Breathing Soliton Dynamics in Laser Cavities

S. Boscolo[1*], J. Peng[2], X. Wu[2], Y. Zhang[2], C. Finot[3], and H. Zeng[2]

[1]Aston Institute of Photonic Technologies, Aston University, Birmingham B4 7ET, United Kingdom
[2]State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China
[3]Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR 6303 CNRS-Université de Bourgogne, 21078 Dijon, France
[*]Corresponding author: s.a.boscolo@aston.ac.uk

Abstract: We review our recent work on the dynamics of breathing solitons in ultrafast fibre lasers, including single breathers and breather molecular complexes, breather explosions, the synchronisation and de-synchronisation of breather complexes, the fractal dynamics of breathers, their transition to chaos, and their control by genetic algorithms.

Mode-locked fibre lasers are well-known to display a rich landscape of "dissipative soliton" dynamics, resulting from the interplay of the nonlinearity with dispersion and dissipation. Breathing solitons, manifesting themselves as localised temporal/spatial structures that exhibit periodic oscillatory behavior, are fundamental modes of many nonlinear physical systems and relate to a wide range of important nonlinear dynamics. First studied experimentally in fibre Kerr cavities and optical micro-resonators, optical breathers have also emerged as a ubiquitous ultra-short pulse regime of passively mode-locked fibre lasers [1-3] thanks to the recent development of real-time measurements. In the present paper, we review our recent experimental and numerical modelling results and advances in this fast-growing research area.

Breathing solitons in mode-locked lasers feature synchronous periodic variations of their spectrum and temporal intensity over cavity round trips. In [1], we have captured such a fast evolutionary behaviour in real time using time-stretch dispersive Fourier transform-based single-shot spectral measurements and spatio-temporal intensity measurements. One of the remarkable properties of dissipative solitons, which are mostly absent in integrable systems, is the ability to form stable multi-soliton bound states, also termed "soliton molecules", and showing similar dynamics to matter molecules. For the first time in experiments with mode-locked fibre lasers, breather-pair molecules have been also generated in the cavity [1]. While soliton pairs constitute the central soliton molecule case, soliton molecules can exist in various isomers, and a large population of solitons can self-assembly into macromolecules and soliton crystals. In [2], we have demonstrated various types of breather molecular complexes (BMCs), including multi-breather molecules, and complexes formed by the binding of two diatomic molecules, or a diatomic and a monoatomic molecule. We have also observed non-equilibrium dynamics of breathers, including collisions and annihilation of an elementary breather within a complex. Soliton explosions are among the most striking nonlinear dissipative phenomena that can manifest in mode-locked lasers. In this regime, a dissipative soliton circulating in the laser cavity experiences an abrupt structural collapse, but within a few round trips returns to its original quasi-stable state. By pushing the analogy in dynamics between stationary and breathing solitons further, we have demonstrated the explosive dynamics of breathers in a mode-locked fibre laser [3].

A laser working in the breather generation regime shows competition between the frequency of the breather oscillations f_b and the cavity repetition frequency f_r. In [4], we have reported frequency locking at Farey fractions of a breather laser by demonstrating for the first time that the winding numbers f_b/f_r show the hierarchy of the Farey tree and the structure of a devil's staircase, in accordance with the predictions from the theory of nonlinear systems with two competing frequencies [5]. The fractal dimension of 0.906 determined from the measured staircase indicates the universal nature of this nonlinear system. The locked breathing frequencies feature narrow linewidth and high signal-to-noise ratio (SNR) and give rise to dense radiofrequency (RF) combs with a line spacing that is not constrained by the cavity length and can reach the sub-megahertz range. The subharmonic breather entrainment is a generalised form of synchronisation wherein a harmonic of the breathing frequency synchronises with the cavity frequency. Our further study [6] has shown for the first time the subharmonic synchronisation and de-synchronisation of breather molecules in a laser cavity, thus opening the possibility to study the dynamics of nonlinear systems with three or more interacting frequencies. In the de-synchronised phase, while the breather molecule as whole is not synchronised to the cavity, lag synchronisation among the constituent breathers is observed. The existence of an intermediate regime between the synchronised and de-synchronised phases is also unveiled, featuring a subharmonic breathing frequency with non-subharmonic sidebands. Furthermore, we have demonstrated that the modulated subharmonic phase of breathing solitons can trigger the onset of chaos in a mode-locked laser, thereby unfolding a new route from solitons to chaos in nonlinear systems [7].

Reaching a specific mode-locked regime in a laser generally involves adjusting multiple control parameters in a high-dimensional space, which is a laborious task if addressed via trial and error. In [8], we have demonstrated, for the first time, the use of genetic algorithms (GAs) for searching and optimising the breather regime in ultrafast lasers. We have designed merit functions relying on the characteristic features of the RF spectrum of the laser output, which are capable to locate various self-starting breather states in the laser, including single breathers with controllable breathing ratio and period, and BMCs with a controllable number of elementary constituents. The use of a GA with a merit function designed to exploit the distinguishing trait of frequency-locked breather states, i.e., a high SNR of the breathing frequency, has been the key to tailoring the frequency locking process in [4].

In conclusion, our work further demonstrates that mode-locked fibre lasers are an ideal test bed for the study of complex nonlinear wave dynamics relevant to a large variety of physical systems.

References

1. Peng, J., S. Boscolo, Z. Zhao *et al.*, "Breathing dissipative solitons in mode-locked fiber lasers," Sci. Adv., Vol. 5, eaax1110, 2019.
2. Peng, J., Z. Zhao, S. Boscolo *et al*., "Breather molecular complexes in a passively mode-locked fibre laser," Laser Photon. Rev., Vol. 15, 2000132, 2021.
3. Peng, J. and H. Zeng, "Experimental observations of breathing dissipative soliton explosions," Phys. Rev. Appl., Vol. 12, 034052, 2019.
4. Wu, X., Y. Zhang, J. Peng *et al.*, "Farey tree and devil's staircase of frequency-locked breathers in ultrafast lasers," Nat. Commun., Vol. 13, 5784, 2022.
5. Jensen, M. H., P. Bak and T. Bohr, "Complete devil's staircase, fractal dimension, and universality of mode-locking structure in the circle map," Phys. Rev. Lett., Vol. 50, 1637, 1983.
6. Wu, X., J. Peng, S. Boscolo *et al.*, "Synchronization, desynchronization, and intermediate regime of breathing solitons and soliton molecules in a laser cavity," Phys. Rev. Lett., Vol. 131, 263802, 2023.
7. Wu, X., J. Peng, S. Boscolo *et al.*, "Observation of optical chaotic solitons and modulated subharmonic route to chaos in mode-locked laser," Submitted, 2024.
8. Wu, X., J. Peng, S. Boscolo *et al.*, "Intelligent breathing soliton generation in ultrafast fibre lasers," Laser Photon. Rev., Vol. 16, 2100191, 2022.