Synchronisation and chaotic dynamics of dissipative breathing solitons

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Breathing solitons exhibiting periodic oscillatory behaviour have emerged as a ubiquitous ultrashort pulse regime of mode-locked fibre lasers thanks to the recent development of real-time measurements. In this paper, we review our recent experimental and numerical modelling results and advances in the area.

The competition between the cavity repetition frequency f_r and the breathing frequency f_b in optical cavities with feedback may lead to a generalised form of synchronisation wherein a harmonic of f_b synchronises with f_r. In [1], we have reported frequency locking at Farey fractions of a breather laser by demonstrating for the first time that the winding numbers f_b/f_r show the hierarchy of the Farey tree and the structure of a devil's staircase, in accordance with the predictions from the theory of nonlinear systems with two competing frequencies. The locked breathing frequencies feature narrow linewidth and high signal-to-noise ratio and give rise to dense radiofrequency combs with a line spacing that is not constrained by the cavity length. Building upon these studies of single breather synchronisation, in [2] we have demonstrated for the first time the subharmonic synchronisation of breather molecules in a laser cavity, thus opening the possibility to study the dynamics of nonlinear systems with three or more interacting frequencies. In the de-synchronised phase, while the breather molecule as whole is not synchronised to the cavity, lag synchronisation among the constituent breathers is observed. We have also revealed the existence of an intermediate state between the synchronised and desynchronised phases featuring self-modulation of the synchronised state, which had not been observed in nonlinear systems before. Furthermore, we have demonstrated that the modulated subharmonic phase of breathing solitons can trigger the onset of chaos in a mode-locked laser, thereby unfolding a new route to chaos in nonlinear systems [3].

[1] J. Peng, X. Wu, Y. Zhang, S. Boscolo, C. Finot, and H. Zeng, "Farey tree and devil's staircase of frequency-locked breathers in ultrafast lasers," *Nat. Commun.* 13, 5784, 2022.
[2] X. Wu, J. Peng, S. Boscolo, C. Finot, and H. Zeng, "Synchronization, desynchronization, and intermediate regime of optical breathing solitons and soliton molecules in a laser cavity," *Phys. Rev. Lett.* 131, 263802, 2023.
[3] H. Kang, A. Zhou, Y. Zhang, X. Wu, B. Yuan, J. Peng, C. Finot, S. Boscolo, and H. Zeng, "Observation of optical chaotic solitons and modulated subharmonic route to chaos in mode-locked laser," Submitted, 2024.