Synchronisation of Breather Molecular Complexes in a Laser Cavity

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Abstract: We report on the experimental and numerical observations of subharmonic synchronisation and desynchronisation of breather molecular complexes in an ultrafast fibre laser. We also unveil an intermediate regime featuring self-modulation of the synchronised state. © 2024 The Author(s)

1. Introduction

Synchronisation, the ubiquitous phenomenon of frequency locking among coupled nonlinear oscillators, has been studied across many disciplines [1]. Breathing solitons, manifesting themselves as localised temporal/spatial structures that exhibit periodic oscillatory behaviour, are fundamental modes of many nonlinear physical systems and relate to a wide range of important nonlinear dynamics. In optics, initially studied in single-pass fibre systems [2], the breathing soliton concept has been extended to passive Kerr cavities [3, 4] and micro-resonators [5] as well as to ultra-fast fibre lasers [6–8]. The studies in [4, 7] have shown that optical cavities can support breather oscillations that are subharmonically entrained to the cavity roundtrip time. This subharmonic breather entrainment, which is a generalised form of synchronisation wherein a harmonic of the breathing frequency f_b synchronises with the cavity frequency f_r, results from the competition between the two intrinsic frequencies to the system f_r and f_b. In [9], we have shown that the winding numbers f_b/f_r of a breather fibre laser feature the hierarchy of a Farey tree and the structure of a devil's staircase.

This work builds upon the studies of single-breather synchronisation in [4, 7, 9] and demonstrates for the first time the subharmonic synchronisation of stable multi-breather bound states – breather molecules – in a laser cavity. In the desynchronised phase, while the breather molecule as whole is not synchronised to the cavity, lag synchronisation among the constituent breathers is observed. The existence of an intermediate regime between the synchronised and desynchronised phases is also unveiled, featuring a subharmonic breathing frequency with non-subharmonic sidebands.

2. Methods, Results, and Discussion

The laser was a standard erbium-doped fibre ring cavity with slightly anomalous dispersion, in which the transfer function of the nonlinear-polarisation evolution-based mode locking was controlled by three wave plates working together with a polarisation beam splitter. The laser repetition frequency was $f_r = 33.39\,\mathrm{MHz}$. The pump strength is the key parameter of this system, where its tuning under given settings of the wave plates may enable switching from stationary to breathing soliton states [6] or to soliton molecules (SMs) with various dynamics [10].

Figure 1(a) shows the evolution of the radio-frequency (RF) spectrum of the laser emission (measured by a radio spectrum analyser) with the pump current. At low currents (up to 102 mA), our laser emits a single soliton pulse per cavity roundtrip (RT) as evidenced by the single frequency component of the RF signal at f_r. Increase of the pump current leads to the generation of a breathing soliton with a short pulsating period of 4 RTs, as revealed by the appearance of the subharmonic narrow peaks located at multiples of $f_b = f_r/4$: the breathing frequency f_b is locked to the cavity repetition frequency [9]. The transition from stationary to breathing soliton correlates to the ubiquitous dynamics known as "Hopf bifurcation". Further pump current increase causes new equally spaced spectral lines to appear symmetrically on both sides of the subharmonic peaks. The separation between these new lines, forming a "modulated subharmonic" structure, is associated with a long pulsating period in the time domain. Pump currents above 106 mA break the frequency locking (the modulated sidebands also vanish), and the breathing frequency continuously drifts as the pump strength varies so that f_b is no longer commensurate with f_r. We term this laser regime "non-subharmonic breathing". For currents above 111 mA, the pulsating regime disappears, being replaced by a short stage of chaotic-like behavior and then the emission of stationary diatomic SMs up to 117 mA, when pulsating pulses are generated again in the form of breathing SMs. Between 120 and 123 mA, subharmonic and modulated subharmonic behaviors are retrieved whereas non-harmonic features are apparent between 117 and 120 mA and then between 123 and 133 mA. Above 133 mA, the diatomic breathing SM switches to a triatomic molecule, and a similar evolution pattern of the RF spectrum from subharmonic to modulated subharmonic and to non-subharmonic breathing is again recorded.

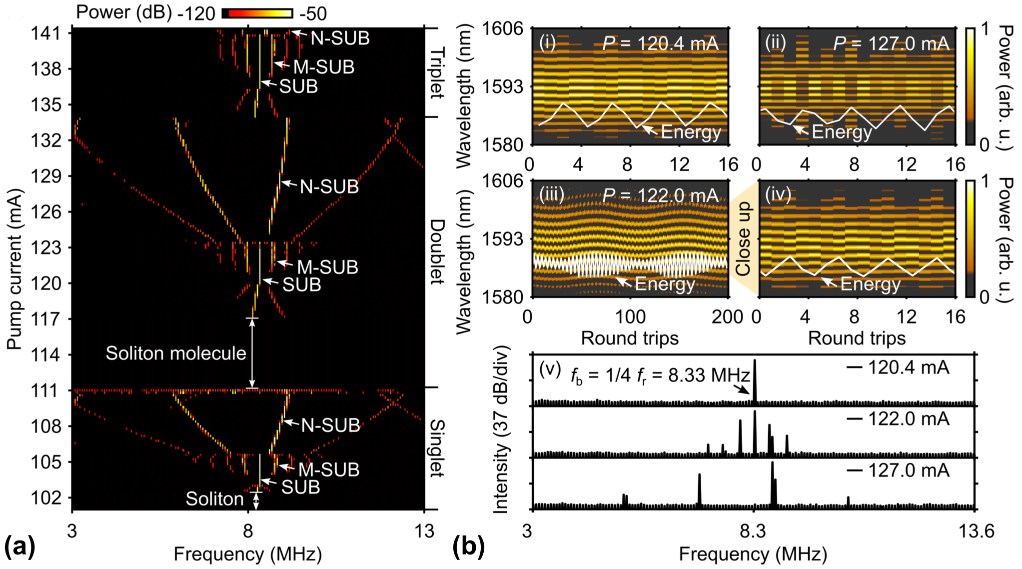

Fig. 1. (a) Map of the laser spectral intensity in the space of RF and pump current, showing phase transitions for single breathers, and diatomic and triatomic breathing SMs. (b) Subharmonic, modulated subharmonic, and non-subharmonic diatomic breathing SMs. (i-iii) Time stretch recording of single-shot optical spectra over consecutive cavity RTs. The white curves denote the energy evolutions. (iv) A magnified version of (iii) showing the short period breathing. (v) RF spectrum measurements for the three breathing regimes.

Complementary spatio-spectral measurements of the laser dynamics in the three phases are summarised in Fig. 1(b) for the example of the diatomic breather molecule. Panels (i-iii) therein show the roundtrip-resolved optical spectra measured by the time-stretch technique [11]. Periodic variations of the spectral intensity across a well-defined period of 4 cavity RTs can be observed for the subharmonic regime, accompanied by corresponding synchronous periodic changes of the pulse energy. By contrast, the non-subharmonic regime shows degraded periodicities in both the optical spectrum and energy. In both regimes, the period of spectral fringes remains almost unchanged over cavity RTs, indicating an almost constant intra-molecular pulse separation. The modulated subharmonic regime features two sets of periodicities with a long period of approximately 88 cavity RTs and a short period of 4 RTs. The details of the corresponding RF spectra in the vicinity of $f_r/4$ (panel (v)) further highlight the differences among the three regimes: the subharmonic state features a single very narrow frequency component located exactly at $f_b = f_r/4$, while a set of equally spaced narrow sidebands on both sides of $f_b = f_r/4$ is observed for the modulated subharmonic case. In sharp contrast to this, the non-subharmonic regime presents much broader spectral lines, confirming the frequency unlocked operation of the laser. The laser dynamics for the triatomic breather molecule generation regime displayed qualitatively similar features.

Our experimental findings were confirmed by numerical simulations of the laser based on a lumped model where each part of the cavity was modelled separately. By changing the polarisation state of the laser in the experiments or, equivalently, by increasing the linear intra-cavity loss in the model, direct synchronisation-desynchronisation transitions (without the intermediate modulated subharmonic state) could also be observed in our breather laser. Such direct transitions are saddle-node bifurcations [1]. The full access to the temporal laser dynamics provided by the simulations revealed that in the desynchronisation regime, although the breather molecules as wholes are not synchronised to the cavity, the constituent breathers are synchronised to each other with a delay, i.e., they show lag synchronisation [1]. Modulated subharmonic breathing SMs also show lag synchronisation. These and further results will be discussed at the conference.

3. Conclusion

We have observed subharmonic and non-subharmonic breathing soliton structures in an ultrafast fibre laser in both experiments and numerical simulations. We have also unveiled the existence of an intermediate state – modulated subharmonic breathing – between the two phases. Our findings open new avenues for the study of nonlinear synchronisation dynamics.

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