Breathing Soliton Dynamics in Ultrafast Fibre Lasers

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ABSTRACT

We review our recent work on the dynamics of breathing solitons in mode-locked fibre lasers, including their fractal dynamics, the synchronisation and de-synchronisation of breather complexes, the transition from breathers to chaos, and the control of breather dynamics generation by genetic algorithms.

Keywords: Nonlinear dissipative systems, breathing solitons, nonlinear synchronisation, fractals, chaos.

1. INTRODUCTION

Mode-locked fibre lasers represent an interesting archetype of dissipative nonlinear systems where the interplay among the effects of nonlinearity, dispersion and energy exchange drives a wealth of complex short-pulse – 'dissipative soliton'– dynamics [1]. Although several such dynamics have been known and studied for many years, their properties have recently received renewed attention because of the development of advanced measurement techniques [2] that enable their single-shot measurement in real time, thus allowing roundtrip-to-roundtrip variations in circulating pulse characteristics to be observed, which is not possible with traditional measurement tools. Breathing solitons, manifesting themselves as localised temporal or spatial structures that exhibit periodic oscillatory behaviour, are found in various subfields of natural science, such as solid-state physics, fluid dynamics, plasma physics, chemistry, molecular biology, and nonlinear optics. In optics, initially studied experimentally in passive Kerr cavities and micro-resonators [3]-[5], the breathing soliton concept has been realised also in ultrafast fibre lasers [6]-[10]. Particularly, in [6],[10] we have reported on the first direct observations of single breathers and the self-organisation of multiple circulating breathers into various types of molecular complexes in a laser cavity, by using real-time detection techniques to resolve the fast, synchronous periodic variations of the spectrum and temporal intensity of breathers over cavity roundtrips.

In this paper, we review our latest results and advances in this fast-growing research area, by reporting on the demonstration of the fractal dynamics of breathers [11], the subharmonic synchronisation and de-synchronisation of breather complexes [12], and the transition from breathing solitons to chaos [13]. Our experimental findings are validated by numerical simulations of a lumped laser model. We also show the possibility of using genetic algorithms (GAs) to generate breather dynamics with controlled characteristics [11],[14].

2. BREATHER LASER DYNAMICS

The excitation of breather oscillations in a laser naturally leads to a second characteristic frequency in the system, i.e., the breathing frequency f_b, which manifests itself as two symmetrical sidebands around the cavity repetition frequency f_r in the radiofrequency (rf) spectrum of the laser emission. The studies in [4],[8] have shown that optical cavity-based systems can support subharmonic entrainment of breather oscillations to the cavity roundtrip (RT) time, which is a generalised form of synchronisation wherein a harmonic of the breathing frequency locks to the cavity frequency, and results from the competition between the two intrinsic frequencies to the system f_r and f_b. The theory of nonlinear systems with two competing frequencies predicts locking or resonances, in which the system locks into a resonant periodic response featuring a rational frequency ratio (winding number), and quasi-periodicity between locked states, following the hierarchy of the Farey tree and the structure of a devil's staircase [15]. While frequency-locking phenomena have been extensively studied theoretically and experimentally in many physical systems including modulated semiconductor lasers in the field of optics [16], in all these systems the second characteristic frequency was added by an external modulation.

In [11], we have established the link between breathing solitons and frequency locking by demonstrating, for the first time, frequency locking at Farey fractions of a breather laser. Fig. 1(a) shows an example of a plot of the measured breathing frequency as a function of the pump power, revealing a devil's staircase. The breathing frequencies of the steps can be related to rational winding numbers through the rf spectra (Fig. 1(b)). Indeed, in a frequency-locked state, the rf spectrum features a finite number n of equally spaced spectral lines below f_r. The most intense line is the breathing frequency, and if this is the m^{th} line from the short-frequency side, then the corresponding winding number is $f_\mathrm{b}/f_\mathrm{r} = m/n$. Importantly, the winding numbers appear from left to right in the order predicted by the Farey tree (inset of Fig. 1(a)), and the width of a given step depends on the level where the associated winding number appears in the Farey tree's hierarchy. The calculated fractal dimension D of the set of gaps is 0.906, which is close to the value of 0.87 expected from a complete devil's staircase [15], thereby indicating the universal nature of this nonlinear system. The locked breathing frequencies feature narrow linewidth and high

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signal-to-noise ratio (SNR; Fig. 1(c)) and are robust against parameter (pump power and polarisation) variations. By slightly changing the intra-cavity loss, we have also observed Farey fractions belonging to different parts of the Farey tree in our laser.

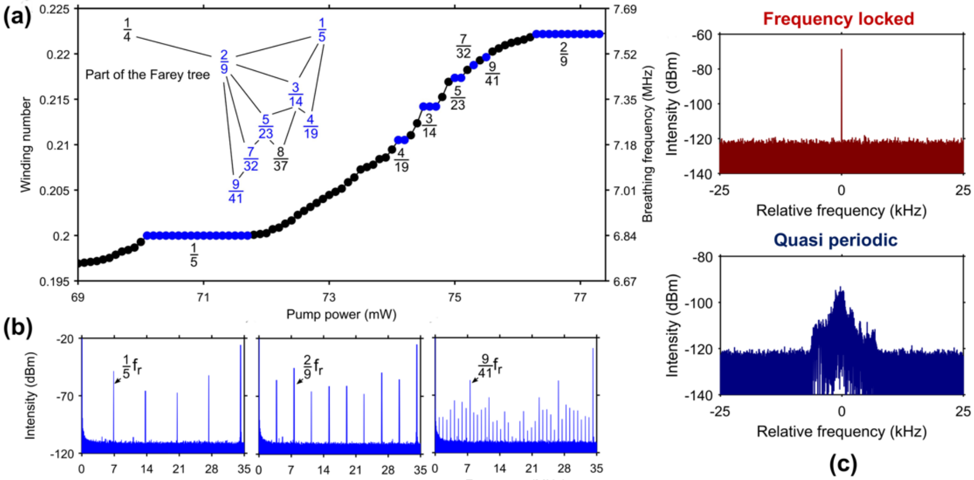

Figure 1. Farey tree and devil's staircase of breather laser. (a) Measured breather frequency (winding number) as a function of the pump power. In the inset is shown the part of the Farey tree containing the observed Farey fractions. (b) Rf spectra for the frequency-locked states corresponding to the winding numbers 1/5, 2/9 and 9/41. (c) Single-mode oscillation of the breathing frequency when frequency locking occurs and unstable multimode oscillation of the breathing frequency in a quasi-periodic state, measured over a span of 50kHz. The reference frequency is one fifth of the fundamental repetition frequency.

Building upon the studies of single-breather synchronisation, in [12], we have demonstrated for the first time the subharmonic synchronisation and desynchronisation of breather molecules in a laser cavity. We have also revealed the existence of an intermediate regime between the synchronised and desynchronised phases, featuring a subharmonic breathing frequency with non-subharmonic sidebands, which has not been found in nonlinear systems before. Figure 2(a) shows the evolution of the rf spectrum of the laser output with the pump current. At low currents, the laser emits a single soliton pulse per cavity RT. Increase of the pump current leads to the generation of a breathing soliton with a short pulsating period of 4 RTs, as revealed by the appearance of the subharmonic narrow peaks located at multiples of $\mathrm{f_b} = \mathrm{f_r}/4$: the breathing frequency is locked to the cavity repetition frequency [11]. Further pump current increase causes new equally spaced spectral lines to appear symmetrically on both sides of the subharmonic peaks. The separation between these new lines, forming a "modulated subharmonic" structure, is associated with a long pulsating period in the time domain. Higher pump currents break the frequency locking (the modulated sidebands also vanish), and the breathing frequency continuously drifts as the pump strength varies so that it is no longer commensurate with $\mathrm{f_r}$. Then the pulsating regime disappears, being replaced by a short stage of chaotic-like behaviour followed by the emission of stationary diatomic soliton molecules. At higher pump currents, pulsating solitons are generated again in the form of breather molecules, which show subharmonic, modulated subharmonic, and non-subharmonic features in succession. Then, the diatomic breather molecule switches to a triatomic molecule, and a similar evolution pattern of the rf spectrum from subharmonic to modulated subharmonic and to non-subharmonic breathing is again recorded. Complementary spatio-spectral measurements of the laser dynamics in the three phases are summarised in Fig. 2(b) for the example of the diatomic breather molecule, evidencing degraded periodicities in both the optical spectrum and energy for the non-subharmonic state, and two sets of periodicities for the modulated subharmonic regime, with a long period of approximately 88 cavity RTs and a short period of 4 RTs. The details of the corresponding rf spectra in the vicinity of $\mathrm{f_r}/4$ further highlight the differences among the three regimes.

The full access to the temporal laser dynamics provided by the simulations revealed that in the desynchronisation regime, although the breather molecules as wholes are not synchronised to the cavity, the constituent breathers are synchronised to each other with a delay, i.e., they show lag synchronisation [17]. Modulated subharmonic breather molecules also show lag synchronisation. By increasing the intra-cavity loss, we have also observed direct synchronisation-desynchronisation transitions (without the intermediate modulated subharmonic state) in our laser, which are saddle-node bifurcations [17].

The non-subharmonic sidebands characteristic of the modulated subharmonic state of breathers are not stable, and, as such, may lead to the onset of chaos. A numerical work using the master-equation approach [18] has shown that solitons generated by mode-locked fibre lasers can contain elements of chaotic dynamics similar to those of a strange attractor in low-dimensional systems. Furthermore, a very recent study [19] has reported a systematic experimental investigation of the transition of solitons to chaos in a mode-locked laser following the scenario of cascaded short- and long-period pulsations through rf spectrum measurements and Lyapunov exponent analysis. Nevertheless, despite these pioneering studies, the experimental research on chaotic dissipative solitons is still at its infancy mainly because measuring ultrafast pulse dynamics with high resolution remains a challenging task. In [13], with the help of real-time detection techniques, we have revealed a new pathway from dissipative solitons to chaos through the modulated subharmonic breathing regime of a fibre laser. By monotonically tuning the cavity gain, we have observed such an unambiguous route to chaos in both experiments and numerical simulations with laser setups relying on two different mode-locking mechanisms, thereby proving its ubiquity in optical resonators and lasers. The chaotic dynamics have been analysed by rf spectrograms, phase portraits, Lyapunov exponent and correlation dimension [20] analyses. Besides the modulated subharmonic route, we have also found that solitons in our laser can transition to chaos following the quasi-periodicity path (Ruelle-Takens scenario).

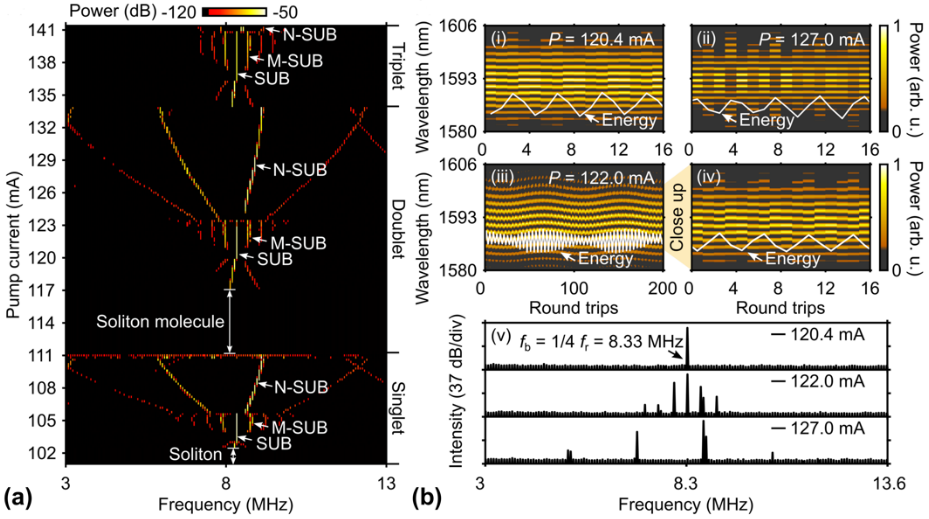

Figure 2. Subharmonic synchronisation and desynchronisation of breather complexes. (a) Measured rf spectrogram of the laser emission as a function of the pump current, showing phase transitions for single breathers, and diatomic and triatomic breather molecules. (b) Subharmonic, modulated subharmonic, and non-subharmonic diatomic breather molecules. (i-iii) Time-stretch recording of single-shot optical spectra over consecutive cavity RTs. The white curves denote the energy evolutions. (iv) A magnified version of (iii) showing the short period breathing. (v) Rf spectrum measurements for the three breathing regimes.

3. INTELLIGENT CONTROL OF BREATHER DYNAMICS GENERATION

Reaching a specific mode-locked regime in a laser generally involves adjusting multiple control parameters in a high-dimensional space, which is a laborious task if addressed via trial and error. Machine-learning tools, especially the use of GAs, have shown promising for the design of smart lasers that can tune themselves into desired operating states [21], but these algorithms have been mainly designed to target regimes of parameter-invariant, stationary pulse generation. In [14], we have demonstrated, for the first time, the use of GAs for searching and optimising the breather regime in ultrafast lasers. The merit function is the key ingredient of any GA as it defines the optimisation target. To drive the laser into the breather mode-locked regime, we have designed a merit function that exploits the ratio of the central band of the rf spectrum of the laser output located at f_{r} to the sidebands located at $f_{\pm 1} = f_{\mathrm{r}} \pm f_{\mathrm{b}}$. By including additional components in the definition of this merit function, we have achieved advanced control of the characteristics of the breather state, such as tuning of the breathing ratio (defined as the ratio of the largest to the narrowest width of the pulse spectrum within a period) and period. Furthermore, by using the merit function for breather mode locking when the pump power is set to a level that favours multi-pulse self-starting of the laser, and subsequently applying pulse count, we have generated different types of breather molecular complexes with a controllable number of elementary constituents.

The use of a GA with a merit function designed to exploit the distinguishing trait of frequency-locked breather states, i.e., a high SNR of the breathing frequency, has been the key to tailoring the frequency locking process in [11].

4. CONCLUSIONS

Our work further demonstrates that mode-locked fibre lasers are an ideal test bed for the study of complex nonlinear dynamics relevant to a large variety of physical systems and holding potential for practical applications. A laser working in the frequency-locked breather regime can generate wide rf combs with a line spacing that is not constrained by the length of the laser cavity and can reach the sub-megahertz range. While single breather oscillations in a laser represent a convenient nonlinear dynamical system to study two-frequency interactions, multi-breather complexes add new degrees of freedom into the system, i.e., the breathing frequencies of the elementary constituents, thereby opening the possibility to study the dynamics of nonlinear systems with three or more interacting frequencies, which is an important topic in nonlinear science [22]. Given the universality of the nonlinear Schrödinger equation and its extensions, it is reasonable to expect that the new modulated subharmonic dynamical state of a breather laser and the new route from solitons to chaos through modulated subharmonic breathing found in our studies will stimulate parallel research in the synchronisation and chaotic dynamics of other nonlinear physical systems. Exploiting the complex dynamics of dissipative solitons eliminates the need for external forces to generate chaos in a mode-locked laser, thus making it an advantageous optical chaos generator for applications. Furthermore, soliton chaos is inherently localised and, therefore, could extend the application areas of chaos theory.

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