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An Economic Optimization Model of an E-Waste Supply Chain Network: Machine Learned Kinetic Modelling for Sustainable Production

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Abstract: *Purpose:* E-waste management (EWM) refers to the operation management of discarded electronic devices, a challenge exacerbated due to overindulgent urbanization. The main purpose of this paper is to amalgamate production engineering, statistical methods, mathematical modelling, supported with Machine Learning to develop a dynamic e-waste supply chain model. *Method Used:* This article presents a multidimensional, cost function-based analysis of the EWM framework structured on three modules including environmental, economic, and social uncertainties in material recovery from an e-waste (MREW) plant, including the production–delivery–utilization process. Each module is ranked using Machine Learning (ML) protocols—Analytical Hierarchical Process (AHP) and combined AHP-Principal Component Analysis (PCA). *Findings:* This model identifies and probabilistically ranks two key sustainability contributors to the EWM supply chain: energy consumption and carbon dioxide emission. Additionally, the precise time window of 400–600 days from the start of the operation is identified for policy resurrection. *Novelty:* Ours is a data-intensive model that is founded on sustainable product designing in line with SDG requirements. The combined AHP-PCA consistently outperformed traditional statistical tools, and is the second novelty. Model ratification using real e-waste plant data is the third novelty. *Implications:* The Machine Learning framework embeds a powerful probabilistic prediction algorithm based on data-based decision making in future e-waste sustained roadmaps.

Keywords: supply chain sustainability; e-waste management; sustainable production; machine learning; kinetic modeling; global optimization

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Citation: Debnath, B.; Chattopadhyay, A.K.; Kumar, T.K. An Economic Optimization Model of an E-Waste Supply Chain Network: Machine Learned Kinetic Modelling for Sustainable Production. *Sustainability* **2024**, *16*, 6491. https://doi.org/10.3390/su16156491

Academic Editor: Janusz Kozinski

Received: 10 May 2024
Revised: 11 June 2024
Accepted: 16 July 2024
Published: 29 July 2024

1. Introduction

E-waste management (EWM) is a global challenge with complexities beyond general waste management [1,2]. Issues like energy efficiency, carbon footprint, availability, heterogeneity, technology management, and overall administration impact the supply chain. To add to the challenge, these factors often evolve asynchronously leading to the randomized time evolution kinetics of an e-waste supply chain network (SCN), which we refer to as uncertainties. E-waste management is a lucrative sector, driven by rapid urbanization and high consumer demand for electronic devices. Such demands have escalated manifold in the past decade [3–6]. The supply-to-demand ratio targets short innovation cycles, affordable pricing, and eye-catching features. A tacit underlier has been the ever-decreasing life span of electronic items [6], a marketing accessory embedded in the supply chain structure to prevent market stagnation. This contributes to higher product obsolescence, as laterally accepted by the United Nations as well, termed as a 'tsunami of electronic waste' [7].

It is a well-known fact that e-waste is the world's fastest growing waste stream [8]. The worldwide generation of e-waste was nearly 62 million metric tons in 2022, which is expected to reach 82 million metric tons by 2030, and 120 million metric tons by 2050 [9,10]. E-waste is a very heterogeneous material and it contains an assortment of materials including metals, polymers, and siliceous materials, including glass [11,12]. E-waste contains a greater volume of metals than natural ores and hence it becomes highly lucrative to recover metals from e-waste [13,14]. Hence, e-waste streams trap a huge amount of metallic and non-metallic resources, which deems the urban mining of e-waste not an option anymore, but rather a necessity [15].

Industrially, Material Recovery from E-waste is mainly accomplished through mechanical recycling, where the metallic fraction is often transported to sister companies or third-party smelters [13,16]. Material Recovery from E-waste (MREW), a contemporary framework, advocates the fact that amalgamating both mechanical and chemical recycling of e-waste under one umbrella can boost urban mining [13,15]. In developing nations, informal sectors preponderate the resource recovery from e-waste chains using rudimentary technologies for metal extraction [17,18]. This results in an inefficient MREW supply chain, which is not a closed loop incumbency, thus affecting business cycles in the longer run [15]. The erratic and uncontrolled use of technology and disposal schemes attribute a strong stochastic element to the e-waste generation process. Typical examples of this relate to heterogeneous material, energy efficiency, secondary emissions, recovery efficiency, supply uncertainty, etc. Hence, the efficiency and the flexibility of the SCN play a pivotal role in determining the profitability profile of an MREW facility. The following subsections outline the published literature on the e-waste supply chain network, its sustainability, and related issues.

1.1. E-Waste Supply Chain Network and Supply Chain Sustainability

1.1.1. General Literature on E-Waste SCN and Application of MCDM Techniques

The SCN of e-waste is a technically complex routine that is even more interesting due to the versatility of the supply chain dynamics [3,4]. There is extensive literature available on e-waste SCN. Hazra et al. [19] analyze e-waste SCN issues in India, which serves as a template for the e-waste menace faced by developing countries. Sharma et al. [20] presented an Analytic Hierarchy Process (AHP)-based approach for optimizing delivery SCN, which focuses on qualitative and quantitative aspects together. AHP was projected as a possible administrative tool for e-waste management focusing on three pillars of sustainability, as well as political and technological aspects [21]. Again, AHP was utilized by Lin et al. [22] to investigate multiple criteria of supply chain management of notebook laptops. They developed a sensitivity model for escalated implementation of supply chain strategies. Recently, Karuppiah et al. [23] utilized a combination of the fuzzy delphi method (FDM), fuzzy AHP, and fuzzy measuring attractiveness by the categorically based evaluation techniques (FMACBETH) and QFD to identify sustainable supply chain strategies. This study integrates the Business Operations model with multiple Multi-Criteria Decision Models (MCDMs) techniques by exploiting the economic theory of duality between production and cost.

1.1.2. Issues and Challenges in E-Waste SCN

The issue of a sustainable e-waste supply chain has been dealt with by focusing on the prerequisites of a sustainable e-waste recycling plant. The outcome of this study primarily addressed requirements from the perspective of production and environmental engineers, although some of that could qualify as operational management issues as well [24]. Wang et al. [25] analyzed the effect of Chinese government subsidy on e-waste SCN of both formal and informal scenarios. The primary revolutionary work on e-waste SCN was crafted by Ghosh et al. [18]. The supply chain mapping executed by them was highly concentrated on BRICS nations, yet the concerns highlighted were surprisingly more widely applicable. They highlighted compliance to the Basel Convention, transboundary movement, informal handling of e-waste, rudimentary processing of e-waste in developing countries, incompetence of formal collection, etc., as major issues. Cruz-Sotelo et al. [26] mapped the e-waste supply chain in Mexico and presented a legal framework for sustainable e-waste management in Mexico. Baidya et al. [27] recognized the drawbacks of the e-waste SCN in India using AHP and proposed an alternative of sustainable SCN framework. Recently, Debnath et al. [3] discussed the sustainability aspects of e-waste supply chain network in detail, and explored the role of new and emerging information and communication technology in greening the supply chain network.

1.1.3. Application of Mathematical Modelling of E-Waste SCN

From the mathematical modeling perspective, Isernia et al. [28] analyzed the efficiency of the reverse SCN of e-waste in Italy within a circular economy framework. They relied on the probability transition matrix method to evaluate the collection efficiency of the collection centers with the threshold targets defined by the European Union. Polat et al. [29] employed fuzzy mathematics to model an e-waste SCN considering sales prices, product weights, costs, and product demands as the fuzzy parameters that reflect the uncertainties in the real-life model much more efficiently. The model developed by Wang et al. [30] is structured on a three-echelon game theoretic supply chain model that estimated the optimal pricing decision and government subsidies with stakeholders, considering the e-waste remanufacturing utilization rate as a key parameter. Moradi et al. [31] developed a sustainable and generic e-waste supply chain network based in London focusing on circular economy. Ghalehkhondabi and Ardjmand [32] used a game theory approach to model an e-waste supply chain with three players including the government, a recycling center, and a collection center, and analyzed different scenarios of material recovery, sustainability, and supply chain profits. Baidya et al. [33] have used a combined AHP-QFD to prioritize different issues and challenges prevailing along the e-waste supply chain. They validated their method findings through case studies in India and China. The study compared the supply chain networks of India and China and discussed the sustainability aspects qualitatively. Recently, Karuppiah and Sankaranarayanan [34] integrated a fuzzy set (FFS) with AHP, and Decision-Making Trial and Evaluation Laboratory (DEMATEL).

1.2. Research Gap and Research Questions

The e-waste market is uncertain and susceptible to market volatility. Hence, it is imperative to identify the sensitive nodes that can reduce uncertainty and maximize profit. Ambiguity in the e-waste SCN needs immediate attention, as it can affect the profitability trend. A MREW facility performing “Mechanical Recycling of E-Waste” has many economic constraints to abide by environmental regulations, an aspect that contributes towards environmental uncertainty. However, this does not rule out the effects of other uncertainties—economic and social uncertainty.

The supply chain literature is replete with examples of applications of Machine Learning (ML) and Deep Learning (DL), including Artificial Intelligence (AI) [35,36]. Mixed integer nonlinear programming [37], fuzzy mathematics [38], robust optimization [39], intelligent algorithms [40], scenario programming [41], stochastic programming [42], and multivariate multi-layered AI [43] have all been used to analyze stochastically driven supply chains. While even e-waste prognosis under an ML/AI route is not an entirely virgin territory, we need to understand that, structurally, an e-waste supply chain network is a reverse flow network. Such networks are inherently complex and heterogeneous [44] and require special treatment and specific models [45]. Typical examples include but are not limited to the implementation of game theory [46], system dynamics modelling approach [47], evolutionary game analysis [48], fuzzy mathematics [29], combined MCDM techniques [23], etc. Since the SCN of e-waste is distinctively different from a generic SCN, there is a clear research gap that can integrate the time evolution of uncertainties evolving from the three pillars of sustainability in a SCN framework. The following research questions arise from the previous analysis:

- How to analyze time evolution kinetics of uncertainties evolving from the three pillars of sustainability and optimize the e-waste SCN?

- How business profit, constrained by incumbencies, is related to the cost kernel?
- How the minimization of cost kernel can provide stable and optimal solutions while addressing the complexities and heterogeneities of the e-waste SCN?
- How does the hybrid AHP-PCA formalism improve the ranking of the variables affecting the cost kernel and what impact will this have on the opertional efficiency and green supply chain practices?

1.3. Objectives and Novelty of the Study

Under the current investigation, we have extended a recent method developed by Debnath et al. [5] for the optimization of a supply chain cost kernel with uncertainty components affecting it. The underlying technique extrapolates the knowledge base from the physics of mechanics, economics of operations, and the mathematics of stochastic processes to analyze a dynamically evolving 'free energy' model. The model used in this present work structurally follows a similar mathematical description as in Debnath et al. [5], but intrinsically differs in the definition of the basic cost function. Three modes of uncertainty, from the three pillars of sustainability, respectively, have been used as inputs in our model. The individual (time-evolving) contributions from the three 'supply lines' are then ranked first through an independent AHP, followed by a cross-verification and benchmarking through Principal Component Analysis (PCA). The impacts of unconstrained and constrained environments are then separately analyzed supporting on the dynamic variables within this structure, leading to a clear identification of the operational windows for a green SCN. The following are our research objectives:

- To develop a model of a dynamic production process, with three pillars of sustainability (economic, environmental, and social), and derive from it a quadratic cost kernel that can optimize business performance that is consistent with SDG.
- To investigate the effects of AHP and combined AHP-PCA to rank and determine the interrelationships of the uncertainty variables on the developed model determining the profitability of the e-waste recycling facility.
- To identify the most delicate nodes and dedicated operational windows for a resilient e-waste SCN by studying the time evolution dynamics of the leading variables.

Our model integrates all possible cost components based on the 'Utilization-to-cradle' regime. One can add or remove components in the cost function and rework the problem. If it is redundant, then the dimension of the resulting matrix in Equations (10) and (13) will be reduced (Section 3.2). The choice is ultimately with the concerned recycler, as, after all, they know their SCN best. This inherent self-sufficiency is the beauty also known as the novelty-cum-flexibility of the model.

2. Materials and Methods

2.1. Concept and Mathematical Background

Supply chain networks can be addressed from the perspective of Life Cycle Assessment (LCA), and the range can vary based on the approach taken, such as cradle-to-cradle, cradle-to-gate, gate-to-gate [5,11] etc. Figure 1 illustrates different boundaries of the supply chain based on LCA concepts. In this study, we consider a generalized version of an e-waste SCN, starting from the consumers and ending with the 3rd party recyclers handling the recovered materials from e-waste, translating to a "utilization-to-cradle" model [5].

The current investigation exclusively develops a cost function-based supply chain model for Material Recovery from E-waste (MREW) facilities catering to both mechanical and chemical recycling. An MREW facility can be defined as an integrated facility that can perform size reduction by implementing mechanical recycling as well as recover materials using the MREW technologies such as pyrolysis, hydro-, pyro-, and bio-metallurgical technologies etc. [13]. The model complements three uncertainty modules evolving from the three pillars of sustainability that affect the supply chain sustainability, each weighted by its weight factor.

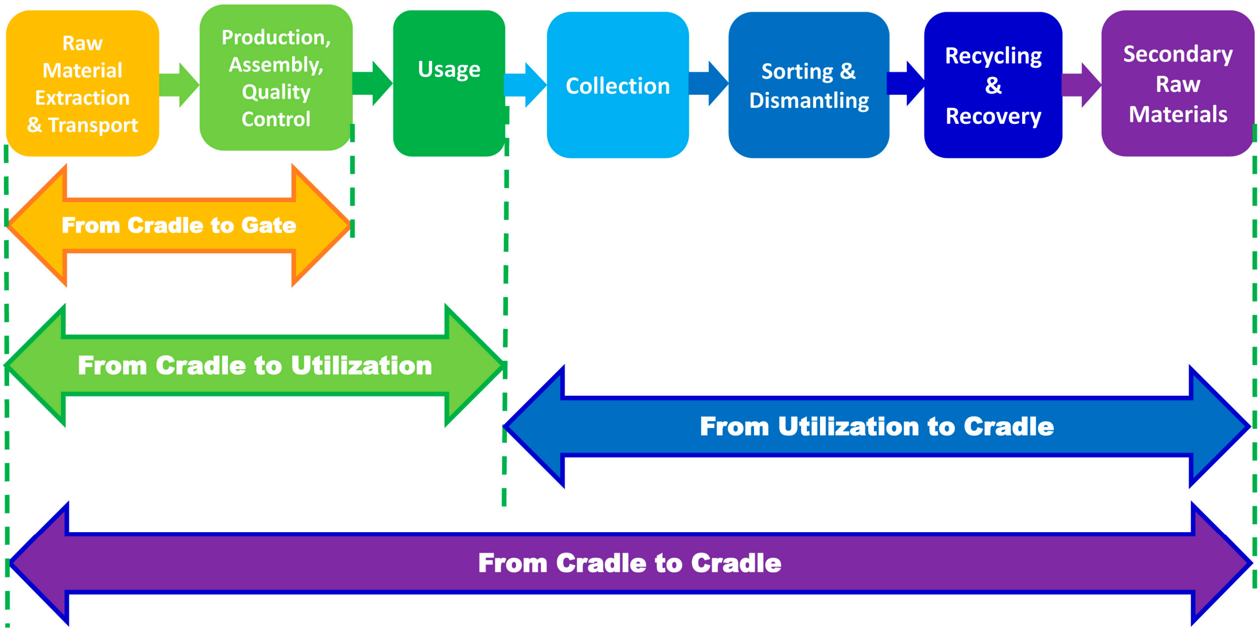


Figure 1. Boundaries of supply chain based on LCA (adapted from reference [5]).

The optimization of a cost kernel is a central feature of many models reviewed in the previous section. Thus, a proper specification of such a cost kernel is quite central to the modeling effort. That cost function specification has two parts, including a theoretical justification and an empirical counterpart. In this study, we draw upon the business economic theories of duality in multiproduct translog production and its dual translog cost function to justify a quadratic cost function. The empirical analysis of production and cost functions received a big boost with the pioneering contributions by Diewert [49] and McFadden [50], establishing the duality between them. The actual specification must reflect the intricate production structure with the underlying separability of production and distribution activities into a hierarchical network. To capture that hierarchical pattern and to derive its specification empirically, we employ the Analytic Hierarchy Process (AHP). The target here is to rank the affecting variables in order of their contribution (through PCA) so that the reweighed cost matrix depicts functional real relationships between the inputs and the potential output on one hand, and the cost of the minimum inputs needed to produce any given output on the other.

2.2. Study Methodology

A new model for e-waste supply chain sustainability has been introduced, which follows the mathematical kernel outlined in Debnath et al. [5]. Here, we have used two popular Machine Learning algorithms, i.e., AHP and PCA, and integrated them to develop a new hybrid AHP-PCA method for ranking the uncertainty variables. The algorithm is detailed later in Section 3.2.3. The overall methodology has been outlined in the running flowchart in Figure 2.

Initially, data on e-waste plants are collected and factors affecting the uncertainty of the e-waste supply chain are identified. Then, the cost function-based model is developed. Then, it is converted into an optimization problem. As detailed above, AHP and hybrid AHP-PCA methods are used to derive the weight factors and the interrelation coefficients. Thereafter, a Hessian matrix H is developed using the second derivatives of the model. $H \geq 0$ defines optimization. This model is dynamically constrained using Lagrange multipliers. The coupled set of equations for both constrained and unconstrained cases are solved using MATLAB 2019b (bvp4c) [5], using appropriate boundary conditions depicting an e-waste recycling plant. Real-life data obtained from an anonymous leading Indian e-waste management company are used for validation purposes. The results enable informed decision making for cleaner production lines leading to a greener supply chain.

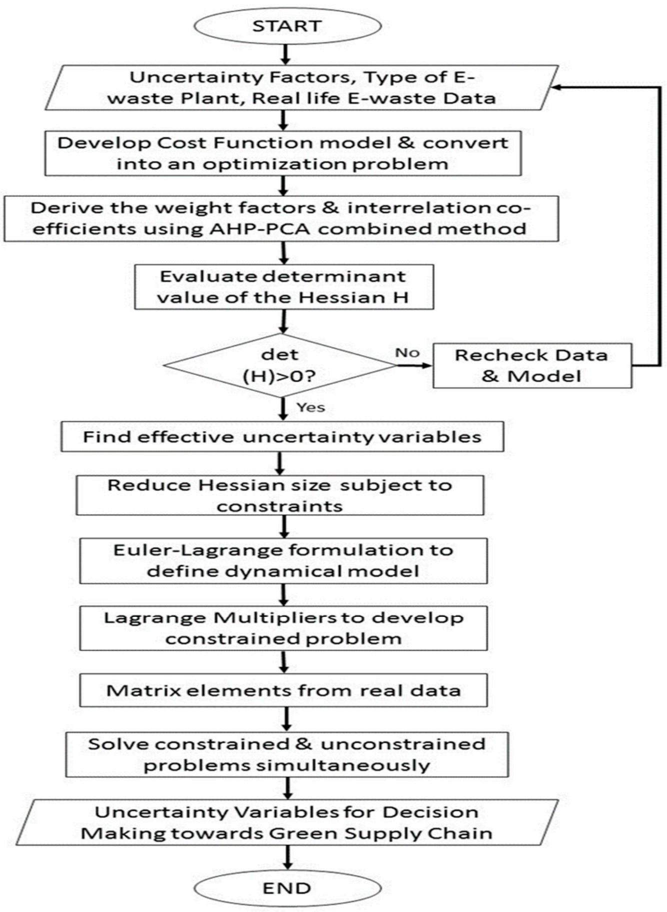


Figure 2. Working flowchart of the problem-to-solution approach.

3. Mathematics and Modeling

3.1. Modelling Approach

3.1.1. Model Assumptions

The assumptions considered in the model are stated below:

(i) Numbers are recorded daily, totaling 300 working days and 10 working hours daily.
(ii) The model is developed considering a Material Recovery from E-waste (MREW) [13] facility performing both mechanical recycling and material recovery (wet processes).
(iii) The cost of recycled products remains constant over time.
(iv) Unit costs remain constant.
(v) Legislative costs and costs towards disposal of hazardous materials in a TSDF remain constant annually.
(vi) The interdependency of the dependent variables has been assumed to be quadratic order accuracy.

3.1.2. Model Descriptions

The model uses three stochastic 'forces' of volatility as inputs, each of which pertains to the e-waste SCN derived from the three pillars of sustainability [5]. We assume, as suggested by the duality theory of production and cost [50], a quadratic cost function-based model, where minimization of the cost function kernel defines the time dynamics of the flow (Equation (1)). The structure resembles that of Eulerian mechanics (Goldstein 1964) where the cost function plays the role of a 'free energy' potential, whose optimized dynamics leads to the paradigmatic Euler–Lagrange model.

The uncertainty in the system arises from different segments of production and distribution functions, starting from uncertainty in the input markets and ending up with uncertainty in the final demand for the main outputs of the firm. We can specify a multi-product, multi-input production relation as follows:

$$g(q_1, q_2, \ldots. q_s) = f(r_1, r_2, \ldots. r_t; l_1, l_2, \ldots l_p; k_1, k_2, \ldots k_u; e), \tag{1}$$

where $q_1, q_2, \ldots. q_n$ are n different outputs produced by the firm including waste products, $r_1, r_2, \ldots. r_t$ are the different raw materials used in production, $l_1, l_2, \ldots l_p$ are p types of human inputs, and $k_1, k_2, \ldots k_u$ are u types of capital goods used in production. Finally, e is the energy input. This relationship is linear in parameters and nonlinear in variables where the left and right sides of Equation (1) are linear in parameters and are chosen to maximize the canonical correlation [51]. Using Shephard's duality, Diewert made a pioneering contribution in establishing a duality relationship between production and cost functions, a translog function being the most prominent one [50]. It is a quadratic approximation of the function expressing the logarithm of outputs as logarithms of all the inputs, keeping one of the outputs, such as the main output of the firm, as the reference or numeraire commodity. Pulley and Braunstein [52] specify a quadratic function in logarithms for a multiproduct cost function. Expression in logarithms is a typical choice by economists as it happens to generalize a very popular production function in economics that is just linear in logarithms. The quadratic cost function can be decomposed into three components including environmental, social, and economic. At each level of decomposition, it could incorporate an uncertain element. The following quadratic cost function can thus be treated as drawn from the duality relations between production and cost, underlying the production of the SMEs we are examining.

$$F = C_{Environmental} + C_{Social} + C_{Economic}, \text{ where} \tag{2}$$

$$C_{Environment} = \sum V_{CO_2} f_1 + \sum E_c f_2 + \zeta\left(\sum W_P f_3 + \sum W_w f_4\right) \tag{3}$$

$$C_{Social} = \sum N_1 f_5 + \sum N_3 f_6 \tag{4}$$

$$C_{Economic} = \sum N_4 f_7 - \sum N_5 f_8 - \sum N_7 f_{10} - \sum N_8 f_{11} - \sum N_9 f_{12} \tag{5}$$

This model has been developed for an e-waste supply chain network. The topic relates to key environmental concerns. The three basic cost functions are a collection of variables from environmental, economic, and social aspects. Equation (3) collectively represents the cost function of the variables that affect the environment directly, i.e., due to the operations, there is an environmental impact. In Equation (5), the cost function of the collective economic variables is defined. For example, in No. of recycled product (N_4), here the variable is basically "product", the adjective "recycled" is added because the product is a yield from the recycling activities, just like No. of Products was present in our earlier model. Again, No. of waste materials being sent to the Treatment, Storage and Disposal Facility (TSDF) (N_8), is an economic indicator because the landfilling in TSDF is chargeable. This cost has been considered in the economic part. Similarly, the social cost function includes two components, i.e., the workers part and the awareness generation part. The overall e-waste awareness is low in India, which is a big problem. Companies whose data have been used in this study conduct regular awareness activities in schools, colleges, and social media, which is the social aspect in the case of e-waste management.

The three function modules outlined in Equations (3) and (4) are derived from the three pillars of sustainability, namely environmental uncertainty (Equation (3)), social uncertainty (Equation (4)), and economic uncertainty (Equation (5)). Each uncertainty function module consists of a linear combination of two or more variables affecting the e-waste supply chain sustainability, categorized as environmental, social, or economic uncertainty, respectively. These variables have been categorically chosen to study and understand each major and minor perturbation along the e-waste SCN. These variables cover a wide range of aspects as they unify the e-waste pollution impacts as a single component module in the utility function. Also, socioeconomic factors are addressed within the same framework, and are all technically constrained by the need to maximize green supply chain deliverables.

Combined with the weight factors (represented by the ϵ_i's and A_i's) derived from AHP and the combined AHP-PCA method (as detailed later), the cost function takes the form below:

$$F = \epsilon_1 (\sum_{i=1}^{N} A_1 V_{CO_2} f_1 + \sum_{i=1}^{N} A_2 E_c f_2 + \sum_{i=1}^{N} \zeta A_3 W_P f_3 + \sum_{i=1}^{N} \zeta A_4 W_w f_4) + \epsilon_2 (\sum_{i=1}^{N} A_5 N_1 f_5 + \sum_{i=1}^{N} A_6 N_3 f_6) \epsilon_3 (\sum_{i=1}^{N} A_7 N_4 f_7 - \sum_{i=1}^{N} A_8 N_5 f_8 - \sum_{i=1}^{N} A_9 N_7 f_{10} - \sum_{i=1}^{N} A_{10} N_8 f_{11} - \sum_{i=1}^{N} A_{11} N_9 f_{12}) \tag{6}$$

Here, ζ is a variable whose value is zero when the e-waste plant performs mechanical recycling only, whereas when the e-waste plant is a MREW facility, the value is fixed at unity.

The interdependencies of the variables are expressed as a linear combination of the dependent variables with quadratic accuracy [53]. Equation (7a–g) represent the mathematical expressions for the interdependency of the variables.

$$V_{CO_2} = V_{CO_2}(N_5, N_7) = a_1 N_5 + a_2 N_7 + a_{12} N_5 N_7 + a'_1 N_5^2 + a'_2 N_7^2 \tag{7a}$$

$$E_C = E_C(N_4, N_5) = b_1 N_4 + b_2 N_5 + b_{12} N_4 N_5 + b'_1 N_4^2 + b'_2 N_5^2 \tag{7b}$$

$$W_P = W_P(N_5) = W_P^o + c_1 N_5 + c_2 N_5^2 \tag{7c}$$

$$W_W = W_W(N_5, W_P) = d_1 N_5 + d_2 W_P + d_{12} N_5 W_P + d'_1 N_5^2 + d'_2 W_P^2 \tag{7d}$$

$$N_3 = N_3(N_4, N_9) = \alpha_1 N_4 + \alpha_2 N_9 + \alpha_{12} N_4 N_9 + \alpha'_1 N_4^2 + \alpha'_2 N_9^2 \tag{7e}$$

$$N_4 = N_4(E_C, N_5) = \beta_1 E_C + \beta_2 N_5 + \beta_{12} E_C N_9 + \beta'_1 E_C^2 + \beta'_2 N_5^2 \tag{7f}$$

$$N_7 = N_7(V_{CO_2}) = \gamma V_{CO_2} \tag{7g}$$

3.2. Uncertainty Analysis

The entire premise of an e-waste portfolio is based on stochastic uncertainty modules that do not allow for absolute prediction of future values. This necessitates appropriate probabilistic approaches to first rank the key contributors and then analyze their interdependence. This is completed using a combination of the Analytic Hierarchy Process (AHP) and the Principal Component Analysis (PCA).

3.2.1. Analytical Hierarchical Process (AHP)

In this paper, two exclusive AHP analyses have been carried out, similar to Debnath et al. [5]. However, the structural specifications differ as the present focus is on e-waste SCN. The first AHP (Figure 3) model addresses three criteria enumerating the three uncertainties occurring from the three pillars of sustainability; the key (three) variables are linked through eleven criteria nodes. The second is a layered AHP (Figure 4) that utilizes the same criteria as the first but has two layers of alternatives. The layers are created in such a way that the structure not only connects the alternatives with the criteria but also the individual interdependencies of the alternatives. The first layer of alternatives consists of those variables that have dependencies on the variables in the second layer (function of a function, i.e., a functional). The second AHP is executed to find the interdependencies, whereas the first AHP is designed to rank the variables. A registered student version of the commercial software package "Super Decision version 2.10" is used for the AHP calculations. The determination of compound and square interdependencies follows the methodology of our previous work. The detailed AHP flowchart is outlined below:

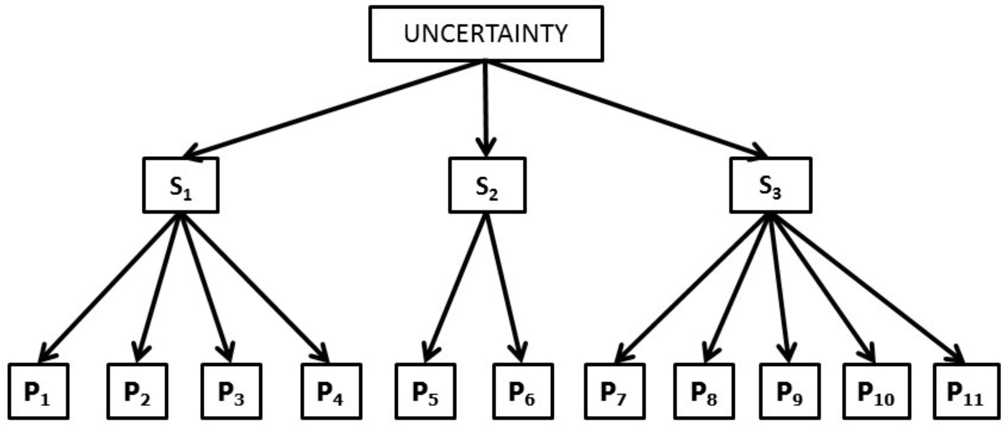


Figure 3. AHP model 1 to determine the general alternative rankings. Environmental uncertainty (S_1), social uncertainty (S_2), and economic uncertainty (S_3) are in the criteria layer. Volume of CO_2 generated (P_1), energy consumption in the processes involved (P_2), water used due to the processes involved (P_3), and wastewater generated in the whole process (P_4) are the alternatives connected to S_1; No. of laborers (P_5) and No. of awareness activities (P_6) are the alternatives connected to S_2; No. of recycled products sold (P_7), No. of operations involved (P_8), No. of logistics involved (P_9), No. of waste materials being send to Treatment, Storage and Disposal Facility (TSDF) (P_{10}), and No. of taxes (P_{11}) are the alternatives connected to S_3.

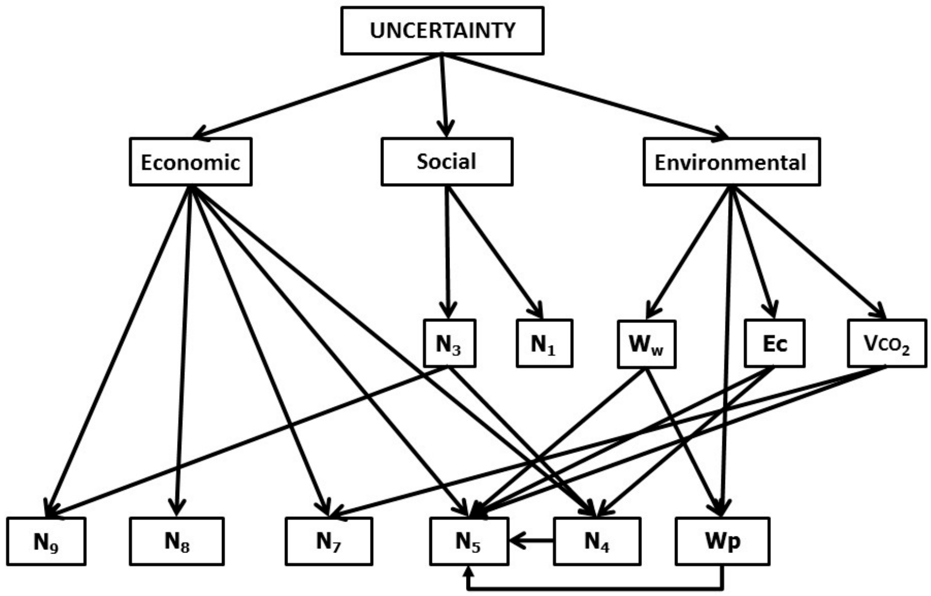


Figure 4. Layered AHP model for determination of interrelationship values.

3.2.2. Multivariate Study—Principal Component Analysis (PCA)

Principal component analysis (PCA) is perhaps the primogenial and one of the well-known multivariate analysis techniques. PCA was first introduced by Pearson in 1901 and later developed by Hotelling by 1933 independently [54]. The fundamental idea of PCA is to reduce the dimensionality of a huge dataset with interrelated variables, increasing interpretability while retaining maximum information [55]. The methodology involves a transformation of the original dataset to a new set of variables also known as the principal components (PCs), which are uncorrelated and ranked where the top few PCs capture the maximum variation present in all the original variables [54].

In the context of supply chains, the PCA has been used for a wide range of multivariate analyses, e.g., damage and fault detection [56], hypothesis testing [56], constrained PCA-based method development [57], chemometrics [58], radiative transfer computational advancement [59], rankings and preferences [55], etc. PCA is also a popular method among supply chain managers for its versatility [60]. In this study, PCA has been used as a hybrid method, combined with the AHP analysis in Step 1, for further refining the values obtained from the generic AHP, specifically to quantify the interdependencies of the dependent variables as well as to measure their relative positions in the coefficient matrix. The method is detailed in the following subsection.

3.2.3. Hybrid AHP-PCA Method

Under the current investigation, a hybrid AHP-PCA method is developed for ranking and finding the interdependencies of the variables of the utility function, the first such approach known. The uniqueness of this method is that it utilizes the best of both generic ranking (PCA) and interdependency calibration (AHP). The super-weighted matrix has been used as an input for the PCA. By doing this, the results obtained using AHP are being cross-checked and verified. This method thus unifies Multi-Criteria Decision Making (MCDM) with statistical approaches. The working algorithm of this method is given below:

Step 1: develop the AHP models with 'm' (here m = 3) criteria and 'n' alternatives (here n = 11), the linear one for alternative rankings and the layered structure for interdependencies.

Step 2: input ratings for the pairwise comparison matrix for both regular and layered AHP models to derive the ranking of the alternatives [61].

Step 3: record the AHP outputs and the super-weighted matrices.

Step 4: reduce the super-weighted matrix obtained from the crown structure AHP into '$p \times p$' matrix (here p = 7).

Step 5: use the '$p \times p$' matrix obtained in Step 4 as input and run PCA.

Step 6: use the principal components as alternative rankings for the variables.

Step 7: replace infinitesimally small values with zero in the correlation matrix obtained from PCA. In this case, we reduce 25 entries to zero while the matrix dimension changes to 7×6.

Step 8: find the norm of the newly developed correlation matrix.

Step 9: divide each element of the new correlation matrix by the norm values.

Step 10: map the matrix developed in Step 9 with the matrix in Step 4 and derive the interdependency factors by matching the positions.

Step 11: if any required values obtained from PCA are zero, then use the equivalent AHP value. Further, normalize it and use the resultant values as weight factors.

For zero entries from the benchmarking table, AHP values are given preference over PCA as with AHP, the rankings are already given using the eigenvalues.

We recall that F is the cost. The lambda values are chosen here through the AHP and AHP-PCA analysis. As was shown in the previous papers [5], these are proportional to the epsilon values.

When PCA is reapplied, the less important alternatives, i.e., the options with the smallest eigenvalues, were simply converted to zeroes (given no weight) to emphasize the prioritized (data or logic-driven) options. In a realistic scenario involving an e-waste supply chain, we may not be allowed to resort to such oversimplification though. This double screening through AHP $\rightarrow$ PCA filters ensures that the finally obtained values offer reliable estimates for relative weight and interdependency factors. The compound and square interdependencies are derived using the methodology of Debnath et al. [5]. The compound interdependencies have been taken as the product of the concerned coefficient, e.g., value of a_{23} = value of $a_2 \times$ value of a_3. The squared coefficient has been taken as the square root of the concerned co-efficient. For example, the value of a'_3 = square root of a_3.

3.2.4. Unconstrained Problem

The central mathematical outline follows the schematic in Debnath et al. [5], leading to a Euler–Lagrange structure [62] that depicts the optimized (from the cost function) time evolution of the interacting variables defining the income–outcome cost matrix. The perturbed dynamics close to the linearly stable fixed points can then be represented by the following dynamical system:

$$\delta\left(\frac{d}{dt}\begin{bmatrix}\frac{\partial F}{\partial V_{CO_2}}\\ \frac{\partial F}{\partial E_C}\\ \frac{\partial F}{\partial W_P}\\ \frac{\partial F}{\partial W_W}\\ \frac{\partial F}{\partial N_1}\\ \frac{\partial F}{\partial N_3}\\ \frac{\partial F}{\partial N_4}\\ \frac{\partial F}{\partial N_5}\\ \frac{\partial F}{\partial N_7}\\ \frac{\partial F}{\partial N_8}\\ \frac{\partial F}{\partial N_9}\end{bmatrix}\right)=\begin{bmatrix}m_{11} & m_{12} & m_{13} & m_{14} & 0 & 0 & m_{17} & m_{18} & m_{19} & 0 & 0\\ m_{21} & m_{22} & m_{23} & m_{24} & 0 & m_{26} & m_{27} & m_{28} & m_{29} & 0 & 0\\ m_{31} & m_{32} & m_{33} & m_{34} & 0 & 0 & m_{37} & m_{38} & m_{39} & 0 & 0\\ m_{41} & m_{42} & m_{43} & m_{44} & 0 & 0 & m_{47} & m_{48} & m_{49} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & m_{62} & 0 & 0 & 0 & m_{66} & m_{67} & 0 & 0 & 0 & 0\\ m_{71} & m_{72} & m_{73} & m_{74} & 0 & m_{76} & m_{77} & m_{78} & 0 & 0 & 0\\ m_{81} & m_{82} & m_{83} & m_{84} & 0 & 0 & m_{87} & 0 & m_{89} & 0 & 0\\ m_{91} & m_{92} & m_{93} & m_{94} & 0 & 0 & 0 & m_{88} & m_{89} & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta W_P\\ \delta W_W\\ \delta N_1\\ \delta N_3\\ \delta N_4\\ \delta N_5\\ \delta N_7\\ \delta N_8\\ \delta N_9\end{bmatrix} \tag{8}$$

Focusing on the leading dynamic variables (V_{CO2}, E_C, N_3, and N_4) for the specific recycler, whose data we seek to compare against, given that the other variables are largely fixed for them, Equation (8) can be easily simplified:

$$\delta\left(\frac{d}{dt}\begin{bmatrix}\frac{\partial F}{\partial V_{CO_2}}\\ \frac{\partial F}{\partial E_C}\\ \frac{\partial F}{\partial N_3}\\ \frac{\partial F}{\partial N_4}\end{bmatrix}\right)=\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta N_3\\ \delta N_4\end{bmatrix} \tag{9}$$

Note that other variables like W_P, W_W, etc., could also have non-trivial contributions for different recyclers, in which case they would have to be considered as well. Also, it is relevant to note that the zero rows ascribed to variables N_1, N_8, and N_9 in Equation (8) above, respectively, relating to the labor capacity (N_1), number of waste materials sent to TSDF (N_8), and tax (N9), remain largely unchanged throughout the operation cycle of the unit, and hence do not contribute to the recycling dynamics. The rearrangement of Equation (9) leads to the following:

$$\frac{d^2}{dt^2}\begin{bmatrix}\rho_1\delta V_{CO_2}\\ \rho_2\delta E_C\\ \rho_3\delta N_3\\ \rho_4\delta N_4\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\\ a_{41} & a_{42} & a_{43} & a_{44}\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta N_3\\ \delta N_4\end{bmatrix} \tag{10}$$

The second-ordered time derivative in Equation (10) mimics an 'underdamped' model of mechanics [5], which from the perspective of a supply chain, represents a 'lightly' constrained SCN where many of the constraints are outliers but not necessarily boundary conditions.

3.2.5. Constrained Problem

The constrained version of the problem is formulated by introducing Lagrange multipliers [62]. This helps in solving the optimization problem without explicit parameterization in terms of the constraint. The values of the individual Lagrange multipliers are considered to be proportional to the epsilon values that replicate the corresponding weightage of the individual uncertainties in the cost function. The Lagrangian '$\mathcal{L}$' is defined as:

$$\mathcal{L} = F - \lambda_1\left(V_{CO_2}f_1 - V\right) - \lambda_2(N_1 f_5 + N_3 f_6 - E) - \lambda_3(N_4 f_7 - R) \tag{11}$$

where λ_i's are the Lagrange multipliers. The realistic system restrictions (constraints) are expressed through the quantities joined with the Lagrange multipliers, which we enforce on the system. We impose three constraints on V, E, and R, which we have been chosen in consultation with the e-waste recycler, on Equation (11): (1) V, the cost associated with CO_2 emission control; (2) E, the maximum expenditure budget accorded for wages of the labors and employees and awareness activities, and (3) R, the maximum revenue target. Overall, this amounts to a suitably recalibrated greener supply chain within viable operation lines.

The constrained version of the problem takes the following form:

$$\delta\left(\frac{d}{dt}\begin{bmatrix}\frac{\partial\mathcal{L}}{\partial V_{CO_2}}\\ \frac{\partial\mathcal{L}}{\partial E_C}\\ \frac{\partial\mathcal{L}}{\partial N_3}\\ \frac{\partial\mathcal{L}}{\partial N_4}\end{bmatrix}\right)=\begin{bmatrix}c_{11}&c_{12}&c_{13}&c_{14}\\c_{21}&c_{22}&c_{23}&c_{24}\\c_{31}&c_{32}&c_{33}&c_{34}\\c_{41}&c_{42}&c_{43}&c_{44}\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta N_3\\ \delta N_4\end{bmatrix} \tag{12}$$

Rearrangement of Equation (12) leads to:

$$\frac{d^2}{dt^2}\begin{bmatrix}\omega_1\delta V_{CO_2}\\ \omega_2\delta E_C\\ \omega_3\delta N_3\\ \omega_4\delta N_4\end{bmatrix}=\begin{bmatrix}c_{11}&c_{12}&c_{13}&c_{14}\\c_{21}&c_{22}&c_{23}&c_{24}\\c_{31}&c_{32}&c_{33}&c_{34}\\c_{41}&c_{42}&c_{43}&c_{44}\end{bmatrix}\begin{bmatrix}\delta V_{CO_2}\\ \delta E_C\\ \delta N_3\\ \delta N_4\end{bmatrix} \tag{13}$$

Equations (10) and (13) are solved using data obtained from an anonymous multi-award winning Indian e-waste recycler company. MATLAB R2019a (bvp4c) was used to solve the system of equations concerning the solutions of the corresponding boundary value problems (Table 1). V_{CO2} values are in tons of carbon dioxide emissions, energy consumption values are in Gigawatt, the number of awareness activities are plain numbers, whereas product sales are in the % of sales target. The initial conditions represent the current status, whereas the boundary conditions represent the targets to be achieved.

Table 1. Boundary conditions for evaluation of results.

Sl.	VCO_2	E_c	N_3	N_4
	Volume of CO_2 (tons of CO_2)	Energy Consumption (% Energy Consumed in GW)	Number of Awareness Activities (No.)	Product Sales (% of sales target)
		Initial Conditions (IC)		
1	1.2	0.002	2	0.3
		Boundary Conditions (BC)		
1	0.82	0.0015	4	0.6

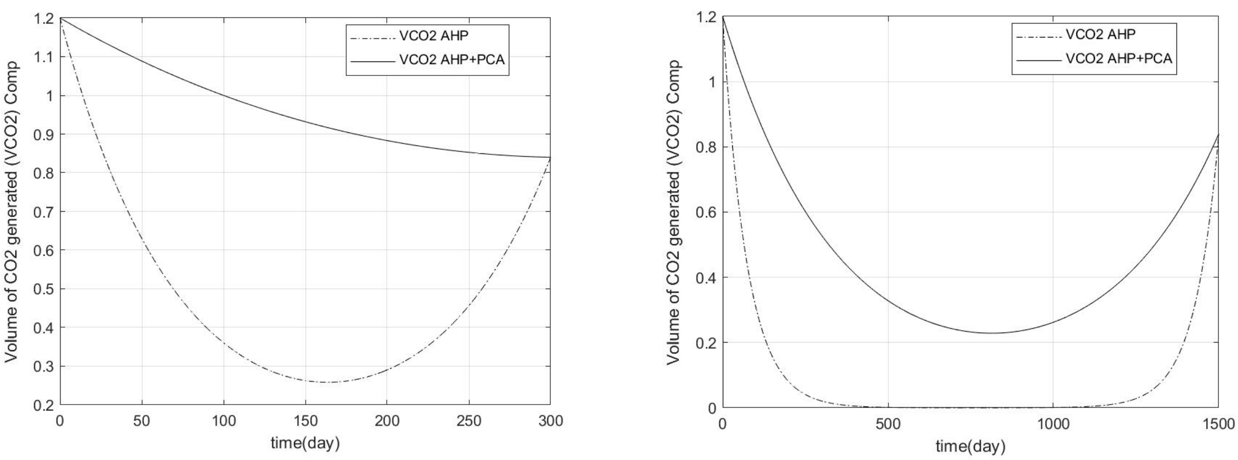
4. Results and Discussion

The impact of the time evolution of the leading variables, both for constrained and unconstrained environments, is discussed below. First, we need to have an essence of what the standalone constrained and unconstrained systems represent. In simple parlance, they jointly characterize the dystopian and utopian case scenarios, respectively. Both cases are ranked using AHP and hybrid AHP-PCA methods and compared. For a real e-waste recycler, the SCN is stochastic and highly sensitive to minor logistic perturbations, technically represented as SCN strategies. Appropriate initial and boundary conditions represent such strategies in our time-varying model. The precise nature of these initial and boundary conditions is subjective and differ for each SCN. The initial conditions represent the present state of the system, while the desired state of the system define the terminal conditions, thus leading to a fixed end point formulation of the problem. It is desirable sometimes to examine how the solution changes with different terminal conditions, and hence it is interesting to solve a variable endpoint problem. Such a solution can identify the trade-offs between the SCN's total cost F and the cost of relaxation of the terminal condition.

This study strategizes how decreasing environmental load with increasing social accountability may still conform to the economic profitability of e-waste recycling plants. The effect of different ranking methods is explored in the chosen variables both in constrained and unconstrained conditions, separately at intervals of one-year, three-year, and five-year timelines. The 1-year results replicate the immediate effect, whereas the 5-year timer represents a long-time effect. The 3-year results provide an understanding of events at intermediate time scales. This intermediate time scale is strategically important because this provides a clear numerical grasp of the state the company is in at that point and offers scopes of strategizing for the future. The time dynamic behavior of the four key variables, e.g., carbon dioxide emission volume (V_{CO2}), energy consumption (E_C), number of awareness activities (N_3), and product sales (N_4) all rely on the hybrid AHP→PCA ranking method outlined in the preceding sections to identify the best performance strategies.

4.1. Volume of Carbon Dioxide Emission

Figure 5a,b compare the time dependence of the volume of CO_2 generated (V_{CO2}) in a constrained environment for 1-year and 5-year scenarios, respectively. In both cases, two ranking methods are used for comparison—the AHP (dash-dotted line) and hybrid AHP-PCA method (solid line). In the 1-year scenario, a parabolic curve is obtained for AHP, whereas a steep curve is obtained for the AHP-PCA method. On the other hand, in the 5-year scenario, a comparatively flat curve is obtained for AHP but a parabolic profile is obtained for the AHP-PCA method. It is clear from the figures that the hybrid AHP-PCA method provides a better ranking than any individual scoring methods (AHP or PCA for us), as the solid-line curves capture the immediate effects much better than the dash-dotted lines. For the 1-year timeline, the AHP results suggest that carbon dioxide emissions will reach a minimum within the fifth and the sixth month and rise again before reaching the boundary value, whereas the hybrid AHP-PCA results suggest that carbon dioxide emissions will smoothly decrease to the targeted value. The AHP curve suggests when to resurrect the strategy change, represented by the point of inflection; whereas the AHP-PCA curve suggests that within a year it is impractical to resurrect a strategy change. We find that the hybrid AHP-PCA results are more realistic, as it is practically impossible to run into a record low-emission figure (~79% of the starting value) within 4 months.



(**a**) (**b**)

Figure 5. Time dependence of volume of carbon dioxide emission (V_{CO2}) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from the simultaneous solution of Equations (9) and (12): (**a**) 1-year time span and (**b**) 5-year time span.

In the 5-year timelines, the hybrid AHP-PCA curve identifies the inflection point (the minima) after 2 years and then converges again to its target value. This suggests that in the long term, it is important to devise a policy change to lower the emissions, i.e., savings in environmental efforts, combining technological efforts in emission reduction, carbon credits, and positive environmental impacts through Corporate Social Responsibility (CSR) and other green activities like greening the supply chain, cleaner production lines, and zero waste efforts. Compared to the hybrid AHP-PCA ranked results, the standalone AHP ranked results fail to capture the 'negative-emission' characteristic of the curve as the curve flattens to zero after the 1st year till towards the end of the fourth year. Unless there is a very enthusiastic supply chain manager, the AHP results will lead to a happy-face decision of offsetting the carbon footprint in the long run. Hence, there may not be a continual improvement unless it is mentioned in the quality (ISO 9000) [63] and environmental (ISO 14000) policy [64] of the recycling company concerned.

Figure 6 depicts the time dependence of the volume of CO_2 generated (V_{CO2}) in a constrained environment for a 3-year year timespan, with the AHP (dash-dotted line) and hybrid AHP-PCA method (solid line) as the ranking methods. This is an example of hierarchical module training, based on Machine Learning inputs from the hybrid AHP $\rightarrow$ PCA model [65]. Similar to the 5-year scenario, a flat curve is obtained for AHP but a parabolic profile is obtained for the AHP-PCA method. In the unconstrained environment, the solutions are non-convergent indicating an unstable system. The curves resemble the ones with 5-year timelines. However, in this case, the hybrid AHP-PCA curve ensures an "operation window" within 400–600 days. This indicates that it is possible to achieve further reduction in emissions driven by a policy change. Hence, it is quite an intelligent approach to set targets by the 3-year timeline for carbon dioxide emission and revise/tailor as required within the next 2 years' timeline. This will allow the supply chain manager to locate operation windows in a less risk-prone period, enabling short-term strategy enforcement towards better-fitted solutions. This also opens opportunities to explore unconventional and newly developed approaches for pilot studies, which is a good way to strengthen the much-required industry–academia bond.

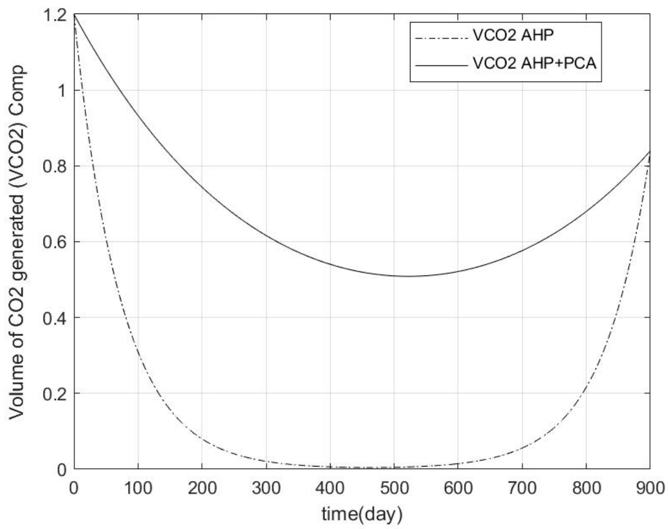


Figure 6. Time dependence of volume of carbon dioxide emission (V_{CO2}) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained for the 3-year scenario from the simultaneous solution of Equations (9) and (12) over 3 years.

4.2. Energy Consumption

In the 1-year timeline, the results from both AHP and hybrid AHP-PCA ranking methods show similar trends. Both curves increase smoothly until they reach the target value. Figure 7a suggests increasing energy consumption, as the mechanical recycling of e-waste is highly energy intensive. In the 5-year timeline (Figure 7b), the AHP curve has a hyperbola shape. The curve shows a steep fall in the beginning, followed by a minimum by the end of the second year, then rising again to a higher value (nearly double) by the end of the fifth year. Alternatively, the hybrid AHP-PCA curve shows a similar trend to its 1-year scenario, which is an increasing profile. We interpret that the system is inherently stochastic, and energy consumption is a very critical and sensitive parameter that needs attention. In the long run, the AHP curve identifies an "operation window" for devising policy changes. A decrease in energy consumption implies an organization heading towards bankruptcy. However, the variations may be attributed to the supply of e-waste demand uncertainty or even an economically damaging pandemic that shuts off the entire work cycle. As the AHP-PCA curve fails to provide any inflection point, we interpret that the current energy policy in the e-waste organization needs immediate attention. Again, this behavior could be attributed to the overestimation of the AHP-PCA method or the underestimation of AHP.

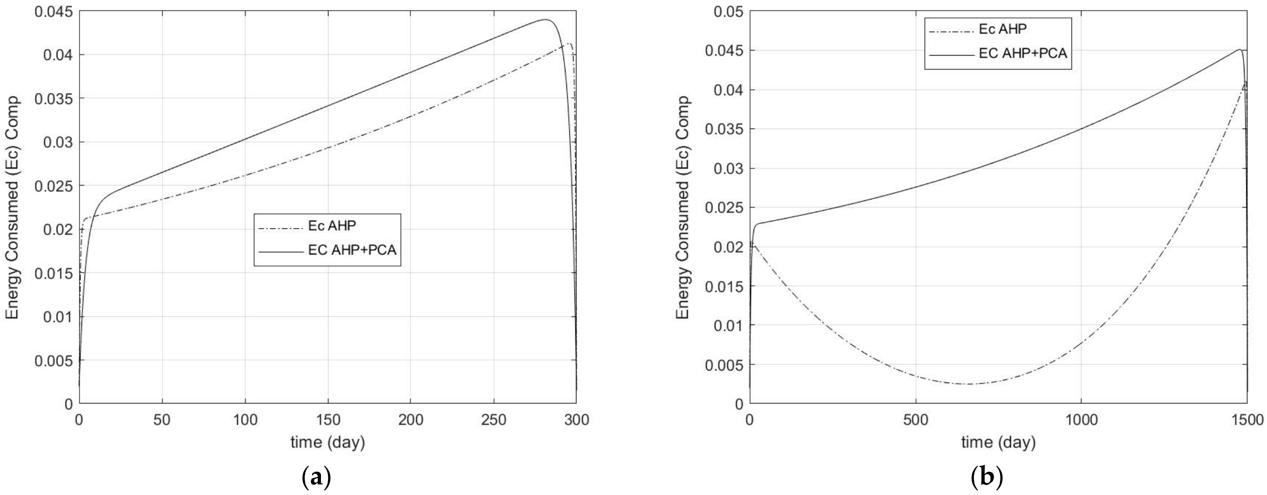
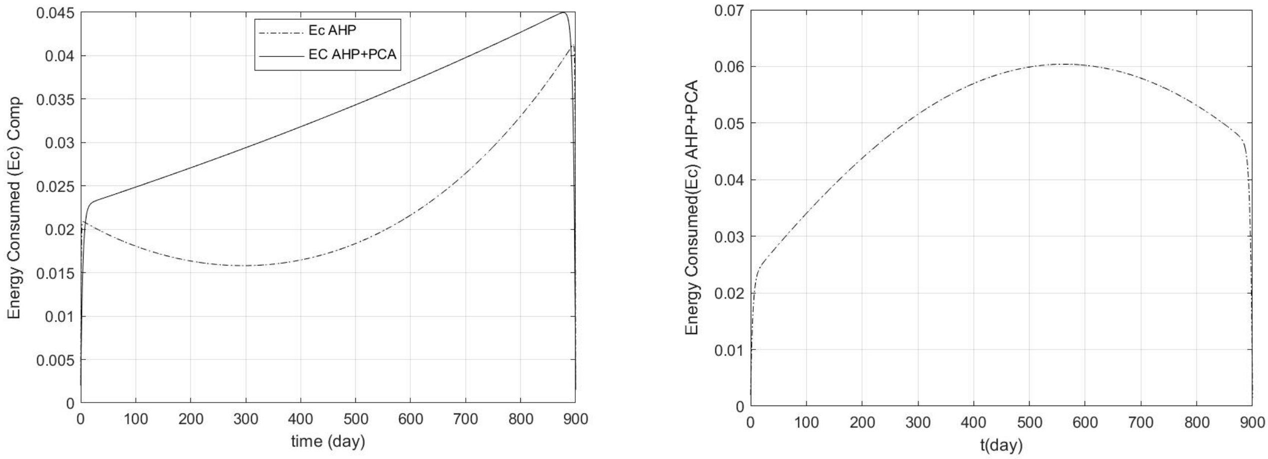


Figure 7. Time dependence of energy consumption (E_c) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from the simultaneous solution of Equations (9) and (12): (**a**) 1-year time span and (**b**) 5-year time span.

In the constrained environment, the hybrid AHP-PCA ranked curve imitates the 1-year AHP ranked curve, which shows an increasing trend (Figure 8a). This is quite realistic, as in reality, the energy consumption of an e-waste recycling plant will increase over time as supply increases, which is an indicator of sustainable business. Alternatively, from a mathematical point of view, perhaps the AHP-PCA method is overestimating the dynamics of the system. On the other hand, the AHP ranked curve (constrained) carries a similar profile to its 5-year appearance. In this case, an operation window within 200–400 days is obtained. Whereas, the hybrid AHP-PCA ranked curve in the unconstrained environment exhibits an increasing parabolic profile, which reaches a maximum within 500–600 days and then reduces. The sensitivity of this parameter is quite high compared to the other cases; hence, such behavior of the curves has appeared. However, comparing the results of constrained and unconstrained cases of the hybrid AHP-PCA method, we interpret that there might be a case of over-prediction in the constrained case. An ML data-powered model like ours also suggests how the perturbations in the terminal conditions at an intermediate period (for a short horizon model, say a 3-year model) for energy consumption alters the cost of CO_2 emissions, thereby providing guidelines on trade-offs between different components of environmental variables.

We should also be cautious about the possibility of recurrent overestimation accruing from our hybrid AHP-PCA ranking method, as has been discussed above. Additionally, both methods are seen to contribute towards parameter sensitivity and are differentially adaptive to the ambient response (AHP is more stable than PCA on this). E-waste recycling facilities performing mechanical recycling operations are highly energy intensive, and hence energy consumption ought to be a critical factor for business sustainability.

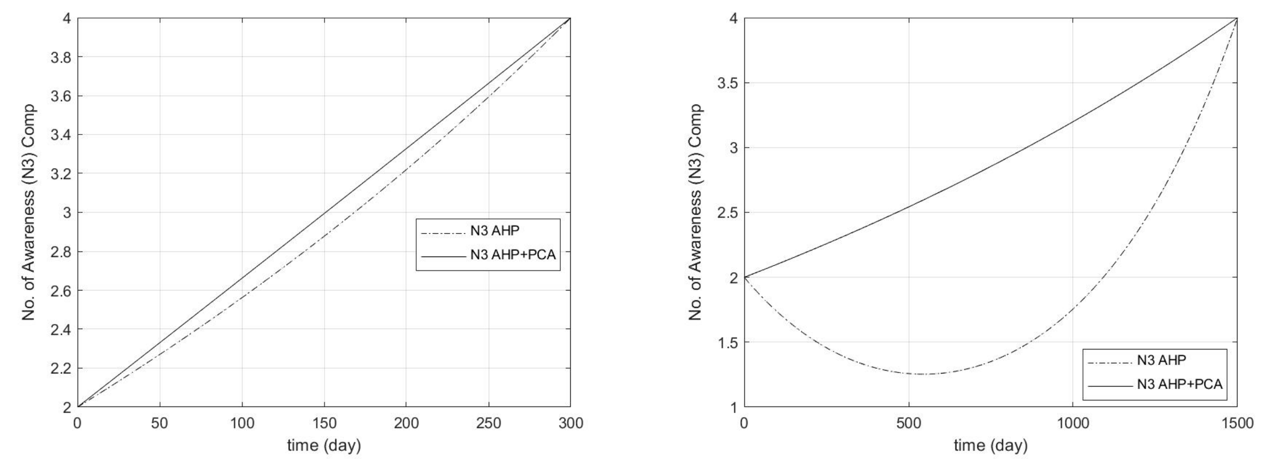


(**a**) (**b**)

Figure 8. Time dependence of energy consumption (E_c): (**a**) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) and (**b**) in an unconstrained environment, using hybrid AHP-PCA for ranking obtained for the 3-year scenario from the simultaneous solutions of Equations (9) and (12) over a 3-year timespan.

4.3. Number of Awareness Activities

In developing countries, the awareness level of e-waste disposal is a big issue and needs proper attention [5,33]. The level of awareness is proportional to the business of an e-waste recycler. Hence, it is in the recycler's best interest that awareness activities need to be taken seriously as well as CSR activity. Such practice is also visible among e-waste recyclers around the globe. The time dependency of several awareness activities (N_3) in a constrained environment is presented in Figure 9a,b, for 1-year and 5-year timespans, respectively. The results obtained using standalone AHP as the ranking method are in dash-dotted lines, whereas the results obtained using the hybrid AHP-PCA method are in solid lines. As shown in the 1-year scenario, both curves depict a smooth increasing profile. While the solid line is almost straight, the dash-dotted one is slightly curved in between. Both curves converge to the same prediction: an increase in awareness activities is helpful.

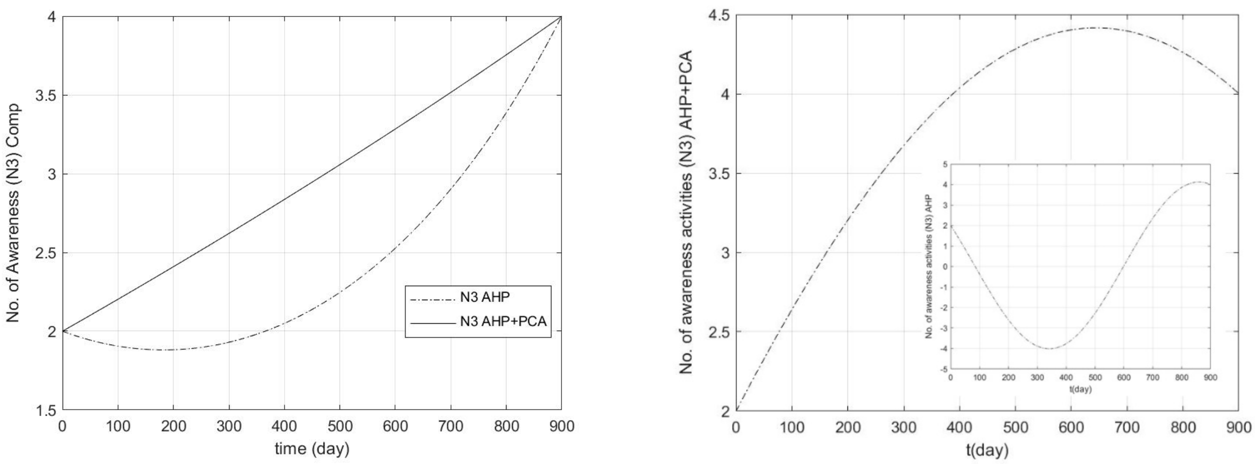


(**a**) (**b**)

Figure 9. Time dependence of number of awareness activities (N_3) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from the simultaneous solution of Equations (9) and (12): (**a**) 1-year time span and (**b**) 5-year time span.

In the long-term scenario (5-year), the dash-dotted curve depicts a minimum around the 500th day and sharply increases to reach the target value. In contrast, the solid line depicts the social responsibility of the recycler with a smoothly increasing curve, which helps the social image and eventually increases the business potential of the recycler. The dash-dotted curve implies that the e-waste recycler might struggle to sufficiently increase the number of awareness activities in the initial years, but after some time they will eventually gear up to increase the number of awareness activities. The minimum is obtained in the second quarter of the second year, which means that is the point when a further decision needs to be taken for higher social accountability based on company policy and budgets. We interpret that the hybrid AHP-PCA model might be giving the perfect fit as the path shown is more realistic.

Figure 10a,b depict the time dependence of the number of awareness activities (N_3), respectively, in constrained and unconstrained environments, respectively, for the 3-year case. In the constrained environment, the solid line exhibits a straight line translating to a realistic scenario. In contrast, the dash-dotted curve toes the trend of the 5-year scenario and hence merits no further discussion. In the unconstrained environment, the AHP ranking method provides a wave-like profile, although the negative minimum is unphysical (as shown in the insets of Figure 10b). On the other hand, the hybrid AHP-PCA method of ranking creates a semi-parabolic profile that offers a maximum value (~4.5) at the 600-day timestamp. This means that in an arbitrage condition, the recycler can keep increasing the awareness activities.



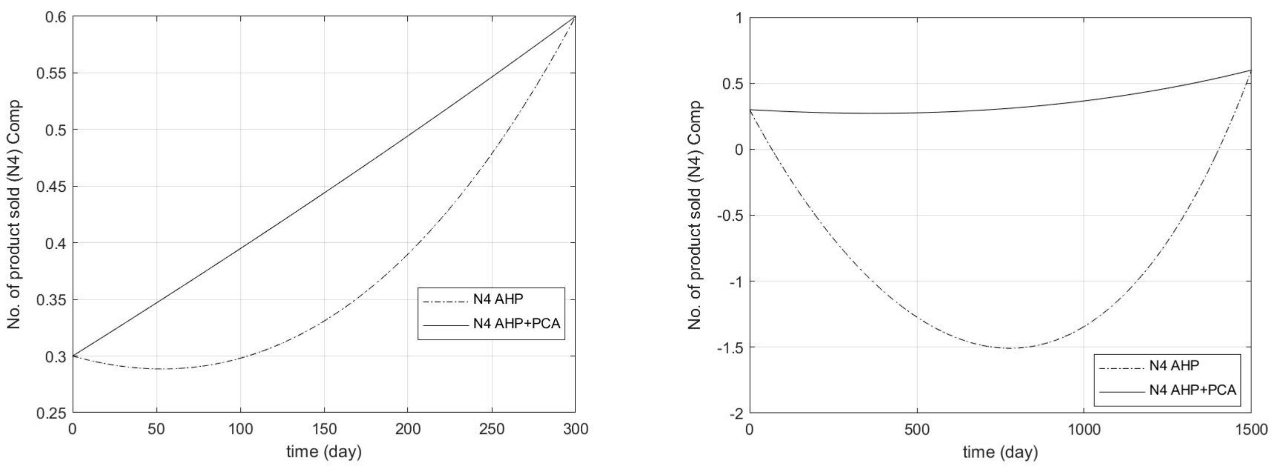
(**a**) (**b**)

Figure 10. Time dependence of number of awareness activities (N_3): (**a**) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) and (**b**) in an unconstrained environment using hybrid AHP-PCA for ranking and AHP for ranking (provided in the insets) in a 3-year time span obtained from simultaneous solutions of Equations (9) and (12).

The awareness activity (N_3) relates to the social uncertainty contributing to the overall cost function. Positive growth in social aspects is always a good deal and the hybrid AHP-PCA ranking method can predict the realistic trends in both constrained and unconstrained scenarios. On the other hand, the standalone AHP method underpredicts and fails to describe the dynamic system both in constrained and unconstrained scenarios. Our model shows that in this case, the decision making on several awareness activities is guided by the budget constraint. Despite the fact that an e-waste recycling facility can survive on its own or operate on break-even mode in an arbitrage condition of continuous supply of e-waste, it is suggested that regular monitoring and the cross-validation of existing policy should be carried out for business sustainability.

4.4. Product Sales

Figure 11a,b compare the time dependence of product sales (N_4) in a constrained environment for 1-year and 5-year scenarios, respectively. In the 1-year scenario, a parabolic curve is obtained for AHP, whereas AHP-PCA predicts a straight line. On the other hand, in the 5-year scenario, a parabolic curve with a negative minimum is obtained for AHP, which is practically unfeasible. On the other hand, the AHP-PCA outcome is a stretched exponential that portrays realistic kinetics. In the 1-year timeline, using AHP (dash-dotted line) as the ranking method predicts that immediately after the beginning, the e-waste plant incurs a loss, as shown by the dipping curve, and starts to peak after 3 months to reach a target value. This parabolic curve (dash-dotted line) suggests that in the current scenario, the company may face some issues at the start. On the other hand, using the hybrid AHP-PCA ranking method (solid line) dictates a straight line. Over the 5-year scenario, the dash-dotted line exhibits a parabolic profile with a minimum at the 750-day timestamp. Clearly, AHP underpredicts the dynamics of the system. In contrast, the solid line demonstrates a smooth increasing curve. The market price volatility of recycled products is a major issue; hence, it is suggested that both methods should be tested in the interest of a greener supply chain with a cleaner production line leading to sustainable business.

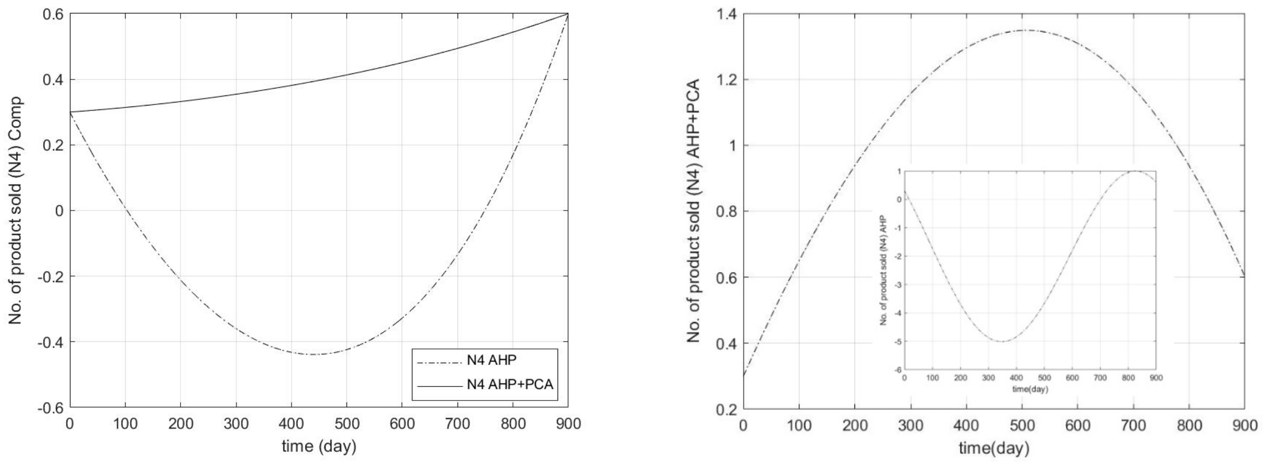


(**a**) (**b**)

Figure 11. Time dependence of product sales (N_4) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) obtained from the simultaneous solution of Equations (9) and (12): (**a**) 1-year time span and (**b**) 5-year time span.

The curves in Figure 12a depict the results of time dependence of product sales (N4) in the constrained environment for the 3-year scenario using AHP (dash-dotted line) and hybrid AHP-PCA (solid line) for ranking. Both the curves display a similar profile to the 5-year results. Hence, the interpretation remains unchanged. Figure 12b illustrates the time dependence of product sales (N4) in the unconstrained environment for the 3-year scenario using the AHP-PCA rankings. The AHP results are presented in the insets of Figure 12b. In this case, the hybrid AHP-PCA method points to interesting outcomes. The results using the AHP ranking method are provided in the insets of Figure 12b. As seen in the previous cases, here the standalone AHP also fails to obtain rational results in an unconstrained scenario (Figure 12b inset). The hybrid AHP-PCA ranked result shows a parabolic curve. This shows the robustness of the hybrid AHP-PCA method, as it captures the system dynamics in the unconstrained scenario. The curve profile suggests that even in arbitrage conditions, the company may not have a steady growth profile.

Economic sustainability is the most important of all from the business perspective (Debnath and Ghosh 2019). In that sense, a product sold (N_4) is the most important parameter that needs to be nurtured for maximum profit. A greener supply chain network with a sustainable production line is a utopian case, but we can always look forward to reaching as close as possible to the target values. That is exactly what these boundary conditions have helped us to do. The alluded case gives an outlook of comparison of both the methods, but the choice of boundary conditions lies in the hand of the supply chain manager of the respective plant. For business sustainability, it is suggested that regular monitoring of critical parameters and policy changes at certain intervals (identified through analysis) will help in greening the supply chain.



(**a**) (**b**)

Figure 12. Time dependence of product sales (N_4): (**a**) in a constrained environment, using AHP for ranking (dash-dotted line) and hybrid AHP-PCA for ranking (solid line) and (**b**) in an unconstrained environment using hybrid AHP-PCA for ranking and AHP for ranking (provided in the insets) for the 3-year scenario obtained from the simultaneous solution of Equations (9) and (12).

5. Conclusions

The constrained dynamics of the e-waste supply chain network is complex. Using a model structured on the paradigmatic Euler–Lagrange architecture, a concept borrowed from physics, the resultant time evolution kinetics of a "Utilization-to-cradle" supply chain network of an e-waste (MREW) facility has been developed using a cost function-based model. The model is weighted against interdependencies of the uncertainty parameters, which are identified using two Machine Learning methods, namely AHP and hybrid AHP-PCA. The boundary value optimization problem is numerically solved using MATLAB 2019b. The outcomes shed light on how ambient (SCN) and environmental factors contrive to create profit–sustainability balance, which is the quintessential challenge of most cash-strapped SMEs.

The model has been validated using anonymous Indian e-waste recyclers data to test the robustness of the SCN model in 1-, 3-, and 5-year timelines, respectively. The boundary conditions replicate the strategy of decreasing environmental load while increasing social accountability together with economic profitability. The time evolution dynamics of the MREW facility, focusing only on mechanical recycling, has been illustrated based on four leading dynamic variables: volume of carbon dioxide generated (V_{CO2}), energy consumption (E_C), number of awareness activities (N_3), and recycled product sales (N_4). The results show that the hybrid AHP-PCA ranking method is superior to the standalone AHP method of ranking. However, in the case of energy consumption (E_C), the hybrid method overestimated the dynamics of the system. The model can also identify the operation windows for the supply chain managers for reinvigoration of the policies. To reduce the carbon dioxide emission, the model predicts the 400–600 day window for policy change. In the case of energy consumption, an operation window of the 200–400th day is obtained. Volume of carbon dioxide and energy consumption emerge as the two most important parameters, while energy consumption is the most sensitive parameter of the system. For awareness activities, the minimum is obtained in the second quarter of the second year, which is the inflection point of decision making. This suggests that for social accountability practice, the decision should be purely guided by the budget constraints to maintain a sustainable business. On the other hand, product sales, serving as the economic descriptor, show an increasing trend. This suggests that the steady growth of e-waste businesses will lead to economic sustainability. The numbers and outputs are likely to change with the change in cases and constraints. This study almost unerringly replicates results seen by real e-waste production facilities and provides a guideline for developing a cleaner production line with a sustainable profitability margin. The findings address SDG 11 and 12 directly, whereas SDG 13, 14, and 15 are addressed indirectly. Future ML-powered studies involving exclusive MREW facilities, carrying out both chemical and mechanical recycling, are presently underway.

Author Contributions: Conceptualization, A.K.C. and B.D.; methodology, A.K.C. and B.D.; software, B.D.; validation, A.K.C., B.D. and T.K.K.; formal analysis, A.K.C. and B.D.; investigation, B.D.; resources, B.D.; writing—original draft preparation, B.D.; writing—review and editing, A.K.C. and T.K.K.; visualization, B.D.; supervision, A.K.C.; project administration, A.K.C.; funding acquisition, A.K.C. and B.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Commonwealth Scholarships Commission (Reference: INCN-2018-52).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in this study are included in the article, further inquiries can be directed to the corresponding author.

Acknowledgments: Both B.D. and A.K.C. acknowledge support received from Aston University.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

Cost components	
F	Cost function
$C_{Environment}$	Cost function component for environmental uncertainty
C_{Social}	Cost function component for social uncertainty
$C_{Economic}$	Cost function component for economic uncertainty
Variables and Parameters	
V_{CO2}	Volume of CO_2 generated
E_C	Energy consumption in the processes involved
W_p	Water used due to the processes involved
W_w	Wastewater generated in the process
N_1	Number of laborers
N_3	Number of awareness activities, e.g., adaptation to information, invisible e-waste, and repair substituting new
N_4	No. of recycled products sold
N_5	No. of operations involved
N_7	No. of logistics involved
N_8	No. of waste materials being send to Treatment, Storage and Disposal Facility (TSDF)
N_9	No. of taxes to be paid
f_1	Unit cost for CO_2 recovery
f_2	Unit cost of energy used
f_3	Unit cost for water used
f_4	Unit cost of wastewater treatment
f_5	Salary of one labor
f_6	Average cost of awareness activity
f_7	Unit revenue earned from product sold
f_8	Unit cost of each operation
f_{10}	Unit cost of logistics
f_{11}	Unit cost for disposal in TSDF
f_{12}	Unit cost of taxes

Weight factors	
ϵ_i's	Weigh factor for the four cost functions
A_i's	Weigh factor for the main parameters
a_i's, a_{ij}'s & a_i's	Interdependency values for V_{CO2}
b_i's, b_{ij}'s & b_i's	Interdependency values for E_C
c_i's	Interdependency values for W_P
d_i's, d_{ij}'s & d_i's	Interdependency values for W_w
α_i's, α_{ij}'s & α_i's	Interdependency values for N_3
β_i's, β_{ij}'s & β_i's	Interdependency values for N_4
γ	Interdependency value for N_7

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