All-fiber fast acousto-optic temporal control of tunable optical pulses

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ABSTRACT

We demonstrate a new all-fiber electrically tunable modulation method which significantly reduces the response time of a Bragg grating acousto-optic modulator. A 4 cm long device is fabricated with a 1 cm grating inscribed in a suspended core fiber. An acoustic pulse train is switched out of phase along the fiber, damping unwanted natural resonant vibrations inside the grating. The device rise time is decreased from 56 to 9 μ s by tuning the duty cycle of the driven electrical signal, contributing to achieve the shortest switching time of 15.6 μ s. This tunable temporal response reveals unique features to change the profile of optical pulses. High pulse modulation depths are achieved employing a compact acousto-optic modulator, pointing to fast switching of all-fiber photonic devices.

1. Introduction

Temporal control of optical pulses has remarkable applications in all-optical signal processing [1], slow and fast light [2–5], pulse shaping [6,7], microwave photonics [8], highresolution strain and temperature fiber sensors [9] and narrow linewidth fiber lasers [10]. The interaction of two counter-propagating optical waves and longitudinal acoustic waves (phonons) in long fiber lengths enables the dynamic tuning of the pulses' rise time by means of stimulated Brillouin scattering [3]. Alternatively, devices employing short fiber lengths are important for practical applications since the pulses' time delay is basically defined by the transit time of the optical signal within the fiber [4,5]. In this case, fiber Bragg gratings (FBGs) are attractive to induce significant temporal delays of the light over short distances (0.6-20 cm) [4,9,11]. Superstructure and π phase shifted FBGs efficiently shape optical pulses in short fiber lengths employing low power sources [5,7–9]. These gratings are inscribed along the fiber with one or multiple π phase shifts, or with the modulated refractive index being apodized with a gaussian or sinc profile [6–8,12]. Permanent modulation of grating properties results in exceptional temporal integrators [6], differentiators [7] and fractional Hilbert transformer (FHT) filters [8] for reconfigurable signal processors [1]. In general, the filter order increases by decreasing the modulation amplitude of the grating refractive index [7]. Nevertheless, the filter order is usually limited due to the grating fabrication complexity [7]. High power optical pulses (gap solitons) have been employed to induce an effectively long optical path length inside the grating [4]. The

optical pulses are delayed by changing the pump power and detuning the pulse in relation to the grating reflection edge.

Overall, most of the proposed devices are usually limited in their ability to rapidly tune the optical properties. In general, high-power pump sources and high voltage-based techniques cause significant temperature-induced changes in the fiber refractive index. Consequently, the device response time is usually limited to the milliseconds range because of the slow material heating and cooling process [13]. Besides, the combination of long fiber lengths, bulk-type modulators and free-space optics may induce undesirable insertion losses, increasing the setup complexity, the number of components, and the costs [2,3,9]. In contrast, all-fiber acousto-optic modulators can change signal optical properties, such as power, wavelength and polarization, with low insertion loss, employing a small number of components [10,14–24]. In particular, the interaction of acoustic waves and FBGs has successfully been employed in dynamic optical filters, fiber sensors and Q-switched and mode-locked fiber lasers [10,15,20,22–24], enabling response times from 40 to 500 μ s [15,17–21].

In general, the temporal response of the acousto-optic devices depends on the damping time of free natural resonant vibrations remaining in the fiber after the switching ON/OFF of the applied acoustic stress (step amplitude variation) [25]. The temporal period of the fundamental sinusoidal wave resonance is determined by the fiber length *L* and material as, $\tau_{\rm R} = 4L(\rho/Y)^{1/2}$, in which, ρ is the density and *Y* is the Young's modulus [25]. This resonance is damped after a transition time, usually given by the rising or falling time of the mechanical stress until reaching a steady state. Reduction of this transition time shortens the pulse duration, increasing for example the maximum repetition rate and output power of acoustically modulated pulsed fiber lasers [15,20]. In this paper, we experimentally and numerically demonstrate a new modulation technique that allows tuning, and significantly reducing, the switching time of an acousto-optic device. Additionally, the amplitude modulation of optical pulses in FBGs caused by the fiber's natural resonant vibrations is investigated in detail.

2. Materials and methods

Fig. 1(a) illustrates the fabricated modulator, basically composed of a piezoelectric transducer (PZT), an acoustic silica horn and a segment of a highly birefringent suspended core fiber (HB-SCF). The HB-SCF in Fig. 1(b) is 124 μ m in diameter, containing two similar pure silica birefringent cores (7.7 μ m × 4 μ m) surrounded by four air holes of 40 - 43 μ m diameter, separated by silica bridges of 2.7 μ m thickness. A 1 cm long FBG is inscribed in the HB-SCF by means of a femtosecond laser using a phase mask interferometer as described in [26]. The HB-SCF's large air holes significantly strengthen the interaction of acoustic waves and the grating in the fiber core [22,24]. The interaction of the HB-SCF's "fast X" and "slow Y" polarization modes with the grating results in two reflected bands centered at $\lambda_X = 1540.7$ nm and $\lambda_Y = 1540.9$ nm (unmodulated spectrum in Fig. 1(c)). The PZT disc is 3 mm in diameter and 2 mm in thickness. The hollow silica horn is tapered in diameter from 1.06 mm to 200 µm along 1.4 cm length (tip of 150 µm inner diameter). The horn is connected to the PZT and to the HB-SCF by means of a glass adhesive. The modulator ends are fixed, resulting in a total device length of 4 cm.

A rectangular pulse train with a voltage amplitude of 10 V and frequency of f = 469 kHz is applied to the PZT with a signal generator (SG). The PZT generates standing longitudinal acoustic waves which are coupled to the fiber by the horn. The acoustically induced strain modulates the grating reflectivity switching ON and OFF the polarization peaks λ_X and λ_Y at twice the applied acoustic frequency (f = 938 kHz) [22]. Fig. 1(c) shows the unmodulated and modulated FBG spectra normalized to the maximum reflectivity, as recorded by the optical spectrum analyzer (OSA). Although it is not visible, the maximum temporal modulation depth of our device might be higher than $\Delta \eta = 90$ % even though the variation of the reflectivity indicated by the OSA only reached 61 % at 10 V; this is because our OSA could not measure the instantaneous modulation depth of the fast modulated optical signal at f = 938 kHz, but only showed its average value in Fig. 1(c) - details about temporal and average reflectivity are shown in [27]. Overall, this spectral modulation indicates a significantly high acousto-optic efficiency for a compact device (4 cm) and low applied voltage (10 V).

One core of the HB-SCF is spliced to the core of a standard single mode fiber (SMF) of a circulator. The temporal response of the grating spectrum is characterized by tuning a CW laser (TL) at λ_x , as illustrated in Fig. 1(c) (green dashed line). The power reflected by the grating is measured with a photodetector (PD) and an oscilloscope (OSC). Acoustic pulse trains are generated by inducing a temporal gap of constant positive polarization of +10 V (duty cycle OFF) in the rectangular electrical signal (a detail of the electrical signal is further shown in Fig. 5). Fig. 1(d) shows the temporal response of the applied electrical pulses (blue signal) and the FBG modulated reflected power at the duty cycle ON (*Dc*) of 89.2 %.



Fig. 1. Illustration of the (a) acousto-optic modulator with the experimental setup. (b) Highly birefringent suspended core fiber (HB-SCF) with a detail of the simulated power distribution of the fast X mode. (c) Measured unmodulated (blue) and modulated (red) spectrum of the fiber Bragg grating (FBG) inscribed in the HB-SCF. (d) Measured electrical pulse train applied to the PZT at f = 469 kHz (blue) and the acoustically modulated optical pulse at the duty cycle of Dc = 89.2 % (red).

The maximum modulation depth $\Delta \eta_t = 1$ is normalized to the maximum voltage of the photodetector, corresponding the peak reflected power of the unmodulated grating. The minimum level $\Delta \eta_t = 0$ corresponds to the combined average of the photodetector's background noise and the high frequency modulated signal at f = 938 kHz. This averaging allowed better evaluation of the temporal properties of the low frequency amplitude modulated optical pulses and fundamental mechanical resonance during the duty cycle OFF. The pulse width Pw_t (full width at half maximum – FWHM) of the reflected optical pulse is tuned by changing the duty cycle from

Dc = 70.1 to 99.4 % (6.4 % steps, 3.8 % last step) at a fixed repetition rate of Rr = 30 Hz (period of T = 33 ms). The pulses are also simulated along the fiber and FBG by means of the finite element method and transfer matrix method, considering the methodology and material parameters described in [16,25]. Fig. 1(b) shows the simulated power distribution of the fundamental fast X mode in the SCF's cores.

3. Results and discussion

Fig. 2(a) shows the temporal modulation of the fast mode X at λ_X for the considered *Dc* range. Although it is not shown, similar results are also observed for the slow mode Y at λ_Y . Note in Fig. 2(b) that the pulse width is linearly tuned from $\Delta Pw_t = 9.9$ to 0.2 ms. The fast mode reflectivity is recovered to 100 % at *Dc* = 70.1 %, gradually decreasing with increasing *Dc*. For low *Dc* values, the time interval between two pulse trains is rather large, allowing the grating reflectivity a full or almost complete recovery. The pulse rise time $\tau_{10-90\%}$ (10 - 90 % of the optical level) is also investigated, gradually decaying from 56 to 44 µs in the considered *Dc* range in Fig. 2(a) (further seen in Fig. 4(a)). From this point, both modulation depth $\Delta \eta$ and rise time $\tau_{10-90\%}$ abruptly decay with increasing *Dc*, as seen respectively in Fig. 2(b) and 4(a).



Fig. 2. (a) Temporal pulse width modulation of the FBG reflected optical power at the wavelength of $\lambda_x = 1540.7$ nm. The temporal widths are tuned by changing the duty cycle ON of the electrical pulse from Dc = 70.1 to 99.4 % at a voltage of 10 V and frequency of f = 469 kHz applied to the PZT. (b) Variation of the pulse width Pw_t and modulation depth $\Delta \eta_t$ for the considered Dc range.

A second experiment is performed to investigate the pulse properties from Dc = 99.83 to 99.98 % (6 arbitrary steps). Fig. 3 shows the simulated (dashed blue curve) and measured (red curve) acoustically modulated optical pulses for the considered Dc range. The simulated pulses are also averaged to emphasize the fundamental resonance peaks and compare with the measured pulses. Note in Fig. 3 that the amplitude modulation caused by the fiber's fundamental acoustic resonance induces peaks on the optical pulse envelope, which are more evident when Dc is switched OFF. The simulated peaks on the pulse's right are used as reference to estimate the full resonance temporal period τ_R , as indicated by the dashed green lines in Fig. 3. Simulated and measured values show good agreement mainly along the pulse fall time of $\tau_F = 13.3 \,\mu$ s which is about one half-period of the resonance's complete sinusoidal cycle ($\tau_F = 0.5 \tau_R$), indicated with the wave's maximum, minimum and node in Fig. 3(b).



Fig. 3. Acousto-optic temporal control and shaping of optical pulses. The pulse rise time is gradually decreased by tuning the duty cycle ON of the applied electrical signal from (a)-(f) Dc = 99.83 to 99.98 %. The rectangular pulses in Fig. 2(a) are shaped into nearly triangular pulses. The fundamental resonance's amplitude modulation inducing peaks on the pulses' envelope is considerably damped with increasing Dc. The resonance temporal period τ_{R} , pulse fall time τ_{F} , and phase-shifted peaks τ_{PS} , are indicated with dashed arrows.

The optical pulse fall time corresponds to the rise time of the acoustic strain when it is switched ON and emitted along the fiber. The measured and simulated resonance period of $\tau_{\rm R} = 26.6 \,\mu s$ agrees well with the analytically calculated $\tau_{\rm R} = 26 \,\mu s$ for the employed fiber length of 3.7 cm (considering $\tau_{\rm R} = 4L(\rho/Y)^{1/2}$, $\rho = 2200 \,\text{kg/m}^2$ and $Y = 72.5 \,\text{GPa}$ [25]).

The fundamental fiber vibration resonance has a sinusoidal wave profile, as indicated by the oscillations in Fig. 3(b). Note that the wave maxima and minima induce peaks and notches in the pulse amplitude. During the switching OFF, the overlap of the out of phase transmitted and

reflected acoustic pulses in the fiber, induces a phase shift in the resonance sinusoidal wave. Note in Fig. 3(a) that the wave period between the central resonance peaks is reduced to $\tau_{R} = 14 \ \mu s$, indicating a phase shift of the left peak of $\tau_{PS} = 12.6 \ \mu s$ (dashed orange arrow). This phase shift reduces the resonance's pulse amplitude modulation, decreasing the rise time of the optical pulses.

The decreasing temporal gaps between pulse trains reduce the peaks of the amplitude modulation. Note in Figs. 3(e) and 3(f) that no peaks are observed on the left side of the pulse. It is because the temporal gap between pulse trains is shorter than the resonance period $\tau_{\rm R}$. Consequently, the resonance modulation is minimized, further smoothing, and decreasing the rise time. However, additional shortening of this temporal gap does not allow the full amplitude recovery of even the main pulse peak, as shown in Fig. 3(f). The resulting pulse distortion induces an apparent pulse lengthening reversion at Dc = 99.98 %.



Fig. 4. (a) Pulse rise time $\tau_{10-90\%}$ for Dc = 70.1-99.98 %. (b) Rise time $\tau_{1-100\%}$ (c) pulse width Pw_t and (d) modulation depth $\Delta \eta_t$ for Dc = 99.83 - 99.98 %.

The pulses' rise time significantly decreases from $\tau_{1-100\%} = 56$ to 9 µs, as shown in Fig. 4(b). Fig. 4(c) shows the variation of the pulse width Pw_t with increasing *Dc*. As expected, the decreasing pulse width is limited by the rising and falling time of the device. In addition, increasing *Dc* does not allow the grating spectrum to completely recover between pulse trains, reducing the modulation depth $\Delta \eta_t$, as shown in Fig. 4(d).

We have also investigated the modulator's shortest response time. Fig. 5 shows the electrical pulse train applied to the PZT and the rise time $\tau_{1-100\%}$ of the optical pulse at Dc = 99.98 %. Both curves are normalized to their individual maximum amplitudes. Note that the PZT is polarized with a positive voltage to shift the phase of consecutive acoustic pulse trains in the fiber during

Dc OFF. The rise time $\tau_{1-100\%} = 9 \ \mu s$ still indicates remaining undamped oscillations in the grating. The delay between optical-electrical signals, $\tau_{DL} = 6.6 \ \mu s$, is related to the acoustic signal generation and propagation from the PZT to the grating, including the travel time along the horn and coupling adhesives. Overall, our achieved switching time of $\tau_{ST} = 15.6 \ \mu s$ is slightly shorter than that obtained with a 20% smaller acousto-optic device [22]. It indicates that the demonstrated modulation technique is promising to reduce the time response of even larger devices, relieving requirements to fabricate complex small acoustic components. Although size reduction of the PZT, acoustic horn, grating and fiber length is still an option to shorten the overall switching time, the decreasing rise time shown here is mainly caused by the damping of the free natural oscillations remaining in the fiber.



Fig. 5. Temporal response of the all-fiber acousto-optic device with details of the electrical pulse applied to the PZT (blue) and the modulated optical signal (red) at a duty cycle of Dc = 99.98 % and frequency of f = 469 kHz. The temporal gap T_M is about 2.5 times the period of the applied electric pulse trains T_p . The sum of the pulse rise time $\tau_{1-100\%}$ and the delay between the optical and electrical signals τ_{DL} defines the overall device's switching time τ_{ST} .

4. Conclusion

We have demonstrated a new fast technique for acousto-optic generation and tuning of acoustically induced pulses in a SCF-FBG. An acoustic pulse train is switched ON/OFF out of phase along the fiber and the temporal response of the grating reflected optical pulses is experimentally and numerically characterized. The pulse's amplitude modulation caused by the natural fiber fundamental acoustic resonance is investigated in detail for the first time.

The good agreement between the experiment, simulations and theory strongly confirms the excitation of natural resonant vibrations in the fiber modulated by an acoustic signal with an abrupt step variating amplitude. The results show that the fundamental resonance induces peaks on the pulse envelope with a temporal period defined by the fiber length and material. This resonance effect is significantly reduced by shortening the temporal gap (duty cycle OFF) between consecutive acoustic pulse trains, which reflect at the modulator ends and interfere out of phase along the grating. For temporal gaps in the order or shorter than the resonance period, the pulse amplitude modulation and peaks are considerably reduced.

The demonstrated technique allows the tunable reduction of the response time of the acoustooptic device. The pulse width is modulated from 9.9 ms to 10 μ s, while keeping high pulse modulation depths. The pulse rise time is tuned from 56 to 9 μ s, contributing to the shortest switching time of 15.6 μ s. To the best of our knowledge, this is the fastest all-fiber acousto-optic device reported. The results indicate a promising technique to shape the pulsed temporal reflection of Bragg gratings, enabling fast modulation of fully reconfigurable all-fiber integrated devices. Further advance of this study might suggest deep investigation of the destructive interference of the remaining resonance vibrations during duty cycle OFF. Optical pulses with narrower widths might be achieved with further damping or canceling of acoustic resonance effects. In this way, pulse shape profiles being adjusted by the amplitude and duty cycle of the applied acoustic signal are expected. It will also enable new opportunities to modulate the power of pulsed fiber lasers.

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