IDENTIFICATION OF NONLINEAR SYSTEMS BY CORRELATION USING PSEUDORANDOM SIGNALS

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Doctor of Philosophy

by

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P R E F A C E

The work reported in this thesis is the result of the research carried out at the Electrical Engineering Department of the University of Aston in Birmingham from 1970 to 1973. Some of the material presented in this dissertation have appeared in the following publications:

1. BARKER, H.A., OBIDEGWU, S.N. and PRADISTHAYON, T:
   'Performance of antisymmetric pseudorandom signals in the measurement of 2nd-order Volterra kernels by crosscorrelation'.
   Proc IEE, Vol 119, No 3, March 1972, pp 353-362.
2. BARKER, H.A. and OBIDEGWU, S.N.:
   'Combined crosscorrelation method for the measurement of 2nd-order Volterra kernels'.
   Proc IEE, Vol 120, No 1, January 1973, pp 114-118.
3. BARKER, H.A. and OBIDEGWU, S.N.:
   'Identification of the dynamics of a nonlinear chemical process by pseudorandom signal testing'.
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4. BARKER, H.A. and OBIDEGWU, S.N.:
   'Effects of nonlinearities on the measurement of weighting functions by crosscorrelation using pseudorandom signals'.
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S U M M A R Y

SUMMARY

This thesis is concerned with the measurement of the characteristics of nonlinear systems by crosscorrelation, using pseudorandom input signals based on m sequences. The systems are characterised by Volterra series, and analytical expressions relating the rth order Volterra kernel to r-dimensional crosscorrelation measurements are derived.

It is shown that the two-dimensional crosscorrelation measurements are related to the corresponding second order kernel values by a set of equations which may be structured into a number of independent subsets. The m sequence properties determine how the maximum order of the subsets for off-diagonal values is related to the upper bound of the arguments for nonzero kernel values. The upper bound of the arguments is used as a performance index, and the performance of antisymmetric pseudorandom binary, ternary and quinary signals is investigated.

The performance indices obtained above are small in relation to the periods of the corresponding signals. To achieve higher performance with ternary signals, a method is proposed for combining the estimates of the second order kernel values so that the effects of some of the undesirable nonzero values in the fourth order autocorrelation function of the input signal are removed.

The identification of the dynamics of two-input, single-output systems with multiplicative nonlinearity is investigated. It is shown that the characteristics of such a system may be determined by crosscorrelation experiments using phase-shifted versions of a common signal as inputs. The effects of nonlinearities on the estimates of system weighting functions obtained by crosscorrelation are also investigated.

Results obtained by correlation testing of an industrial process are presented, and the differences between theoretical and experimental results discussed for this case.

LIST OF PRINCIPAL SYMBOLS

p = prime

$GF(p)$ = Galois field

$a_1, a_2, a_3, a_4, a_5, C, S, D,$
I = elements of $GF(p)$

$\{S_i\}$ = m sequence in $GF(p)$

N = period of S_i

$f(D)$ = characteristic polynomial of $\{S_i\}$

$f^*(D)$ = reciprocal polynomial of $f(D)$

$f(-D)$ = related polynomial of $f(D)$

$f^*(-D)$ = related polynomial of $f^*(D)$

n = order of $f(D)$

$X(I)$ = real number mapped from I in $GF(p)$

$\{x_i\}$ = pseudorandom sequence

P = period of $\{x_i\}$

$x(t)$ = pseudorandom signal, system input signal

y_i = system output sequence

$y(t)$ = system output signal

$d, i, j, k, m, n, p, q, r, u,$
$V, J, K, N, P, Q, R, I, L, M$ = integers

t, τ, λ = time

T = sampling period

T_D = time delay

T_1, T_2 = time constants

$\omega_j(\tau_1, \tau_2, \ldots, \tau_j)$ = j^{th} order Volterra kernel of system

R = upper limit of $i_1, i_2, \ldots, i_j$ for nonzero $\omega_j(i_1T, i_2T, \ldots, i_jT)$

$\phi(i_1, i_2, \ldots, i_j)$ = j^{th} order autocorrelation function of $\{x_i\}$

$\theta(\tau_1, \tau_2, .., \tau_j)$ = j^{th} order autocorrelation function of $x(t)$

$e(JT,KT)$ = crosscorrelation measurement of $w_2(JT,KT)$

r = number of dependent $e(JT,KT)$

R_r = performance index in direct crosscorrelation

R_m = upper limit of R_r, diagonal limit

Q_r = performance index in combined crosscorrelation

Q_m = upper limit of Q_r

$[\ ]$ = matrix or vector

Z_r = r x r matrix

$[I]$ = unit matrix

$w_1(IT)$ = first order Volterra kernel, weighting function

$e_1(IT)$ = estimate of $w_1(IT)$

$e_2(IT)$ = error due to second order nonlinearity

$e_3(IT)$ = error due to third order nonlinearity

Δk = increment of k

$\{W_j\}$ = weighting function estimates

$z(t)$ = postulated output signal

$\{z_i\}$ = postulated output sequences

$\{w_j\}$ = weighted crosscorrelation between the postulated output and the test sequence

δ_{jk} = Kronecker delta function

U = unit step function

C O N T E N T S

CHAPTER ONE

INTRODUCTION

1. INTRODUCTION

1.1 Nonlinear Systems

The techniques of identification of linear systems are now well developed, but unfortunately no physical system is linear over a sufficiently wide operating range. There are two main methods of describing a nonlinear system: either in terms of its differential equation or as an explicit expression of the output in functional form. Direct differential equation solutions give little insight into the behaviour of systems other than the specific system analysed. Functional methods, on the other hand, apply to a large class of systems.

In the functional representation, the response, y(t), of a time invariant nonlinear system to an input, x(t), can be expressed as:

$$\begin{aligned} y(t) = & \int_{-\infty}^{\infty} w_1(\tau_1)x(t-\tau_1)d\tau_1 \\ & + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 \\ & + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 \\ & + \ldots\ldots\ldots\ldots \\ = & \sum_{j=1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} w_j(\tau_1,\tau_2,\ldots,\tau_j)x(t-\tau_1)x(t-\tau_2)\ldots x(t-\tau_j)\,d\tau_1 d\tau_2\ldots \\ & \ldots d\tau_j \end{aligned} \tag{1.1}$$

In this form of representation, the system is characterised by the set of functions, $w_j(\tau_1,\tau_2,\ldots,\tau_j)$, known as the Volterra kernels of the system. The kernels are properties of the system alone and are not dependent on the nature of the input. Once the kernels are determined, then the system is completely characterised; the output may be computed for any given input.

The use of the Volterra series in the analysis of nonlinear systems is a generalisation of the well known convolution integral used in linear system analysis. The Volterra functional technique is a powerful method for nonlinear system analysis and synthesis because of its generality and the explicit input-output relations expressed by a functional.

1.2 Historical Review

The properties of functionals were first studied by Volterra[1] at the beginning of the twentieth century. Volterra himself applied functionals to the theory of elasticity.

Wiener[2,3] used functional theory in the solution of a Brownian motion and circuit problems. A systematic attempt to apply functional methods to nonlinear systems stems from the more recent work of Wiener[4], who used them to obtain a canonical representation of nonlinear systems.

Brilliant[5] gave a rigorous mathematical description of the theory of Volterra functionals. He introduced an algebra of functionals and also indicated how the kernels might be measured in practice.

George[6] developed the relation between the Volterra kernels of combined systems comprised of cascading, multiplying adding or a feedback combination of any two systems and the Volterra kernels of the two individual systems. He introduced the use of the association of variables when multidimensional Laplace transform is employed.

Zames[7] used the operator algebra to consider general feedback systems and he obtained an iterative expansion of the feedback operator. He later used these reults to study nonlinear distortions in feedback amplifiers[8].

Barrett[9] used functional power series to analyse systems subjected to transient, steady state and random inputs. He outlined the uses of multidimensional transform theory and applied this to the analysis of cascade and inverse filters as well as to the analysis of feedback circuits.

Flake[10] developed a method of solution applicable to nonlinear systems with or without zero initial conditions. This work has been extended by Bansal[11], who has also developed a general theory for the analysis of time-varying systems.

Stark[12] characterised the human pupil, a complex neurological system, by the first two Volterra kernels and experimentally evaluated the response of the pupil to a pseudorandom light excitation. Some of his results merely confirmed known facts but others led to the formulation of new theories that are likely to give deeper insight into the pupillary system. Stark's work has shown that bioengineers may find functional approach a valuable method in the analysis and synthesis of diverse biological systems.

More recent work on the applications of functional methods includes those of Narayanan[13] who used the Volterra series to study the intermodulation distortion of transistor feedback amplifiers, and Goldman[14] who presented, with the aid of the Volterra series analysis, a general mathematical description of the crosstalk interference created in a communications system and subsequently isolated the intelligible portion of the crosstalk.

1.3 Kernel Measurement

One of the main problems in the application of the functional theory is the explicit determination of the kernels. Analytical procedures for evaluating these kernels have been considered by McFee[15], George[6] and Flake[10]. An experimental method for measuring the Wiener kernels, suggested by Wiener himself[4] and developed by Lee and Schetzen[16], involves cross-correlation using Gaussian white noise input. The orthogonality property of the Wiener G-functionals makes it possible to determine the j^{th} order kernel by crosscorrelating the output signal with a j-dimensional product formed from the input.

The crosscorrelation technique of Lee and Schetzen will be explored in the subsequent work but neither the Wiener representation of nonlinear systems nor the input Gaussian white noise will be used. The Volterra series is preferred because it requires less terms to represent a given nonlinear system than the Wiener series. Furthermore, it is easier to interpret Volterra kernels in physical terms than the corresponding Wiener kernels. In fact, the only justification for the use of the Wiener kernels is the orthogonality of their outputs for a Gaussian white noise input.

Although Volterra series representation of a nonlinear system is an infinite one, most physical systems of practical importance can be characterised by the first few terms of the series. If the system is linear, all the terms of the Volterra series are zero except the first, and the series reduces to a convolution integral. In this case, the first order Volterra kernel or system weighting function may be computed from the results of step or sinusoidal testing or by crosscorrelating the output signal with an appropriate input perturbation, as shown schematically in fig. 1.1. The crosscorrelation experiment, which may be performed on-line without significantly affecting the normal operation of the system, gives better results than the step test when noise is present, and it is in theory faster than sine wave testing.

If the system under investigation is not linear, the first order kernel may still be obtained by crosscorrelating the output signal with the input signal. The second order kernel may be obtained by crosscorrelating the output signal with a two-dimensional product formed from the input, and the third order kernel may be obtained by crosscorrelating the output signal with a three-dimensional product formed from the input. In general, the j^{th} order kernel may be measured by crosscorrelating the output signal with a j-dimensional product formed from the input as shown schematically in fig. 1.2. The values of any kernels determined in this manner will not normally be accurate due to correlation between the input signal and the contributions to the output of other kernels present in the system. The accuracy of the results is therefore dependent on the nature of the nonlinearity and on the input signal.

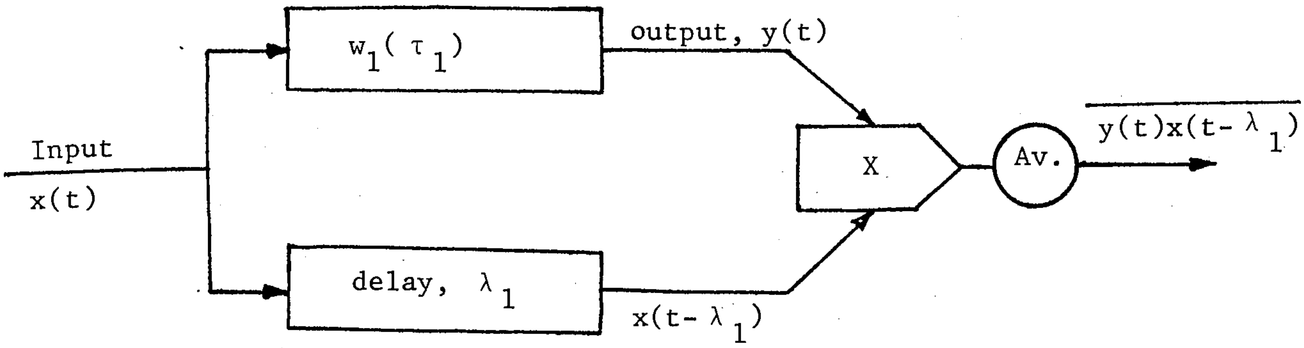


Fig.1.1. Evaluation of the first order kernel by crosscorrelation.

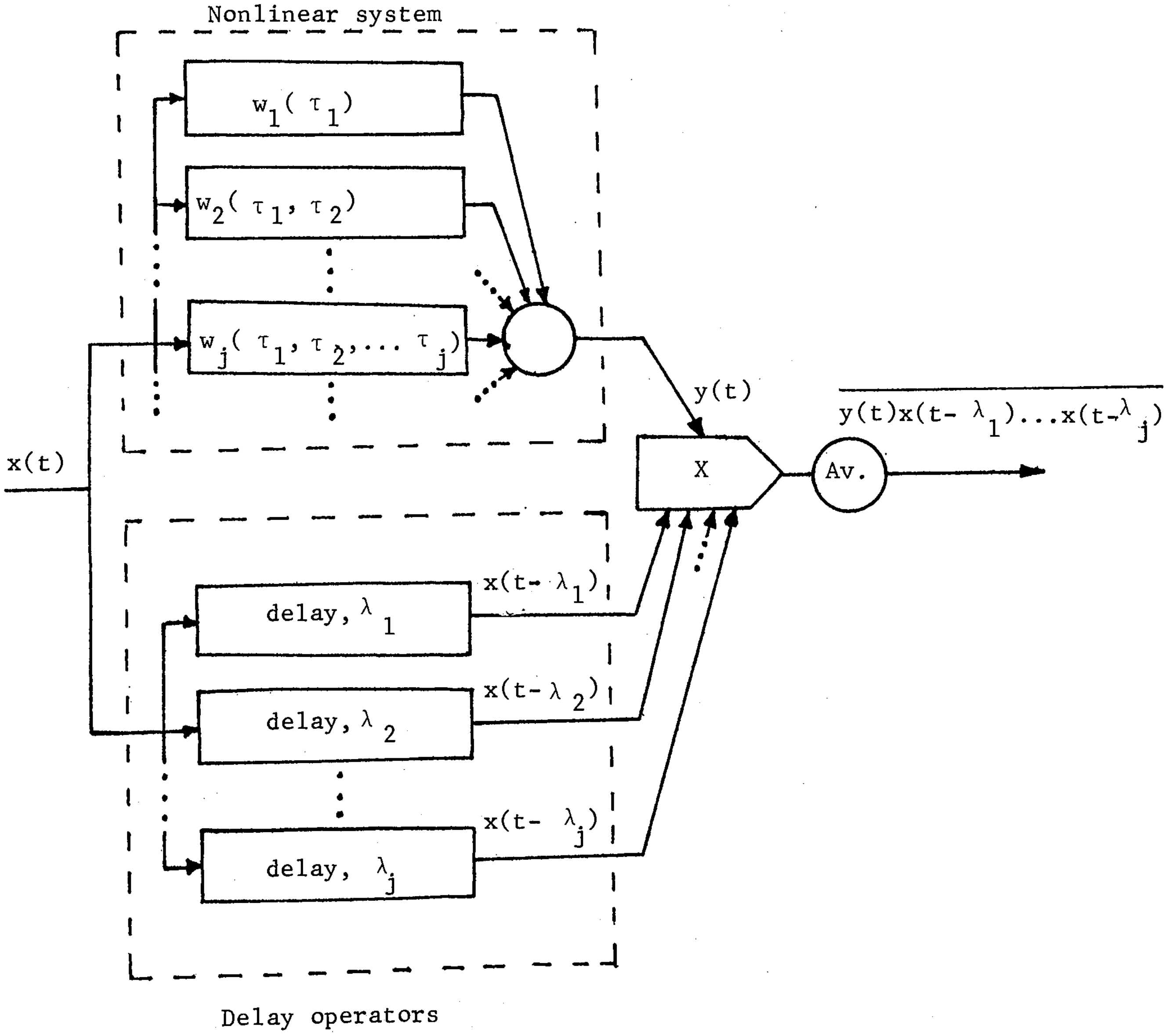


Fig. 1.2. Evaluation of the jth order kernel by crosscorrelation.

1.4 The Input Signal

The higher order autocorrelation function properties of Gaussian white noise make it an ideal input signal for correlation experiments, but unfortunately, no white noise, whose power spectrum is flat for all frequencies, is physically realisable and therefore band limited noise has to be used. Such signals have been successfully used in system identification[12,17]. However, although approximate white noise generators can be constructed from certain physical sources, it is difficult to generate a flat power spectrum at low frequencies, and it is not easy to subject a random waveform to a constant delay. The requirement of an accurate wideband analogue multiplier presents further difficulties. For accurate and consistent results, a lengthy period of integration is required. These difficulties which are encountered with conventional physical noise sources have led to a search for possible substitutes.

Of the known white noise signal approximants[18], the pseudorandom signals based on maximal length (m) sequences are the most suitable for system identification. These signals and their delayed versions are simple to generate. They are periodic and therefore it is sufficient to carry out the correlation only over the signal period. Moreover since the signals always assume a constant value over a known interval of time, multiplication can be replaced by algebraic addition. These advantages have made pseudorandom signals very popular in linear system identification[19,20].

For nonlinear systems, the pseudorandom signals present serious problems due to the fact that their higher order autocorrelation functions, unlike those of white noise, contain undesirable nonzero values. This behaviour, which was first observed by Gyftopoulos and Hooper[21,22], has been investigated by Simpson[23] and Ream[24] but the most important contribution in this field was made by Barker and Pradisthayon[25], who presented analytical expressions for calculating the higher order autocorrelation functions of pseudorandom signals of any number of levels. The undesirable nonzero values were shown to exist whenever there was linear dependence between members of the m sequence.

1.5 Outstanding Problems

Although it is well known that in the measurement of system weighting functions by crosscorrelation, the presence of higher order kernels introduces errors in the result[4], and this has been manifest in many practical crosscorrelation experiments[26,27,28,20], the exact nature of these errors, which are due to correlation between the input signal and the higher order terms of the Volterra series, has not hitherto been explained. This is probably due to the fact that until recently, analytical expressions for the higher order correlation moments of pseudorandom signals were not available.

One of the most important conclusions from the discovery of Barker and Pradisthayon[25] is that some pseudorandom signals are better than others in the identification of nonlinear systems. A need therefore exists to isolate and tabulate these superior signals.

There is still a tendency to assume that the higher order autocorrelation functions of pseudorandom signals are the same as those of Gaussian white noise[29]. While this assumption greatly simplifies the mathematics involved in the computation of the Volterra kernels, it may lead to serious errors.

The expressions for the higher order autocorrelation functions of pseudorandom signals are cumbersome mainly because of the undesirable nonzero values. Any correlation technique which will eliminate or even reduce the effects of these nonzero values is therefore useful.

Many practical systems have more than one input and one output. An extension of the correlation techniques to nonlinear multi-variable systems is also a useful contribution.

Accurate identification of a number of engineering processes such as gas chromatography[30] and direction dependent systems[31] has not yet been achieved, and therefore further work in these areas is necessary.

1.6 The Scope of the Present Investigation

The present work aims at solving some of the problems mentioned in section 1.5. The known expressions for the higher order autocorrelation functions of pseudorandom signals have been adapted to give results which are useful in system identification. The pseudorandom signals which yield optimum performance are identified and tabulated, and new properties of these signals are derived and used to obtain improved performance in the measurement of the second order Volterra kernels. The use of pseudorandom signals to identify two-input systems, which has been hitherto confined to linear processes, is extended to nonlinear systems. The effects of nonlinearities on the estimation of system weighting function by crosscorrelation using pseudorandom binary or ternary signals are studied in detail and the general results obtained are used to explain previously unexplained effects in continuous gas chromatography experiments, and to identify a system with direction dependent dynamic characteristics. The first and second order kernels of a nonlinear, noisy industrial process are also successfully identified.

CHAPTER 2

THEORY OF IDENTIFICATION OF NONLINEAR SYSTEMS BY CROSSCORRELATION USING PSEUDORANDOM SIGNALS

2. THEORY OF IDENTIFICATION OF NONLINEAR SYSTEMS BY CROSSCORRELATION USING PSEUDORANDOM SIGNALS

2.1 Introduction

This chapter describes how pseudorandom signals based on maximal length (m) sequences are derived from the Galois field elements. Emphasis is given to binary and ternary signals which have greater practical importance than others. The higher order autocorrelation functions of a pseudorandom signal and its corresponding sampled-data are defined, and the relationship between the two correlation moments is established.

The general expression for an r-dimensional crosscorrelation function from which the r^{th} order Volterra kernel may be obtained is derived, and shown to be the same for either discrete or continuous crosscorrelation. The one- and two-dimensional crosscorrelations are treated in greater depth. Analytical expressions for the errors encountered in the measurement of the first order kernel are given. The limitations of pseudorandom signals in the measurement of second order kernels of both single-input and two-input nonlinear systems are discussed and their performance in these applications established.

2.2 Pseudorandom signals based on m sequences

When $q = p^m$ 2.1

where p is a prime and m is a positive integer, a Galois field, GF(q) with q elements $a_1, a_2, \ldots, a_q$ may be defined. If S_i and C_j are elements of GF(q), the recurrence relationship

$$C_0S_i + C_1S_{i-1} + \ldots + C_nS_{i-n} = 0 \qquad C_0C_n \neq 0 \qquad 2.2$$

defines a sequence $\{S_i\}$ which has the characteristic polynomial

$$f(D) = C_0 + C_1D + \ldots + C_nD^n \qquad 2.3$$

When this polynomial is irreducible and the sequence has the maximum possible period

$$N = q^n - 1 \qquad 2.4$$

then f(D) is said to be primitive and $\{S_i\}$ is known as a maximal-length or m sequence[32].

If each field element a_j of GF(q) is mapped into a real number $X(a_j)$, then $\{S_i\}$ is mapped into the pseudorandom sequence $\{x_i\}$ in the set of real numbers $X(a_1)$, $X(a_2)\ldots X(a_q)$. The corresponding pseudorandom signal x(t), with value x_i in the interval $iT \leq t \leq (i+1)T$ is defined by means of the unit step function U(t) as

$$x(t) = \sum_{i=-\infty}^{\infty} x_i \left[U(t-iT) - U(t-<i+1>T)\right] \qquad 2.5$$

2.3 The preferred types of signals

The three main types of signals which will be considered as appropriate test signals are the pseudorandom binary signals and the antisymmtric pseudorandom binary and ternary signals based on m sequences. These signals, by comparison with the five or higher level pseudorandom signals based on m sequences, are simple to generate and use. Moreover, as will be shown in Chapter 3, the performance of ternary signals in the measurement of second order kernels is superior to that of quinary signals of comparable period, and the performance of higher level signals will be even worse because the undesirable nonzero values in the higher order autocorrelation functions of pseudorandom signals become more numerous as the number of levels of the signal increases.

2.3.1 Pseudorandom binary and antisymmetric pseudorandom binary signals

When m=1 and p=2 in equation 2.1, the elements 0,1 of GF(2) may be mapped into real numbers X(0) and X(1) so that X(0) =-X(1). If $x_i = X(S_i)$ then $\{S_i\}$ is mapped into the pseudorandom binary sequence $\{x_i\}$ with period $P=N=2^n-1$,[23] but if $x_i = (-1)^i X(S_i)$, $\{S_i\}$ is mapped into the antisymmetric pseudorandom binary sequence $\{x_i\}$ with period P=2N.[24] The corresponding pseudorandom signals are obtained by passing the sequences through a zero order hold.

2.3.2 Pseudorandom ternary signals

When m=1 and p=3, the elements of GF(3) may be taken as -1,0,1 and mapped into real numbers X(-1), X(0) and X(1) so that X(-1) = -X(1) and X(0) = 0. If $x_i = X(S_i)$ then $\{S_i\}$ is mapped into the antisymmetric ternary sequence $\{x_i\}$ with period $P=N=3^n-1$.[25]

2.3.3 Pseudorandom quinary signals

When m=1 and p=5, the elements of GF(5) may be taken as -2,-1,0,1,2 and mapped into real numbers X(-2), X(-1), X(0), X(1) and X(2) so that X(-2) = -X(2), X(-1) = -X(1) and X(0) = 0. If $\{S_i\}$ is mapped into $\{x_i\}$, the latter becomes an antisymmetric quinary sequence with period $P=N=5^n-1$.[25]

2.4 Higher order autocorrelation functions

The r^{th} order autocorrelation function of a pseudorandom sequence $\{x_i\}$ is defined by

$$\phi(i_1,i_2,\ldots,i_r)=\frac{1}{P}\sum_{i=0}^{P-1} x_{i-i_1}x_{i-i_2}\ldots x_{i-i_r} \qquad 2.6$$

and the r^{th} order autocorrelation function of a pseudorandom signal $x(t)$ is defined by

$$\theta(\tau_1,\tau_2,\ldots,\tau_r)=\frac{1}{PT}\int_0^{PT} x(t-\tau_1)x(t-\tau_2)\ldots x(t-\tau_r)dt \qquad 2.7$$

These two correlation moments are equal at the sampling instants $\tau=iT$; furthermore since $x(t)$ is constant in any interval $iT\leq t\leq(i+1)T$, the continuous autocorrelation function is linear in any τ_j in any interval $iT\leq\tau_j\leq(i+1)T$, so from equations 2.5, 2.6, and 2.7,

$$\theta(\tau_1,\tau_2,\ldots,\tau_r)=\sum_{i_1=-\infty}^{\infty}\sum_{i_2=-\infty}^{\infty}\ldots\sum_{i_r=-\infty}^{\infty}\phi(i_1,i_2,\ldots,i_r)$$

$$\times\prod_{k=1}^{r}\left[1-\frac{(\tau_k-i_kT)}{T}\right]\left[U(\tau_k-\langle i_k-1\rangle T)-U(\tau_k-\langle i_k+1\rangle T)\right] \qquad 2.8$$

2.5 r-dimensional crosscorrelation function

The r^{th} order kernel of a nonlinear system may be determined by crosscorrelating the system output with an r-dimensional product formed from the input. As shown below, if the input is delayed by multiples of the sampling interval, both continuous and discrete crosscorrelations yield the same result.

2.5.1 Continuous crosscorrelation

The crosscorrelation function of the output signal y(t) and the r-dimensional product of the input signal delayed by multiples of T, $x(t-J_1T)x(t-J_2T)\dots x(t-J_rT)$, is obtained from equations 1.1, 2.7 and 2.8 as

$$\frac{1}{PT}\int_0^{PT} y(t)\prod_{h=1}^{r} x(t-J_hT)dt$$

$$= \frac{1}{PT}\int_0^{PT}\left[\sum_{j=1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} w_j(\tau_1,\tau_2,\dots,\tau_j)\right.$$

$$\left. x\prod_{k=1}^{j} x(t-\tau_k)d\tau_k\right]\prod_{h=1}^{r} x(t-J_hT)dt$$

$$= \sum_{j=1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} w_j(\tau_1,\tau_2,\dots,\tau_j)\left[\frac{1}{PT}\int_0^{PT}\prod_{k=1}^{j} x(t-\tau_k)\right.$$

$$\left. x\prod_{h=1}^{r} x(t-J_hT)dt\right]d\tau_1 d\tau_2\dots d\tau_j$$

$$= \sum_{j=1}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} w_j(\tau_1,\tau_2,\dots,\tau_j)\,\theta(\tau_1,\tau_2,\dots,\tau_j,J_1T,$$

$$J_2T,\dots,J_rT)d\tau_1 d\tau_2\dots d\tau_j$$

$$= \sum_{j=1}^{\infty}\sum_{i_1=-\infty}^{\infty}\sum_{i_2=-\infty}^{\infty}\dots\sum_{i_j=-\infty}^{\infty}\phi(i_1,i_2,\dots,i_j,J_1,J_2,\dots,J_r)$$

$$x\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} w_j(\tau_1,\tau_2,\dots,\tau_j)\prod_{k=1}^{j}\left[1-\frac{(\tau_k-i_kT)}{T}\right]$$

$$x\left[U(\tau_k-\langle i_k-1\rangle T)-U(\tau_k-\langle i_k+1\rangle T)\right]d\tau_1 d\tau_2\dots d\tau_j$$

$$= \sum_{j=1}^{\infty} \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \phi(i_1,i_2,\ldots,i_j,J_1,J_2,\ldots,J_r)$$

$$\times \int_{(i_1-1)T}^{(i_1+1)T} \int_{(i_2-1)T}^{(i_2+1)T} \cdots \int_{(i_j-1)T}^{(i_j+1)T} w_j(\tau_1,\tau_2,\ldots,\tau_j) \prod_{k=1}^{j} \left[1- \frac{(\tau_k-i_kT)}{T} \right]$$

$$d\tau_1 d\tau_2 \ldots d\tau_j$$

The above expression gives a weighted estimate of $w_j(\tau_1,\tau_2,\ldots,\tau_j)$. The weighting is symmetrical about $i_1T,i_2T,\ldots,i_jT$, so if the kernel is expressed as a Taylor series about this point and if T is sufficiently small for the second and higher terms in the Taylor series to be ignored, then

$$\frac{1}{PT} \int_0^{PT} y(t) \prod_{h=1}^{r} x(t-J_hT)dt$$

$$= \sum_{j=1}^{\infty} T^j \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \phi(i_1,i_2,\ldots,i_j,J_1,J_2,\ldots,J_r)$$

$$\times\, w_j(i_1T,i_2T,\ldots,i_jT) \qquad 2.9$$

For physical systems, $w_j(i_1T,i_2T,\ldots,i_jT)=0$ for $i_1,i_2,\ldots,i_j<0$, and since the input pseudorandom signal is chosen such that $w_j(i_1T,i_2T,\ldots,i_jT)=0$ for $i_1T,i_2T,\ldots,i_jT>RT$ where $R<N$, the lower and upper limits in the inner summation signs in equation 2.9 can be changed from $-\infty$ to 0 and from $+\infty$ to R. Physical systems will also contain a finite number of terms and therefore the upper limit in the outer summation sign will be n, the number of terms in the Volterra series.

2.5.2 Discrete crosscorrelation

Assuming that the output sequence $\{y_i\}$ is obtained by intersampling the signal y(t) so that

$$y_i = y(\langle i+\tfrac{1}{2}\rangle T) \qquad 2.10$$

then, from Barker's synchronous sampling theorem for nonlinear systems,[33] the discrete equivalent of equation 1.1 is

$$y_i = \sum_{j=1}^{\infty} \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \int_{(i_1-\frac{1}{2})T}^{(i_1+\frac{1}{2})T} \int_{(i_2-\frac{1}{2})T}^{(i_2+\frac{1}{2})T} \cdots \int_{(i_j-\frac{1}{2})T}^{(i_j+\frac{1}{2})T} w_j(\tau_1, \tau_2, \ldots, \tau_j) \prod_{k=1}^{j} x_{i-i_k} d\tau_k \qquad 2.11$$

From equations 2.11 and 2.6, the crosscorrelation between the output sequence y_i and the r-dimensional product formed from the input sequence is given by

$$\frac{1}{P} \sum_{i=0}^{P-1} y_i \prod_{h=1}^{r} x_{i-J_h}$$

$$= \frac{1}{P} \sum_{i=0}^{P-1} \sum_{j=1}^{\infty} \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \int_{(i_1-\frac{1}{2})T}^{(i_1+\frac{1}{2})T} \int_{(i_2-\frac{1}{2})T}^{(i_2+\frac{1}{2})T} \cdots \int_{(i_j-\frac{1}{2})T}^{(i_j+\frac{1}{2})T} w_j(\tau_1, \tau_2, \ldots, \tau_j) \prod_{k=1}^{j} x_{i-i_k} d\tau_k \prod_{h=1}^{r} x_{i-J_h}$$

$$= \sum_{j=1}^{\infty} \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \int_{(i_1-\frac{1}{2})T}^{(i_1+\frac{1}{2})T} \int_{(i_2-\frac{1}{2})T}^{(i_2+\frac{1}{2})T} \cdots \int_{(i_j-\frac{1}{2})T}^{(i_j+\frac{1}{2})T} w_j(\tau_1, \tau_2, \ldots, \tau_j) \frac{1}{P} \sum_{i=0}^{P-1} \prod_{k=1}^{j} x_{i-i_k} d\tau_k \prod_{h=1}^{r} x_{i-J_h}$$

$$= \sum_{j=1}^{\infty} \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \int_{(i_1-\frac{1}{2})T}^{(i_1+\frac{1}{2})T} \int_{(i_2-\frac{1}{2})T}^{(i_2+\frac{1}{2})T} \cdots \int_{(i_j-\frac{1}{2})T}^{(i_j+\frac{1}{2})T}$$

$$w_j(\tau_1,\tau_2,\ldots,\tau_j)\phi(i_1,i_2,\ldots,i_j,J_1,J_2,\ldots,J_r)d\tau_1 d\tau_2\ldots d\tau_j$$

$$= \sum_{j=1}^{\infty} \sum_{i_1=-\infty}^{\infty} \sum_{i_2=-\infty}^{\infty} \cdots \sum_{i_j=-\infty}^{\infty} \phi(i_1,i_2,\ldots,i_j,J_1\ J_2,\ldots,J_r)$$

$$\int_{(i_1-\frac{1}{2})T}^{(i_1+\frac{1}{2})T} \int_{(i_2-\frac{1}{2})T}^{(i_2+\frac{1}{2})T} \cdots \int_{(i_j-\frac{1}{2})T}^{(i_j+\frac{1}{2})T} w_j(\tau_1,\tau_2,\ldots,\tau_j)d\tau_1 d\tau_2\ldots d\tau_j$$

2.12

The weighting is again symmetrical about $i_1T, i_2T, \ldots, i_jT$. If T is sufficiently small for second variations in $w_j(\tau_1,\tau_2,\ldots,\tau_j)$ to be ignored in regions of dimension T about any point, then for physical systems, equation 2.12 reduces to

$$\frac{1}{P}\sum_{i=0}^{P-1} y_i \prod_{h=1}^{r} x_{i-J_h}$$

$$= \sum_{j=1}^{n} T^j \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \cdots \sum_{i_j=0}^{R} \phi\ (i_1,i_2,\ldots,i_j,J_1,J_2,\ldots J_r)w_j(i_1T,i_2T,\ldots i_jT)$$

2.13

This is the same as the expression obtained for continuous crosscorrelation.

2.6 Correlation expressions for the systems investigated

The expression for the r-dimensional crosscorrelation function given in equation 2.13 involves all the kernels of the system. The ease with which the r^{th} order kernel is extracted from this equation depends on the nature of the system and on the higher order autocorrelation function properties of the input signal.

2.6.1 One-dimensional crosscorrelation

When a system is represented by more than the first two terms of the Volterra series, it is not possible to make an error-free measurement of the first order kernel by simply crosscorrelating the output signal with any input pseudorandom signal. These errors are fully investigated in chapter 6. The systems considered are those which can be represented by the first three terms of the Volterra series but in the case of antisymmetric binary and ternary input signals, the results presented are valid no matter how many even order Volterra terms are present. This is because all the odd-order correlation moments of antisymmetric pseudorandom signals are zero.

In the estimation of the first order kernel or weighting function of a system in which it is assumed that all the kernels decay to zero after a time RT, where R is less than the period of binary signals or half the period of antisymmetric binary or ternary signals, continuous crosscorrelation measurements are denoted by

$$e(IT) = \frac{1}{PT^2\phi(0,0)} \int_0^{PT} y(t)x(t-IT)dt \qquad 2.14$$

and discrete crosscorrelation measurements are denoted by

$$e(IT) = \frac{1}{PT\phi(0,0)} \sum_{i=0}^{P-1} y_i x_{i-I} \qquad 2.15$$

so from equations 2.9 and 2.13,

$$e(IT) = \frac{1}{\phi(0,0)} \sum_{j=1}^{3} T^{j-1} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \cdots \sum_{i_j=0}^{R}$$

$$\times \ \phi(i_1,i_2,\ldots,i_j,I)\ w_j(i_1T,i_2T,\ldots,i_jT)$$

$$= \sum_{j=1}^{3} e_j(IT) \qquad 2.16$$

where

$$e_j(IT) = \frac{1}{\phi(0,0)} T^{j-1} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \cdots \sum_{i_j=0}^{R} \phi(i_1,i_2,\ldots,i_j,I)$$

$$\times \ w_j(i_1T,i_2T,\ldots,i_jT)$$

When the second and third order kernels are absent, equation 2.16 reduces to the well known expression for evaluating the impulse response of linear systems. If the second order autocorrelation function of the input sequence is zero within the range RT except at $i_1=I=0$, then from equation 2.16, the first order kernel $w_1(IT)$ is given by

$$e_1(IT) = \frac{1}{\phi(0,0)} \sum_{i_1=0}^{R} w_1(i_1T)\ \phi(i_1,I)$$

$$= \frac{1}{\phi(0,0)} \sum_{i_1=0}^{R} w_1(i_1T)\,\phi(0,0)\ \delta_{i_1 I}$$

$$= w_1(IT) \qquad 2.17$$

while $e_2(IT)$ which is given by

$$e_2(IT) = \frac{T}{\phi(0,0)} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} w_2(i_1T, i_2T)\,\phi(i_1, i_2, I) \qquad 2.18$$

and $e_3(IT)$ which is given by

$$e_3(IT) = \frac{T^2}{\phi(0,0)} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \sum_{i_3=0}^{R} w_3(i_1T, i_2T, i_3T)\,\phi(i_1, i_2, i_3, I) \qquad 2.19$$

are errors due to second and third order nonlinearities respectively.

2.6.2 Two dimensional crosscorrelation - single input system

Antisymmetric pseudorandom signals are used for the measurement of second order kernels because the odd-order autocorrelation functions of such signals are zero and therefore the effects of the first and higher odd-order kernels are absent from the crosscorrelation result. Furthermore, if the system under investigation contains no even order kernels except the second, the two-dimensional crosscorrelation involves the second term alone. If JT and KT are the amount by which the input signal is delayed, then from eqns. 2.9 and 2.13,

$$\frac{1}{PT} \int_0^{PT} y(t)x(t-JT)x(t-KT)dt$$

$$= T^2 \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \phi\,(i_1, i_2, J, K) w_2(i_1T, i_2T)$$

$$= \frac{1}{P} \sum_{i_1=0}^{P-1} y_i x_{i-J} x_{i-K} \qquad 2.20$$

Now continuous two-dimensional crosscorrelation measurements are denoted by

$$e(JK,KT) = \frac{1}{2PT^3 \phi(J,K,J,K)} \int_0^{PT} y(t)x(t-JT)x(t-KT)dt \qquad 2.21$$

and discrete two-dimensional crosscorrelation measurements are denoted by

$$e(JK,KT) = \frac{1}{2PT^2 \phi(J,K,J,K)} \sum_{i=0}^{P-1} y_i x_{i-J} x_{i-K} \qquad 2.22$$

So from eqn. 2.20,

$$e(JT,KT) = \frac{1}{2\phi(J,K,J,K)} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \phi(i_1,i_2,J,K) w_2(i_1T,i_2T) \qquad 2.23$$

$$\text{If } \phi(i_1,i_2,J,K) = \phi(J,K,J,K)(\delta_{i_1J}\delta_{i_2K}+\delta_{i_1K}\delta_{i_2J}) \qquad 2.24$$

$$0 \leq i_1, i_2, J \neq K \leq R$$

where δ_{jk} is the kronecker delta eqn. 2.23 becomes

$$e(JT,KT) = \frac{1}{2\phi(J,K,J,K)} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \phi(J,K,J,K)$$

$$x(\delta_{i_1J}\delta_{i_2K} + \delta_{i_1K}\delta_{i_2J}) w_2(i_1T,i_2T)$$

$$= \tfrac{1}{2}\left[w_2(JT,KT) + w_2(KT,JT)\right]$$

$$= w_2(JT,KT) \quad \text{by symmetry} \qquad 2.25$$

Thus provided eqn. 2.24 is satisfied, each crosscorrelation measurement e(JK,KT) yields the corresponding value $w_2(JT,KT)$ of a second order kernel directly. Since the fourth order autocorrelation function of a pseudorandom signal is dependent on the characteristic polynomial,[25] signals of the same level and order obey eqn. 2.24 for different values of i_1, i_2, J, K. The index of performance of a signal with the criterion that each crosscorrelation measurement yields the corresponding kernel directly may be defined as the upper bound R_1 of R beyond which eqn. 2.24 cannot be satisfied. By comparing the values of R_1 for all signals of a particular class, those pseudorandom signals for which R_1 is greatest may be selected as most suitable for second order kernel measurements.

Unfortunately the greatest value of R_1 for any class of signals is small in relation to the signal period. This difficulty may be partly overcome by extending eqn. 2.24 to include the nonzero off-diagonal values in the fourth order autocorrelation functions. The simplest extension is to allow pairs of off-diagonal measurements $e(J_1T,K_1T)$, $e(J_2T,K_2T)$ to yield corresponding pairs of values $w_2(J_1T,K_1T)$, $w_2(J_2T,K_2T)$ of second order kernel. From equation 2.23, this requires that

$$\phi(i_1,i_2,J_1,K_1)=\phi(J_1,K_1,J_1,K_1)(\delta_{i_1J_1}\delta_{i_2K_1}+\delta_{i_1K_1}\delta_{i_2J_1})$$

$$+\phi(J_2,K_2,J_1,K_1)(\delta_{i_1J_2}\delta_{i_2K_2}+\delta_{i_1K_2}\delta_{i_2J_2})$$

$$\phi(i_1,i_2,J_2,K_2)=\phi(J_1,K_1,J_2,K_2)(\delta_{i_1J_1}\delta_{i_2K_1}+\delta_{i_1K_1}\delta_{i_2J_1})$$

$$+\phi(J_2,K_2,J_2,K_2)(\delta_{i_1J_2}\delta_{i_2K_2}+\delta_{i_1K_2}\delta_{i_2J_2}) \qquad 2.26$$

for all $0\leq i_1,i_2,J_1\neq K_1\neq J_2\neq K_2\leq R$, in which case

$$\begin{bmatrix} e(J_1T,K_1T) \\ e(J_2T,K_2T) \end{bmatrix} = Z_2 \begin{bmatrix} w_2(J_1T,K_1T) \\ w_2(J_2T,K_2T) \end{bmatrix} \text{ and therefore } \begin{bmatrix} w_2(J_1T,K_1T) \\ w_2(J_1T,K_1T) \end{bmatrix} = Z_2^{-1} \begin{bmatrix} e(J_1T,K_1T) \\ e(J_2T,K_2T) \end{bmatrix}$$

where
$$Z_2 = \begin{bmatrix} 1 & \dfrac{\phi(J_2,K_2,J_1,K_1)}{\phi(J_1,K_1,J_1,K_1)} \\ \dfrac{\phi(J_1,K_1,J_2,K_2)}{\phi(J_2,K_2,J_2,K_2)} & 1 \end{bmatrix}$$

The upper bound R_2 of R beyond which eqn. 2.26 cannot be satisfied is the index of performance with the criterion that pairs of off-diagonal measurements yield corresponding pairs of second order kernel values, except when the matrix Z_2 is singular, in which case, the values $w_2(J_1T,K_1T)$ and $w_2(J_2T,K_2T)$ are not seperable, and the performance index R_2 is equal to R_1.

In general, the criterion that sets of r off-diagonal measurements yield corresponding sets of r values of a second order kernel gives a performance index R_r, which is the upper bound of R in

$$\phi(i_1,i_2,J_j,K_j) = \sum_{k=1}^{r} \phi(J_k,K_k,J_j,K_j)(\delta_{i_1J_k}\delta_{i_2K_k} + \delta_{i_1K_k}\delta_{i_2J_k}) \qquad 2.27$$

$$j = 1,2,\ldots,r$$

for all $0 \leq i_1,i_2,J_1 \neq K_1 \neq J_2 \neq K_2 \neq \ldots \neq J_r \neq K_r \leq R$ provided that the r x r matrix

$$Z_r = \begin{bmatrix} \dfrac{\phi(J_k,K_k,J_j,K_j)}{\phi(J_j,K_j,J_j,K_j)} \end{bmatrix} \qquad 2.28$$

with element $\dfrac{\phi(J_k,K_k,J_j,K_j)}{\phi(J_j,K_j,J_j,K_j)}$ in the j^{th} row and K^{th} column is not singular.

$\phi\,(J_k,K_k,J_j,K_j)$ is nonzero when a linear relationship

$$a_1S_{i-J_k} + a_2S_{i-K_k} + a_3S_{i-J_j} + a_4S_{i-K_j} = 0 \qquad 2.29$$

exists between S_{i-J_k}, S_{i-K_k}, S_{i-J_j} and S_{i-K_j} for nonzero a_1,a_2,a_3,a_4 in GF(P).

If $\left[e(J_jT,K_jT)\right]$ is the r vector with element $e(J_jT,K_jT)$ in the j^{th} row, and if $\left[w_2(J_jT,K_jT)\right]$ is the corresponding r vector with element $w_2(J_jT,K_jT)$ in the j^{th} row, then

$$\left[e(J_jT,K_jT)\right] = Z_r\left[w_2(J_jT,K_jT)\right] \text{ and } \left[w_2(J_jT,K_jT)\right] = Z_r^{-1}\left[e(J_jT,K_jT)\right] \qquad 2.30$$

$$0\leq J_1\neq K_1\neq J_2\neq K_2\neq\ldots\neq J_r\neq K_r\leq R\leq R_r$$

If Z_r is singular, the values $w_2(J_1T,K_1T)$, $w_2(J_2T,K_2T),\ldots,$ $w_2(J_rT,K_rT)$ are not separable and the performance index R is simply given by $R_r=R_{r-1}$.

Since diagonal terms of $\phi\,(i_1,i_2,J,K)$ with $i_1=i_2$ are excluded from equation 2.27, off-diagonal terms cannot appear in $\phi(i_1,i_2,J,J)$ for any $0\leq i_1\neq i_2,J\leq R_r$, so

$$\phi(i_1,i_2,J,J) = \sum_{k=0}^{R_r} \phi\,(k,k,J,J)\,\delta_{i_1K}\,\delta_{i_2K}$$

and diagonal measurements with J=K in equations 2.21, 2.22 and 2.23 give

$$e(JT,JT) = \frac{1}{2\phi(J,J,J,J)} \sum_{k=0}^{R} \phi\,(K,K,J,J)w_2(KT,KT)$$

$$J=0,1,\ldots,R\leq R_r$$

If $\left[e(JT,JT)\right]$ is the R+1 vector with element e(JT,JT) in the $(J+1)^{th}$ row and if $\left[w_2(JT,JT)\right]$ is the corresponding R+1 vector with element $w_2(JT,JT)$ in the $(J+1)^{th}$ row, and if the matrix Y defined by

$$Y = \left[\frac{\phi(K,K,J,J)}{2\phi(J,J,J,J)}\right] \qquad 2.31$$

with element $\frac{\phi(K,K,J,J)}{2\phi(J,J,J,J)}$ in the $(J+1)^{th}$ row and $(K+1)^{th}$ column is not singular,

$$\left[e(JT,JT)\right] = Y\left[w_2(JT,JT)\right] \text{ and } \left[w_2(JT,JT)\right] = Y^{-1}\left[e(JT,JT)\right] \qquad 2.32$$

If Y is singular, then the diagonal values $w_2(0,0)$, $w_2(T,T)$,..., $w_2(RT,RT)$ are not separable.

2.6.3 Two dimensional crosscorrelation - two input system

A block diagram of a system with a single output y(t) and two inputs $x_1(t)$ and $x_2(t)$ is shown in fig. 2.1. Provided that the nonlinearity of this system does not exceed the second order the input-output relationship may be given by the Volterra series as

$$\begin{aligned} y(t) = &\int_{-\infty}^{\infty} w_1(\tau_1)x_1(t-\tau_1)d\tau_1 + \int_{-\infty}^{\infty} w_2(\tau_2)x_2(t-\tau_2)d\tau_2 \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_1(\tau_1,\tau_2)x_1(t-\tau_1)x_1(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_2(\tau_1,\tau_2)x_2(t-\tau_1)x_2(t-\tau_2)d\tau_1 d\tau_2 \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_{12}(\tau_1,\tau_2)x_1(t-\tau_1)x_2(t-\tau_2)d\tau_1 d\tau_2 \end{aligned} \qquad 2.33$$

where $w_1(\tau_1)$, $w_2(\tau_2)$, $w_1(\tau_1,\tau_2)$, $w_2(\tau_1,\tau_2)$ and $w_{12}(\tau_1,\tau_2)$ are the system kernels. Equation 2.33 is the most general case of this type; subsequent work will be confined to systems with only

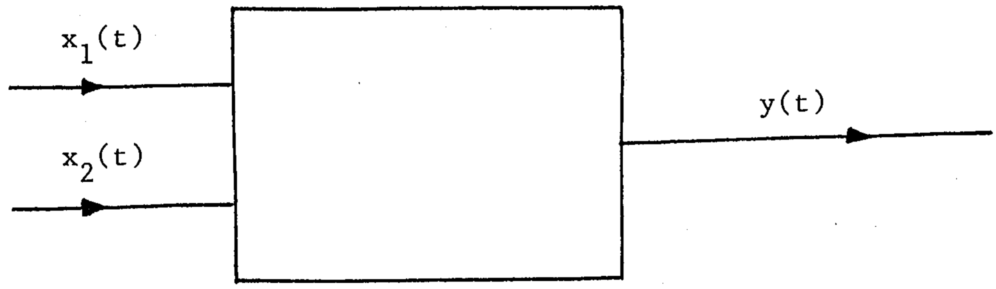


Fig. 2.1. Two-input, single-output system

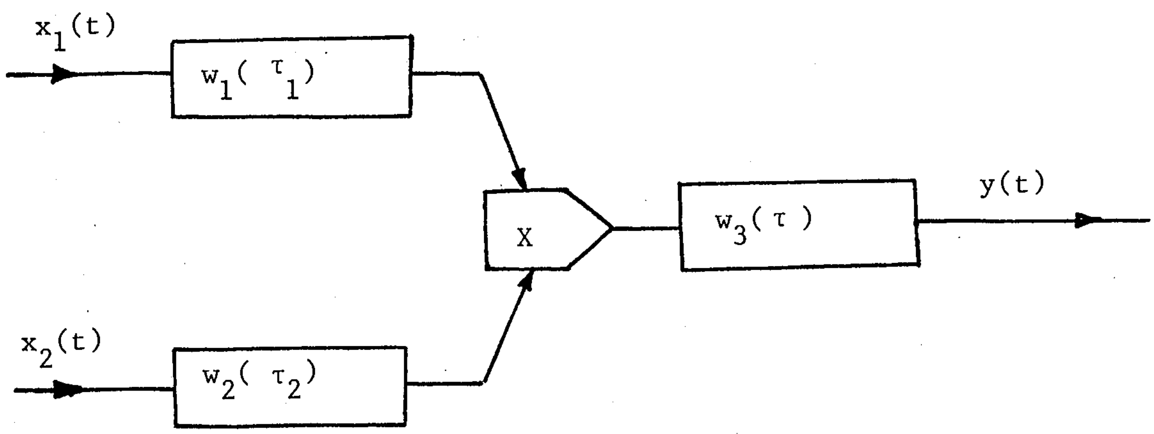


Fig.2.2. System with multiplicative nonlinearity.

multiplicative nonlinearity, a typical example of which is shown in fig. 2.2. This consists of two linear dynamics followed by a multiplier and another linear system. The output y(t) of the system of fig. 2.2 is given by

$$y(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_{12}(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 \qquad 2.34$$

where the second order kernel $w_{12}(\tau_1,\tau_2)$ is given by

$$w_{12}(\tau_1,\tau_2) = \int_0^{\tau_1 \text{ or } \tau_2} w_3(\lambda)w_1(\tau_1-\lambda)w_2(\tau_2-\lambda)d\lambda \qquad 2.35$$

To determine this second order kernel by crosscorrelation, the same pseudorandom signal may be used in the two input channels except that one will be a delayed version of the other. If x(t) and x(t-VT) are the input signals, then two-dimensional crosscorrelation gives

$$\left.\begin{array}{c} \frac{1}{PT}\int_0^{PT} y(t)x(t-JT)x(t-VT-KT) \\ \text{or} \\ \frac{1}{P}\sum_{i=0}^{P-1} y_i x_{i-J} x_{i-V-K} \end{array}\right\} = T^2 \sum_{i_1=0}^{R}\sum_{i_2=0}^{R} w_{12}(i_1T, i_2T) \times\phi(i_1, i_2+V, J, K+V) \qquad 2.36$$

The estimates e(JT,KT) of the second order kernel $w_{12}(JT,KT)$ are therefore obtained by continuous or discrete crosscorrelation as

$$e(JT,KT) = \frac{1}{\phi(J,K+V,J,K+V)} \sum_{i_1=0}^{R}\sum_{i_2=0}^{R} w_{12}(i_1T,i_2T)\phi(i_1,i_2+V,J,K+V) \qquad 2.37$$

If $\phi(i_1,i_2+V,J,K+V) = \phi(J,K+V,J,K+V)\delta_{i_1J}\delta_{i_2K}$ 2.38

$$0 \leq i_1, i_2, J, K \leq R$$

then $e(JT,KT) = w_{12}(JT,KT)$. 2.39

Thus the dynamics of a two-input, one-output system with multiplicative nonlinearity can be accurately determined by crosscorrelation provided eqn. 2.38 is satisfied. The index of performance of a signal with the criterion that each crosscorrelation measurement $e(JT,KT)$ gives the corresponding kernel $w_{12}(JT,KT)$ directly may be defined as the upper bound R_{T1} of R beyond which eqn. 2.38 cannot be satisfied. The most suitable signals in this application are those with highest values of R_{T1}.

Equation 2.38 implies that no three or four term linear relationships must exist within the regions 0 to R and V to V+R. It is possible to remove this restriction. For example, a three term linear relationship involving the arguments J_1, K_1+V and K_2+V may be allowed provided

$$\phi(i_1,i_2+V,J_1,K_1+V) = \phi(J_1,K_1+V,J_1,K_1+V)\delta_{i_1J_1}\delta_{i_2K_1}$$

$$+ \phi(J_1,K_2+V,J_1,K_1+V)\delta_{i_1J_1}\delta_{i_2K_2}$$

and

$$\phi(i_1,i_2+V,J_1,K_2+V) = \phi(J_1,K_1+V,J_1,K_2+V)\delta_{i_1J_1}\delta_{i_2K_1}$$

$$+ \phi(J_1,K_2+V,J_1,K_2+V)\delta_{i_1J_1}\delta_{i_2K_2} \qquad 2.40$$

$$0 \leq i_1, i_2, J_1, K_1, K_2 \leq R$$

in which case, from eqns. 2.37 and 2.40,

$$\begin{bmatrix} e(J_1T,K_1T) \\ e(J_1T,K_2T) \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{\phi(J_1,K_2+V,J_1,K_1+V)}{\phi(J_1,K_1+V,J_1,K_1+V)} \\ \dfrac{\phi(J_1,K_1+V,J_1,K_2+V)}{\phi(J_1,K_2+V,J_1,K_2+V)} & 1 \end{bmatrix} \begin{bmatrix} w_{12}(J_1T,K_1T) \\ w_{12}(J_1T,K_2T) \end{bmatrix} \qquad 2.41$$

The kernels $w_{12}(J_1T,K_1T)$ and $w_{12}(J_1T,K_2T)$ are easily obtained by solving eqn. 2.41. If a four term linear relationship involving the arguments J_1,J_2,K_1+V,K_2+V is allowed, then provided that

$$\phi(i_1,i_2+V,J_1,K_1+V) = \phi(J_1,K_1+V,J_1,K_1+V)\delta_{i_1J_1}\delta_{i_2K_1}$$

$$+\phi(J_2,K_2+V,J_1,K_1+V)\delta_{i_1J_2}\delta_{i_2K_2}$$

and

$$\phi(i_1,i_2+V,J_2,K_2+V) = \phi(J_1,K_1+V,J_2,K_2+V)\delta_{i_1J_1}\delta_{i_2K_1}$$

$$+\phi(J_2,K_2+V,J_2,K_2+V)\delta_{i_1J_2}\delta_{i_2K_2}$$

$$0\leq i_1,i_2,J_1,K_1,J_2,K_2\leq R \qquad 2.42$$

the pairs of second order kernel values $w_{12}(J_1T,K_1T)$ and $w_{12}(J_2T,K_2T)$ are obtained from the measurements $e(J_1T,K_1T)$ and $e(J_2T,K_2T)$ as

$$\begin{bmatrix} w_{12}(J_1T,K_1T) \\ w_{12}(J_2T,K_2T) \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{\phi(J_2,K_2+V,J_1,K_1+V)}{\phi(J_1,K_1+V,J_1,K_1+V)} \\ \dfrac{\phi(J_1,K_1+V,J_2,K_2+V)}{\phi(J_2,K_2+V,J_2,K_2+V)} & 1 \end{bmatrix}^{-1} \begin{bmatrix} e(J_1T,K_1T) \\ e(J_2T,K_2T) \end{bmatrix}$$

The upper limit of R for which either eqn. 2.40 or 2.42, but not both, is satisfied is the performance index R_{T2}.

CHAPTER 3

SECOND ORDER KERNEL MEASUREMENT - DIRECT METHOD

3. SECOND ORDER KERNEL MEASUREMENT - DIRECT METHOD

3.1 Introduction

The performance criteria and indices of pseudorandom signals in the measurement of second order Volterra kernels of single-input, single-output nonlinear systems were defined in the preceding chapter. The performance indices are governed by the undesirable nonzero values or the so-called anomalies in the fourth order autocorrelation functions of the input signal. These nonzero values are determined by polynomial division.

In this chapter, methods of evaluating the performance indices are explained. These involve numerous lengthy polynomial divisions and other mathematical manipulations but it is shown that the task may be facilitated by the exploitation of some useful properties of pseudorandom signals. The performance indices of all pseudorandom binary, ternary and quinary signals which are likely to be used in practice have been calculated and the signals with superior performance are tabulated.

All the relevant relationships of the m sequence members for one of the tabulated polynomials were derived to show how the second order kernel values could be obtained by the proposed technique. A nonlinear system of known dynamics was simulated in a digital computer and perturbed in turn by two pseudorandom ternary signals having different performance indices. The second order kernel values of the system were obtained by two dimensional cross-correlation and the results obtained with the two signals compared. All the crosscorrelation functions were in agreement with theoretical predictions. The importance of choosing appropriate signals for satisfactory identification of nonlinear systems is demonstrated by these results.

3.2 Binary Signals

The fourth order correlation moment of an antisymmetric pseudorandom binary signal has the value $\{X(1)\}^4(-1)^{i_1+i_2+J+K}$ when S_{i-i_1}, S_{i-i_2}, S_{i-J} and S_{i-K} are linearly dependent and the value $\{X(1)\}^4(-1)^{i_1+i_2+J+K+1}/N$ when they are not. For sufficiently large N, the latter value may be neglected and the fourth order autocorrelation functions may then be divided into a number of subsets as given in eqn. 2.27.

The value of r is not necessarily the same for each subset but since the matrix Z_r in eqn. 2.28 is equal to $(-1)^{J_K+K_K+J_J+K_J}$ which is singular except when r=1, R_1 is the only index of performance for antisymmetric binary signals. R_1 is the upper bound of R beyond which eqn. 2.27 cannot be satisfied when r=1, in which case, eqn. 2.29 can have no solutions other than j=k=1. The values of a_1, a_2, a_3 and a_4 in eqn. 2.29 are unity for binary signals. The performance index R_1 is therefore obtained by ensuring that there is no linear relationship of the form

$$S_{i-J} + S_{i-K} + S_{i-L} + S_{i-I} = 0 \quad \text{mod } 2 \qquad 3.1$$

for $0 \leq J \neq K \neq L < I \leq R_1 \leq N-1$

The required performance index is equal to I_{min-1} where I_{min} is the minimum value of I given by eqn. 3.1 for $0 \leq J \neq K \neq L < I \leq N-1$. All the linear relationships in the form of eqn. 3.1 are obtained by dividing the polynomial $D^J+D^K+D^L$ by the characteristic polynomial f(D) until a single term remainder D^I is obtained.[34]

To illustrate the polynomial division method, suppose it is desired to find the value of I given that J=0, K=1, L=9 and $f(D)=1+D^3+D^5$. The polynomial division may be performed as shown in fig. 3.1.

D^0	D^1	D^2	D^3	D^4	D^5	D^6	D^7	D^8	D^9	D^{10}	D^{11}	D^{12}	D^{13}	D^{14}
1	1	0	0	0	0	0	0	0	1	0	0	0	0	0
1	0	0	1	0	1									
	1	0	1	0	1	0								
	1	0	0	1	0	1								
			1	1	1	1	0	0						
			1	0	0	1	0	1						
				1	1	0	0	1	1					
				1	0	0	1	0	1					
					1	0	1	1	0	0				
					1	0	0	1	0	1				
							1	0	0	1	0	0		
							1	0	0	1	0	1		
												1		

Figure 3.1. Division of $D^0+D^1+D^9$ by the characteristic polynomial $f(D) = 1 + D^3 + D^5$

The single term remainder occurs at the 12^{th} delay and therefore $S_i+S_{i-1}+S_{i-9}+S_{i-12}=0$.

Determination of performance indices is a time consuming operation. Complete solution of equation 3.1 requires (N-3)(N-2)(N-1) polynomial divisions. Thus to determine R_1 for a binary signal of order 11, over eight thousand million polynomial divisions are needed. Fortunately this number can be drastically reduced by exploiting some properties of maximal length pseudorandom signals.

In the first place, the number of polynomial divisions can be reduced by a sixth by making use of the commutative law, for if the equation $S_{i-I} = S_{i-A} + S_{i-B} + S_{i-C}$ is satisfied for A=J, B=K and C=L, the same equation is also satisfied for the following cases: A=J, B=L, C=K; A=K, B=J, C=L; A=K, B=L, C=J; A=L, B=J, C=K; and A=L, B=K, C=J. It is therefore sufficient to solve equation 3.1 for $0 \leq J < K < L \leq N-2$.

Secondly, by the shift and add property, if $S_{i-I} = S_{i-J} + S_{i-K} + S_{i-L}$ then for any positive integer M, $S_{i-(I+M)} = S_{i-(J+M)} + S_{i-(K+M)} + S_{i-(L+M)}$. Since the minimum value of I is of interest and since (I+M) is always greater than I, it is sufficient to consider only cases for which J=0. In other words, no polynomial divisions need be performed with the shifted values of J, K and L, in which case, the number of polynomial divisions is reduced to (N-2)(N-3)/2.

In fact, this figure can be further reduced by taking as the upper limit of equation 3.1 a variable R which assumes an initial value of N-1 but decreases as the calculations proceed. Another way of minimising the computational time is to terminate the polynomial division as soon as it becomes obvious that the answer will lie outside the limit imposed by the previous calculations. For instance, if the first two values of K and L, say K_1, K_2 and L_1, L_2 in equation 3.14 yield the results

$$S_i + S_{i-K_1} + S_{i-L_1} = S_{i-I_1}$$

$$S_i + S_{i-K_2} + S_{i-L_2} = S_{i-I_2}$$

then since the minimum value of I is required, if I_2 is greater than I_1, the second equation is of no relevance. Unfortunately, there is no way of telling before commencing the polynomial division whether I_1 will be less than I_2, but this fact can be established during the division, and as soon as it is known, the calculations are discontinued for the particular values of K and L.

If R is the minimum of all previous I, only subsequent values of I less than R are of interest. The upper limit of L is now R-2 and therefore evaluation of R_1 requires solving the equation

$$S_i + S_{i-K} + S_{i-L} + S_{i-I} = 0 \quad \text{mod } 2 \qquad 3.2$$

$$I \leq K < L < I < R \leq N - 1$$

The possible steps which can be adopted in determining R_1 are summarised below:

Step 1: The variables R, K and L are assigned initial values of N-1, 1 and 2 respectively.

Step 2: The value of I, if any, satisfying equation 3.2 is obtained by dividing the polynomial $D^0+D^K+D^L$ by the characteristic polynomial f(D) until a single term remainder is obtained. If I is outside the required region, Step 3 is performed; otherwise, before Step 3, R is made equal to I.

Step 3: If L is less than R-1, L is incremented by one and Step 2 is repeated. If L is equal to R-1 and K is less than R-2, K is incremented by one, L is assigned the value of K+1 and the procedure is repeated starting from Step 2. When L is equal to R-1 and K is equal to R-2, the process is terminated and the performance index is given by R_1=R-1.

To illustrate the technique, suppose R_1 is required for a pseudorandom signal whose characteristic polynomial is given by $1+D^3+D^5$. When K=1 and L=2,3..., the following relationships are obtained

$$S_i + S_{i-1} + S_{i-2} + S_{i-22} = 0$$
$$S_i + S_{i-1} + S_{i-3} + S_{i-26} = 0$$
$$S_i + S_{i-1} + S_{i-4} + S_{i-10} = 0$$
$$S_i + S_{i-1} + S_{i-5} + S_{i-29} = 0$$
$$S_i + S_{i-1} + S_{i-6} + S_{i-25} = 0$$
$$S_i + S_{i-1} + S_{i-7} + S_{i-23} = 0$$
$$S_i + S_{i-1} + S_{i-8} + S_{i-18} = 0$$
$$S_i + S_{i-1} + S_{i-9} + S_{i-12} = 0$$

Further increment of L is not necessary because from the third equation above, R=10.

When K=2 and L=3,4,5,...,9, the following equations are obtained

$$S_i + S_{i-2} + S_{i-3} + S_{i-7} = 0$$
$$S_i + S_{i-2} + S_{i-4} + S_{i-13} = 0$$
$$S_i + S_{i-2} + S_{i-5} + S_{i-16} = 0$$
$$S_i + S_{i-2} + S_{i-6} + S_{i-21} = 0$$

It is not necessary to perform more divisions for higher values of L because from equation 1 above, R=7.

The remaining relevant values of K and L yield the following results

$$S_i + S_{i-3} + S_{i-4} + S_{i-18} = 0$$
$$S_i + S_{i-3} + S_{i-5} = 0$$
$$S_i + S_{i-3} + S_{i-6} + S_{i-19} = 0$$
$$S_i + S_{i-4} + S_{i-5} + S_{i-17} = 0$$
$$S_i + S_{i-4} + S_{i-6} + S_{i-14} = 0$$
$$S_i + S_{i-5} + S_{i-6} + S_{i-8} = 0$$

It is obvious from the above relations that R is still 7 and therefore the performance index R_1 is given by

$$R_1 = R-1 = 6$$

A program for computing R_1 for any pseudorandom binary signal is given in Appendix 1.1. Since the performance index of a sequence $\{S_i\}$ is equal to that of the reverse sequence $\{S_{-i}\}$, which is also an m sequence, for which the characteristic polynomial, $f^*(D)$, is the reciprocal polynomial of f(D) given by

$$f^*(D) = C_n + C_{n-1}D + \ldots + C_0D^n \qquad 3.3$$

only half of all m sequences of a particular order are investigated.

The performance index R_1 has been computed for all antisymmetric pseudorandom binary signals with characteristic polynomials of order 2 to 11, and the characteristic polynomials of each order for which the performance indices R_1 are greatest are given in Table 3.1.

Since the matrix Y in equation 2.31 is equal to $\left[\frac{1}{2}\right]$ which is singular except when J=K=0, the diagonal values of a second-order kernel are not separable by antisymmetric pseudorandom binary signals. The use of these signals in this application is therefore restricted to the measurement of off-diagonal values.

3.3 Ternary Signals

The ternary signals' fourth order autocorrelation functions $\phi(i_1,i_2,J,K)$ for $0 \leq i_1,i_2,J \neq K \leq R \leq R_m$, where R_m is a limit beyond which diagonal terms appear, may be divided into a number of subsets given by eqn. 2.27. The fourth order correlation moments are zero except when eqn. 2.29 is satisfied, in which case,

Table 3.1

Characteristic Polynomials of Antisymmetric Pseudorandom Binary Signals with Greatest Performance Indices.

Order	Coefficients of characteristic polynomials f(D), $f^*(D)$												Period	Performance Indices	
n	c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	2N	R_m	R_1
2	1	1	1										6	1	1
3	1	0	1	1									14	2	3
4	1	0	0	1	1								30	3	4
5	1	0	1	1	1	1							62	7	5
	1	0	0	1	0	1								4	6
6	1	1	0	0	1	1	1						126	10	6
	1	0	1	1	0	1	1							7	8
7	1	1	0	0	1	0	1	1					254	20	9
	1	0	1	0	0	1	1	1						13	10
	1	0	0	1	1	1	0	1						9	10
8	1	1	0	0	0	0	1	1	1				510	26	8
	1	1	1	0	0	1	1	1	1					26	8
	1	0	1	0	0	1	1	0	1					22	13
	1	0	0	0	1	1	1	0	1					20	13
	1	0	0	1	0	1	0	1	1					12	13
9	1	0	0	0	1	1	0	0	1	1			1022	60	12
	1	0	1	0	0	0	0	1	1	1				18	19
10	1	1	0	0	1	0	0	1	1	1	1		2046	82	19
	1	0	1	1	1	1	1	1	0	1	1			25	27
11	1	0	1	1	1	0	1	1	0	1	1	1	4094	142	26
	1	0	1	1	1	0	1	0	1	1	1	1		38	36

The performance index R_m is defined in Chapter 6.

$$\phi(J_j,K_j,J_j,K_j) = \frac{4 \times 3^{n-2}\,[X(1)]^4}{N} \qquad 3.4$$

and

$$\phi(J_k,K_k,J_j,K_j) = \frac{2 \times 3^{n-2}\,[X(1)]^4}{N}\, a_1 a_2 a_3 a_4 \qquad 3.5$$

If the matrix Z_r in eqn. 2.28 is not singular, the performance index R_r is obtained as the upper bound of R beyond which eqn. 2.27 cannot be satisfied for the largest subset of $\phi(i_1,i_2,J,K)$ with r members. In this situation, eqn. 2.29 may have at most r solutions K=1,2,...,r for any j=1,2,...,r.

Since diagonal terms are excluded from eqn. 2.27, it is necessary that for all subsets, no linear relationship of the form

$$a_J S_{i-J} + a_K S_{i-K} + a_L S_{i-L} = 0 \quad \text{mod } 3 \qquad 3.6$$

exist between any S_{i-J}, S_{i-K} and S_{i-L} for nonzero a_J, a_K and a_L in GF(3) for $0 \leq J \neq K \neq L \leq R_r \leq R_m < \frac{N}{2}$. The diagonal limit R_m is given by $R_m = L_{min} - 1$ where L_{min} is the minimum value of L given by

$$a_0 S_i + a_K S_{i-K} = a_L S_{i-L} \quad \text{mod } 3 \qquad 3.7$$

$$1 \leq K < L < R \leq \frac{N}{2} - 1$$

The above equation is solved by dividing the polynomial $a_0 D^0 + a_K D^K$ by the characteristic polynomial f(D) until a single term remainder $a_L D^L$ is obtained. Of the four possible combinations of nonzero a_0, a_K in eqn. 3.7, only two yield non-redundant equations, so for each value of K, it is sufficient to consider only two polynomials $D^0 + D^K$ and $D^0 - D^K$.

A general computer program for calculating R_m is given in Appendix 1.2 and the values of R_m for some pseudorandom ternary signals of order 2 to 8 are given in table 3.2.

Order	Coefficients of characteristic Polynomials f(D)	Period	Performance indices																				Limit
n	c_0 c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8	N	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}	R_{19}	R_{20}	R_m
2	1 1 -1	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1 1 -1 1	26	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	1 1 1 -1 -1) 1 -1 1 1 -1)	80	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	1 0 1 -1 -1 1 1 1 0 1 1 1	242	7 5	7 6	9 7	9 8	10 8	10 9	10 10	11 11	11 11	11 11	11 12	11 12	11 12	12 12	12 12	12 12	12 12	12 12	12 12	12 12	14 16
6	1 1 0 -1 -1 0 -1) 1 -1 0 1 -1 0 -1)	728	7	8	13	13	16	16	17	17	18	18	18	18	18	18	18	18	19	19	19	19	36
	1 1 1 0 1 -1 -1) 1 -1 1 0 1 1 -1)		7	8	10	11	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
	1 1 0 1 -1 1 -1) 1 -1 0 -1 -1 -1 -1)		9	10	12	13	16	17	17	17	17	18	18	18	18	18	18	18	18	18	18	18	18
	1 0 0 1 0 1 -1) 1 0 0 -1 0 -1 -1)		5	8	13	14	17	17	17	18	18	18	19	19	19	19	20	20	20	20	20	20	24
7	1 0 0 1 1 1 0 1 1 1 -1 1 1 -1 -1 1 1 1 -1 -1 1 1 -1 1 1 0 1 0 0 -1 -1 1	2186	18 11 10 10	22 16 15 14	24 16 21 15	25 25 21 19	26 27 23 21	26 27 28 25	26 28 28 25	26 29 29 25	26 31 29 29	26 31 30 29	26 31 31 30	26 31 31 33	26 31 31 33	26 31 33 33	26 31 34 33	26 31 34 34	26 31 34 34	26 31 34 35	26 31 34 36	26 31 35 36	26 50 37 36
8	1 0 -1 -1 0 -1 0 -1 -1) 1 0 -1 1 0 1 0 1 -1)	6560	21	29	37	39	39	43	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
	1 -1 0 1 -1 0 -1 0 -1) 1 1 0 -1 -1 0 -1 0 -1)		15	23	34	35	44	46	47	47	47	48	48	49	49	49	50	50	50	50	51	53	105
	1 -1 -1 -1 0 -1 1 1 -1) 1 1 -1 1 0 1 1 -1 -1)		18	26	27	41	42	47	49	49	49	52	52	54	54	54	55	57	57	57	57	57	65
	1 0 -1 -1 -1 -1 -1 0 -1) 1 0 -1 1 -1 1 -1 0 -1)		19	25	34	39	39	45	45	48	48	48	48	53	53	54	54	54	54	54	55	55	63
	1 -1 0 0 1 0 1 1 -1) 1 1 0 0 1 0 1 -1 -1)		12	20	27	32	32	34	42	43	47	49	52	53	53	53	53	53	54	54	54	54	112
	1 -1 -1 -1 0 1 -1 -1 -1) 1 1 -1 1 0 -1 -1 -1 -1)		26	35	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
	1 0 0 -1 -1 -1 0 -1 -1) 1 0 0 1 -1 1 0 1 -1)		18	29	38	42	42	43	46	47	48	49	49	50	50	51	51	51	51	51	51	51	51
	1 0 0 0 1 -1 0 1 -1) 1 0 0 0 1 1 0 -1 -1)		26	33	37	43	43	45	45	47	49	51	51	51	51	52	52	54	54	54	57	57	71
	1 -1 -1 -1 -1 -1 -1 1 -1) 1 1 -1 1 -1 1 -1 -1 -1)		19	30	30	42	44	46	49	52	52	52	52	53	53	53	53	53	53	53	53	53	53
	1 0 1 -1 0 0 0 1 -1) 1 0 1 1 0 0 0 -1 -1)		17	21	28	28	35	35	42	42	45	49	49	50	53	54	54	56	56	56	58	58	83

Table 3.2. Pseudorandom ternary signals with greatest performance indices.

The first step in evaluating R_r is to obtain all relationships of the form

$$a_1 S_{i-J_1} + a_2 S_{i-K_1} + a_3 S_{i-J_j} + a_4 S_{i-K_j} = 0 \mod 3$$

$$j = 2,3,\ldots,r \qquad 3.8$$

$$0 \leq J_1 \neq K_1 \neq J_j \neq K_j \leq R_m$$

The coefficient a_4 and the integer K_j are obtained by dividing $a_1 D^{J_1} + a_2 D^{K_1} + a_3 D^{J_j}$ by the characteristic polynomial until a single term remainder is obtained.

Of the eight possible combinations of nonzero a_1, a_2, and a_3 in eqn. 3.8, only four of them result in non-redundant equations. Therefore for each set of J_1, K_1 and J_j, division is performed only on the four polynomials $D^{J_1} + D^{K_1} + D^{J_j}$, $D^{J_1} + D^{K_1} - D^{J_j}$, $D^{J_1} - D^{K_1} + D^{J_j}$ and $D^{J_1} - D^{K_1} - D^{J_j}$.

In view of the shift and add property and the commutative law, polynomial divisions need only be performed for $J_1=0$ and $1 \leq K_1 \neq J_j < K_j \leq R_m$. To illustrate this point, all the non-shifted values of J_1, K_1 which are linearly dependent on J_j, K_j are given in table 3.3 for a pseudorandom ternary signal whose characteristic polynomial is $1+D^2-D^3-D^4+D^5$ and whose diagonal limit is 14. Let the notation

$$(J_1,K_1) \rightarrow (J_2,K_2)\ (J_3,K_3)\ldots(J_r,K_r)$$

denote the fact that the pair J_1,K_1 is linearly dependent on the pairs J_j,K_j; $j=2,3,\ldots,r$.

For all the shifted values of J_1 and K_1, the corresponding values of J_j and K_j are obtainable from table 3.3. For instance, suppose dependent pairs are required for $J_1=4$ and $K_1=9$, table 3.3 gives the result for $J_1 = 4-4 = 0$ and $K_1 = 9-4 = 5$ as $(0,5) \rightarrow (8,11)$ $(3,14)$. Therefore by the shift and add property

J_1	K_1	J_2	K_2	J_3	K_3	J_4	K_4	J_5	K_5	J_6	K_6	J_7	K_7
0	1	3	8	6	10	2	12	11	14				
0	2	4	10	1	12	8	13						
0	3	1	8	9	11	5	14						
0	4	7	8	6	9	2	10						
0	5	8	11	3	14								
0	6	4	9	1	10	11	13						
0	7	4	8	11	12								
0	8	1	3	4	7	5	11	2	13	9	14		
0	9	4	6	3	11	10	12	8	14				
0	10	2	4	1	6	9	12						
0	11	5	8	3	9	7	12	6	13	1	14		
0	12	1	2	9	10	7	11						
0	13	2	8	6	11								
0	14	3	5	8	9	1	11						

<u>Table 3.3</u> Ternary Sequence with Characteristic Polynomial $1+D^2-D^3-D^4+D^5$; Values of J_1, K_1 Linearly Dependent on J_j, K_j.

$(4,9) \rightarrow (12,15)\ (7,18)$.

Since at least one of the arguments in each pair (12,15) and (7,18) is greater than R_m, none of the pairs gives rise to a four term linear relationship within the required region. However from line 1 of table 3.3, $(0,1) \rightarrow (3,8)$ so by the shift and add property and the commutative law, $(4,9) \rightarrow (1,2)$. Also from line 6, $(0,6) \rightarrow (4,9)$ so by the commutative law, $(4,9) \rightarrow (0,6)$. Therefore for $J_1=4$ and $K_1=9$, the J_j and K_j values in the required region are given by

$(4,9) \rightarrow (1,2)\ (0,6)$.

Once all the values of K_j which are dependent on J_1, K_1 and J_j are found, R_r can be calculated by finding the maximum value of R for which

$$(J_K,K_K) \rightarrow (J_1,K_1)\ (J_2,K_2)\dots(J_r,K_r)$$

$$K = 1,2,\dots,r$$

$$0 \leq J_1 \neq K_1 \neq J_2 \neq K_2 \dots \neq J_r \neq K_r \leq R \leq R_m$$

For instance, suppose R_3 is required for a ternary signal with characteristic polynomial $1+D^2-D^3-D^4+D^5$. It is obvious from line 1 of table 3.3 that if the above relationship is to be satisfied for $J_1=0$ and $K_1=1$, R must be reduced from R_m to 11, in which case,

$(0,1) \rightarrow$	$(0,1)$	$(3,8)$	$(6,10)$
$(3,8) \rightarrow$	$(0,1)$	$(3,8)$	-
$(6,10) \rightarrow$	$(0,1)$	-	$(6,10)$

which satisfies the required criterion. The test must now be carried out for all the shifted values of (0,1). The first shift gives

(1,2) →	(1,2)	(4,9)	(6,11)			
(4,9) →	(1,2)	(4,9)	-	(0,6)		
(6,11) →	(1,2)	-	(6,11)	-		
(0,6) →	-	(4,9)	-	-	(1,10)	
(1,10) →	-	-	-	(0,6)	(1,10)	(5,7)
(5,7) →	-	-	-	-	(1,10)	(5,7)

Thus within R=11, there are 6 pairs of values. To reduce this number to 3, R must be decreased to 9, in which case,

(1,2) →	(1,2)	(4,9)	-
(4,9) →	(1,2)	(4,9)	(0,6)
(0,6) →	-	(4,9)	(0,6)

which satisfies the criterion.

It can be shown that eqn. 2.29 is satisfied for the remaining shifted values of (0,1) which are (2,3), (3,4)...,(8,9). The other nonshifted values of (J_1,K_1) which are (0,2), (0,3)..,(0,9) and all their shifted counterparts also satisfy the criterion. Therefore for the sequence under consideration, R_3=9.

A computer program for calculating the performance indices is given in Appendix 1.3, and the performance indices R_r for values of r from 1 to 20 have been computed for all ternary signals with characteristic polynomials of order 2 to 8; the characteristic polynomials of each order for which each performance index is greatest are given in table 3.2. The reciprocal polynomials $f^*(D)$ give exactly the same performance indices as f(D) and are not tabulated.

The off-diagonal values of a 2nd-order kernel for which $R \leq R_r$ in eqn. 2.23 are obtained from the corresponding off-diagonal measurements using eqn. 2.30 for each subset. The diagonal values of such a kernel are obtained from the corresponding diagonal measurements using eqn. 2.32, which reduces to a simple form for a ternary signal. Since ϕ (J,J,J,J) is given by[25]

$$\phi\,(J,J,J,J) = \frac{3^{n-1}}{N} \sum_{I_1=-1}^{1} \{\, X(I_1)\,\}^4 = \frac{2 \times 3^{n-1}}{N} \{X(1)\}^4 \qquad 3.9$$

and ϕ(K,K,J,J) is given by eqn. 3.4, then

$$\left[\frac{\phi\,(K,K,J,J)}{2\ \phi(J,J,J,J)}\right] = \frac{1}{6}\left[I\right] + \left[\frac{1}{3}\right]$$

so that, since the matrices are (R+1) x (R+1),

$$\left[\frac{\phi\,(K,K,J,J)}{2\ \phi(J,J,J,J)}\right]^{-1} = 6\left[I\right] - \left[\frac{12}{2R+3}\right]$$

and therefore eqn. 2.32 reduces to

$$w_2(JT,JT) = 6e(JT,JT) - \frac{12}{2R+3} \sum_{K=0}^{R} e(KT,KT) \qquad 3.10$$

$$J=0,1,\ldots,R$$

3.4 Quinary Signals

The performance of antisymmetric pseudorandom quinary signal depends on the properties of the fourth order autocorrelation function of the corresponding pseudorandom sequence which have been considered by Barker and Pradisthayon.[25] The performance indices R_r are obtained by procedures based on polynomial division in a similar way to that described in section 3.3, and have been computed for values of r from 1 to 20 for all quinary signals with characteristic polynomials of order 2 to 5. The characteristic polynomials of each order for which each performance index is greatest are given in Table 3.4.

Table 3.4.

Characteristic polynomials of pseudorandom quinary signals with greatest performance indices.

order n	Coefficients of characteristic polynomials: $f(D)$ (c_o c_1 c_2 c_3 c_4 c_5)	$f^*(D)$ (c_o c_1 c_2 c_3 c_4 c_5)	Period N	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}	R_{19}	R_{20}	Limit R_m
2	1 1 2 1 2 -2	1 -2 -2 1 -1 2	24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1 1 -1 -2 1 -1 -1 2	1 -2 2 2 1 2 2 -2	124	2	3	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4	1 1 2 1 2 1 2 -2 -2 2 1 -2 -2 2 2 1 -1 2 -1 2	1 -2 1 -2 -2 1 -1 -1 1 -2 1 1 -1 -1 -2 1 2 1 2 -2	624	4	5	5	6	6	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	9
5	1 1 0 -2 2 2 1 -1 0 2 2 -2	1 1 -1 0 -2 -2 1 -1 -1 0 -2 2	3124	9	10	12	12	14	14	14	15	15	15	15	15	15	15	15	15	15	15	15	15	15
	1 2 0 2 -1 2 1 -2 0 -2 -1 -2	1 2 1 0 1 -2 1 -2 1 0 1 2		6	7	10	11	13	13	13	13	14	14	15	15	15	15	15	15	16	16	16	16	28

3.5 Example : General Nonlinear System

In the following example, relationships are obtained for the determination of the values $w_2(JT,KT)$ of a 2nd-order kernel $w_2(\tau_1, \tau_2)$ for which

$$w_2(\tau_1,\tau_2) = 0 \qquad \tau_1, \tau_2 > 0, \quad \tau_1, \tau_2 > 13T$$

by means of an antisymmetric pseudorandom ternary signal x(t) with levels X(1), 0 and -X(1), period 728T and characteristic polynomial

$$f(D) = 1 + D + D^3 - D^4 + D^5 - D^6$$

From eqns. 2.21, 2.22 and 3.4, off-diagonal measurements e(JT,KT) with J≠K are obtained by continuous or discrete crosscorrelation as

$$e(JT,KT) = \frac{1}{648T^3 \{X(1)\}^4} \int_0^{728T} y(t)x(t-JT)x(t-KT)dt$$

or 3.11

$$e(JT,KT) = \frac{1}{648T^2 \{X(1)\}^4} \sum_{i=0}^{727} y_i x_{i-J} x_{i-K}$$

All linear relationships in the form of eqn. 2.29 for $0 \leq J_k, K_k, J_j, K_j \leq 13$, are given by changes of order in each equation of

$$S_i + S_{i-1} + S_{i-6} + S_{i-10} = 0$$
$$S_i - S_{i-2} + S_{i-3} + S_{i-11} = 0$$
$$S_i - S_{i-5} - S_{i-10} - S_{i-12} = 0$$
$$S_{i-1} + S_{i-2} + S_{i-7} + S_{i-11} = 0$$
$$S_{i-1} - S_{i-3} + S_{i-4} + S_{i-12} = 0$$
$$S_{i-1} - S_{i-6} - S_{i-11} - S_{i-13} = 0$$
$$S_{i-2} - S_{i-3} + S_{i-8} + S_{i-12} = 0$$
$$S_{i-2} - S_{i-4} + S_{i-5} + S_{i-13} = 0$$
$$S_{i-3} + S_{i-4} + S_{i-9} + S_{i-13} = 0$$

and from eqns. 3.4 and 3.5, eqn. 2.30 may then be obtained for each subset. For 1 subset with 4 members, the required relationships are

$$\begin{bmatrix} e(T,6T) \\ e(0,10T) \\ e(5T,12T) \\ e(11T,13T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 2 & -1 & 0 \\ 0 & -1 & 2 & 0 \\ -1 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} w_2(T,6T) \\ w_2(0,10T) \\ w_2(5T,12T) \\ w_2(11T,13T) \end{bmatrix}$$

$$\begin{bmatrix} w_2(T,6T) \\ w_2(0,10T) \\ w_2(5T,12T) \\ w_2(11T,13T) \end{bmatrix} = \frac{2}{5} \begin{bmatrix} 6 & -4 & -2 & 3 \\ -4 & 6 & 3 & 2 \\ -2 & 3 & 4 & -1 \\ 3 & -2 & -1 & 4 \end{bmatrix} \begin{bmatrix} e(T,6T) \\ e(0,10T) \\ e(5T,12T) \\ e(11T,13T) \end{bmatrix}$$

For 6 subsets with 3 members, the required relationships are of the form

$$\begin{bmatrix} e(0,3T) \\ e(T,7T) \\ e(2T,11T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(0,3T) \\ w_2(T,7T) \\ w_2(2T,11T) \end{bmatrix} \qquad \begin{bmatrix} w_2(0,3T) \\ w_2(T,7T) \\ w_2(2T,11T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 3 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 4 \end{bmatrix} \begin{bmatrix} e(o,3T) \\ e(T,7T) \\ e(2T,11T) \end{bmatrix} \quad \text{...3.12}$$

the other subsets obtained by relationships identical to eqn. 3.12 being

$$\begin{bmatrix} w_2(0,11T) \\ w_2(8T,12T) \\ w_2(2T,3T) \end{bmatrix} \begin{bmatrix} w_2(T,4T) \\ w_2(2T,8T) \\ w_2(3T,12T) \end{bmatrix} \begin{bmatrix} w_2(T,12T) \\ w_2(9T,13T) \\ w_2(3T,4T) \end{bmatrix} \begin{bmatrix} w_2(2T,5T) \\ w_2(3T,9T) \\ w_2(4T,13T) \end{bmatrix} \begin{bmatrix} w_2(6T,13T) \\ w_2(2T,7T) \\ w_2(T,11T) \end{bmatrix}$$

For 7 subsets with 2 members, the required relationships are of the form

$$\begin{bmatrix} e(0,2T) \\ e(3T,11T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(0,2T) \\ w_2(3T,11T) \end{bmatrix} \qquad \begin{bmatrix} w_2(0,2T) \\ w_2(3T,11T) \end{bmatrix} = \tfrac{2}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} e(0,2T) \\ e(3T,11T) \end{bmatrix} \quad \text{...3.13}$$

the other subsets obtained by relationships identical to eqn. 3.13 being

$$\begin{bmatrix} w_2(T,3T) \\ w_2(4T,12T) \end{bmatrix} \begin{bmatrix} w_2(2T,4T) \\ w_2(5T,13T) \end{bmatrix} \begin{bmatrix} w_2(0,5T) \\ w_2(10T,12T) \end{bmatrix} \begin{bmatrix} w_2(4T,5T) \\ w_2)2T,13T) \end{bmatrix} \begin{bmatrix} w_2(5T,10T) \\ w_2(0,12T) \end{bmatrix} \begin{bmatrix} w_2(6T,11T) \\ w_2(T,13T) \end{bmatrix}$$

For 5 subsets with 2 members, the required relationships are of the form

$$\begin{bmatrix} e(0,T) \\ e(6T,10T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(0,T) \\ w_2(6T,10T) \end{bmatrix}, \quad \begin{bmatrix} w_2(0,T) \\ w_2(6T,10T) \end{bmatrix} = \tfrac{2}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} e(0,T) \\ e(6T,10T) \end{bmatrix} \qquad \ldots 3.14$$

the other subsets obtained by relationships identical to eqn. 3.14 being

$$\begin{bmatrix} w_2(T,1T) \\ w_2(7T,11T) \end{bmatrix} \begin{bmatrix} w_2(0,6T) \\ w_2(T,10T) \end{bmatrix} \begin{bmatrix} w_2(3T,8T) \\ w_2(2T,12T) \end{bmatrix} \begin{bmatrix} w_2(4T,9T) \\ w_2(3T,13T) \end{bmatrix}$$

For the 45 remaining values $w_2(JT,KT)$ with $J < K$, $w_2(JT,KT)=e(JT,KT)$.

For the 91 symmetrical values $w_2(JT,KT)$ with $J > K$, $w_2(JT,KT)=w_2(KT,JT)$.

From eqns. 2.21, 2.22 and 3.9, diagonal measurements are obtained by continuous or discrete crosscorrelation as

$$e(JT,JT) = \frac{1}{972T^3\{X(1)\}^4} \int_0^{728T} y(t) \left[x(t-JT) \right]^2 dt$$

or

$$e(JT,JT) = \frac{1}{972T^2\{X(1)\}^4} \sum_{i=0}^{727} y_i (x_{i-J})^2$$

and from eqn. 3.10,

$$w_2(JT,JT) = 6e(JT,JT) - \frac{12}{29} \sum_{K=0}^{13} e(KT,KT) \qquad J=0,1,\ldots,13$$

3.6 Example: Simulated Second Order System

To test the validity of the equations developed in the preceding sections, the system shown in Fig. 3.2 was simulated on a digital computer. The second order kernel of this system was determined by two-dimensional crosscorrelation and compared with the theoretical result which may be shown to be:

$$w_2(JT,KT) = A_1^2 \, (1 - e^{-T/T_1})^2 \, e^{-(J+K-2)T/T_1} \qquad 3.15$$

$$J,K=1,2,3,\ldots$$

The linear component of the system shown in Fig. 3.2 is given by

$$w_1(t) = \frac{A_1}{T_1} e^{-t/T_1} \qquad t > 0 \qquad 3.16$$

$$= 0 \qquad \text{otherwise}$$

where T_1 is a time constant and A_1 is a constant. The Z transform of the zero-order-hold and the linear component is given by

$$\frac{U(z)}{x(z)} = \frac{z-1}{z} \mathcal{Z} \frac{A_1/T_1}{S(S+{}^1/T_1)}$$

$$= \frac{A_1(1-e^{-T/T_1})}{z-e^{-T/T_1}}$$

$$= \frac{C_1}{z-C_2} \qquad 3.17$$

where $C_1 = A_1(1-e^{-T/T_1})$ and $C_2 = e^{-T/T_1}$.
The simulation diagram of the system of Fig. 3.2 is therefore as shown in Fig. 3.3.

All the tests were carriedout using two of the pseudorandom ternary signals given in table 3.2 so as to show the desirability of a judicious choice of signal in system identification. The first

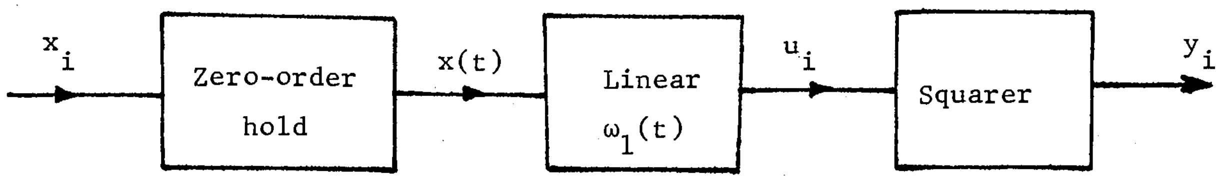


Fig. 3.2 Block diagram of the system to be investigated.

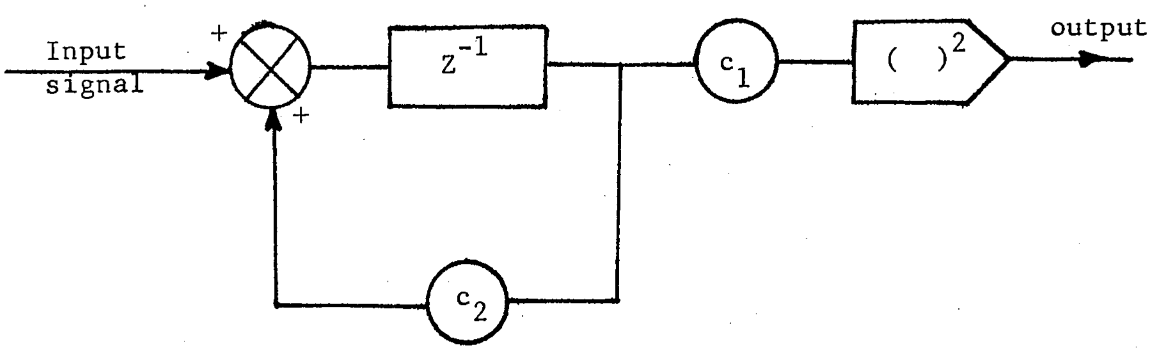


Fig.3.3 Simulation diagram of the system shown in Fig.3.2.

signal was derived from a sequence with characteristic polynomial $f_A(D)$ given by

$$f_A(D) = 1 + D - D^3 - D^4 - D^6 \qquad 3.18$$

and the second signal was derived from an m sequence with characteristic polynomial, $f_B(D)$, given by

$$f_B(D) = 1 + D - D^3 - D^4 - D^6 - D^8 \qquad 3.19$$

The constant A_1 and the sampling interval T were taken as unity. The computer program which was used for the simulation of the nonlinear system and the subsequent crosscorrelation is given in Appendix 1.4.

The results were plotted along lines parallel to the diagonal as suggested by Kadri.[35] This was achieved by making K in $w_2(JT,KT)$ equal to J+c where c is a constant. When the results are plotted in this way, rather than along lines parallel to the time delay axis, less 'anomalies' are encountered, and the errors due to these undesirable nonzero values in the fourth order autocorrelation functions of the input pseudorandom signal vary in a smooth manner as the delay point moves along an axis parallel to the diagonal.

Provided $w_2(JT, \langle J+c\rangle T) = 0$ for $JT, \langle J+c\rangle T > RT$, error-free identification of $w_2(JT, \langle J+c\rangle T)$ can be achieved if eqn. 3.8 is not satisfied within the shaded and unshaded areas shown in Fig. 3.4. Any 'anomalies' in the unshaded area are forward or positive values which lie within the region 0 to N/2, while those in the shaded area are backward to negative values which lie within the region $^N/2 + 1$ to N. The backward values may be obtained from the forward values by the shift and add property.

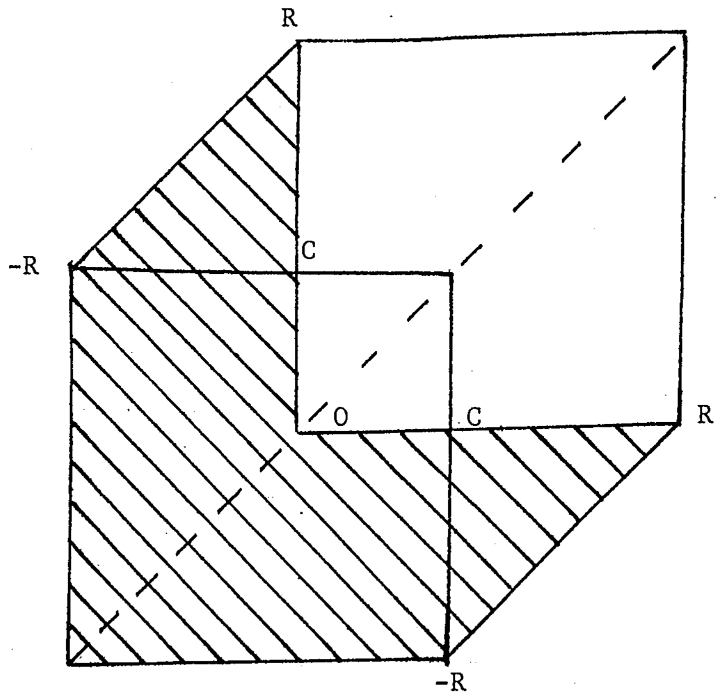


Fig.3.4. Area required to be free of 'anomalies' for correct identification of second order kernels.

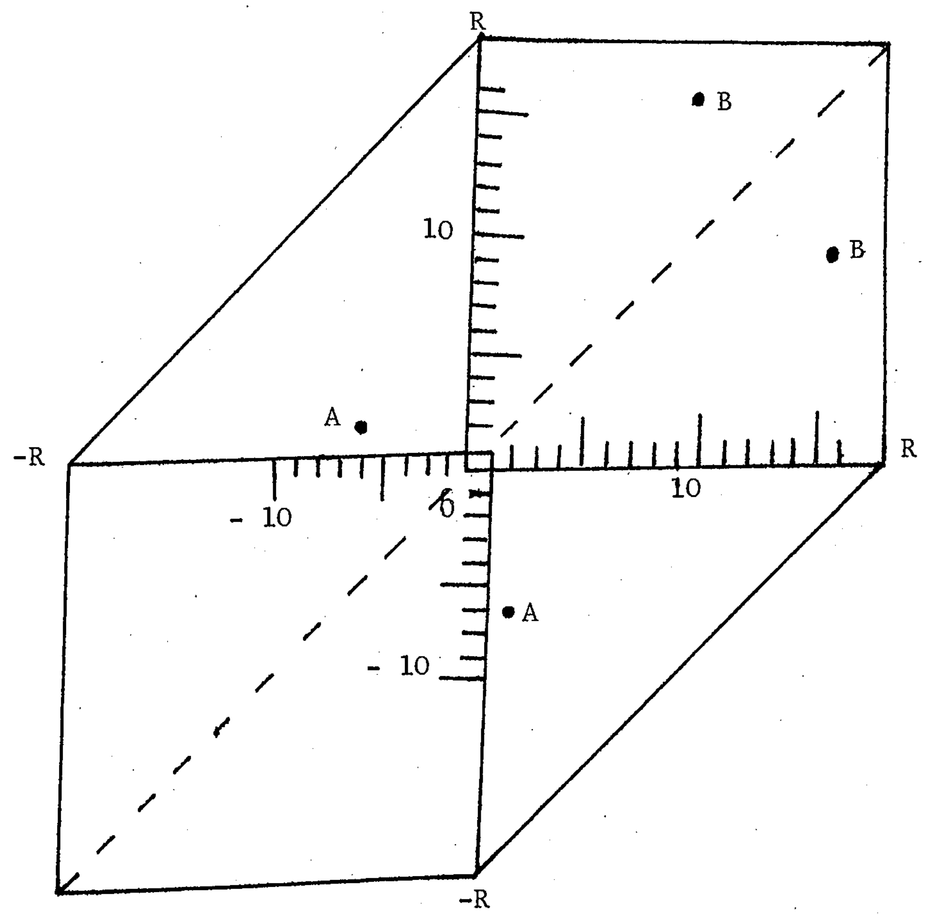


Fig. 3.5. 'Anomalies' for C = 1, $f(D) = 1 + D - D^3 - D^4 - D^6$.

Equation 3.8 may be rewritten as

$$a_1S_{i-J} + a_2S_{i-(J+c)} + a_3S_{i-K} + a_4S_{i-L} = 0 \qquad 3.20$$

It is sufficient to consider only the cases for which J=0 since all the other results may be obtained by shifting. If all the points J, J+c, K and L occur within the significant portion of the second order kernel, they will result on a bias whose sign is obtained from eqn. 3.5 as the product of a_1, a_2, a_3 and a_4. If three of the points occur within the significant portion of the kernel and the fourth, say L, occurs outside the settling time, there will be a positive or negative spike at the point LT. This point corresponds to (L-c)T on the susequent graphs since these have assumed c as the origin.

The second order kernel values computed by two-dimensional crosscorrelation are shown in dotted lines and for the purpose of comparison, the true second order kernel values calculated from eqn. 3.15 are shown on continuous lines.

Fig. 3.6 which shows the results obtained with the polynomial $f_A(D)$ and time constant T_1 = 5 sec, contains plots of the second order kernel for c=1,2,...,10. The differences between the kernel values obtained by two-dimensional crosscorrelation and the true values calculated from eqn. 3.15 are due to the existence of linear relationships in the form of eqn. 3.20. A computer program for computing these relationships is given in Appendix 1.5. For J=0, the relevant values of c,K,L,a_1,a_2,a_3 and a_4 which satisfy eqn. 3.20 are shown in Table 3.5. The significant nonzero values in the fourth order autocorrelation functions of the input signal which cause the

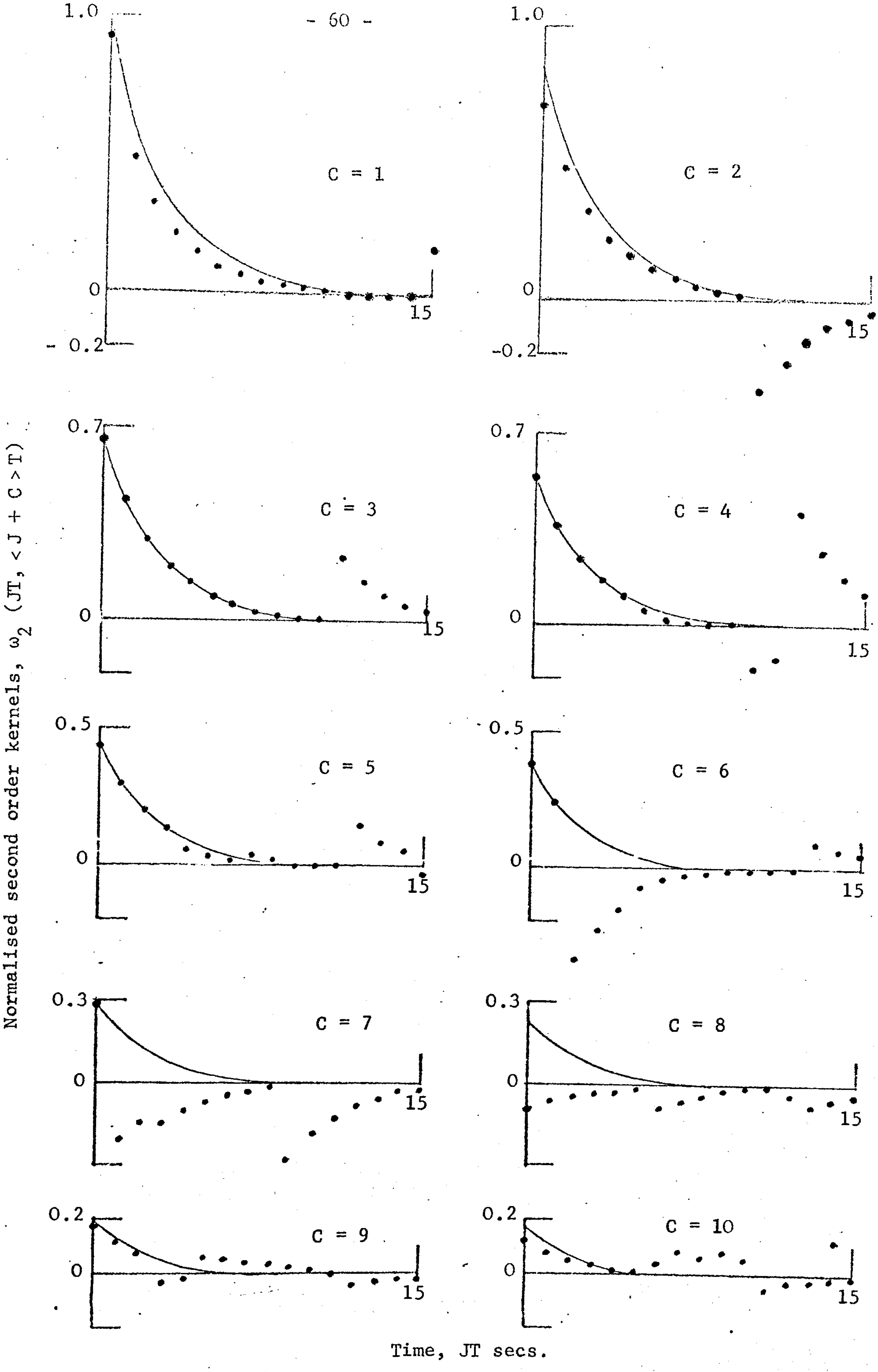


Fig. 3.6. Second order kernel measurements of the system of Fig.3.2 using the polynomial $1 + D - D^3 - D^4 - D^6$ and $T_1 = 5$ seconds.

____ theoretical result, ...experimental values

Table 3.5

Values of c, K, L, a_1, a_2, a_3 and a_4 which Satisfy the Relationship $a_1S_1+a_2S_{i-c}+a_3S_{i-K}+a_4S_{i-L}=0$ for the Polynomial $1+D-D^3-D^4-D^6$ used in Example 3.6.

c	shifted c	K	L	a_1	a_2	a_3	a_4	I=L-c	Sign of $a_1a_2a_3a_4$	c+K+L
1		10	16	1	-1	-1	1	15	+	odd
	7	2 6	-6 8	1 1	1 1	-1 1	-1 -1 }		-	odd
2		9	12	1	1	-1	-1	10	-	odd
	8	1 6	-6 7	1 1	-1 -1	1 1	-1 1 }		-	odd
3		9	14	1	-1	1	-1	11	+	even
4		8	14	1	-1	1	1	10	-	even
		15	16	1	-1	-1	1	12	+	odd
5		1	18	1	1	1	-1	13	-	even
		8	20	1	-1	1	1	15	-	odd
		10	17	1	-1	-1	1	12	+	even
6		7	8	1	1	1	-1	2	-	odd
		10	19	1	-1	1	-1	13	+	odd
7			21	-1	1		-1	14	+	even
		6	8	1	1	1	-1	1	-	odd
		12	16	1	-1	-1	-1	9	-	odd
8		5	20	1	1	-1	1	12	-	odd
		6	7	1	-1	1	1		-	odd
		9	21	1	1	-1	1	13	-	even
		4	14	1	1	-1	1	6	-	even
9		2	12	1	-1	1	1	3	-	odd
		3	14	1	1	-1	-1	5	+	even
		8	21	1	-1	1	1	12	-	even
10		12	21	1	-1	-1	-1	11	-	odd
		6	19	1	1	-1	-1	9	+	odd
		5	17	1	-1	-1	1	7	+	even
		1	16	1	-1	-1	1	6	+	odd
	12	7 2	-2 9	1 1	1 1	-1 1	-1 -1 }		-	odd

computed curves to differ from the true ones may be obtained from this table. For c=1, the two principal relationships given in table 3.5 are shown in Fig. 3.5.

The 'anomaly' at A causes a negative exponential to commence at the point (-6+c)T or -5T and this exponential biases the second order kernel negatively. The 'anomaly' at B causes a positive exponential to start at the point (16-c)T or 15T. Since the true second order kernel has decayed to zero by this point, only the 'anomalous' effects will be observed from the point 15T. All the other discrepancies between the experimental and theoretical results can be similarly explained.

For the polynomial $f_B(D)$, which as shown in Table 3.2 has better performance indices than $f_A(D)$, the only significant relationship within the region of interest is

$$s_i - S_{i-8} - S_{i-11} - S_{i-16} = 0 \qquad 3.21$$

Therefore the computed second order kernels should be the same as the theoretical ones except for the case when c=8. These predictions are borne out by the graphs shown in Fig. 3.7.

Figs. 3.8 and 3.9 show the graphs obtained by doubling the time constant. These curves are generally worse than the previous ones because of the intrusion of more 'anomalies'.

3.7 Conclusions

The second order kernels $w_2(\tau_1, \tau_2)$ of nonlinear systems described by Volterra series in which no other even-order kernels are present may be measured without interaction with the odd-order kernels by continuous or discrete crosscorrelation using antisymmetric signals based on antisymmetric pseudorandom sequences with sampling

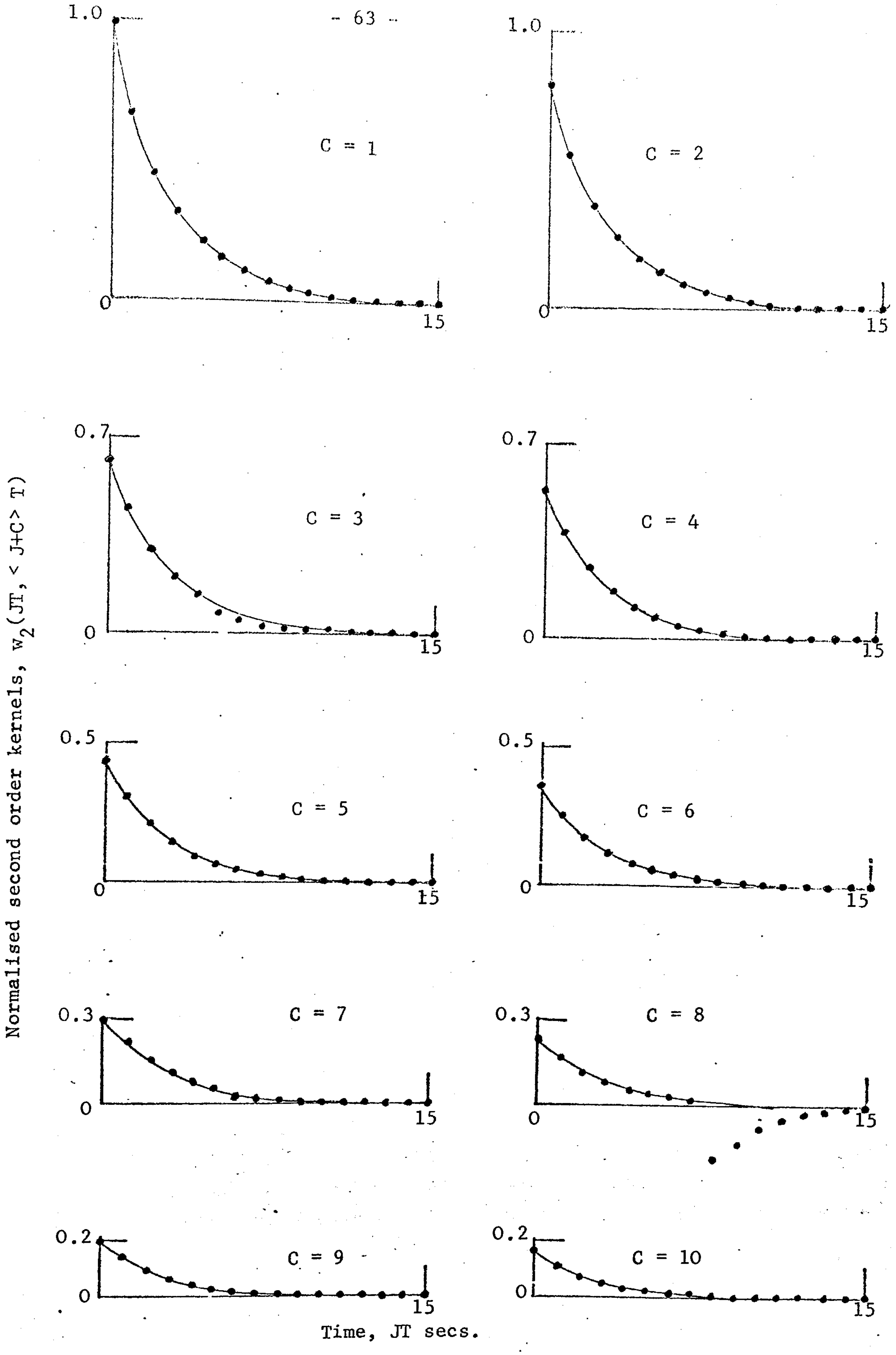


Fig. 3.7. Second order kernel measurements of the system of of fig.3.2 using the polynomial $1+D-D^3-D^4-D^6-D^8$ and T_1 = 5 seconds.

______ theoretical result, ... experimental values.

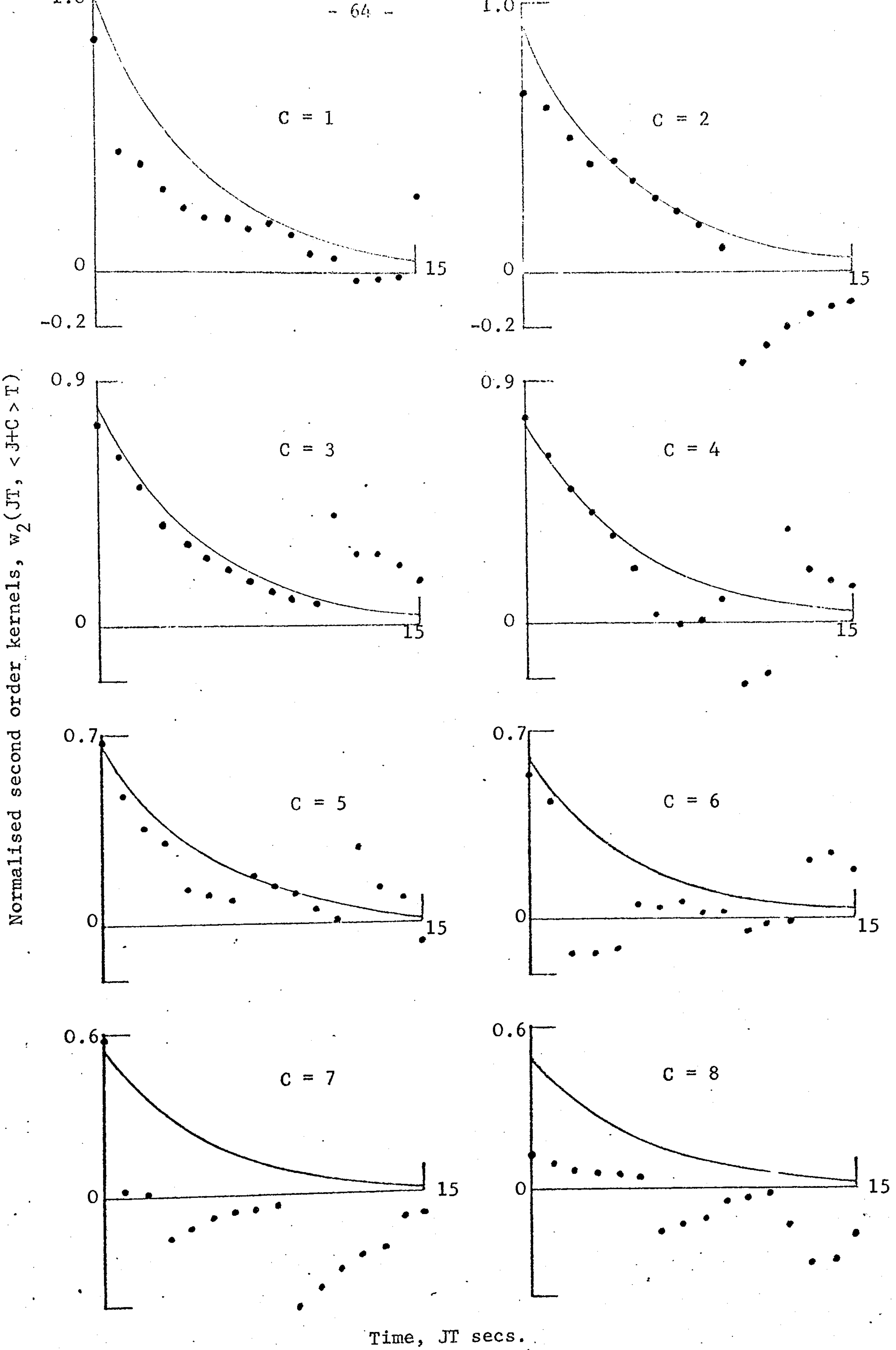


Fig.3.8. Second order kernel measurements of the system of fig.3.2 using the polynomial $1+D-D^3-D^4-D^6$ and $T_1 = 10$ seconds.

——— theoretical result, ... experimental values.

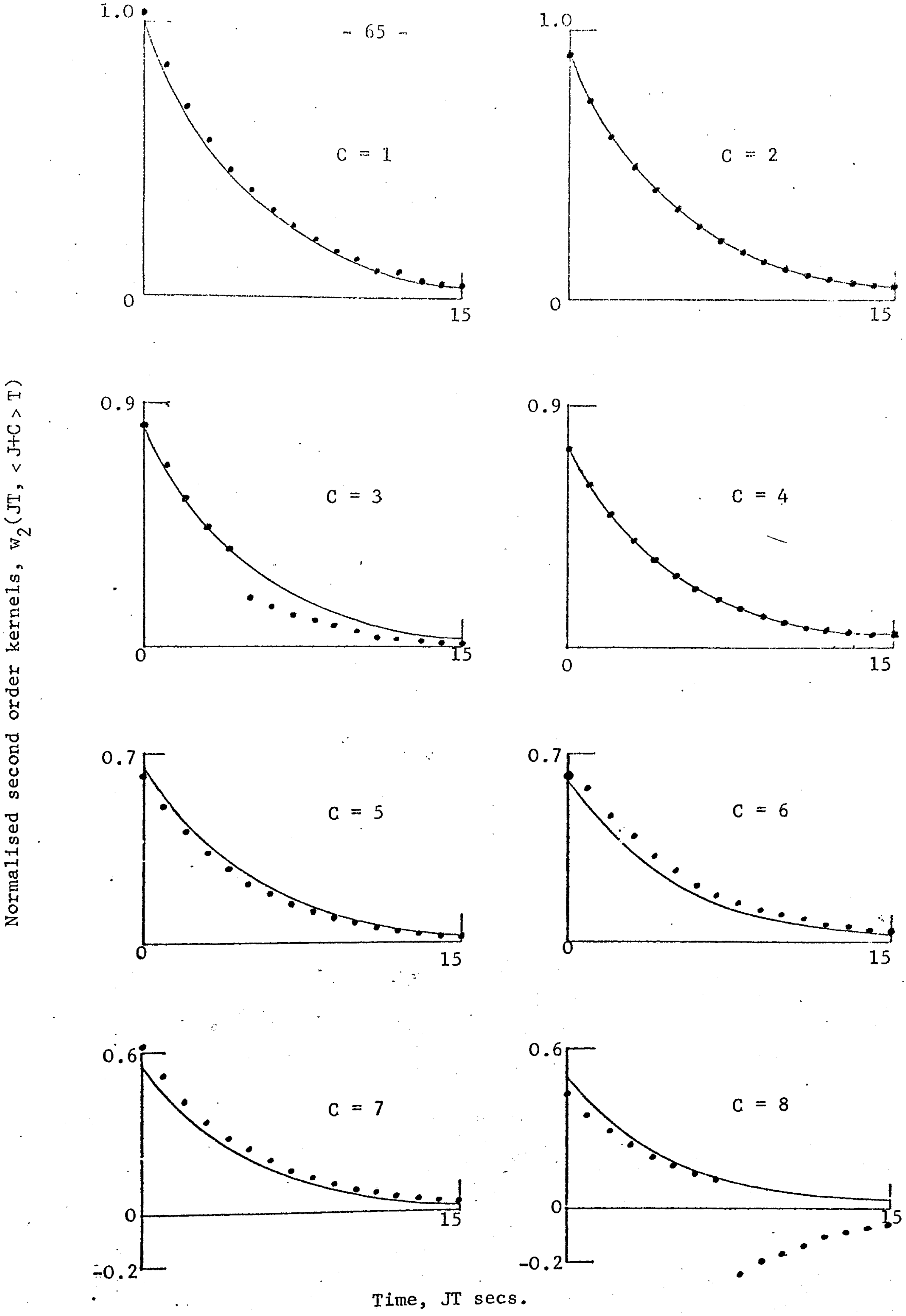


Fig.3.9. Second order kernel measurements of the system of fig.3.2 using the polynomial $1+D-D^3-D^4-D^6-D^8$ and T_1 = 10 seconds.

______ theoretical result, ...experimental values.

period T. Indices of performance for a signal in this application are the upper limits R_r of J and K for which the set of all off diagonal values $w_2(JT,KT)$ of a second order kernel may be obtained in subsets of not more than r members from corresponding subsets in the set of off-diagonal measurements e(JT,KT). The set of diagonal values $w_2(JT,JT)$ is then obtained from the corresponding set of diagonal measurements e(JT,JT) for all $0 \leq J \leq R_r$.

Pseudorandom signals of the same level and order vary widely in their performance. To facilitate the selection of suitable signals, the characteristic polynomials of all binary, ternary and quinary antisymmetric pseudorandom signals with periods less than 8000T which have the greatest performance indices R_r for values of r from 1 to 20 have been tabulated. The methods of obtaining these performance indices have been fully described.

The performance indices R_1 represent an absolute limit of performance for binary signals; furthermore these signals cannot measure the diagonal values of second order kernels, and therefore, their use in this application is deprecated. The performance indices for quinary signals are all less than those for ternary signals of comparable period because of the increased density of nonzero values in their fourth order correlation moments, and the performance indices for pseudorandom signals with more than five levels are likely to be even less for the same reason. It is therefore recommended that ternary signals with characteristic polynomials chosen from those tabulated here be used in this application.

One of the tabulated ternary signals was used to illustrate the application of the proposed technique, and the results of a simulated nonlinear system were in complete agreement with theory.

CHAPTER 4.

SECOND ORDER KERNEL MEASUREMENT - COMBINED METHOD

4.1 Introduction

4.2 Related pseudorandom ternary signals

4.3 Combined crosscorrelation

4.4 Evaluation of performance indices

4.5 Combined crosscorrelation with other pseudorandom signals

4.6 Example - General nonlinear system

4.7 Example - Simulated second order system

4.8 Conclusions

4. SECOND ORDER KERNEL MEASUREMENT - COMBINED METHOD

4.1 Introduction

In this chapter, a method is proposed for obtaining improved performance in the measurement of 2nd order Volterra kernel by combining the results of crosscorrelation experiments using related pseudorandom ternary signals so as to cancel the effects of some of the undesirable values in their 4th order autocorrelation functions. Relationships between pairs of pseudorandom ternary signals, and their 4th order autocorrelation functions which make them suitable for this purpose, are established, and their performance in this application is investigated. The performance with this combined crosscorrelation method is shown to be generally superior to that with the direct crosscorrelation method, within common limits imposed by the intrusion of nonzero diagonal values.

4.2 Related Pseudorandom Ternary Signals

The 2nd order autocorrelation function of the pseudorandom ternary sequence $\{x_i\}$ is

$$\phi_x(J,K) = \frac{1}{N}\sum_{i=0}^{N-1} x_{i-J}x_{i-K} = \frac{1}{N}\, 2 \times 3^{n-1}\left[X(1)\right]^2(-1)^r\,\delta_{JK\pm\frac{rN}{2}} \qquad 4.1$$

and, for the related ternary sequence $\{u_i\}$, defined by

$$u_i = (-1)^i x_i \qquad 4.2$$

the 2nd order autocorrelation function is therefore

$$\phi_u(J,K) = \frac{1}{N}\sum_{i=0}^{N-1} u_{i-J}u_{i-K}$$

$$= \frac{1}{N}(-1)^{J+K}\sum_{i=0}^{N-1} x_{i-J}x_{i-K}$$

$$= \frac{1}{N}\, 2 \times 3^{n-1} \left[X(1) \right]^2 (-1)^{J+K+r}\ \delta_{JK \pm \frac{rN}{2}}$$

$$= \frac{1}{N}\, 2 \times 3^{n-1} \left[X(1) \right]^2 (-1)^{r(1 \pm N/2)}\ \delta_{JK \pm \frac{rN}{2}} \qquad 4.3$$

Now the half period N/2 is even or odd with n, and so

$$\phi_u(J,K) = \frac{1}{N}\, 2 \times 3^{n-1} \left[X(1) \right]^2 \delta_{JK \pm \frac{rN}{2}} \qquad \text{for n odd}$$

or

$$\phi_u(J,K) = \frac{1}{N}\, 2 \times 3^{n-1} \left[X(1) \right]^2 (-1)^r\ \delta_{JK \pm \frac{rN}{2}}$$

$$= \phi_x(J,K) \qquad \text{for n even} \qquad 4.4$$

Therefore, when n is even, as is subsequently assumed, $\{u_i\}$ is also a pseudorandom ternary sequence, and, from the derivation of such sequences as described in section 2.3, if $\{x_i\}$ has the characteristic polynomial

$$f(D) = c_0 + c_1 D + c_2 D^2 + \ldots + c_n D^n \qquad 4.5$$

then $\{u_i\}$ has the characteristic polynomial

$$c_0 - c_1 D + c_2 D^2 - \ldots + (-1)^n c_n D^n = f(-D) \qquad 4.6$$

This shows that when a polynomial f(D) in GF(3) is of even order and primitive, so is the polynomial f(-D).

The importance of the sequences $\{x_i\}$ and $\{u_i\}$ in this application lies in the relationship between their 4th order autocorrelation functions $\phi_u(i_1, i_2, J, K)$ and $\phi_x(i_1, i_2, J, K)$, given by

$$\phi_u(i_1,i_2,J,K) = \frac{1}{N}\sum_{i=0}^{N-1} u_{i-i_1}u_{i-i_2}u_{i-J}u_{i-K}$$

$$= \frac{1}{N}(-1)^{i_1+i_2+J+K}\sum_{i=0}^{N-1} x_{i-i_1}x_{i-i_2}x_{i-J}x_{i-K}$$

$$= (-1)^{i_1+i_2+J+K}\,\phi_x(i_1,i_2,J,K) \qquad 4.7$$

4.3 Combined Crosscorrelation

Estimates $e_u(JT,KT)$ of the 2nd order kernel values $w_2(JT,KT)$ may be obtained by crosscorrelation using the pseudorandom ternary signal u(t) as the input signal. The performance of u(t) is similar to that of x(t) except that their fourth order autocorrelation functions, which are always equal in magnitude, are opposite in sign when the sum of the arguments is odd.

If the estimates $e_x(JT,KT)$ and $e_u(JT,KT)$ are combined to give an average estimate e(JT,KT), then, from eqns. 3.4, 2.23 and 4.7,

$$e(JT,KT) = \tfrac{1}{2}\Big[e_x(JT,KT) + e_u(JT,KT)\Big]$$

$$= \frac{N}{8\text{x}3^{n-2}\big[X(1)\big]^4}\sum_{i_1=0}^{R}\sum_{i_2=0}^{R}\tfrac{1}{2}\Big[\phi_x(i_1,i_2,J,K)+\phi_u(i_1,i_2,J,K)\Big] \text{ x } w_2(i_1T,i_2T)$$

$$= \frac{N}{8\text{x}3^{n-2}\big[X(1)\big]^4}\sum_{i_1=0}^{R}\sum_{i_2=0}^{R}\psi(i_1,i_2,J,K)w_2(i_1T,i_2T)$$

4.8

where
$$\psi(i_1,i_2,J,K) = \tfrac{1}{2}\Big[\phi_x(i_1,i_2,J,K)+\phi_u(i_1,i_2,J,K)\Big]$$
$$= \tfrac{1}{2}\Big[1+(-1)^{i_1+i_2+J+K}\Big]\phi_x(i_1,i_2,J,K) \qquad 4.9$$

The function $\psi(i_1,i_2,J,K)$ is therefore structurally equivalent to the 4th order autocorrelation functions $\phi_x(i_1,i_2,J,K)$ and $\phi_u(i_1,i_2,J,K)$, except that it is zero when i_1+i_2+J+K is odd. The set of functions $\psi(i_1,i_2,J,K)$ for $0 \leq i_1,i_2,J \neq K \leq R \leq Q_m$, where Q_m is a limit beyond which nonzero diagonal terms appear in $\psi(i_1,i_2,J,K)$ at some $i_1=i_2$, may be divided into a number of subsets of the form

$$\psi(i_1,i_2,J_j,K_j)=\sum_{K=1}^{q}\psi(J_k,K_k,J_j,K_j)(\delta_{i_1J_k}\delta_{i_2K_k}+\delta_{i_1K_k}\delta_{i_2J_k})$$

$$j = 1,2,\ldots,q$$

$$0 \leq i_1,i_2,J_1 \neq K_1 \neq J_2 \neq K_2 \neq \ldots \neq J_q \neq K_q \leq R \leq Q_m$$

$$J_k+K_k+J_j+K_j \quad \text{even} \qquad 4.10$$

and $\psi(J_k,K_k,J_j,K_j)$ is nonzero when S_{i-J_k}, S_{i-K_k}, S_{i-J_j}, S_{i-K_j} are all linearly dependent and $J_k+K_k+J_j+K_j$ is even.

The value of q is not necessarily the same for each subset, and an index of performance for the pseudorandom ternary signals u(t) and x(t) in the combined crosscorrelation method is the upper bound Q_r of R for which all $q \leq r$. The second order kernel values for which $R \leq Q_r \leq Q_m$ are then obtained by inverting subsets of not more than r equations of the form

$$e(J_jT,K_jT)=\frac{N}{4x3^{n-2}\left[X(1)\right]^4}\sum_{K=1}^{q}\psi(J_k,K_k,J_j,K_j)w_2(J_kT,K_kT) \qquad 4.11$$

$$j=1,2,\ldots,q$$

or, for diagonal values, from

$$w_2(JT,JT) = 4e(JT,JT) - \frac{8}{2R+3} \sum_{K=0}^{R} e(KT,KT) \qquad 4.12$$

$$J=0,1,\ldots,R \leq Q_m$$

4.4 Evaluation of Performance Indices

The performance index R_m is the upper limit of i_1, i_2, $J \neq K$ beyond which nonzero diagonal terms appear in $\phi_x(i_1,i_2,J,K)$ at some $i_1=i_2$, and the performance index Q_m is the upper limit of $i_1,i_2,J\neq K$, beyond which nonzero diagonal terms appear in $\psi\,(i_1,i_2,J,K)$ at some $i_1=i_2$. The function $\phi_x(L,L,J,K)$ is therefore zero for all $0 \leq L,J\neq K \leq R_m$, and $\psi\,(L,L,J,K)$ must also be zero for all $0 \leq L,J\neq K \leq R_m$, so $Q_m \geq R_m$. $\phi_x(L,L,J,K)$ must be nonzero for some $0 \leq L,J\neq K \leq R_m+1$, and this can only occur when two members of J, K and L take the extreme values 0 and R_m+1, while the third member takes some intermediate value P_m, so that

$$\begin{aligned} &\phi_x(0,P_m,R_m+1,R_m+1) \neq 0 \\ &\phi_x(0,R_m+1,P_m,P_m) \neq 0 \qquad 0 < P_m \leq R_m \qquad 4.13 \\ &\phi_x(P_m,R_m+1,0,0) \neq 0 \end{aligned}$$

From equation 4.9,

$$\psi\,(0,P_m,R_m+1,R_m+1) = \tfrac{1}{2}\left[1+(-1)^{P_m}\right] \phi_x(0,P_m,R_m+1,R_m+1) \qquad 4.14$$

$$\psi\,(0,R_m+1,P_m,P_m) = \tfrac{1}{2}\left[1+(-1)^{R_m+1}\right] \phi_x(0,R_m+1,P_m,P_m) \qquad 4.15$$

$$\psi\,(P_m,R_m+1,0,0) = \tfrac{1}{2}\left[1+(-1)^{P_m+R_m+1}\right] \phi_x(P_m,R_m+1,0,0) \qquad 4.16$$

and, whatever the values of P_m and R_m, at least one of the equations 4.14 to 4.16 is nonzero, so $\psi\,(L,L,J,K)$ is nonzero for some $0 \leq L,J\neq K \leq R_m+1$ and therefore

$$Q_m = R_m.$$

The performance of the combined crosscorrelation method is therefore the same as that of the normal crosscorrelation method in respect of the exclusion of diagonal values, and the performance indices R_r and Q_r of each method have the common upper bound R_m.

The performance index Q_r is the upper bound of R for which all $q \leq r$ in equation 4.10. Nonzero $\psi(J_k,K_k,J_j,K_j)$, which are the same as those of $\phi(J_k,K_k,J_j,K_j)$ given in equations 3.4 and 3.5, occur when eqn. 2.29 is satisifed and $J_k+K_k+J_j+K_j$ is even. The performance indices Q_r are obtained by procedures based on polynomial division. In view of the relationship between x(t) and u(t), the computer program used to determine R_r can, with minor modifications, also be used to calculate Q_r.

The performance indices Q_r, for values of r from one to 20 have been computed for all pseudorandom ternary signals based on m sequences with characteristic polynomials of order four, six an eight, and the characteristic polynomials and performance indices of all pseudorandom ternary signals for which the performance indices Q_r are greatest are given in table 4.1. The performance indices of ternary signals with characteristic polynomials $f^*(D)$ and $f^*(-D)$ are the same as those of f(D) and f(-D).

Table 4.1 shows that, by appropriate choice of the input pseudorandom ternary signals, improved performance in cases likely to be of practical interest may be obtained by means of the combined crosscorrelation method. This is particularly important in view of the fact that improvements in performance obtained by increasing r beyond eight or nine are unlikely to justify the concomitant increases in the complexity of obtaining the 2nd order kernel values from the estimates.

Table 4.1

Characteristic polynomials of related pseudorandom ternary signals with greatest performance indices

Order	Coefficients of characteristic polynomials		Period	Performance indices																				Limit
	f(D)	f(-D)																						
n	c_0 c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8	c_0 c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8	N	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}	Q_{13}	Q_{14}	Q_{15}	Q_{16}	Q_{17}	Q_{18}	Q_{19}	Q_{20}	Q_m
4	1 1 1 -1 -1	1 -1 1 1 -1	80	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
6	1 1 0 -1 -1 0 -1	1 -1 0 1 -1 0 -1	728	13	13	17	19	19	20	22	22	23	23	24	24	24	24	25	25	25	25	25	25	36
	1 1 1 0 1 -1 -1	1 -1 1 0 1 1 -1		8	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
	1 1 0 1 -1 1 -1	1 -1 0 -1 -1 -1 -1		10	13	17	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
	1 0 0 1 0 1 -1	1 0 0 -1 0 -1 -1		5	8	13	14	17	19	20	21	22	22	22	22	22	23	23	24	24	24	24	24	24
8	1 0 -1 -1 0 -1 0 -1 -1	1 0 -1 1 0 1 0 1 -1	6560	34	37	39	41	41	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
	1 -1 0 1 -1 0 -1 0 -1	1 1 0 -1 -1 0 -1 0 -1		33	43	46	51	59	62	63	63	64	71	72	73	73	74	74	74	75	75	75	75	105
	1 -1 -1 -1 0 -1 1 1 -1	1 1 -1 1 0 1 1 -1 -1		22	38	51	57	57	58	58	59	59	60	60	61	61	62	62	63	63	64	64	65	65
	1 0 -1 -1 -1 -1 -1 0 -1	1 0 -1 1 -1 1 -1 0 -1		25	39	44	57	61	63	63	63	63	63	63	63	63	63	63	63	63	63	63	63	63
	1 -1 0 0 1 0 1 1 -1	1 1 0 0 1 0 1 -1 -1		20	27	32	32	34	57	58	60	68	68	69	70	70	70	71	71	71	72	72	72	112
	1 -1 -1 -1 0 1 -1 1 -1	1 1 -1 1 0 -1 -1 -1 -1		28	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
	1 0 0 -1 -1 -1 0 -1 -1	1 0 0 1 -1 1 0 1 -1		18	32	38	47	49	51	51	51	51	51	51	51	51	51	51	51	51	51	51	51	51
	1 0 0 0 1 -1 0 1 -1	1 0 0 0 1 1 0 -1 -1		26	33	37	43	43	45	45	47	50	51	51	51	52	53	54	54	57	57	60	60	71
	1 -1 -1 -1 -1 -1 -1 1 -1	1 1 -1 1 -1 1 -1 -1 -1		19	30	35	42	53	53	53	53	53	53	53	53	53	53	53	53	53	53	53	53	53
	1 0 1 -1 0 0 0 1 -1	1 0 1 1 0 0 0 -1 -1		17	21	28	28	35	35	42	42	45	49	49	50	53	54	54	57	57	57	58	58	83

4.5 Combined Crosscorrelation with other Pseudorandom Signals

The combined crosscorrelation method is applicable to all three or higher level pseudorandom signals. For a quinary signal with characteristic polynomial, f(D), for example, a related pseudorandom quinary signal with characteristic polynomial f(-D) exists when f(D) is of any order, and further related signals with characteristic polynomials f(2D) and f(-2D) exist when f(D) is of even order. Although it is possible, by combined crosscorrelation using two or more related pseudorandom signals with more than three levels, to cancel the effects of more undesirable nonzero values in their 4th order autocorrelation functions than is possible with pseudorandom ternary signals, these values are so dense that the resulting performance indices never exceed the greatest performance indices of ternary signals of comparable period. It is therefore concluded that signals with more than three levels are not to be preferred to pseudorandom ternary signals in this application.

4.6 Example - General Nonlinear System

In this example, relationships are obtained for the determination of the values $w_2(JT,KT)$ of a 2nd order kernel $w_2(\tau_1,\tau_2)$ for which

$$w_2(\tau_1, \tau_2) = 0 \qquad \begin{array}{l} \tau_1, \tau_2 < 0 \\ \tau_1, \tau_2 > 57T \end{array} \qquad 4.17$$

by means of the combined crosscorrelation method using pseudorandom ternary signals x(t) and u(t) with levels X(1), X(0), X(-1), periods 6560T and characteristic polynomials

$$f(D) = 1-D-D^2-D^3-D^5+D^6+D^7-D^8 \qquad 4.18$$

$$f(-D) = 1+D-D^2+D^3+D^5+D^6-D^7-D^8$$

Estimates obtained by continuous or discrete crosscorrelation, using x(t) and u(t) as the input signals, are given by eqn. 3.11 with e(JT,KT) replaced by $e_x(JT,KT)$ or $e_u(JT,KT)$. The combined crosscorrelation estimates are then given by eqn. 4.8.

All linear relationships in the form of eqn. 2.29 for $0 \leq J_k, K_k, J_j, K_j \leq 57$ and $J_k+K_k+J_j+K_j$ even are given by changes of order in each equation of

$$S_{i-p}+S_{i-p-1}+S_{i-p-40}+S_{i-p-43} = 0 \qquad p=0,1,\dots,14$$

$$S_{i-q}+S_{i-q-18}-S_{i-q-19}-S_{i-q-27} = 0 \qquad q=0,1,\dots,30$$

$$S_{i}-S_{i-43}+S_{i-44}+S_{i-57} = 0$$

$$S_{i}+S_{i-25}-S_{i-52}+S_{i-57} = 0$$

$$S_{i-p}+S_{i-p-2}+S_{i-p-11}+S_{i-p-23} = 0 \qquad p=0,1,\dots,34$$

$$S_{i-q}-S_{i-q-18}+S_{i-q-40}-S_{i-q-52} = 0 \qquad q=0,1,\dots,5$$

$$S_{i-p}+S_{i-p-28}+S_{i-p-47}-S_{i-p-49} = 0 \qquad p=0,1,\dots,8$$

$$S_{i-q}+S_{i-q-14}-S_{i-q-28}-S_{i-q-54} = 0 \qquad q=0,1,\dots,3$$

$$S_{i-p}+S_{i-p-6}+S_{i-p-9}-S_{i-p-41} = 0 \qquad p=0,1,\dots,16$$

$$S_{i-q}+S_{i-q-30}-S_{i-q-33}-S_{i-q-55} = 0 \qquad q=0,1,2$$

$$S_{i-p}-S_{i-p-25}-S_{i-p-51}-S_{i-p-56} = 0 \qquad p=0,1$$

$$S_{i-q}+S_{i-q-7}-S_{i-q-22}+S_{i-q-39} = 0 \qquad q=0,1,\dots,18$$

$$S_{i-p}-S_{i-p-8}+S_{i-p-23}+S_{i-p-43} = 0 \qquad p=0,1,\dots,14$$

$$S_{i-q}+S_{i-q-17}+S_{i-q-42}+S_{i-q-51} = 0 \qquad q=0,1,\dots,6$$

4.19

Equation 4.11 and its inverse are obtained for each subset as follows:

(a) 13 subsets with 4 members

For four subsets with four members, the required relationships are

$$\begin{bmatrix} e(47T,49T) \\ e(0,28T) \\ e(14T,54T) \\ e(15T,57T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(47T,49T) \\ w_2(0,28T) \\ w_2(14T,54T) \\ w_2(15T,57T) \end{bmatrix} \qquad 4.20$$

$$\begin{bmatrix} w_2(47T,49T) \\ w_2(0,28T) \\ w_2(14T,54T) \\ w_2(15T,57T) \end{bmatrix} = \frac{2}{5} \begin{bmatrix} 4 & 3 & -2 & 1 \\ 3 & 6 & -4 & 2 \\ -2 & -4 & 6 & -3 \\ 1 & 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} e(47T,49T) \\ e(0,28T) \\ e(14T,54T) \\ e(15T,57T) \end{bmatrix}$$

the other subsets obtained by relationships identical to eqn. 4.20 being

$$\begin{bmatrix} w_2pT, \langle p+49 \rangle T) \\ w_2(\langle p+28 \rangle T, \langle p+47 \rangle T) \\ w_2(\langle p+46 \rangle T, \langle p+55 \rangle T) \\ w_2(\langle p+4 \rangle T, \langle p+21 \rangle T) \end{bmatrix} \qquad p = 0,1,2$$

The second order kernels for the remaining 9 subsets with 4 members are obtained by solving the equations:

$$\begin{bmatrix} e(51T,56T) \\ e(0,25T) \\ e(52T,57T) \\ e(T,26T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(51T,56T) \\ w_2(0,25T) \\ w_2(52T,57T) \\ w_2(T,26T) \end{bmatrix}$$

$$\begin{bmatrix} e(16T,22T) \\ e(25T,57T) \\ e(18T,40T) \\ e(0,52T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 1 \\ 0 & -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(16T,22T) \\ w_2(25T,57T) \\ w_2(18T,40T) \\ w_2(0,52T) \end{bmatrix}$$

$$\begin{bmatrix} e(44T,57T) \\ e(0,43T) \\ e(8T,23T) \\ e(T,40T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(44T,57T) \\ w_2(0,43T) \\ w_2(8T,23T) \\ w_2(T,40T) \end{bmatrix}$$

$$\begin{bmatrix} e(\langle p+29\rangle T, \langle p+31\rangle T) \\ e(\langle p+40\rangle T, \langle p+52\rangle T) \\ e(pT, \langle p+18\rangle T) \\ e(\langle p+19\rangle T, \langle p+27\rangle T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(\langle p+29\rangle T, \langle p+31\rangle T) \\ w_2(\langle p+40\rangle T, \langle p+52\rangle T) \\ w_2(pT, \langle p+18\rangle T) \\ w_2(\langle p+19\rangle T, \langle p+27\rangle T) \end{bmatrix}$$

$$p = 0,1,\ldots,5$$

(b) 89 subsets with 3 members

For 30 subsets with 3 members, the required relationships are of the form

$$\begin{bmatrix} e(43T,44T) \\ e(25T,52T) \\ e(0,57T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(43T,44T) \\ w_2(25T,52T) \\ w_2(0,57T) \end{bmatrix}$$

4.21

$$\begin{bmatrix} w_2(43T,44T) \\ w_2(25T,52T) \\ w_2(0,57T) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} e(43T,44T) \\ e(25T,52T) \\ e(0,57T) \end{bmatrix}$$

the other subsets obtained by relationships identical to equation 4.21 being

$$\begin{bmatrix} w_2(\langle p+2\rangle T, \langle p+11\rangle T) \\ w_2(\langle p T, \langle p+23\rangle T) \\ w_2(\langle p+8\rangle T, \langle p+43\rangle T) \end{bmatrix} \qquad \begin{bmatrix} w_2(\langle q+2\rangle T, \langle q+41\rangle T) \\ w_2(\langle q+1\rangle T, \langle q+44\rangle T) \\ w_2(\langle q+9\rangle T, \langle q+24\rangle T) \end{bmatrix}$$

$$p = 0,1,\ldots,14 \qquad\qquad q = 0,1,\ldots,13$$

For 14 subsets with 3 members, the required relationships are of the form

$$\begin{bmatrix} e(17T,42T) \\ e(0,51T) \\ e(25T,56T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(17T,42T) \\ w_2(0,51T) \\ w_2(25T,56T) \end{bmatrix}$$

$$\begin{bmatrix} w_2(17T,42T) \\ w_2(0,51T) \\ w_2(25T,56T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 3 & -2 & -1 \\ -2 & 4 & 2 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} e(17T,42T) \\ e(0,51T) \\ e(25T,56T) \end{bmatrix} \qquad 4.22$$

the other subsets obtained by relationships identical to eqn. 4.22 being

$$\begin{bmatrix} w_2(18T,43T) \\ w_2(T,52T) \\ w_2(26T,57T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle p+15\rangle T, \langle p+55\rangle T) \\ w_2(\langle p+1\rangle T, \langle p+29\rangle T) \\ w_2(\langle p+48\rangle T, \langle p+50\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle q+26\rangle T, \langle q+37\rangle T) \\ w_2(\langle q+28\rangle T, \langle q+49\rangle T) \\ w_2(\langle qT, \langle q+47\rangle T) \end{bmatrix}$$

$$p = 0,1,2 \qquad\qquad q = 0,1,\ldots,8$$

For 23 subsets with 3 members the required relationships are

$$\begin{bmatrix} e(42T,51T) \\ e(0,17T) \\ e(24T,43T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} w_2(42T,51T) \\ w_2(0,17T) \\ w_2(24T,43T) \end{bmatrix}$$

4.23

$$\begin{bmatrix} w_2(42T,51T) \\ w_2(0,17T) \\ w_2(24T,43T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 4 & -2 & -2 \\ -2 & 3 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} e(42T,51T) \\ e(0,17T) \\ e(24T,43T) \end{bmatrix}$$

the other subsets obtained by relationships identical to eqn. 4.23 being

$$\begin{bmatrix} w_2(\langle p+43 \rangle T, \langle p+52 \rangle T) \\ w_2(\langle p+1 \rangle T, \langle p+18 \rangle T) \\ w_2(\langle p+25 \rangle T, \langle p+44 \rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle q+18 \rangle T, \langle q+27 \rangle T) \\ w_2(\langle q T, \langle q+19 \rangle T) \\ w_2(\langle q+16 \rangle T, \langle q+39 \rangle T) \end{bmatrix}$$

$p = 1,2,3 \qquad\qquad q = 0,1,\ldots,18$

For 16 subsets with 3 members the required relationships are

$$\begin{bmatrix} e(0,6T) \\ e(9T,41T) \\ e(2T,24T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(0,6T) \\ w_2(9T,41T) \\ w_2(2T,24T) \end{bmatrix}$$

4.24

$$\begin{bmatrix} w_2(0,6T) \\ w_2(9T,41T) \\ w_2(2T,24T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} e(0,6T) \\ e(9T,41T) \\ e(2T,24T) \end{bmatrix}$$

the other subsets obtained by relationships identical to eqn. 4.24 being

$$\begin{bmatrix} w_2(\langle p+1\rangle T, \langle p+7\rangle T) \\ w_2 \langle p+10\rangle T, \langle p+42\rangle T) \\ w_2(\langle p+3\rangle T, \langle p+25\rangle T) \end{bmatrix} \qquad p = 1,2,\ldots,15$$

For 6 subsets with 3 members the required relationships are

$$\begin{bmatrix} e(18T,52T) \\ e(0,40T) \\ e(T,43T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(18T,52T) \\ w_2(0,40T) \\ w_2(T,43T) \end{bmatrix}$$

$$\begin{bmatrix} w_2(18T,52T) \\ w_2(0,40T) \\ w_2(T,43T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} e(18T,52T) \\ e(0,40T) \\ e(T,43T) \end{bmatrix} \qquad 4.25$$

the other subsets obtained by relationships identical to equation 4.25 being

$$\begin{bmatrix} w_2(\langle p+19\rangle T, \langle p+53\rangle T) \\ w_2(\langle p+1\rangle T, \langle p+41\rangle T) \\ w_2(\langle p+2\rangle T, \langle p+44\rangle T) \end{bmatrix} \qquad p = 1,2,\ldots,5.$$

(c) 240 subsets with 2 members.

For 167 subsets with 2 members the required relationships are

$$\begin{bmatrix} e(17T,26T) \\ e(15T,38T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} w_2(17T,26T) \\ w_2(15T,38T) \end{bmatrix} \qquad 4.26$$

$$\begin{bmatrix} w_2(17T,26T) \\ w_2(15T,38T) \end{bmatrix} = \tfrac{2}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} e(17T,26T) \\ e(15T,38T) \end{bmatrix}$$

the other subsets obtained by relationships identical to equation 4.26 being

$$\begin{bmatrix} w_2(17T,51T) \\ w_2(0,42T) \end{bmatrix} \quad \begin{bmatrix} w_2(pT,\langle p+14\rangle T) \\ w_2(\langle p+28\rangle T,\langle p+54\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle p+14\rangle T,\langle p+28\rangle T) \\ w_2(\langle pT,\langle p+54\rangle T) \end{bmatrix}$$

$$p = 0,1,\ldots,3$$

$$\begin{bmatrix} w_2(\langle q+30\rangle T,\langle q+33\rangle T) \\ w_2(qT,\langle q+55\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle q+33\rangle T,\langle q+55\rangle T) \\ w_2(\langle qT,\langle q+30\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle q+30\rangle T,\langle q+55\rangle T) \\ w_2(qT,\langle q+33\rangle T) \end{bmatrix}$$

$$q = 0,1,2.$$

$$\begin{bmatrix} w_2(\langle p+44\rangle T,\langle p+45\rangle T) \\ w_2(\langle p+26\rangle T,\langle p+53\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle p+37\rangle T,\langle p+46\rangle T) \\ w_2(\langle p+19\rangle T,\langle p+38\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle p+19\rangle T,\langle p+41\rangle T) \\ w_2(\langle p+1\rangle T,\langle p+53\rangle T) \end{bmatrix}$$

$$p = 0,1,\ldots,4$$

$$\begin{bmatrix} w_2(\langle p+19\rangle T,\langle p+44\rangle T) \\ w_2(\langle p+2\rangle T,\langle p+53\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle q+6\rangle T,\langle q+46\rangle T) \\ w_2(\langle q+7\rangle T,\langle q+49\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle mT,\langle m+1\rangle T) \\ w_2(\langle m+40\rangle T,\langle m+43\rangle T) \end{bmatrix}$$

$$p = 0,1,\ldots,4 \qquad q = 0,1,\ldots,7 \qquad m = o,1,\ldots,14$$

$$\begin{bmatrix} w_2(\langle p+18\rangle T,\langle p+19\rangle T) \\ w_2(pT,\langle p+27\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(\langle p+25\rangle T,\langle p+33\rangle T) \\ w_2(\langle p+6\rangle T,\langle p+24\rangle T) \end{bmatrix} \quad p=0,1,\ldots,24$$

$$\begin{bmatrix} w_2(mT,\langle m+11\rangle T) \\ w_2(\langle m+2\rangle T,\langle m+23\rangle T) \end{bmatrix} \quad \begin{bmatrix} w_2(qT,\langle q+2\rangle T) \\ w_2(\langle q+11\rangle T,\langle q+23\rangle T) \end{bmatrix} \quad \begin{matrix} m=0,1,\ldots,25 \\ q=0,1,\ldots,28 \end{matrix}$$

For 73 subsets with 2 members the required relationships are

$$\begin{bmatrix} e(43T,57T) \\ e(0,44T) \end{bmatrix} = \tfrac{1}{2} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} w_2(43T,57T) \\ w_2(0,44T) \end{bmatrix} \qquad 4.27$$

$$\begin{bmatrix} w_2(43T,57T) \\ w_2(0,44T) \end{bmatrix} = \tfrac{2}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} e(43T,57T) \\ e(0,44T) \end{bmatrix}$$

the other subsets obtained by relationships identical to equation 4.27 being

$$\begin{bmatrix} w_2(7T,22T) \\ w_2(0,39T) \end{bmatrix} \quad \begin{bmatrix} w_2(<p+51>T,<p+53>T) \\ w_2(<p+4>T,<p+32>T) \end{bmatrix} \quad \begin{bmatrix} w_2(<q+6>T,<q+9>T) \\ w_2(0,<q+41>T) \end{bmatrix}$$

$p=0,1,\ldots,4$ $\qquad$ $q=0,1,\ldots,16$

$$\begin{bmatrix} w_2(mT,<m+9>T) \\ w_2(<m+6>T,<m+41>T) \end{bmatrix} \quad \begin{bmatrix} w_2(mT,<m+22>T) \\ w_2(<m+7>T,<m+39>T) \end{bmatrix} \quad \begin{bmatrix} w_2(<m+25>T,<m+51>T) \\ w_2(mT,<m+56>T) \end{bmatrix}$$

$m = 0,1$

$$\begin{bmatrix} w_2(<p+23>T,<p+38>T) \\ w_2(<p+16>T,<p+55>T) \end{bmatrix} \quad \begin{bmatrix} w_2(<q+31>T,<q+50>T) \\ w_2(<q+3>T,<q+52>T) \end{bmatrix} \qquad p=0,1,2 \quad q=0,1,\ldots,5$$

$$\begin{bmatrix} w_2(pT,<p+8>T) \\ w_2(<p+23>T,<p+43>T) \end{bmatrix} \quad \begin{bmatrix} w_2(qT,<q+7>T) \\ w_2(<q+22>T,<q+39>T) \end{bmatrix} \qquad p=0,1,\ldots,14 \quad q=0,1,\ldots,18$$

For the 854 remaining values of $w_2(JT,KT)$ with $J<K$,

$$w_2(JT,KT) = e(JT,KT) \qquad 4.28$$

For the 1653 symmetrical values, $w_2(JT,KT)$ with $J>K$,

$$w_2(JT,KT) = w_2(KT,JT) \qquad 4.29$$

For the 58 diagonal values $w_2(JT,JT)$, from equation 4.12,

$$w_2(JT,JT) = 4e(JT,JT) - \frac{8}{117}\sum_{K=0}^{57} e(KT,KT) \qquad 4.30$$

$$J=0,1,\ldots,57$$

Determination of the values of the same 2nd order kernel by direct crosscorrelation using either of these pseudorandom ternary signals involves considerably more calculation. For 1451 of the

1653 values, $w_2(JT,KT)$, with $J < K$, it is necessary to establish relationships for 3 subsets with 16 members, 1 subset with 15 members, 1 subset with 13 members, 2 subsets with 12 members, 5 subsets with 11 members, 11 subsets with 10 members, 13 subsets with 9 members, 10 subsets with 8 members, 34 subsets with 7 members, 20 subsets with 6 members, 43 subsets with 5 members, 28 subsets with 4 members, 56 subsets with 3 members and 68 subsets with 2 members. The 202 remaining values of this type, the 1653 symmetrical values and the 58 diagonal values are determined through equations analogous to eqns. 4.28 - 4.30.

4.7 Example - Simulated Second Order System

These tests were conducted with the system and polynomials employed in Chapter 3, section 3.6. The related polynomials of those of equations 3.18 and 3.19 are given respectively by

$$f_A(-D) = 1 - D + D^3 - D^4 - D^6$$

and

$$f_B(-D) = 1 - D + D^3 - D^4 - D^6 - D^8$$

The computer program that performs the nonlinear simulation and combined crosscorrelation is given in Appendix 1.4. Theory indicates that combined crosscorrelation would eliminate the 'anomalous' effects present in Figs. 3.6, 3.7, 3.8 and 3.9 when J+J+c+K+L in eqn. 3.20 (table 3.5) is odd. This is confirmed by the graphs in figs. 4.1, 4.2, 4.3 and 4.4 which show that with the combined method, all the second order kernels can be accurately identified when signals based on the polynomials $f_B(D)$ and $f_B(-D)$ are used, and that a marked improvement in the measurement of the kernels is achieved

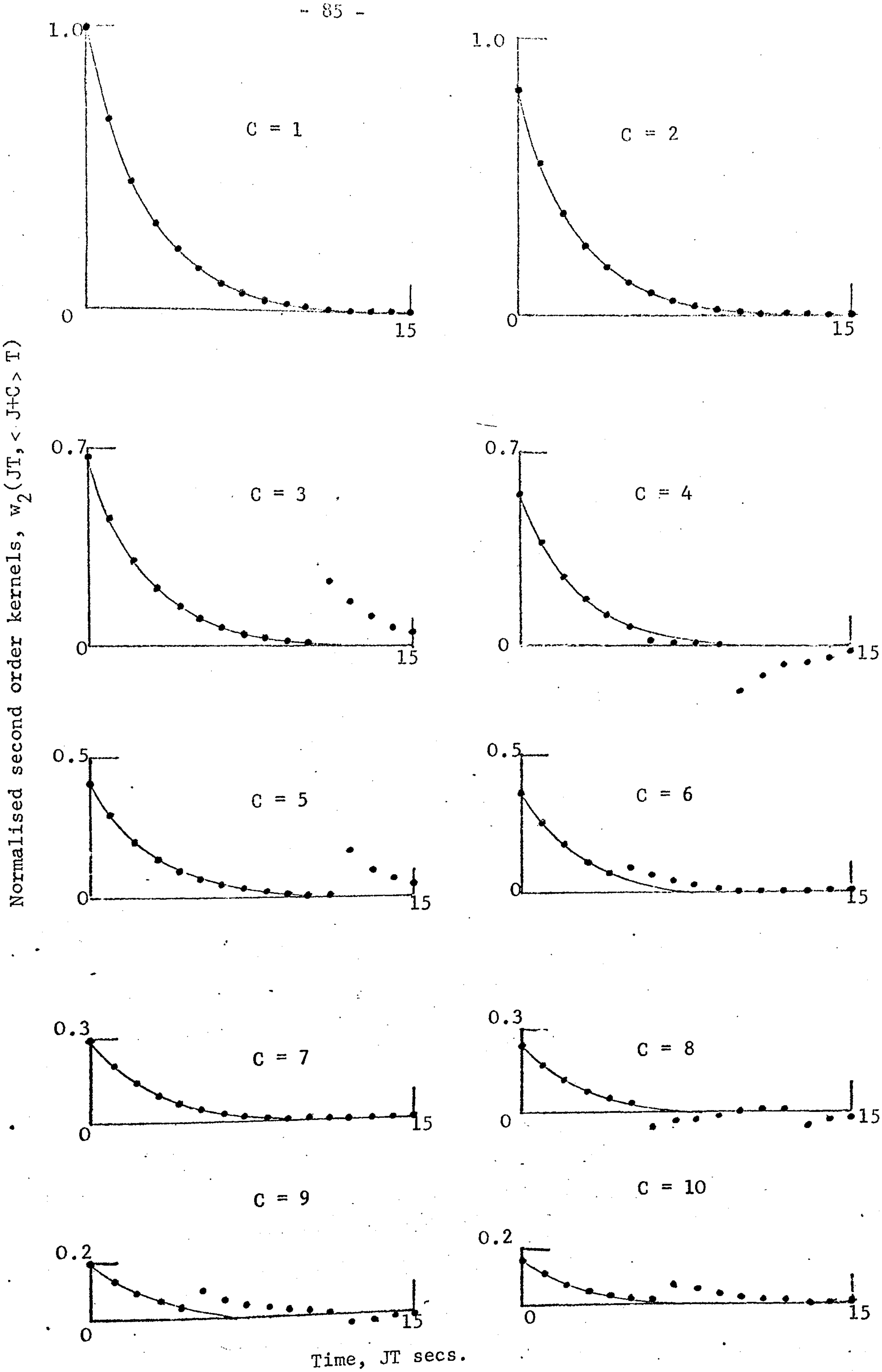


Fig.4.1. Second order kernel measurements of the system of fig.3.2 using the related polynomials $1+D-D^3-D^4-D^6$ and $1-D+D^3-D^4-D^6$, T_1 = 5 seconds.

______ theoretical result, ...experimental values.

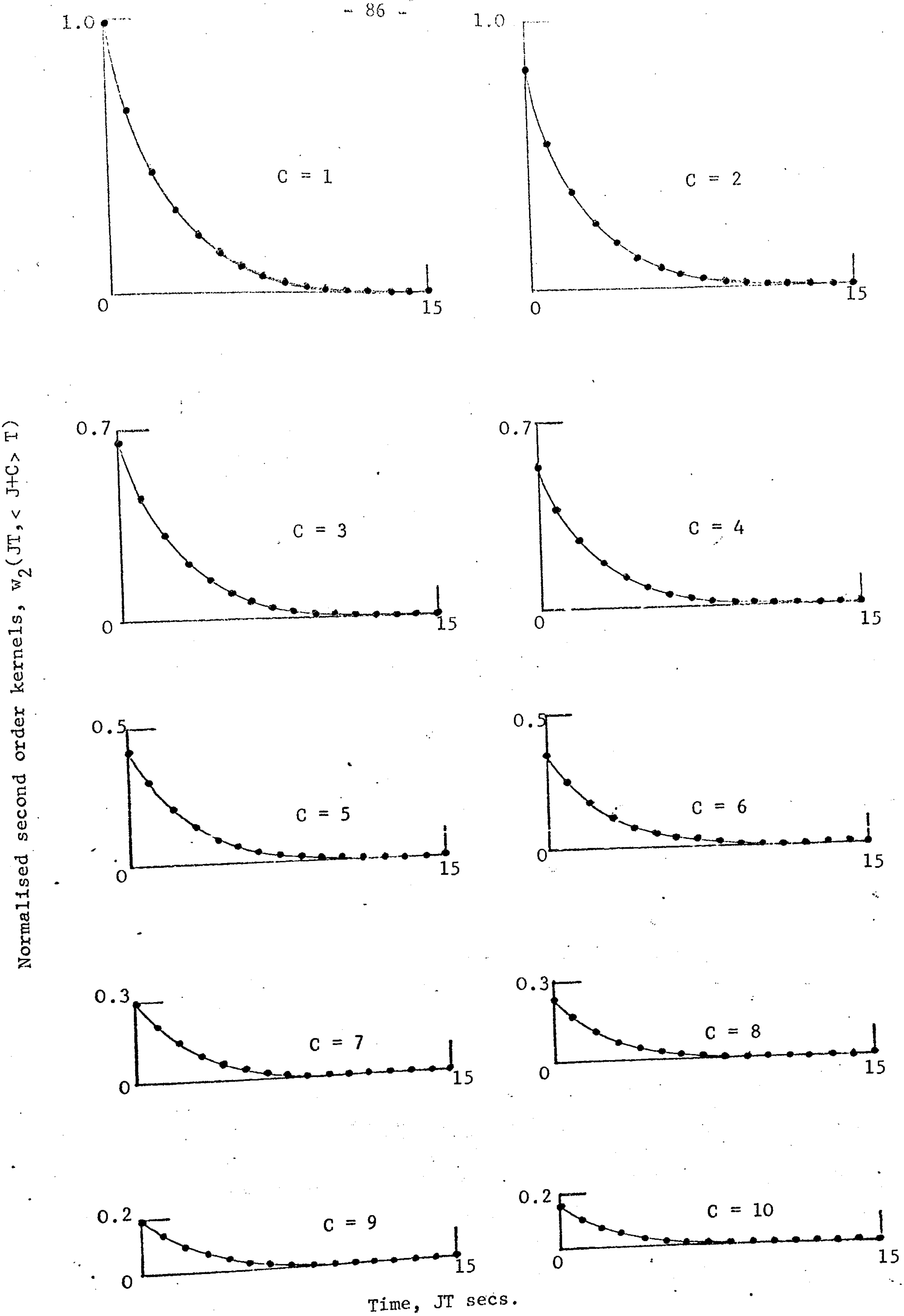


Fig.4.2. Second order kernel measurements of the system of fig.3.2 using related polynomials $1+D-D^3-D^4-D^6-D^8$ and $1-D+D^3-D^4-D^6-D^8$, T_1 = 5 seconds.

——— theoretical result, ... experimental values.

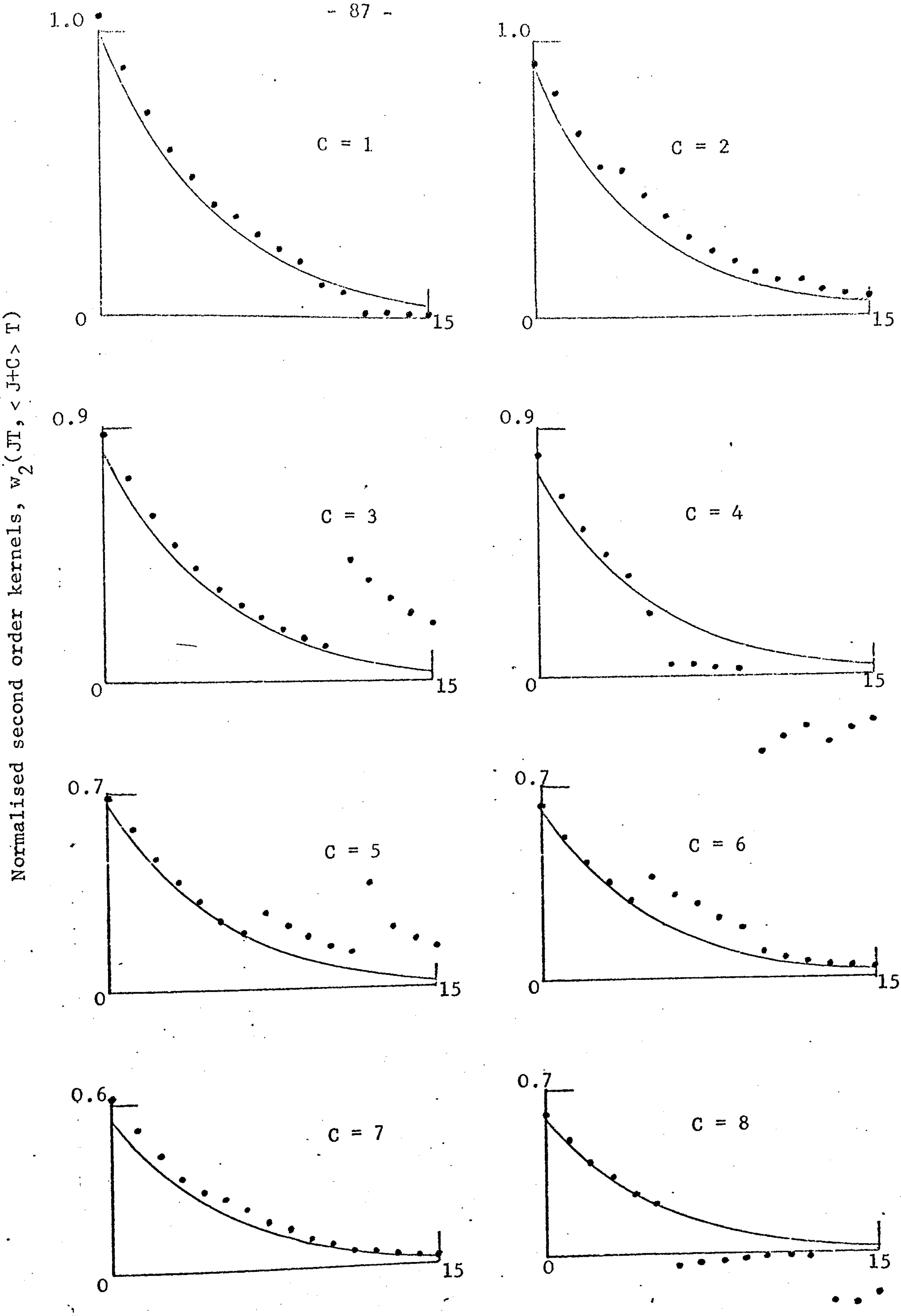


Fig.4.3. Second order kernel measurements of the system of fig.3.2 using related polynomials $1+D-D^3-D^4-D^6$ and $1-D+D^3-D^4-D^6$, T_1 = 10 seconds.

——— theoretical result, ...experimental values.

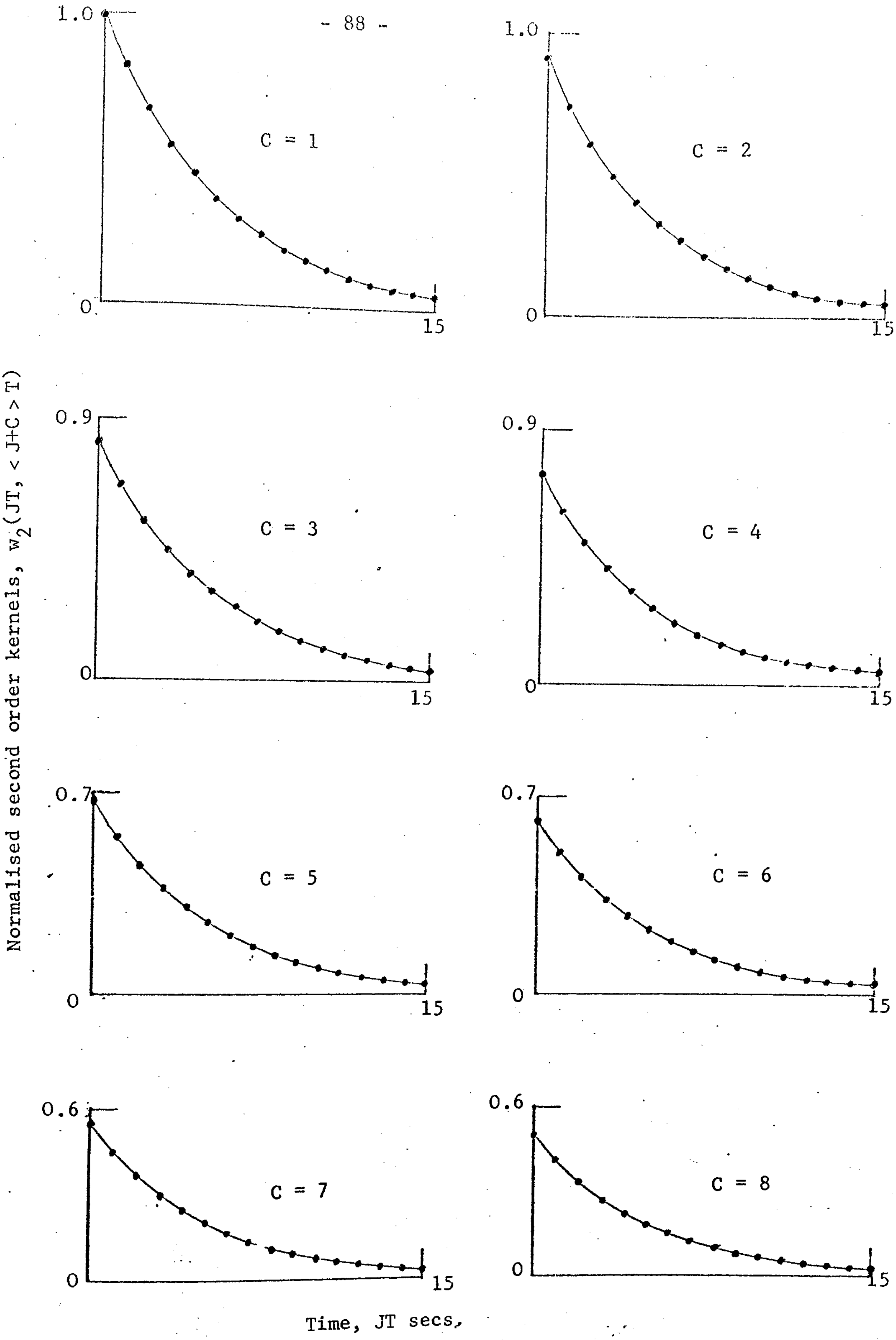


Fig.4.4 Second order kernel measurements of the system of fig.3.2 using related polynomials $1+D-D^3-D^4-D^6-D^8$ and $1-D+D^3-D^4-D^6-D^8$, T_1 = 10 seconds.

_____ theoretical result, ... experimental values when signals derived from the polynomials $f_A(D)$ and $f_A(-D)$ are employed. The superiority of the combined crosscorrelation method over the direct method can be seen by comparing these graphs with the corresponding curves in section 3.6.

4.8 Conclusions

When the polynomial f(D) in GF(3) is of even order and primitive, so is the polynomial f(-D), and the pseudorandom ternary sequences and signals of which these are the characteristic polynomials are closely related. In particular, one sequence may be obtained from the other by reversing the sign of every other member, and the fourth order correlation moment of one sequence has the same structure as that of the other, the values of these functions being equal when the sum of the arguments is even, and equal in magnitude but opposite in sign when the sum of the arguments is odd.

If estimates of the values of the second order kernel of a nonlinear system are obtained by crosscorrelation experiments in which related pseudorandom ternary signals are used as the system input signal, then addition of these estimates results in the cancellation of the effects of those undesirable nonzero values of the fourth order autocorrelation functions for which the sum of the arguments is odd. This combined crosscorrelation therefore results in improved performance without a concomitant increase in the complexity of obtaining the second order kernel values from the estimates.

Although the upper bound of the indices of performance for the combined crosscorrelation method are the same as those for the direct crosscorrelation method, significant improvements of performance may be obtained within these limits, as demonstrated by the two examples given. The characteristic polynomials and performance indices of those pseudorandom ternary signals for which the performance indices are greatest have been tabulated.

CHAPTER 5

TWO-INPUT, SINGLE-OUTPUT SYSTEMS

5. TWO-INPUT, SINGLE-OUTPUT SYSTEMS

5.1 Introduction

The majority of the work on identification using pseudorandom signals has been confined to single-input, single-output linear systems. Many practical systems however are multivariable. Some systems have two or more desired inputs and one or more desired outputs. Others have a single desired input signal but one or more undesirable input signals; when this unwanted disturbance is of sufficient magnitude, it adversely affects the system performance. Determination of the dynamics of multivariable systems using pseudorandom signals has been hitherto limited to linear systems.[43,44,45]

The case considered here is a two-input, one-output system with multiplicative nonlinearity. Both inputs are the same pseudorandom ternary signal except that one is a delayed version of the other. It was shown in Chapter 2 that if the delay is carefully chosen, each crosscorrelation measurement yields the corresponding kernel directly provided that the system settling time does not exceed a predetermined value but this value may be increased by allowing three or four term linear relationships to exist between the m sequence members within the region of interest. The combined crosscorrelation method is also investigated and compared with the direct method. The most suitable pseudorandom ternary signals for these measurements are tabulated.

5.2 Direct Crosscorrelation Method

When a pseudorandom ternary signal x(t) and its delayed counterpart x(t-VT) are the perturbations of a two-input, single-output system with multiplicative nonlinearity, the system dynamics may be determined by crosscorrelating the output signal with a product of the two input signals. As shown in section 2.6.3, each crosscorrelation measurement yields the corresponding kernel directly provided equation 2.38 is obeyed. This equation is satisfied if the following linear relationships do not exist:

$$a_1 S_{i-I} + a_2 S_{i-J} + a_3 S_{i-(K+V)} = 0 \qquad 5.1$$

$$a_1 S_{i-(I+V)} + a_2 S_{i-(J+V)} + a_3 S_{i-K} = 0 \qquad 5.2$$

$$a_1 S_{i-I} + a_2 S_{i-J} + a_3 S_{i-(K+V)} + a_4 S_{i-(L+V)} = 0 \qquad 5.3$$

$$0 \leqslant I,J,K,L \leqslant R$$

The performance index R_{T1} is the maximum value of R for which none of the above equations is satisfied. R_{T1} may be determined in two stages.

5.2.1 Three term linear dependence

The first stage in the evaluation of the performance index R_{T1} and the delay, V is to determine two regions $0 \leqslant A \leqslant R$ and $V \leqslant B \leqslant V+R$ which are free of three term linear dependence. This case is illustrated in fig. 5.1.

Regions A and B are free of three term linear dependence if the following two conditions are satisfied:

(i) No one value in B is linearly dependent on any two values in A.

(ii) No one value in A is linearly dependent on any two values in B.

As a consequence of the shift and add property of pseudorandom ternary signals, condition (ii) is satisfied if no one value in

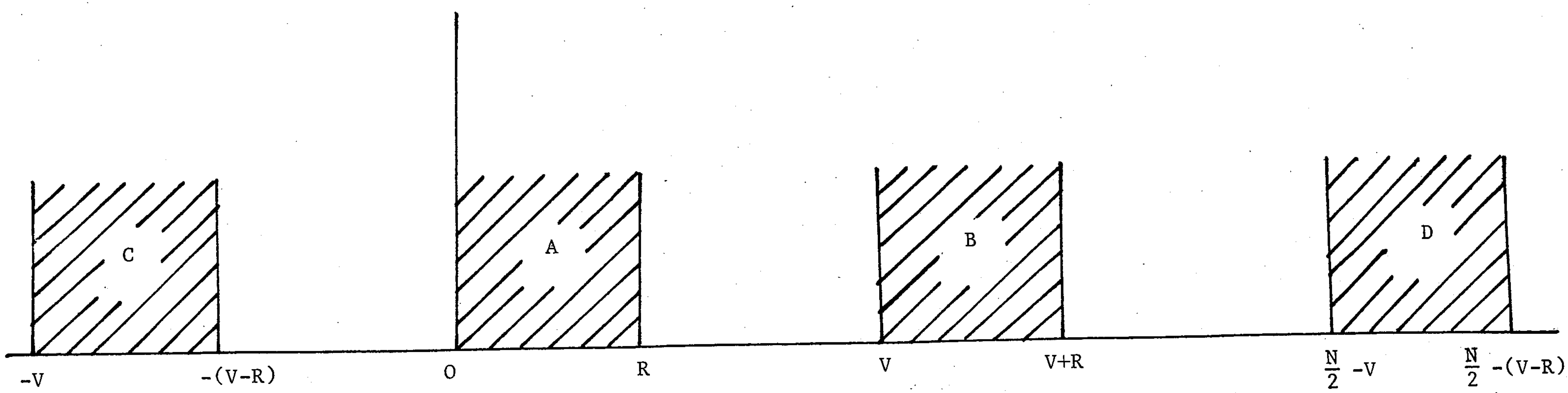


Fig.5.1. Regions required to be free of linear dependence.

region C is linearly dependent on any two values in region A, i.e. if no one value in region D is linearly dependent on two values in region A. Thus conditions (i) and (ii) may be expressed in the following manner:

(iii) No two values in A must be linearly dependent on a value in B or D. When calculating the performance indices, a considerable saving of effort is achieved by using condition (iii) rather than (i) and (ii).

The investigation must be conducted for all possible values of the delay, V. The least value of V is R+1; as V increases region B moves right while region D moves left and they coincide when $V = \frac{N}{4}$. Thus it is only necessary to take $V=R+1, R+2, \ldots, \frac{N}{4}$, since if a solution is found for V an identical solution also exists for $\frac{N}{2} - V$.

The values of R and V which satisfy condition (iii) may be found by determining all values of K which obey the equation

$$a_1 S_{i-I} + a_2 S_{i-J} + a_3 S_{i-K} = 0 \qquad 5.4$$

$$0 \leq I \neq J \leq R$$

and ensuring that these K values do not lie in regions B or D. It is the V values corresponding to the greatest R that are of importance and these may be obtained as follows:

Step 1: A reasonable value of R is chosen.

Step 2: For I=0, the polynomials D^I+D^J and D^I-D^J are divided by the characteristic polynomial, f(D), until single term remainders $a_1 D^{K_1}$ and $a_2 D^{K_2}$ are found. These are kept for subsequent use.

Step 3: The K_1 and K_2 values for all the other combinations of I and J in eqn. 5.4 are obtained by shifting the appropriate values in step 2.

Step 4: For $V=R+1, R+2, \ldots, \frac{N}{4}$, the regions B and D (fig. 5.1) are examined to see whether they contain any values of K_1 or K_2. If three term linear dependence exists for all values of V, a lower value of R is chosen and the procedure is repeated starting from step 2.

Step 5: If any values of V satisfy condition (iii), R is incremented by one; step 2 and subsequent steps are repeated. In this way, the greatest value of R, R_V, which obeys condition (iii) and the corresponding values of V are obtained. These are used in the next stage of the investigation.

5.2.2 Four term linear dependence

For all the values of V obtained in section 5.2.1, it is necessary to ensure that no pairs of values in region A are linearly dependent on pairs of values in region B. This is achieved by ensuring that no relationship of the type

$$a_1S_{i-I} + a_2S_{i-J} + a_3S_{i-K} + a_4S_{i-L} = 0 \qquad 5.5$$

exists for all $0 \leq I \neq J \leq R \leq R_V$

and $V \leq K \neq L \leq V+R \leq V+R_V$.

The value of L may be determined by dividing the polynomial $a_1D^I+a_2D^J+a_3D^K$ by f(D) until a single term remainder a_4D^L is obtained. However, a more efficient procedure may be adopted by splitting the four term linear dependence into two pairs, a pair of which lies in region A and the other in B.

Suppose

$$a_1S_{i-I} + a_2S_{i-J} = a_5S_{i-M}$$

$$0 \leq I \neq J \leq R \qquad 5.6$$

and

$$a_3S_{i-K} + a_4S_{i-L} = a_6S_{i-N}$$

$$V \leq K \neq L \leq V+R \qquad 5.7$$

then if $a_5 = -a_6$ and $M = N$, equations 5.6 and 5.7 give

$$a_1S_{i-I} + a_2S_{i-J} + a_3S_{i-K} + a_4S_{i-L} = 0$$

which is the same as equation 5.5. Thus four term linear dependence may be determined by dividing the polynomials $a_1D^I + a_2D^J$ and $a_3D^K + a_4D^L$ by f(D); if the single term remainders M and N are equal then S_{i-I}, S_{i-J}, S_{i-K} and S_{i-L} are linearly dependent, if M is not equal to N, no linear dependence exists. The elegance of this technique is that the time consuming polynomial division need not be performed at all, for all the values of I, J, M, K, L and N can easily be obtained by shifting the values stored in step 2 of section 5.2.1.

The performance index R_{T1} is the maximum value of R for which all M and N in equations 5.6 and 5.7 are unequal for all I, J, K and L in eqn. 5.5. By means of the computer program in Appendix 1.6, the performance index R_{T1} and the corresponding delay, V were computed for all pseudorandom ternary signals of order 5, 6, 7 and 8, and the signals with superior performance are given in table 5.1. The reciprocal polynomials, $f^*(D)$ have the same performance as f(D) and are not tabulated.

The performance index R_{T2} is the maximum value of R for which any one of the following conditions is met:

Performance indices	Order n	Period N	f(D) c_o	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	f(-D) c_o	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	Value of performance indices	Delays, V
R_{T1}	5	242	1	0	1	-1	0	1													4	36
			1	0	-1	-1	1	1													4	42
			1	1	1	-1	1	1													4	47
	6	728	1	0	-1	1	1	1	-1			1	0	-1	-1	1	-1	-1			7	182
			1	0	1	1	1	-1	-1			1	0	1	-1	1	1	-1			7	182
			1	0	0	1	0	1	-1			1	0	0	-1	0	-1	-1			7	24,25
			1	0	0	0	0	1	-1			1	0	0	0	0	-1	-1			7	55,56,170,171,172,173,178, 179,180,181,182,
	7	2186	1	1	1	0	1	1	-1	1											12	43
	8	6560	1	0	0	0	0	-1	0	0	-1	1	0	0	0	0	1	0	0	-1	20	1414
			1	-1	1	-1	-1	1	-1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	20	60,61,62,63,64,65,
R_{T2}	5	242	1	0	0	0	-1	1													5	24,28
	6	728	1	0	0	0	0	1	-1			1	0	0	0	0	-1	-1			8	34,35
			1	1	1	1	1	1	-1			1	-1	1	-1	1	-1	-1			8	58,59,80,81
	7	2186	1	0	1	0	1	1	0	1											13	396
	8	6560	1	0	0	0	0	-1	0	0	-1	1	0	0	0	0	1	0	0	-1	21	1415,1416,1417,1418
Q_{T1}	6	728	1	1	1	1	-1	-1	-1			1	-1	1	-1	-1	1	-1			11	182
	8	6560	1	-1	0	0	0	-1	-1	-1	-1	1	1	0	0	0	1	-1	1	-1	24	1302,1303
			1	-1	0	1	0	-1	-1	1	-1	1	1	0	-1	0	1	-1	-1	-1	24	1632,1633
			1	0	-1	0	-1	0	-1	-1	-1	1	0	-1	0	-1	0	-1	1	-1	24	972,973,974,975,976, 977,978
Q_{T2}	6	728	1	1	1	1	-1	-1	-1			1	-1	1	-1	-1	1	-1			11	182
	8	6560	1	-1	-1	0	1	-1	1	-1	-1	1	1	-1	0	1	1	1	1	-1	25	886
			1	0	-1	0	-1	0	-1	-1	-1	1	0	-1	0	-1	0	-1	1	-1	25	973
			1	-1	0	0	0	-1	-1	-1	-1	1	1	0	0	0	1	-1	1	-1	25	1304
			1	-1	0	0	-1	1	1	1	-1	1	1	0	0	-1	-1	1	-1	-1	25	1552,1553
			1	-1	0	1	0	-1	-1	1	-1	1	1	0	-1	0	1	-1	-1	-1	25	1632,1633,1634

Table 5.1. Best pseudorandom ternary signals for identifying two-input, single-output nonlinear systems.

(a) One value in region A is linearly dependent on two values in region B.

(b) One value in region B is linearly dependent on two values in region A.

(c) Two values in region A are linearly dependent on two values in region B.

The shifted counterparts, if any, of the above values are not regarded as additional relationships. R_{T2} and the corresponding delays V are computed by procedures similar to those employed in determining R_{T1}. The pseudorandom ternary signals whose R_{T2} are greatest are also given in table 5.1.

5.3 Combined Crosscorrelation Method

The performance indices R_{T1} and R_{T2} may be too small for certain applications. In such cases, combined crosscorrelation method involving two related pseudorandom ternary signals with characteristic polynomials f(D) and f(-D) may be used. If the signal x(t) is injected into one of the two system inputs and the delayed signal x(t-VT) into the other, the estimate $e_x(JT,KT)$ of the system kernel is obtained by two dimensional crosscorrelation as indicated in section 2.6.3. Using related signals u(t) and u(t-VT) defined by $u_i=(-1)^i x_i$, the estimate $e_u(JT,KT)$ may be similarly obtained. By averaging the two results, an estimate e(JT,KT) of the system kernel $w_{12}(JT,KT)$ is obtained from eqns. 2.37, 3.4 and 4.7 as

$$e(JT,KT) = \tfrac{1}{2}\left[e_x(JT,KT) + e_u(JT,KT)\right]$$

$$= \frac{N}{4x3^{n-2}X^4(1)} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \psi(i_1,i_2+V,J,K+V)w_{12}(i_1T,i_2T) \qquad 5.8$$

where

$$\psi(i_1, i_2+V, J, K+V) = \tfrac{1}{2}\left[\phi_x(i_1, i_2+V, J, K+V) + \phi_u(i_1, i_2+V, J, K+V)\right]$$

$$= \tfrac{1}{2}\left[1+(-1)^{i_1+i_2+J+K}\right]\phi_x(i_1, i_2+V, J, K+V)$$

5.9

The function $\psi(i_1, i_2+V, J, K+V)$ is the same as the function $\phi_x(i_1, i_2+V, J, K+V)$ except that the former is zero when $i_1+i_2+V+J+K+V$ is odd. Therefore the combined method will at worst give the same performance as the direct method and at best result in significant improvement.

The performance index Q_{T1}, which is the upper bound of R for which each combined crosscorrelation measurement gives the corresponding kernel directly, and the performance index Q_{T2}, which is the upper bound of R for which pairs of combined crosscorrelation measurements yield corresponding kernels, have been computed for all pseudorandom ternary signals of order 6 and 8. The signals with best performance indices are given in table 5.1. The reciprocal polynomials $f^*(D)$ and $f^*(-D)$ have the same performance as f(D) and f(-D) and are not tabulated.

Table 5.1 shows that useful additions to the direct-method performance indices are achieved by the combined crosscorrelation method. In both methods, however, the improvements obtained by including the three or four term linear relationships are very small and are unlikely to offset the effort involved in obtaining the kernels from the estimates.

5.4 Conclusions

It has been shown that if a two-input, single-output system is tested with a pseudorandom ternary signal in one input and the same signal delayed by a certain amount at the other input, the system dynamics may be determined by two dimensional crosscorrelation. The accuracy of the result is affected by the undesirable nonzero values in the fourth order autocorrelation functions of the input signal. Since these nonzero values are not the same for all signals, some pseudorandom ternary signals will give better results than others. These signals with superior performance have been identified and tabulated. The combined crosscorrelation method involving two related pseudorandom ternary signals was also considered and shown to be a useful method of determining the characteristics of systems which could not be correctly identified by the direct crosscorrelation method.

CHAPTER 6

EFFECTS OF NONLINEARITIES ON THE MEASUREMENT OF WEIGHTING FUNCTIONS

6 EFFECTS OF NONLINEARITIES ON THE MEASUREMENT OF WEIGHTING FUNCTIONS.

6.1 Introduction

In the measurement of linear system weighting functions by crosscorrelation, it is well known that the presence of nonlinearities introduces errors in the estimates.[4] This is clearly evident from the experimental results presented in the next chapter, which is typical of many results obtained in a diverse number of practical plants.[19,20] Although the identification of these nonlinearities has been the subject of considerable study[36], the investigation of these effects on the linear estimates when pseudorandom signals are used has been restricted to a few specific cases.[31,37]

A more general form of problem, in which it is assumed that the behaviour of a system which includes nonlinearities may be represented by a Volterra functional series, is considered here. Explicit results are given for systems with second and third order nonlinearities which are tested by pseudorandom signals derived from binary and ternary m sequences. As shown in section 2.6.1, these results are dependent on the second, third and fourth order autocorrelation functions of the input signal.

It is shown that the principal errors are of two distinct types, a systematic error which is the same for all pseudorandom signals of a common type, and an unsystematic error which depends on relationships between members of the m sequence from which the pseudorandom signal is derived. The unsystematic error may be removed from a range of interest extending over the settling time of the system by an appropriate choice of test signal, and those pseudorandom signals most suitable for this purpose are identified. Three examples, one of which is taken from results obtained in a practical system, are used to illustrate and validate the analytical expressions obtained.

6.2 Estimation using Pseudorandom Binary Signals

The second order autocorrelation function of a pseudorandom binary sequence mapped from the elements of GF(2) as indicated in section 2.3.1 is given by

$$\phi(i_1,i_2) = \begin{cases} X^2(1) & i_1=i_2 \\ \dfrac{-X^2(1)}{N} & i_1\neq i_2 \end{cases} \qquad 0 \leq i_1,i_2 < N$$

$$= -\frac{X^2(1)}{N}\left[1 - 2^n \delta_{i_1 i_2}\right] \qquad 0 \leq i_1,i_2 < N \qquad 6.1$$

The third order autocorrelation function is given by

$$\phi(i_1,i_2,i_3) = \begin{cases} -X^3(1) & S_{i-i_1} + S_{i-i_2} + S_{i-i_3} = 0 \\ \dfrac{X^3(1)}{N} & \text{All other cases.} \end{cases}$$

$$= \frac{X^3(1)}{N}\left[1 - 2^n \sum \delta_{i_1 J}\,\delta_{i_2 K}\,\delta_{i_3 H}\right] \qquad 6.2$$

where the summation is taken over all J,K,H for which

$$S_{i-J} + S_{i-K} + S_{i-H} = 0 \qquad \text{mod } 2 \qquad 6.3$$

$$0 \leq J\neq K\neq H < N$$

The fourth order autocorrelation function is given by

$$\phi(i_1,i_2,i_3,i_4) = \begin{cases} X^4(1) & \left.\begin{array}{l} i_1=i_2=i_3=i_4 \\ i_1=i_2, i_3=i_4\neq i_1 \\ i_1=i_3, i_2=i_4\neq i_1 \\ i_1=i_4, i_2=i_3\neq i_1 \\ S_{i-i_1}+S_{i-i_2}+S_{i-i_3}+S_{i-i_4}=0 \end{array}\right] \\ -\dfrac{X^4(1)}{N} & \text{all other cases} \end{cases}$$

$$= -\frac{X^4(1)}{N}\Big[1 - 2^n\{(\delta_{i_1i_2}\delta_{i_3i_4} + \delta_{i_1i_3}\delta_{i_2i_4} + \delta_{i_1i_4}\delta_{i_2i_3}) - 2\,\delta_{i_1i_2}\delta_{i_1i_3}\delta_{i_1i_4} + \sum \delta_{i_1J}\delta_{i_2K}\delta_{i_3L}\delta_{i_4H}\}\Big] \qquad 6.4$$

where the summation is taken over all J,K,L,H for which

$$S_{i-J} + S_{i-K} + S_{i-L} + S_{i-H} = 0 \qquad \text{mod } 2$$

$$0 \leq J\neq K\neq L\neq H < N \qquad 6.5$$

The weighting function estimates and errors due to second and third order nonlinearities are obtained by substituting eqns. 6.1, 6.2 and 6.4 into eqn. 2.16. The constant term $(-1)^j X^{j+1}(1)/N$ in $\phi(i_1,i_2,\ldots,i_j,I)$ results in a constant bias in $e_j(IT)$ which is given by

$$b_j = -\frac{\{-TX(1)\}^{j-1}}{N} \sum_{i_1=0}^{R}\sum_{i_2=0}^{R}\cdots\sum_{i_j=0}^{R} w_j(i_1T,i_2T,\ldots,i_jT) \qquad 6.6$$

6.2.1 Linear estimate

The linear estimate obtained from eqns. 2.16, 6.1 and 6.6 is the well known result

$$e_1(IT) = b_1 + \frac{2^n}{N} w_1(IT) \qquad 0 \leq I < N \qquad 6.7$$

and it is necessary to remove the constant bias b_1 to obtain the required value $w_1(IT)$.

6.2.2 Error due to second order nonlinearities

This is obtained from eqns. 2.16, 6.2 and 6.6 as

$$e_2(IT) = b_2 - \frac{2^n}{N} TX(1) \sum w_2(JT,KT) \qquad 0 \leq I < N \qquad 6.8$$

where the summation is taken over $0 \leq J,K \leq R$ for which

$$S_{i-J} + S_{i-K} + S_{i-I} = 0 \qquad \text{mod } 2 \qquad 6.9$$

Since J and K may be taken in any order, eqn. 6.8 may be written as

$$e_2(IT) = b_2 - \frac{2^n}{N} TX(1)\ 2 \sum w_2(JT,KT)$$

$$0 \leq I < N \qquad 6.10$$

where the summation is now taken over $0 \leq J < K \leq R$ in eqn. 6.9.

Apart from the bias, the error due to the second order nonlinearities is unsystematic because the values of J, K and I for which eqn. 6.9 is satisfied are dependent on the characteristic polynomial of the pseudorandom sequence. Furthermore the existence or non-existence of solutions of eqn. 6.9 for a particular value of I does not necessarily imply the existence or non-existence of solutions for adjacent values of I, and therefore the error does not vary smoothly with I.

Since the values of $w_1(IT)$ are required only for $0 \leq I \leq R$, if eqn. 6.9 is not satisfied for any I within this range of interest, the error due to second order nonlinearities is reduced to the constant bias b_2 which may be removed with the constant bias b_1 of the linear estimate. For a given m sequence, the upper bound R_m of R beyond which eqn. 6.9 becomes satisfied for some $0 \leq I \leq R$ is a measure of the limit beyond which unsystematic errors due to second order nonlinearities cannot be eliminated when the corresponding pseudorandom binary signal is used for determining the first order kernel. R_m is obtained by polynomial division techniques which are described in Chapter 3, and a computer program for this purpose is given in Appendix 1.7. The characteristic polynomials of order 2 to 11 corresponding to the pseudorandom binary signals for which the value of R_m is greatest are given in table 3.1. The reciprocal polynomials $f^*(D)$ have the same performance as f(D) and are not tabulated.

6.2.3 Error due to third order nonlinearities

From eqns. 2.16, 6.4 and 6.6,

$$e_3(IT) = b_3 + \frac{2^n}{N}\left[TX(1)\right]^2 \left[3 \sum_{j=0}^{R} w_3(IT,jT,jT) - 2w_3(IT,IT,IT) + \sum w_3(JT,KT,LT)\right]$$

$$= b_3 + \frac{2^n}{N}\left[TX(1)\right]^2 \left[\frac{3}{T}\int_0^\infty w_3(IT,\tau,\tau)d\tau - 2w_3(IT,IT,IT) + \sum w_3(JT,KT,LT)\right]$$

$$0 \leq I < N \qquad 6.11$$

where the summation is taken over $0 \leq J \neq K \neq L \leq R$ for which

$$S_{i-J} + S_{i-K} + S_{i-L} + S_{i-I} = 0 \qquad \text{mod } 2 \qquad 6.12$$

Since the first three terms of eqn. 6.12 may be taken in any order, eqn. 6.11 may be written as

$$e_3(IT) = b_3 + \frac{2^n}{N}\left[TX(1)\right]^2 \left[\frac{3}{T}\int_0^\infty w_3(IT,\tau,\tau)d\tau - 2w_3(IT,IT,IT) + 6\sum w_3(JT,KT,LT)\right]$$

$$0 \leqslant I < N \qquad 6.13$$

where the summation is now taken over $0 \leqslant J < K < L \leqslant R$ in eqn. 6.12. The first term in eqn. 6.13 is a constant bias, the second and third terms are systematic errors which are always present irrespective of the characteristic polynomial of the input signal, while the last term is the unsystematic error which may be reduced to zero if eqn. 6.12 is not satisfied within the settling time of the system.

The characteristic polynomials of order 2 to 11 corresponding to the pseudorandom binary signals for which the value of R_1, the upper bound of R beyond which eqn. 6.12 becomes satisfied for some $0 \leqslant I \leqslant R$, is greatest are given in table 3.1.

6.3 Estimation using Antisymmetric Pseudorandom Binary Signals

The second, third and fourth order autocorrelation functions of antisymmetric pseudorandom binary sequences mapped from the elements of GF(2) as indicated in section 2.3.1 are given respectively by

$$\phi(i_1,i_2) = \begin{cases} X^2(1) & i_1=i_2 \\ -X^2(1)(-1)^{i_1+i_2}/N & i_1\neq i_2 \end{cases} \qquad 0\leqslant i_1,i_2 < N$$

$$= \frac{-X^2(1)(-1)^{i_1+i_2}}{N}\left[1 - 2^n\delta_{i_1i_2}\right] \qquad 0\leqslant i_1,i_2 < N \qquad 6.14$$

$$\phi(i_1,i_2,i_3) = 0 \qquad 6.15$$

$$\phi(i_1,i_2,i_3,i_4) = \begin{cases} \left.\begin{array}{l} \\ X^4(1) \\ \\ \end{array}\right] & \begin{array}{l} i_1=i_2=i_3=i_4 \\ i_1=i_2, i_3=i_4\neq i_1 \\ i_1=i_3, i_2=i_4\neq i_1 \\ i_1=i_4, i_2=i_3\neq i_1 \end{array} \\ X^4(1)(-1)^{i_1+i_2+i_3+i_4} & S_{i-i_1}+S_{i-i_2}+S_{i-i_3}+S_{i-i_4}=0 \\ -\dfrac{X^4(1)}{N}(-1)^{i_1+i_2+i_3+i_4} & \text{all other cases} \end{cases}$$

$$= -\frac{X^4(1)}{N}\Big[(-1)^{i_1+i_2+i_3+i_4} - 2^n \{(\delta_{i_1i_2}\delta_{i_3i_4} + \delta_{i_1i_3}\delta_{i_2i_4} + \delta_{i_1i_4}\delta_{i_2i_3}) - 2\delta_{i_1i_2}\delta_{i_1i_3}\delta_{i_1i_4} + (-1)^{i_1+i_2+i_3+i_4}\sum \delta_{i_1J}\delta_{i_2K}\delta_{i_3L}\delta_{i_4H}\}\Big] \qquad 6.16$$

where the summation is taken over all J,K,L,H in eqn. 6.5

Since the third order correlation moment is zero, there is no error due to second order nonlinearities. The term $-X^4(1)(-1)^{i_1+i_2+\ldots+i_j+I}/N$ in $\phi(i_1,i_2,\ldots,i_j,I)$ for odd values of j results in a term $(-1)^I \beta_j$ in $e_j(IT)$ where β_j is a constant given by

$$\beta_j = -\frac{1}{N}\Big[TX(1)\Big]^{j-1} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \cdots \sum_{i_j=0}^{R} (-1)^{i_1+i_2+\ldots+i_j} w_j(i_1T, i_2T, \ldots, i_jT) \qquad j \text{ odd} \qquad 6.17$$

6.3.1 <u>Linear estimate</u>

From eqns. 2.16, 6.14 and 6.17,

$$e_1(IT) = (-1)^I \beta_1 + \frac{2^n}{N} w_1(IT) \qquad 6.18$$

To obtain the weighting function $w_1(IT)$ it is necessary to remove the oscillatory term $(-1)^I\beta_1$ which is very small in comparison to $w_1(IT)$.

6.3.2 Error due to third order nonlinearities

From eqns. 2.16, 6.16 and 6.17, the error due to third order nonlinearities is given by

$$e_3(IT) = (-1)^I\beta_3 + \frac{2^n}{N}\left[TX(1)\right]^2 \left[\frac{3}{T}\int_0^\infty w_3(IT,\tau,\tau)d\tau - 2w_3(IT,IT,IT) + 6\sum(-1)^{J+K+L+I}w_3(JT,KT,LT)\right] \qquad 0 \leq I < N \qquad 6.19$$

where the summation is taken over $0 \leq J < K < L \leq R$ in eqn. 6.12.

The systematic error obtained when using antisymmetric pseudorandom binary signals is identical to that given by the ordinary binary signals while the unsystematic error components have the same magnitude but may take the opposite sign. The conditions under which unsystematic errors may be removed are the same for both signals, so the antisymmetric pseudorandom binary signals with superior performance in this respect are those for which the value of R_1 is greatest in table 3.1.

6.4 Estimation using Ternary Signals

For pseudorandom sequences mapped from the elements of GF(3) as stated in section 2.3.2, all odd-order autocorrelation functions are zero and therefore there is no error due to the second or any other even-order nonlinearities. The second and fourth order correlation moments are given respectively by

$$\phi(i_1,i_2) = \begin{cases} \frac{2.3^{n-1}}{N}X^2(1) & i_1=i_2 \\ 0 & i_1\neq i_2 \end{cases} \qquad 0 \leq i_1,i_2 < \frac{N}{2}$$

$$= \frac{2.3^{n-1}}{N} X^2(1) \delta_{i_1 i_2} \qquad 0 \leq i_1, i_2 < \frac{N}{2} \qquad 6.20$$

and

$$\phi(i_1,i_2,i_3,i_4) = \begin{cases} \frac{6.3^{n-2}}{N} X^4(1) & i_1=i_2=i_3=i_4 \\ \frac{4.3^{n-2}}{N} X^4(1) & \left\{ \begin{array}{l} i_1=i_2, i_3=i_4 \neq i_1 \\ i_1=i_3, i_2=i_4 \neq i_1 \\ i_1=i_4, i_2=i_3 \neq i_1 \end{array} \right. \\ \frac{2.3^{n-2}}{N} X^4(1) a_1 a_2 a_3 & a_1 S_{i-i_1} + a_2 S_{i-i_2} + a_3 S_{i-i_3} + S_{i-i_4} = 0 \quad \text{(a)} \\ \frac{2.3^{n-2}}{N} X^4(1) a_4 a_5 & -a_4 S_{i-i_1} - a_5 S_{i-i_2} + S_{i-i_3} + S_{i-i_4} = 0, i_3 = i_4 \quad \text{(b)} \\ \frac{2.3^{n-2}}{N} X^4(1) a_4 & a_4 S_{i-i_1} - a_5 S_{i-i_2} - a_5 S_{i-i_3} + S_{i-i_4} = 0, i_2 = i_3 \quad \text{(c)} \\ \frac{2.3^{n-2}}{N} X^4(1) a_5 & -a_4 S_{i-i_1} - a_4 S_{i-i_2} + a_5 S_{i-i_3} + S_{i-i_4} = 0, i_1 = i_2 \quad \text{(d)} \\ 0 & \text{all other cases} \end{cases} \qquad 6.21$$

Equation 6.21 may be written as

$$\phi(i_1,i_2,i_3,i_4) = \frac{3^{n-2} X^4(1)}{N} \Big[4(\delta_{i_1 i_2} \delta_{i_3 i_4} + \delta_{i_1 i_3} \delta_{i_2 i_4}$$

$$+ \delta_{i_1 i_4} \delta_{i_2 i_3}) - 6 \delta_{i_1 i_2} \delta_{i_1 i_3} \delta_{i_1 i_4}$$

$$+ 2 \sum a_1 a_2 a_3 \delta_{i_1 J} \delta_{i_2 K} \delta_{i_3 L} \delta_{i_4 H} + 2 \sum a_4 a_5 \delta_{i_1 M} \delta_{i_2 N} \delta_{i_3 G} \delta_{i_4 G}$$

$$+ 2 \sum a_4 \delta_{i_1 M} \delta_{i_2 N} \delta_{i_3 N} \delta_{i_4 G} + 2 \sum a_5 \delta_{i_1 M} \delta_{i_2 M} \delta_{i_3 N} \delta_{i_4 G} \Big] \qquad 6.22$$

where the summations are taken over all $J,K,L,H,M,N,G,a_1,a_2,a_3,a_4,a_5$ for which

$$a_1S_{i-J}+a_2S_{i-K}+a_3S_{i-L}+S_{i-H} = 0 \qquad \text{mod } 3 \qquad 6.23$$

$$0 \leq J \neq K \neq L \neq H \leq R$$

$$a_4S_{i-M}+a_5S_{i-N}+S_{i-G}=0 \qquad \text{mod } 3 \qquad 6.24$$

$$0 \leq M \neq N \neq G \leq R$$

6.4.1 Linear estimate

The linear term obtained from eqns. 2.16 and 6.20 is the well known result

$$e_1(IT) = w_1(IT) \qquad 6.25$$

6.4.2 Error due to third order nonlinearities

Substituting eqn. 6.22 into eqn. 2.19 and noting that the first three terms in eqns. 6.21(a) and 6.21(b) may be ordered in six distinct ways but that those in eqns. 6.21(c) and 6.21(d) may be ordered in only three distinct ways, the error due to third order nonlinearities is obtained as

$$\begin{aligned} e_3(IT) = \left[TX(1)\right]^2 \Bigg[& \frac{2}{T}\int_0^\infty w_3(IT,\tau,\tau)d\tau - w_3(IT,IT,IT) \\ & + 2\sum a_1a_2a_3w_3(JT,KT,LT) + 2\sum a_4a_5w_3(MT,NT,IT) \\ & + \sum a_4w_3(MT,NT,NT) + \sum a_5w_3(MT,MT,NT)\Bigg] \end{aligned}$$

$$0 \leq I < \frac{N}{2} \qquad 6.26$$

where the summations are taken over $J,K,L,M,N,a_1,a_2,a_3,a_4,a_5$ for which

$$\begin{aligned} & a_1S_{i-J}+a_2S_{i-K}+a_3S_{i-L}+S_{i-I} = 0 \\ & a_4S_{i-M}+a_5S_{i-N}+S_{i-I} = 0 \\ & 0 \leq J < K < L \leq R, \quad 0 \leq M < N \leq R; \quad J,K,L,M,N \neq I \end{aligned} \qquad 6.27$$

The first two terms in eqn. 6.26 are systematic errors and the rest are unsystematic errors. Since the last three terms of eqn. 6.26 add up to zero when a_4 and a_5 are both negative, three term linear dependence in the form of eqn. 6.24 does not always result in an error. All unsystematic errors may be removed from the region of interest by a judicious choice of the input signal. For a given ternary signal, the upper bound R_1 of R beyond which eqn. 6.27 becomes satisfied for $0 \leq I \leq R$ is a measure of the limit below which unsystematic errors due to third order nonlinearities are eliminated. The characteristic polynomials of order 2 to 8 corresponding to the pseudorandom ternary signals for which the value of R_1 is greatest is given in table 3.2.

6.5 Examples

The validity of the results derived in this chapter will be demonstrated by three examples. The first is a simple hypothetical system chosen so as to show clearly the systematic and unsystematic errors, and the precise structural dependence of the latter on fundamental m sequence properties. The second example analyses a process with direction dependent dynamic characteristics and shows that such a process is a particular case of the general nonlinear system represented by Volterra series. The third example explains the nonlinearities observed in continuous gas chromatography experiments.

6.5.1 System with a simple weighting function

The system chosen is shown in fig. 6.1. The weighting function, which is shown in fig. 6.2, has the equation

$$w_j(\tau_1,\tau_2,\ldots,\tau_j) = \prod_{k=1}^{j} 0.3 \left[U(\tau_k-0.5)-U(\tau_k-4.5)\right] \qquad j = 1,2,3$$

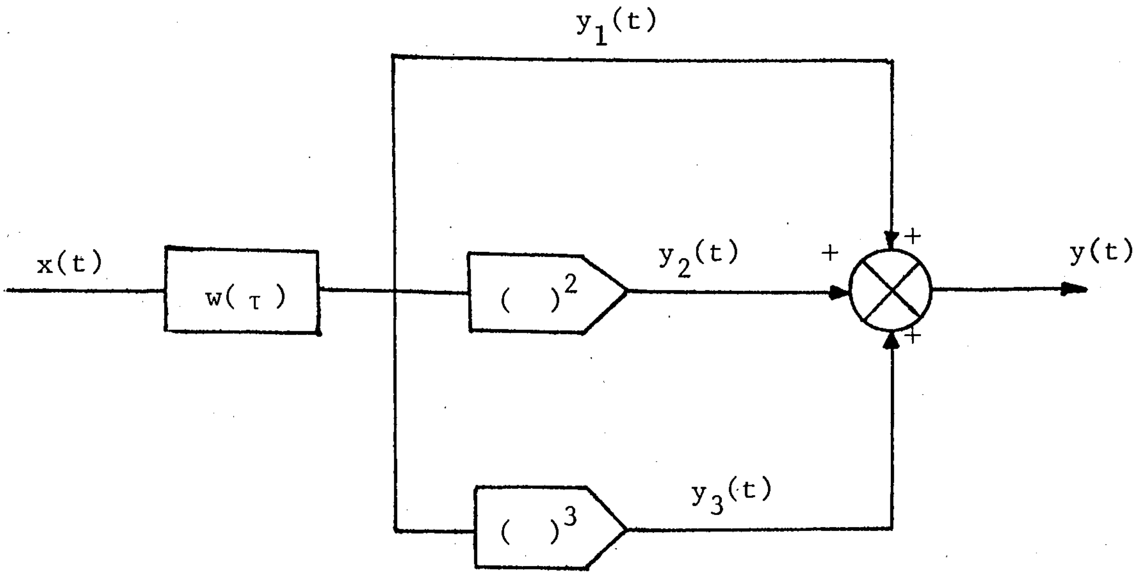


Fig.6.1. System used in example 6.5.1.

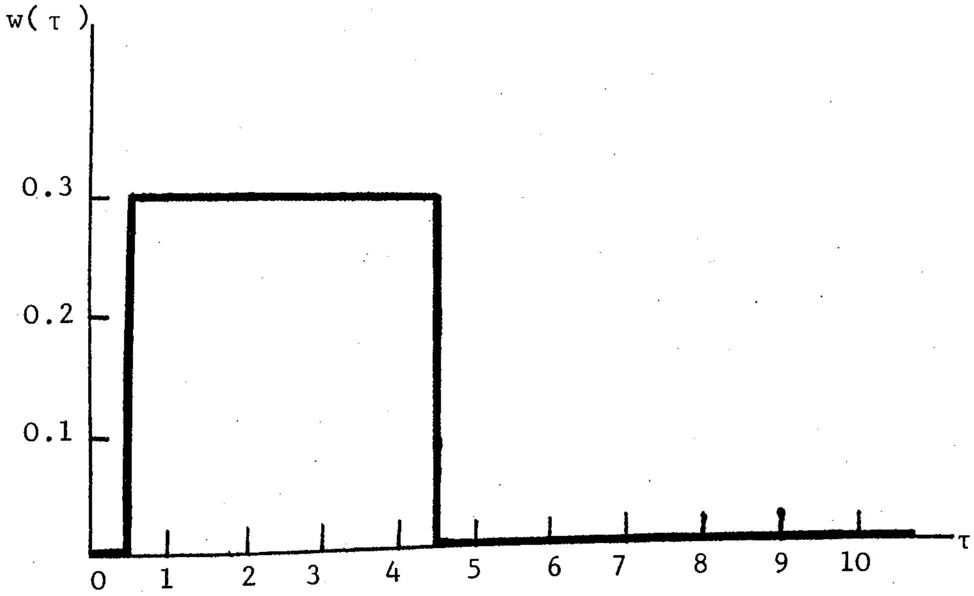


Fig.6.2. Weighting function used in example 6.5.1

where $U(\tau)$ is the unit step function. For both binary and ternary pseudorandom signals, the bit interval, T, and the amplitude X(1) of the signal x(t) are equal to unity. The linear estimate and errors due to second and third order nonlinearities are obtained by separately crosscorrelating the outputs $y_1(t)$, $y_2(t)$ and $y_3(t)$ of fig. 6.1 with x(t) using the computer programs in Appendices 1.8, 1.9 and 1.10.

(a) Pseudorandom binary input signal.

For the signal with characteristic polynomial $1 + D^2 + D^5 + D^6 + D^7$, the results are as shown in fig. 6.3.

The linear component $e_1(I)$ has the predicted bias $-\frac{4x0.3}{127} = -0.01$ with a value of $\frac{126}{127} \times 0.30$ added when I=1,2,3,4 as expected. The error $e_2(I)$ due to the second order nonlinearity has the predicted bias $\frac{(4x0.3)^2}{127} = 0.01$, with unsystematic errors with the predicted value $-\frac{128}{127} \times 2 \times 0.3^2 = -0.18$ added when I=15,16,17,29, 30,38. Since R_m is 13 for this signal and $w_2(J,K) \neq 0$ for J,K=1,2,3,4 only, no unsystematic errors are expected when I=0,1,...,13, and none occur. The occurrence of unsystematic errors for the other values of I may be predicted from the following relationships which exist between the m sequence members:

$$S_{i-1-j} + S_{i-2-j} + S_{i-15-j} = 0 \qquad j=0,1,2$$

$$S_{i-1-k} + S_{i-3-k} + S_{i-19-k} = 0 \qquad k=0,1$$

$$S_{i-1} + S_{i-4} + S_{i-38} = 0 \qquad \text{mod } 2$$

The error $e_3(I)$ due to the third order nonlinearity has the predicted bias $-\frac{(4x0.3)^3}{127} = -0.01$, with systematic errors with the predicted value $\frac{128}{127}(3x4x0.3^3 - 2x0.3^3) = 0.27$ added when I=1,2,3,4 as expected, and

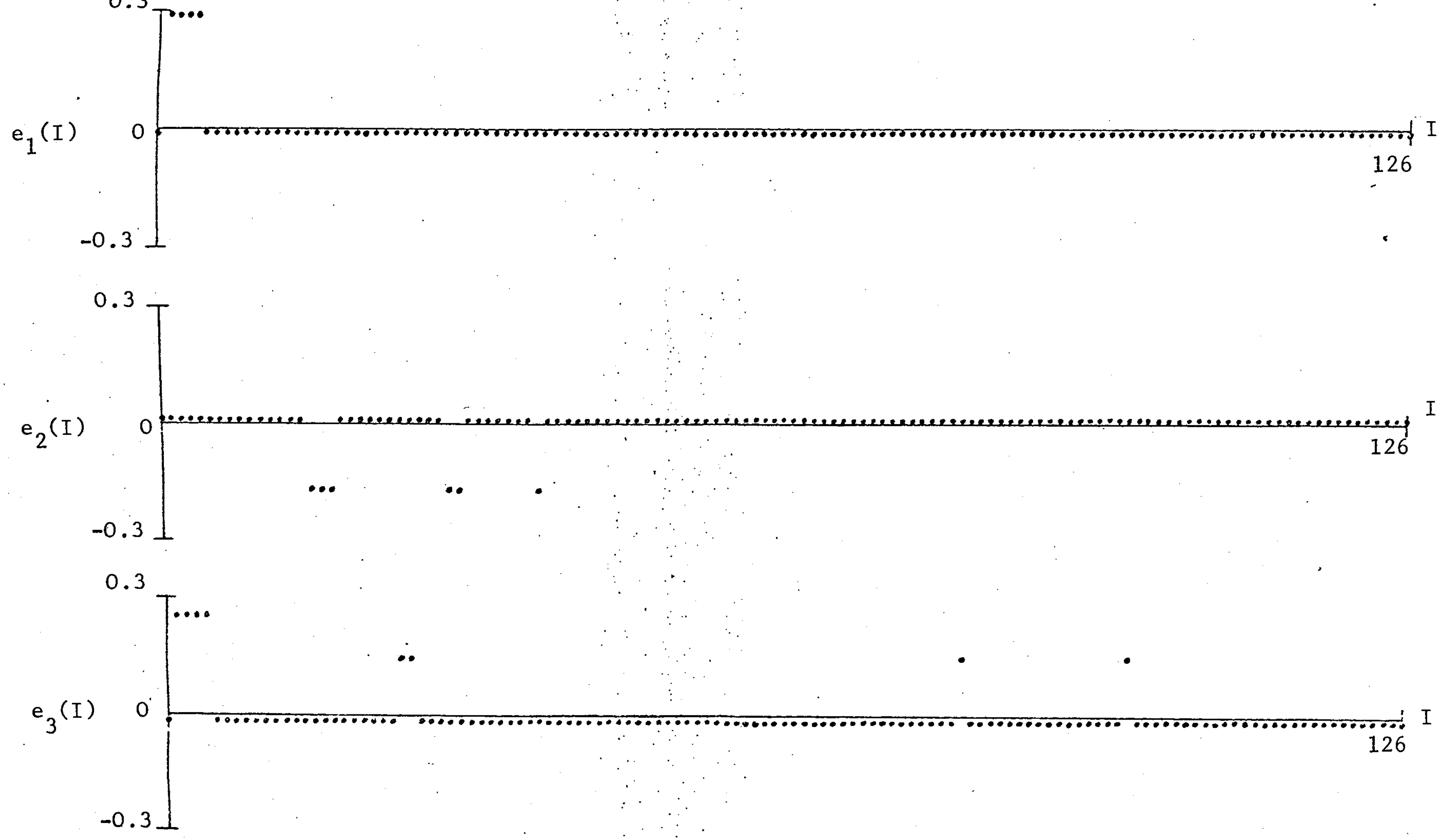


Fig. 6.3. Linear estimate and errors due to second-and third-order nonlinearities for pseudorandom binary input signal.

unsystematic errors with the predicted value $\frac{128}{127} \times 6 \times 0.3^3 = 0.16$ added when I=24,25,81,98. Since R_1 is 10 for this signal, and $w_3(J,K,L) \neq 0$ for J,K,L=1,2,3,4 only, no unsystematic errors are expected when I=0,1,...,10, and none occur. The occurrence of unsystematic errors for the other values of I may be predicted from the following relationships which exist between the m sequence members

$$S_{i-1-j} + S_{i-2-j} + S_{i-3-j} + S_{i-24-j} = 0 \qquad j=0,1$$

$$S_{i-1} + S_{i-2} + S_{i-4} + S_{i-81} = 0$$

$$S_{i-1} + S_{i-3} + S_{i-4} + S_{i-98} = 0 \qquad \text{mod } 2$$

(b) Antisymmetric pseudorandom binary signals.

When the input is an antisymmetric pseudorandom binary signal with the characterstic polynomial given in the preceding section, the results are as shown in fig. 6.4, except that the error $e_2(I)$ due to the second order nonlinearity is zero as expected, and is omitted.

The oscillatory term of the linear component is zero as predicted for the particular $w_1(\tau)$ used in this case, and the first order kernel values of 0.3 occur when I=1,2,3,4 as expected.

In the error $e_3(I)$ due to third order nonlinearity, the oscillatoryterm is zero as predicted for the particular $w_3(\tau_1, \tau_2, \tau_3)$ used in this case. The systematic errors with the predicted value of 0.27 occur when I=1,2,3,4 as expected, and unsystematic errors with the predicted value of 0.16 occur when I=24,25,81 and 88 as expected.

(c) Pseudorandom ternary input signal.

When the input is a pseudorandom ternary input signal with characteristic polynomial $1 + D^2 - D^3 - D^4 + D^5$, the results are as shown in fig. 6.5, except that the error $e_2(I)$ due to the second order

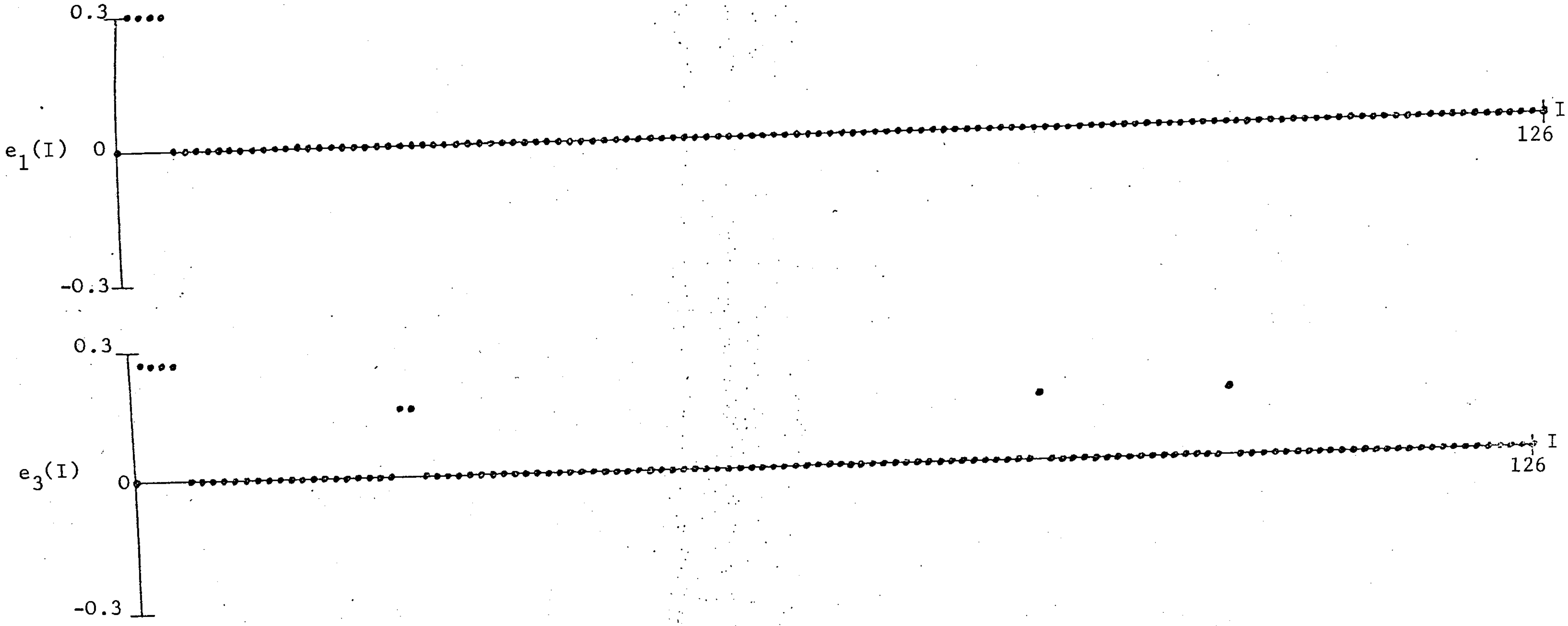


Fig. 6.4. Linear estimate and error due to third-order nonlinearity for antisymmetric pseudorandom binary input signal.

nonlinearity is zero as expected and is omitted.

The linear component, $e_1(I)$ has the value 0.3 when I=1,2,3,4 and zero elsewhere as expected.

The error $e_3(I)$ due to the third order nonlinearity consists of the systematic component with the predicted value $2x4x0.3^3-o.3^3=0.19$ when I=1,2,3,4, and unsystematic component with the predicted values $\pm 2x0.3^3 = \pm 0.5$ when I=9,13,14,16,17,20,21,45,59,63,75,77,80,81,83,88, 96,97,101,102,103,113. Since the value of R_1 for this signal is 7 and $w_3(J,K,L)\neq 0$ for J,K,L=1,2,3,4 only, no unsystematic errors are expected when I=0,1,...,7, and none occur. The occurrence of unsystematic errors for the other values of I may be predicted from the following relationships which exist between the m sequence members:

$$-S_{i-1}-S_{i-2}+S_{i-4}+S_{i-9} = 0$$

$$-S_{i-1-j}-S_{i-2-j}+S_{i-3-j}+S_{i-13-j} = 0 \qquad j = 0,1$$

$$S_{i-1-k}+S_{i-2-k}+S_{i-3-k}+S_{i-20-k} = 0 \qquad k = 0,1$$

$$S_{i-1}+S_{i-2}+S_{i-4}+S_{i-45} = 0$$

$$S_{i-1}-S_{i-2}+S_{i-4}+S_{i-63} = 0$$

$$-S_{i-1}-S_{i-3}+S_{i-4}+S_{i-75} = 0$$

$$-S_{i-1}+S_{i-3}+S_{i-4}+S_{i-77} = 0$$

$$-S_{i-1-j}-S_{i-2-j}+S_{i-3-j}+S_{i-80-j} = 0 \qquad j = 0,1$$

$$-S_{i-1}-S_{i-3}-S_{i-4}+S_{i-83} = 0$$

$$-S_{i-1}+S_{i-3}-S_{i-4}+S_{i-88} = 0$$

$$-S_{i-1-k}+S_{i-2-k}+S_{i-3-k}+S_{i-96-k} = 0 \qquad k = 0,1$$

$$S_{i-1}-S_{i-2}-S_{i-4}+S_{i-113} = 0$$

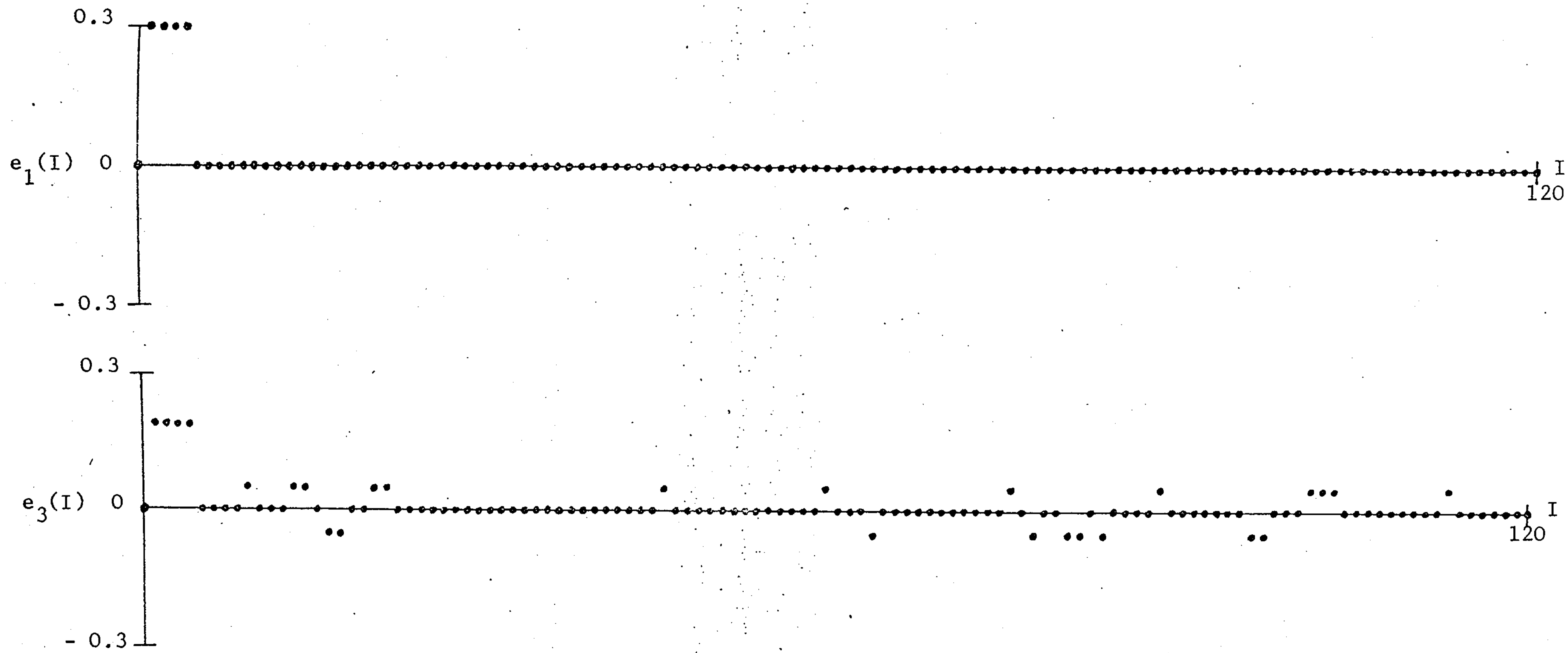


Fig. 6.5. Linear estimate and error due to third-order nonlinearity for antisymmetric pseudorandom ternary input signal.

$-S_{i-1-j}-S_{i-3-j}+S_{i-16-j} = 0$ $\qquad j=0,1$

$S_{i-1}+S_{i-4}+S_{i-59} = 0$

$S_{i-1-k}+S_{i-2-k}+S_{i-101-k} = 0$ $\qquad k=0,1,2$

There are only 6 other relationships of the required form which exist for $0 \leq J,K,L \leq 4$ and $0 \leq I \leq 121$, and these are

$-S_{i-1-j}+S_{i-3-j}+S_{i-50-j} = 0$ $\qquad j=0,1$

$-S_{i-1-k}+S_{i-2-k}+S_{i-71-k} = 0$ $\qquad k=0,1,2$

$S_{i-1}-S_{i-4}+S_{i-90} = 0$

but in all the above cases, the unsystematic error is predicted to be zero for the particular $w_2(\tau_1, \tau_2, \tau_3)$ used in this case, and therefore no unsystematic errors are expected when I=50,51,71,72,73,90 and none occur.

6.5.2 Processes with direction dependent dynamic responses

In this section, the use of a pseudorandom binary signal with levels X(0) = -1 and X(1) = 1 and characteristic polynomial $1+D^3+D^5$ to identify the characteristics of a first order system whose dynamics is dependent on the direction of the input signal is investigated. It is shown that the result which is obtained by direct crosscorrelation can be achieved analytically by the use of the formulae derived in this chapter. The analytical approach results in a better understanding of the behaviour of this particular type of nonlinear system.

The system investigated, which is the same as that considered by Godfrey and Briggs[31], is described by the equations

$$T_u\dot{y} + y = x \qquad \dot{y} \text{ positive} \qquad 6.28$$

$$T_D\dot{y} + y = x \qquad \dot{y} \text{ negative} \qquad 6.29$$

where x is the input, y is the output and T_u, T_D are time constants.

Since the input signal level is ± 1, eqns. 6.28 and 6.29 can be combined into a single equation

$$\frac{1}{rx + w}\dot{y} + y = x \qquad 6.30$$

where $r = \frac{1}{2}(\frac{1}{T_u} - \frac{1}{T_D})$ and

$$w = \frac{1}{2}(\frac{1}{T_u} + \frac{1}{T_D})$$

The block diagram of eqn. 6.30, which is given in fig. 6.6, represents a nonlinear system which may be represented by Volterra series. If the fourth and higher order terms are negligible, then the system is characterised by the first three Volterra kernels given in the S domain by[11]

$$w_1(S) = 1 - \frac{S}{S+w}$$

$$w_2(S_1,S_2) = \frac{rS_2}{(S_2+w)(S_1+S_2+w)} \qquad 6.31$$

$$w_3(S_1,S_2,S_3) = -\frac{rS_3}{(S_3+w)(S_2+S_3+w)(S_1+S_2+S_3+w)}$$

or in the Z domain by[38]

$$w_1(Z) = 1 - \frac{Z-1}{Z-e^{-wT}}$$

$$w_2(Z_1,Z_2) = \frac{rTe^{-wT}(Z_2-1)}{(Z_2-e^{-wT})(Z_1Z_2-e^{-wT})} \qquad 6.32$$

$$w_3(Z_1,Z_2,Z_3) = -\frac{r^2T^2e^{-wT}(Z_3-1)(Z_2Z_3 + e^{-wT})}{2(Z_3-e^{-wT})(Z_2Z_3-e^{-wT})(Z_1Z_2Z_3-e^{-wT})}$$

The corresponding time domain expressions are

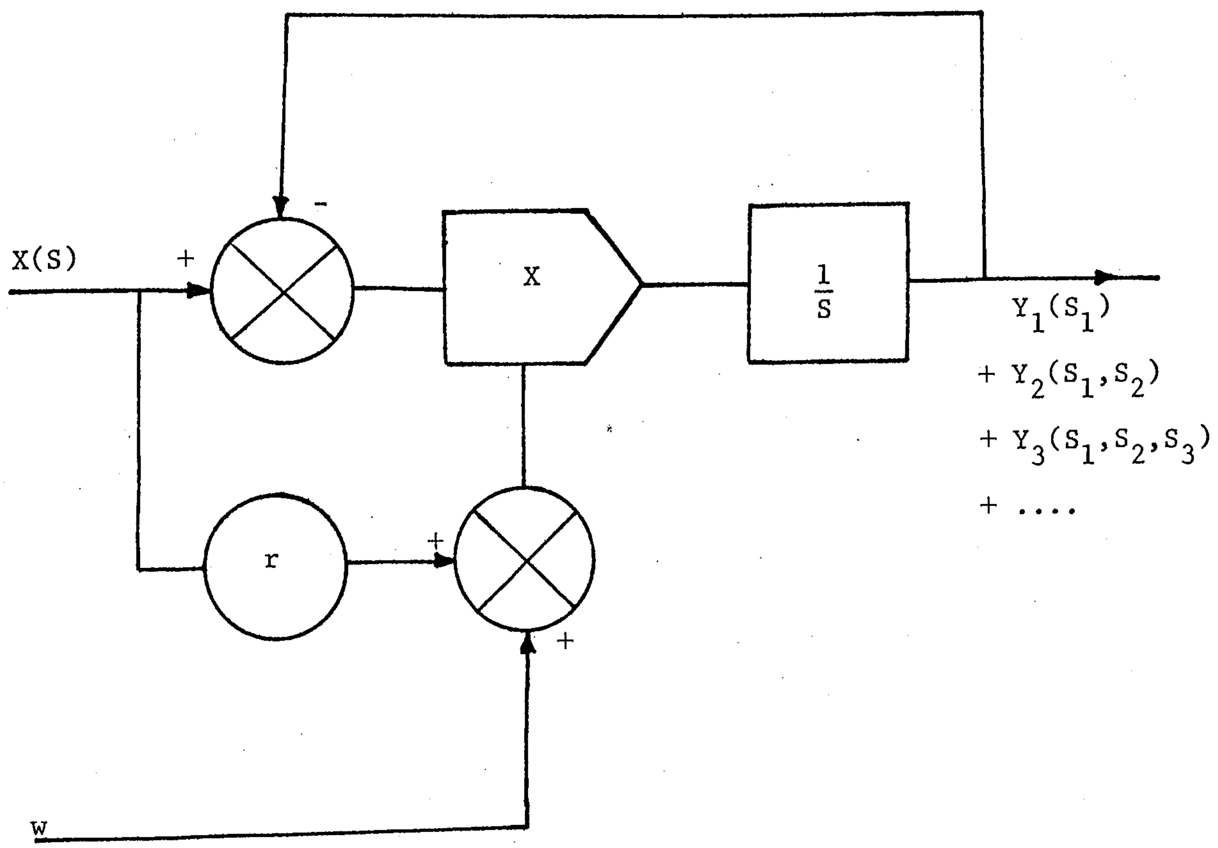


Fig.6.6. Block diagram of a system with direction dependent dynamic characteristics.

$$w_1(i_1T) = (1 - e^{-wT})e^{-(i_1-1)wT} \qquad i_1 > 0$$

$$w_2(i_1T, i_2T) = -\frac{rT}{2}(1 - e^{-wT})e^{-(i_1-1)wT} \qquad i_1 > i_2 > 0$$

$$w_3(i_1T, i_2T, i_3T) = \frac{r^2T^2}{6}(1 - e^{-wT})e^{-(i_1-1)wT} \qquad i_1 > i_2 \geq i_3 > 0 \qquad ..6.33$$

$$w_3(i_1T, i_1T, i_1T) = -\frac{r^2T^2}{2}e^{-i_1wT} \qquad i_1 > 0$$

From the preceding equations, the bias, linear term and errors due to the second and third order nonlinearities can be computed.

(a) Constant bias.

From eqns. 6.6 and 6.33, the constant bias is given by

$$b_j = -\frac{\left[-TX(1)\right]^{j-1}}{N} \sum_{i_1=0}^{R} \sum_{i_2=0}^{R} \cdots \sum_{i_j=0}^{R} w_j(i_1T, i_2T, \ldots, i_jT)$$

$$= -\frac{\left[-TX(1)\right]^{j-1}}{N} \left[w_j(Z_1, Z_2, \ldots, Z_j) \right]_{Z_1=Z_2\ldots=Z_j=1}$$

$$= \begin{cases} -\frac{1}{N} & j=1 \\ 0 & j>1 \end{cases} \qquad 6.34$$

(b) The linear component.

This is obtained from eqns. 6.7 and 6.33 as

$$e_1(IT) = -\frac{1}{N} + \frac{2^n}{N}(1 - e^{-wT})e^{-(I-1)wT} \qquad 0 < I < N \qquad 6.35$$

and is plotted in fig. 6.7 for T=1, $r=\frac{1}{8}$, $w=\frac{3}{8}$, N=31 and n=5. These parameter values are used in all the subsequent graphs.

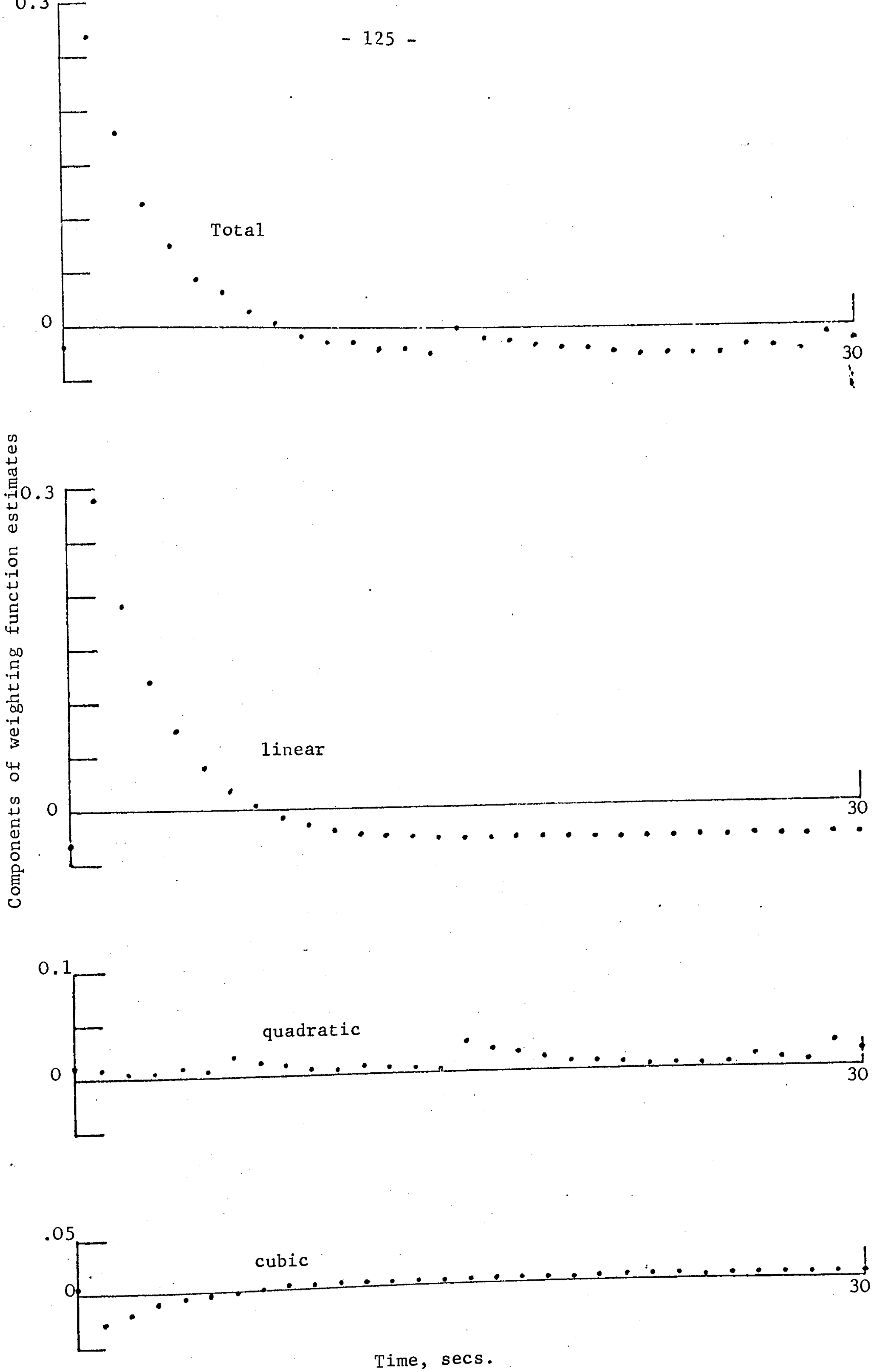


Fig. 6.7. Direction dependent system: weighting function estimate and its components.

(c) Error due to 2nd order nonlinearity.

This error is obtained from eqns. 6.8 and 6.34 as

$$e_2(IT) = -\frac{2.2^n T}{N} \sum w_2(JT,KT) \qquad 6.36$$

where the summation is taken over $0 \leq J < K \leq R$ in eqn. 6.9.
For each value of I satisfying eqn. 6.9, there is a contribution $e_2'(IT)$ to the second order nonlinearity obtained from eqns. 6.36 and 6.33 as

$$e_2'(IT) = N^{-1}2^n rT^2(1-e^{-wT})e^{-(k-1)wT}$$

$$= F_2 e^{-(k-1)wT} \qquad 6.37$$

where $F_2 = N^{-1}2^n rT^2(1-e^{-wT})$

The values of J, K and I which obey eqn. 6.9 may be divided into two groups, namely the main terms and the shifted or subsidiary terms. Once the main terms are evaluated by polynomial division, the subsidiary terms are easily obtained by using the shift and add property of pseudorandom signals. The main terms which contribute significantly to the total second order nonlinearity in the present example are given by

$$S_{i-1} + S_{i-k} + S_{i-I_k} = 0 \qquad 6.38$$

where k=2,3,...,13 and I_k=15,29,6,26,4,11,17,20,25,7,24 and 21 respectively. For each main relationship in the form of eqn. 6.38, there are N-k-1 subsidiary relations. Taking a typical case of k=10, the main relationship

$$S_{i-1} + S_{i-10} + S_{i-25} = 0$$

contributes the value $e_2'(25T) = F_2 e^{-9T}$ to the second order nonlinearity while the 20 shifted relations

$$S_{i-1-j} + S_{i-10-j} + S_{i-25-j} = 0 \qquad j=1,2,...,20$$

contribute the values $e_2'(26T) = F_2e^{-10T}$, $e_2'(27T) = F_2e^{-11T}, \ldots,$ $e_2'(30T) = F_2e^{-14T}$, $e_2'(0) = F_2e^{-15T}, \ldots, e_2'(13T) = F_2e^{-28T}$ respectively. This case is shown graphically in fig. 6.8. The total second order nonlinearity which is plotted in fig. 6.7 is the sum of all such contributions.

(d) Error due to 3rd order nonlinearity.

From eqns. 6.13, 6.32 and 6.33, the systematic error is obtained as

$$\Big[e_3(IT)\Big]_s = \frac{2^n}{N} T^2 \left[3 \sum_{j=0}^{R} w_3(IT,jT,jT) - 2w_3(IT,IT,IT) \right]$$

$$= \frac{2^n T^2}{N} \left[-\frac{r^2T^2}{2} \left\{ \frac{1+e^{-wT}}{1-e^{-wT}} - I(1-e^{-wT}) \right\} e^{-(I-1)wT} + r^2T^2e^{-IwT} \right]$$

$$= -\frac{2^n r^2 T^4}{2N} \left[e^{-wT} \frac{(1+e^{-wT})}{1-e^{-wT}} - (I-1)(1-e^{-wT}) \right] e^{-(I-1)wT}$$

6.39

and the unsystematic error is given by

$$\Big[e_3(IT)\Big]_u = \frac{6.2^n T^2}{N} \sum w_3(JT,KT,LT) \qquad 6.40$$

where the summation is taken over $0 \leq J < K < L \leq R$ in eqn. 6.12. For every value of I obeying eqn. 6.12, there is a contribution $\Big[e_3'(IT)\Big]_u$ to the unsystematic 3rd order nonlinearity which is obtained from eqns. 6.40 and 6.33 as

$$\Big[e_3'(IT)\Big]_u = N^{-1}2^n r^2 T^4 (1-e^{-wT}) e^{-(L-1)wT}$$

$$= F_3 e^{-(L-1)wT} \qquad 6.41$$

where $F_3 = N^{-1}2^n r^2 T^4 (1-e^{-wT})$

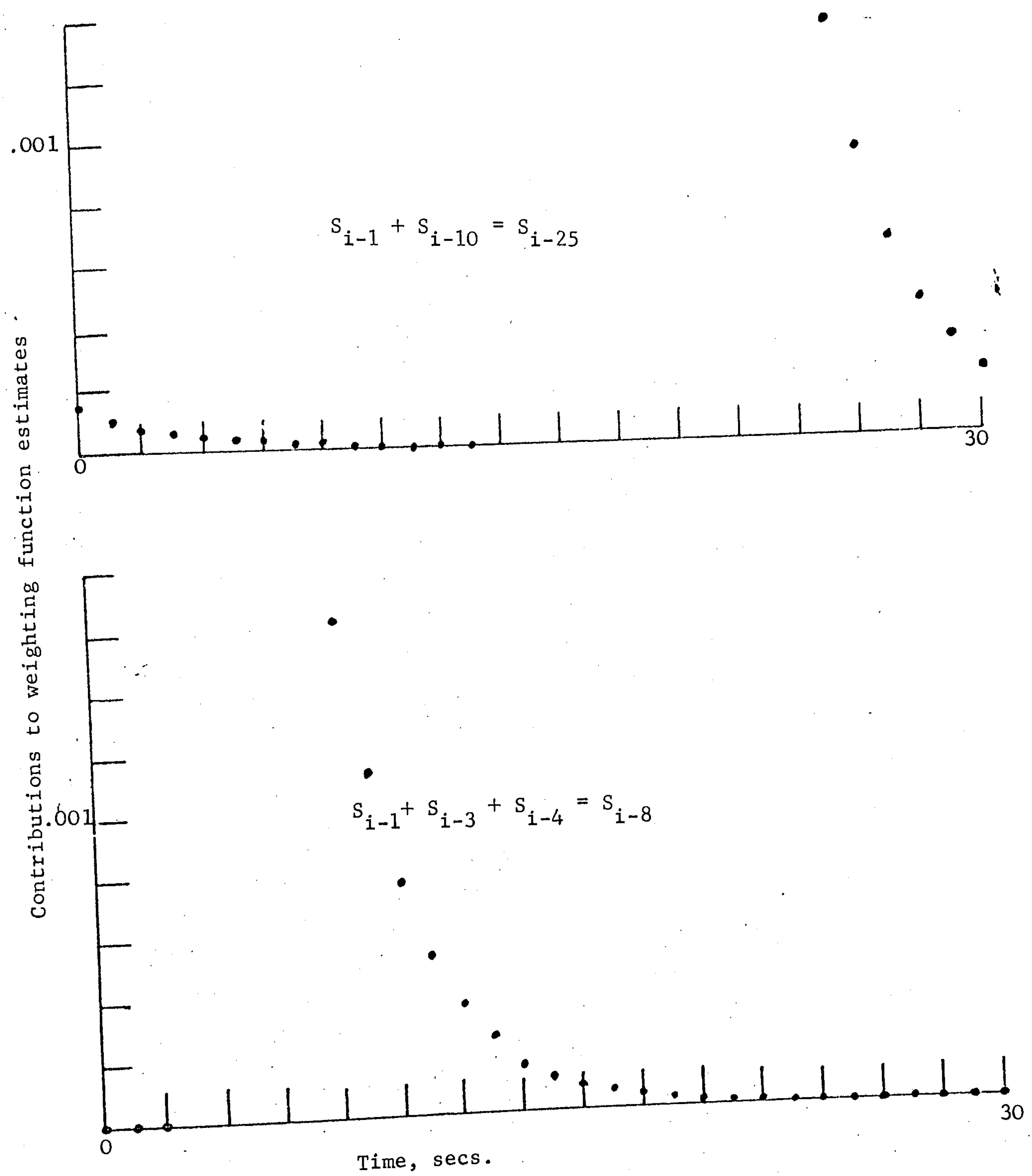


Fig. 6.8. Direction dependent system: typical contributions from 3 and 4 term linear relationships and their shifted values.

The main relationships which give significant contribution to the unsystematic component in the problem under consideration are

$$S_{i-1} + S_{i-K} + S_{i-L_K} + S_{i-I_K} = 0 \qquad 6.42$$

where K=2,3,4,5,6,7,8,9

L_2=3,4,...,10 and I_2=23,27,11,30,26,24,19,13 respectively

L_3=4,5,...,10 and I_3=8,14,17,22,4,21,18 respectively

L_4=5,7,8,9 and I_4=19,20,3,11 respectively

L_5=6,7...,10 and I_5=18,15,25,27,17 respectively

L_6=7,8,9,10 and I_6=9,29,7,14 respectively

L_7=8,9,10 and I_7=13,6,24 respectively

L_8=9,10 and I_8=28,26 respectively

L_9=10 and I_9=16

For each main relationship in the form of eqn. 6.42, there are $N=L_K-1$ subsidiary relations. In a typical case of $L_3=4$, the main relationship

$$S_{i-1} + S_{i-3} + S_{i-4} + S_{i-8} = 0$$

contributes the term $\left[e_3'(8T)\right]_u = F_3 e^{-3wT}$ while the 26 shifted relations

$$S_{i-1-r} + S_{i-3-r} + S_{i-4-r} + S_{i-8-r} = 0$$

$$r = 1,2,\ldots,26$$

contribute the following values:

$$\left[e_3'(9T)\right]_u = F_3 e^{-4wT}, \quad \left[e_3'(10T)\right]_u = F_3 e^{-5wT}, \ldots,$$

$$\left[e_3'(30T)\right]_u = F_3 e^{-25wT}, \quad \left[e_3'(0)\right]_u = F_3 e^{-26wT}, \ldots, \left[e_3'(3T)\right]_u$$

$$= F_3 e^{-29wT}.$$

This case is illustrated in fig. 6.8. The total unsystematic cubic error is the sum of all the contributions due to the four term relationships in eqn. 6.42 and their shifted values. The total cubic nonlinear effect is the sum of the systematic and unsystematic components. These are plotted in fig. 6.9.

The sum of the linear, quadratic and cubic terms which is shown in fig. 6.7 compares favourably with the result obtained by injecting the pseudorandom signal into the direction dependent system and crosscorrelating the output signal with the input signal. This fact is illustrated very clearly by table 6.1. Column 2 of table 6.1(a) is the response of the direction dependent system to the input pseudorandom binary signal. The next three columns, which are obtained by simulating eqn. 6.32 in the manner suggested by Schetzen[39], are the responses of the linear, quadratic and cubic components. The sixth, seventh and eighth columns are the errors obtained if it is assumed that the system is represented by the first, the first two and the first three terms of the Volterra series respectively. Columns two to five of table 6.1(b) are obtained by crosscorrelating the corresponding data in table 6.1(a) with the input signal. The linear, quadratic and cubic terms shown in table 6.1(b) are the same as the theoretical results of fig. 6.7. The computer program which was used to obtain the data in table 6.1 is given in Appendix 1.11.

An important feature of this system and many other practical processes subjected to pseudorandom signal testing is the error due to systematic cubic nonlinearity which, in this example, contributes up to 10% of the total crosscorrelation function. Since this contribution varies smoothly with time, it is all too easy when

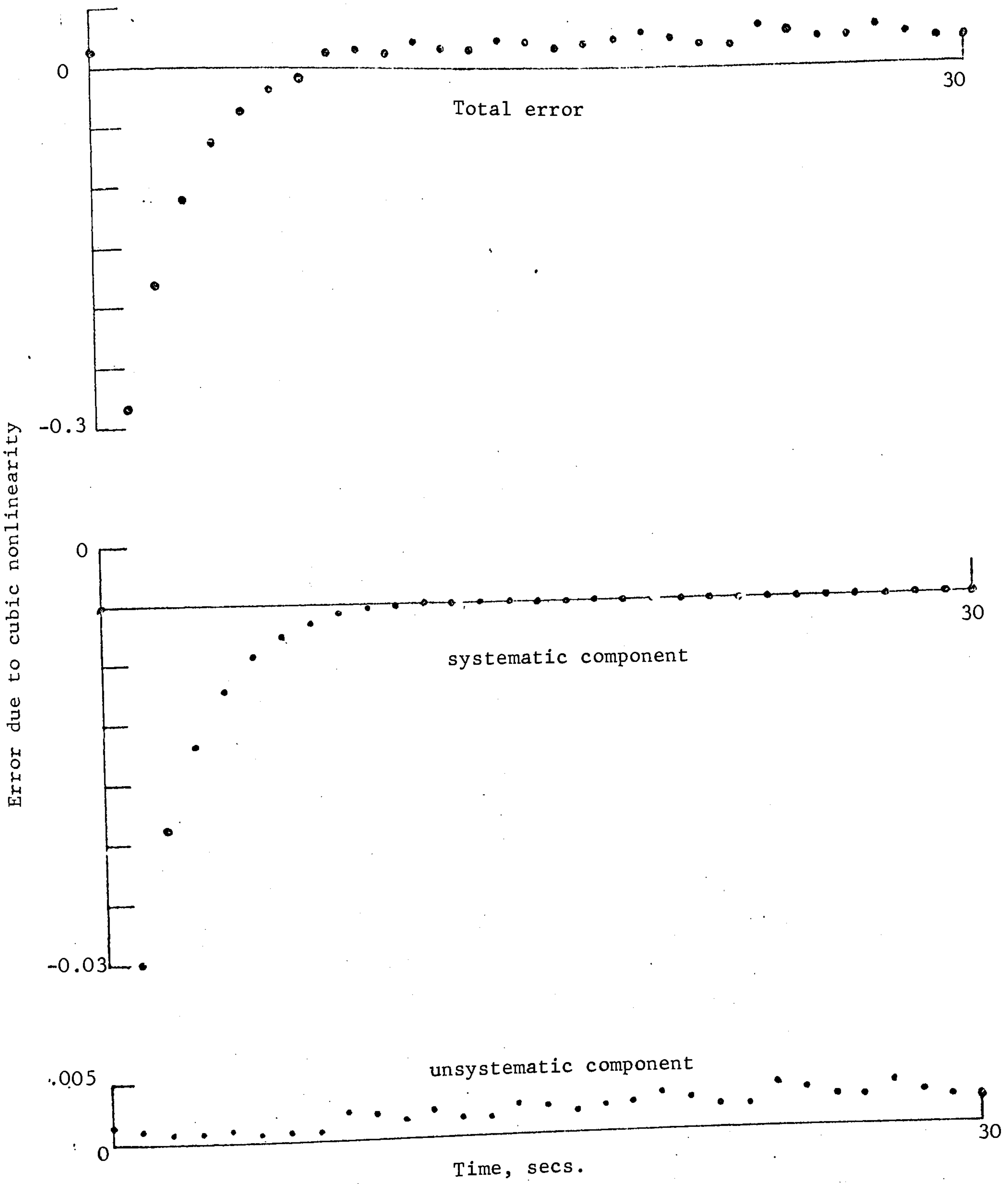


Fig.6.9. Direction dependent system: error due to cubic nonlinearity and its components.

POLYNOMIAL 100101
OUTPUT OF DIRECTION DEPENDENT SYSTEM

NO	TOTAL T	LINEAR L	QUADRATIC Q	CUBIC C	ERROR L	ERROR L+Q	ERROR L+Q+C
0	-0.024	-0.365	0.299	0.038	0.341	0.042	0.0039
1	0.379	0.062	0.323	-0.007	0.317	-0.006	0.0011
2	0.623	0.355	0.303	-0.038	0.268	-0.035	0.0031
3	0.772	0.557	0.263	-0.055	0.215	-0.049	0.0066
4	0.861	0.696	0.219	-0.063	0.166	-0.053	0.0100
5	0.916	0.791	0.177	-0.064	0.125	-0.051	0.0123
6	0.492	0.231	0.275	-0.019	0.261	-0.014	0.0052
7	0.162	-0.154	0.295	0.017	0.316	0.021	0.0041
8	-0.095	-0.419	0.275	0.042	0.324	0.048	0.0066
9	0.336	0.025	0.311	-0.003	0.311	-0.000	0.0024
10	0.597	0.330	0.298	-0.034	0.267	-0.030	0.0035
11	0.244	-0.086	0.319	0.009	0.330	0.011	0.0016
12	0.541	0.254	0.312	-0.027	0.288	-0.025	0.0021
13	0.722	0.487	0.279	-0.049	0.235	-0.044	0.0052
14	0.831	0.647	0.236	-0.061	0.184	-0.052	0.0087
15	0.426	0.132	0.304	-0.012	0.294	-0.010	0.0029
16	0.652	0.404	0.283	-0.039	0.248	-0.035	0.0045
17	0.287	-0.035	0.315	0.005	0.322	0.007	0.0018
18	0.567	0.288	0.306	-0.029	0.279	-0.027	0.0026
19	0.221	-0.114	0.321	0.013	0.335	0.014	0.0013
20	-0.049	-0.391	0.296	0.041	0.342	0.045	0.0042
21	-0.260	-0.582	0.256	0.057	0.322	0.066	0.0089
22	-0.423	-0.713	0.212	0.063	0.289	0.077	0.0137
23	0.137	-0.177	0.293	0.016	0.314	0.021	0.0046
24	-0.115	-0.434	0.272	0.041	0.320	0.048	0.0069
25	-0.311	-0.611	0.235	0.054	0.301	0.065	0.0108
26	0.205	-0.107	0.300	0.008	0.312	0.012	0.0037
27	-0.061	-0.387	0.283	0.036	0.325	0.042	0.0055
28	0.356	0.047	0.314	-0.007	0.309	-0.005	0.0022
29	0.610	0.345	0.297	-0.037	0.264	-0.033	0.0037
30	0.253	-0.076	0.320	0.008	0.329	0.009	0.0015
31	-0.024	-0.365	0.299	0.038	0.341	0.042	0.0039

(a)

POLYNOMIAL 100101
CROSSCORRELATION FUNCTION OF DIRECTION DEPENDENT SYSTEM

NO	TOTAL T	LINEAR L	QUADRATIC Q	CUBIC C	ERROR L	ERROR L+Q	ERROR L+Q+C
0	-0.019	-0.032	0.011	0.002	0.013	0.002	0.0001
1	0.270	0.291	0.008	-0.028	-0.021	-0.028	-0.0001
2	0.178	0.190	0.005	-0.018	-0.012	-0.017	0.0001
3	0.114	0.120	0.004	-0.010	-0.006	-0.010	0.0004
4	0.075	0.073	0.009	-0.006	0.003	-0.006	-0.0000
5	0.043	0.040	0.006	-0.003	0.003	-0.003	0.0003
6	0.032	0.017	0.017	-0.001	0.015	-0.002	-0.0006
7	0.014	0.002	0.013	-0.000	0.012	-0.001	-0.0001
8	0.002	-0.009	0.009	0.002	0.011	0.002	0.0001
9	-0.008	-0.016	0.006	0.002	0.009	0.002	0.0004
10	-0.015	-0.021	0.004	0.002	0.007	0.002	0.0007
11	-0.014	-0.025	0.007	0.003	0.010	0.003	0.0004
12	-0.019	-0.027	0.005	0.002	0.008	0.003	0.0007
13	-0.022	-0.029	0.003	0.002	0.006	0.003	0.0008
14	-0.024	-0.030	0.002	0.003	0.006	0.004	0.0008
15	-0.000	-0.031	0.029	0.003	0.030	0.001	-0.0018
16	-0.010	-0.031	0.020	0.002	0.022	0.001	-0.0008
17	-0.013	-0.031	0.017	0.003	0.019	0.002	-0.0005
18	-0.017	-0.032	0.012	0.003	0.014	0.003	0.0000
19	-0.020	-0.032	0.008	0.003	0.012	0.004	0.0003
20	-0.021	-0.032	0.008	0.003	0.011	0.003	0.0004
21	-0.023	-0.032	0.006	0.003	0.009	0.003	0.0006
22	-0.025	-0.032	0.004	0.002	0.007	0.003	0.0008
23	-0.025	-0.032	0.003	0.004	0.008	0.005	0.0007
24	-0.025	-0.032	0.003	0.003	0.007	0.004	0.0008
25	-0.025	-0.032	0.003	0.003	0.007	0.004	0.0008
26	-0.018	-0.032	0.011	0.003	0.014	0.003	0.0001
27	-0.020	-0.032	0.008	0.004	0.012	0.004	0.0003
28	-0.023	-0.032	0.005	0.003	0.009	0.004	0.0005
29	-0.008	-0.032	0.023	0.003	0.024	0.002	-0.0010
30	-0.014	-0.032	0.016	0.003	0.018	0.002	-0.0003
31	-0.019	-0.032	0.011	0.002	0.013	0.002	0.0001

(b)

Table 6.1 Output sequences and weighting function estimates.

interpreting experimental crosscorrelation results to overlook it and assume, in the absence of any appreciable unsystematic errors, that the system under investigation is linear.

6.5.3 Gas chromatography

This section explains the second order nonlinear effects observed by Godfrey and Devenish[30] in their continuous gas chromatography experiments using a 127 bit pseudorandom binary signal whose characteristic polynomial is $1+D^3+D^4+D^5+D^7$. One of the results for the argon carrier and air sample which is reproduced in fig. 6.10(b), shows two main peaks due to the presence of oxygen and nitrogen in the air sample and other subsidiary peaks due to nonlinearity. The true system weighting function given in fig. 6.10(a), shows that significant values of the oxygen peak occur from digit 57 to 62 and those of nitrogen peak from digit 74 to 82. Since the two biggest values of the weighting function of fig. 6.10(a) occur at digits 77 and 78 and since

$$S_{i-77} + S_{i-78} + S_{i-87} = 0$$

a significant second order nonlinear effect should exist in digit 87 of fig. 6.10(b) and this in fact is the case. The other nonlinear effects at digits 14, 20,42, 70,97,104 and 117 may be accounted for by the following linear relationships

$$S_{i-60} + S_{i-79} + S_{i-14} = 0$$

$$S_{i-59} + S_{i-80} + S_{i-20} = 0$$

$$S_{i-77} + S_{i-80} + S_{i-42} = 0$$

$$S_{i-58} + S_{i-75} + S_{i-42} = 0$$

$$S_{i-60} + S_{i-61} + S_{i-70} = 0$$

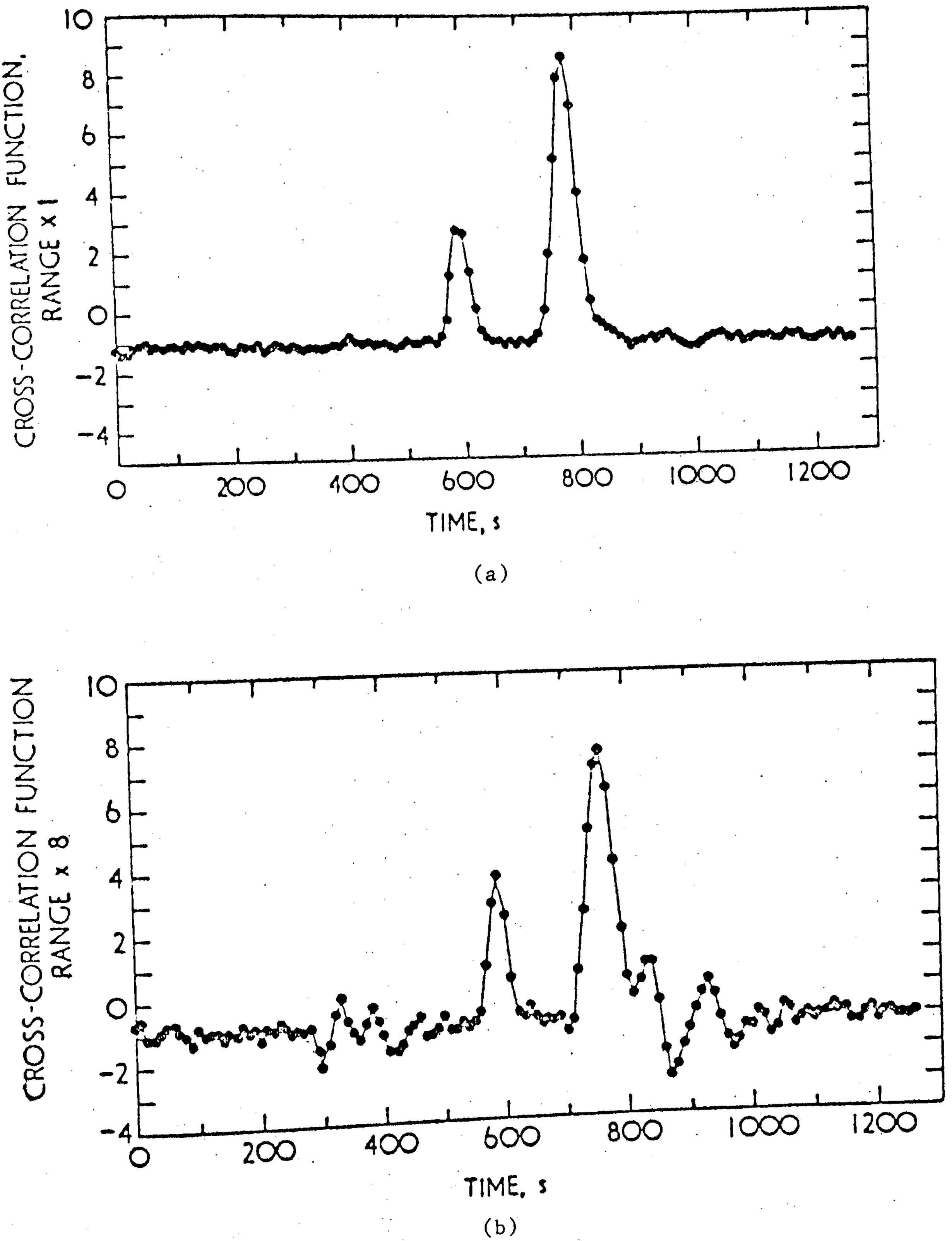


Fig. 6.10. Weighting function estimates of a continuous gas chromatography with argon carrier and air sample. (By courtesy of Godfrey and Kevenish, reference 30, page 230, figs. 6 and 7).

$$S_{i-57} + S_{i-61} + S_{i-97} = 0$$

$$S_{i-77} + S_{i-79} + S_{i-97} = 0$$

$$S_{i-60} + S_{i-82} + S_{i-97} = 0$$

$$S_{i-61} + S_{i-75} + S_{i-104} = 0$$

$$S_{i-77} + S_{i-81} + S_{i-117} = 0$$

It is essential to eliminate nonlinear effects from the crosscorrelation functions obtained in continuous gas chromatography tests because these effects present two major problems. In the first place, they alter the size and distort the shape of the weighting function which consists of a number of peaks. Since the position and magnitude of these peaks are measures of the substances present and their relative proportions, nonlinearity may lead to incorrect identification of the substances. Secondly nonlinearity may result in the appearance of additional peaks which may be erroneously interpreted as due to some substances present in the gas being analysed. It is hoped that the use of signals with superior performance given in tables 3.1 and 3.2 together with the results of this chapter will obviate these problems.

6.6 Conclusions

The effects of nonlinearities on system weighting function estimates obtained by crosscorrelation using pseudorandom signals depend on the kernels in a Volterra series representation of the nonlinearities, and on the higher order autocorrelation functions of the signals. Explicit results have been given for systems with second and third order nonlinearities which are tested by pseudorandom binary, antisymmetric pseudorandom binary and ternary signals.

With a pseudorandom binary signal, both second and third order nonlinearities give errors in the weighting function estimate. The error due to second order nonlinearities consists of a constant bias, similar to that observed in purely linear systems, together with an error which is unsystematic in the sense that it depends on relationships between members of the m sequence from which the signal is derived and is therefore neither smoothly varying nor the same for any other signal. By a judicious choice of signal, this unsystematic error may be removed from a range of interest which extends over the system settling time. This unexpected result could be extremely useful in certain applications. The error due to third order nonlinearities consists of a constant bias, a systematic error which is the same for all pseudorandom binary signals with common amplitude and bit interval, and another unsystematic error, which may also be removed from the range of interest.

With an antisymmetric pseudorandom binary signal, only third order nonlinearities give an error in the weighting function estimate, and this consists of an oscillatory term, similar to that observed in purely linear systems, a systematic error which is identical to that obtained with the corresponding pseudorandom binary signal, and an unsystematic error with components which have the same magnitude as those obtained with the corresponding pseudorandom binary signal, but which may take the opposite sign. The performance of an antisymmetric pseudorandom binary signal in removing the unsystematic error from the range of interest is identical to that of the corresponding pseudorandom binary signal.

With an antisymmetric pseudorandom ternary signal, only third order nonlinearities give an error in the weighting function estimate, and this consists of a systematic and an unsystematic error.

The systematic error is similar in nature, but not identical, to that obtained with an antisymmetric pseudorandom binary signal with the same amplitude and bit interval, while the unsystematic error consists of components which are much more numerous than those obtained with a binary signal.

In all cases, the results obtained have considerable significance, both for the design of correlation experiments so as to remove unsystematic errors from the range of interest, and for the interpretation of estimates obtained from correlation experiments in which the effects of nonlinearities are present.

Three illustrative examples have been given: the first was chosen so as to demonstrate clearly the nature of the errors due to the nonlinearities, while the second and third examples show how the results of this chapter could help in the correct identification of industrial processes.

CHAPTER 7

IDENTIFICATION OF THE DYAMICS OF A CHEMICAL PLANT

7 IDENTIFICATION OF THE DYNAMICS OF A CHEMICAL PLANT

7.1 Introduction

This chapter describes the practical determination of the dynamics of an ammonium nitrate synthesising process using the data obtained by Mr. M. Connell as part of an undergraduate project under the supervision of Professor H.A. Barker. The experiment had been performed before the theoretical results of Chapters 2, 3 and 6 were known, and therefore the input pseudorandom signal used was not one of those with superior performance. Nevertheless, the pH measurements obtained from the plant are adequate for illustrating how the characteristics of a practical nonlinear process may be determined by crosscorrelation, and the importance of choosing appropriate input signal for correlation experiments.

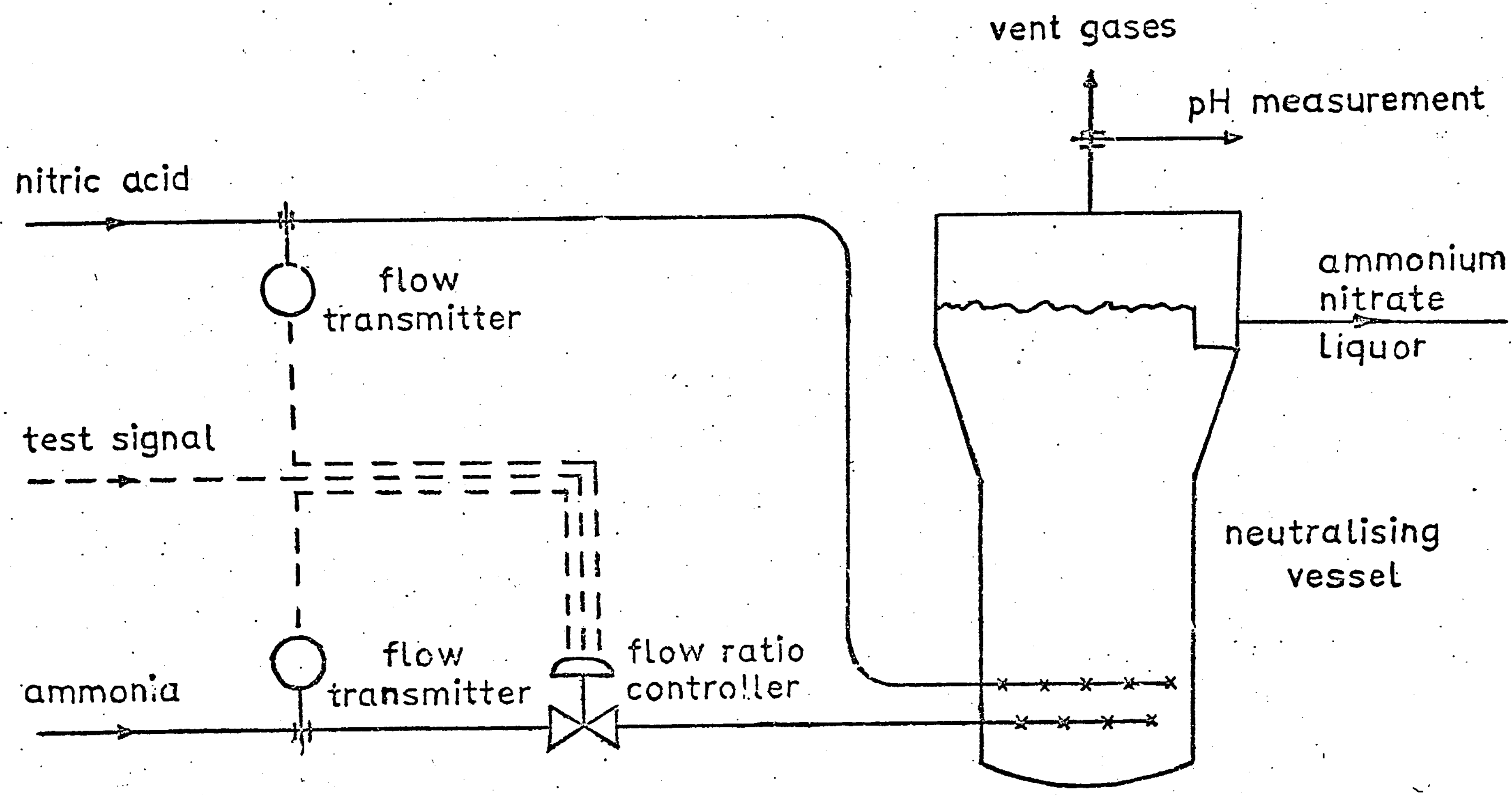
Since the relationship between the input variable, ammonia/nitric acid flow ratio, and the output variable, reactor vapour pH, was inherently nonlinear, the input variable was perturbed by a pseudorandom test signal based on a ternary m sequence in order that its antisymmetric property would, in the subsequent determination of the system weighting function by one-dimensional crosscorrelation, automatically remove the effects of even-order nonlinearities.[40] Low frequency noise in the form of constant, linear quadratic and cubic drifts was eliminated by crosscorrelating over two periods of the reference phase of the input signal, using a linear weighting which involved the ratio of the characteristic constants of the reference phase defined through its correlation with quadratic and cubic signals.[41] The resulting weighting function estimates exhibited structural deviations due to unwanted components but these were largely accounted for by a component due to a cubic nonlinearity.

The use of antisymmetric pseudorandom ternary signal also made the determination of the system's second order kernels possible.[22] These kernels were computed by two-dimensional crosscorrelation but they contained errors due mainly to the presence of numerous undesirable nonzero values in the fourth order autocorrelation functions of the input signal.

7.2 System Description

The system investigated was the neutralisation process of an ammonium nitrate synthesising plant at the Nobel Division of Imperical Chemical Industries Limited. Those features of the process with which the investigation was concerned are shown in fig. 7.1. A neutralising vessel of 8600 kg capacity is supplied with 60% strength nitric acid at 8800 kg/hr and ammonia gas at 1300 kg/hr. The product, 80% strength ammonium nitrate liquor, leaves the vessel through an overflow.

The pH of the liquor must be maintained within the range 4 to 5; at lower values the mixture becomes explosive while at higher values ammonia boils off and therefore the process becomes uneconomic. In addition, it is important to maintain a constant pH if the composition of the final product is to be homogeneous. There are practical difficulties in measuring the liquor pH directly; therefore this is obtained by inference through measurements of the pH of the neutraliser vent gases dissolved in water. Under constant operating conditions, this is directly related to the liquor pH, but because of the enrichment of the ammonia vapour at liquor/vapour interface, the pH measurement by this method is required to be maintained within the range 10 to 10.8.



Ammonium nitrate synthesising process

FIG. 7.1

During normal operation, the required pH value is maintained by a control system in which proportional feedback of the pH measurement is used to change the ratio of ammonia to nitric acid flowrate by means of a flow ratio controller in the ammonia supply. For the purposes of the investigation, this feedback was removed and the test signal inserted as a perturbation of the flow ratio; since the nitric acid flow rate was nominally constant during the experiment, this is equivalent to a perturbation of the ammonia flowrate.

The perturbation of ammonia flowrate results in fluctuation of the pH, and it is the dynamic relationship between these which is of interest. The logarithmic relationship between pH and ion concentration makes the output variable a nonlinear function of the input variable. The pH measurement is also corrupted by noise which, as expected in a process of this nature, is considerable and contains pronounced low frequency components in the form of drift.

7.3 Experimental Design

The determination of the first order kernel in a system where the output contains a significant contribution from nonlinearities and a considerable amount of noise presents a difficult problem. The approach adopted here is to use as the test signal a pseudorandom ternary signal based on a ternary m sequence; the inverse-repeat property of such a signal ensures that in the subsequent estimation of the weighting function by one-dimensional crosscorrelation, the effect of all even order nonlinearities is automatically removed. The use of such a signal also provides the possibility of determining the second order kernels by two-dimensional crosscorrelation.

One of the important requirements for accurate system identification by correlation is that the half-period of the ternary signal must exceed the system's settling time which, in this case, was estimated from a step test to be about 30 minutes. A pseudorandom ternary signal with a half-period of 36.4 minutes was therefore selected. This was achieved by choosing a 728 length sequence with a bit interval of 6 seconds.

The characteristic polynomial, f(D), of the sequence is given by

$$f(D) = 1 - D^2 - D^3 + D^4 - D^5 - D^6 \qquad 7.1$$

and a six stage shift register for generating the sequence is shown schematically in fig. 7.2. The initial states of the register were chosen so that the signal started at a desired reference phase. A digital computer was programmed to generate the pseudorandom sequences, and a number of periods of the signal were transcribed onto a channel of a magnetic tape recorder through a digital-analogue converter. A portion of the signal is shown in fig. 7.3.

The input signal was applied through an electro-pneumatic transducer to the flow ratio controller in the ammonia supply to the neutraliser, and the signal amplitude adjusted so that the resulting pH measurements did not exceed the permitted range of 10 to 10.8. A perturbation of $\pm$143 kg/hr about the nominal ammonia flowrate of 1300 kg/hr was found to be suitable.

After steady-state conditions were established, five successive periods of pH measurements, for which a proportional electrical signal was available, were recorded on to another channel of the magnetic tape

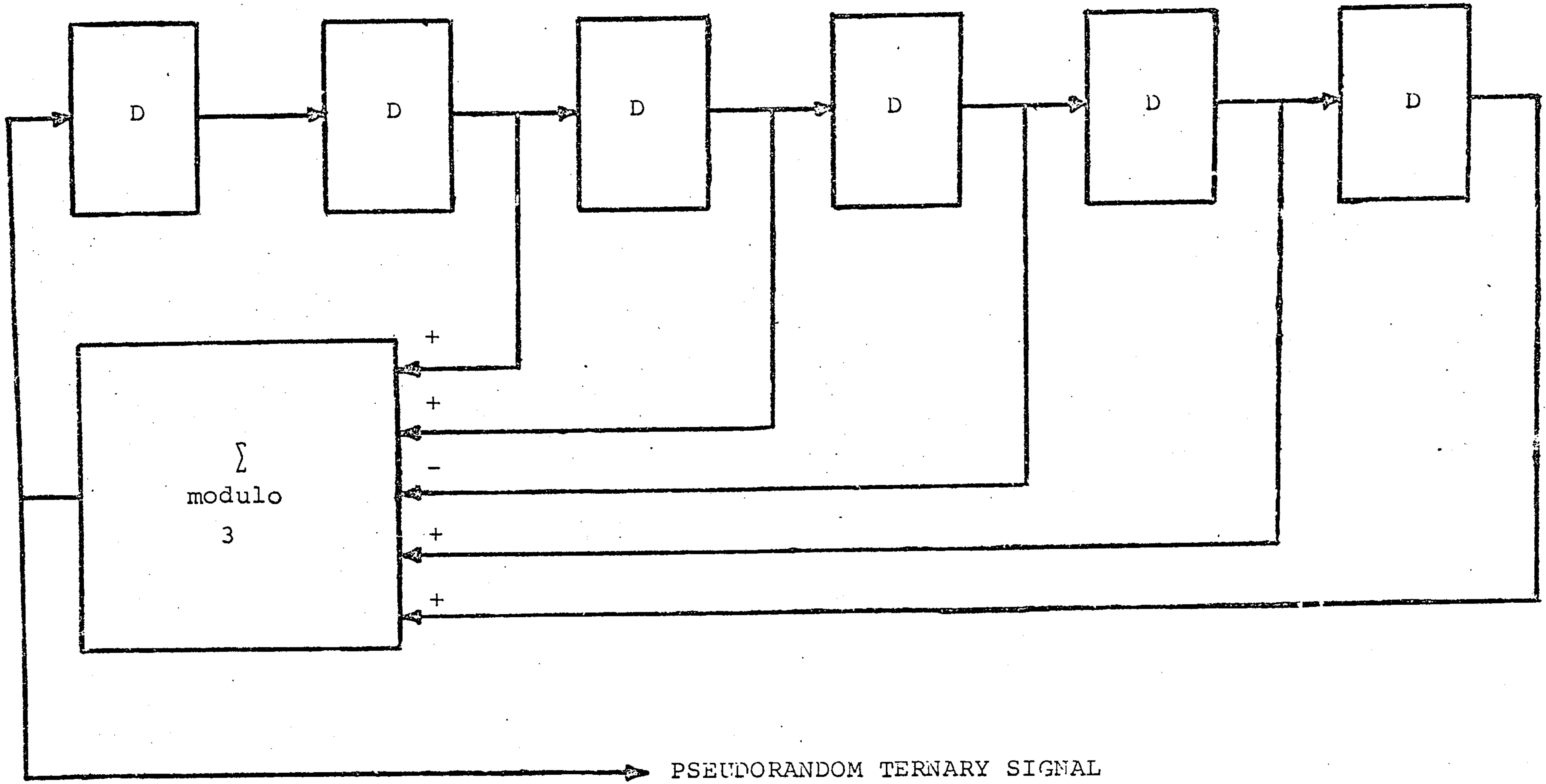
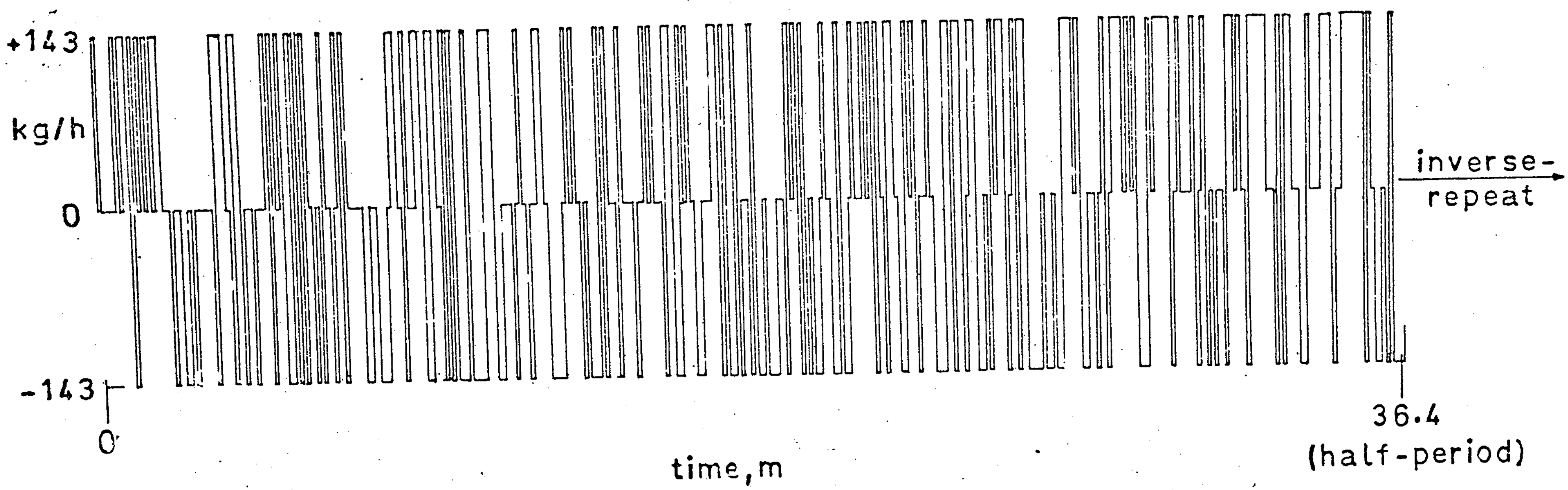


Fig. 7.2. Generation of pseudorandom ternary signal.



Pseudorandom ternary signal as ammonia flowrate perturbation

FIG. 7.3

recorder. An analogue-digital converter was then used to sample the recorded data at the 6 seconds bit interval of the input pseudorandom signal for subsequent analysis by digital computer.

The five cycles of pH deviations due to the pseudorandom signal perturbation are shown in fig. 7.4. This output signal is not antisymmetric thus indicating the presence of even-order nonlinearities, and the presence of noise in the system is shown by dissimilarities between successive periodic output values.

7.4 Determination of the First Order Kernel

The pH measurements shown in fig. 7.4 are in discrete form so the first order kernel or weighting function is obtained in the form of samples at 6 seconds interval by one-dimensional crosscorrelation. Since all the odd-order autocorrelation functions of a ternary sequence are zero, the effects of all even-order nonlinearities are absent from the crosscorrelation function. The errors due to constant, linear, quadratic and cubic drifts are removed or minimised by commencing the correlation at the reference phase of the input sequence and by using drift-correction techniques.[41,42] If $\{x_i\}$ is the ternary sequence of the test signal reference phase samples, a weighted reference phase sequence $\{r_i\}$ of twice the period of $\{x_i\}$ may be obtained by the following relation:

$$r_i = \begin{cases} x_i \left[0.3584 + \dfrac{i}{728} \right] & i = 0,1,2,\ldots,727 \\ x_{i-728} \left[1.6416 - \dfrac{i-728}{728} \right] & i = 728,729,\ldots,1455 \end{cases} \qquad 7.2$$

The weighting function estimates $\{W_j\}$ are then obtained from eqns. 2.15 and 6.20 as

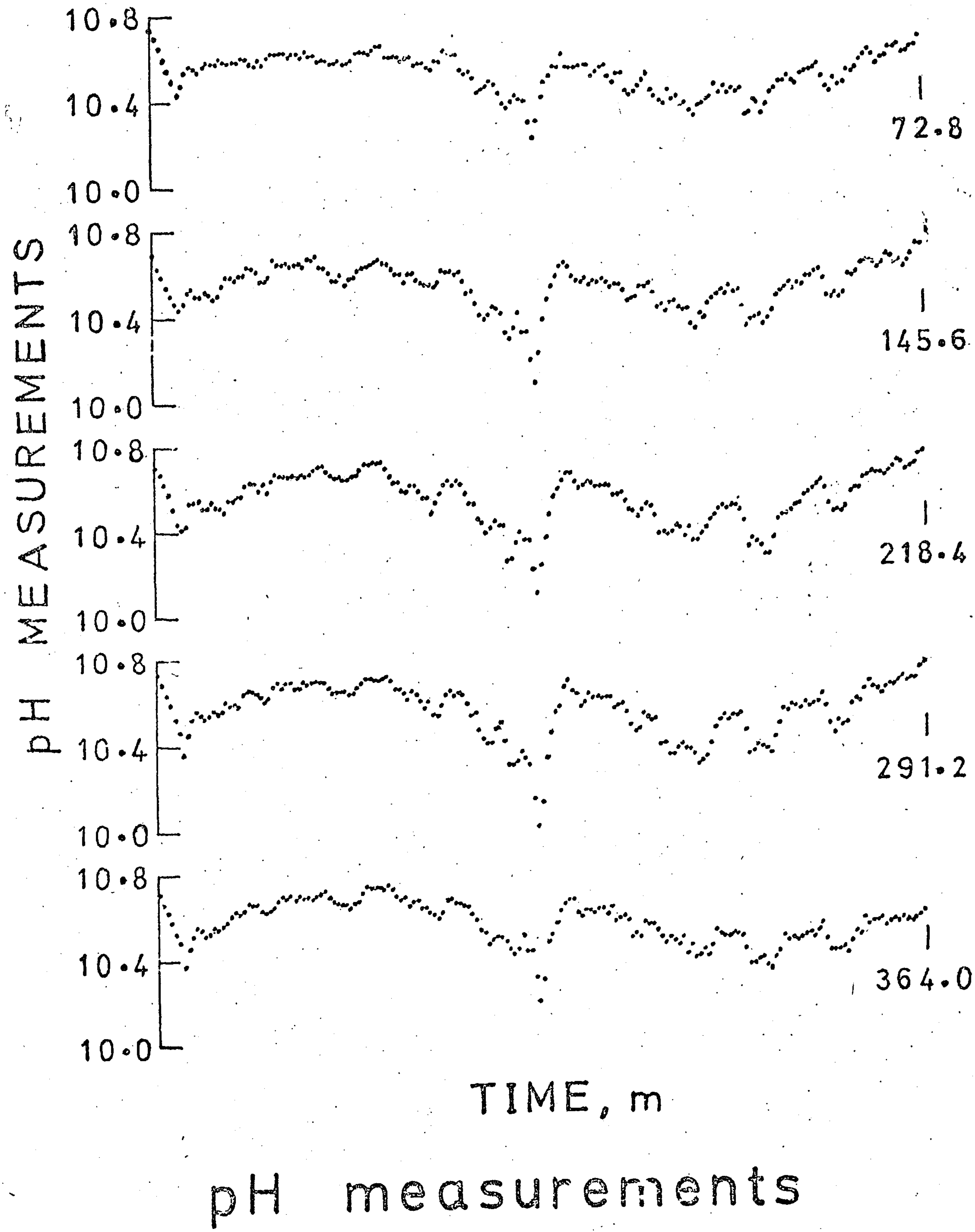

10·8
10·4
10·0
72·8
145·6
218·4
291·2
364·0
pH MEASUREMENTS
TIME, m
pH measurements

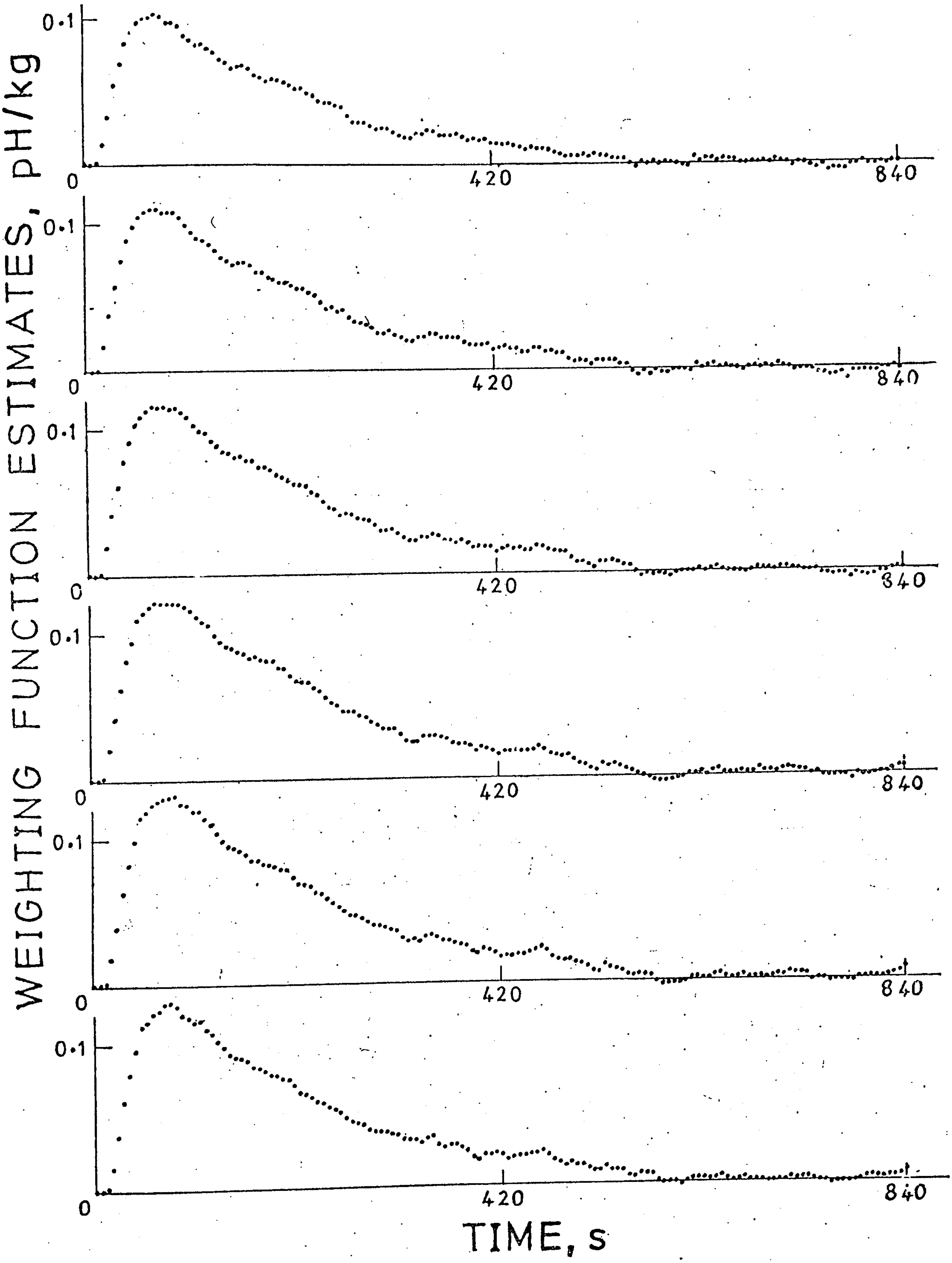
FIG. 7.4

$$W_j = \frac{3}{4 \cdot 143^2 \cdot \frac{729}{600}} \sum_{i=0}^{1455} r_i y_{i+j} \qquad 7.3$$

$$j = 0,1,2,\ldots,2184$$

where $\{y_i\}$ is the sequence of input samples. A computer program to evaluate $\{W_j\}$ from eqns. 7.3 and 7.2 is given in Appendix 1.12. Since there are five cycles of pH measurements, performing the correlation over two cycles results in three periods of crosscorrelation function but because of the antisymmetric property of the input pseudorandom ternary sequence, these three periods in fact yield six weighting function estimates, with every alternate function reversed in sign. The six weighting function estimates are shown in fig. 7.5. Although these functions extend over the half period of the input ternary signal, which is 363 x 6 seconds, they have been truncated at 140 x 6 seconds; the remaining values of the estimates are not significant because the system settling time is evidently less than the assessed value. The weighting function estimates have a substantial common form of structure which may be envisaged as a small variation about a smooth curve. This variation is due to the effects of residual odd-order nonlinearities.

To proceed further with the analysis, it is necessary to postulate a specific structure for the system. If it is assumed that the greater part of the nonlinear effects are due to low order terms, then the plant dynamics may be represented by the first three terms of the Volterra series. Since the most significant nonlinear effects occur as a result of the instantaneous nonlinear relationship between



Weighting function estimates

FIG. 7.5

pH and ion concentration which follows the reaction dynamics, a possible model for the system is shown in fig. 7.6. It is assumed that the second and third order kernels are given by

$$w_2(\tau_1, \tau_2) = k_2 w_1(\tau_1) w_1(\tau_2)$$
$$w_3(\tau_1, \tau_2, \tau_3) = k_3 w_1(\tau_1) w_1(\tau_2) w_1(\tau_3) \qquad 7.4$$

An expression for the first order kernel which might be reasonably expected for a system of this nature and which is generally in conformity with the graphs of fig. 7.5 is given by

$$w_1(\tau_1) = K_1 U(\tau_1 - T_D)\left(e^{-\tau_1/T_1} - e^{-\tau_1/T_2}\right) \qquad 7.5$$

where: τ_1 is time, s

$w_1(\tau_1)$ is the proposed weighting function, pH/kg

$U(\tau_1)$ is the unit step function

K_1 is a constant, pH/kg

T_D is a time delay, s

T_1, T_2 are time constants, s, with $T_1 > T_2$.

The part of the system output which contributes to the weighting function estimates is given by

$$Z(t) = \frac{1}{3600}\int_{-\infty}^{\infty} w_1(\tau_1)x(t-\tau_1)d\tau_1 + K_3\left[\frac{1}{3600}\int_{-\infty}^{\infty} w_1(\tau_1)x(t-\tau_1)d\tau_1\right]^3 \qquad 7.6$$

where τ_1 is time, s

$Z(t)$ is the postulated output, pH

$x(t)$ is the ternary test signal, kg/hr

K_3 is a constant, pH^{-2}

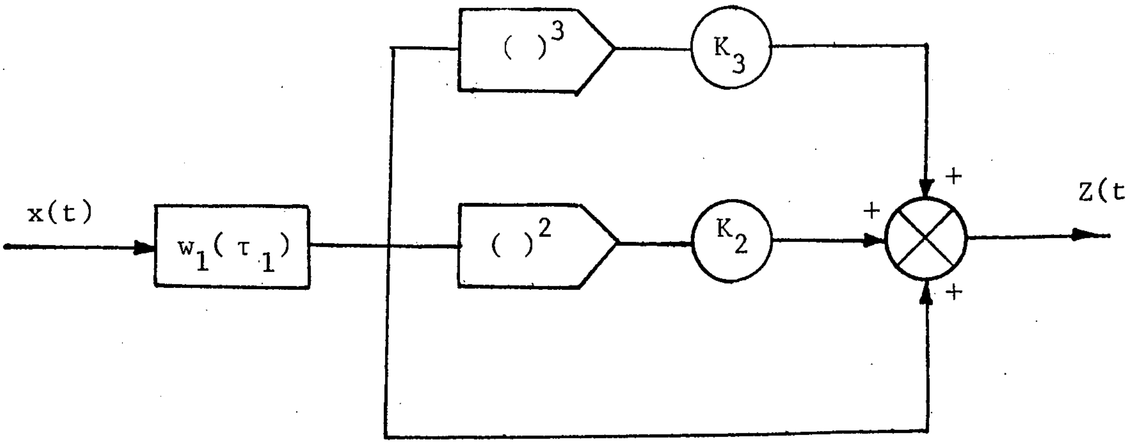


Fig. 7.6. Structure of postulated nonlinear model.

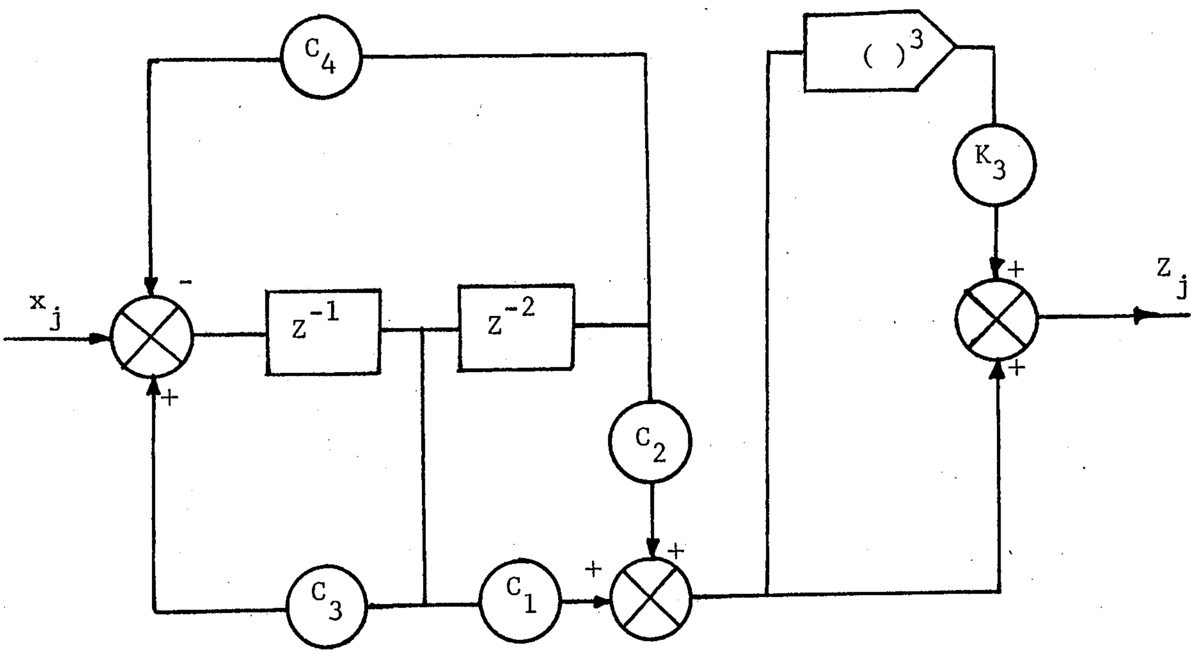


Fig. 7.7. Simulation diagram for the postulated nonlinear system with the quadratic term omitted.

The quadratic term is omitted from eqn. 7.6 because the third order autocorrelation function of the input ternary signal is zero.

Since the plant output data is in sampled form, the postulated output must, for meaningful comparisons, also be available at the same sampling interval. The sequence of postulated output samples $\{ z_i \}$ may be obtained by injecting the pseudorandom ternary sequence, which is generated in a digital computer, into the simulated system model. Only that part of the nonlinear model which contributes to the weighting function estimates need be considered. The Z transform of the linear dynamics and the zero-order-hold is given by

$$w_1(z) = \frac{z-1}{z} \mathcal{Z} \frac{w_1(s)}{s}$$

$$= \frac{z-1}{z} \mathcal{Z} \frac{K_1(\frac{1}{T_2} - \frac{1}{T_1})}{s(s+\frac{1}{T_1})(s+\frac{1}{T_2})}$$

$$= \frac{(z-1)}{z} K(\frac{1}{T_2} - \frac{1}{T_1}) \left[\frac{zT_1T_2}{z-1} + \frac{zT_2}{(\frac{1}{T_2} - \frac{1}{T_1})(z-e^{-T/T_2})} - \frac{zT_1}{(\frac{1}{T_2} - \frac{1}{T_1})(z-e^{-T/T_1})} \right]$$

$$= \frac{c_1z^{-1}+c_2z^{-2}}{1-c_3z^{-1}+c_4z^{-2}} \qquad 7.7$$

where T = sampling period

$$c_1 = K_1\left[T_1(1-e^{-T/T_1}) - T_2(1-e^{-T/T_2})\right]$$

$$c_2 = K_1\left[T_2 e^{-T/T_1}(1-e^{-T/T_2}) - T_1 e^{-T/T_2}(1-e^{-T/T_1})\right]$$

$$c_3 = e^{-T/T_1} + e^{-T/T_2}$$

$$c_4 = e^{-T\left(\frac{1}{T_1} + \frac{1}{T_2}\right)}$$

A simulation diagram for calculating the output samples which contribute to the weighting function estimates is therefore as shown in fig. 7.7.

The weighted crosscorrelation $\{w_j\}$ between the postulated output $\{z_i\}$ and the test sequence is given by

$$w_j = \frac{3}{4 \cdot 143^2 \cdot \frac{729}{600}} \sum_{i=0}^{1455} r_i Z_{i+j} \qquad 7.8$$

$$j = 0,1,2,\ldots,140$$

The optimum values of the parameters K_1, T_D, T_1, T_2 and K_3 are obtained by two curve-fitting programs which match the crosscorrelation $\{w_j\}$ to each weighting function estimate $\{W_j\}$ until the percentage error

$$100\left[\frac{\sum_{j=0}^{140}(W_j-w_j)^2}{\sum_{j=0}^{140} W_j^2}\right]^{\frac{1}{2}}$$

becomes a minimum. The program of Appendix 1.13 optimises each parameter in an iterative fashion until it is no longer possible to improve the result by varying any of the parameters. More accurate values of K_1, T_1, T_2 and K_3 are obtained by the program in Appendix 1.14, which calculates the squared error for each of the 81 possible combinations of T_1, $T_1 \pm \Delta T_1$, T_2, $T_2 \pm \Delta T_2$, K_1, $K_1 \pm \Delta K_1$, K_3, $K_3 \pm \Delta K_3$; the set that gives minimum error is used for the next iteration, and the iterations are terminated when an absolute minimum error is achieved.

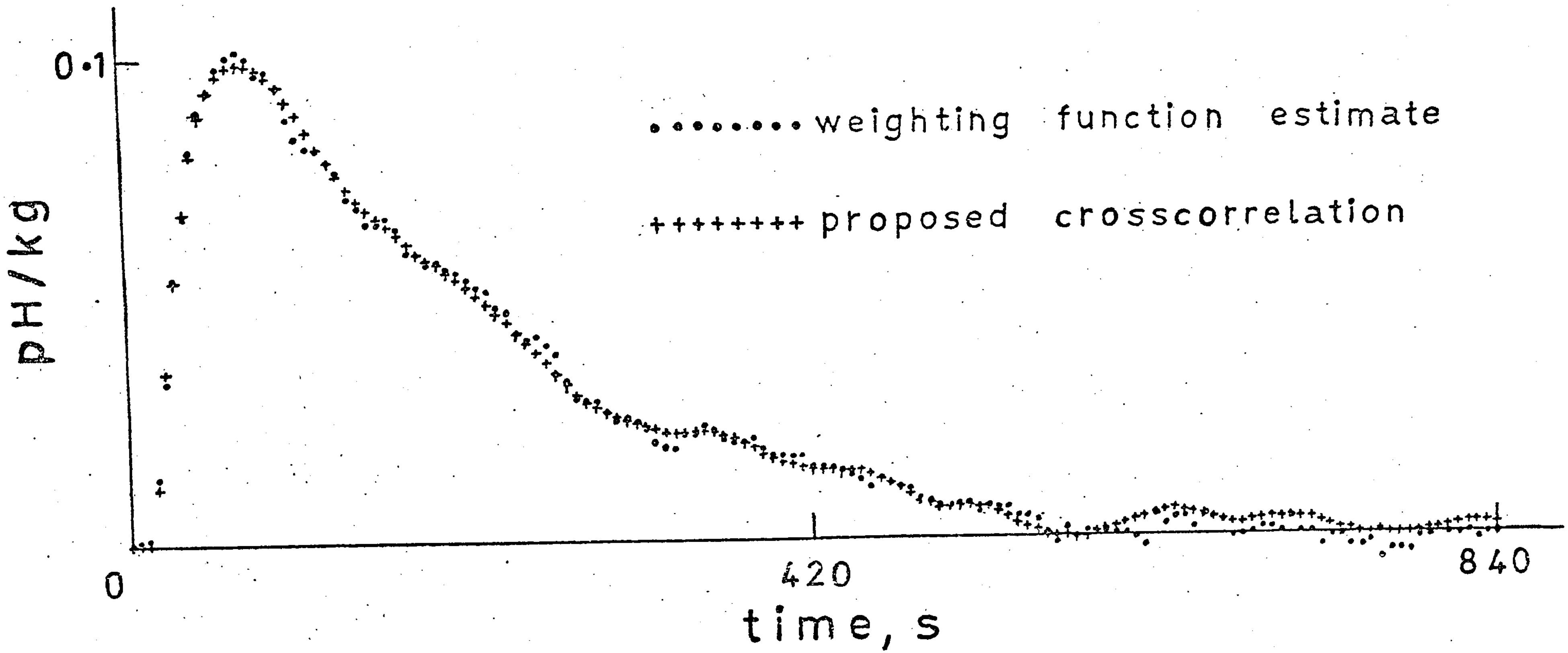
The optimum values of K_1, T_D, T_1, T_2 and K_3 are given in table 7.1. A comparison shown in fig. 7.8 illustrates how the characteristic structure of the weighting function estimates is largely accounted for by the proposed form of linear dynamics plus the cubic nonlinearity, with appropriate choice of parameters. The linear system component or the first order kernel and the error due to cubic nonlinearity are shown seperately in fig. 7.9. Also shown in the same figure is a small residual error, the presence of which could not be explained. A more sophisticated nonlinear model might possibly account for most of this residual error. The error due to cubic nonlinearity is made up of systematic and unsystematic components and these are shown on the enlarged graphs in fig. 7.10.

7.5 Determination of the Second Order Kernel

The second order Volterra kernels may be determined by crosscorrelating the pH measurements with two-dimensional product formed from the input sequence. In this operation, the linear and cubic terms of the Volterra series are respectively multiplied by the third and fifth order autocorrelation functions of the input pseudorandom ternary sequence and since these correlation moments are zero, the two-dimensional crosscorrelation involves the 2nd term alone. An alternative approach which is adopted here, involves the extraction

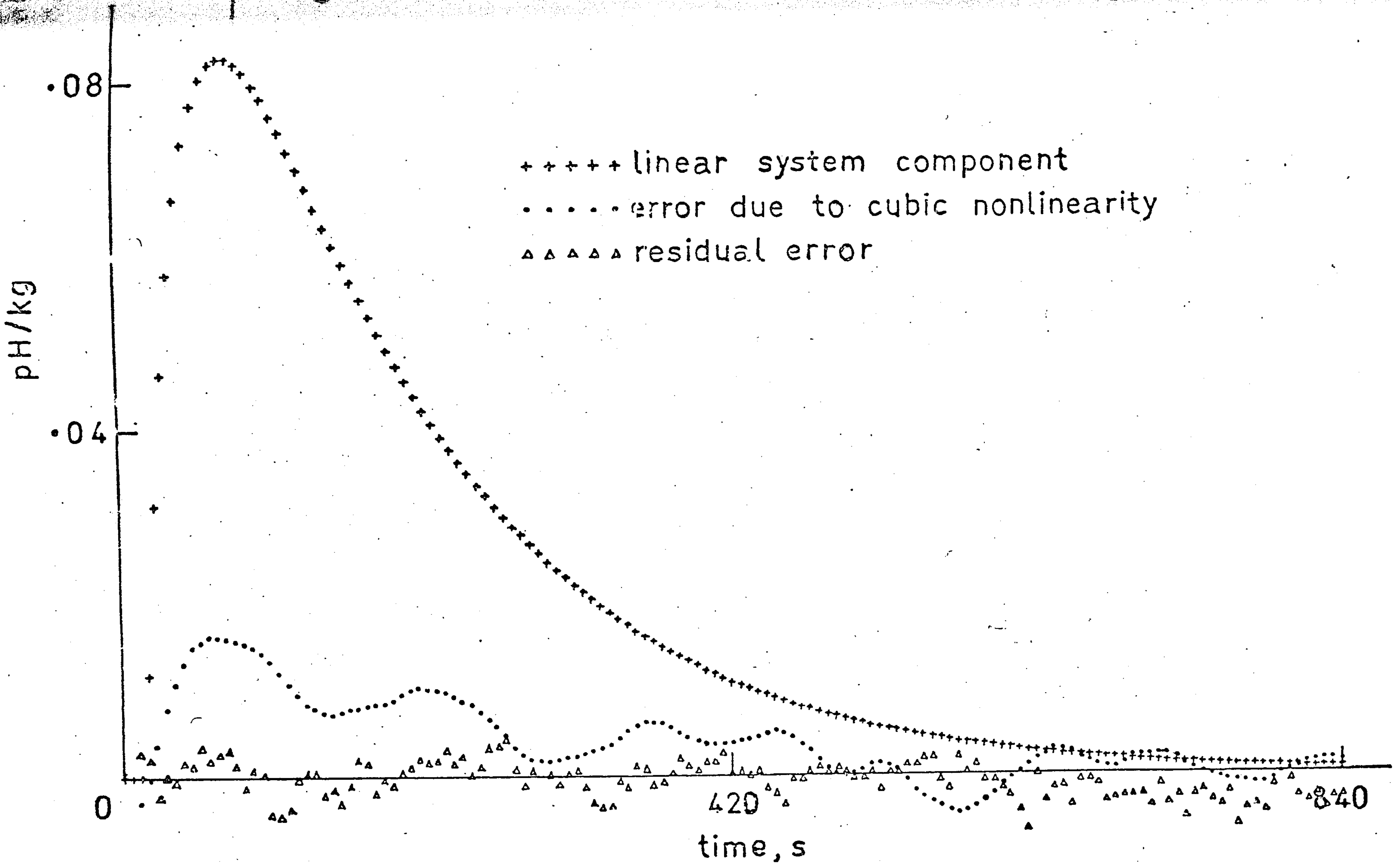
Estimate	Linear dynamic parameters				Quadratic	Cubic	% Errors
	K_1	T_D	T_1	T_2	K_2	K_3	
1	0.151	12	154	29.2	-3.49	11.2	5.4
2	0.174		146	30.6		10.3	6.3
3	0.194		150	32.0		6.3	5.5
4	0.207		149	34.2		7.4	5.0
5	0.208		155	32.6		5.8	4.7
6	0.194		166	30.2		4.4	4.8

Table 7.1   Optimum Parameter Values



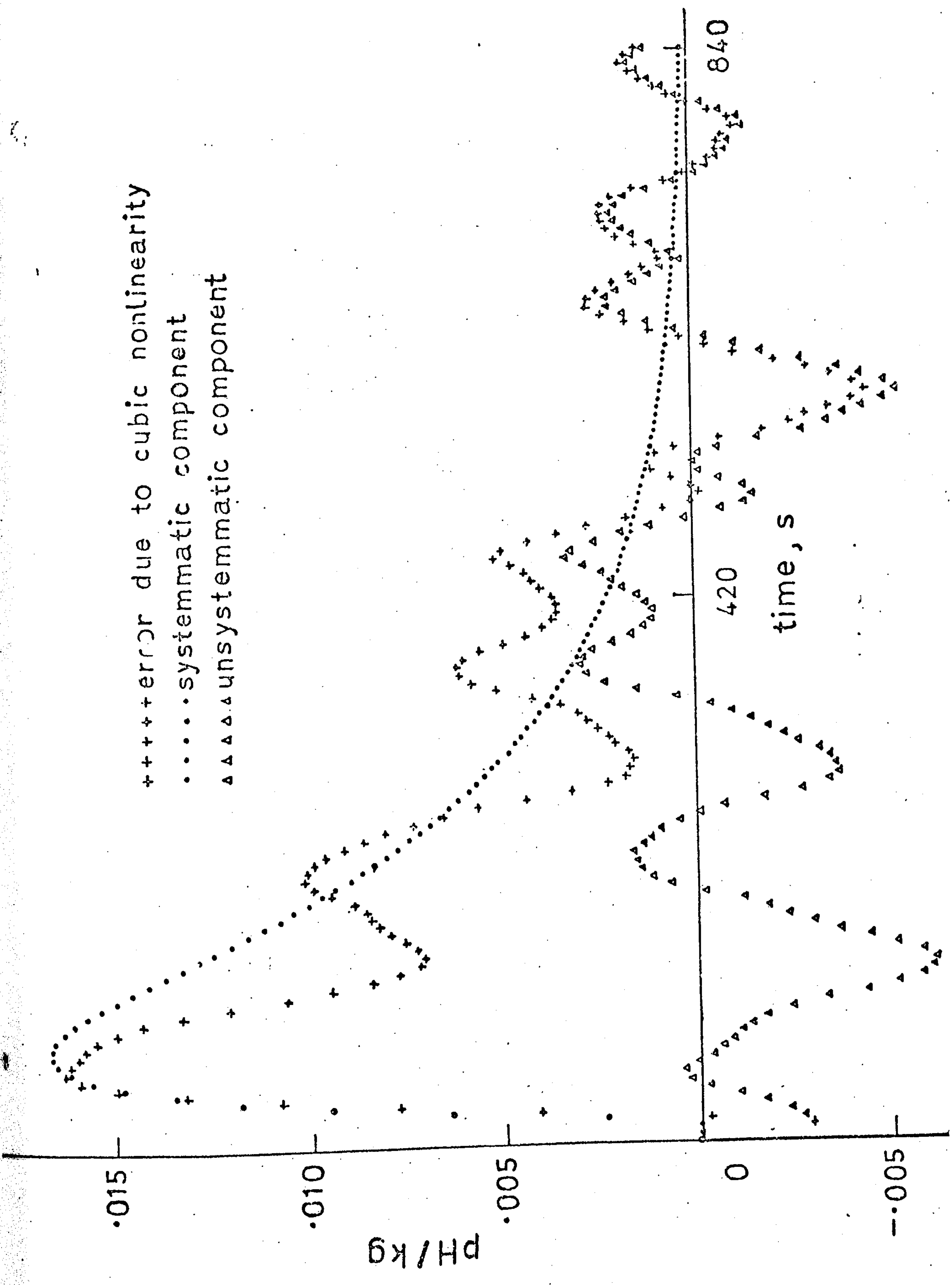
Comparison between weighting function estimate and proposed crosscorrelation

FIG. 7.8



Components of weighting function estimate

FIG. 7.9



Error due to cubic nonlinearity and its components

FIG. 7.10

of even order components from the pH measurements before the two-dimensional crosscorrelation. This approach has two main advantages. Since only a half cycle of even order data is required for determining the second order kernels, the five cycles of pH measurements may be reduced to a half cycle of even order terms by a procedure that eliminates most of the drift present in the original data. Furthermore, by comparing this even-order data with the quadratic output from the plant model (fig. 7.6), the validity of the assumption that the plant dynamics can be represented by the first three Volterra kernels can be ascertained.

From the five periods of pH measurements $\{y_i\}$, four and a half cycles of even-order data $\{f_i\}$ were obtained from the relationship:

$$f_i = \tfrac{1}{2}\left[y_i + y_{i+364}\right] \qquad 7.9$$

$$i = 1,2,3,\ldots,3276$$

No odd-order terms are present in the output values $\{f_i\}$ and, if there were no drift and other errors, the nine half cycles which make up $\{f_i\}$ would have been identical. Only four cycles of data were needed in the subsequent analysis so the last four cycles of $\{f_i\}$ were used since they contained less unwanted components than the first four cycles. The four periods of even order data were reduced, in three steps, to a drift free half period by using the weighting

$$g_i = \frac{i}{364J}\, h_i + \left(1 - \frac{i}{364J}\right) y_{i+364J} \qquad 7.10$$

where $J = 4,2,1$

$i = 1,2,3,\ldots,364J$

$$\text{and } h_i = \begin{cases} f_{i+364} & \text{when } J=4 \\ \text{previous values of } g_i & \text{when } J=2 \end{cases}$$

If the assumption that the second order Volterra kernel is the only significant even order term is correct, then apart from a constant bias, the even-order output data $\{g_i\}$ should be nearly the same as the quadratic output of fig. 7.6. By means of the curve fitting program of Appendix 1.13, these two output data were found to have the best fit when $K_2 = -3.49$. A comparison shown in fig. 7.11 illustrates how the quadratic component of the postulated nonlinear model nearly matches the even-order components of the pH measurements. The difference between the two, which is quite small, is possibly due to a quartic nonlinearity.

From equations 2.22 and 3.4, the estimates e(JT,KT) of the second order kernel $w_2(JT,KT)$ are obtained from the half cycle of even-order output sequences $\{g_i\}$ by two-dimensional crosscorrelation as

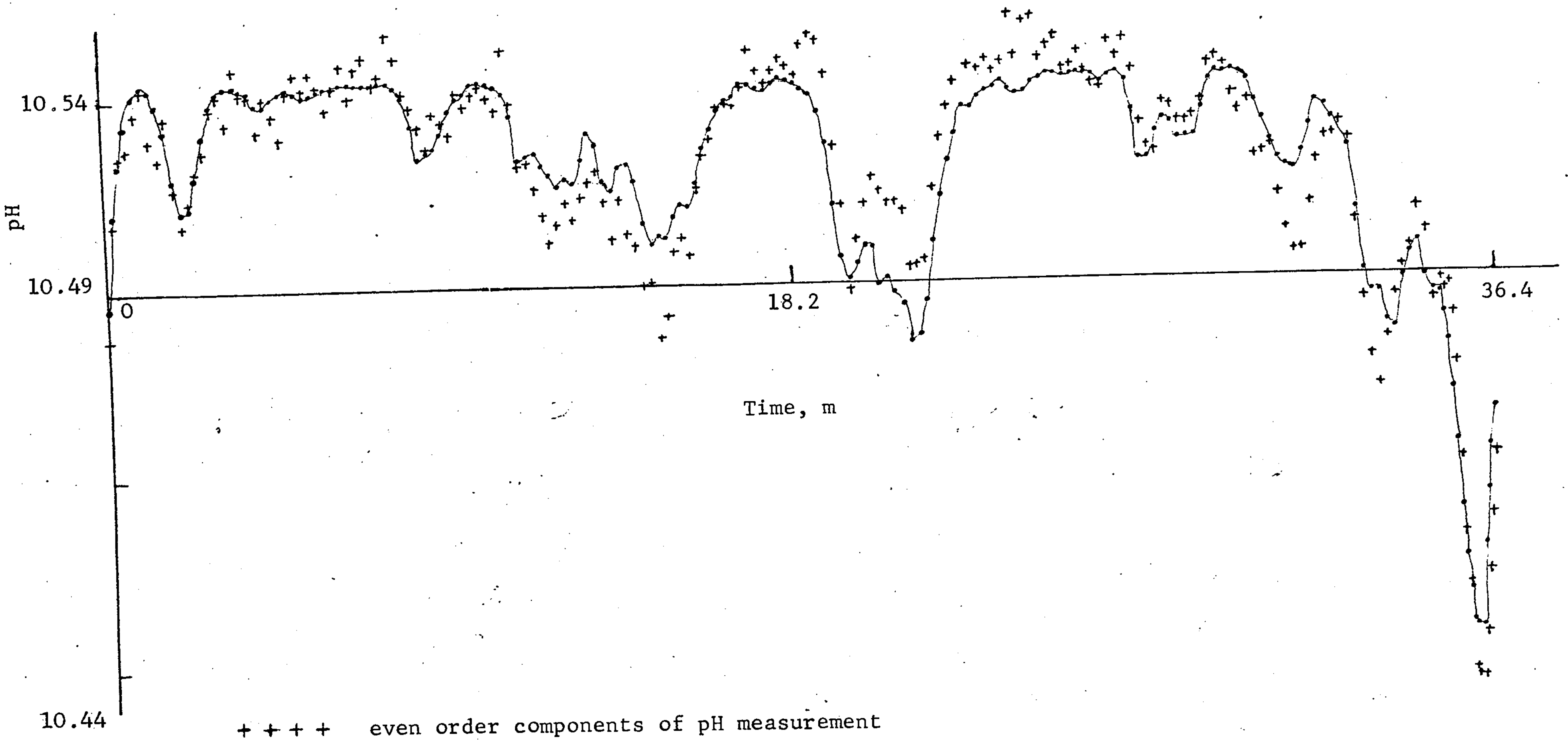
$$e(JT,KT) = \frac{2}{8x3^4T^2X^4(1)} \sum_{i=0}^{363} g_i x_{i-J} x_{i-K} \qquad 7.11$$

These estimates, which were calculated using the computer program in Appendix 1.15, are plotted in dotted lines in Fig. 7.12 for J=0,1,2,...,60, and K=J+C where C=1,2,...,16. The second order kernels of the postulated nonlinear model also shown in fig. 7.12 in continuous lines, are obtained from equations 7.4 and 7.5 as

$$w_2(JT, \langle J+C\rangle T) = K_2 w_1(JT) w_1(\langle J+C\rangle T) \qquad 7.12$$

where

$$w_1(JT) = K_1T_1(1-e^{-T/T_1})e^{-(J-1)T/T_1} - K_1T_2(1-e^{-T/T_2})e^{-(J-1)T/T_2}$$



+ + + + even order components of pH measurement

•••••••• quadratic output component of the postulated nonlinear model.

Fig. 7.11. Comparison between even order components of pH measurement and quadratic output component of the postulated nonlinear model

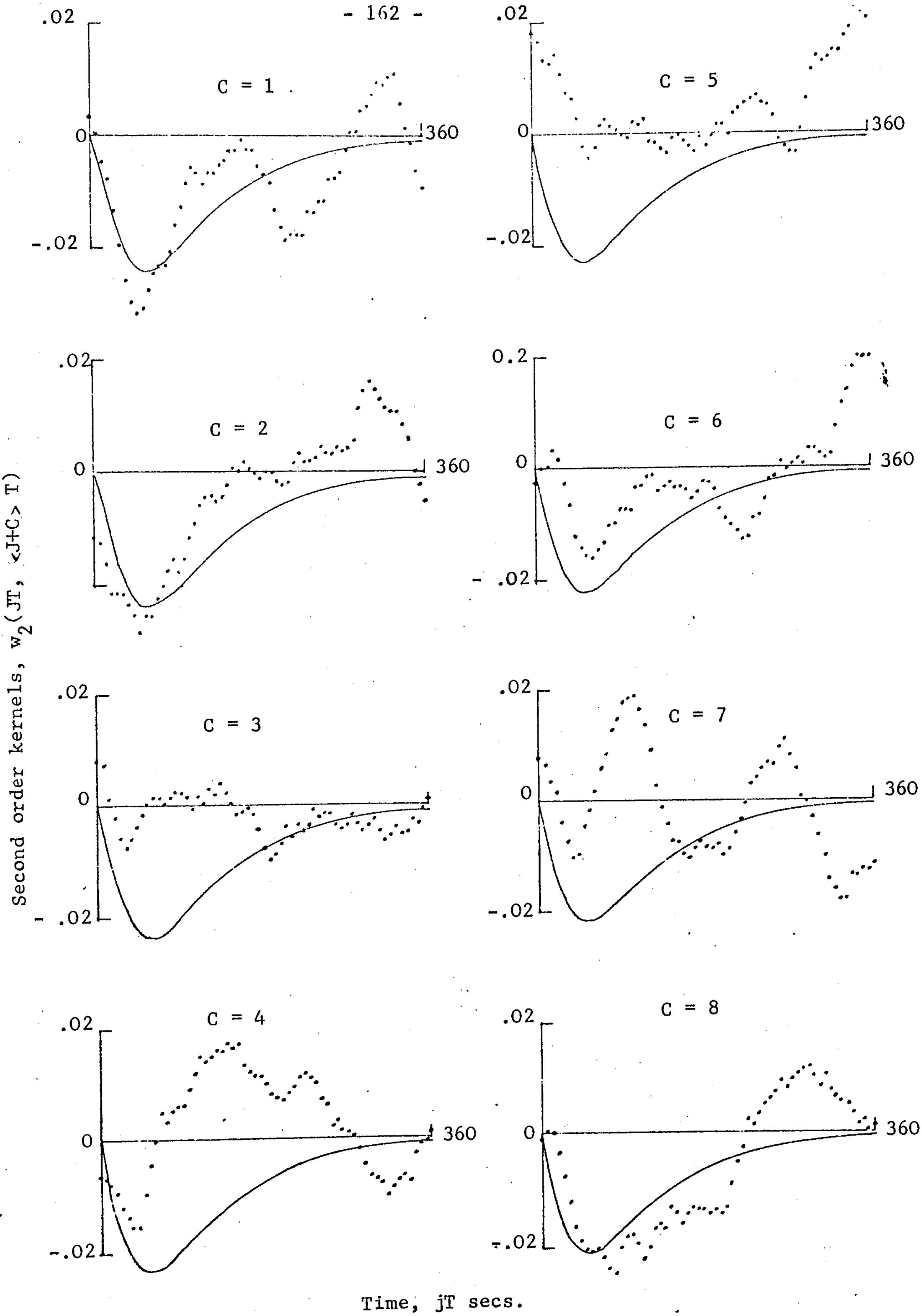


Fig. 7.12(a) Second order kernels of ammonium nitrate synthesising process.

.... kernels obtained by two-dimensional crosscorrelation on the plant data.

_______ True kernels of the postulated model.

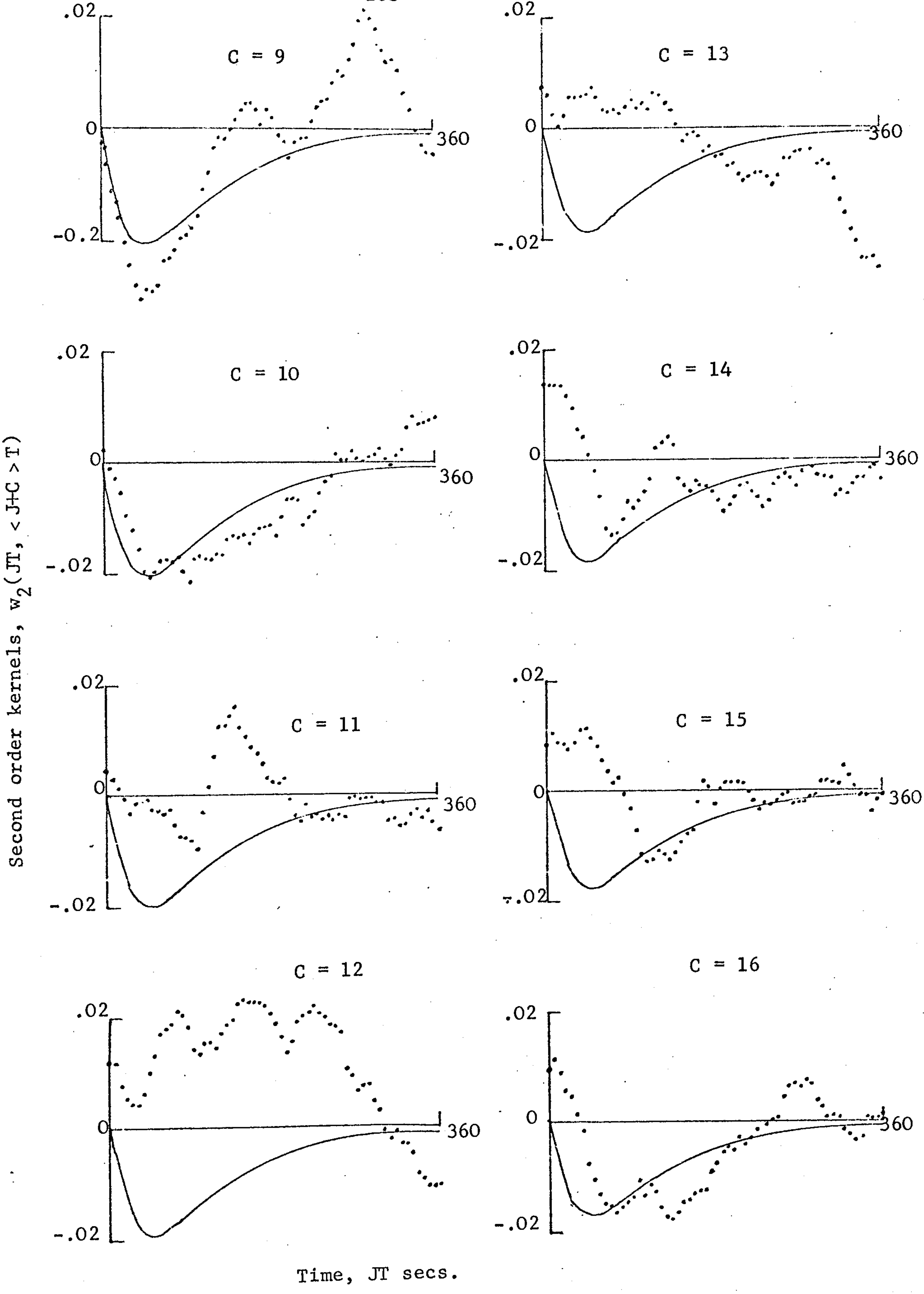


Fig. 7.12(b) Second order kernels of ammonium nitrate synthesising process.

.... Kernels obtained by two-dimensional crosscorrelation on the plant data.

——— True kernels of the postulated model.

$$w_1(\langle J+C\rangle T) = K_1T_1(1-e^{-T/T_1})e^{-(J+C-1)T/T_1}$$

$$-K_1T_2(1-e^{-T/T_2})e^{-(J+C-1)T/T_2}$$

There are considerable discrepancies between the second order kernel estimates of the ammonium nitrate plant obtained by two-dimensional crosscorrelation and the corresponding kernels of the postulated model calculated from equation 7.12. These differences are mainly due to the undesirable nonzero values in the fourth order autocorrelation functions of the input ternary sequence. This fact is supported by the curves in fig. 7.13 which are enlargements of the graphs of fig. 7.12 for C=2 and 4, except that the two-dimensional crosscorrelation function between the input sequence and the postulated second order output is also plotted. This function is nearly the same as the corresponding result obtained using the even-order plant data. The difference between these and the true kernels of the postulated model given by eqn. 7.12 may be accounted for by the undesirable nonzero values in the fourth order autocorrelation functions of the input sequence which can be determined by polynomial division.

Since the performance indices R_1 and R_m of the input pseudorandom ternary signal are only 8 and 16 respectively, these nonzero values are quite numerous within the settling time of the second order kernel, for instance, when C=2, the major nonzero values are given by the following relationships:

$$S_i - S_{i-2} - S_{i-17} = 0$$

$$S_i + S_{i-2} + S_{i-4} - S_{i-34} = 0$$

$$S_i - S_{i-2} - S_{i-8} + S_{i-31} = 0$$

$$S_i + S_{i-2} - S_{i-4} - S_{i-39} = 0$$

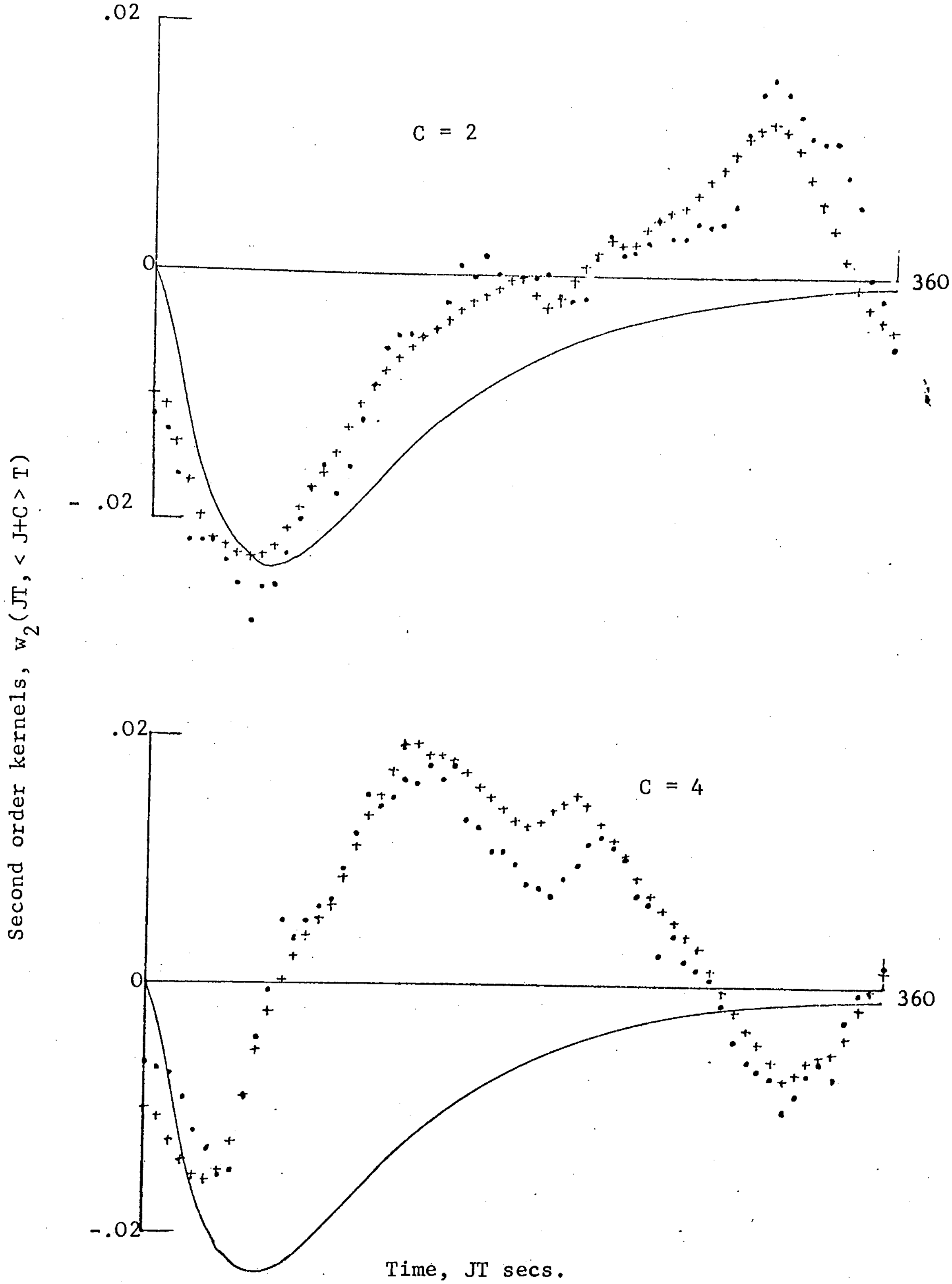


Fig. 7.13. Enlarged second order kernels.

.... Kernels obtained by two-dimensional crosscorrelation on the plant data.

+ + + + Kernels obtained by two-dimensional crosscorrelation on the postulated model.

———— True kernels of the postulated model.

$$S_i + S_{i-2} - S_{i-14} - S_{i-18} = 0$$

$$S_i + S_{i-2} - S_{i-15} + S_{i-37} = 0$$

$$S_i - S_{i-2} - S_{i-15} + S_{i-32} = 0$$

$$S_i + S_{i-2} + S_{i-16} - S_{i-48} = 0$$

$$S_i - S_{i-2} - S_{i-16} - S_{i-59} = 0$$

$$S_i - S_{i-2} - S_{i-19} - S_{i-34} = 0$$

$$S_i - S_{i-2} - S_{i-18} + S_{i-60} = 0$$

$$S_i - S_{i-2} - S_{i-19} - S_{i-34} = 0$$

$$S_i + S_{i-2} - S_{i-23} - S_{i-57} = 0$$

$$S_i - S_{i-2} - S_{i-26} - S_{i-40} = 0$$

$$S_i + S_{i-2} - S_{i-27} + S_{i-40} = 0$$

$$S_i + S_{i-2} + S_{i-31} + S_{i-53} = 0$$

$$S_i + S_{i-2} + S_{i-34} + S_{i-39} = 0$$

$$S_i - S_{i-2} + S_{i-36} - S_{i-56} = 0$$

$$S_i - S_{i-2} + S_{i-43} + S_{i-44} = 0$$

$$S_i - S_{i-2} + S_{i-47} - S_{i-52} = 0$$

$$S_i + S_{i-2} - S_{i-52} + S_{i+4} = 0$$

$$S_i - S_{i-2} + S_{i-3} - S_{i+6} = 0$$

$$S_i - S_{i-2} + S_{i-18} + S_{i+9} = 0$$

$$S_i - S_{i-2} + S_{i-16} + S_{i+10} = 0$$

$$S_i - S_{i-2} - S_{i-22} + S_{i+13} = 0$$

$$S_i + S_{i-2} - S_{i-22} + S_{i+15} = 0$$

$$S_i - S_{i-2} - S_{i-12} - S_{i+18} = 0$$

$$S_i - S_{i-2} - S_{i+3} - S_{i+22} = 0$$

It is therefore not surprising that some of the experimental second order kernels bear little resemblance to the true values. A far better result could have been obtained if a signal with superior performance indices had been used to perturb the ammonium flowrate.

7.6 Conclusions

Estimates of the linear dynamics of a neutralisation process in which the output contains significant contributions from both nonlinearities and noise, have been obtained by the method described. The effects of even order nonlinearities and the principal components of low frequency drift were removed by the use of a pseudorandom test signal based on a ternary m sequence, together with the use of a weighted crosscorrelation based on the reference phase of the test signal. The resulting estimates nevertheless contained structural deviations, to account for which it was postulated that the estimates were due to a particular form of linear dynamics together with a cubic nonlinearity. By appropriate choice of parameters, the postulated crosscorrelations were matched to the weighting function estimates to a reasonable degree of accuracy.

The form of linear dynamics which largely accounted for the behaviour of the system consisted of a delay of 12 seconds and two cascaded lags with time constants of about 30 seconds and 150 seconds. Both the delay, which may be attributed to the transport time of liquor and vapour in pipes, and the shorter time constant, which may be attributed to a lag in pH measurement due to the method of dissolving the neutraliser vent gases in water, have values of the order expected. The longer time constant however is about twenty times less than that which would be expected for the dynamic response of a neutraliser with 8600 kg capacity at a production rate of 10100 kg/hr. This implies that the response of vapour pH in the neutraliser to changes in ammonia flowrate is very much faster than the corresponding response of liquor pH, an unexpected result which might be used to advantage in improving the control of the system, and possibly that of similar systems.

The second order output components of the proposed model were shown to be nearly the same as the even-order components of the plant output data from which the second order kernels of the system were computed by two-dimensional crosscorrelation. The crosscorrelation functions were however at variance with the true second order kernels due to the numerous undesirable nonzero values in the fourth order autocorrelation functions of the input signal. These results re-enforce the point already made concerning the need for a judicious choice of signal in nonlinear system identification.

CHAPTER 8

CONCLUSIONS

CHAPTER 8

CONCLUSIONS

New and improved techniques have been developed for the identification of nonlinear systems which can be represented by Volterra series, using as the input pseudorandom signals based on m sequences. Exact analytical expressions relating the Volterra kernels to the crosscorrelation measurements have been obtained.

The indices of performance for antisymmetric pseudorandom signals in the measurement of second order kernels of single-input, single-output nonlinear systems are the upper limits R_r of J and K for which the set of all off-diagonal values $\omega_2(JT, KT)$ of a second order kernel may be obtained in subsets of not more than r members from corresponding subsets in the set of off-diagonal measurements e(JT, KT). The set of diagonal values $\omega_2(JT, JT)$ is then obtained from the corresponding set of diagonal measurements e(JT, JT) for all $0 \leq J \leq R_r$.

There are wide variations in performance of antisymmetric pseudorandom signals, even between signals with the same number of levels and with characteristic polynomials of the same order. It is therefore essential that the most suitable signals be chosen for the second order kernel measurements. To facilitate this choice, the performance indices of all the 336 antisymmetric binary signals of order 2 to 11, all the 560 ternary signals of order 2 to 8 and all the 352 quinary signals of order 2 to 5 have been calculated, and the signals with the greatest performance indices R_r for values of r from 1 to 20 have been tabulated. The results obtained indicate that the pseudorandom ternary signals surpass all the others in this application.

It has been shown that for every even order pseudorandom ternary sequence with characteristic polynomial f(D), there is a related sequence with primitive polynomial f(-D). These related polynomials have been used to improve the performance of pseudorandom ternary signals in the measurement of second order Volterra kernels by partly overcoming the limitations imposed by the undesirable nonzero values in the fourth order autocorrelation functions of ternary signals.

One of the properties of the sequences derived from f(D) and f(-D) is that their fourth order autocorrelation functions are always equal in magnitude but they are opposite in sign when the sum of the arguments is odd. Thus if estimates of the values of the second order kernel of a nonlinear system are obtained by crosscorrelation experiments in which related ternary signals are used as the system input signal, then addition of these estimates results in the cancellation of the effects of those undesirable values of the fourth order autocorrelation functions for which the sum of the arguments is odd. Although the upper limits of the indices of performance for the combined cross-correlation method are the same as those for the direct crosscorrelation method, significant improvements in performance are obtained within these limits.

The performance indices of all the 376 ternary and related ternary signals of order 4, 6 and 8 have been calculated and the signals with the best performance tabulated. The superiority of the combined crosscorrelation method over the direct method has been demonstrated by two examples.

The identification of two-input, single-output systems, hitherto confined to linear systems, has been extended to nonlinear systems. It has been shown that the characteristics of a two-input system with multiplicative nonlinearity may be determined by crosscorrelation experiments in which the two input signals have different phases but are derived from the same characteristic polynomial, and that each crosscorrelation measurement yields the corresponding kernel directly if certain requirements are met. The use of related pseudorandom ternary signals has also been considered, and the most suitable signals for the combined or direct crosscorrelation method have been identified.

The effects of nonlinearities on system weighting function estimates obtained by crosscorrelation have been investigated, and explicit results have been given for systems with second and third order nonlinearities which are tested by pseudorandom binary, antisymmetric pseudorandom binary and ternary signals.

For pseudorandom binary signals, the error due to second order nonlinearities consists of a small constant bias together with an error which is unsystematic in the sense that it depends on relationships between members of the m sequence from which the signal is derived and therefore is neither smoothly varying nor the same for any other signal. By a judicious choice of signal, this unsystematic error may be removed from a range of interest which extends over the system settling time, an unexpected result which could be useful in certain applications.

The error due to third order nonlinearities for all the three groups of signals consists of a small constant or oscillatory term, an unsystematic error which may be removed from the region of interest by choosing a suitable test signal, and a systematic error which cannot be removed before crosscorrelation.

The conditions under which all unsystematic errors may be removed have been given, and the signals with the greatest rejection of these errors have been isolated. The significance of the results obtained may be seen from the few illustrative examples given, which include accurate identification of a system whose dynamics is dependent on the direction of the input signal, and the explanation of the nonlinear effects observed in gas chromatography experiments.

The first and second order kernels of a neutralisation process which was perturbed by a pseudorandom ternary signal have been obtained by the correlation methods described. The one-dimensional cross-correlation function contained errors due mainly to systematic and unsystematic third order nonlinearities, and the two-dimensional crosscorrelation functions were imperfect chiefly because of the undesirable nonzero values in the fourth order autocorrelation functions of the input ternary signal. Removal of these errors resulted in accurate values of the kernels of the chemical process.

The applications of the results presented in this thesis include the following areas: accurate identification of the second order Volterra kernels of single-input or two-input nonlinear systems by direct or combined crosscorrelation method, designing correlation experiments so as to remove unsystematic errors from the weighting function estimates, interpretation of the weighting function estimates obtained from crosscorrelation experiments in which the effects of nonlinearities are present, and choosing the most suitable signals for correlation tests.

A further development of this project, which may result in a simplified method for kernel measurements, is the identification of higher order kernels by one-dimensional crosscorrelation. This possibility, which was first suggested by Kadri[29], is confirmed by the results of chapter six. Other developments of this project include the determination of the third and higher order Volterra kernels by direct and combined crosscorrelation, extension of the work on multivariable systems to cover more nonlinearities and a greater number of inputs and outputs, and a continuing search for deterministic signals whose higher order autocorrelation function properties are as close as possible to those of Gaussian white noise.

APPENDICES

COMPUTER PROGRAMS

```
C       PROGRAM FA
C       COMPUTATION OF R1 FOR BINARY SIGNALS
        INTEGER R,G,H,P
        DIMENSION P(2,12)
        ICO=0
        WRITE(2,4)
    4   FORMAT(//27H FOUR TERM INDEPENDENT AREA/
     1  4H  NO,3X,2HR1,2X,10HPOLYNOMIAL/
     2  4H ---,2X,3H---,2X,10H----------)
    6   READ(3,8) N2,(P(1,1),I=1,N2)
    8   FORMAT(I3,1X,12I1)
        N=N2-1
        NP=2**N-1
        DO 86 JJ=1,2
        GOTO(24,12),JJ
C       RECIPROCAL POLYNOMIAL
   12   DO 16 IR=1,N
        MAN=P(1,N2)
        IS=N2+1
   14   IS=IS-1
        P(1,IS)=P(1,IS-1)
        IF((IS-1).NE.IR) GOTO 14
   16   P(1,IS-1)=MAN
   24   R=NP
C       POLYNOMIAL DIVISION - 4 TERM RELATION
   32   G=1
   34   G=G+1
        IF(G.GT.(R-2)) GOTO 82
   36   H=G
   38   H=H+1
        IF(H.GT.(R-1)) GOTO 34
        INK=0
        DO 52 I=2,N2
   52   P(2,I)=0
        P(2,1)=1
        IF(G.LE.N2) P(2,G)=1
        IF(H.LE.N2) P(2,H)=1
   56   CONTINUE
        IF(P(2,1).NE.0) GOTO 64
        INK=INK+1
        DO 58 I=2,N2
   58   P(2,I-1)=P(2,I)
        P(2,N2)=0
        IF((G.EQ.(INK+N2)).OR.(H.EQ.(INK+N2))) P(2,N2)=1
        GOTO 56
   64   JZ=0
        DO 72 I=1,N2
        P(2,I)=P(2,I)-P(1,I)+2
   66   IF(P(2,I).LT.2) GOTO 68
        P(2,I)=P(2,I)-2
        GOTO 66
   68   IF(P(2,I).EQ.0) GOTO 72
        JZ=JZ+1
        KN=INK+I
   72   CONTINUE
        IF((JZ.EQ.0).OR.(KN.GT.R)) GOTO 38
        IF((JZ.GT.1).OR.(KN.LT.H)) GOTO 56
        R=KN
        GOTO 38
   82   ICO=ICO+1
        R=R-2
        WRITE(2,84) ICO,R,(P(1,I),I=1,N2)
   84   FORMAT(I4,I5,2X,12I1)
   86   CONTINUE
        GOTO 6
        STOP
```

APPENDIX 1.2

```
C       PROGRAMS FA31 & FA32
C       TERNARY SIGNAL : DIAGONAL LIMIT
        INTEGER F,G,P1,P2,Q
        DIMENSION MA(12)
        COMMON MA,N2,LS
        LS=3
        GOTO 52
        WRITE(2,4)
    4   FORMAT(////4H  NO,5X,1HF,5X,10HPOLYNOMIAL/
     1  4H ---,4X,3H---,4X,10H----------/)
    6   READ(3,8) ICOUNT,NUM,N2
    8   FORMAT(3I3)
   52   READ(3,53) N2,(MA(I),I=1,N2)
   53   FORMAT(I1,9I1)
        WRITE(2,54) (MA(I),I=1,N2)
   54   FORMAT(//////////5X,10HPOLYNOMIAL,2X,9I1)
        WRITE(2,55)
   55   FORMAT(/1X,3H1D1,3X,4HAD G,4X,4HBD J,3X,11H(N/2+1+G-J)/
     1  4H ---,3X,5H-----,3X,5H-----,2X,11H-----------/)
        N=N2-1
        NP=3**N-1
        GOTO 13
        DO 48 IS=1,NUM
        F=NP/2
        READ(3,12) (MA(I),I=1,N2)
   12   FORMAT(12I1)
   13   G=1
   14   G=G+1
        IF(G.GT.(NP/2)) GOTO 52
        CALL SUB3(1,G,P1,P2)
        GOTO 14
        F=MIN0(F,P1,P2,NP/2+1+G-P1,NP/2+1+G-P2)
        GOTO 14
   34   G=G-2
        IF(F.LT.G) F=G
   36   F=F-2
        ICOUNT=ICOUNT+1
   46   FORMAT(I4,I6,5X,12I1)
   48   WRITE(2,46) ICOUNT,F,(MA(I),I=1,N2)
        GOTO 6
        STOP
        END


        SUBROUTINE SUB3(KX,KY,KP1,KP2)
        DIMENSION MA(12),MB(4321),KN(2)
        COMMON MA,N2,LS
        N=N2-1
        LZ=LS-1
        NP=LS**N-1
        NFP=NP/LZ
        KB=1
        KA=0
        DO 46 ICYCLE=1,2
        KA=KA+1
        KM=NP
        DO 4 I=1,KM
    4   MB(I)=0
        MB(KX)=KA
        MB(KY)=KB
        KL=KX
        IF(KL.LT.KY) KL=KY
        KK=KX
        IF(KK.GT.KY) KK=KY
        KK=KK-1
    6   KK=KK+1
   12   IF(MB(KK).EQ.0) GOTO 6
        KM=N2+KK-1
        J=0
        L=0
        JL=MB(KK)
        DO 44 I=KK,KM
        L=L+1
        MB(I)=MB(I)-JL*MA(L)+LS*LS
   16   IF(MB(I).LT.LS) GOTO 25
        MB(I)=MB(I)-LS
        GOTO 16
   25   IF(MB(I).EQ.0) GOTO 44
        J=J+1
        KN(ICYCLE)=I
        KD=MB(I)
   44   CONTINUE
        IF(J.GT.1) GOTO 6
        IF(KN(ICYCLE).GT.NFP) KN(ICYCLE)=KN(ICYCLE)-NFP
        KN7=KN(ICYCLE)
        KAY=NP/2+1+KY-KN7
        WRITE(2,45) KB,KX,KA,KY,KD,KN7,KAY
   45   FORMAT(I2,1HD,I1,3H + ,I1,1HD,I3,3H = ,I1,1HD,I3,I8)
   46   CONTINUE
        WRITE(2,47)
   47   FORMAT( )
        KP1=KN(1)
        KP2=KN(2)
        RETURN
        END
```

APPENDIX 1.3

```
C       PROGRAM U1
C       M SEQUENCES - COMPUTATION OF PERFORMANCE INDICES
        INTEGER G,Z,W
        DIMENSION W(90,84),Z(3,112),MA(12),MB(149)
        COMMON W,Z,MA,MB
        LA8=90
        LA9=84
        LS=2
        LAD=LS-1
        LID=LAD*LAD
      4 READ(3,6) N2,L,(MA(IR),IR=1,N2)
      6 FORMAT(2I3,1X,12I1)
        NOM=1
        NO2=2*NOM
        NOP=NO2+2
        WRITE(2,8) (MA(IR),IR=1,N2)
      8 FORMAT(////5X,10HPOLYNOMIAL,3X,9I1)
        L=L+2
        DO 14 IS=1,NOP
        DO 14 IT=1,L
     14 W(IT,IS)=L
        DO 16 IT=1,3
        DO 16 IS=1,112
     16 Z(IT,IS)=L
        WRITE(2,17)
     17 FORMAT(/20X,13HSTORED VALUES/20X,13H-------------)
     18 G=1
     22 G=G+1
        DO 24 IS=1,NOP
     24 Z(1,IS)=L
        IKG=G-1
        WRITE(2,25) IKG,(W(IKG,IR),IR=1,NO2)
     25 FORMAT(/4H G =,I3,4X,8(2I3,1X)/11X,8(2I3,1X)/11X,8(2I3,1X))
     26 IF(G.GT.(L-1)) GOTO 104
        I=1
     28 I=I+1
        IF(I.EQ.G) GOTO 62
C       CASE OF I LESS THAN G
        IJI=I
        IGA=G
        KAL=1
        IRA=1
C       TEST OF STORED VALUES FOR 'ANOMALIES'
     32 NA=NOP
        NEW=L
        DO 36 IS=2,NO2,2
        IF(W(IJI,IS).EQ.IGA) NEW=W(IJI,IS-1)
        IF(W(IJI,IS-1).EQ.IGA) NEW=W(IJI,IS)
        IF(NEW.LE.IJI) GOTO(36,36,34,34),KAL
     34 IF(NEW.GE.L) GOTO 36
        NA=NA+2
        IF(NA.GT.112) GOTO 198
        Z(IRA,NA-1)=I
        Z(IRA,NA)=NEW
     36 NEW=L
     38 IF(NA.EQ.NOP) GOTO(28,42,118,148),KAL
C       ARRANGEMENT OF 'ANOMALIES' IN ASCENDING ORDER
     42 JCO=NA
        DO 46 IS=2,NA,2
        DO 44 IT=2,JCO,2
        I2=IT+2
        IF(Z(IRA,I2).GE.Z(IRA,IT)) GOTO 44
        KH1=Z(IRA,IT-1)
        KH2=Z(IRA,IT)
        Z(IRA,IT-1)=Z(IRA,I2-1)
        Z(IRA,IT)=Z(IRA,I2)
        Z(IRA,I2-1)=KH1
        Z(IRA,I2)=KH2
     44 CONTINUE
     46 JCO=JCO-2
        L=MIN0(Z(IRA,NOP),L)
        GOTO(48,48,116,156),KAL
     48 IF(L.LE.G) GOTO 102
C       TRANSFER OF 'ANOMALIES' INTO W ARRAY
        DO 58 IS=2,NO2,2
        IF(Z(1,IS).GE.W(G,IS)) GOTO 58
        IF(IS.EQ.NO2) GOTO 56
        NO3=NO2
     54 W(G,NO3)=W(G,NO3-2)
        W(G,NO3-1)=W(G,NO3-3)
        NO3=NO3-2
        IF(NO3.GT.IS) GOTO 54
     56 W(G,IS)=Z(1,IS)
        W(G,IS-1)=I
        IF(W(G,IS).GE.L) W(G,IS-1)=W(G,IS)
     58 CONTINUE
        GOTO(28,62),KAL
C       CASE OF I GREATER THAN G
C       POLYNOMIAL DIVISION PERFORMED
     62 I=I+1
        IF(I.GT.(L-2)) GOTO 22
        L12=L+12
        KA=1
        KUT=0
        DO 84 KB=1,LAD
        DO 82 KC=1,LAD
        DO 64 IS=1,L12
     64 MB(IS)=0
```

```
C       PROGRAM U1 (CONTINUED)
   64   MB(IS)=0
        MB(1)=KA
        MB(G)=KB
        MB(I)=KC
        KR=0
        KL=MAX0(G,I)
   66   KR=KR+1
        IF(MB(KR).EQ.0) GOTO 66
        KM=N2+KR-1
        JZ=0
        MJ=0
        JL=MB(KR)
        DO 74 IS=KR,KM
        MJ=MJ+1
        MB(IS)=MB(IS)-JL*MA(MJ)+LS*LS
   68   IF(MB(IS).LT.LS) GOTO 72
        MB(IS)=MB(IS)-LS
        GOTO 68
   72   IF(MB(IS).EQ.0) GOTO 74
        JZ=JZ+1
        KN=IS
        KD=MB(IS)
   74   CONTINUE
        IF(KR.GT.(L-1)) GOTO 76
        IF((JZ.GT.1).OR.(KM.LT.KL)) GOTO 66
   76   IF(JZ.GT.1) KN=999
        IF(JZ.EQ.0) KN=888
        IF(KM.LT.KL) KN=777
        GOTO 78
C       FOR NO CANCELLATION GOTO 78
        IF(KN.GE.L) GOTO 78
        IU1=1+G+I+KN
        IU2=IU1-(IU1/2)*2
        IF(IU2.EQ.0) GOTO 78
        KN=666
   78   KUT=KUT+1
   82   Z(2,KUT)=KN
   84   CONTINUE
        MIN=L+L
        DO 94 IS=1,LID
   94   MIN=MIN0(Z(2,IS),MIN)
        IF((MIN.LT.I).OR.(MIN.GT.L)) GOTO 62
        NA=NOP
        DO 96 IS=1,LID
        IF(Z(2,IS).GT.L) GOTO 96
        NA=NA+2
        IF(NA.GT.92) GOTO 198
        Z(1,NA)=Z(2,IS)
   96   CONTINUE
        KAL=2
        IRA=1
        GOTO 42
  102   L=G
  104   G=1
        LA3=0
        IF(NOM.EQ.0) GOTO 192
  106   G=G+1
        IF(G.GT.(L-1)) GOTO 192
        DO 108 IS=1,NOP
  108   Z(1,IS)=W(G,IS)
        Z(2,1)=1
        Z(2,2)=G
        I=1
        LG=G
        LA7=1
        GOTO 216
  112   I=I+1
        LG=LG+1
        IF(LG.GT.(L-1)) GOTO 106
        Z(2,1)=I
        Z(2,2)=LG
        DO 114 IS=1,NOP
  114   Z(1,IS)=Z(1,IS)+1
        LA7=2
  216   CONTINUE
        IF(LA3.EQ.0) GOTO(122,113),LA7
        LA1=0
        LA2=LA9
        DO 222 IM=1,LA3
        LA1=LA1+1
        IF(LA1.LE.LA8) GOTO 218
        LA1=LA1-LA8
        LA2=LA2-2
  218   CONTINUE
  221   IF((Z(2,1).EQ.W(LA1,LA2-1)).AND.(Z(2,2).EQ.W(LA1,LA2)))GOTO112
  222   CONTINUE
        GOTO(122,113),LA7
  113   IJI=I
        IGA=LG
        KAL=3
        IRA=1
        GOTO 32
  116   IF(L.GT.LG) GOTO 118
        L=LG
        GOTO 106
  118   CONTINUE
  122   NET=0
  123   LIA=1
        DO 124 IS=2,NO2,2
  124   IF(Z(1,IS).LT.L) NET=NET+1
  126   DO 128 IS=1,NO2
  128   Z(2,IS+2)=Z(1,IS)
        IF(Z(2,2).GE.L) GOTO 112
  131   CONTINUE
```

```
C       PROGRAM U1 (CONTINUED)
  131   CONTINUE
        MIK=0
        DO 132 IS=2,NOP,2
  132   IF(Z(2,IS).GE.L) MIK=MIK+1
        GOTO 136
        GOTO(136,133),LIA
  133   WRITE(2,134) (Z(2,IR),IR=1,NOP)
  134   FORMAT(/9(2I3,1X)/9(2I3,1X)/9(2I3,1X))
  136   IE=0
        LLA=1
  142   IE=IE+2
        IF(Z(2,IE).GE.L) GOTO 188
  144   IG=Z(2,IE)-Z(2,IE-1)+1
        ID=1
        DO 146 IS=1,NOP
  146   Z(3,IS)=W(IG,IS)
  147   LTE=L
  148   IF(IG.EQ.Z(2,IE)) GOTO 158
        ID=ID+1
        IG=IG+1
        DO 152 IS=1,NOP
  152   Z(3,IS)=Z(3,IS)+1
        IJ1=ID
        IGA=IG
        KAL=4
        IRA=3
        GOTO 32
  156   IF(L.LT.IG) L=IG
        GOTO 148
  158   CONTINUE
  165   IF(L.NE.LTE) GOTO 179
  166   DO 168 IS=2,NO2,2
        IF(Z(3,IS).LT.L) GOTO 169
  168   CONTINUE
        Z(2,IE)=L
        Z(2,IE-1)=L
        GOTO 136
  169   LLA=2
        GOTO(172,170),LIA
  170   WRITE(2,171) ID,IG,(Z(3,IR),IR=1,NO2)
  171   FORMAT(I4,1X,I3,2H =,8(2I3,1X)/10X,8(2I3,1X)/10X,8(2I3,1X))
  172   DO 188 IS=2,NO2,2
        IF(Z(3,IS).GE.L) GOTO 188
        DO 174 IT=2,NOP,2
        IF(IT.EQ.IE) GOTO 174
        IF((Z(3,IS).EQ.Z(2,IT)).AND.(Z(3,IS-1).EQ.Z(2,IT-1)))GOTO188
  174   CONTINUE
        IF(MIK.NE.0) GOTO 184
        GOTO(176,175),LIA
  175   WRITE(2,134) (Z(2,IR),IR=1,NOP)
  176   L=Z(3,IS)
        DO 178 IW=2,NOP,2
  178   L=MAX0(Z(2,IW),L)
  179   GOTO(126,181),LIA
  181   WRITE(2,182) L
  182   FORMAT(/2X,15HAREA REDUCED TO,I4/)
        GOTO 126
  184   DO 186 IT=2,NOP,2
        IF(Z(2,IT).LT.L) GOTO 186
        Z(2,IT)=Z(3,IS)
        Z(2,IT-1)=Z(3,IS-1)
        MIK=MIK-1
        GOTO 188
  186   CONTINUE
  188   CONTINUE
        IF(IE.LT.NOP) GOTO 142
        GOTO(112,191),LLA
  189   WRITE(2,190)
  190   FORMAT(1H0)
  191   CONTINUE
        LA1=LA3
        LA2=LA9
        DO 232 IM=4,NOP,2
        IF(Z(2,IM).GE.L) GOTO 232
  226   IF((LA1+1).LE.LA8) GOTO 228
        IF(LA2.EQ.(NOP+2)) GOTO 112
        LA1=LA1-LA8
        LA2=LA2-2
        GOTO 226
  228   LA1=LA1+1
        LA3=LA3+1
        W(LA1,LA2)=Z(2,IM)
        W(LA1,LA2-1)=Z(2,IM-1)
  232   CONTINUE
        GOTO 112
  192   L=L-2
        NOM=NOM+1
        WRITE(2,194) NOM,L
  194   FORMAT(5X,1HR,I2,2H =,I3///)
        IF(NOM.EQ.1) GOTO 4
        NOM=NOM-2
        NO2=2*NOM
        NOP=NO2+2
        L=L+2
        G=1
  196   G=G+1
        IF(G.GT.(L-1)) GOTO 104
        L=MIN0(W(G,NOP),L)
        IF(L.GT.G) GOTO 196
        GOTO 102
  198   STOP
        END
```

APPENDIX 1.4

```
C       PROGRAM CCM
C       2ND ORDER KERNEL, LINEAR-SQUARE SYSTEM
C       DIRECT AND COMBINED CROSSCORRELATION METHOD
C       POLYNOMIALS 110220202 AND 120120202, N=6560
C       R1=15,Q1=33,RM=105, USE T1=5T AND T1=10T
C       POLYNOMIALS 1102202 AND 1201202, N=728,R1=7,Q1=13,RM=36
        INTEGER U,A,P,FB,C
        DIMENSION U(2,3380),A(2,3380),P(8),B(2)
        COMMON U,A,P,B
        AMP=50.0
        T=1.0
        T1=6.0
    4   READ(3,6) T1,KAY1,KAY2
    6   FORMAT(F5.1,2I3)
        C2=EXP(-T/T1)
        C1=AMP*(1.0-C2)
        DEN=4.0*3.0**4
        TS=T1*T1
        WN=C1*C1*EXP(-T/T1)
        DO 26 NN=1,2
        X1A=0.0
        DO 8 I=1,6
    8   P(I)=1
C       GENERATION OF PRS
        DO 26 J=1,928
   10   PONE=P(1)
        IF(NN.EQ.2) GOTO 11
        FB=P(1)+P(2)-P(4)-P(5)+6
        GOTO 12
   11   FB=P(1)-P(2)+P(4)-P(5)+6
   12   IF(FB.LT.2) GOTO 14
        FB=FB-3
        GOTO 12
C       CALCULATION OF THE OUTPUT
   14   X1B=PONE+X1A*C2
        JJ=J-364
        IF(JJ.LT.1) GOTO 24
        Y=(C1*X1A)**2*TS
        U(NN,JJ)=P(1)
        A(NN,JJ)=Y
   24   X1A=X1B
        DO 25 L=1,5
   25   P(L)=P(L+1)
        P(6)=FB
   26   CONTINUE
C       CROSSCORRELATION OVER A HALF PERIOD
        DO 46 C=KAY1,KAY2
        WRITE(2,28) C,T1
   28   FORMAT(//2X,41H2ND ORDER KERNEL OF LINEAR-SQUARE SYSTEM,,
     1 4H C =,I2,4H T1=,F4.1/
     2 4H  NO,2X,5HFIRST,5X,6HSECOND,4X,8HCOMBINED,2X,10HCALCULATED/
     3 6X,8HESTIMATE,2X,8HESTIMATE,2X,8HESTIMATE,2X,6HRESULT/
     4 4H  --,2X,8H--------,2X,8H--------,2X,8H--------,
     5 2X,10H----------)
C       FORMULA U(I)*U(I+C)*A(I+C+J)
        DO 46 JJ=1,21
        J=JJ-1
        DO 34 NN=1,2
        Y=0.0
        DO 32 I=1,364
        IF((U(NN,I).EQ.0).OR.(U(NN,I+C).EQ.0)) GOTO 32
        LY=U(NN,I)*U(NN,I+C)*A(NN,I+C+J)
        YL=FLOAT(LY)/TS
        Y=Y+YL
   32   CONTINUE
        Y=(Y/DEN)/WN
   34   B(NN)=Y
        BC=(B(1)+B(2))/2.0
C       CALCULATED VALUE
        W=0.0
        IF(J.EQ.0) GOTO 36
        JC=2*J+C-2
        CJ=JC
        W=C1*C1*EXP(-CJ*T/T1)/WN
   36   WRITE(2,38) J,B(1),B(2),BC,W
   38   FORMAT(I4,F9.3,3F10.3)
   46   CONTINUE
        GOTO 4
        STOP
        END
```

APPENDIX 1.5

```
C       PROGRAM PDN
C       POLYNOMIAL DIVISION: 3 OR 4 TERM RELATIONSHIP
        INTEGER C
        DIMENSION MA(2,9),MG(2,3)
        LS=3
        LAD=LS-1
   4    READ(3,6) N2,LP,LQ,(MA(1,IR),IR=1,N2)
   6    FORMAT(I2,2I3,1X,9I1)
        NPF=(LS**(N2-1)-1)/2
        WRITE(2,8) (MA(1,IR),IR=1,N2)
   8    FORMAT(////2X,12HPOLYNOMIAL =,1X,9I1)
        DO 58 JC=2,LP
        C=JC-1
        MG(1,1)=1
        MG(1,2)=JC
        DO 54 IC=1,LQ
        MG(1,3)=IC
        DO 48 KB=1,LAD
        DO 47 KC=1,LAD
        MG(2,1)=1
        MG(2,2)=KB
        MG(2,3)=KC
        INK=0
        DO 22 IR=1,N2
  22    MA(2,IR)=0
        DO 26 IR=1,3
        IF(MG(1,IR).GT.N2) GOTO 26
        MG1=MG(1,IR)
        MA(2,MG1)=MG(2,IR)
  26    CONTINUE
  27    CONTINUE
        IF(MA(2,1).NE.0) GOTO 34
        INK=INK+1
        DO 28 IR=2,N2
  28    MA(2,IR-1)=MA(2,IR)
        DO 32 IR=1,3
        IF(MG(1,IR).NE.(INK+N2)) GOTO 32
        MA(2,N2)=MG(2,IR)
        GOTO 27
  32    CONTINUE
        MA(2,N2)=0
        GOTO 27
  34    JZ=0
        MA1=MA(2,1)
        DO 42 IR=1,N2
        MA(2,IR)=MA(2,IR)-MA1*MA(1,IR)+9
  36    IF(MA(2,IR).LT.LS) GOTO 38
        MA(2,IR)=MA(2,IR)-LS
        GOTO 36
  38    IF(MA(2,IR).EQ.0) GOTO 42
        JZ=JZ+1
        KD=MA(2,IR)
        KN=INK+IR
  42    CONTINUE
        IF(KN.GT.(LQ+1)) GOTO 47
        IF((JZ.GT.1).OR.((INK+N2).LT.MAX0(JC,IC))) GOTO 27
        JN=KN-1
        KID=IC-1
        IF(JZ.EQ.1) GOTO 45
        WRITE(2,43) KB,C,KC,KID
  43    FORMAT(1X,6H1D 0 +,I2,1HD,I3,2H +,I2,1HD,I3,4H = 0)
        GOTO 47
  45    WRITE(2,46) KB,C,KC,KID,KD,JN
  46    FORMAT(1X,6H1D 0 +,I2,1HD,I3,2H +,I2,1HD,I3,2H =,I2,1HD,I3)
  47    CONTINUE
  48    CONTINUE
  54    CONTINUE
        WRITE(2,56)
  56    FORMAT(/)
  58    CONTINUE
        GOTO 4
        STOP
        END
```

APPENDIX 1.6

```
C        PROGRAM MNA
C        TWO INPUT SYSTEM - PERFORMANCE INDICES
         INTEGER G,R,Q,Z1,Z2
         DIMENSION MA(2,9),M1(81,41),M2(81,41),Z1(543,2),Z2(543)
         COMMON MA,M1,M2,Z1,Z2
         LS=3
    4    READ(3,6) N2,R,(MA(1,IR),IR=1,N2)
    6    FORMAT(2I3,1X,9I1)
         R=R+1
         NPF=(LS**(N2-1)-1)/2
         NHA=(NPF+1)/2
         LY=92
         WRITE(2,8) (MA(1,IR),IR=1,N2)
    8    FORMAT(///2X,10HPOLYNOMIAL,2X,9I1)
C        POLYNOMIAL DIVISION
         KAL=1
         MAN=5
         G=1
   14    G=G+1
   16    CONTINUE
         WRITE(2,9)
    9    FORMAT(/)
   17    DO 46 KB=1,2
   18    INK=0
         DO 22 IR=1,N2
   22    MA(2,IR)=0
         MA(2,1)=1
         IF(G.LE.N2) MA(2,G)=KB
   24    IF(MA(2,1).NE.0) GOTO 32
         INK=INK+1
         DO 26 IR=2,N2
   26    MA(2,IF-1)=MA(2,IR)
         MA(2,N2)=0
         IF(G.EQ.(INK+N2)) MA(2,N2)=KB
         GOTO 24
   32    JZ=0
         MAI=MA(2,1)
         DO 38 IR=1,N2
         MA(2,IR)=MA(2,IR)-MAI*MA(1,IR)+9
   34    IF(MA(2,IR).LT.LS) GOTO 36
         MA(2,IR)=MA(2,IR)-LS
         GOTO 34
   36    IF(MA(2,IR).EQ.0) GOTO 38
         JZ=JZ+1
         KD=MA(2,IR)
         KN=INK+IR
   38    CONTINUE
         IF((JZ.GT.1).OR.((INK+N2).LT.G)) GOTO 24
         IF(KN.LE.NPF) GOTO 42
         KN=KN-NPF
         KD=LS-KD
   42    CONTINUE
         IF(MAN.NE.5) GOTO 46
          WRITE(2,44) KB,G,KD,KN
   44    FORMAT(2X,4H1D 1,3H + ,I1,1HD,I2,3H = ,I1,1HD,I4)
   46    M1(KB,G)=KN
         IF(G.EQ.R) GOTO(62,212),KAL
C        SHIFTED VALUES
   48    LIM=(R-G+1)*2
         DO 54 IR=3,LIM
         IRH=(IR+1)/2
         IG=IRH+G-1
         M1(IR,G)=M1(IR-2,G)+1
         IF(M1(IR,G).GT.NPF) M1(IR,G)=M1(IR,G)-NPF
         GOTO 54
         WRITE(2,52) IRH,IG,M1(IR,G)
   52    FORMAT(3X,1HD,I2,3H + ,1X,1HD,I2,3H = ,1X,1HD,I4)
   54    CONTINUE
         GOTO 14
C        3 TERM RELATION
   62    Q=R-1
         KAT=0
   64    Q=Q+1
         READ(3,63) Q
   63    FORMAT(I5)
   65    IF(Q.GT.NHA) GOTO 4
   66    J1=Q+1
         J2=J1+R-1
         K1=NPF-Q+1
         K2=K1+R-1
         GOTO(68,216),KAL
   68    JAN=0
         MR=(R-1)*2
         DO 78 G=2,R
         DO 76 IR=1,MR
         IF((M1(IR,G).GE.J1).AND.(M1(IR,G).LE.J2)) GOTO 69
         IF((M1(IR,G).LT.K1).OR.(M1(IR,G).GT.K2)) GOTO 76
   69    CONTINUE
         IRH=(IR+1)/2
         IG=IRH+G-1
         IF(MAN.NE.5) GOTO 74
         WRITE(2,72) LY,R,Q,IRH,IG,M1(IR,G)
   72    FORMAT(/2X,1HR,I2,2H =,I3,5H, Q =,I5/
        1 2X,1HD,I4,4H + D,I4,4H = D,I5)
   74    JAN=JAN+1
         IF(JAN.EQ.LY) GOTO 64
         GOTO 78
   76    CONTINUE
   78    MR=MR-2
C        Q OR M2 ARRAY
   84    MR=(R-1)*2
         DO 88 G=2,R
         DO 86 IR=1,MR
         M2(IR,G)=M1(IR,G)+Q
```

```
C       PROGRAM MNA (CONTINUED)
        M2(IR,G)=M1(IR,G)+Q
        IF(M2(IR,G).GT NPF) M2(IR,G)=M2(IR,G)-NPF
   86   CONTINUE
   88   MR=MR-2
        GOTO(102,242),KAL
C       4 TERM RELATION
  102   DO 118 KEY=2,R
        DO 118 IS=1,2
        MR=(R-1)*2
        DO 118 G=2,R
        DO 116 IR=1,MR
        IF(M1(IS,KEY).NE.M2(IR,G)) GOTO 116
        L1=1
        IF(MAN.NE.5) GOTO 108
        L2=KEY
        L3=(IR+1)/2+Q
        L4=L3+G-1
        WRITE(2,106) L1,L2,L3,L4
  106   FORMAT(2H D,I3,4H + D,I3,4H = D,I4,4H + D,I4)
  108   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 64
  116   CONTINUE
  118   MR=MR-2
C       COMBINATIONS OF SHIFTED M1 ARRAY
  122   MRG=R-1
        DO 128 KEY=2,R
        DO 128 IS=1,2
        MR=(R-1)*2
        DO 128 G=2,MRG
        DO 126 IR=3,MR
        IF(M2(IS,KEY).NE.M1(IR,G)) GOTO 125
        IF(MAN.NE.5) GOTO 124
        LA1=(IR+1)/2
        L2=LA1+G-1
        L3=Q+1
        L4=L3+KEY-1
        WRITE(2,106) LA1,L2,L3,L4
  124   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 64
  125   CONTINUE
  126   CONTINUE
  128   MR=MR-2
        KAT=KAT+1
        Z1(KAT,1)=Q
        Z1(KAT,2)=JAN
        GOTO 64
C       FINAL RESULT
  142   CONTINUE
        IR=R-1
        IF(KAT.NE.0) GOTO 162
        IF(KAL.EQ.1) GOTO 152
  143   IR=R-2
        WRITE(2,144) LY,IR,(Z2(IS),IS=1,KATE)
  144   FORMAT(/2X,1HR,I2,2H =,I4/(10I6))
        GOTO 150
        WRITE(2,9)
  150   LY=LY+1
        IF(LY.GT.2) GOTO 4
        KAL=1
        R=R-1
        GOTO 62
  152   R=R-2
        IR=R-1
        GOTO 62
        WRITE(2,154) IR
  154   FORMAT(/2X,15HAREA REDUCED TO,I4)
        GOTO 62
  162   KAL=2
        KATE=KAT
        DO 166 IR=1,KAT
  166   Z2(IR)=Z1(IR,1)
        GOTO 172
        WRITE(2,168) KAT,IR,((Z1(IR,IS),IS=1,2),IR=1,KAT)
  168   FORMAT(/2X,6HKATE =,I3,4X,3HR =,I3/(10I6))
C       INCREMENT OF AREA
  172   MR=R*2
        IRP=R+1
        DO 178 G=2,R
        IRH=MR/2
        IS=MR-1
        DO 176 IR=IS,MR
        M1(IR,G)=M1(IR-2,G)+1
        IF(M1(IR,G).GT.NPF) M1(IR,G)=M1(IR,G)-NPF
        GOTO 176
        WRITE(2,52) IRH,IRP,M1(IR,G)
  176   CONTINUE
  178   MR=MR-2
        R=R+1
        G=R
        GOTO 16
  212   IMI=0
        KAT=0
  214   IMI=IMI+1
        IF(IMI.GT.KATE) GOTO 142
        Q=Z1(IMI,1)
        IF(Q.LT.R) GOTO 214
        JAN=Z1(IMI,2)
        GOTO 66
C       3 TERM TEST
  216   CONTINUE
        MR1=R-1
        DO 220 KEY=2,MR1
        DO 220 IS=1,2
        IF(M1(IS,KEY).EQ.J2) GOTO 217
        IF(M1(IS,KEY) NE.K2) GOTO 220
  217   IF(MAN.NE.5) GOTO 218
```

```
C       PROGRAM MNA (CONTINUED)
  217   IF(MAN.NE.5) GOTO 218
        WRITE(2,72) LY,R,Q,L1,KEY,M1(IS,KEY)
  218   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 214
  220   CONTINUE
        MR=(R-1)*2
        DO 226 G=2,R
        IS=MR-1
        DO 224 IR=IS,MR
        IF((M1(IR,G).GE.J1).AND.(M1(IR,G).LE.J2)) GOTO 221
        IF((M1(IR,G).LT.K1).OR.(M1(IR,G).GT.K2)) GOTO 224
  221   IF(IR.LT.3) GOTO 222
        IF((M1(IR-2,G).GE.J1).AND.(M1(IR-2,G).LE.J2)) GOTO 224
        IF((M1(IR-2,G).GE.K1).AND.(M1(IR-2,G).LE.K2)) GOTO 224
  222   CONTINUE
        IRH=(IR+1)/2
        IG=IRH+G-1
        IF(MAN.NE.5) GOTO 223
        WRITE(2,72) LY,R,Q,IRH,IG,M1(IR,G)
  223   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 214
  224   CONTINUE
  226   MR=MR-2
        GOTO 84
C       Q OR M2 ARRAY
  242   CONTINUE
C       4 TERM TEST - PART ONE
        MR=(R-1)*2
        DO 248 G=2,R
        DO 246 IS=1,2
        DO 246 IR=1,MR
        IF(M1(IS,R).NE.M2(IR,G)) GOTO 246
        IF(MAN.NE.5) GOTO 240
        L3=(IR+1)/2+Q
        L4=L3+G-1
        WRITE(2,106) L1,R,L3,L4
  240   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 214
  246   CONTINUE
  248   MR=MR-2
C       4 TERM TEST - PART TWO
        MR1=R-1
        MR=(R-1)*2
        DO 318 G=2,MR1
        DO 316 IS=1,2
        DO 316 IR=1,MR
        IF(M2(IS,R).NE.M1(IR,G)) GOTO 316
        IF(MAN.NE.5) GOTO 312
        LA1=(IR+1)/2
        L2=LA1+G-1
        L3=Q+1
        L4=L3+R-1
        WRITE(2,106) LA1,L2,L3,L4
  312   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 214
  316   CONTINUE
  318   MR=MR-2
C       4 TERM TEST - PART THREE
        DO 258 KEY=2,MR1
        DO 258 IS=1,2
        MR=(R-1)*2
        DO 256 G=2,MR1
        MIS=MR-1
        DO 254 IR=MIS,MR
        IF(M1(IS,KEY).NE.M2(IR,G)) GOTO 253
        IF(MAN.NE.5) GOTO 250
        L3=(IR+1)/2+Q
        L4=L3+G-1
        WRITE(2,106) L1,KEY,L3,L4
  250   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 214
  253   CONTINUE
  254   CONTINUE
  256   MR=MR-2
  258   CONTINUE
C       4 TERM TEST - PART FOUR
        DO 268 KEY=2,MR1
        DO 268 IS=1,2
        MR=(R-1)*2
        DO 266 G=2,MR1
        MIS=MR-1
        DO 264 IR=MIS,MR
        IF(M2(IS,KEY).NE.M1(IR,G)) GOTO 263
        IF(MAN.NE.5) GOTO 260
        LA1=(IR+1)/2
        L2=LA1+G-1
        L3=Q+1
        L4=L3+KEY-1
        WRITE(2,106) LA1,L2,L3,L4
  260   JAN=JAN+1
        IF(JAN.EQ.LY) GOTO 214
  263   CONTINUE
  264   CONTINUE
  266   MR=MR-2
  268   CONTINUE
        KAT=KAT+1
        Z1(KAT,1)=Q
        Z1(KAT,2)=JAN
        GOTO 214
        STOP
        END
```

APPENDIX 1.7

```
C       PROGRAM DV
C       BINARY SIGNALS: PERFORMANCE INDEX, R0
        INTEGER F,G,P
        DIMENSION MA(2,12)
        ICO=0
        WRITE(2,4)
      4 FORMAT(//25H DIAGONAL VALUES FOR PRBS/
       1 25H ------------------------/
       2 4H  NO,3X,1HF,3X,10HPOLYNOMIAL/
       3 4H ---,2X,3H---,2X,10H----------)
      6 READ(3,8) N2,(MA(1,IR),IR=1,N2)
      8 FORMAT(I3,1X,12I1)
        N=N2-1
        NP=2**N-1
        DO 64 JJ=1,2
        GOTO(13,9),JJ
C       RECIPROCAL POLYNOMIAL
      9 DO 12 IR=1,N
        MAN=MA(1,N2)
        IS=N2+1
     10 IS=IS-1
        MA(1,IS)=MA(1,IS-1)
        IF((IS-1).NE.IR) GOTO 10
     12 MA(1,IS-1)=MAN
     13 F=31
        G=1
     14 G=G+1
        IF(G.GT.(F-1)) GOTO 52
        INK=0
        DO 22 IR=2,N2
     22 MA(2,IR)=0
        MA(2,1)=1
        IF(G.GT.N2) GOTO 26
        MA(2,G)=1
     26 CONTINUE
        IF(MA(2,1).NE.0) GOTO 34
        INK=INK+1
        DO 28 IR=2,N2
     28 MA(2,IR-1)=MA(2,IR)
        MA(2,N2)=0
        IF(G.EQ.(INK+N2)) MA(2,N2)=1
        GOTO 26
     34 JZ=0
        DO 42 IR=1,N2
        MA(2,IR)=MA(2,IR)-MA(1,IR)+2
     36 IF(MA(2,IR).LT.2) GOTO 38
        MA(2,IR)=MA(2,IR)-2
        GOTO 36
     38 IF(MA(2,IR).EQ.0) GOTO 42
        JZ=JZ+1
        KD=MA(2,IR)
        KN=INK+IR
     42 CONTINUE
        IF((JZ.GT.1).OR.(KN.LT.G)) GOTO 26
        IF(KN.GT.NP) KN=KN-NP
        WRITE(2,51) G,KN
     51 FORMAT(3H D1,3H + ,1HD,I2,3H = ,1HD,I3)
        GOTO 14
     52 F=F-2
        GOTO(54,58),JJ
     54 ICO=ICO+1
        WRITE(2,56) ICO,F,(MA(1,IR),IR=1,N2)
     56 FORMAT(I4,I5,2X,12I1)
        GOTO 64
     58 WRITE(2,62) F,(MA(1,IR),IR=1,N2)
     62 FORMAT(I9,2X,12I1)
     64 CONTINUE
        GOTO 6
        STOP
        END
```

```
C       PROGRAM RI
C       RECTANGULAR INPUT - BINARY M SEQUENCE
        INTEGER X,A,P,FD,FB
        DIMENSION X(1530),Y(1530),A(510),B(510),P(11),FD(12)
        COMMON X,Y,A,B
    2   READ(3,4) NS2,LAY,(FD(IR),IR=1,NS2)
    4   FORMAT(2I2,1X,12I1)
        NS=NS2-1
        NS1=NS-1
        NP=2**NS-1
        INK=3*NP
        AK=0.3
        T=1.0
        DO 6 J=1,NS
    6   P(J)=1
C       GENERATION OF PRBS - FULL AND HALF BITS
        DO 32 J=1,INK
        NJ=P(1)
        IF(NJ.EQ.0) NJ=-1
        I=J*2
C       FULL BIT
   12   X(I-1)=NJ
C       HALF BIT
        X(I)=NJ
   16   FB=0
        DO 22 IR=1,NS
        IFB=FD(IR)+1
        GOTO(22,18),IFB
   18   FB=FB+P(IR)
   22   CONTINUE
   24   IF(FB.LT.2) GOTO 26
        FB=FB-2
        GOTO 24
   26   DO 28 L=2,NS
   28   P(L-1)=P(L)
   32   P(NS)=FB
C       CALCULATION OF THE OUTPUT
        INK=5*NP
        LA=2*LAY+1
        DO 54 J=1,INK
        MR=0
        DO 52 IR=2,LA
        I=J+IR
   52   MR=MR+X(I)
        Y(J)=AK*T/2.0*FLOAT(MR)
   54   CONTINUE
C       DATA FOR CORRELATION
        INK=4*NP
        I=0
        DO 56 J=2,INK,2
        I=I+1
        A(I)=X(J)
        B(I)=Y(J-1)
   56   CONTINUE
        WRITE(2,58) (FD(IR),IR=1,NS2)
   58   FORMAT(/2X,10HPOLYNOMIAL,1X,12I1)
        WRITE(2,59) LAY,AK
   59   FORMAT(1X,36HCROSSCORRELATION - BINARY M SEQUENCE/
       1 1X,32HINPUT-RECTANGULAR WAVE OF WIDTH,I2,2HT,,6HHEIGHT,F5.1/
       2 3X,2HNO,1X,6HLINEAR,1X,9HQUADRATIC,2X,5HCUBIC/
       3 3X,2H--,1X,6H------,1X,9H---------,2X,5H-----)
C       CROSSCORRELATION
        LSTART=1
        DO 84 J=1,NP
        JJ=J-1
        Y1=0.0
        Y2=0.0
        Y3=0.0
        DO 68 L=1,NP
        LTEST=A(LSTART)+2
        GOTO(64,68,66),LTEST
   64   Y1=Y1-B(L)
        Y2=Y2-B(L)**2
        Y3=Y3-B(L)**3
        GOTO 68
   66   Y1=Y1+B(L)
        Y2=Y2+B(L)**2
        Y3=Y3+B(L)**3
   68   LSTART=LSTART+1
        Y1=Y1/FLOAT(NP)
        Y2=Y2/FLOAT(NP)
        Y3=Y3/FLOAT(NP)
        WRITE(2,74) JJ,Y1,Y2,Y3,A(J)
   74   FORMAT(I5,F7.2,F8.2,F9.2,I6)
        LSTART=J+1
   84   CONTINUE
        WRITE(2,86)
   86   FORMAT(//////)
        GOTO 2
        STOP
        END
```

```
C       PROGRAM RI2
C       RECTANGULAR INPUT - INVERSE REPEAT PRBS
        INTEGER X,P,FD,FB
        DIMENSION X(2040),Y(1020),P(11),FD(12)
C       Y(1020)=Y(2*NP)
        COMMON X,Y
      2 READ(3,4) NS2,LAY,(FD(IR),IR=1,NS2)
      4 FORMAT(2I2,1X,12I1)
        NS=NS2-1
        NPF=2**NS-1
        NP=NPF+NPF
        INK=2*NP
        AK=0.3
        T=1.0
        DO 6 J=1,NS
      6 P(J)=1
C       GENERATION OF PRBS - FULL AND HALF BITS
        DO 32 J=1,INK
        NJ=P(1)
        IF(NJ.EQ.0) NJ=-1
C       INVERSE REPEAT PROPERTY
        IF((J-(J/2)*2).EQ.0) NJ=-NJ
        I=J*2
C       FULL BIT
     12 X(I-1)=NJ
C       HALF BIT
        X(I)=NJ
     16 FB=0
        DO 22 IR=1,NS
        IFB=FD(IR)+1
        GOTO(22,18),IFB
     18 FB=FB+P(IR)
     22 CONTINUE
     24 IF(FB.LT.2) GOTO 26
        FB=FB-2
        GOTO 24
     26 DO 28 L=2,NS
     28 P(L-1)=P(L)
     32 P(NS)=FB
C       CALCULATION OF OUTPUT
        LA=2*LAY+1
        DO 54 J=1,INK
        MR=0
        DO 52 IR=2,LA
        I=J+IR
     52 MR=MR+X(I)
        Y(J)=AK*T/2.0*FLOAT(MR)
     54 CONTINUE
        WRITE(2,56) (FD(IR),IR=1,NS2)
     56 FORMAT(/2X,10HPOLYNOMIAL,1X,12I1)
        WRITE(2,58) LAY,AK
     58 FORMAT(1X,37HCROSSCORRELATION - ANTISYMMETRIC PRBS/
     1 1X,32HINPUT-RECTANGULAR WAVE OF WIDTH ,I2,2HT,,6HHEIGHT,F5.1/
     2 3X,2HNO,3X,6HLINEAR,2X,9HQUADRATIC,2X,5HCUBIC/
     3 3X,2H--,3X,6H------,2X,9H---------,2X,5H-----)
C       CROSSCORRELATION
        LSTART=2
        DO 84 J=1,NPF
        JJ=J-1
        Y1=0.0
        Y2=0.0
        Y3=0.0
        DO 68 L=1,INK,2
        LTEST=X(LSTART)+2
        GOTO(64,68,66),LTEST
     64 Y1=Y1-Y(L)
        Y2=Y2-Y(L)**2
        Y3=Y3-Y(L)**3
        GOTO 68
     66 Y1=Y1+Y(L)
        Y2=Y2+Y(L)**2
        Y3=Y3+Y(L)**3
     68 LSTART=LSTART+2
        PN=NP
        Y1=Y1/PN
        Y2=Y2/PN
        Y3=Y3/PN
        J2=J*2
        WRITE(2,74) JJ,Y1,Y2,Y3
     74 FORMAT(I5,3F9.3)
        LSTART=J2+2
     84 CONTINUE
        WRITE(2,86)
     86 FORMAT(//////)
        GOTO 2
        STOP
        END
```

```
C       PROGRAM RI3
C       RECTANGULAR WEIGHTING FUNCTION - TERNARY SIGNAL
        INTEGER X,P,FD,FB,FE
        DIMENSION X(2912),Y(1456),P(8),FD(9),FE(8)
        COMMON X,Y
      2 READ(3,4) NS2,LAY,(FD(IR),IR=1,NS2)
      4 FORMAT(2I2,1X,9I1)
        NS=NS2-1
        NP=3**NS-1
        NPF=NP/2
        DO 92 IR=1,NS
     92 FE(IR)=FD(IR)
C       LAST TERM IN POLYNOMIAL MUST BE TWO
        IF(FD(NS2).EQ.2) GOTO 96
        DO 94 IR=1,NS
        FE(IR)=FE(IR)*2
     94 IF(FE(IR).EQ.4) FE(IR)=1
     96 CONTINUE
        INK=2*NP
        AK=0.3
        T=1.0
        DO 6 J=1,NS
      6 P(J)=1
C       GENERATION OF SEQUENCE - FULL AND HALF BITS
        DO 32 J=1,INK
        NJ=P(1)
        I=J*2
C       FULL BIT
        X(I-1)=NJ
C       HALF BIT
        X(I)=NJ
     16 FB=0
        DO 22 IR=1,NS
        IFB=FE(IR)+1
        GOTO(22,18,19),IFB
     18 FB=FB+P(IR)
        GOTO 22
     19 FB=FB-P(IR)
     22 CONTINUE
        FB=FB+9
     24 IF(FB.LT.2) GOTO 26
        FB=FB-3
        GOTO 24
     26 DO 28 L=2,NS
     28 P(L-1)=P(L)
     32 P(NS)=FB
C       CALCULATION OF OUTPUT
        LA=2*LAY+1
        DO 54 J=1,INK
        MR=0
        DO 52 IR=2,LA
        I=J+IR
     52 MR=MR+X(I)
        Y(J)=AK*T/2.0*FLOAT(MR)
     54 CONTINUE
        WRITE(2,56) (FD(IR),IR=1,NS2)
     56 FORMAT(/2X,10HPOLYNOMIAL,1X,9I1)
        WRITE(2,58) LAY,AK
     58 FORMAT(1X,33HCROSSCORRELATION - TERNARY SIGNAL/
       1 1X,32HINPUT-RECTANGULAR WAVE OF WIDTH ,I2,2HT,,6HHEIGHT,F5.1/
       2 3X,2HNO,3X,6HLINEAR,2X,9HQUADRATIC,2X,5HCUBIC/
       3 3X,2H--,3X,6H------,2X,9H---------,2X,5H-----)
C       CROSSCORRELATION
        LSTART=2
        DO 84 J=1,NPF
        JJ=J-1
        Y1=0.0
        Y2=0.0
        Y3=0.0
        DO 68 L=1,INK,2
        LTEST=X(LSTART)+2
        GOTO(64,68,66),LTEST
     64 Y1=Y1-Y(L)
        Y2=Y2-Y(L)**2
        Y3=Y3-Y(L)**3
        GOTO 68
     66 Y1=Y1+Y(L)
        Y2=Y2+Y(L)**2
        Y3=Y3+Y(L)**3
     68 LSTART=LSTART+2
        PN=2.0*3.0**(NS-1)
        Y1=Y1/PN
        Y2=Y2/PN
        Y3=Y3/PN
        J2=J*2
        J3=J2+NP
        WRITE(2,74) JJ,Y1,Y2,Y3,X(J2),X(J3)
     74 FORMAT(I5,3F9.3,I6,I3)
        LSTART=J2+2
     84 CONTINUE
        WRITE(2,86)
     86 FORMAT(//////)
        GOTO 2
        STOP
        END
```

```
C       PROGRAM DDS4
C       DIRECTION DEPENDENT SYSTEM
        INTEGER U,FB,P,FD
        DIMENSION U(382),Y(4,391),P(11),FD(12),KD(12)
        COMMON U,Y
C       READ IN 6 100101
    2   READ(3,4) NS2,(FD(IR),IR=1,NS2)
    4   FORMAT(I2,1X,12I1)
C       RECIPROCAL POLYNOMIAL
        K=NS2
        DO 7 I=1,NS2
        KD(I)=FD(K)
    7   K=K-1
        NS=NS2-1
        NP=2**NS-1
        NP1=NP+1
        NP2=NP+NP+2
        NP3=3*NP
        T=1.0
        W=3.0/(8.0*T)
        R=1.0/(8.0*T)
        C1=EXP(-W*T)
C       GENERATION OF PRBS
        DO 14 J=1,NS
   14   P(J)=1
        DO 32 J=1,NP3
        NJ=P(1)
        IF(NJ.EQ.0) NJ=-1
        U(J)=NJ
   16   FB=0
        DO 22 IR=1,NS
        IFB=KD(IR)+1
        GOTO (22,18),IFB
   18   FB=FB+P(IR)
   22   CONTINUE
   24   IF(FB.LT.2) GOTO 26
        FB=FB-2
        GOTO 24
   26   DO 28 L=2,NS
   28   P(L-1)=P(L)
   32   P(NS)=FB
C       CALCULATION OF OUTPUT
C       INITIAL CONDITION OF STATE VARIABLES
        XAB=0.0
        X1A=0.0
        Y1A=0.0
        Y2A=0.0
        Z1A=0.0
        Z2A=0.0
        Z3A=0.0
        DO 58 ICYCLE=1,3
        FE=0.0
        DO 56 J=1,NP3
        UJ=U(J)
C       TOTAL OUTPUT
        IF(J.EQ.1) GOTO 38
        IF(U(J).EQ.U(J-1)) GOTO 38
        XAB=Y(1,J)
        FE=0.0
   38   FE=FE+1.0
        IFB=U(J)+2
        GOTO (44,46,42),IFB
   42   Y(1,J+1)=UJ-(UJ-XAB)*EXP(-(W+R)*T*FE)
        GOTO 46
   44   Y(1,J+1)=UJ-(UJ-XAB)*EXP(-(W-R)*T*FE)
   46   CONTINUE
   52   CONTINUE
C       FIRST ORDER TERM
        X1B=UJ+X1A*C1
        Y(2,J)=(1.0-C1)*X1A
        X1A=X1B
C       SECOND ORDER TERM
        Y2B=UJ+Y2A*C1
        Y1B=(Y2B-Y2A)*UJ+Y1A*C1
        Y(3,J)=R*T*C1*Y1A
        Y1A=Y1B
        Y2A=Y2B
```

```
C       PROGRAM DDS4 (CONTINUED)
        Y2A=Y2B
C       THIRD ORDER TERM
        Z3B=UJ+C1*Z3A
        Z2B=(Z3B-Z3A)*UJ+C1*Z2A
        Z1B=(Z2B+C1*Z2A)*UJ+C1*Z1A
        Y(4,J)=-R*R*T*T*C1*Z1A/2.0
        Z1A=Z1B
        Z2A=Z2B
        Z3A=Z3B
   56   CONTINUE
        XAB=Y(1,NP+1)
        Y(1,1)=Y(1,NP+1)
   58   CONTINUE
C       PRINTOUT OF THE OUTPUT
        WRITE(2,61) (FD(IR),IR=1,NS2)
   61   FORMAT(//11H POLYNOMIAL,2X,12I1)
        WRITE(2,62)
   62   FORMAT(37H OUTPUT OF DIRECTION DEPENDENT SYSTEM)
        WRITE(2,64)
   64   FORMAT(4H  NO,3X,5HTOTAL,2X,6HLINEAR,2X,9HQUADRATIC,2X,
     1  5HCUBIC,4X,5HERROR,3X,5HERROR,3X,5HERROR/
     2  9X,1HT,6X,1HL,9X,1HQ,8X,1HC,8X,1HL,6X,3HL+Q,4X,5HL+Q+C/
     3  4H ---,3X,5H-----,2X,6H------,2X,9H---------,2X,5H-----,
     4  4X,5H-----,3X,5H-----,3X,5H-----)
        DO 68 J=1,NP1
        JJ=J-1
        Y2=Y(1,J)-Y(2,J)
        Y3=Y2-Y(3,J)
        Y4=Y3-Y(4,J)
        WRITE(2,66) JJ,(Y(I,J),I=1,4),Y2,Y3,Y4
   66   FORMAT(I4,2F8.3,F10.3,F8.3,F9.3,F8.3,F9.4)
   68   CONTINUE
C       CROSSCORRELATION
        WRITE(2,61) (FD(IR),IR=1,NS2)
   72   WRITE(2,74)
   74   FORMAT(39H CROSSCORRELATION FUNCTION OF DIRECTION,
     1  17H DEPENDENT SYSTEM)
        WRITE(2,64)
        LSTART=1
        PN=NP
        DO 98 J=1,NP1
        JJ=J-1
        Y1=0.0
        Y2=0.0
        Y3=0.0
        Y4=0.0
        DO 88 L=1,NP
        LTEST=U(L)+2
        GOTO(84,88,86),LTEST
   84   Y1=Y1-Y(1,LSTART)
        Y2=Y2-Y(2,LSTART)
        Y3=Y3-Y(3,LSTART)
        Y4=Y4-Y(4,LSTART)
        GOTO 88
   86   Y1=Y1+Y(1,LSTART)
        Y2=Y2+Y(2,LSTART)
        Y3=Y3+Y(3,LSTART)
        Y4=Y4+Y(4,LSTART)
   88   LSTART=LSTART+1
        Y1=Y1/PN
        Y2=Y2/PN
        Y3=Y3/PN
        Y4=Y4/PN
        Y5=Y1-Y2
        Y6=Y5-Y3
        Y7=Y6-Y4
        WRITE(2,66) JJ,Y1,Y2,Y3,Y4,Y5,Y6,Y7
        LSTART=J+1
   98   CONTINUE
        STOP
        END
```

```
C       PROGRAM TV
C       CHEMICAL PLANT DATA: WEIGHTING FUNCTION ESTIMATES
C       WEIGHTING TO REMOVE CUBIC DRIFT
        INTEGER A,P,FB
        DIMENSION A(3640),S(1456),P(6)
        COMMON A,S
        READ(3,7) A
      7 FORMAT(14I5)
C       TO CONVERT DATA TO PH, DIVIDE BY 1900.0
C       STEADY STATE PH VALUE = 10.54
        P(1)=1
        P(2)=0
        P(3)=0
        P(4)=0
        P(5)=0
        P(6)=1
C       AMPLITUDE OF INPUT SIGNAL = 142.8814 = 143 KG/HR
C       GENERATION OF 2 PERIODS OF WEIGHTED PRS
        DO 28 J=1,1456
        IF(J.GT.728) GOTO 12
        S(J)=FLOAT(P(1))*(0.3584+FLOAT(J-1)/728.0)
        GOTO 14
     12 S(J)=FLOAT(P(1))*(1.6416-FLOAT(J-729)/728.0)
     14 FB=-P(6)-P(4)-P(3)+P(5)+P(1)+6
     16 IF(FB.LT.2) GOTO 18
        FB=FB-3
        GOTO 16
     18 DO 22 L=1,5
     22 P(L)=P(L+1)
     28 P(6)=FB
C       CROSSCORRELATION
        LSTART=1
        DO 36 J=1,2184
        Y=0.0
        DO 34 L=1,1456
        Y=Y+FLOAT(A(LSTART))*S(L)
     34 LSTART=LSTART+1
        A(J)=IFIX((Y/729.0)*15.0)
C       TO CONVERT TO PH/KG, DIVIDE BY 9049.1604
     36 LSTART=J+1
        DO 38 N=1,364
        N3=728+N
        N5=1456+N
        A(N)=-A(N)
        A(N3)=-A(N3)
     38 A(N5)=-A(N5)
        GOTO 41
        WRITE(4,40) (A(IR),IR=1,2184)
     40 FORMAT(1X,14I5)
     41 WRITE(2,42)
     42 FORMAT(17H IMPULSE RESPONSE/
       1 32H WEIGHTING TO REMOVE CUBIC DRIFT/
       2 32H -------------------------------//
       3 4H  NO,6X,1H1,7X,1H2,7X,1H3,7X,1H4,7X,1H5,7X,1H6/
       4 4H ---,4X,5H-----,3X,5H-----,3X,5H-----,3X,5H-----,3X,
       5 5H-----,3X,5H-----/)
        K=364
        DO 55 N=1,K
        N2=N+K
        N3=N2+K
        N4=N3+K
        N5=N4+K
        N6=N5+K
     53 FORMAT(I4,6I8)
     55 WRITE(2,53) N,A(N),A(N2),A(N3),A(N4),A(N5),A(N6)
        STOP
```

```
C       PROGRAM CF
C       CURVE FITTING
        INTEGER U,P,FB,A
        DIMENSION A(6,364),B(1456),U(728),P(6),C(4),D(4),N(364),H(6,4)
        COMMON A,B,U,N
        READ(3,5) ((H(IR,JR),JR=1,4),IR=1,6)
    5   FORMAT(4F9.4)
        PAUSE
    8   P(1)=1
        P(2)=0
        P(3)=0
        P(4)=0
        P(5)=0
        P(6)=1
        DO 18 J=1,728
        U(J)=P(1)
        FB=-P(6)-P(4)-P(3)+P(5)+P(1)+6
   12   IF(FB.LT.2) GOTO 14
        FB=FB-3
        GOTO 12
   14   DO 16 L=1,5
   16   P(L)=P(L+1)
   18   P(6)=FB
   21   READ(3,22) ((A(IR,JR),JR=1,364),IR=1,6)
   22   FORMAT(14I5)
        D(1)=0.1
        D(2)=1.0
        D(3)=0.0002
        D(4)=0.2
   26   DO 84 IT=1,6
        DO 2 KT=1,4
    2   C(KT)=H(IT,KT)
C       C(1),C(3)=LINEAR,CUBIC GAINS; C(2),C(4)=TIME CONSTANTS
C       D(J)=AMOUNT BY WHICH C(J) INCREASES
C       GAIN VALUES IN THE TEXT ARE OBTAINED AS FOLLOWS:
C       LINEAR GAIN=C(1)*3600.0/(143.0*1900.0)
C       CUBIC GAIN=C(3)*1900.0**2/(C(1)**3)
        INI=1
        JEC=140
        T=6.0
        E2=0.0
        EMIN=3640000.0
   34   DO 82 IU=1,4
        CAT=1.0
        IC=1
   36   CONTINUE
        GOTO(42,38),INI
   38   C(IU)=C(IU)+D(IU)*CAT
   42   T1=C(2)
        T2=C(4)
        CT1=1.0-EXP(-T/T1)
        CT2=1.0-EXP(-T/T2)
        C1=T1*CT1-T2*CT2
        C2=T2*CT2*EXP(-T/T1)-T1*CT1*EXP(-T/T2)
        C3=EXP(-T/T1)+EXP(-T/T2)
        C4=EXP(-T/T1-T/T2)
```

```
C       PROGRAM CF (CONTINUED)
        C4=EXP(-T/T1-T/T2)
        X1A=0.0
        X2A=0.0
        DO 48 J=1,728
        X1B=X2A
        X2B=-C4*X1A+C3*X2A+FLOAT(U(J))
        J1=J-364
        IF(J1.LE.0) GOTO 44
        B1=C2*X1A+C1*X2A
        B(J1)=-B1
        B(J1+364)=B1
        B(J1+728)=-B1
        B(J1+1092)=B1
   44   X1A=X1B
        X2A=X2B
   48   CONTINUE
        LSTART=1
C       CROSSCORRELATION
        DO 64 J=1,JEC
        Y=0.0
        DO 56 L=1,728
        B1=B(LSTART)
        LTEST=U(L)+2
        GOTO (52,56,54),LTEST
   52   Y=Y-C(1)*B1-C(3)*B1*B1*B1
        GOTO 56
   54   Y=Y+C(1)*B1+C(3)*B1*B1*B1
   56   LSTART=LSTART+1
        Y=(Y/729.0)*30.0
        N(J)=Y
   62   LSTART=J+1
   64   CONTINUE
        E1=E2
        E2=0.0
        DO 66 IR=2,JEC
        J1=IR+2
   66   E2=E2+FLOAT((A(IT,J1)-N(IR))**2)
        E2=E2/FLOAT(JEC-1)
        WRITE(2,68) (C(IR),IR=1,4),E2
   68   FORMAT(2F9.2,F9.4,2F9.2)
        GOTO(78,72),INI
   72   IF(E2-E1) 76,82,74
   74   E2=E1
        C(IU)=C(IU)-D(IU)*CAT
        IF(IC.EQ.2) GOTO 82
        CAT=-1.0
   76   IC=2
   78   INI=2
        GOTO 36
   82   CONTINUE
        IF((EMIN-E2).LT.0.0001) GOTO 84
        EMIN=E2
        GOTO 34
   84   CONTINUE
        STOP
        END
```

APPENDIX 1.14

```
C       PROGRAM ACF
C       CURVE FITTING - MORE ACCURATE METHOD
        INTEGER U,P,FB,A
        DIMENSION U(728),P(6),N(160),A(364),D(4),C(4),B(1456)
        DIMENSION Q(4,30),CJ(810),H(6,4)
        COMMON A,B,P,U,N,CJ,Q
        READ(3,4) ((H(IR,JR),JR=1,4),IR=1,6)
        READ(3,5) JI
    4   FORMAT(4F9.4)
    5   FORMAT(I4)
        PAUSE
    8   P(1)=1
        P(2)=0
        P(3)=0
        P(4)=0
        P(5)=0
        P(6)=1
        DO 18 J=1,728
        U(J)=P(1)
        FB=-P(6)-P(4)-P(3)+P(5)+P(1)+6
   12   IF(FB.LT.2) GOTO 14
        FB=FB-3
        GOTO 12
   14   DO 16 L=1,5
   16   P(L)=P(L+1)
   18   P(6)=FB
        ITI=0
   19   DO 22 IS=1,JI
        READ(3,20) A
   20   FORMAT(14I5)
        ITI=ITI+1
   22   CONTINUE
        DO 23 KT=1,4
   23   Q(KT,2)=H(ITI,KT)
   24   ICOUNT=0
        ICT=0
        FMIN=1000000.0
        EMIN=1000000.0
        INI=-1
        JAY=2
        D(1)=0.1
        D(2)=1.0
        D(3)=0.0002
        D(4)=0.2
   25   JA=JAY-1
        INI=INI+1
        DO 26 IT=1,4
        Q(IT,JAY-1)=Q(IT,JAY)-D(IT)
        Q(IT,JAY+1)=Q(IT,JAY)+D(IT)
   26   CONTINUE
        WRITE(2,32)
   32   FORMAT(/)
        T=6.0
        E2=0.0
        JA3=JA+2
        DO 72 IA=JA,JA3
        DO 72 IB=JA,JA3
  104   DO 72 IC=JA,JA3
        DO 72 ID=JA,JA3
  108   IF(INI.EQ.0) GOTO 38
        KA=1
        NET=0
        DO 36 KAL=1,INI
        KA3=KA+2
        DO 34 M1=KA,KA3
        DO 34 M2=KA,KA3
  114   DO 34 M3=KA,KA3
        DO 34 M4=KA,KA3
  118   NET=NET+1
        IF(Q(1,M1).NE.Q(1,IA)) GOTO 34
        IF(Q(2,M2).NE.Q(2,IB)) GOTO 34
  124   IF(Q(3,M3).NE.Q(3,IC)) GOTO 34
        IF(Q(4,M4).NE.Q(4,ID)) GOTO 34
  128   E2=CJ(NET)
        GOTO 65
   34   CONTINUE
   36   KA=KA+3
   38   T1=Q(2,IB)
        T2=Q(4,ID)
        CT1=1.0-EXP(-T/T1)
        CT2=1.0-EXP(-T/T2)
        C1=T1*CT1-T2*CT2
```

```
C       PROGRAM ACF (CONTINUED)
        C1=T1*CT1-T2*CT2
        C2=T2*CT2*EXP(-T/T1)-T1*CT1*EXP(-T/T2)
        C3=EXP(-T/T1)+EXP(-T/T2)
        C4=EXP(-T/T1-T/T2)
        X1A=0.0
        X2A=0.0
        DO 48 J=1,728
        X1B=X2A
        X2B=-C4*X1A+C3*X2A+FLOAT(U(J))
        J1=J-364
        IF(J1.LE.0) GOTO 44
        B1=C2*X1A+C1*X2A
        B(J1)=-B1
        B(J1+364)=B1
        B(J1+728)=-B1
        B(J1+1092)=B1
   44   X1A=X1B
        X2A=X2B
   48   CONTINUE
        LSTART=1
   51   DO 64 J=1,140
        Y=0.0
        DO 56 L=1,728
        B1=B(LSTART)
        LTEST=U(L)+2
        GOTO(52,56,54),LTEST
   52   Y=Y-Q(1,IA)*B1-Q(3,IC)*B1*B1*B1
        GOTO 56
   54   Y=Y+Q(1,IA)*B1+Q(3,IC)*B1*B1*B1
   56   LSTART=LSTART+1
        Y=(Y/729.0)*30.0
        N(J)=Y
   62   LSTART=J+1
   64   CONTINUE
        E1=E2
        E2=0.0
        DO 66 IR=2,140
        J1=IR+2
   66   E2=E2+FLOAT((A(J1)-N(IR))**2)
        E2=E2/139.0
   65   ICT=ICT+1
        CJ(ICT)=E2
        IF(E2.GE.EMIN) GOTO 67
        EMIN=E2
        C(1)=Q(1,IA)
        C(2)=Q(2,IB)
        C(3)=Q(3,IC)
        C(4)=Q(4,ID)
        IMI=ICT
   67   WRITE(2,68) ICT,Q(1,IA),Q(2,IB),Q(3,IC),Q(4,ID),E2
   68   FORMAT(I5,2F9.2,F9.4,2F9.2)
   72   CONTINUE
        IF(ABS(EMIN-FMIN).LT.0.00001) GOTO 99
        FMIN=EMIN
        WRITE(2,82) IMI,(C(IR),IR=1,4),EMIN
   82   FORMAT(/I5,2F9.2,F9.4,2F9.2/)
        ICOUNT=ICOUNT+1
        IF(ICOUNT.LE.9) GOTO 83
        DO 87 IR=1,4
   87   Q(IR,2)=C(IR)
        GOTO 24
   83   JAY=JAY+3
        DO 84 IR=1,4
   84   Q(IR,JAY)=C(IR)
        GOTO 25
   99   JI=1
        ITI=0
        GOTO 19
        STOP
        END
```

```
C       PROGRAM SOKM
C       AMMONIUM NITRATE DATA
C       SECOND ORDER KERNEL MEASUREMENT
        INTEGER U,P,FB,C
        DIMENSION A(364),U(1456),P(6),K(364)
        COMMON A,K,U,P,N
        T=6.0
        SA=142.8814/3600.0
        DEN=(8.0*3.0**4)*SA**4*T**2
        READ(3,8) K
    8   FORMAT(14I5)
        DO 254 I=1,364
  254   A(I)=FLOAT(K(I))/10000.0
        GOTO 9
        DO 10 I=1,362
   10   A(I)=FLOAT(K(I+2))/1900.0
        A(363)=-0.0634875
        A(364)=-0.0471278
    9   P(1)=1
        P(2)=0
        P(3)=0
        P(4)=0
        P(5)=0
        P(6)=1
        DO 18 J=1,1820
        JJ=J-364
        IF(JJ.LT.1) GOTO 11
        U(JJ)=P(1)
   11   FB=-P(6)-P(4)-P(3)+P(5)+P(1)+6
   12  IF(FB.LT.2) GOTO 14
        FB=FB-3
        GOTO 12
   14   DO 16 L=1,5
   16   P(L)=P(L+1)
   18   P(6)=FB
C       CROSSCORRELATION
        DO 46 KC=1,16
        C=KC
C       FORMULA U(I-J)*U(I-C-J)*A(I)
        WRITE(2,24) C
   24   FORMAT(//2X,41HAMMONIUM NITRATE PLANT - 2ND ORDER KERNEL,
     1  5H, C =,I2)
        DO 46 JJ=1,61
        J=JJ-1
        Y=0.0
        DO 34 I=1,364
        IF((U(I+728-J).EQ.0).OR.(U(I+728-C-J).EQ.0)) GOTO 34
        YL=FLOAT(U(I+728-J))*SA*FLOAT(U(I+728-C-J))*SA*A(I)
        Y=Y+YL
   34   CONTINUE
        Y=(Y/DEN)
        WRITE(2,38) J,Y
   38   FORMAT(I4,F12.6)
   46   CONTINUE
        STOP
        END
```

REFERENCES

REFERENCES

1. Volterra, V. : 'Theory of functionals and of integral and integrodifferential equations'. Blackie and Son, London, 1930.

2. Wiener, N. : 'Differential space'. J.Math and Phys., 2, 1923, pp.131-174.

3. Wiener, N. : 'Response of a nonlinear device to noise'. Radiation Laboratory, MIT, Cambridge, Mass., Report No.V16S, 1942.

4. Wiener, N. : 'Nonlinear problems in random theory'. The Technological Press, Cambridge, Mass., and John Wiley, New York, 1958.

5. Brilliant, M.B. : 'Theory of the analysis of nonlinear systems', MIT Research Laboratory of Electronics, Cambridge, Mass., Tech. Report 345, March,1958.

6. George, D.A. : 'Continuous nonlinear systems'. ibid., Tech. Report 355, July, 1959.

7. Zames, G. : 'Nonlinear Operators for system analysis'. ibid., Report No.370, 1960.

8. Zames, G. : 'Functional analysis applied to nonlinear feedback systems'. IEEE Trans. on Circuit Theory, Sept. 1963, pp.392-404.

9. Barrett, J.F. : 'The use of functionals in the analysis of nonlinear systems'. Journal of Electronics and Control, Vol.15, No.6, December, 1963, pp.567-615.

10. Flake, R.H. : 'Volterra series representation of nonlinear systems'. Trans. AIEE, January, 1963, pp.330-335.

11. Bansal, V.S. : 'Volterra functional analysis of nonlinear systems'. Ph.D. Thesis, Dept. of Electronics and Electrical Engineering, University of Glasgow,1970.

12. Stark, L. : 'The pupillary control: its nonlinear adaptive and stochastic engineering design characteristics'. Automatica, vol.5, pp.655-676, Pergammon Press, 1969.

13. Narayanan, S. : 'Application of Volterra series to intermodulation distortion analysis of transistor feedback amplifiers'. IEEE Transactions on Circuit Theory, vol.CT-17, No.4, pp.518-527, November 1970.

14. Goldman, J. : 'A Volterra series description of crosstalk interference in communication systems'. The Bell System Technical Journal, Vol.52, No.5, pp.649-668, May - June, 1973.

15. Mc Fee, R. : 'Determining the response of nonlinear systems to arbitrary inputs'. AIEE Transactions, Part II, Applications and Industry, vol.80, September 1961,pp189-193.

16. Lee, Y.W. and Schetzen, M. : 'Measurement of the Wiener kernels of a nonlinear system by crosscorrelation'. International Journal of Control, Vol.II, No.3, September 1965, pp.237-254.

17. Widnall, W.S. : 'Measurement of a second-degree Wiener kernel in a nonlinear system by crosscorrelation'. MIT Research Laboratory of Electronics, Quarterly progress report 65, April, 1962.

18. Barker, H.A. and Raeside, D. : 'Linear modelling of multivariable systems with pseudorandom binary input signals'. Automatica, Vol.4, pp.393-416, Pergammon Press, 1968.

19. Nikiforuk, P.N. and: Gupta, M.M. 'A bibliography of the properties, generation and control systems applications of shift-register sequences'. Int.J. Control, 9, 217-234, 1969.

20. Godfrey, K.R. : 'The application of pseudorandom sequences to industrial processes and nuclear power plant'. 2nd IFAC Symp. Identification and process parameter estimation, Prague, Section 7.1, Part II, June 1970.

21. Gyftopoulos, E.P. and Hooper, R.J. : 'Signals for transfer-function measurement in nonlinear systems' in 'Noise analysis in nuclear systems'. USAEC Symposium series 4, TID - 7679, 1964.

22. Hooper, R.J. and Gyftopoulos, E.P. : 'On the measurement of characteristic kernels of a class of nonlinear systems', in 'Neutron noise, wave and pulse propagation'. USAEC Conference Report 660206, 1967.

23. Simpson, H.R. : Statistical properties of a class of pseudorandom sequences'. Proc.IEE, Vol.113, No.12, pp.2075 - 2080, 1966.

24. Ream, N. : 'Nonlinear identification using inverse-repeat m sequences'. ibid., vol.117, No.1, pp.213-218, 1970.

25. Barker, H.A. and Pradisthayon, T. : 'High-order autocorrelation functions of pseudorandom signals based on m sequences'. ibid, vol.117, No.9, pp.1857-1863, 1970.

26. Harland, G.E., Panko, M.A., Gill, K.F. and Schwarzenbach, J. : 'Pseudorandom signal testing applied to a diesel engine'. Control, February, 1969, pp.137 - 140.

27. Chang, J.A., Owen, A.G., Zaman, M. and Griffin, A.W.J. : 'Dynamic modelling of a Four Stand cold rolling steel mill using multilevel pseudorandom sequences'. Measurement and Control, vol.1, April 1968, pp. T80-T84.

28. Godfrey, K.R. and Devenish, M. : 'An experimental investigation of continuous gas chromatography using pseudorandom binary sequence'. ibid., vol.2, June 1969, pp.228-233.

29. Kadri, F.L. : 'Nonlinear plant identification by crosscorrelation'. IFAC Symposium on digital simulation of continuous processes, Gyor, Hungary, 1971.

30. Godfrey, K.R. and Devenish, M. : 'An experimental investigation of continuous gas chromatography using pseudorandom binary sequence'. Measurement and Control, vol.2, June 1969, pp.228-233.

31. Godfrey, K.R. and Briggs, P.A.N. : 'Identification of processes with direction-dependent dynamic responses'. Proc.IEE, 1972, vol.119, No.12, pp.1733-1739.

32. Zierler, N. : 'Linear recurring sequences'. J.Soc. Indust. Appl. Math, 1959, 7, pp.31-48.

33. Barker, H.A. : 'Synchronous sampling theorem for nonlinear systems'. Electron. Lett. 1969, 5, p.657.

34. Davies, A.C. : 'Calculations relating to delayed m sequences'. Electron. Lett., 1968, 4, pp.291-292.

35. Kadri, F.L. : 'Extensions to the stochastic functional analysis of nonlinear control systems'. Ph.D. thesis, University of Wales Institute of Science and Technology, Cardiff, May 1973.

36. Simpson, R.J. and Power, H.M. : 'Correlation techniques for the identification of nonlinear systems'. Trans. Inst. Meas. and Control, 1972, 5, pp.316 -321.

37. Godfrey, K.R. and Murgatroyd, W. 'Input transducer errors in binary crosscorrelation experiments'. Proc. IEE, 1965, vol.112, No.3, pp.565-573.

38. Barker, H.A. and Ambati, S. : 'Nonlinear sampled data system analysis by multidimensional Z transforms'. Proc.IEE, vol.119, No.9, September 1972, pp.1407-1413.

39. Schetzen, M. : 'Synthesis of a class of non-linear systems'. Int.J.Control, 1965, vol.1, No.1, pp.401-414.

40. Godfrey, K.R. : 'Three level m sequences'. Electronic Letters, 1966,vol.2, pp.241-242.

41. Barker, H.A. : 'Reference phase of pseudorandom signals'. Proc.IEE, 1969, vol.116, pp.429-435.

42. Macleod, C.J. : 'Elimination of the effect of low frequency disturbances on process identification'. Int. J. Control, 1972, vol.15, No.5, pp.839-847.

43. Briggs, P.A.N. and Godfrey, K.R. : 'Pseudorandom signals for the dynamic analysis of multivariable systems'. Proc.IEE, vol.113, No.7, July 1966, pp.1259-1267.

44. Cummins, J.D. : 'The simultaneous use of several pseudorandom binary sequences in the identification of linear multivariable dynamic systems'. Report No. AEEW-M 507, Control and Instrumentation Division, Atomic Energy Establishment, Winfrith, Dorchester, Dorset 1965.

45. Ream, N. : 'Testing a 2-input linear system with periodic binary sequences'. Proc.IEE, vol.114, No.2, February 1967, pp.305-308.

PUBLICATIONS

Performance of antisymmetric pseudorandom signals in the measurement of 2nd-order Volterra kernals by crosscorrelation

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Indexing terms: Identification, Nonlinear systems, Correlation methods

ABSTRACT

The paper is concerned with the measurement of the 2nd-order kernel in a Volterra-series representation of a nonlinear system by continuous or discrete crosscorrelation using an antisymmetric pseudorandom input signal derived from an m sequence. It is shown that the crosscorrelation measurements are related to the corresponding kernel values by a set of equations which may be structured into a number of independent subsets. The m-sequence properties determine how the maximum order of the subsets for off-diagonal values is related to the upper bound of the arguments for nonzero kernel values, which is used as an index of performance. The performance of signals derived from binary, ternary and quinary m sequences is investigated, and the characteristic polynomials and performance indexes of signals with superior performance are tabulated. Comparison of the results obtained demonstrates the advantages of ternary signals in this application, and an example is used to illustrate the solution of a typical problem.