Review of Industrial Organization
https://doi.org/10.1007/s11151-024-09991-9

Cartel Damages Claims, Passing-On, and Passing-Back

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Abstract

Firms can mitigate the harm of an input cartel by passing on some of the higher cost to their customers by raising their own prices. Recent damages claims have highlighted that firms may also respond by reducing the prices that are paid to their suppliers of complementary inputs; the firm thereby passes back some harm upstream. To provide guidance for practitioners as to how such effects together affect the division of the harm, we derive the equilibrium 'passing-on' and 'passing-back' effects in a successive oligopolies model where one of two inputs is cartelised. We show that the passing-back effect is larger when there is greater market power in the complementary input sector. This reduces the passing-on effect. The complementary input suppliers can incur substantial harm, and the harm that is inflicted on the cartel's direct and/or indirect purchasers can thereby be reduced.

Keywords Damages · Cartel overcharge · Cost pass-through · Vertically related markets

JEL Classification D43 · K21 · L13 · L40

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Published online: 12 September 2024

1 Introduction

Cartel victims in many jurisdictions can sue for damages to compensate them for the harm they have suffered; but calculating that harm can be fraught with difficulties. One difficulty arises from the need to estimate the 'overcharge': how much more the buyers paid for each unit as compared with a competitive outcome. Another difficulty arises due to the so-called "passing-on effect", where the harm to downstream firms—the 'direct purchasers'—depends not only on the overcharge but also on whether the direct purchasers raised their own prices to their customers: the 'indirect purchasers'.

Two high-profile damages claims in the UK (that will be reviewed in Sect. 2) have highlighted a further complication that relates to a "passing-back effect": when downstream firms mitigate the harm of an input cartel by paying lower prices to their suppliers of other complementary inputs. These passing-back effects have two main implications for the harm of an input cartel: First, the complementary input suppliers can experience some harm that would have otherwise fallen entirely on the direct and/or indirect purchasers. Second, the harm that is inflicted on the direct and indirect purchasers may be reduced, because they benefit from the lower complementary input prices.

Passing-back effects have yet to be investigated in the literature, so there are a number of important policy questions that are currently unanswered: What market conditions affect the size of the passing-back effect? How is the passing-back effect related to the overcharge and the passing-on effect? How does the combination of the passing-on and passing-back effects influence the division of the harm among the direct purchasers, indirect purchasers, and complementary input suppliers?

In a successive oligopolies model where downstream firms must source two inputs in fixed proportions to produce differentiated products, we analyse the effects of an input cartel when downstream firms can pass the overcharge: (i) on to their customers; and (ii) back to the complementary input suppliers by paying them lower prices. The cartel is modelled as an exogenous price increase of one input. We derive the equilibrium passing-on and passing-back effects and show how they influence the division of the harm among the market participants. We also demonstrate how this depends upon the concentration of the complementary input sector and the downstream cost pass-through rate.

1.1 Our Main Results

To help summarise our main results, Fig. 1 illustrates the harm of an input cartel. Figure 1a shows the simple case where there is only a passing-on effect: the input cartel raises direct purchasers' costs from c to c', so their prices rise from p to p', which reduces the industry quantity from Q to Q'. The overcharge harm is caused by an overcharge of $(c' - c)$ paid by direct purchasers on the Q' units produced. However, $(p' - p)$ of this harm per unit is passed on to indirect purchasers. The volume harm is associated with the loss in volume, $Q - Q'$, where direct purchasers incur additional harm of $(p - c)$ per unit lost and indirect purchasers also lose the associated consumer surplus.

Figure 1b depicts the more interesting case where there is market power in the complementary input sector. Here, the complementary input suppliers expect direct purchasers to pass on some of the overcharge to indirect purchasers, so they anticipate that the quantity demanded of the final products will fall. This in turn will lead to a decrease in demand for the complementary input, so its price falls. This passing-back effect limits the increase in the downstream firms' marginal cost to $(c'' - c)$, so the rise in the downstream price is limited to $(p'' - p)$, and there is a smaller loss in volume, $Q - Q''$. Consequently, the existence of the passing-back effect reduces the passing-on effect.

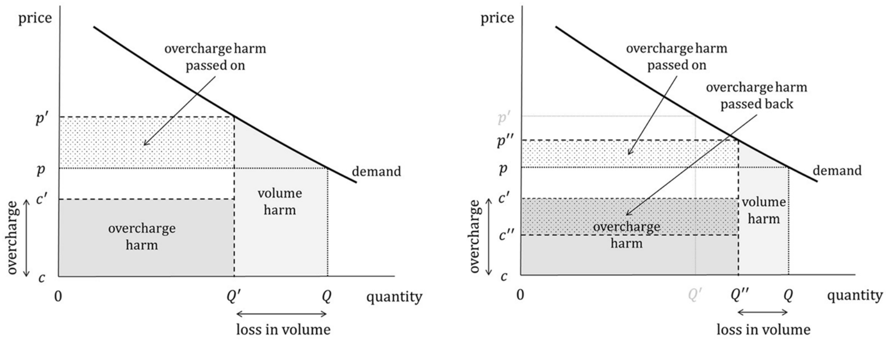
We show that the passing-back effect is larger when the complementary input sector is more concentrated. The reason is that the price of the complementary input is more responsive to changes in demand, so an input cartel that decreases demand for the complementary input leads to a larger fall in the input's price. This in turn leads to a smaller passing-on effect. This arises because the larger decrease in the price of the complementary input offsets more of the increase in downstream marginal cost that results from the overcharge.

The larger passing-back effect causes the complementary input suppliers to incur a greater share of the overcharge and total harm when the complementary input sector is more concentrated (where the total harm is the sum of the overcharge and volume harm). Furthermore, the smaller passing-on effect reduces the direct and/or indirect purchasers' shares of the overcharge and total harm.

In contrast, the downstream cost pass-through rate affects only the passing-on effect and not the passing-back effect. Thus, a higher downstream cost pass-through rate reduces the direct purchasers' share of the overcharge harm and total harm at the expense of the complementary input suppliers and/or indirect purchasers.

In an extension, we demonstrate that our successive oligopolies model can be used to determine quickly the results of other market settings with different contractual arrangements. Our focus is on a setting in which the complementary input prices are determined by negotiation as modelled by the symmetric Nash bargaining solution. We show that when the complementary input suppliers have greater bargaining power there is a greater passing-back effect and a smaller passing-on effect. Again, the reason is that the price of the complementary input is more responsive to changes in demand. We also explain how bargaining power affects the division of the harm.

Our results have two important implications for damages claims in practice: First, passing-back effects can inflict significant harm on complementary inputs suppliers. Hence, there is a case for them to be encouraged (or even allowed in some jurisdictions) to sue for compensation. Our results show that this case is stronger when complementary input suppliers have greater market power. Second, since a passing-back effect can reduce the harm inflicted on direct/indirect purchasers, it may need to be estimated when calculating their damages. This would involve analysing competition between the complementary input suppliers, even though they may not be part of the trial.



(a) harm from an overcharge of $c' - c$ with passing on (from p to p')

(b) harm from an overcharge of $c' - c$ with passing back (from c' to c'') and passing on (from p to p'')

Fig. 1 The harm caused by an input cartel

The rest of the paper is structured as follows: In Sect. 2, we provide an overview of damages claims in the EU and UK and discuss the related literature. In Sect. 3, we present the successive oligopolies model and characterise the equilibrium. In Sect. 4, we investigate the implications of the equilibrium passing-on and passing-back effects. In Sect. 5, we extend our analysis to a setting where the price of the complementary input is determined by negotiation. In Sect. 6, we discuss the robustness of our results to substitution possibilities among the inputs and non-linear pricing. We offer concluding remarks in Sect. 7. All proofs are relegated to the Appendix.

2 Further Context and Related Literature

In this section, we first provide an overview of damages claims in the EU and UK. Then we discuss the related theoretical literatures on damages and cost-pass through.

2.1 Damages Claims in the EU and UK

In Europe, after an infringement of competition law it has now "become normal for the victims of cartels to bring 'follow-on' actions for damages" (Whish & Bailey, 2021, p.311). This represents a significant change from 10 years ago, when such actions were rare. Part of the reason for this change is due to the adoption of the Damages Directive by the EU in November 2014,[1] with its key features later being incorporated into UK law following the UK's withdrawal from the EU.[2]

[1] Directive 2014/104/EU of the European Parliament and of the Council on certain rules governing actions for damages under national law for infringements of the competition law provisions of the Member States and of the European Union, *Official Journal of the European Union* L 349, 5 November, 2014.

[2] For further details, see Coulson and Blacklock (2021).

The Damages Directive states that anyone "who has suffered harm caused by an infringement of competition law is able to claim and to obtain full compensation for that harm".[3] Full compensation is defined as placing a victim "in the position in which that person would have been had the infringement of competition law not been committed".[4] Furthermore, full compensation "shall not lead to overcompensation, whether by means of punitive, multiple or other types of damages".[5]

The compensatory principle at the heart of the Damages Directive implies that claimants can be direct or indirect purchasers of the infringers, and there is nothing to stop suppliers to the infringers from being claimants either. Moreover, it also follows that compensation can be claimed not just for the harm that is associated with the units bought/sold—the overcharge harm—but also for the harm that is associated with the loss in volume: the volume harm. However, in practice the latter is more difficult to prove, so estimating the overcharge harm is often an important part of damages claims.

This approach contrasts with the US in two main ways: First, only direct purchasers can sue for damages in the US at the federal level (although indirect purchasers can sue in some states). Second, successful US claimants can obtain 'treble damages', which are usually estimated as three times the overcharge harm.[6]

Due to difficulties in achieving full compensation, the European Commission published two guidance documents to assist practitioners and national courts: First, European Commission (2013) primarily discusses the methods to quantify the overcharge. Second, European Commission (2019) discusses the methods of quantifying how much of an overcharge has been passed on. Neither of these guidance documents address the quantification challenges that relate to the passing-back effects, because such issues have only recently come to light as the case law in the UK has evolved.

The passing-back effect was first mentioned in a damages claim brought by Sainsbury's, a leading UK supermarket, against Mastercard.[7] Sainsbury's claimed that it had been overcharged by Mastercard for "Merchant Services". In response, Mastercard argued, amongst other things, that Sainsbury's would have passed-on any overcharge.

In its June 2020 judgment, the Supreme Court of the UK outlined the four ways in which a buyer—in this case, Sainsbury's as the merchant—could respond to an overcharge (emphasis added):

[3] *Sup.* note 1, Article 3, paragraph 1.

[4] *Id.* Article 3, paragraph 2.

[5] *Id.*, Article 3, paragraph 3.

[6] For more discussion of the US approach, see Schinkel et al. (2008), Verboven and van Dijk (2009), Boone and Müller (2012), and Harrington (2017).

[7] Judgment in *Sainsbury's Supermarkets Ltd (Respondent) v. Visa Europe Services LLC and others (Appellants); Sainsbury's Supermarkets Ltd and others (Respondents) v. Mastercard Incorporated and others (Appellants)* Supreme Court, UKSC 24, 17 June, 2020.

> (i) a merchant can do nothing in response to the increased cost and thereby suffer a corresponding reduction of profits or an enhanced loss; or (ii) the merchant can respond by reducing discretionary expenditure on its business such as by reducing its marketing and advertising budget or restricting its capital expenditure; or *(iii) the merchant can seek to reduce its costs by negotiation with its many suppliers;* or (iv) the merchant can pass on the costs by increasing the prices which it charges its customers.[8]

It concluded that the merchants were entitled to claim the overcharge as the measure of their loss; but "if there is evidence that they have adopted either option (iii) or (iv) or a combination of both to any extent, the compensatory principle mandates the court to take account of their effect".[9] This judgment indicates that damages should take account of the sizes of: the overcharge; the passing-on effect (option iv); and the passing-back effect (option iii).

The importance of the passing-back effect for complementary inputs is further demonstrated in the claim that was brought by Royal Mail and BT against DAF and that was heard at the UK Competition Appeal Tribunal (CAT).[10] Royal Mail and BT claimed that they had been overcharged for trucks that they had purchased from DAF between 1997 and 2011 due to, amongst other things, DAF price-fixing with other truck manufacturers: MAN; Volvo/Renault; Daimler; and Iveco. DAF was granted permission to bring a defence concerned with a passing-back effect on the claimants' purchases of bodies and trailers, which are complements to trucks because they attach to trucks and contain the cargo.[11]

In the trial, DAF argued that Royal Mail and BT recovered 6% and 25% of the overcharge from passing-back effects, respectively,[12] but the claimants argued no such effects existed.[13] In its Judgment, the CAT accepted that passing-back effects were theoretically plausible because "trailer/body suppliers might have responded to the fall in demand [...] by reducing their selling prices and margins".[14] Furthermore, it noted that trailer/body suppliers "would consequently have a potential damages claim against the truck suppliers".[15] However, it ultimately dismissed the defence, because DAF failed to establish the existence of passing-back effects in these cases.[16]

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8 *Id.*, paragraph 205.

9 *Id.* paragraph 206.

10 *Royal Mail Group Limited v. DAF Trucks Limited & Others*; *BT Group PLC & Others v DAF Trucks Limited & Others*, Judgment: Expert Evidence and Amendment, Competition Appeal Tribunal, 1284-1290/5/7/18 (T), 13 May 2021.

11 *Id.* paragraph 21.

12 *Royal Mail Group Limited v. DAF Trucks Limited & Others*;; *BT Group PLC & Others v DAF Trucks Limited & Others*, Judgment, Competition Appeal Tribunal, 1284-1290/5/7/18 (T), 7 February 2023, paragraph 491.

13 *Id.* paragraph 492.

14 *Id.* paragraph 508.

15 *Id.* paragraph 490.

16 *Id.* paragraph 509.

The implications of passing-back effects are currently under-researched because—in addition to such effects not being addressed in the guidance documents mentioned above—they have also not been investigated in the academic literature (which we will discuss below). The model developed in this paper goes some way to address this gap. It should also be of interest for scholars and practitioners outside of the UK, because it is still important to understand potential sources of inaccuracy in damages estimates within jurisdictions where the case law currently differs from the UK.

2.2 Damages Literature

There are two main strands to the academic literature on damages: The first investigates the accuracy of estimates of the harm inflicted on the victims. The second analyses how damages can lead to unintended consequences.[17] Our paper is related to the former; consequently we focus on that. All of the papers analyse passing-on effects and do not consider passing-back effects.

Similar to our approach, many papers model an input cartel as an exogenous increase in the marginal costs of downstream firms. Hellwig (2007) explains the effects on the profits of direct purchasers. Kosicki and Cahill (2006) focus on the harm that is incurred by indirect purchasers. Verboven and van Dijk (2009) show how the total harm that is inflicted on direct purchasers' profits can be estimated as a discount on the overcharge. Basso and Ross (2010) show the inaccuracy of using the overcharge harm to estimate damages for direct purchasers. Boone and Müller (2012) show how the share of the total harm between direct and indirect purchasers could be calculated.

Unlike us, some other papers model an input cartel as an exogenous decrease in competition. In particular, Han et al. (2008) consider a vertical industry with many different levels and shows how a cartel at one level inflicts harm on all other downstream and upstream firms. Bet et al. (2021) analyse how the harm that is inflicted on a downstream firm depends upon whether its downstream rival is vertically integrated or not.

2.3 Cost-Pass Through Literature

The cost pass-through literature aims to uncover the determinants of the (direct) effect of an increase in marginal cost on equilibrium prices (e.g., Anderson et al., 2001; Bulow & Pfleiderer, 1983; Seade, 1985; Ritz, 2024; and Weyl & Fabinger, 2013). Our analysis contributes to this literature by showing how an exogenous downstream cost increase is passed on when some endogenous input suppliers have market power. We show that the direct effect of an increase in marginal cost on the downstream price is offset by an additional smaller indirect effect that is associated with the decrease in the equilibrium price of an input.

[17] See, for example, Harrington (2004), Schinkel et al. (2008), and Bodnar et al. (2023).

A related literature analyses cost pass-through in vertically related markets when there is an exogenous increase in the marginal costs of upstream firms (e.g., Adachi & Ebina, 2014a and 2014b; and Gaudin, 2016). Our paper differs in that our focus is on an exogenous increase in the marginal costs of downstream firms (interpreted as a price rise of one input). We analyse the resultant equilibrium effects on the downstream price and another endogenous input price.

3 Model

3.1 Basic Assumptions

Suppose that there is a market in which $n \geq 1$ downstream firms (henceforth retailers) wish to sell differentiated products to final consumers. The production of each product requires a retailer to combine two complementary inputs: A and B. Specifically, one unit of each final product always requires φ_A and φ_B units of inputs A and B, respectively.[18] For a given input $S = \{A, B\}$, let w_{Si} be the price that retailer $i = \{1, ..., n\}$ pays for each unit of input S. Thus, if we let κ denote the marginal costs associated with retailing, it follows that the constant marginal cost of retailer i is $c_i \equiv \kappa + \Sigma_S \varphi_S w_{Si}$.[19]

Without loss of generality, suppose that a cartel fixes the price of input A. Following much of the literature, the cartel is modelled by an exogenous price increase and we wish to analyse the equilibrium effects on the downstream and input B sectors. To determine the equilibrium prices of input B and the final products, we analyse a successive oligopoly model in which the quantities and prices of input B are determined first, and then the quantities and prices of the final products are determined.

To ensure downstream competition among the differentiated products is modelled as generally as possible, we use the conjectural variations approach. This allows us to nest various forms of competition that span from monopoly to no market power, which includes Cournot, Bertrand and perfect competition. In the input B sector, we assume that there are $m \geq 1$ homogeneous suppliers that compete in quantities. Given that input B suppliers are undifferentiated, we can analyse the full competitive spectrum from monopoly ($m = 1$) to no market power ($m \to \infty$), without the need for conjectural variations.

Let q_i represent the quantity of retailer i, where $\boldsymbol{q} = \left(q_1, \ldots, q_i, \ldots, q_n\right)$ and $Q \equiv \Sigma_i q_i$. Suppose the inverse demand function of retailer i's product is:

$$p_i(\boldsymbol{q}) = v - \frac{1}{\beta}\left[(1-\sigma)nq_i + \sigma\left(q_i + \sum_{j \neq i} q_j\right)\right], \tag{1}$$

[18] This implies that retailers face a Leontief (or perfect complements) production function. In section 6.1, we discuss the implications of other production functions, where the inputs are imperfect complements.

[19] In section 6.2, we discuss the implications of non-linear pricing.

where: $v > 0$; $\beta > 0$; and $\sigma \in [0, 1]$ represents the degree of substitutability between the products. The products are independent when $\sigma = 0$ and are increasingly substitutable as σ rises; they are perfect substitutes when $\sigma = 1$. As per Shubik and Levitan (1980), this inverse demand function can be derived by maximising the following net surplus function of a representative consumer with respect to q_i:

$$CS(\boldsymbol{q}) = \sum_{i=1}^{n} \left(v - p_i\right) q_i - \frac{n}{2\beta}\left[(1-\sigma)\sum_{i=1}^{n} q_i^2 + \frac{\sigma}{n}\left(\sum_{i=1}^{n} q_i\right)^2\right].$$

An advantage of this demand system is that it isolates the competition effect of product differentiation, because there is no market expansion effect. To see this, note that if $q_i = q$ for all i, then $p_i(\boldsymbol{q}) = v - \frac{nq}{\beta}$ for all i.[20]

Finally, let $\varphi_S x_{Sk}$ represent the quantity of input S sold by supplier k where $X_S \equiv \varphi_S \Sigma_k x_{Sk}$. Given that the demand for each input is derived from the demand for the final products, in equilibrium we must have that $X_S = \varphi_S Q$ for any S. Let $c_B \geq 0$ represent the marginal cost of input B. All fixed costs are normalised to zero. We restrict attention to symmetric subgame perfect equilibria, where $x_{Bk} = x_B$ for all k, and $w_{Si} = w_S$ and $q_i = q$ for all i. We drop subscripts when there is no ambiguity.

3.2 Equilibrium Analysis

We first solve for the downstream equilibrium: Retailer i's profit function is $\pi_{Ri}(\boldsymbol{q}) = \left(p_i(\boldsymbol{q}) - c_i\right) q_i$. Suppose that when retailer i changes q_i by a small amount the retailer conjectures that its rivals will change their quantities by $\frac{\partial q_j}{\partial q_i} = \frac{\theta - 1}{n-1}$ for all $j \neq i$. Thus, the conduct parameter is $\sum_{j \neq i} \frac{\partial q_j}{\partial q_i} = \theta - 1$; and the first-order condition of retailer i is:

$$\frac{\partial \pi_{Ri}(\boldsymbol{q})}{\partial q_i} = p_i(\boldsymbol{q}) - c_i + \left(\frac{\partial p_i(\boldsymbol{q})}{\partial q_i} + (\theta - 1)\frac{\partial p_i(\boldsymbol{q})}{\partial q_j}\right) q_i = 0, \tag{2}$$

where $\frac{\partial p_i(\boldsymbol{q})}{\partial q_i} = -\frac{(1-\sigma)n+\sigma}{\beta}$ and $\frac{\partial p_i(\boldsymbol{q})}{\partial q_j} = -\frac{\sigma}{\beta}$ from (1). Substituting into (2) and imposing symmetry yields each retailer's symmetric equilibrium quantity:

$$q^*(c) = \frac{\beta(v - c)}{n\left[2 - \sigma\left(1 - \frac{\theta}{n}\right)\right]}. \tag{3}$$

The Cournot outcome is obtained from (2) when $\theta = 1 \equiv \theta^c$; and the local monopoly outcome occurs when $\theta = n \equiv \theta^m$.[21] The perfectly competitive outcome requires $\theta = -\frac{n(1-\sigma)}{\sigma} \equiv \theta^p$. This can be interpreted as a setting where each differentiated

[20] For more details on the demand system, see Choné and Linnemer (2020).

[21] The latter is often referred to as the perfect collusion outcome; but it could also be interpreted as when retailers have exclusive territories.

product is sold by at least two homogeneous retailers that compete in prices. The Bertrand outcome is derived by $\theta = \frac{1}{1+\frac{\sigma}{1-\sigma}\frac{n-1}{n}} \equiv \theta^b \in (0, 1)$ for any $\sigma \in (0, 1)$ and $n \geq 2$.[22] Note that θ^p and θ^b equal the usual value of 0 when the products are homogeneous ($\sigma = 1$). Furthermore, $\theta^b = \theta^c = 1$ when products are independent ($\sigma = 0$), so both Bertrand and Cournot yield the monopoly outcome – despite $\theta^m > 1$ – because $\lim_{\sigma\to 0}\frac{\partial p_i(\boldsymbol{q})}{\partial q_j} = 0$.

Now consider the input B sector: Given that $Q^*(c) = nq^*(c)$ units will be bought by final consumers in equilibrium, it follows that $X_B = \varphi_B nq^*(c)$ units of input B will be demanded by retailers. Substituting $c = \kappa + \varphi_A w_A + \varphi_B w_B$ into $X_B = \varphi_B nq^*(c)$ and rearranging yields the inverse demand curve for input B:

$$w_B\left(w_A, X_B\right) = \frac{1}{\varphi_B}\left[v - \kappa - \varphi_A w_A - \frac{X_B}{\varphi_B}\left(\frac{2 - \sigma\left(1 - \frac{\theta}{n}\right)}{\beta}\right)\right]. \tag{4}$$

Thus, the profit function of input B supplier k is $\pi_{Bk}\left(X_B\right) = \left(w_B\left(w_A, X_B\right) - c_B\right)\varphi_B x_{Bk}$.

Proposition 1 derives the equilibrium prices in both sectors for a given w_A:

Proposition 1 *For all* $v > \kappa + \varphi_A w_A + \varphi_B c_B$ *and* $\theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]$, *the equilibrium price of input B is*:

$$w_B^*(w_A, m) = c_B + \frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{\varphi_B(m+1)} \in \left[c_B, \frac{v - \kappa - \varphi_A w_A + \varphi_B c_B}{2\varphi_B}\right], \tag{5}$$

so the marginal cost of each retailer is $c(w_A, w_B^*(w_A, m)) = \kappa + \varphi_A w_A + \varphi_B w_B^*(w_A, m)$. *The downstream equilibrium price is*:

$$p^*(w_A, w_B^*(w_A, m), \sigma, n, \theta) = v - \frac{v - c(w_A, w_B^*(.))}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} \in \left[c(w_A, w_B^*(.)), \frac{v + c(w_A, w_B^*(.))}{2}\right]. \tag{6}$$

The downstream equilibrium price equals marginal cost when the downstream sector is perfectly competitive: $p^*(., \theta^p) = c(w_A, w_B^*(.))$; and it takes the usual form under local monopoly: $p^*(., \theta^m) = \frac{v+c(w_A, w_B^*(.))}{2} > c(w_A, w_B^*(.))$. Under Cournot and Bertrand competition with $n \geq 2$, the price is at the monopoly level when the products are independent ($\sigma = 0$). When products are substitutes – $\sigma > 0$ – then $p^*\left(., \theta^b\right) < p^*(., \theta^c)$ because price competition is more intense than is quantity competition.

Similarly, the equilibrium price of input B is equal to marginal cost when the number of input B suppliers tends to infinity: $\lim_{m\to\infty} w_B^*(w_A, m) = c_B$. It rises above

22 Details are available upon request.

c_B as the number of input B suppliers decreases, and it equals the monopoly level when there is only one input B supplier ($m = 1$).

4 The Effects of an Input Cartel

In this section, we investigate the equilibrium effects of a rise in the unit price of input A from w_A to $w'_A = w_A + \Delta$. We interpret this as the result of an input cartel so that $\Delta > 0$ represents the overcharge.[23] We first analyse the effects on the prices downstream ("passing-on effect") and the prices upstream in the input B sector ("passing-back effect"). We then consider how the passing-on and passing-back effects influence the share of the overcharge harm that is incurred by the other market participants, before doing the same for the total harm that takes into account the volume harm.

Given our input cartel interpretation, it follows that the retailers are the "direct purchasers" of input A, final consumers are the "indirect purchasers" of input A, and the suppliers of input B are the "complementary input suppliers". To simplify notation, we write all expressions as a function of w_A only, so (for example) $w_B^*(w_A, m) \equiv w_B^*(w_A)$ and $p^*(w_A, w_B^*(w_A, m), \sigma, n, \theta) \equiv p^*(w_A)$.

4.1 Passing-On and Passing-Back

The effects on the equilibrium prices follow immediately from Proposition 1:

Corollary 1 *For all $v > \kappa + \varphi_A w'_A + \varphi_B c_B$ and $\theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]$, an increase in the price of input A from w_A to $w'_A = w_A + \Delta$ decreases the equilibrium price of input B*:

$$w_B^*(w'_A) - w_B^*(w_A) = -\frac{\varphi_A \Delta}{\varphi_B(m+1)} \leq 0, \tag{7}$$

where the inequality is strict $\forall m < \infty$, and strictly increases the downstream equilibrium price:

$$p^*(w'_A) - p^*(w_A) = \frac{m}{m+1} \frac{\varphi_A \Delta}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} > 0. \tag{8}$$

To understand the intuition, first consider the case of $m \to \infty$. Here, the increase in w_A has no effect on the price of input B – as $\lim_{m\to\infty} w_B^*(w_A, m) = c_B$ – so the increase in the downstream price is similar to that analysed by other papers in the literature. Thus, the magnitude of the downstream price rise is determined by the

[23] It is worth noting that our analysis applies to any increase in the price of input A, regardless of the source of the price increase. It can also be applied to a price decrease for input A as well.

marginal cost increase – $\varphi_A \Delta$ – multiplied by the downstream cost pass-through rate, which from (6) is:

$$\frac{\partial p^*}{\partial c} = \frac{1}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} \equiv \tau \in \left[\frac{1}{2}, 1\right] \quad \forall \theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]. \tag{9}$$

Given that this case is well understood, our focus henceforth is on $m < \infty$.

When $m < \infty$, an increase in w_A will also affect $w_B^*(w_A)$. Specifically, input B suppliers will expect retailers to pass on a proportion of the cost rise to final consumers and that this will reduce both the quantity demanded of the final products and demand for input B. Consequently, holding quantities constant, the price of input B will decrease by $-\frac{\varphi_A \Delta}{\varphi_B}$ from (4). However, this incentivises input B suppliers to reduce their quantities, which will raise the price of input B by $\frac{\varphi_A \Delta}{\varphi_B} \frac{m}{m+1}$. Summing these two effects yields (7). *This is the passing-back effect.*[24]

The decrease in the price of input B will also limit the resultant rise in $p^*(w_A)$. The reason is that, while the pass-through rate is the same as in (9), the associated rise in retail marginal costs is reduced, since:

$$c(w'_A) - c(w_A) = \varphi_A \Delta - \varphi_B \left(\frac{\varphi_A \Delta}{\varphi_B (m+1)}\right) \in \left[\frac{\varphi_A \Delta}{2}, \varphi_A \Delta\right]. \tag{10}$$

The first term on the right-hand side is the increase in cost that is associated with the rise in w_A; and the second term is the decrease that is associated with the fall in $w_B^*(w_A, m)$ in (7). Thus, (8) is given by the multiple of: the total increase in marginal costs, in (10); and the downstream cost pass-through rate, in (9). *This is the passing-on effect.*

Having identified the passing-on and passing-back effects, let us next consider how they vary with market characteristics:

Proposition 2 *For any given downstream pass-through rate*, τ, *as the input B sector increases in concentration* (*i.e.*, *as m falls*):

(*i*) *the passing-back effect gets larger*: $w_B^*(w'_A) - w_B^*(w_A)$ *is more negative*; *and*

(*ii*) *the passing-on effect gets smaller*: $p^*(w'_A) - p^*(w_A)$ *is less positive.*

Intuitively, when the input B sector is more concentrated, the decrease in demand for input B reduces the industry equilibrium quantity to a smaller extent. The reason

[24] When the input B suppliers have no market power ($m \to \infty$), the two effects are the same size and hence there is no passing-back effect: $\lim_{m\to\infty}\left(w_B^*(w'_A) - w_B^*(w_A)\right) = 0$. However, it is worth noting that there could still be a passing-back effect when the input B suppliers have no market power if, in contrast to our model, the market for input B is perfectly competitive with an upward-sloping supply curve. In that case, the fall in the quantity demanded of the final products will reduce the demand for input B, and this in turn will lead to a contraction in the supply of input B and a lower competitive price.

is that while each supplier reduces its quantity by more, there are fewer of them. Consequently, $w_B^*(w_A)$ falls to a greater extent, and the passing-back effect is larger. The larger passing-back effect implies that the increase in the retail marginal cost is smaller, because the rise in w_A is offset more by the larger decrease in $w_B^*(w_A)$. Consequently, $p^*(w_A)$ rises to a smaller extent, so the passing-on effect is smaller.

The effects of the other parameters that capture the intensity of downstream competition – n, σ, and θ – operate through the downstream pass-through rate: τ. So, to avoid unnecessary duplication, we present the results in terms of τ and then explain how τ is affected by n, σ, and θ below:

Proposition 3 *For any $m \geq 1$, as the downstream cost pass-through rate, τ, increases*:

i) the passing-back is unchanged: $w_B^*(w_A') - w_B^*(w_A)$ *is constant*; *and*

ii) the passing-on effect gets larger: $p^*(w_A') - p^*(w_A)$ *is more positive*.

Proposition 3 implies that any change that raises τ will increase the passing-on effect yet will not change the passing-back effect. This includes changing to a more intense form of competition: $\frac{\partial \tau}{\partial \theta} < 0$. Furthermore, under Cournot ($\theta^c = 1$) and Bertrand competition ($\theta^b = \frac{1}{1+\frac{\sigma}{1-\sigma}\frac{n-1}{n}}$), a higher τ can also result from more downstream firms – $\frac{\partial \tau}{\partial n} > 0$ – or from an increase in product substitutability: $\frac{\partial \tau}{\partial \sigma} > 0$.

4.2 Shares of the Overcharge Harm

In this subsection, we analyse how the overcharge harm is divided among the market participants when there is a passing-back effect. Note that an overcharge of Δ implies that $\varphi_A \Delta Q^*\left(w_A'\right)$ represents the (industry) overcharge harm, so that $\varphi_A \Delta$ is the (industry) overcharge harm per unit of the final products. We start by finding each market participant's proportion of the overcharge harm per unit of the final products:

The overcharge harm per unit of the final products that is incurred by direct purchasers is given by $p^*(w_A) - c\left(w_A\right) - [p^*(w_A') - c\left(w_A'\right)]$. Substituting for $c(.)$ from Proposition 1 and then manipulating yields:

$$\varphi_A(w_A' - w_A) - [p^*(w_A') - p^*(w_A)] + \varphi_B[w_B^*(w_A') - w_B^*(w_A)]. \tag{11}$$

The first term is the overcharge that direct purchasers pay on φ_A units of input A, where $\varphi_A(w_A' - w_A) = \varphi_A \Delta$. The second term is the extent to which the overcharge is offset by the passing-on effect for each unit of the final products, where from (8):

$$[p^*(w_A') - p^*(w_A)] = \varphi_A \Delta \left[\frac{m}{m+1}\tau\right] \equiv \varphi_A \Delta \left[\Omega_F(m, \tau)\right] > 0. \tag{12}$$

The third term is the extent to which the overcharge is offset by the passing-back effect for φ_B units of input B, where from (7):

$$\varphi_B[w_B^*(w_A') - w_B^*(w_A)] = -\varphi_A\Delta\left[\frac{1}{m+1}\right] \equiv -\varphi_A\Delta\left[\Omega_B(m)\right] < 0. \tag{13}$$

Substituting (12) and (13) into (11) yields the overcharge harm per unit of the final products on direct purchasers, given by:

$$\varphi_A\Delta\left[1 - \Omega_B(m) - \Omega_F(m,\tau)\right] = \varphi_A\Delta\left[\frac{m}{m+1}(1-\tau)\right] \equiv \varphi_A\Delta\left[\Omega_R(m,\tau)\right] \geq 0. \tag{14}$$

Thus, the terms in square brackets in (12)-(14)—$\Omega_B(m)$, $\Omega_F(m,\tau)$ and $\Omega_R(m,\tau)$—represent the proportions of the overcharge harm *per unit of input A* that is incurred by the indirect purchasers, complementary input suppliers, and direct purchasers, respectively.

Our next task is to show how the division of the overcharge harm varies with the market characteristics. However, before we focus on the proportions, notice that the (industry) overcharge harm—$\varphi_A\Delta Q^*\left(w_A'\right)$—is greater when there is a higher downstream pass-through rate—τ—and when there are more input B suppliers: m. The reason is that $Q^*\left(w_A'\right) = nq^*\left(w_A'\right)$ is strictly increasing in τ and m, because the downstream equilibrium price falls. For the former, it is due to more intense downstream competition (in terms of θ, n or σ); and for the latter it is due to a lower $w_B^*(w_A')$ reducing the marginal cost of retailers.

We now consider how the division of the overcharge harm changes as the input B sector becomes more concentrated:

Proposition 4 *For any given* $\tau \in \left[\frac{1}{2}, 1\right]$, *as the input B sector increases in concentration—as m falls towards* 1*—the proportion of the overcharge harm*:

(*i*) *Decreases for direct purchasers*, $\frac{\partial\Omega_R}{\partial m} \geq 0$, *towards* $\frac{1-\tau}{2}$;

(*ii*) *Strictly decreases for indirect purchasers*, $\frac{\partial\Omega_F}{\partial m} > 0$, *towards* $\frac{\tau}{2}$; *and*

(*iii*) *Strictly increases for complementary input suppliers*, $\frac{\partial\Omega_B}{\partial m} < 0$, *towards* $\frac{1}{2}$.

Intuitively, when the input B sector is more concentrated—as m decreases—the passing-back effect is larger, so the complementary input suppliers' share of the overcharge harm increases: $\frac{\partial\Omega_B}{\partial m} < 0$. Furthermore, the passing-on effect is smaller, because the marginal cost of direct purchasers increases to a smaller extent. Thus, indirect buyers incur a smaller proportion of the overcharge harm: $\frac{\partial\Omega_F}{\partial m} > 0$. Similarly, direct purchasers also incur a smaller proportion of the overcharge harm due to the benefit from the larger passing-back effect dominating the cost that relates to the smaller passing-on effect: $\frac{\partial\Omega_R}{\partial m} \geq 0$.

We next consider the impact of the downstream cost pass-through rate: τ:

Proposition 5 *For any* $m \geq 1$, *as downstream cost pass-through rate increases towards* 1, *the proportion of the overcharge harm*:

(i) *Strictly decreases for direct purchasers*, $\frac{\partial \Omega_R}{\partial \tau} < 0$, *towards* 0;

(ii) *Strictly increases for indirect purchasers*, $\frac{\partial \Omega_F}{\partial \tau} > 0$, *towards* $\frac{m}{(m+1)} \geq \frac{1}{2}$; *and*

(iii) *Remains unchanged for complementary input suppliers*, $\frac{\partial \Omega_F}{\partial \tau} = 0$, *at* $\frac{1}{m+1} \leq \frac{1}{2}$.

As τ increases from $\frac{1}{2}$, the passing-on effect increases, so direct buyers incur a smaller proportion of the overcharge harm—$\frac{\partial \Omega_R}{\partial \tau} < 0$—and indirect buyers incur a greater proportion: $\frac{\partial \Omega_F}{\partial \tau} > 0$. The passing-back effect is unaffected by τ, so the complementary input suppliers incur the same proportion of the (larger) overcharge harm: $\frac{\partial \Omega_F}{\partial \tau} = 0$. Thus, when $\tau = 1$, direct purchasers will experience no harm, because they pass on any harm to indirect buyers. However, the indirect purchasers will still share the harm with the complementary input suppliers, for any $m < \infty$ —even though $\tau = 1$—due to the passing-back effect.

Before moving on, let us emphasise that the input B suppliers can incur a large share of the overcharge harm and so the shares of direct and indirect purchasers can be smaller than they would be otherwise. Table 1 shows how the proportions of the overcharge harm vary with the number of input B suppliers (m) and the downstream pass-through rate (τ). It includes three levels of τ: (i) full pass-through ($\tau = 1$), which arises when downstream is perfectly competitive; (ii) monopoly pass-through ($\tau = \frac{1}{2}$); and (iii) an intermediate pass-through ($\tau = \frac{3}{4}$) which is consistent with, for example, either Bertrand competition where $n = 2$ and $\sigma = \frac{4}{5}$ or Cournot competition where $n = 3$ and $\sigma = 1$.

Table 1 shows that when the input B sector is unconcentrated ($m \rightarrow \infty$), all of the overcharge harm is incurred by the direct and indirect purchasers. As it becomes more concentrated—as m decreases—the input B suppliers incur a greater proportion of the overcharge harm (other things equal); and as a consequence the direct and indirect purchasers incur less. When there is a monopoly input B supplier ($m = 1$), it incurs 50% of the overcharge harm. This adds weight to the argument that the complementary input suppliers should be able to sue for compensation—especially when the complementary input sector is concentrated.

In contrast, the shares of the direct and indirect purchasers decrease as m falls. For any $m \geq 1$, the proportion of the purchasers' joint overcharge harm is $1 - \tau$ for direct purchasers and τ for indirect purchasers. Thus, when $\tau = \frac{1}{2}$ and $m = 1$ (where the share of the complementary input suppliers is $\frac{1}{2}$), direct purchasers incur only $\left(1 - \frac{1}{2}\right) \times \frac{1}{2} = \frac{1}{4}$ of the overcharge harm, whereas they would incur $\frac{1}{2}$ of the overcharge harm if there is no passing-back effect. This indicates that taking account of any passing-back effect will be important in calculating the appropriate compensation for purchasers—especially when the complementary input sector is concentrated.

4.3 Shares of the Total Harm

In this subsection, we analyse how the total harm is divided by the market participants when there is a pass-backing effect. The overcharge harm captures only a proportion of the total harm when demand is downward sloping, because it does not take into account the loss in volume.[25] Thus, we first need to derive expressions of the volume harm and then sum this with the overcharge harm to find expressions of the total harm:

For an increase in marginal cost from $c\left(w_A\right)$ to $c\left(w'_A\right)$, it follows from (3) that the loss in volume:

$$Q^*(w_A) - Q^*\left(w'_A\right) = \frac{\beta\left[c\left(w'_A\right) - c\left(w_A\right)\right]}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} > 0. \tag{15}$$

This is strictly positive because demand is downward sloping. The volume harm that is inflicted on direct purchasers is the loss in volume multiplied by the retail price–cost margin before the cost increase: $\left[p^*\left(w_A\right) - c\left(w_A\right)\right]\left[Q^*(w_A) - Q^*\left(w'_A\right)\right]$. The volume harm that is incurred by the complementary input suppliers can be calculated in a similar way.

In contrast, the volume harm that is experienced by indirect purchasers amounts to the lost consumer surplus, which is given by $\frac{1}{2}\left[p^*\left(w'_A\right) - p^*\left(w_A\right)\right]\left[Q^*(w_A) - Q^*\left(w'_A\right)\right]$.

Denoting

$$\Psi(\Delta) \equiv \left(2 + \frac{\varphi_A \Delta}{v - \kappa - \varphi_A w'_A - \varphi_B c_B}\right),$$

where $\Psi(\Delta) > 2$ for all $v > \kappa + \varphi_A w'_A + \varphi_B c_B$, we next derive expressions of the total harm for each market participant:

Proposition 6 *For all* $v > \kappa + \varphi_A w'_A + \varphi_B c_B$ *and* $\theta \in \left[-\frac{n(1-\sigma)}{\sigma}, n\right]$, *an increase in the price of input A from* w_A *to* $w'_A = w_A + \Delta$ *inflicts*:

(*i*) *Total harm on direct purchasers of*

$$n\left[\pi^*_R\left(w_A\right) - \pi^*_R\left(w'_A\right)\right] = \Psi(\Delta)\Omega_R(m,\tau)\varphi_A \Delta Q^*\left(w'_A\right) \geq 0; \tag{16}$$

(*ii*) *Total harm on the complementary input suppliers of*

$$m\left[\pi^*_B\left(w_A\right) - \pi^*_B\left(w'_A\right)\right] = \Psi(\Delta)\Omega_B(m)\varphi_A \Delta Q^*\left(w'_A\right) > 0; \tag{17}$$

(*iii*) *Total harm on the indirect purchasers of*

[25] If demand were perfectly inelastic, there would be no loss in volume so the overcharge harm would amount to the total harm.

$$CS^*\left(w_A\right) - CS^*\left(w'_A\right) = \frac{\Psi(\Delta)}{2}\Omega_F(m,\tau)\varphi_A\Delta Q^*\left(w'_A\right) > 0;\text{ and} \tag{18}$$

(*iv*) *total industry harm of*

$$\mathbb{H} \equiv \Psi(\Delta)\left[1 - \frac{1}{2}\Omega_F(m,\tau)\right]\varphi_A\Delta Q^*\left(w'_A\right) > 0. \tag{19}$$

Proposition 6 shows that the total harm can be written as a function of the overcharge harm that each market participant incurs. The reason is that the volume harm for each is proportionate to the overcharge harm. Summing the total harm inflicted on all market participants yields the total industry harm: $\mathbb{H}$.

Note that for direct purchasers and complementary input suppliers $\Psi(\Delta)$ represents the ratio by which their total harm exceeds their overcharge harm for all $\tau \in \left[\frac{1}{2}, 1\right)$. Thus, it represents the extent to which damages would fall short of full compensation if just the overcharge harm was used to calculate them. Given that $\Psi(\Delta)$ exceeds 2, it follows that the total harm is more than twice the size of the overcharge harm, so the underestimation can be substantial. It also implies that the volume harm is greater than the overcharge harm for both direct purchasers and complementary input suppliers.[26]

In contrast, for indirect purchasers, the ratio to which the total harm exceeds the overcharge charge harm is $\frac{1}{2}\Psi(\Delta)$ for all $\tau \in \left[\frac{1}{2}, 1\right]$. This implies that the overcharge harm for indirect purchasers captures more of the total harm than for direct purchasers or the complementary input suppliers. Note that this ratio for indirect purchasers does not necessarily exceed 2, so it is unclear whether the volume harm exceeds the overcharge harm or not.

By dividing (16), (17), and (18) by (19), it follows that the shares of the total industry harm that is incurred by direct purchasers, indirect purchasers, and complementary input suppliers are, respectively:

$$\frac{n\left[\pi_R^*\left(w_A\right) - \pi_R^*\left(w'_A\right)\right]}{\mathbb{H}} = \frac{\Omega_R(m,\tau)}{1 - \frac{1}{2}\Omega_F(m,\tau)} \equiv \Phi_R(m,\tau) \geq \Omega_R(m,\tau); \tag{20}$$

$$\frac{CS^*\left(w_A\right) - CS^*\left(w'_A\right)}{\mathbb{H}} = \frac{\frac{1}{2}\Omega_F(m,\tau)}{1 - \frac{1}{2}\Omega_F(m,\tau)} \equiv \Phi_F(m,\tau) < \Omega_F(m,\tau);\text{ and} \tag{21}$$

[26] For $\tau = 1$, the retail price equals marginal cost, so the overcharge harm and total harm incurred by retailers is zero. Consequently, in this case, the overcharge harm is not an underestimation of the total harm for retailers. However, the total harm still exceeds the overcharge harm for the complementary input suppliers when $\tau = 1$, and the ratio of the two remains $\Psi(\Delta)$.

Table 1 Share of the overcharge harm

m	$\forall\tau$	$\tau=\frac{1}{2}$		$\tau=\frac{3}{4}$		$\tau=1$	
	Input B sellers	Direct buyers	Indirect buyers	Direct buyers	Indirect buyers	Direct buyers	Indirect buyers
∞	0.000	0.500	0.500	0.250	0.750	0.000	1.000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	0.091	0.455	0.455	0.227	0.682	0.000	0.909
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
5	0.167	0.417	0.417	0.208	0.625	0.000	0.833
4	0.200	0.400	0.400	0.200	0.600	0.000	0.800
3	0.250	0.375	0.375	0.188	0.563	0.000	0.750
2	0.333	0.333	0.333	0.167	0.500	0.000	0.667
1	0.500	0.250	0.250	0.125	0.375	0.000	0.500

$$\frac{m\left[\pi_B^*\left(w_A\right)-\pi_B^*\left(w_A'\right)\right]}{\mathbb{H}}=\frac{\Omega_B(m)}{1-\frac{1}{2}\Omega_F(m,\tau)}\equiv\Phi_B(m,\tau)>\Omega_B(m). \tag{22}$$

This shows that the direct purchasers and the complementary input suppliers incur a greater proportion of the total industry harm than the overcharge harm. In contrast, the indirect purchasers incur a smaller proportion. The reason is that the volume harm is relatively less costly for the indirect purchasers because, since demand is downward sloping, each extra unit lost is valued less by indirect purchasers.

Before analysing how the division of the total industry harm varies with the market characteristics, we consider how the total industry harm varies with them:

Proposition 7 *The total industry harm is greater when the input B sector is less concentrated* (*when m is higher*)—$\frac{\partial\mathbb{H}}{\partial m}>0$—*or when the downstream pass-through rate is higher*: $\frac{\partial\mathbb{H}}{\partial\tau}>0$.

When the input B sector is less concentrated or the downstream pass-through rate is higher, there is a larger overcharge harm—$\varphi_A\Delta Q^*\left(w_A'\right)$—but there can be larger or smaller volume harm, $\left[\frac{1}{2}\left(p^*\left(w_A'\right)+p^*\left(w_A\right)\right)-\kappa-\varphi_A w_A-\varphi_B c_B\right]\left[Q^*\left(w_A\right)-Q^*\left(w_A'\right)\right]$. The reason for the latter is that while there is a larger loss in volume—$\left[Q^*\left(w_A\right)-Q^*\left(w_A'\right)\right]$—there is the opposing effect of a smaller welfare loss per unit that is caused by lower $p^*\left(w_A'\right)$ and $p^*\left(w_A\right)$. Thus, the total industry harm always increases, because the larger overcharge harm always dominates any counteracting decrease in the volume harm.

Let us now analyse how the division of the total industry harm varies with the market characteristics, starting with the concentration in the input B sector:

Proposition 8 *For any given* $\tau \in \left[\frac{1}{2}, 1\right]$, *as the input B sector increases in concentration—as m decreases towards* 1*—the proportion of the total industry harm*:

(i) *decreases for direct purchasers*, $\frac{\partial \Phi_R}{\partial m} \geq 0$, *towards* $\frac{2(1-\tau)}{4-\tau} \leq \frac{2}{7}$;

(ii) *strictly decreases for indirect purchasers*, $\frac{\partial \Phi_F}{\partial m} > 0$, *towards* $\frac{\tau}{4-\tau} \geq \frac{1}{7}$; *and*

(iii) *strictly increases for complementary input suppliers*, $\frac{\partial \Phi_B}{\partial m} < 0$, *towards* $\frac{2}{4-\tau} \geq \frac{4}{7}$.

As m decreases, the larger passing-back effect and the smaller passing-on effect decreases the total harm that is experienced by direct and indirect purchasers. Furthermore, they both experience smaller shares of the (smaller) total industry harm, because their total harm decreases at a faster rate than does the total industry harm. In contrast, the larger passing-back effect and the smaller passing-on effect increases the total harm inflicted on the complementary input suppliers. Consequently, they get a larger share of the (smaller) total industry harm.

Next, consider the effects of the downstream cost pass-through rate: τ:

Proposition 9 *For any* $m \geq 1$, *as the downstream cost pass-through rate rises towards* 1, *the proportion of the total industry harm*:

(i) *Strictly decreases for direct purchasers*, $\frac{\partial \Phi_R}{\partial \tau} < 0$, *towards* 0;

(ii) *Strictly increases for indirect purchasers*, $\frac{\partial \Phi_F}{\partial \tau} > 0$, *towards* $\frac{m}{2+m} \geq \frac{1}{3}$; *and*

(iii) *Strictly increases for complementary input suppliers*, $\frac{\partial \Phi_B}{\partial \tau} > 0$, *towards* $\frac{2}{2+m} \leq \frac{2}{3}$.

As τ rises, the constant passing-back effect and the larger passing-on effect increases the total harm that is experienced by the indirect purchasers and complementary input suppliers. Furthermore, they both experience a larger share of the (larger) total industry harm, because the total harm that they experience increases at a faster rate than the total industry harm. In contrast, the constant passing-back effect and the larger passing-on effect decreases the total harm that is experienced by the direct purchasers. Consequently, they experience a smaller share of the (larger) total industry harm.

Table 2 shows how the proportions of the total industry harm that is experienced by the market participants varies with the number of input B suppliers (m) for the same three downstream pass-through rates (τ) from Table 1.

Table 2 shows a similar pattern to Table 1: When the input B sector is unconcentrated ($m \to \infty$), direct and indirect purchasers experience all of the total industry harm. As the complementary input sector becomes more concentrated, the share of input B suppliers rises—especially as the complementary input sector tends to monopoly. When there is a monopoly input B supplier ($m = 1$), it experiences over $\frac{1}{2}$ and up to $\frac{2}{3}$ of the total industry harm—depending on the

Table 2 Share of the industry total harm

m	$\tau = \frac{1}{2}$			$\tau = \frac{3}{4}$			$\tau = 1$		
	Input B sellers	Direct buyers	Indirect buyers	Input B sellers	Direct buyers	Indirect buyers	Input B sellers	Direct buyers	Indirect buyers
∞	0.000	0.667	0.333	0.000	0.400	0.600	0.000	0.000	1.000
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10	0.118	0.588	0.294	0.138	0.345	0.517	0.167	0.000	0.833
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5	0.211	0.526	0.263	0.242	0.303	0.455	0.286	0.000	0.714
4	0.250	0.500	0.250	0.286	0.286	0.429	0.333	0.000	0.667
3	0.308	0.462	0.231	0.348	0.261	0.391	0.400	0.000	0.600
2	0.400	0.400	0.200	0.444	0.222	0.333	0.500	0.000	0.500
1	0.571	0.286	0.143	0.615	0.154	0.231	0.667	0.000	0.333

downstream pass-through rate. This again indicates that compensation for complementary input suppliers could be substantial.

As before, the shares of the direct and indirect purchasers decrease as m falls. For any $m \geq 1$, the proportion of the purchasers' joint total harm is $\frac{2(1-\tau)}{2-\tau}$ for direct purchasers and $\frac{\tau}{2-\tau}$ for indirect purchasers. When $\tau = \frac{1}{2}$ and $m = 1$ (where the share of the complementary input suppliers is $\frac{4}{7}$), direct purchasers experience only $\left(1 - \frac{4}{7}\right) \times \frac{2}{3} = \frac{2}{7}$ of the total industry harm, whereas they would incur $\frac{2}{3}$ of the overcharge harm if there is no passing-back effect. This again demonstrates that taking account of any passing-back effect will be important in determining the appropriate compensation for purchasers.

5 Extension: Passing-Back by Negotiation

In this section, we extend our analysis to a situation where the prices of input B are determined by negotiation. We do so for two reasons: First, we wish to show how our successive oligopolies model can be helpful in quickly determining the results of other market settings. Second, reducing input prices "by negotiation" was the language that was used in the UK Supreme Court's judgment in *Sainsbury's v Mastercard* (see Sect. 2).

5.1 Basic Assumptions

Assume that there are two retailers—$n = 2$—and two suppliers of input B: $m = 2$. In contrast to before, suppose that each retailer is exclusively supplied by one supplier and that the retailer-supplier pairs bargain over input B prices simultaneously. Then, after observing the outcomes of the bargains, each retailer determines its quantity and price. Given the exclusive relationships, all firms' outside options equal zero. All other assumptions are the same as in Sect. 2.1. Despite possible asymmetries in downstream marginal costs, the conduct parameter has the same values for Cournot and Bertrand competition as before, so we restrict attention to those cases ($\theta^c = 1$ and $\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$).[27]

5.2 Equilibrium Analysis

Starting downstream, we can use (2) for $i = \{1, 2\}$ to obtain the equilibrium quantity of retailer i for any given c_i and c_j:

[27] Further details about the Bertrand conduct parameter are available upon request.

$$q_i^*(c_i, c_j) = \frac{\beta}{2\left(2 - \sigma\left(1 - \frac{\theta}{2}\right)\right)}\left[v - c_i + (c_j - c_i)\left(\frac{\frac{\sigma}{2(1-\sigma)}}{2 + \frac{\sigma\theta}{2(1-\sigma)}}\right)\right], \tag{23}$$

where if $c_i = c_j$, then (23) collapses to (3) with $n = 2$. Denoting $\mathbf{q}^* \equiv \{q_i^*(c_i, c_j), q_j^*(c_j, c_i)\}$, we let $\pi_{Ri}^*(w_{Bi}, w_{Bj}) = (p_i(\mathbf{q}^*) - c_i)q_i^*(c_i, c_j)$ and $\pi_{Si}^*(w_{Bi}, w_{Bj}) = (w_{Bi} - c_B)\varphi_B q_i^*(c_i, c_j)$ represent the equilibrium profits of retailer i and its supplier for a given w_{Bi} and w_{Bj}, respectively.

Given the two negotiations over input B prices are conducted simultaneously, each retailer-supplier pair will treat their rival-pair's price of input B as given during the negotiations. Following Dobson and Waterson (1997 and 2007), the symmetric equilibrium input B price can then be obtained from the symmetric Nash bargaining solution. For the negotiation between retailer i and its supplier over w_{Bi}, it is characterised by:

$$w_{Bi}^* = \arg\max_{w_{Bi}} \left[\pi_{Si}^*\left(w_{Bi}, w_{Bj}^*\right)\right]^{\gamma} \left[\pi_{Ri}^*\left(w_{Bi}, w_{Bj}^*\right)\right]^{1-\gamma}, \tag{24}$$

where $\gamma \in [0, 1]$ represents the supplier's bargaining power relative to its retailer. When $\gamma = 0$, retailer i has all of the bargaining power; and when $\gamma = 1$, the supplier has all of the bargaining power. The first-order condition of (24) can be expressed as:

$$\frac{\gamma}{\pi_{Si}^*(w_{Bi}, w_{Bj}^*)} \frac{\partial \pi_{Si}^*(w_{Bi}, w_{Bj}^*)}{\partial w_{Bi}} = -\frac{1-\gamma}{\pi_{Ri}^*(w_{Bi}, w_{Bj}^*)} \frac{\partial \pi_{Ri}^*(w_{Bi}, w_{Bj}^*)}{\partial w_{Bi}}. \tag{25}$$

Proposition 10 *For all* $v > \kappa + \varphi_A w_A + \varphi_B c_B$, $\gamma \in [0, 1]$ *and* $\theta \in \left\{\frac{1}{1+\frac{\sigma}{2(1-\sigma)}}, 1\right\}$, *the symmetric Nash bargaining solution yields a unique equilibrium input B price*:

$$w_B^N(w_A, \gamma, \sigma, \theta) = c_B + \frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{\varphi_B[\mu^*(\gamma, \sigma, \theta) + 1]} \in \left[c_B, \frac{v - \kappa - \varphi_A w_A + \varphi_B c_B}{2\varphi_B}\right], \tag{26}$$

where $\mu^*(\gamma, \sigma, \theta) \equiv \left(\frac{2-\gamma}{\gamma}\right)\left(1 + \frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right) \geq 1$ *such that the retailers' marginal cost is* $c(w_A, w_B^N(w_A, \gamma, \sigma, \theta)) = \kappa + \varphi_A w_A + \varphi_B w_B^N(w_A, \gamma, \sigma, \theta)$. *The downstream equilibrium price is*:

$$p^*(w_A, w_B^N(w_A, \gamma, \sigma, \theta), \sigma, \theta) = v - \frac{v - c(w_A, w_B^N(.))}{2 - \sigma\left(1 - \frac{\theta}{2}\right)} \in \left[c(w_A, w_B^N(.)), \frac{v + c(w_A, w_B^N(.))}{2}\right]. \tag{27}$$

The negotiated equilibrium price of input B equates the supplier's and retailer's weighted concession costs measured as a proportion of the gains from agreement. For the supplier, this is given by the left-hand side of (25); and for the retailer, it is the right-hand side. The intuition is that when, say, the left-hand side is smaller than the right-hand side, it is relatively less costly for the supplier to concede to a lower unit price than it is for the retailer to concede to a higher unit price. Consequently, the retailer bargains more aggressively than the supplier, which leads to a reduction in the unit price until (25) is balanced.

When the supplier has no bargaining power—$\gamma = 0$—the retailer extracts all of the surplus, so $w^N(.) = c_B$.[28] As the supplier's bargaining power rises (and the retailer's falls), it becomes relatively less costly for the retailer to concede to a higher price, so $w^N(.)$ rises. When the supplier has all of the bargaining power—$\gamma = 1$—the negotiated price maximises the supplier's profits.

5.3 The Effects of an Input Cartel

We now analyse the effects of an input cartel. Note that the equilibrium prices in (26) and (27) are the same as in (5) and (6), respectively, except that $\mu^*(\gamma, \sigma, \theta)$ replaces m. Thus, we need only substitute $\mu^*(\gamma, \sigma, \theta)$ for m throughout section 4 to derive expressions of the various effects and harms. We again simplify notation by writing expressions as a function of w_A only.

Let us first consider how the passing-on and passing-back effects vary with the supplier's bargaining power: γ. When the suppliers of input B have no bargaining power—$\gamma = 0$—there is no passing-back effect (since $\mu^*(0, \sigma, \theta) \to \infty$); and consequently input B suppliers will incur none of the overcharge harm or total harm. However, noting that $\frac{\partial \mu^*}{\partial \gamma} < 0$ and drawing upon Proposition 2, we can state the following for $\gamma > 0$:

Proposition 11 *For any given downstream pass-through rate, τ, as the suppliers' bargaining power, γ, rises:*

(*i*) *The passing-back effect gets larger: $w_B^N(w_A') - w_B^N(w_A)$ is more negative; and*

(*ii*) *The passing-on effect gets smaller: $p^*(w_A') - p^*(w_A)$ is less positive.*

When each input B supplier has greater bargaining power, the negotiated price is closer to the supplier's profit-maximising price than to the retailer's profit-maximising price. Consequently, the negotiated price is more responsive to changes in demand, because the retailer's preferred price of c_B is independent of demand. Thus, the passing-back effect is larger because an increase in w_A causes a greater fall in $w_B^N(.)$. This in turn reduces the passing-on effect because each retailer's marginal cost increases to a smaller extent.

Next, we can use $\frac{\partial \mu^*}{\partial \gamma} < 0$ and Proposition 4 to understand how bargaining power affects the division of the overcharge harm: As each supplier's bargaining power

[28] This is equivalent to the case where each retailer-supplier pair are vertically integrated.

rises, the proportions of the overcharge harm that is incurred by the direct and indirect purchasers strictly decreases: $\frac{d\Omega_R}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\Omega_R}{\partial m} < 0$ and $\frac{d\Omega_F}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\Omega_F}{\partial m} < 0$; and the input B suppliers' share strictly increases: $\frac{d\Omega_B}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\Omega_B}{\partial m} > 0$. This follows from a larger passing-back effect and a smaller passing-on effect.

Similarly, with regards to the total harm, we can use $\frac{\partial\mu^*}{\partial\gamma} < 0$ and Proposition 7 to see that the total industry harm is smaller when each input B supplier has greater bargaining power: $\frac{d\mathbb{H}}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\mathbb{H}}{\partial m} < 0$. Moreover, from Proposition 8, when the supplier's bargaining power is stronger, the proportions of the total harm for direct and indirect purchasers are smaller: $\frac{d\Phi_R}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\Phi_R}{\partial m} < 0$ and $\frac{d\Phi_F}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\Phi_F}{\partial m} < 0$; and the complementary input suppliers' share is larger: $\frac{d\Phi_B}{d\gamma} = \frac{\partial\mu^*}{\partial\gamma}\frac{\partial\Phi_B}{\partial m} > 0$.

One difference in this bargaining model is that the passing-back effect is now a function of the degree of product substitutability—σ—and the form of competition downstream: θ. The reason is that these parameters endogenously affect the bargaining positions of the negotiating retailer and supplier, where $\frac{\partial\mu^*}{\partial\sigma} > 0$ and $\frac{\partial\mu^*}{\partial\theta} < 0$. Nevertheless, the effect of product substitutability on the passing-on effect has the same sign as before—regardless of whether downstream competition is in prices or quantities:

Proposition 12 *For any $\gamma > 0$, under Cournot ($\theta^c = 1$) and Bertrand competition ($\theta^b = \frac{1}{1+\frac{\sigma}{2(1-\sigma)}}$), as the degree of product substitutability, σ, increases*:

(*i*) *The passing-back effect gets smaller*: $w_B^*(w_A') - w_B^*(w_A)$ *is less negative*; *and*

(*ii*) *The passing-on effect gets larger*: $p^*(w_A') - p^*(w_A)$ *is more positive.*

Intuitively, when the products are closer substitutes, a cost disadvantage relative to its rival is more costly for each retailer, so they bargain more aggressively. The negotiated price is closer to the retailer's preferred price of c_B, and it is therefore less responsive to changes in demand. Thus, an increase in w_A leads to a smaller passing-back effect. As a result, the increase in downstream marginal cost is larger, because the rise in w_A is offset less by the fall in $w_B^N(.)$. Consequently, the passing-on effect is larger because, in addition to a higher pass-through rate when products are closer substitutes—$\frac{\partial\tau}{\partial\sigma} > 0$—there is also a greater cost increase to pass on.

An implication of the above is that product substitutability affects the overcharge harm and total harm through both the pass-through rate—τ—and $\mu^*(\gamma, \sigma, \theta)$. This can complicate some of the effects of product substitutability on the various harms. For instance, as products become closer substitutes, the proportion of the overcharge harm on the complementary input suppliers is smaller—$\frac{d\Omega_B}{d\sigma} = \frac{\partial\mu^*}{\partial\sigma}\frac{\partial\Omega_B}{\partial m} < 0$—due to the smaller passing-back effect; and the proportion on indirect purchasers is larger, due to a greater cost being passed on at a higher rate: $\frac{d\Omega_F}{d\sigma} = \frac{\partial\mu^*}{\partial\sigma}\frac{\partial\Omega_F}{\partial m} + \frac{\partial\tau}{\partial\sigma}\frac{\partial\Omega_F}{\partial\tau} > 0$.

The change to direct purchasers' share depends upon how much they lose from the smaller passing-back effect and how much they gain from the larger passing-on effect: $\frac{d\Omega_R}{d\sigma} = \frac{\partial \mu^*}{\partial \sigma}\frac{\partial \Omega_R}{\partial m} + \frac{\partial \tau}{\partial \sigma}\frac{\partial \Omega_R}{\partial \tau}$. When the supplier's bargaining power is low, the passing-back effect is close to zero, so the direct purchasers' share falls with product substitutability, due to the larger passing-on effect: $\frac{d\Omega_R}{d\sigma} < 0$. When the supplier's bargaining power is high, there is a U-shaped relationship between product substitutability and the direct purchaser's share. This implies that when products are close substitutes the gain from the larger passing-on effect is dominated by the loss from the smaller passing-back effect.[29]

6 Discussion

In this section, we discuss the robustness of our results to substitution possibilities among the inputs and to non-linear pricing.

6.1 Input Substitution Possibilities

Throughout the paper, we have assumed that downstream firms face a Leontief (or perfect complements) production function, where producing one unit of each final product always requires φ_A and φ_B units of inputs A and B, respectively. We now discuss the implications of relaxing this assumption to consider other production functions where the inputs are imperfect complements. In this case, there will be substitution possibilities among the inputs. These substitution possibilities can arise for various production functions: including Cobb-Douglas; constant elasticity of substitution (CES); or translog.

If there are substitution possibilities among the inputs, then following an increase in w_A and a resultant fall in w_B, retailers could substitute away from input A and use more units of input B. This would limit the decrease in demand for input B and incentivise a smaller reduction in w_B. Consequently, substitution possibilities among the inputs would reduce the magnitude of the passing-back effect compared with the main model. Furthermore, the increase in the marginal cost of downstream firms would be greater, because the increase in w_A will be offset less by the smaller decrease in w_B. Thus, the smaller passing-back effect would lead to a larger passing-on effect.

The above indicates that the complementary input suppliers may still incur some of the harm when there are substitution possibilities among the inputs. However, the smaller passing-back effect implies that less harm will be passed back to input B suppliers. Consequently, the harm that is inflicted on the complementary input suppliers will be greatest when the inputs are perfect complements and must be used in fixed proportions, like in our main model.

[29] Similar results apply for the division of the total harm: $\frac{d\Phi_B}{d\sigma} < 0$; $\frac{d\Phi_F}{d\sigma} < 0$; and $\frac{d\Phi_F}{d\sigma}$ is non-monotonic.

6.2 Non-Linear Pricing

Up to this point, we assumed that the retailers can buy from any of the input B suppliers at a uniform linear price. Such an approach is applicable to markets where upstream firms are relatively undifferentiated or where there is limited scope for them to price discriminate. In this subsection, we now explain how input B suppliers can still incur some harm if they use non-linear prices. For simplicity, we consider the case of a monopoly input B supplier ($m = 1$) that sets a two-part tariff: a constant price per unit—w_B—and a fixed fee: F_B.

In the presence of a two-part tariff, the equilibrium downstream price will depend upon w_B but not F_B, because F_B is a fixed cost. Thus, for a given w_A, the monopoly input B supplier can avoid exacerbating double marginalisation by setting w_B equal to its marginal cost: c_B. It can then extract the maximised total retail profits from the downstream firms through F_B. Consequently, following an increase in w_A, the retailers will pass on a proportion of the cost rise to their customers, as normal, and retail profits will fall. In response, the monopoly input B supplier will have to reduce F_B to extract the smaller retail profits while maintaining the level of its unit price: $w_B = c_B$.

The above implies that some of the harm of an input cartel will still be passed back to a monopoly input B supplier with a two-part tariff. Given that the monopoly input supplier extracts all of the retail profits from the downstream firms, it follows that its total harm is equivalent to the change in retail profits. In contrast, the retailers—the direct purchasers—will not incur any harm, because they receive zero profits whether the cartel is active or not. Indirect purchasers will still incur some harm due to the passing-on effect. However, in this case, the passing-on effect will not be reduced by the change to the input B tariff, because the monopoly input B supplier does not change w_B.

7 Conclusion

We have analysed the equilibrium effects of an input cartel when the cartel's direct purchasers can pass on the overcharge to their customers and/or pass it back to their suppliers of other complementary inputs. We showed that the passing-back effect is larger and the passing-on effect is smaller when the complementary input sector is more concentrated or when the complementary input suppliers have greater bargaining power. We also showed how this affects the division of the harm among the direct and indirect purchasers and the complementary input suppliers.

Our results have two important implications for damages claims in practice: First, the complementary input suppliers can experience significant harm as a consequence of the passing-back effect. Hence, there is a case for them to have the same rights to sue for compensation as direct and indirect purchasers. Second, deriving the true harm that is inflicted on direct and/or indirect buyers is also likely to involve estimating the size of any passing-back effect in many cases. This will involve developing an understanding of competition in the complementary input sector—even when the complementary input suppliers are not part of the litigation.

Appendix: Proofs

Proof of Proposition 1 Let $\varphi_B \Sigma_{l \neq k} x_l^B$ denote the quantity of input B that is produced by all of supplier k's $m-1$ rivals. Substitute $X^B = \varphi_B x_k^B + \varphi_B \Sigma_{l \neq k} x_l^B$ and (4) into $\pi_{Bk}(X_B) = (w_B(w_A, X_B) - c_B)\varphi_B x_{Bk}$ and maximise with respect to x_k^B to find supplier k's best response function:

$$\varphi_B x_k^B \left(\varphi_B \Sigma_{l \neq k} x_l^B\right) = \varphi_B \left[\frac{\beta}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} \left(\frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{2} \right) - \frac{\Sigma_{l \neq k} x_l^B}{2} \right]. \tag{28}$$

In a symmetric equilibrium, where $\varphi_B x_k^B = \varphi_B x_B^*$ for all k, it follows that input B supplier k's rivals will produce $\varphi_B \Sigma_{l \neq k} x_l^B = (m-1)\varphi_B x_B^*$. Substituting $\varphi_B x_k^B = \varphi_B x_B^*$ and $\varphi_B \Sigma_{l \neq k} x_l^B = \varphi_B (m-1) x_B^*$ into (28) and then rearranging yields:

$$\varphi_B x_B^*(w_A) = \varphi_B \frac{\beta}{2 - \sigma\left(1 - \frac{\theta}{n}\right)} \left(\frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{m+1} \right).$$

The total quantity of input B that is produced is $X_B^* = \varphi_B m x_B^*(w_A)$. In equilibrium, we require that $X_B^* = \varphi_B n q^*(c)$, so that the retailers demand φ_B units of input B for each unit of the final products that they sell. Substituting $\varphi_B m x_B^*(w_A) = \varphi_B n q^*(c)$ and rearranging shows that $w_B^*(w_A, m)$ is as claimed. Substituting $q^*\left(c\left(w_A, w_B^*(.)\right)\right)$ into (1) shows that $p^*\left(w_A, \sigma, n, \theta\right)$ is as claimed. □

Proof of Proposition 2 Differentiating (7) and (8) with respect to m yields $\frac{\partial\left(w_B^*(w_A') - w_B^*(w_A)\right)}{\partial m} = \frac{\varphi_A \Delta}{\varphi_B (m+1)^2} > 0$ and $\frac{\partial\left(p^*(w_A') - p^*(w_A)\right)}{\partial m} = \frac{\varphi_A \Delta}{(m+1)^2}\tau > 0$, respectively. The former implies that the price change is more negative as m falls, so the passing-back effect larger; and the latter implies that the price change is less positive as m falls, so the passing-on is smaller. □

Proof of Proposition 3 Differentiating (7) and (8) with respect to τ yields $\frac{\partial\left(w_B^*(w_A') - w_B^*(w_A)\right)}{\partial \tau} = 0$ and $\frac{\partial\left(p^*(w_A') - p^*(w_A)\right)}{\partial \tau} = \frac{\varphi_A \Delta m}{(m+1)} > 0$, respectively. The former implies the passing-back effect is independent of τ; and the latter implies that the price change is more positive as τ rises, so the passing-on effect becomes larger. □

Proof of Proposition 4 Differentiating $\Omega_R(m, \tau)$, $\Omega_F(m, \tau)$, and $\Omega_B(m)$ with respect to m yields: $\frac{\partial \Omega_R}{\partial m} = \frac{1-\tau}{(m+1)^2} \geq 0$; $\frac{\partial \Omega_F}{\partial m} = \frac{\tau}{(m+1)^2} > 0$; and $\frac{\partial \Omega_B}{\partial m} = -\frac{1}{(m+1)^2} < 0$, respectively. □

Proof of Proposition 5 Differentiating $\Omega_R(m, \tau)$, $\Omega_F(m, \tau)$, and $\Omega_B(m)$ with respect to τ yields: $\frac{\partial \Omega_R}{\partial \tau} = -\frac{m}{(m+1)} < 0$; $\frac{\partial \Omega_F}{\partial \tau} = \frac{m}{(m+1)} > 0$; and $\frac{\partial \Omega_B}{\partial \tau} = 0$, respectively. □

Proof of Proposition 6 The total harm that is inflicted on retailers equals:

$$\varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right) + \left[p^*\left(w_A\right) - c\left(w_A\right)\right]\left[Q^*\left(w_A\right) - Q^*\left(w_A'\right)\right],$$

where the first term is the overcharge harm and the second is the volume harm. Substituting (6) and (15) in for the second term yields:

$$\begin{aligned}\left[p^*\left(w_A\right) - c\left(w_A\right)\right]\left[Q^*\left(w_A\right) - Q^*\left(w_A'\right)\right] =& \left[(1-\tau)\left(v - c\left(w_A\right)\right)\right]\left[\beta\tau\left(c\left(w_A'\right) - c\left(w_A\right)\right)\right] \\ =& (1-\tau)\left(\frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right) Q^*\left(w_A'\right)\varphi_A \Delta \frac{m}{m+1} \\ =& \varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right)\left(\frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right),\end{aligned}$$

since $c\left(w_A'\right) - c\left(w_A\right) = \varphi_A \Delta \frac{m}{m+1}$ and recall that $\tau = \frac{1}{2 - \sigma\left(1 - \frac{\theta}{n}\right)}$. Then summing the first term with the second yields (16) as:

$$\begin{aligned}n\left[\pi_R^*\left(w_A\right) - \pi_R^*\left(w_A'\right)\right] =& \varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right)\left[1 + \frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right] \\ =& \varphi_A \Delta \Omega_R(m,\tau) Q^*\left(w_A'\right)\Psi(\Delta),\end{aligned}$$

since $v - c\left(w_A\right) = \left(\frac{m}{m+1}\right)\left(v - \kappa - \varphi_A w_A - \varphi_B c_B\right)$.

The total harm to each input B supplier equals:

$$\varphi_B\left[w_B^*\left(w_A'\right) - w_B^*\left(w_A\right)\right] Q^*\left(w_A'\right) + \varphi_B\left[w_B^*\left(w_A\right) - c_B\right]\left[Q^*\left(w_A\right) - Q^*\left(w_A'\right)\right],$$

where again the first term is each input B supplier's overcharge harm and the second term is the volume harm. Substituting (7) and (15) in for the second term yields:

$$\begin{aligned}\varphi_B\left[w_B^*\left(w_A\right) - c_B\right]\left[Q^*\left(w_A\right) - Q^*\left(w_A'\right)\right] =& \left[\frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{m+1}\right]\left[\beta\tau\left(c\left(w_A'\right) - c\left(w_A\right)\right)\right] \\ =& \frac{\varphi_A \Delta}{m+1}\left(\frac{m}{m+1}\right)\left(\frac{v - \kappa - \varphi_A w_A - \varphi_B c_B}{v - c\left(w_A'\right)}\right) Q^*\left(w_A'\right) \\ =& \varphi_A \Delta \Omega_B(m)\left(\frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right) Q^*\left(w_A'\right).\end{aligned}$$

Then summing the first term with the second term yields (17) as:

$$\begin{aligned}m\left[\pi_B^*\left(w_A\right) - \pi_B^*\left(w_A'\right)\right] =& \varphi_A \Delta \Omega_B(m) Q^*\left(w_A'\right)\left[1 + \frac{v - c\left(w_A\right)}{v - c\left(w_A'\right)}\right] \\ =& \varphi_A \Delta \Omega_B(m) Q^*\left(w_A'\right)\Psi(\Delta).\end{aligned}$$

The total harm to consumers equals:

$$\left[p^*\left(w'_A,\right)-p^*\left(w_A\right)\right]Q^*\left(w'_A\right)+\frac{1}{2}\left[p^*\left(w'_A\right)-p^*\left(w_A\right)\right]\left[Q^*\left(w_A\right)-Q^*\left(w'_A\right)\right],$$

where the first term is the overcharge harm to final consumers and the second term is the volume harm. Substituting (8) and (15) for the second term yields:

$$\begin{aligned}\frac{1}{2}\left[p^*\left(w'_A\right)-p^*\left(w_A\right)\right]\left[Q^*\left(w_A\right)-Q^*\left(w'_A\right)\right]=&\frac{\varphi_A\Delta}{2}\Omega_F(m,\tau)\left[\beta\tau\left(c\left(w'_A\right)-c\left(w_A\right)\right)\right]\\=&\frac{\varphi_A\Delta}{2}\Omega_F(m,\tau)\left(\frac{c\left(w'_A\right)-c\left(w_A\right)}{v-c\left(w'_A\right)}\right)Q^*\left(w'_A\right).\end{aligned}$$

Then summing the first term with the second term yields (18) as:

$$\begin{aligned}CS^*\left(w_A\right)-CS^*\left(w'_A\right)=&\varphi_A\Delta\Omega_F(m,\tau)Q^*\left(w'_A\right)\left[1+\frac{1}{2}\left(\frac{c\left(w'_A\right)-c\left(w_A\right)}{v-c\left(w'_A\right)}\right)\right]\\=&\varphi_A\Delta\Omega_F(m,\tau)Q^*\left(w'_A\right)\frac{1}{2}\Psi(\Delta).\end{aligned}$$

Finally, summing (16), (17), and (18) shows that (19) is as claimed. □

Proof of Proposition 7 Differentiating $\mathbb{H}$ with respect to m and τ yields:

$$\frac{\partial\mathbb{H}}{\partial m}=\frac{2\tau}{(m+1)^2}\left(1-\frac{m}{m+1}\tau\right)\frac{\Psi(\Delta)}{2}\varphi_A\Delta\beta\left(v-\kappa-\varphi_A w'_A-\varphi_B c_B\right)>0;$$

and

$$\frac{\partial\mathbb{H}}{\partial\tau}=\frac{2m}{(m+1)^2}\left(1-\frac{m}{m+1}\tau\right)\frac{\Psi(\Delta)}{2}\varphi_A\Delta\beta\left(v-\kappa-\varphi_A w'_A-\varphi_B c_B\right)>0,$$

respectively. □

Proof of Proposition 8 Differentiating (20), (21), and (22) with respect to m yields: $\frac{\partial\Phi_R}{\partial m}=\frac{4(1-\tau)}{[2+m(2-\tau)]^2}\geq 0$; $\frac{\partial\Phi_F}{\partial m}=\frac{2\tau}{[2+m(2-\tau)]^2}>0$; and $\frac{\partial\Phi_B}{\partial m}=-\frac{2(2-\tau)}{[2+m(2-\tau)]^2}<0$. □

Proof of Proposition 9 Differentiating (20), (21), and (22) with respect to τ yields: $\frac{\partial\Phi_R}{\partial\tau}=-\frac{2m(m+2)}{[2+m(2-\tau)]^2}<0$; $\frac{\partial\Phi_F}{\partial\tau}=\frac{2m(m+1)}{[2+m(2-\tau)]^2}>0$; and $\frac{\partial\Phi_B}{\partial\tau}=\frac{2m}{[2+m(2-\tau)]^2}>0$, respectively. □

Proof of Proposition 10 To solve for the symmetric Nash bargaining solution, we must find appropriate terms and substitute them for each of the terms in (25). First, if $w_{Bi}=w_{Bj}\equiv w_B$, such that $c_i=c_j=c$, it follows that for any $v>c$:

$$\pi^*_{Ri}\left(w_B, w_B\right) = (v-c)\left(1 - \frac{1}{2-\sigma\left(1-\frac{\theta}{2}\right)}\right)\frac{\beta(v-c)}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)}; \tag{29}$$

and

$$\pi^*_{Si}\left(w_B, w_B\right) = \varphi_B\left(w_B - c_B\right)\frac{\beta(v-c)}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)}. \tag{30}$$

Furthermore, substituting $q^*_i\left(c_i, c_j\right)$ and $q^*_j\left(c_j, c_i\right)$ into (1) yields:

$$p_i(\mathbf{q}^*) = v - \frac{1}{2-\sigma\left(1-\frac{\theta}{2}\right)}\left[v - c_i + \left(c_i - c_j\right)\frac{\sigma}{2}\left(1 - \frac{1}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right] \equiv p^*_i\left(c_i, c_j\right). \tag{31}$$

Then it follows from $\pi^*_{Ri}\left(w_{Bi}, w_{Bj}\right) = \left(p_i(\mathbf{q}^*) - c_i\right)q^*_i\left(c_i, c_j\right)$ that:

$$\begin{aligned}\frac{\partial\pi^*_{Ri}\left(w_B, w_B\right)}{\partial w_{Bi}} &= -\frac{\partial c_i}{\partial w_{Bi}}\left[\left(1 - \frac{\partial p^*_i}{\partial c_i}\right)q^*_i\left(c_i, c_j\right) - \left(p^*_i\left(c_i, c_j\right) - c_i\right)\frac{\partial q^*_i}{\partial c_i}\right]_{c_i=c_j=c} \\ &= -\frac{\varphi_B\beta(v-c)}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)^2}\left[\frac{\sigma}{2}\left(1 - \frac{1}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right) + \left(1-\sigma\left(1-\frac{\theta}{2}\right)\right)\left(2 + \frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right];\end{aligned} \tag{32}$$

and from $\pi^*_{Si}\left(w_{Bi}, w_{Bj}\right) = \left(w_{Bi} - c_B\right)\varphi_B q^*_i\left(c_i, c_j\right)$ that:

$$\begin{aligned}\frac{\partial\pi^*_{Si}\left(w_B, w_B\right)}{\partial w_{Bi}} &= \left[\varphi_B q^*_i\left(c_i, c_j\right) + \left(w_B - c_B\right)\varphi_B\frac{\partial c_i}{\partial w_{Bi}}\frac{\partial q^*_i}{\partial c_i}\right]_{c_i=c_j=c} \\ &= \frac{\varphi_B\beta}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)}\left[v - c - \left(w_B - c_B\right)\varphi_B\left(1 + \frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right],\end{aligned} \tag{33}$$

given that $\frac{\partial c_i}{\partial w_{Bi}} = \varphi_B$; $\frac{\partial p^*_i}{\partial c_i} = \frac{1}{2-\sigma\left(1-\frac{\theta}{2}\right)}\left[1 - \frac{\sigma}{2}\left(1 - \frac{1}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right)\right]$; and $\frac{\partial q^*_i}{\partial c_i} = -\frac{\beta}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)}\left[1 + \frac{\frac{\sigma}{2(1-\sigma)}}{2+\frac{\sigma\theta}{2(1-\sigma)}}\right]$. Substituting (29), (30), (32), and (33) into (25) and rearranging yields:

$$\varphi_B\left(w_B - c_B\right)\left(\left(1 + \frac{\sigma}{4(1-\sigma)+\sigma\theta}\right) + (1-\gamma)\left(1 + \frac{\sigma\left(1 - \frac{2(1-\sigma)}{4(1-\sigma)+\sigma\theta}\right)}{2\left(1-\sigma\left(1-\frac{\theta}{2}\right)\right)}\right)\right) = \gamma(v-c).$$

Finally, substituting for $c = \kappa + \varphi_A w_A + \varphi_B w_B$ and rearranging shows that $w^N_B(w_A, \gamma, \sigma, \theta)$ is as claimed. Substituting $c_i = c_j = c\left(w_A, w^N_B(.)\right)$ into (31) shows that $p^*(w_A, w^N_B(.), \sigma, \theta)$ is as claimed. □

Proof of Proposition 11 Evaluating m at $\mu^*(\gamma,\sigma,\theta)$ in (7) and (8) and totally differentiating with respect to γ yields: $\frac{d(w_B^*(w_A')-w_B^*(w_A))}{d\gamma}=\frac{\partial\mu^*}{\partial\gamma}\frac{\partial(w_B^*(w_A')-w_B^*(w_A))}{\partial m}=\frac{\partial\mu^*}{\partial\gamma}\frac{\varphi_A\Delta}{\varphi_B(\mu^*+1)^2}<0$; and $\frac{d(p^*(w_A')-p^*(w_A))}{d\gamma}=\frac{\partial\mu^*}{\partial\gamma}\frac{\partial(p^*(w_A')-p^*(w_A))}{\partial m}=\frac{\partial\mu^*}{\partial\gamma}\frac{\varphi_A\Delta}{(\mu^*+1)^2}\tau<0$, respectively, where $\frac{\partial\mu^*}{\partial\gamma}=-\frac{2}{\gamma^2}\left(1+\frac{\sigma}{4(1-\sigma)+\sigma\theta}\right)<0$. The former implies that the price change is more negative as γ rises, so the passing-back effect becomes larger; and the latter implies that the price change is less positive, so the passing-on effect becomes smaller. □

Proof of Proposition 12 Evaluating m at $\mu^*(\gamma,\sigma,\theta)$ in (7) and (8) and totally differentiating with respect to σ yields: $\frac{d(w_B^*(w_A')-w_B^*(w_A))}{d\sigma}=\frac{\partial\mu^*}{\partial\sigma}\frac{\partial(w_B^*(w_A')-w_B^*(w_A))}{\partial m}=\frac{\partial\mu^*}{\partial\sigma}\frac{\varphi_A\Delta}{\varphi_B(\mu^*+1)^2}>0$ and $\frac{d(p_B^*(w_A')-p_B^*(w_A))}{d\sigma}=\frac{\partial(p_B^*(w_A')-p_B^*(w_A))}{\partial\sigma}+\frac{\partial\mu^*}{\partial\sigma}\frac{\partial(p_B^*(w_A')-p_B^*(w_A))}{\partial m}=\frac{\partial(p_B^*(w_A')-p_B^*(w_A))}{\partial\sigma}+\frac{\partial\mu^*}{\partial\sigma}\frac{\varphi_A\Delta}{(\mu^*+1)^2}\tau>0$, respectively, where $\frac{\partial(p_B^*(w_A')-p_B^*(w_A))}{\partial\sigma}=\frac{2-\theta-\sigma\frac{\partial\theta}{\partial\sigma}}{2\left(2-\sigma\left(1-\frac{\theta}{2}\right)\right)^2}>0$ and $\frac{\partial\mu^*}{\partial\sigma}=\frac{(2-\gamma)\left(4-\sigma\frac{\partial\theta}{\partial\sigma}\right)}{\gamma(4(1-\sigma)+\sigma\theta)^2}>0\ \forall\gamma>0$ since $\frac{\partial\theta}{\partial\sigma}\le 0$ for $\theta^c=1$ and $\theta^b=\frac{1}{1+\frac{\sigma}{2(1-\sigma)}}\in(0,1)$. The former implies that the input B price change is less negative as σ increases, so the passing-back becomes smaller; and the latter implies that the price change is more positive, so the passing-on effect becomes larger. □

Acknowledgments We are grateful for comments from Anna Rita Bennato, Chris M. Wilson, the editor (Lawrence White), and an anonymous referee. Any mistakes are our own. A motivating case used in this paper is a damages claim in the UK that related to the EU trucks cartel for which James Harvey acted as the economic expert for the claimants. The theoretical model that is presented in this paper has evolved from a simpler framework developed by Luke Garrod.

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