by (A3) and (A4) and

$$\text{erfc}(z) = (2\pi)^{-\frac{1}{2}} \int_z^\infty \exp(-t^2/2)dt.$$

In the above equation for the conditional probability it is noted that

$$(y_1^2 + y_3^2)^{\frac{1}{2}} = \hat{v},$$

the estimated channel gain, so that the conditional probability can be expressed as

$$P^e_{\hat{a}_1, \hat{a}_2} = P^e_{\hat{v}}(\hat{v}).$$

REFERENCES

1. P.A. Bello, 'Characterization of Randomly Time-Variant Linear Channels', IEEE Trans. Commun. Systems, 360-393, December 1963.

2. P.A. Bello, 'Time-Frequency Duality', IEEE Trans. Inform. Theory, 18-33, January 1964.

3. P.A. Bello, 'On the measurement of a channel correlation function', IEEE Trans. Inform. Theory, 229-239, September 1962.

4. P.A. Bello, 'The influence of fading spectrum on the binary error probabilities of incoherent and differentially coherent matched filter receivers', IRE Trans. Commun. Systems, 160-168, June 1962.

5. P.A. Bello and B.D. Nelin, 'The effect of frequency selective fading on the binary error probabilities of incoherent and differentially coherent matched filter receivers', IEEE Trans. Commun. Systems, 170-186, June 1963.

6. P.A. Bello, 'Bounds on the error probability of FSK and PSK receivers due to non-Gaussian noise in fading channels', IEEE Trans. Inform. Theory, 315-326, July 1966.

7. P.A. Bello, 'Error probabilities due to atmospheric noise and flat fading in HF ionospheric communication systems', IEEE Trans. Commun. Technol., 266-279, September 1965.

8. P.A. Bello, 'A Troposcatter channel model', IEEE Trans. Commun. Technol., 130-137, April 1969.

9. T. Kailath, 'Correlation detection of signals perturbed by a random channel', IRE Trans. Inform. Theory, 361-366. June 1960.

10. T. Kailath, 'Measurements on time-variant communication channels', IEEE Trans. Inform. Theory, 229-236, September 1962.

11 T. Kailath, 'Time-variant communication channels', IEEE Trans. Inform. Theory, 233-237, October 1963.

12. J.C. Hancock, 'Optimum performance of self-adaptive systems operating through a Rayleigh fading medium', IEEE Trans. Commun. Systems, 443-453, December 1963.

13. R. Price, 'The detection of signals perturbed by scatter and noise', IRE Trans. Inform. Theory, 163-170, September 1954.

14. R. Price, 'Optimum detection of random signals in noise, with application to scatter-multipath communication, 1', IRE Trans. Inform. Theory, 125-135, December 1956.

15. R. Price, 'Communication Technique for multipath channels', Proc. IRE, 555-570, March 1958.

16. R. Price, 'Error probabilities for adaptive multichannel reception of binary signals', IEEE Trans. Inform. Theory, 305-316, September 1962.

17. G.L. Turin, 'On optimal diversity reception', IRE Trans. Inform. Theory, 154-166, July 1961.

18. G.L. Turin, 'On optimal diversity reception, II', IRE Trans. Inform. Theory, 22-31, March 1962.

19. G.L. Turin, 'Error probabilities for binary symmetric ideal reception through nonselective slow fading and noise', Proc. IRE, 1603-1619, September 1958.

20. G.L. Turin, 'Some computations of error rates for selectively fading multipath channels', N.E.C., 1959, 431-440.

21. G.L. Turin, 'Communication through noisy, random-multipath channels', IRE Convention, 154-166, 1956.

22. J.N. Pierce, 'Error probabilities for a certain spread channel', IEEE Trans. Commun. Systems, 120-122, March 1964.

23. J.N. Pierce, 'Theoretical limitations on frequency and time diversity for fading binary transmissions', IRE Trans. Commun. Systems, 186-189, June 1961.

24. J.N. Pierce, 'Approximate error probabilities for optimal diversity combining', IEEE Trans. Commun. Systems, 352-354, September 1963.

25. J.N. Pierce and S. Stein, ' Multiple diversity with non-independent fading', Proc. IRE, 89-104, January 1960.

26. R.G. Gallager, 'Characterization and measurement of time and frequency spread channels', MIT Lincoln Laboratory Tech. Rep.352, Defense Documumentation Center, AD443715, April 30, 1964.

27. W.L. Root, 'On the measurement and use of time-varying communication channels', Information and Control, 390-422, August 1965.

28. D.G. Brennan, 'Linear diversity combining techniques', Proc. IRE, 1075-1102, June 1959.

29. H.B. Voelcker, 'Phase shift keying in fading channels', Proc.Inst. Elec. Engrs., part B, 31-38, January 1960.

30. R.S. Kennedy and I.L. Lebour, 'Signal design for dispersive channels', IEEE spectrum, 231-237, March 1964.

31. R.S. Kennedy, 'Fading dispersive communication channels', New York, Wiley, 1969.

32. R.S. Kennedy, 'Performance limitations of dispersive fading channels', ICMCI, Tokyo, Japan, 1964, Part 3, 61.

33. R. Esposito, D. Middleton and J.A. Mullen, 'Advantage of amplitude and phase adaptivity in the detection of signals subject to slow Rayleigh fading', IEEE Trans. Inform Theory, 473-481, October 1965.

34. R. Esposito, 'Error probabilities for the Nakagami channel', IEEE Trans. Inform. Theory, 145-148, January 1967.

35. R.K. Kwan and O. Shimbo, 'The effect of fading on the performance of a multihop PCM radio system', IEEE Trans. Commun. Technol., 804-810, December 1970.

36. A.T. Ural and A.H. Haddad, 'A binary sequential communication scheme with information feedback', IEEE Trans. Commun. Technol., 423-429, June 1972.

37. J.L. Marier, Jr., and T.Y. Young, 'A signalling scheme for frequency-shift-keying channels with noiseless feedback', IEEE Trans. Commun. Technol., 218-221, April 1972.

38. S.S. Lavenberg, 'Repetitive signalling using a noisy feedback channel', IEEE Trans. Inform. Theory, 269-278, May 1971.

39. U. Mengali, 'Phase noncoherent reception of binary FSK signals using post-decision feedback', IEEE Trans. Commun. Technol., 85-88. February 1971.

40. R.J.F. Fang, 'Sequential block orthogonal signalling with two noiseless decision feedbacks', IEEE Trans. Commun. Technol., 339-344, August 1970.

41. S.S. Lavenberg, 'Feedback communication using orthogonal signals', IEEE Trans. Inform. Theory, 478-483, July 1969.

42. S. Butman, 'A general formulation of linear feedback communication systems with solutions', IEEE Trans. Inform. Theory, 392-400, May 1969.

43. A.J. Kramer, 'Improving communication reliability by use of an intermittent feedback channel',IEEE Trans. Inform. Theory, 52-60, January 1969.

44. M.J. Ferguson, 'Optimal signal design for sequential signalling over a channel with feedback', IEEE Trans. Inform. Theory, 331-340, March 1968.

45. M. Hecht and M. Schwartz, 'M-ary sequential detection for amplitude-modulated signals in one and two dimensions', IEEE Trans. Commun. Technol., 669-675, October 1968.

46. G.L. Turin, 'More on uncertainty feedback: the band-limited case', IEEE Trans. Inform. Theory, 321-324, March 1968.

47. J.F. Hayes, 'Studies of sequential detection systems with uncertainty feedback', IEEE Trans. Inform. Theory, 63-70, January 1968.

48. G.L. Turin, 'Comparison of sequential and nonsequential detection systems with uncertainty feedback', IEEE Trans. Inform. Theory, 5-8, January 1966.

49. P. Elias, 'Networks of gaussian channels with application to feedback systems', IEEE Trans. Inform. Theory, 493-501, July 1967.

50. G.L. Turin, 'Signal design for sequential detection systems with feedback', IEEE Trans. Inform. Theory, 401-408, July 1965.

51. J.P.M. Schalkwigk and T. Kailath, 'A coding scheme for additive noise channels with feedback - 'art I: no bandwidth constraint',IEEE Trans. Inform. Theory, 172-183, April 1966.

52. J.P.M. Schalkwijk, 'Coding scheme for additive noise channels with feedback - part II: band-limited signals', IEEE Trans. Inform. Theory, 183-189, April 1966.

53. J.P.M. Schalkwijk, 'Center-of-gravity information feedback', IEEE Trans. Inform. Theory, 324-331, March 1968.

54. J.P.M. Schalkwijk, ' Recent developments in feedback communications', Proc. IEEE, 1242-1249, July 1969.

55. A.D. Wyner, 'On the Schalkwijk-Kailath coding scheme with a peak energy constraint,' IEEE Trans. Inform. Theory, 129-134, January 1968.

56. L.A. Shepp, J.K. Wolf, A.D. Wyner and J Ziv, 'Binary communication over the Gaussian channel using feedback with a peak energy constraint, 'IEEE Trans. Inform. Theory, 476-478, July 1969.

57. J.P.M. Schalkwijk and M.E. Barron, 'Sequential signalling under a peak power constraint', IEEE Trans. Inform. Theory, 278-282, May 1971.

58. F.E. Glave, 'Communication over fading dispersive channels with feedback', IEEE Trans. Inform. Theory, 142-150, January 1972.

59. J.F. Hayes, 'Adaptive feedback communications', IEEE Trans. Commun. Technol., 29-34, February 1968.

60. J.K. Cavers, 'Variable-rate transmission for Rayleigh fading channels', Commun., 15-22, February 1972.

61. T.A. Skinner and J.K. Cavers, 'Selective diversity for Rayleigh fading channels with a feedback link', IEEE Trans. Commun. 117-126, February 1973.

62. V.O. Hentinen, 'Error performance for adaptive transmission on fading channels', IEEE Trans. Commun., 1331-1337, September 1974.

63. G.D. Hingorani and J.C. Hancock, 'A transmitted reference system for communication in random or unknown channels', IEEE Trans. Commun. Technol., 293-301, September 1965.

64. N.J. Bershad, 'Optimum binary FSK for transmitted reference systems over Rayleigh fading channels', IEEE Trans. Commun. Technol., 784-790, December 1966.

65. C.K. Rushforth, 'Transmitted-reference techniques for random or unknown channels', IEEE Trans. Inform. Theory, 39-42, January 1964.

66. W.B. Davenport and W.L. Root, 'Random Signals and Noise', Mc Graw-Hill, New York, 1958.

67. Harry L. Van Trees, 'Detection, Estimation and Modulation Theory', Part 1, John Wiley, New York, 1968.

68. R. Price, 'Error probabilities for adaptive multichannel reception of binary signals', IEEE Trans. Inform. Theory, 305-316, September 1962.

69. J.G. Proakis, P.R. Drouilhet and R. Price, 'Performance of coherent detection systems using decision-directed channel measurement', IEEE Trans. Commun. Systems, 54-63, March 1964.

70. M.Schwartz, W.R. Bennett and S. Stein, 'Communication Systems and techniques', Mc Graw Hill, New York, 1966.

71. Emanuel Parzen, 'Modern probability theory and its applications', John Wiley, New York, 1967.

72. R. Srinivasan, 'Feedback rate control in fading channels'. Electronic Letters, IEE, 23rd August, 1973.

73. R. Srinivasan and R.L. Brewster, 'Feedback communication in fading channels', IEEE Trans. Commun., 93, January 1974.

74. R. Srinivasan, 'Feedback communication in fading channels - II', under review by IEEE Trans. Commun.

75. R. Srinivasan and R.L. Brewster, 'Feedback Communications in fading channels - III, under review by IEEE Trans. Commun.

76. G.F. Montgomery, 'Intermittent Communication with a fluctuating signal', Proc. IRE, 1678-1684, December 1957.

77. Proc. IRE, 1642-1733, December 1957. (Papers on JANET).

78. H.C.A. Van Duuren, 'Error probability and transmission speed on circuits using error detection and automatic repetition of signals', IRE Trans. Commun. Systems, 38-50, March 1961.

79. A.L. Kirsch, P.R. Gray and D.W. Hanna, Jr., 'Field-test results of the AN/GSC-10 (KATHRYN) digital data terminal', IEEE Trans. Commun. Technol., 118-128, April 1969.

80. J.A.Betts, R.S. Broom, S.J. Cook and J.G. Clark, 'Use of pilot tones for real-time channel estimation of h.f. data circuits', Proc. IEE, 887-896, September 1975.