IMA Journal of Management Mathematics (2024) **00**, 1–24
https://doi.org/10.1093/imaman/dpae028

A mathematical model for potash supply chain management with a strategic logistics perspective

Mohammad A. Shbool
Industrial Engineering Department, School of Engineering, The University of Jordan, Queen Rania Al Abdullah Street, Amman 11942, Jordan

Ammar Al-Bazi[†]
Aston Business School, Aston University, Birmingham, 295 Aston Express Way, B4 7UP, UK
Faculty of Business, Sohar University, Swehra Area, Al Salem Street, Sohar, Sultanate of Oman
[†]Corresponding author. Email: albazia@aston.ac.uk

and

Nibal T. Albashabsheh
Industrial Engineering Department, School of Engineering, The University of Jordan, Queen Rania Al Abdullah Street, Amman 11942, Jordan

[Received on 26 February 2024; accepted on 15 October 2024]

Accepted by: M. Zied Babai

This paper introduces a novel Integer Linear Programming model designed to enhance the efficiency and sustainability of the potash supply chain, a crucial element supporting global agriculture. The developed mathematical optimization model focuses on fleet selection (private/outsource) and incorporates the concept of 'inter-warehouse collaboration', which addresses key logistics considerations. Integrating mining, processing, storage and transportation, the model encompasses decision variables like extracted carnallite amount, production, storage levels and shipped potash amount. Illustrated through a case study on the Arab Potash Company in Jordan, the results showcase the model's proficiency in meeting local and international market demands. The model ensures resilient and sustainable supply chain performance by emphasizing logistics optimization, particularly in fleet selection. The study attains the highest 'warehouse-to-warehouse' support for Standard and Granular potash types in the international demand scenario, contributing to efficient production planning and fleet management. In conclusion, the presented mathematical model is a valuable tool for potash industry stakeholders, offering insights for strategic decision-makers involved in production planning and fleet management.

Keywords: supply chain management; mathematical optimization; 'warehouse-to-warehouse' support; fleet selection; sensitivity analysis.

1. Introduction

Potash supply chain companies face several challenges impacting their operations, profitability and ability to meet customer demand. Some of the key challenges by potash supply chain companies include but are not limited to production constraints, which production can be impacted by a range of factors, including weather patterns, labour shortages and equipment failures (Seifi *et al.*, 2021a). The potash suppliers also consider regulatory and environmental constraints, such as potash mining and the cost of inputs, such as water and energy, and create compliance costs for potash supply chain companies (Ushakova *et al.*, 2023).

Other factors include transportation and logistics of potash, which is typically produced in remote locations and must be transported to end-users across long distances (Bouffard & Boggis, 2018). This can create challenges in terms of transportation logistics, including access to transportation infrastructure, shipping costs, and potential delays or disruptions in transit. One of the challenges faced is using company (private) trucks, which incur substantial additional costs compared to the potential benefits of outsourcing trucking services. Companies' trucks may require custom maintenance, training for specialized operators and facility modifications to accommodate their unique design. Consequently, these factors contribute to elevated operational and capital expenditures, impacting the overall cost-effectiveness of the potash supply chain.

In contrast, outsourcing trucking services to external providers presents an alternative solution that merits exploration. Third-party logistics (3PL) providers often possess diverse trucks optimized for various terrains and industries, leading to sharing the logistics emission reduction costs of the 3PL with other supply chain components (Li *et al.*, 2024). Utilizing such external expertise could lead to cost savings, improved efficiency and a more streamlined transportation process for the potash supply chain.

Hence, potash supply chain companies face various challenges that require careful management and strategic planning. Companies that can effectively manage these challenges can gain a competitive advantage and position themselves for long-term success in the industry.

Therefore, this paper develops a new mathematical optimization model to investigate potash supply chain management for one of the largest potash suppliers in the Middle East. The purpose of the model is to optimize the performance of the entire supply chain, including mining, production and storage, with a strategic logistics focus. This focus includes fleet selection (private/outsourced) trucks and the 'warehouse-to-warehouse support' concept essential for the potash industry's competitiveness, sustainability and overall efficiency in the increasingly competitive global market.

The contribution of this paper is summarized as follows:

- Developing a new mathematical model aims to optimize decision variables such as extracted carnallite amounts, production, storage and shipping of potash specifically tailored for optimizing the potash supply chain.
- Introducing the 'inter-warehouse collaboration' concept for 'warehouse-to-warehouse support' in potash supply chains.
- Allow potash supply chain companies to determine the most efficient and cost-effective way to transport potash from production facilities to end-users while considering the incurred potash storage and transportation costs.

This paper is organized as follows. Section 2 presents previous potash supply chain modelling work, including similar configurations and used approaches. The potash supply chain mathematical model is described in Section 3. Section 4 presents a case study based on one of the largest potash suppliers in the Middle East. Section 5 includes results analysis and discussion of different production, inventory and transportation plans, followed by the conclusion and future work in the final section.

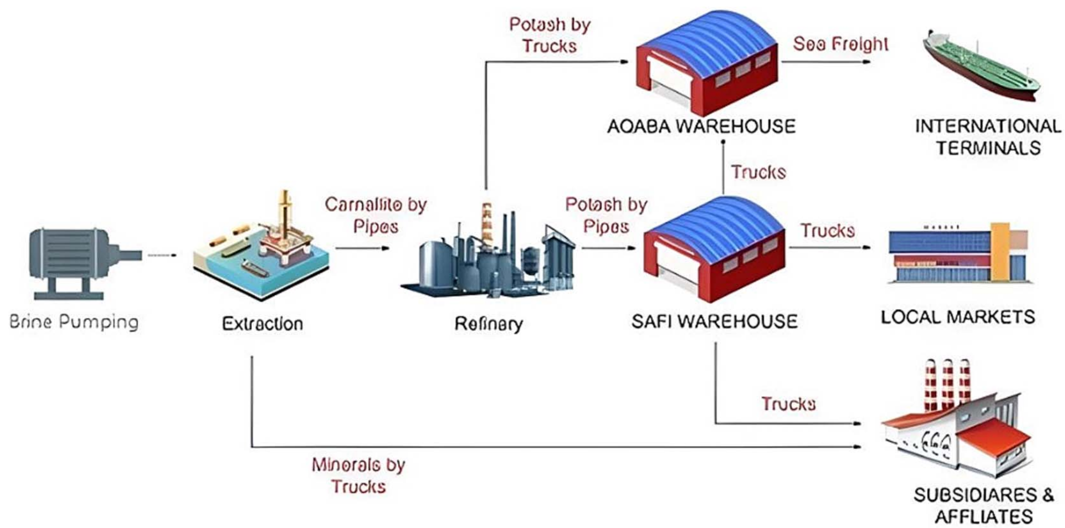

FIG. 1 The APC Potash supply chain.

2. Problem description

In this study, one of the largest potash producers in the Middle East, the Arab Potash Company (APC), is considered. The company produces three types of potash, Standard, Fine and Granular, in their refinery plant in Ghor Al-Safi.

The extraction stage begins with pumping mineral-rich brine directly from the Dead Sea, which contains a mixture of salts, including magnesium, sodium and potassium chlorides. This brine is transported to large solar evaporation ponds, where the water gradually evaporates under the intense sun, which crystallizes these minerals. Potassium chloride, a key component, is harvested from the ponds and transported via pipes to a refinery for potash processing. Meanwhile, non-potassium minerals are transported by trucks to subsidiaries and affiliates for fertilizer production or cosmetics, though the study does not cover this part.

Now, the density of the pre-carnallite is increased and adjusted contentiously in the harvesters until it achieves a satisfactory carnallite production, where it is harvested as a slurry beneath the brine. Afterwards, it will be delivered to the refinery plant for further processing and extraction of potassium compounds from carnallite. The schematic representation of the APC supply chain under study is shown in Fig. 1.

After the production stage is completed at the refinery, the potash produced needs to be moved and stored in two warehouses; the first is located in Ghor Al-Safi, where potash is transported using special pipes with infinite capacity and negligible cost. Al-Safi warehouse in the Dead Sea mainly satisfies the local market demands, subsidiaries (not considered in this study), and international demand using either company and/or outsourced trucks. Al-Safi can also fulfil any shortages in the Aqaba warehouse under the ‘warehouse-to-warehouse support’ concept using company trucks to export potash products from Aqaba using sea freights to international terminals such as Malaysia, China and India. The second one is in Aqaba, where company trucks transport products from the refinery plant to the Aqaba warehouse. The distance between the two warehouses is much shorter than from the refinery to the Aqaba warehouse.

Trucks from the company fleet (private) or rented (outsourced) can be used whenever there is an overload or when the outsourced solution is an excellent cost-effective option. The Aqaba warehouse is used only to satisfy international demand, using containers that depart from the Aqaba port via sea freight and head to international terminals such as China, India, Malaysia and Egypt.

Therefore, this study investigates the entire APC's supply chain echelons and its dynamic behaviour to develop a mathematical optimization model that considers all the related constraints, with a focus made on the fleet selection process for the best solution to the problem, including supply chain efficiency, sustainability and performance.

3. Literature review

This section explores the current state of potash supply chain management research and provides insights into the various approaches, models and frameworks used in such supply chains. Furthermore, it offers an overview of the petroleum supply chain. Its structure shares some structural similarities with that of the potash sector, including Extraction and Production, processing facilities, transportation, global transportation and storage facilities.

In potash supply chain optimization research, Bouffard & Boggis (2018) developed a Simplified Linear Integrated Capacity Estimate (SLICE) model aimed at optimizing the production capacity of the Jansen Potash project. Their model focused on assessing and quantifying risks and uncertainties, determining the range of production capacity under these variable conditions. Jamaludin *et al.* (2024) explored resilience strategies for supply chains in a volatile, uncertain, complex and ambiguous (VUCA) environment. They developed a theoretical framework integrating engineering, business and environmental perspectives to analyse resilience and sustainability. Using Harzing's Publish software, they identified trends and gaps in the literature on sustainable supply chain management. Seifi *et al.* (2021a) developed a mixed-integer linear program for shift scheduling in a German potash mine, involving the simultaneous assignment of machines and workers for short-term (work shift) production, where drill-and-blast mining operations are assigned to both machines and workers. Seifi *et al.* (2021b) developed a mixed-integer linear programming (MILP) model to design a block selection and sequencing problem with a quality-oriented objective, aiming to achieve consistent potash extraction based on its potassium content. The goal was to ensure a homogeneous output in terms of the quality of the extracted potash in one of the biggest German potash mines. Schulze *et al.* (2016) developed a mixed-integer linear model to solve a scheduling problem in potash mining, where a block excavation sequence must be determined while considering a limited number of underground machines and safety-related restrictions. The objective was to minimize the maximum completion time of excavations or the makespan. Ushakova *et al.* (2023) summarized the main features and challenges of underground mining for water-soluble ores and potassium fertilizer production using one of the world's largest potash deposits. The study considered the material composition of waste, its disposal, mining issues related to salt solubility and the risks of groundwater inflow, which can lead to mine flooding. Hanandeh (2022) used **partial least squares** for data processing to investigate the influence of **supply chain integration (SCI)** on customer satisfaction in the potash supply chain in Jordan. The study found that **enterprise resource planning (ERP) systems**, business strategy and suppliers significantly impact SCI and customer satisfaction. The proposed model integrates SCI into its framework, ensuring supply chain decisions positively affect customer satisfaction by optimizing service levels and response times. Rabbani *et al.* (2022) developed a multi-objective, multi-product, multi-period mathematical model for sustainable phosphorus supply chain management in an uncertain environment. Given the potential adverse effects of the phosphorus supply chain on the environment and human health, a sustainable and resilient supply chain network for the fertilizer industry is designed to address the associated environmental, social and economic challenges of phosphorus management.

Given the structural and logistical similarities between potash and other complex supply chains, including transportation and storage, such as those in the petroleum industry, it is valuable to draw on insights from broader supply chain management literature. Therefore, relevant literature in the petroleum industry is reviewed to provide further valuable insights into supply chain management and distribution. This literature includes but is not limited to Said *et al.* (2024), who applied both the Analytical Hierarchy Process (AHP) and the Supply Chain Operations Reference (SCOR) model to assess risks related to road transport in petroleum supply chains. The AHP method enables prioritization of risk factors by assigning weights based on their potential impact on supply chain performance. Alnaqbi *et al.* (2023) developed a stochastic programming model for tactical supply chain planning in the oil and gas industry in the Middle East, aiming to minimize the total supply chain cost. The supply chain consists of four echelons: production facilities, processing plants, gathering centres and demand terminals. It also includes constraints related to inventory balance, capacity limitations at oil depots and gas flow through existing routes. Alnaqbi *et al.* (2022) developed a MILP model to determine optimal investment levels and implement efficient operational strategies within the oil and gas supply chain. This supply chain includes four echelons: production facilities, processing plants, gathering centres and demand terminals. The model incorporates various constraints, including capacity, demand satisfaction, inventory management and sustainability. Zhang *et al.* (2019) developed a MILP model to minimize the total supply chain costs, considering the transportation of oil products. The supply chain includes two refineries and tank depots, with a transportation network linking these components. The transportation process incorporates three standard land-based modes: pipelines, railways and highways. Lima *et al.* (2021) developed a MILP model to optimize strategic and tactical planning costs for the downstream oil supply chain. The model includes refineries, warehouse locations, local markets and multiple transportation modes. Liquid fuels are transported by pipeline, tanker ships or tank trucks from refineries to warehouses, where they are stored in tanks. Ghatee & Zarrinpoor (2022) developed a multi-objective sustainable model for designing an oil supply chain and optimizing economic, social and environmental goals. The supply chain components include oil wells, gas injection wells, production units, refineries, gas injection centres, distribution centres, and export and import terminals. The model also considers production and transportation modes, such as roads and pipelines, as well as inventory constraints.

Yu *et al.* (2020) proposed a continuous-time scheduling model for managing pipeline distribution and depot inventory. The model aims to minimize transportation costs for delivering oil from the refinery to depots, average inventory costs for storage tanks and blending costs for final oil products. Wang *et al.* (2019) developed a mathematical model to simulate the refined products supply chain, accounting for demand increases, production decreases, refinery disruptions and pipeline interruptions. The model includes transport constraints (railways, pipelines, waterways, highways), material balance constraints and capacity constraints to reflect the distribution process accurately. Li *et al.* (2020) addressed the refined oil secondary distribution problem with a two-stage stochastic programming model. This model determines the replenishment quantities for each petrol station based on existing stock, available supply from depots and transportation schedules. The objective is to minimize the total distribution cost of the first stage and the expected storage and surplus costs of the second stage.

This study's proposed mathematical optimization model introduces significant advancements in potash supply chain management that sharply differentiate it from previous approaches outlined in the above literature. Unlike existing models focusing on specific aspects such as extraction, production capacity, risk mitigation, scheduling, transportation or sustainability, our model integrates these components within a more comprehensive framework. It uniquely addresses gaps in the current research by incorporating inter-warehouse collaboration and flexible fleet selection features not previously considered in potash supply chain literature. The holistic approach of our model optimizes the entire supply chain, from extraction and refinery processes to logistics, ensuring efficient and cost-effective potash delivery to both local and international markets. This comprehensive approach provides professionals with robust decision-making tools that optimize costs, resources and logistics, setting a new benchmark for potash supply chain optimization.

Our model equips professionals with an innovative toolset beyond traditional approaches, allowing for more strategic and integrated supply chain decisions. By enabling inter-warehouse collaboration and flexible transportation options, professionals can achieve greater efficiency in meeting market demands, ultimately enhancing customer satisfaction and reducing operational costs. This paradigm shift in supply chain management empowers stakeholders to make data-driven decisions that align with both economic and sustainability goals, ensuring a competitive edge in the global market.

4. Developing potash supply chain mathematical model

This section presents the development of our mathematical model for potash supply chain management. The fundamental assumptions that guide the model development are presented to balance abstraction and realism, allowing the model to capture the essential elements of the potash supply chain while maintaining practical applicability to real-world scenarios. The model assumption is presented in the next section.

4.1. *Model assumption*

- All parameters, such as transportation and holding costs, production capacities and demand, are deterministic and known with certainty.
- A single objective function is considered, including minimizing transportation costs, maximizing revenue or minimizing total lead time.
- Warehouse-to-Warehouse support is essential and allowed.
- Linear relationships between variables include linear transportation costs per unit distance or production costs.
- The potash shipments cannot be split, meaning a shipment must be made as a whole and cannot be divided among multiple destinations.
- There is no special storage requirement for different potash types during the storage process.
- Extracted minerals must be transported simultaneously to the refinery as there is no storage facility at the Dead Sea.
- No backlogs or shortages are allowed in the supply chain, meaning demand is always fully met.
- Fleet types include company and outsourced trucks.
- A fixed network structure for the supply chain means the locations of mines, refineries, warehouses and customers do not change during the planning horizon.
- The potash supply chain operates in a steady-state manner without significant disruptions or unexpected events.

- The pipe capacity for transporting potash from the refinery to the SAFI warehouse is unlimited. This applies to the transported carnallite pipe from the extraction site to the refinery.

The assumptions made in our potash supply chain optimization model are designed to achieve a balance between abstraction and practical applicability. Specifically, the assumption that all parameters, including transportation and holding costs, production capacities and demand, are deterministic and known with certainty allows the model to focus on optimizing the supply chain under a controlled and stable environment. This deterministic approach simplifies the mathematical formulation, enabling clearer insights into cost minimization, warehouse management and transporter selection.

However, the reliance on deterministic parameters and fixed demand limits the model's applicability. Real-world supply chains, including potash, are subject to uncertainties such as fluctuating market demand, transportation delays, supply disruptions and variable production yields. The model may not fully capture actual operations' inherent risks and variability by assuming fixed demand and deterministic parameters. This could result in less-than-ideal decision-making practice when applied in environments characterized by high uncertainty or frequent disruptions.

Regarding possible limitations and impact of stochastic elements, introducing stochastic elements into the model could better represent real-world scenarios where parameters like demand, production rates and transportation costs fluctuate over time. For example, supply chain disruptions due to geopolitical and regional conflict, natural disasters, or market volatility could significantly impact the model's performance. If these stochastic factors were incorporated, the model would need to account for potential deviations from the expected values, leading to a more complex yet realistic optimization process. However, including stochastic elements in future potash supply chain optimization models will be one of the recommendations of this study.

4.2. *The model*

This section develops and explains an Integer Linear Programming Model for the best APC potash supply chain management. The supply chain components, including the logistic parts between each component, are modelled, and presented as constraints along with the 'inter-warehouse collaboration' requirement. The definition of sets, variables and parameters used in the model is given as below:

Sets and Indices

L	Set of all locations in the potash supply chain, $l \in L; l = 1, \ldots, n$
L_1	Set of warehouses, $L_1 \subseteq L$
L_2	Set of Local markets, $L_2 \subseteq L$
L_3	Set of international markets, $L_3 \subseteq L$
n	Given the location of the production facility, $n \in L$
P	Set of potash types, $p \in P$, $p = 1$ (fine), $p = 2$ (standard), $p = 3$ (granular)
T	Set of time periods in the planning horizon, $t \in T$
M	Set of transportation mode $m \in M$, $m = 1$ (pipes), $m = 2$ (company trucks), $m = 3$ containers), $m = 4$ (outsourced trucks)

Input Parameters

E	Extraction capacity of carnallite (ton/period)
C_p	Potash plant capacity of potash type p (ton/period)
D_{plt}	Demand of potash type $p \in P$ at market $l \in (L_2 \cup L_3)$ at the time period $t \in T$ (ton)
S_l	Storage capacity at facility $l \in L$ (ton)
$T_{ll'm}$	Transportation cost of moving potash from facility $l \in L$ to facility $l' \in L$, $l' \neq l$, by transportation mode $m \in M$ (JD/ton)
φ	Extracting cost (per unit) of carnallite (JD/ton)
α	potash yield from the extracted carnallite (unitless)
V_p	Production cost (per unit) of potash type $p \in P$ (JD/ton)
H_{pl}	Unit inventory holding cost of potash type $p \in P$ at facility $l \in (n \cup L_1)$ (JD/ton. Period)
$\gimel_{pl}$	The selling price of potash type $p \in P$ at market $l \in (L_2 \cup L_3)$ (JD/ton)
W	Waste percentage associated with potash storage (per period)
F	Recommended difference between fine and granular production (ton/period)
β_{plt}	Minimum strong limit of potash type $p \in P$ at warehouse $l \in L_1$ at time period $t \in T$ (ton)
$O_{ll'm}$	Maximum amount of potash can be transported between facility $l \in (\mathrm{n} \cup L_1)$ to facility $l' \in (L_1 \cup L_2 \cup L_3)$ using transported mode $m \in M$ (ton/period)

Decision Variables

X_t	Extracted amount of carnallite at time period $t \in T$ (ton)
Y_{pt}	Amount of potash type $p \in P$ produced at time period $t \in T$ (ton)
Z_{plt}	Amount of potash type $p \in P$ stored at facility $l \in (n \cup L_1)$ from time period $t \epsilon T$ to the next time period (ton)
$U_{pll'mt}$	Amount of potash type $p \in P$ shipped from facility $l \in (\mathrm{n} \cup L_1)$ to facility $l' \in (L_1 \cup L_2 \cup L_3)$ by transportation mode $m \in M$ at time period $t \in T$ (ton)

The ILP optimization model is presented below.

$$
\begin{aligned}
\mathit{Maximization} &\sum_{l\in L_1} \sum_{l'\in(L_2\cup\ L_3)} \sum_{m\in M} \sum_{p\in P} \sum_{t\in T} \gimel_{pl} U_{pll'mt} - \sum_{t\in T} \varphi X_t - \sum_{t\in T} \sum_{p\in P} \left(V_p * Y_{pt}\right) \\
&- \sum_{l\in \frac{L}{L_3}} \sum_{l'\in(L_2\cup\ L_3):l'\neq l} \sum_{m\in M} \sum_{p\in P} \sum_{t\in T} T_{ll'm} U_{pll'mt} - \sum_{t\in T} \sum_{p\in P} \sum_{\in(n\cup L_1)} H_{pl} Z_{plt} \\
&- \sum_{t\in T} \sum_{p\in P} \sum_{l\in(n\cup L_1)} W\left(\varphi + V_p\right) Z_{plt}
\end{aligned}
$$

Subject to:

$$\alpha X_t = \sum_{p=1}^{P} Y_{pt} \qquad \forall t \in T \tag{1}$$

$$\sum_{l' \in (L_1 \cup L_2 \cup L_3)} \sum_{m \in M} U_{pnl'mt} + Z_{pnt} = Y_{pt} + (1 - W)\, Z_{pn,t-1} \qquad \forall p \in P, \quad t \in T \tag{2}$$

$$\sum_{l' \in (L_2 \cup L_3)} \sum_{m \in M} U_{pnl'mt} + Z_{plt} = \sum_{m \in M} U_{pnlmt} + (1 - W)\, Z_{pl,t-1} \qquad \forall l \in L_1, p \in P, \quad t \in T \tag{3}$$

$$D_{plt} = \sum_{l' \in (n \cup L_1)} \sum_{m \in M} U_{pl'lmt} \qquad \forall l \in \left(L_2 \cup \ L_3\right), p \in P, \ t \in T \tag{4}$$

$$X_t \le E \qquad \forall t \in T \tag{5}$$

$$Y_{pt} \le C_p \qquad \forall p \in P, t \in T \tag{6}$$

$$\sum_{p \in P} Z_{plt} \le S_l \qquad \forall l \in \left(n \cup L_1\right), t \in T \tag{7}$$

$$Z_{plt} \ge \beta_{plt} \qquad \forall l \in L_1, p \in P, t \in T \tag{8}$$

$$\sum_{p \in P} \sum_{m \in M/\{4\}} U_{pll'mt} \le \sum_{m \in M} O_{ll'm} \qquad \forall l \in \left(n \cup L_1\right), l' \in \left(L_2 \cup \ L_3\right), t \in T \tag{9}$$

$$X_t \ge 0 \qquad \forall t \in T \tag{10}$$

$$Y_{pt} \ge 0 \qquad \forall p \in P, t \in T \tag{11}$$

$$Z_{plt} \ge 0 \qquad \forall l \in \left(n \cup L_1\right), p \in P, t \in T \tag{12}$$

$$U_{pll'mt} \ge 0 \qquad \forall l \in \left(n \cup L_1\right), l' \in \left(L_2 \cup \ L_3\right), p \in P, t \in T, m \in M \tag{13}$$

The objective function comprises revenue from potash sales and supply chain costs. The first term is the total revenue from the sale of potash to both domestic and international terminals. The remaining terms relate to the costs associated with the supply chain. The second term is the extracting cost of carnallite from Dead Sea slurry using harvesters. The third term is the production cost of potash products (three grades) using carnallite. The cost of transporting potash between facilities is reflected in the fourth term. The fifth term is the inventory holding cost of potash at the refinery and warehouses. The last term accounts for the cost of potash that is produced but lost during potash production.

Constraint (1) links potash production to the corresponding carnallite via the potash yield. Constraints (2)–(4) describe flow balance at time–space locations for the potash supply chain, where the left-hand side of each equation represents flow out and the right-hand side represents flow into a node. Constraints (2) and (3) are the potash flow balance constraints at the refinery and warehouses, respectively. Potash flow balance constraints at the markets are enforced by constraint (4). This constraint ensures that the amount of potash shipped monthly to the market (flow in) matches the demand (flow out).

Fig. 2 APC extraction and refinery facilities—Dead Sea.

Constraint (5) is the extraction capacity constraint. This guarantees that the amount extracted (carnallite) is limited by the extraction capacity, determined by the amount of salts in the Dead Sea and the harvesters' capacity. The refinery's production capacity in each time period is reflected in constraint (6). Constraint (7) is a capacity constraint on potash inventory at the warehouses and refineries. Constraint (8) ensures the maintenance of minimal inventory levels at warehouses. Constraint (9): The amount of potash transported between facilities in the supply chain does not exceed route capacity. Constraints (10)–(13) enforce non-negativity.

5. Case study

5.1. *Company background*

In this section, The APC will be considered a case study to justify the applicability of the proposed optimization model. APC is a key player in the global potash industry, contributing significantly to producing and supplying essential nutrients for agricultural purposes. Established in 1956, the company operates in the Dead Sea region of Jordan, extracting potash and other minerals from the rich deposits in the area. As a major exporter of potash, APC plays a crucial role in supporting global agriculture. The company's products enhance soil fertility, thereby improving crop yields worldwide. The strategic location of APC near the Dead Sea provides a unique advantage, allowing efficient extraction and processing of potash resources. See Fig. 2 for the APC Dead Sea site.

5.2. *Data collection*

We have interviewed two managers from different departments at the APC to discuss the company's supply chain structure, material flow and constraints, including material balance, extraction capacity, production and demand constraints, and the route's capacity. All the related data, including selling prices, demands and all types of costs, from extraction to transportation costs, the number of available trucks and containers and their capacities, and the route's capacity, have been collected.

Three types of potash products, 2 warehouses and 49 terminals of both local and international markets were considered in this case study. The company has 92 trucks utilized for potash transportation; 80 trucks have a capacity of 26 tons, and 12 trucks have a capacity of 70 tons; all are working 30 days per month (See Appendix B). The trucks, which have a capacity of 26 tons, transport potash from

TABLE 1 *The APC collected data*

Parameter	Value
Extraction cost (JD/ton)	$\varphi = 4.09$
Extraction capacity (ton/month)	$E = 1{,}125{,}000$
Production cost of fine Potash (JD/ton)	$V_1 = 100$
Production cost of standard Potash (JD/ton)	$V_2 = 75.93$
Production cost of granular Potash (JD/ton)	$V_3 = 57.14$
Production capacity of fine Potash at refinery (ton/month)	$C_1 = 63{,}000$
Production capacity of standard Potash at refinery (ton/month)	$C_2 = 108{,}000$
Production capacity of granular Potash at refinery (ton/month)	$C_3 = 21{,}000$
Unit inventory holding cost of fine Potash at Ghor Al-Safi warehouse (JD/(ton. month))	$H_{11} = 69.72$
Unit inventory holding cost of standard Potash at Ghor Al-Safi warehouse (JD/(ton. month))	$H_{21} = 81.21$
Unit inventory holding cost of granular Potash at Ghor Al-Safi warehouse (JD/(ton. month))	$H_{31} = 76.71$
Unit inventory holding cost of fine Potash at Aqaba warehouse (JD/(ton. month))	$H_{12} = 87.15$
Unit inventory holding cost of standard Potash at Aqaba warehouse (JD/(ton. month))	$H_{22} = 101.52$
Unit inventory holding cost of granular Potash at Aqaba warehouse (JD/(ton. month))	$H_{33} = 95.89$
Storage capacity of Ghor Al-Safi warehouse (ton)	$S_1 = 100{,}000$
Storage capacity of Aqaba warehouse (ton)	$S_2 = 285{,}000$
Waste percentage (per month)	$W = 0.07$
Potash yield from the extracted carnallite	$\alpha = 0.18$
Transportation cost from refinery to Ghor Al-Safi warehouse by pipes (JD/ton)	$T_{111} = 0$ (negligible)
Transportation cost from refinery to Aqaba warehouse by company trucks (JD/ton)	$T_{122} = 10$
Transportation cost from refinery to Ghor Al-Safi warehouse by outsourcing trucks (JD/ton)	$T_{114} = 8.5$
Transportation cost from refinery to Aqaba warehouse by outsourcing trucks (JD/ton)	$T_{124} = 15$
Transportation cost from Ghor Al-Safi to Aqaba warehouse using rented trucks (JD/ton)	10

the refinery to the Aqaba warehouse. At the same time, trucks with a capacity of 70 tons transport potash from the Al-Safi warehouse to the local terminal. The maximum amount of Potash that can be transported by company trucks is 87,600 tons per month. The collected data are presented in Table 1.

The average monthly demand for potash is another important issue that the researchers dealt with, and it was collected from historical company data. The international market significantly drives the demand for all potash types due to their considerable size compared to the local market. The minimum storage of each type of potash at each warehouse is dynamic and equal to 10% of the demand (See Appendix A). See Fig. 3 below for the monthly potash demand for 2019. It is worth mentioning that the demand for potash is seasonal, coinciding with planting seasons.

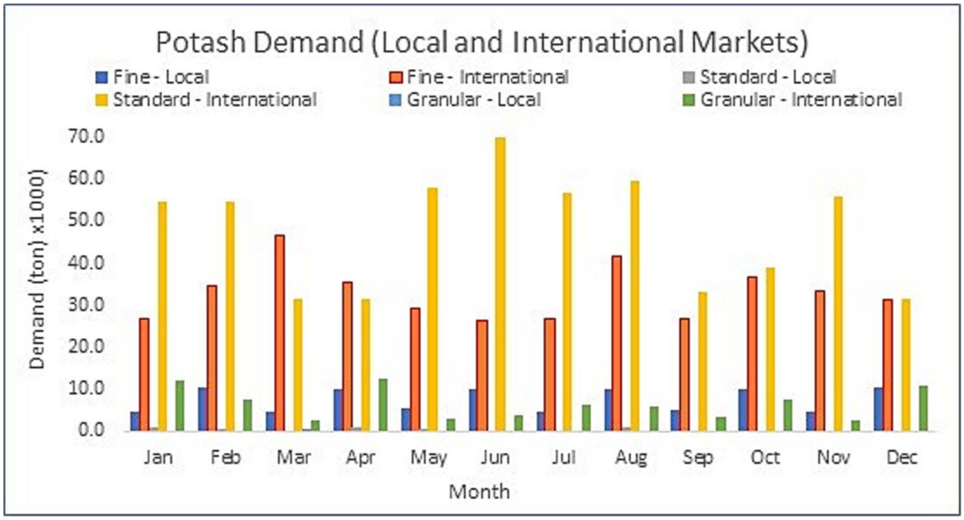

FIG. 3 Monthly demand of potash for local and international markets.

In Fig. 3, demand for the Granular type is higher for the international market than the local one, with an average monthly quantity of (12.5) thousand tons in April. It is worth mentioning that the highest demand is recorded on the Standard type, with an average monthly amount of (71.3) thousand tons in June. In contrast, the local market records the highest rate of the Fine grade with an average monthly quantity among all grades of (10.6) thousand tons in December, while the minimal demand for potash is recorded on the Granular type as this type has the lowest agriculture consumption rate in Jordan's agriculture industry.

The next section will discuss the key results of the optimization model, including the best production, storage and transportation plans.

6. Results analysis and discussion

The model is implemented using IBM ILOG CPLEX Studio and solved using CPLEX 12.10.0 with default settings on a desktop computer with a 3.4 GHz processor. The resulting model base case includes 7543 continuous variables and 2065 constraints. The optimal solution for the case is found in 18 s.

6.1. *The optimal APC results*

After running the model, optimal results were obtained, including production, storage, transportation quantities, and the costs of potash production, extraction, holding, transportation, and minor waste costs. Figure 4 shows the total cost of the potash product.

Figure 4 shows the annual total potash cost contributions by category: production accounts for 53%, storage 17%, transportation 15%, extraction 14% and waste 1%. Logistics costs, covering transportation and storage, represent 32% of the total, which is high for non-value-added activities. Figure 5 provides a monthly breakdown of costs, from extraction to transportation.

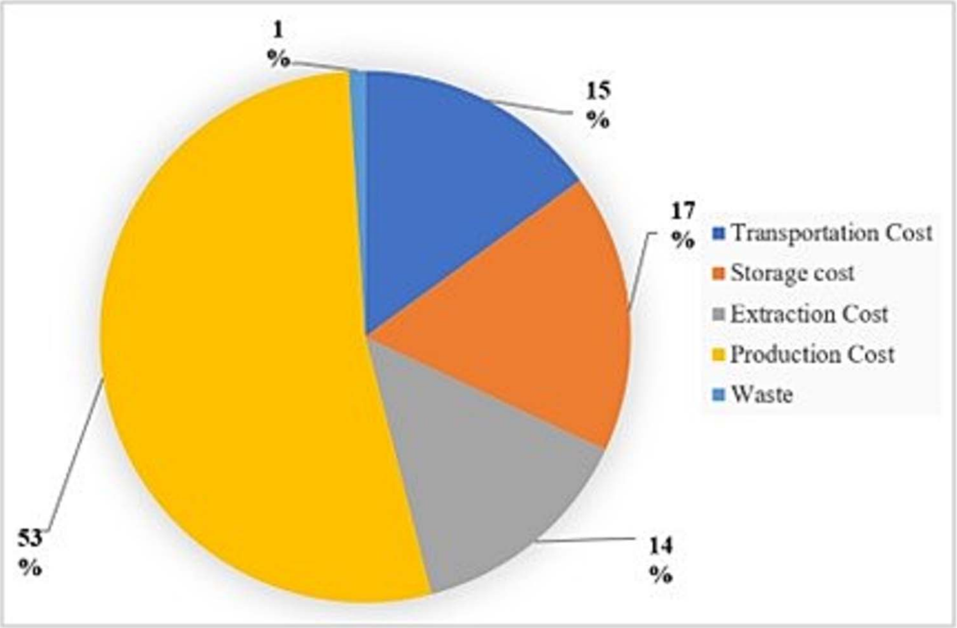

FIG. 4 Potash costs (in percentage).

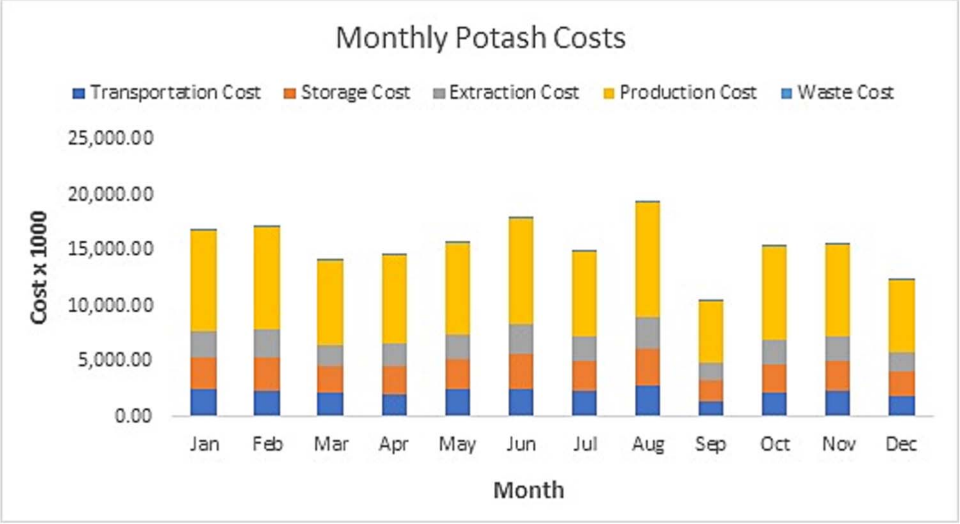

FIG. 5 Monthly potash costs in 2019.

Figure 5 shows that production is the main cost in potash production, followed by extraction, both of which are fixed and cannot be reduced. The focus is on reducing transportation costs, which will, in turn, lower storage costs when applying the inter-warehouse collaboration concept. Potash storage costs

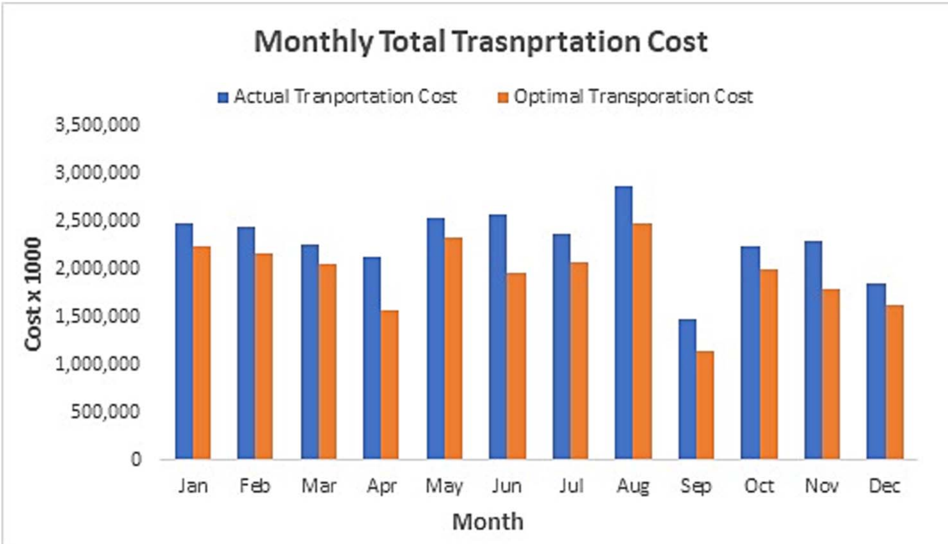

Fig. 6 Monthly transportation costs in 2019.

are high due to the need for proper conditions, such as ventilation, humidity control and protection from precipitation. In Jordan, high summer temperatures exacerbate moisture issues, potentially degrading the material, which is why APC avoids long-term storage. The study will focus on optimizing transportation costs, outsourcing and using company trucks to minimize costs while maintaining service efficiency. See Fig. 6 for monthly transportation costs.

Figure 6 shows that cost differences fluctuate from month to month. In August, significant gaps between actual and optimal costs are observed due to increased transportation between the refinery and Aqaba. June and July also show higher costs than other months, though still lower than August. This figure highlights areas where the company can optimize transportation costs, with a 15% reduction target across the year being both realistic and impactful.

The production vs demand satisfaction will be discussed below, illustrating how support is provided by the Ghor Al-Safi warehouse (in Grey) via trucks. International demand for Fine Potash products must be met monthly through sea freight at international terminals. Figure 7 demonstrates how demand for this type of potash can be satisfied.

Figure 7 illustrates that utilizing support from the SAFI warehouse (Grey) for transporting Fine Potash via trucks offers an efficient way to reduce transportation costs, achieving a 79.3% warehouse-to-warehouse contribution. SAFI's truck use proves more economical than refinery vehicles, aligning with cost-efficient supply chain practices.

Support from the Aqaba warehouse (Red) in September and December, covering 0.7% of Fine Potash supply, addresses seasonal demand fluctuations, with the remaining 20% transported directly from the refinery. This ensures a stable supply chain during these months.

Domestic demand for Fine Potash remains low due to its agricultural use (see Fig. 8 for a production, demand and inventory plan).

Figure 8 highlights the key role of the SAFI warehouse (blue) in the Fine Potash supply chain. The product, transported from the refinery via pipes, depends on the warehouse during specific months. In

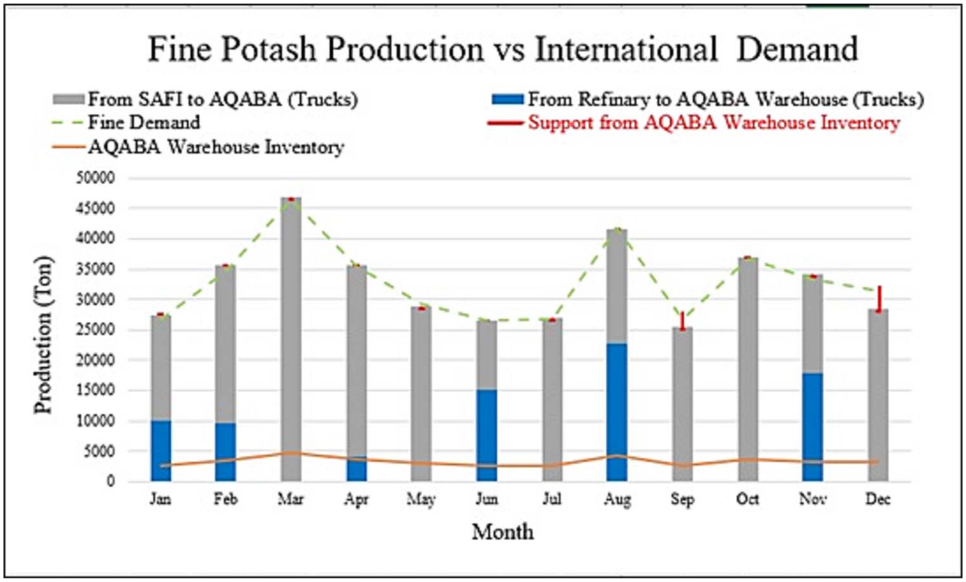

FIG. 7 Fine Potash production vs international demand in 2019.

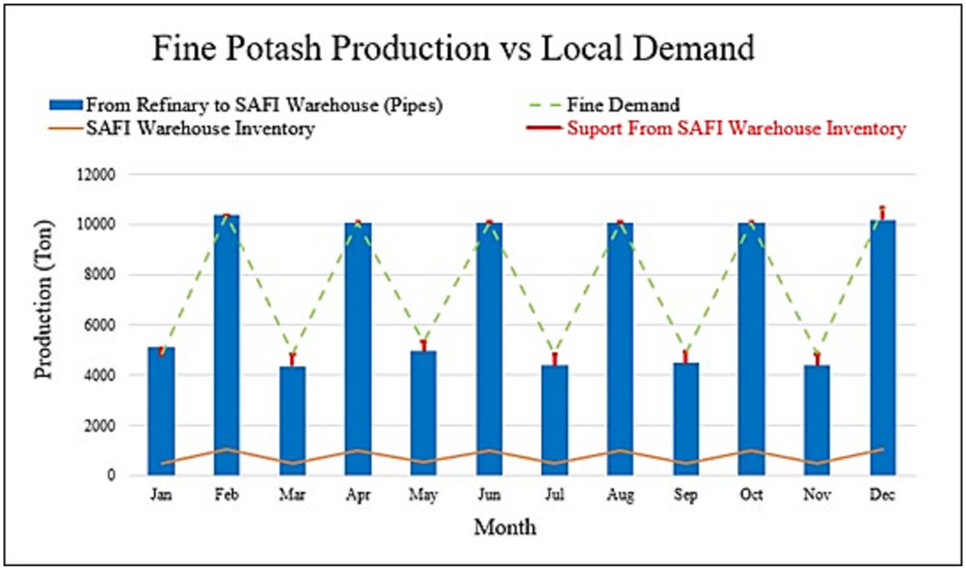

FIG. 8 Fine Potash production vs local demand in 2019.

March, May, July, September, November and December, the SAFI warehouse inventory (red) addresses supply gaps, ensuring a consistent and reliable product supply to meet market demands. The warehouse's inventory is crucial during these medium-risk periods, preventing shortages and strengthening supply

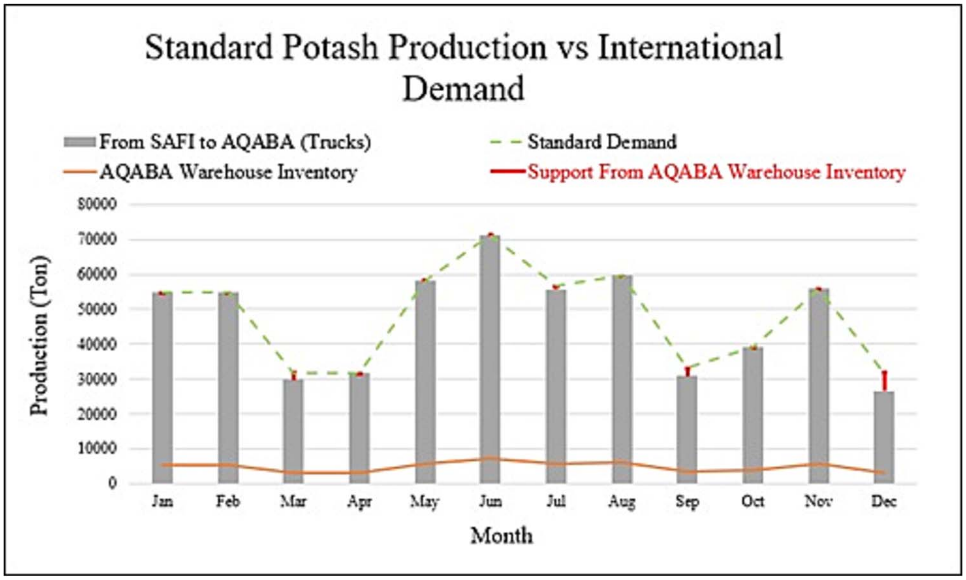

FIG. 9 Standard Potash production vs international demand in 2019.

chain resilience. By maintaining potash reserves, the warehouse minimizes disruptions and ensures timely delivery. In 2019, 3% of Fine Potash inventory supported local demand.

The Standard Potash product is exported internationally to meet global market needs, as shown in Fig. 9.

Figure 9 reveals that 98% of the requested supply of standard potash stored at the Aqaba warehouse was supported by the SAFI warehouse via trucks. This operation could benefit from a more economical distribution method than using vehicles directly from the refinery. A 2% inventory was used to meet the total international demand for standard potash. The high demand from international consumers highlights the need for a reliable supply chain. A key finding is that the SAFI warehouse intervened to cover supply shortages of the final potash product during March, July and December. Fluctuating local demand for standard potash is observed across these months, as shown in Fig. 10.

Figure 10 highlights a distinctive consumption pattern, with varying demand in January, April and August, and no demand in the intervening months. This fluctuation negates the need for sustained inventory levels due to the absence of year-round demand. Potash stored in April and August was transferred to the Aqaba warehouse as part of 'warehouse-to-warehouse support'.

The pipeline network used to transport potash from the refinery to the SAFI warehouse in January, April and August offers a cost-effective solution, reducing long-term storage and holding costs. In February, lower demand was managed by drawing from the SAFI warehouse inventory at 3% support, ensuring uninterrupted product availability without the financial burden of excess stock.

Global demand for Granular Potash, shipped from international ports, is shown in Fig. 11, outlining the optimal plan using local resources.

This figure displays similar behaviour to the Granular Potash production, demand and storage plan shown in Fig. 11, with the key difference being higher international demand than Standard Potash. Demand fluctuated sharply throughout the year, with the AQABA warehouse contributing 3% to cover

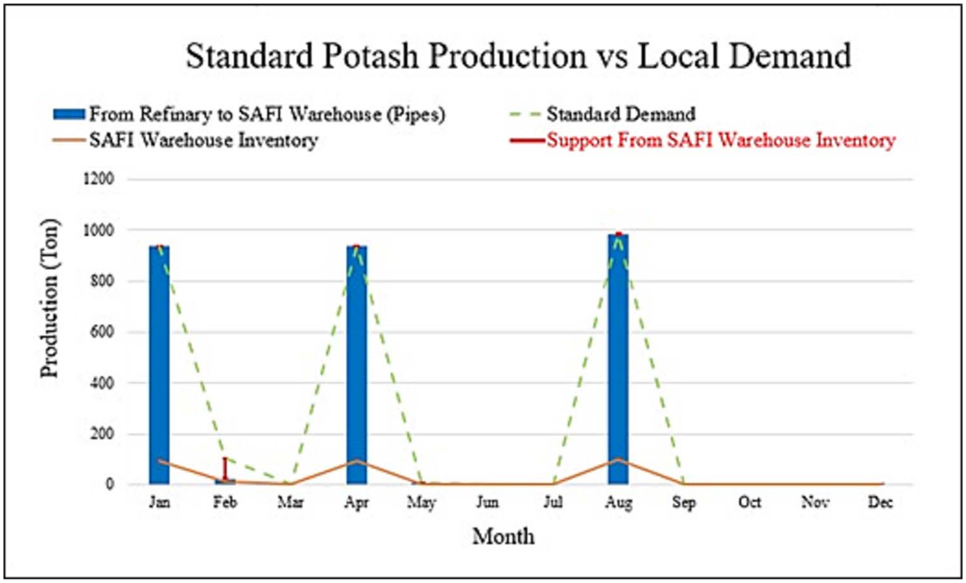

FIG. 10 Standard Potash production vs local demand in 2019.

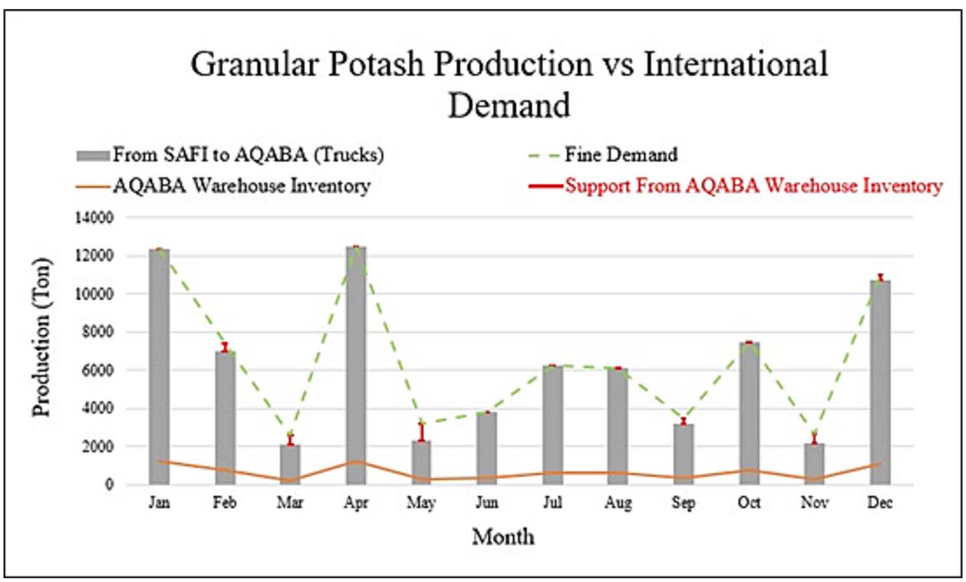

FIG. 11 Granular Potash production vs international demand in 2019.

supply shortages in February, March, May, September, November and December, while 97% of support came from truck deliveries. Local demand for Granular Potash remains weak, as it is not widely used in Jordan. Figure 12 outlines the market and optimal plan to meet this demand.

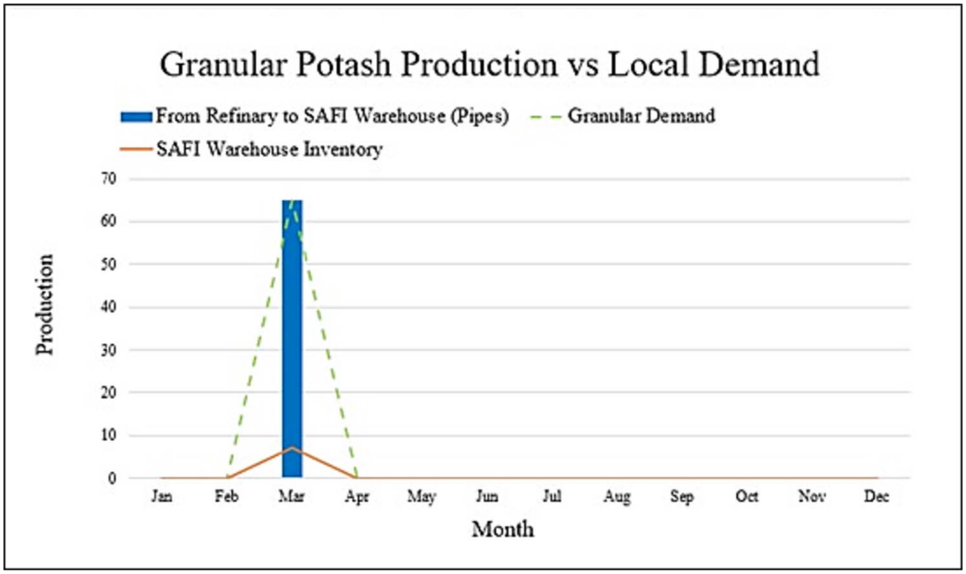

FIG. 12 Granular Potash production vs local demand in 2019.

The figure shows a single local demand for 65 tons of Granular Potash in March, intended for domestic agriculture in Jordan, explaining the lack of inventory and avoidance of holding costs. In April, seven tons were moved from Safi to Aqaba to meet part of the international demand. The model for outsourced truck use is based on the amount of potash to be transported and the cost per ton (5 JD), determining the number of trucks hired each month, as shown in Fig. 13.

Figure 13 shows that the maximum number of hired trucks (utilized) occurs in January, May and November due to the large amount of potash transported during these months across the supply chain. In contrast, the minimum number of hired trucks is observed in September and December because of the limited amount of potash required for transport. The company utilized its trucks when it was more cost-effective than outsourcing transportation to other 3PLs. The highest utilization of company trucks occurs in August, as the cost of using an outsourced truck this month is higher than in others, alongside the significant amount of potash transported.

6.2. *Sensitivity analysis study*

Other mathematical models mentioned in the literature review section would not have direct comparison factors with the same model and its experimental setting suggested in this study. Hence, they might give a biassed judgement. Therefore, a sensitivity analysis study is carried out to verify the performance of the developed system under changing variables in terms of the Key Performance Indicators (KPIs).

In this study, the behaviour of the developed mathematical model in response to different levels of the company and outsourcing vehicle costs is of utmost interest to test the model's performance. The sensitivity analysis explores the robustness and accuracy of the developed model outcomes under different company and outsourced vehicle costs. Monitoring the relationship among these parameters and the impact of their variations on the potash amount transported helps identify the inputs that cause significant uncertainty in the KPIs and for practical model analysis.

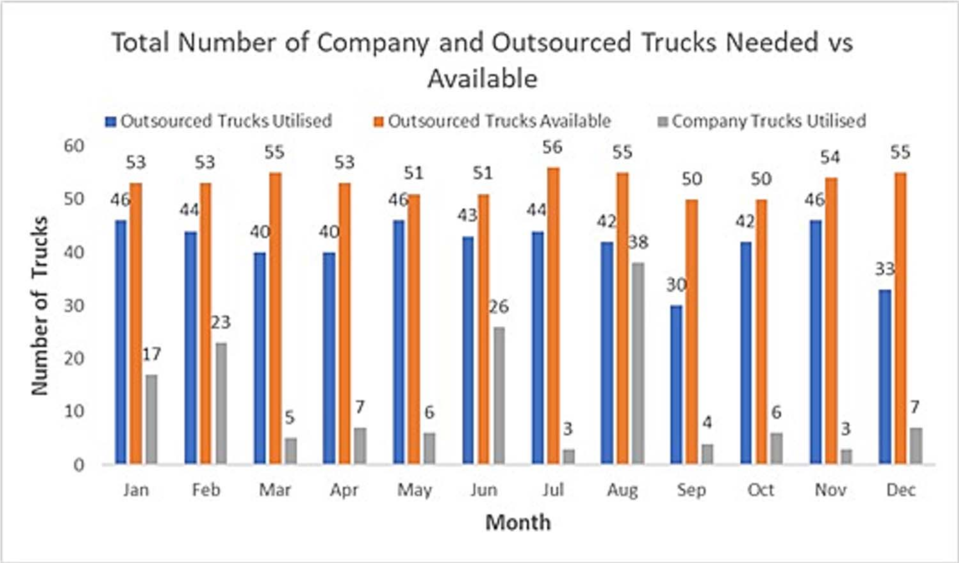

Fig. 13 Total number of outsourced vs company trucks in 2019.

The primary findings of the developed model reveal a significant reliance on outsourced trucks rather than direct transport utilizing the potash company's truck fleet. This atrributes to efficient transportation not only ensures the timely delivery of products to customers but also plays a key role in maintaining competitiveness, optimizing operations and preserving profit margins in a dynamic global market.

6.2.1. *Impact of the outsourced truck costs.* The impact of the outsourced truck costs on the utilization of outsourced and/or company vehicles is crucial so that the potash company can decide on the truck source to transport the potash to warehouses and local markets. Figure 14 shows the impact of outsourcing trucks on the selected truck source and the amount of potash to be transported to the Aqaba warehouse.

Figure 14 shows that the potash company tends to utilize outsourced trucks for transporting potash to its warehouses and local markets when the transportation cost per ton is 10 JD or lower. Conversely, if the cost surpasses this threshold, the company needs to decide whether to utilize its transportation to deliver potash to various local markets or outsource the logistics service to a 3PL company.

6.2.2. *Impact of the company truck (insource) costs.* The impact of insourced cost represented by the company trucks on selecting the truck source and the total amount of potash transported to and between warehouses and the local market is analysed as presented in Fig. 15.

Figure 15 shows that the potash company tends to use its trucks to transport potash to its warehouses and local markets if the transportation cost per ton is 10 JD or less. If the cost exceeds this range, the company prefers to outsource their transportation to other 3PL companies to avoid any maintenance and transportation costs incurred and to assist in risk mitigation related to the delivery process of potash to different local markets. This is consistent with the analysis shown in Fig. 14.

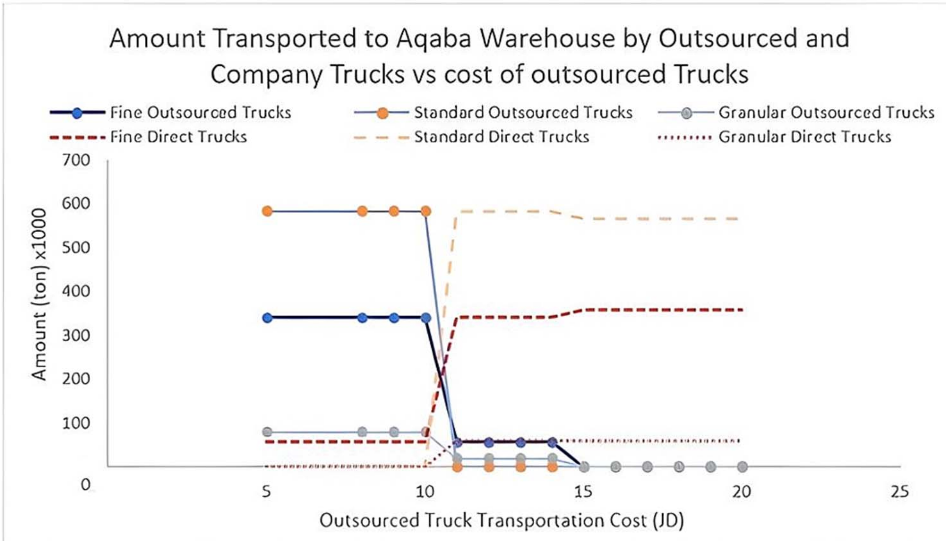

FIG. 14 Truck source and transported amount vs outsourced costs in 2019.

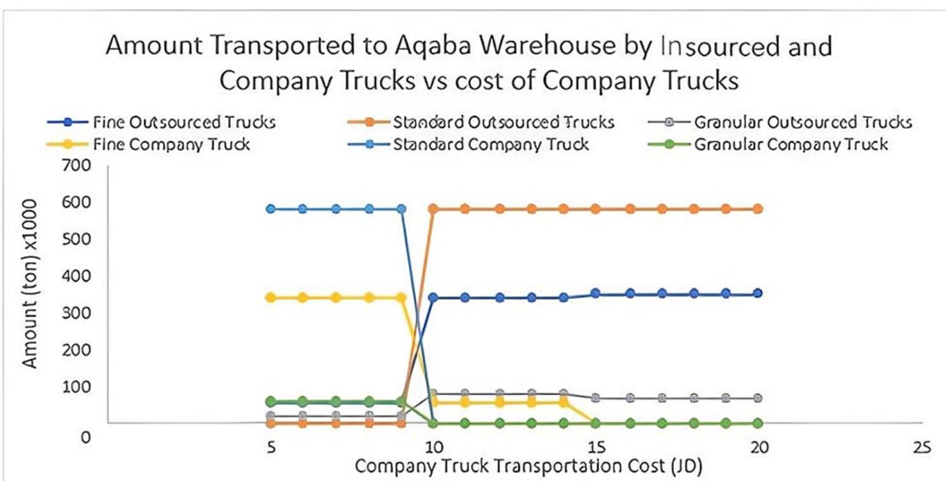

FIG. 15 Truck source and transported amount vs insourced costs in 2019.

7. Managerial insights and practical implications

The development of a novel mathematical optimization model for potash supply chain management offers valuable managerial insights and practical implications for industry stakeholders. The incorporation of fleet selection (private/outsource) as the core of the logistics, coupled with the innovative concept of ‘inter-warehouse collaboration’, addresses critical aspects of potash supply chain dynamics. These strategic considerations become essential tools for decision-makers seeking to enhance operational efficiency. The comprehensive cost considerations involving mining, production, storage and transportation provide a holistic financial perspective crucial for effective decision-making. The extensive computational experiments that validate the model's efficacy highlight its robustness in generating optimized production and storage plans. This capability becomes a key asset for managers aiming to meet both local and international demands across diverse potash product types. The evident warehouse-to-warehouse contribution support, particularly in scenarios involving Standard and Granular Potash production versus international demand, signifies the potential for collaborative practices to optimize the supply chain.

Conversely, the observed lower warehouse exchange in the Fine Potash production scenario implies precise considerations in different product contexts. Managers can utilize these insights to tailor supply chain strategies based on product types and international demand scenarios. The adoption of pipes for transporting all potash product types from the refinery to the Ghor Al-Safi warehouse to meet local demand reflects an innovative and efficient logistical solution. This approach contributes to reducing transportation costs and streamlining operations, providing a practical example for industry players seeking sustainable and cost-effective transportation solutions. Furthermore, the recorded highest inventory support in the Standard Potash production versus international demand scenario underscores the model's ability to optimize inventory levels, ensuring an efficient response to local demand. This insight is particularly valuable for managers looking to balance production, storage and local distribution effectively.

The potash supply chain management analysis also presents several key managerial implications with significant cost-saving potential. Transportation costs, which account for 15% of total production expenses, can be managed effectively through strategic fleet selection, either by using insourced company trucks or outsourcing to third-party logistics (3PL) providers. Outsourcing becomes favourable when transportation costs exceed 10 JD per ton, offering flexibility and avoiding needing specialized truck investments. Additionally, inter-warehouse collaboration, where different warehouses contribute to production and distribution, reduces transportation costs; for instance, the SAFI warehouse provided 79.30% of Fine Potash and 98% of Standard Potash production. Using pipes for potash transfer further enhances logistical efficiency by minimizing truck usage and costs, especially in local markets.

Moreover, the supply chain model's sensitivity to transportation cost changes enables agile decision-making, allowing managers to quickly adapt fleet strategies to market fluctuations and optimize cost management. These managerial insights are crucial for optimizing operations and maintaining competitiveness in the potash industry.

8. Conclusions and future research

In this study, a new mathematical optimization model for potash supply chain management was developed. This model considered fleet selection (private/outsource) as the logistics core of the potash supply chain along with the 'inter-warehouse collaboration' concept in response to the potash supply chain modelling requirements being investigated. Our model also considered various costs, including mining, production, storage and transportation.

Through extensive computational experiments, the model justified its ability to produce optimized production and storage plans to satisfy local and international demands for the three types of potash products.

The warehouse-to-warehouse contribution support was evident in both Standard and Granular potash production vs international demand scenario at a contribution level equal to 98%. Less warehouse exchange was noticed in the Fine potash production vs international demand at a contribution of 79.30%.

All potash product types were moved from the refinery to the Ghor Al-Safi warehouse using pipes for local demand. The highest inventory support to satisfy the local demand was recorded in the Standard Potash production vs international demand scenario at 2% inventory support.

A sensitivity analysis is conducted to identify the impact of transportation prices on fleet selection and the requested amount of transported potash. This demonstrated the effectiveness of our model in optimizing different aspects of the potash supply chain. It was concluded that if the transportation cost is 10 JD or less, the company hires outsourced trucks to transport their potash products to customers, avoiding maintenance and other transportation costs.

However, we acknowledge certain limitations in our current model, such as demand forecasting and supply network representation simplifications. Additionally, the model assumes static parameters, which may not fully capture the dynamic nature of the potash industry. To address these issues and further enhance the applicability of our model, we encourage researchers to develop a dynamic version of the mathematical model that incorporates stochastic elements to reflect better real-world scenarios where parameters like demand, production rates and transportation costs fluctuate over time to account for changing parameters and conditions within the potash supply chain. Nonetheless, considering stochastic elements represents a promising direction for future research to enhance the model's applicability in dynamic and unpredictable supply chain environments.

Acknowledgements

The authors would like to thank the Arab Potash Company (APC) for their involvement and support in this study. We also thank the fleet planners and warehouse managers for their invaluable support in justifying and validating the proposed optimization model.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data availability

The authors confirm that the data supporting the findings of this study are available in the article and the Appendix.

Disclosure statement

No potential conflict of interest was reported by the author(s).

References

Alnaqbi, A., Dweiri, F. & Chaabane, A. (2022) Impact of horizontal mergers on supply chain performance: the case of the upstream oil and gas industry. *Comput. Chem. Eng.*, **159**, 107659.

Alnaqbi, A., Trochu, J., Dweiri, F. & Chaabane, A. (2023) Tactical supply chain planning after mergers under uncertainty with an application in oil and gas. *Comput. Ind. Eng.*, **179**, 109176.

Bouffard, S. C. & Boggis, P. (2018) Stochastic optimization of the Jansen potash production and logistics chain. *Miner. Process. Extr. Metall. Rev.*, **40**, 207–217.

Ghatee, A. & Zarrinpoor, N. (2022) Designing an oil supply chain network considering sustainable development paradigm and uncertainty. *Chem. Eng. Res. Des.*, **184**, 692–723.

Hanandeh, A. (2022) Uncertain Supply Chain Management Factors affecting supply chain integration and customer satisfaction. *Uncertain Supply Chain Manag.*, **10**, 1037–1040. http://doi.org/10.5267/j.uscm.2022.2.008.

Jamaludin, S. Z. H. S., Mustapa, M., Ramly, Z. M. & Muhammad, N. (2024) Resilience strategies in sustainable supply chain management: bibliometric analysis and future research directions. *Built Environ. J.*, **21**, 204–223. https://doi.org/10.24191/bej.v21i2.976.

Li, Z., Zhang, Y. & Zhang, G. (2020) Two-stage stochastic programming for the refined oil secondary distribution with uncertain demand and limited inventory capacity. *IEEE Access*, **8**, 119487–119500.

Li, B., Zhang, H., Wang, M., Han, S. & Peng, S. (2024) Cooperative strategies of emission reduction in the 3PL-led supply chain. *IMA J. Manag. Math.*, **35**, 595–614.

Lima, C., Relvas, S. & Barbosa-Póvoa, A. (2021) Designing and planning the downstream oil supply chain under uncertainty using a fuzzy programming approach. *Comput. Chem. Eng.*, **151**, 107373.

Rabbani, M., Molana, S. M. H., Sajadi, S. M. & Davoodi, M. H. (2022) Sustainable fertiliser supply chain network design using evolutionary-based resilient, robust stochastic programming. *Comput. Ind. Eng.*, **174**, 108770.

Said, A. M., Mahdi, D. & Zina, Z. (2024) Applying AHP and SCOR to assess road transport risk in the petroleum supply chain. *Supply Chain Forum: An International Journal*. **25**, 380–406. https://doi.org/10.1080/16258312.2024.2328510.

Schulze, M., Rieck, J., Seifi, C. & Zimmermann, J. (2016) Machine scheduling in underground mining: an application in the potash industry. *OR Spectrum.*, **38**, 365–403. https://doi.org/10.1007/s00291-015-0414-y.

Seifi, C., Schulze, M. & Zimmermann, J. (2021a) A new mathematical formulation for a potash-mine shift scheduling problem with a simultaneous assignment of machines and workers. *Eur. J. Oper. Res.*, **292**, 27–42.

Seifi, C., Schulze, M. & Zimmermann, J. (2021b) Solution procedures for block selection and sequencing in flat-bedded potash underground mines. *OR Spectrum.*, **43**, 409–440.

Ushakova, E., Perevoshchikova, A., Menshikova, E., Khayrulina, E., Perevoshchikov, R. & Belkin, P. (2023) Environmental aspects of potash mining: a case study of the Verkhnekamskoe potash deposit. *Mining*, **3**, 176–204.

Wang, B., Zhang, H., Yuan, M., Guo, Z. & Liang, Y. (2019) Sustainable refined products supply chain: a reliability assessment for demand-side management in primary distribution processes. *Energy Sci. Eng.*, **8**, 1029–1049.

Yu, L., Chen, M. & Xu, Q. (2020) Simultaneous scheduling of multi-product pipeline distribution and depot inventory management for petroleum refineries. *Chem. Eng. Sci.*, **220**, 115618.

Zhang, W., Li, Z., Liao, Q., Zhang, H., Wang, B., Huang, S., Xu, N. & Liang, Y. (2019) A stochastic linear programming method for the reliable oil products supply chain system with hub disruption. *IEEE Access*, **7**, 124329–124340.

Appendix A. The minimum inventory level required monthly for each type of potash per warehouse in 2019.

	Ghor Al-Safi Warehouse			Aqaba Warehouse		
	Fine	Standard	Granular	Fine	Standard	Granular
Jan	485	94	0	2670	5464	1232
Feb	1036	10	0	3459	5464	740
Mar	485	0	7	4650	3172	257
Apr	1008	94	0	3549	3167	1248
May	535	1	0	2926	5820	317
Jun	1008	0	0	2657	7135	382
Jul	485	0	0	2670	5660	627
Aug	1008	99	0	4178	5960	609
Sep	492	0	0	2677	3313	345
Oct	1008	0	0	3665	3900	750
Nov	485	0	0	3324	5582	265
Dec	1064	0	0	3149	3172	1100

Appendix B. The logistics part of the potash supply chain with a focus on company trucks in 2019.

	Potash amount from the refinery to Aqaba	Number of trips	Number of trucks from refinery to Aqaba (Capacity 26)	Number of trucks from Safi to local terminal (Capacity 70)			Total trucks from Al-Safi to local terminal	Total number of utilized trucks
	Fine			Fine	Standard	Granular		
Jan	9961.81	383.15	13	3	1	0	4	17
Feb	9595.17	369	13	8	2	0	10	23
Mar	0	0	0	3	0	2	5	5
Apr	0	0	0	6	1	0	7	7
May	0	0	0	4	2	0	6	6
Jun	15091.62	580	20	6	0	0	6	26
Jul	0	0	0	3	0	0	3	3
Aug	22712.70	873	30	6	2	0	8	38
Sep	0	0	0	4	0	0	4	4
Oct	0	0	0	6	0	0	6	6
Nov	0	0	0	3	0	0	3	3
Dec	0	0	0	7	0	0	7	7
Maximum no. of company trucks			80	12				