An advanced data analytic approach for reallocating green gas emissions in cap-and-trade context

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Abstract

Due to rapid economic growth in countries, global warming has been a major challenge for many years. Cap-and-trade is recognized as the most effective method for controlling and decreasing greenhouse gas emissions. In this paper, we develop a data envelopment analysis (DEA) model to reallocate permitted emissions among decision making units (DMUs). The model considers the dependency between the production of desirable and undesirable outputs in the cap-and-trade context. In addition, in current mathematical modelling the undesirable output is addressed as a by-product of a desirable output. Moreover, the sum of efficiency scores of all DMUs is maximized. Furthermore, to address the alternative optimal solutions in the cap-and-trade context, the efficiency interval of DMUs is calculated. Finally, to show the capabilities of the developed model a case study is presented.

Keywords: Cap-and-trade; Data envelopment analysis (DEA); Efficiency; Reallocation; Undesirable outputs.

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1. Introduction

Greenhouse gas emissions and consuming fossil fuels are key challenges of sustainable development (Wang et al. 2018). Over the last half-century, greenhouse gas emissions have been increased significantly and reached a worrisome figure of 80% (Momeni et al. 2019). Increasing greenhouse gas emissions not only lead to ecological problems but also is a serious threat to human lives (Xu et al. 2021). To confront these threats, many countries have approved some protocols (Wang and Wu, 2021). Based on the Kyoto protocol approved in 1997, some countries committed to decrease greenhouse gas emissions (Nicoletti et al. 2015). Paris Agreement in 2015 is another significant preventive measure to reduce greenhouse gas emissions. In this Agreement, 195 countries committed to decrease greenhouse gas emissions (Momeni et al. 2019).

Allocation of emission permits is one of the most effective strategies for reducing greenhouse gas emissions (Ni and Shu, 2015). In allocating emissions permits, the cap-and-trade policy is used (Ji et al. 2017). Emission permits are considered as permissions for organizations to emit extra CO_2 more than their caps (Song et al. 2015). Controlling pollutions, improving ecological management, and enhancing companies' operations are distinctive advantages of allocating emission permits. As such, developing methods for allocating emission permits should be taken into account (Jaber 2013). However, allocating emission permits has key challenges, including dealing with multiple criteria and their effect on efficiency scores. Having said that, the allocation of emission permits has received much attention and is considered a major challenge by scholars and decision-makers (Yu and Cao 2020). Cap-and-trade is one of the most effective approaches for allocating emission permits (Qin et al. 2020). The "cap" means the total allowed greenhouse gas emissions and "trade" denotes that the organizations are allowed to trade their emission allowances with each other (Chu et al. 2021). This approach is broadly applied by governments. In the cap-and-trade approach, carbon emission permits are allocated to each organization and then they decide to trade permits based on their actual use (Sun et al. 2014). However, some traditional schemes of cap-and-trade are not effective and lead to higher transaction costs and lower ecological efficiency (Momeni et al. 2019). Thus, it is essential to develop a method for allocating caps to companies (Ji et al. 2017).

There are different methods to allocate emission permits (caps) to decision making units (DMUs). Among them, data envelopment analysis (DEA) can deal with cap-and-trade policy (Sun and Li, 2020). However, some DEA axioms and production technologies should be developed when the mechanisms are adopted (Chu et al. 2021). In DEA, greenhouse gas emissions are considered undesirable outputs and should be minimized. In this paper, the total amount of permitted CO_2 (as an undesirable output) is considered as a cap and DMUs should produce less than or equal to the cap. Otherwise, they have to buy emission permissions from other DMUs. The main objective of this paper

is to develop a new DEA model to reallocate CO_2 emissions in a cap-and-trade context. The contributions of this paper are as follows:

- This study considers the dependency between the desirable and undesirable outputs in the cap-and-trade context.
- In this paper, the undesirable output is considered a by-product of a desirable output.
- In this paper, the sum of efficiency scores of all DMUs is maximized.
- To address the alternative optimal solutions in the cap-and-trade context, the efficiency interval of DMUs is calculated.
- A case study is provided.

The reset of this study is as follows: In the next section, related works are discussed. In Section 3, we propose the model followed by a case study in Section 4. In Section 5, conclusions are presented.

2. Literature review

2.1. Methods for greenhouse gas emissions permits

Cap-and-trade methods have received considerable attention (Dhavale and Sarkis, 2015). Cap-and-trade, tradable emission permits (TEP), and pollution tax are some known methods for trading greenhouse gas emissions in the market (Dong et al. 2016). TEP is a system in European Union (EU) started in 2005 to reduce greenhouse gas emissions in the energy sector and encompasses more than 1000 plants in the EU. Emission tax is another method for ecological protection. Imposing a heavy tax on the production of greenhouse gas emissions is a suitable method for improving global warming (Chan, 2020). Cap-and-trade is recognized as the most popular approach for decreasing greenhouse gas emissions (Du et al. 2015). In this approach, concentration is on the optimal allocation of greenhouse gas emissions. The caps are determined by governments (Dong et al. 2016). Based on some regulations, organizations can buy or sell greenhouse gas emissions quotas in the market if they produce more or less than the caps (Zheng et al. 2021).

2.2. Approaches for emissions allocation

Grandfathering is one of the traditional approaches for allocating greenhouse gas emissions (Wang and Zhou, 2017). Nevertheless, this approach is not effective as allocations are not determined fairly among organizations (Peng et al. 2021). Periodic reallocation is also a conventional method for allocating emissions. In this method, current and new organizations are treated fairly to allocate emissions, and organizations need to adjust their outputs for getting more permits (Wang and Zhou,

2017). Nonetheless, applying this approach is not only complicated but also costly (Zhao et al. 2018). Output-based allocation (OBA) is another approach for emissions allocation (Sun and Li, 2020). However, this approach focuses only on outputs and ignores inputs (Wu et al. 2016). To address this issue, DEA was proposed in which both inputs and outputs are taken into consideration (Lee, 2019).

Assuming undesirable outputs, Lozano et al. (2009) proposed a DEA model for reallocating emission permits. Their model includes multi-stages for minimizing undesirable outputs. Wang et al. (2013) presented a DEA model for allocating permitted emissions with zero-sum. The results showed that provinces should adopt dissimilar mitigation strategies. Sun et al. (2014) applied DEA for allocating emission permits in industrial companies to control the emission level. However, their model cannot deal with trade among companies (DMUs). Feng et al. (2015) presented a two-stage approach based on DEA for mitigating side effects of allocating permitted emissions. To do so, they developed centralized allocation models based on conventional assumptions of DEA. Then, they developed two compensation schemes for allocation plans. Wu et al. (2016) proposed a DEA-based method for allocating emission decrease plans. They presented the idea of satisfaction level and applied it to maximize satisfaction level. Wu et al. (2018) developed a DEA model for allocating emissions permits. Momeni et al. (2019) proposed a centralized DEA model for reallocating emissions permits in the cap-and-trade framework. Their proposed model takes all DMUs into account. Chu et al. (2021) proposed a DEA model for the efficiency measurement of organizations and addressed the cap-and-trade mechanism. The model improves the potentials of organizations for decreasing CO_2 emission and increasing productivity.

2.3 Research Gap

This study aims to formulate a cap-and-trade DEA model for the fair allocation of undesirable outputs. Undesirable outputs are produced along with the production of desirable outputs. Undesirable outputs can be considered as by-product of desirable outputs. Thus, production of undesirable outputs is inevitable. To the best of our knowledge, there is no paper to take into account the dependency of desirable and undesirable outputs in the cap-and-trade context. Moreover, considering all the inefficiencies of inputs and outputs are needed for the efficiency evaluation in the cap-and-trade context. Also, dealing with alternative optimal solutions is an important challenge in the cap-and-trade context. To address this issue, in this paper, the efficiency intervals are calculated in the cap-and-trade context. Here, we introduce a centralized DEA model to reallocate and reduce the sum of the undesirable outputs among DMUs.

3. Proposed model

Here, we introduce and formulate a new DEA model in the cap-and-trade context. Table 1 lists the used notations in this paper.

Table 1. The notations

x	An input vector	s^+	Surplus variable for outputs
y	An output vector	λ	Intensity variable
g	A vector of direction	α	Variable used for changes in input
s^-	Slack variable for inputs	β	Variable used for changes in output
F	A function between the desirable and undesirable outputs	i	The input index
δ	Efficiency score	r	The output index
ρ	Objective function value	j, p	Index of DMUs
m	Number of inputs	s	Number of outputs
n	Number of DMUs	DMU_p	The DMU under evaluation

Model (1) is the additive model, which considers variable returns to scale (Bardhan et al., 1996).

$$
\begin{aligned}
&Max\ \rho^0 = \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
&s.t. \\
&\textstyle\sum_{j=1}^{n} \lambda_j x_{ij} = x_{io} - s_i^-, && i = 1, \dots, m, \\
&\textstyle\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + s_r^+, && r = 1, \dots, s, \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1, \\
&s_i^- \geq 0, s_r^+ \geq 0, && i = 1, \dots, m; r = 1, \dots, s, \\
&\lambda_j \geq 0, && j = 1, \dots, n.
\end{aligned}
\tag{1}
$$

Model (1) considers all possible improvements in the inputs and outputs. In Model (1), DMUs are classified into two classes, Pareto efficient and inefficient DMUs. In the proposed model, since for all DMUs, there is a cap on the undesirable output, a centralized approach is used. Note that DMUs can interact with each other so that the total undesirable output of DMUs does not exceed the cap. In

other words, DMU_j can buy and use the undesirable output quota of other DMUs. In the proposed method, the optimal inputs and outputs are determined by taking into account the conditions set by managers. We consider the dependency of undesirable and desirable outputs. In other words, the undesirable outputs are considered as by-products of desirable outputs. Due to the bad nature of the undesirable outputs, we try to reduce the undesirable outputs. The directional distance function (DDF) is used to recommend improvements in inputs and outputs. For instance, consider the *i*th input and the *r*th output of DMU$_j$, respectively, as x_{ij} and y_{rj}. Moreover, consider y_{1j} as an undesirable output, which is a by-product of the desirable output y_{2j}. The rest elements of the output vector, $\{y_{3j}, \dots, y_{sj}\}$, are desirable outputs. Also, in natural direction vector, decreases in the inputs and increases in the desirable outputs are considered, $g = (g^x, g^y) = (-x, y)$.

Given the capability of Model (1) in distinguishing the Pareto efficient and inefficient DMUs, we consider the changes in inputs and outputs. In Expression (2), α and β represent changes in inputs and outputs, respectively. Note that changes in the undesirable outputs, β_1, are considered free in sign as y_{1j} and y_{2j} are dependent. These changes are included in the model using β_1, which is free in sign as well. Free changes in signs of the undesirable output can lead to a decrease or increase of the undesirable output.

One of the main goals of decision-makers is to increase the desirable outputs and to decrease the undesirable outputs. Note that the relationship between the desirable and undesirable outputs (the "F") is determined by the decision-maker. The proposed DDF model is based upon Slack-based measure. Therefore, we have

$$
\begin{aligned}
& x_{ip} - \alpha_{ip} x_{ip}, && i = 1, \dots, m;\ p = 1, \dots, n \\
& y_{1p} + \beta_{1p} y_{1p}, && r = 1;\ p = 1, \dots, n \\
& y_{2p} + \beta_{2p} y_{2p}, && r = 2;\ p = 1, \dots, n \\
& y_{rp} + \beta_{rp} y_{rp}, && r = 3, \dots, s;\ p = 1, \dots, n \\
& \alpha_{ip} \geq 0, \beta_{2p} \geq 0, && i = 1, \dots, m;\ p = 1, \dots, n \\
& \beta_{rp} \geq 0,\ \beta_{1p} \text{ free}, && r = 3, \dots, s; p = 1, \dots, n
\end{aligned}
\tag{2}
$$

In model (3), the maximum improvements of inputs and outputs are suggested. The objective function represents the sum of input and output improvements of all DMUs.

$$
\begin{aligned}
&Max\ \rho^1 = \sum_{p=1}^{n}\left(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left(\beta_{1p} + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}\right)\right)\\
&s.t.\\
&\sum_{j=1}^{n}\lambda_j^p x_{ij} = x_{ip} - \alpha_{ip}x_{ip}, && i = 1,\dots,m;\ p = 1,\dots,n,\\
&\sum_{j=1}^{n}\lambda_j^p y_{1j} = y_{1p} + \beta_{1p}y_{1p}, && r = 1;\ p = 1,\dots,n\\
&\sum_{j=1}^{n}\lambda_j^p y_{2j} = y_{2p} + \beta_{2p}y_{2p}, && r = 2;\ p = 1,\dots,n\\
&\sum_{j=1}^{n}\lambda_j^p y_{rj} = y_{rp} + \beta_{rp}y_{rp}, && r = 3,\dots,s;\ p = 1,\dots,n\\
&\sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_j^p y_{1p} \le A\\
&\sum_{j=1}^{n}\lambda_j^p y_{1p} \le F^p\left(\sum_{j=1}^{n}\lambda_j^p y_{2p}\right) && p = 1,\dots,n\\
&\alpha_{ip} \ge 0, \beta_{2p} \ge 0, \beta_{rp} \ge 0, && i = 1,\dots,m;\ r = 2,\dots,s\\
&\beta_{1p}\ \text{free in sign}\\
&\lambda_j^p \ge 0, && j = 1,\dots,n;\ p = 1,\dots,n
\end{aligned}
\tag{3}
$$

In the objective function of model (3), $\frac{1}{3}$ is used as three sets of outputs that are considered. The first is y_{1p}, the second is y_{2p}, and the third is y_{rp}, for $r = 3,\dots,s$. In the first set of constraints, input reductions are considered and in the third and fourth set of constraints, the output increments are considered. In the second set of constraints, the undesirable outputs are changed using β_1. Note that β_1 is free in sign as the undesirable outputs should be allowed to be freely increased or decreased. Consider the fifth constraint. According to the defined cap, undesirable outputs of all DMUs should be less than or equal to the cap, which is defined by decision maker. The sixth constraint shows relationship between the production of desirable and undesirable outputs. In most of the times, the undesirable outputs are by-products of desirable outputs. Thus, any increase in production of desirable output may increase the undesirable outputs. Conversely, if it is desired to decrease the production of undesirable outputs, production of desirable outputs should be decreased to some extent. Assume F^p to be a mathematical function that describes the relationship between production of desirable and undesirable outputs for each DMU_p.

For dealing with free variables such as β_{1p}, the following relationships can be caonsiderd.

$$
\beta_{1p}^{+} = \begin{cases}\beta_{1p}, & \beta_{1p} \ge 0,\\ 0, & \beta_{1p} \le 0,\end{cases}
\qquad
\beta_{1p}^{-} = \begin{cases}0, & \beta_{1p} \ge 0,\\ -\beta_{1p}, & \beta_{1p} \le 0,\end{cases}
\qquad \forall p = 1,\dots,n
\tag{4}
$$

we have

$$
\beta_{1p} = \begin{cases}(\beta_{1p}) - (0) \ge 0, & \beta_{1p} \ge 0\\ (0) - (\beta_{1p}) \ge 0, & -\beta_{1p} \ge 0\end{cases}
\qquad \forall p = 1,\dots,n
\tag{5}
$$

Note that, according to (4), product of variables β_{1p}^{+} and β_{1p}^{-} equals to 0. Consider β_{1p}. We have

$$\beta_{1p}^{+}\beta_{1p}^{-} = \begin{cases} (\beta_{1p})(0) = 0, & \beta_{1p} \geq 0 \\ (0)(-\beta_{1p}) = 0, & \beta_{1p} \leq 0 \end{cases} \quad \forall p = 1, \dots, n \qquad \square \qquad (6)$$

Remark 1: Given Expression (4), the signs of β_{1p}^{+} and β_{1p}^{-} are symmetric. Therefore, relation (6) is always valid. Thus, according to Bazaraa et al. (2008), Expression (6) is redundant and can be removed from the model. As a result, Model (7) is as follows:

$$\begin{aligned}
&Max\ \rho^2 = \sum_{p=1}^{n}\left(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}\right)\right) \\
&s.t. \\
&\sum_{j=1}^{n}\lambda_j^p x_{ij} = x_{ip} - \alpha_{ip}x_{ip}, && i = 1, \dots, m;\ p = 1, \dots, n \\
&\sum_{j=1}^{n}\lambda_j^p y_{1j} = y_{1p} + (\beta_{1p}^{+} - \beta_{1p}^{-})y_{1p}, && r = 1; p = 1, \dots, n \\
&\sum_{j=1}^{n}\lambda_j^p y_{2j} = y_{2p} + \beta_{2p}y_{2p}, && r = 2; p = 1, \dots, n, \\
&\sum_{j=1}^{n}\lambda_j^p y_{rj} = y_{rp} + \beta_{rp}y_{rp}, && r = 3, \dots, s; p = 1, \dots, n, \\
&\sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_j^p y_{1p} \leq A \\
&\sum_{j=1}^{n}\lambda_j^p y_{1p} \leq F^p\left(\sum_{j=1}^{n}\lambda_j^p y_{2p}\right) && p = 1, \dots, n \\
&\alpha_{ip} \geq 0, \beta_{rp} \geq 0, && i = 1, \dots, m; r = 2, \dots, s, \\
&\beta_{1p}^{-} \geq 0, \beta_{1p}^{+} \geq 0, && r = 1; p = 1, \dots, n, \\
&\lambda_j^p \geq 0, && j = 1, \dots, n;\ p = 1, \dots, n
\end{aligned} \qquad (7)$$

The first set of constraints of model (7) shows input reductions, which can be considered as $x_{ip}(1 - \alpha_{ip})$ for all *i* and *p*. The third and fourth set of constraints show output increases. They can be also considered as $y_{2p}(1 + \beta_{2p})$ and $y_{rp}(1 + \beta_{rp})$, respectively. The second set of constraints show free changes for the outputs. According to Expression (6) and Remark 1, β_{1p}^{+} and β_{1p}^{-} cannot take positive values at the same (Bazaraa et al., 2008). Therefore, either increase or decrease can occur in the optimal solution of model (7). The fifth constraint emphasizes that the sum of undesirable outputs should be less than the cap. In the sixth constraint, the F^p shows the relationship between the desirable and undesirable outputs. This function, for each DMU_p, is defined by decision maker. Also, the F^p may vary from a DMU to another DMU. Note that to have linear Model (7), F^p is considered as a linear function.

Theorem 1: In every feasible solution of model (7), we have $0 \leq \alpha_{ip} \leq 1,\ 0 \leq \beta_{1p}^{-} \leq 1,\ \forall p, \forall r.$

Proof: According to model (7), we have

$$\sum_{j=1}^{n} \lambda_j^p x_{ij} = x_{ip} - \alpha_{ip} x_{ip}, \qquad i = 1, \ldots, m; p = 1, \ldots, n \qquad (8)$$

Since $\sum_{j=1}^{n} \lambda_j^p x_{ij} \geq 0 \ \rightarrow\ x_{ip} - \alpha_{ip} x_{ip} \geq 0 \ \rightarrow (1 - \alpha_{ip}) x_{ip} \geq 0 \rightarrow (1 - \alpha_{ip}) \geq 0, \forall i, \forall p,$ (9)

Thus, $\alpha_{ip} \leq 1, \quad \forall i, \forall p,$ (10)

Now, according to (4), all the variables are non-negative. Thus,

$0 \leq \alpha_{ip} \leq 1, \quad \forall i, \forall p,$ (11)

Consider the second set of constraints in Model (7).

$$\sum_{j=1}^{n} \lambda_j^p y_{1j} = y_{1p} + (\beta_{1p}^{+} - \beta_{1p}^{-}) y_{1p}, \quad r = 1; p = 1, \ldots, n \qquad (12)$$

Note that both β_{1p}^{+} and β_{1p}^{-} cannot take positive values. Therefore, assuming $\beta_{1p}^{+} = 0$ and $\beta_{1p}^{-} > 0$, we have

$$\sum_{j=1}^{n} \lambda_j^p y_{1j} \geq 0 \ \rightarrow y_{1p} - \beta_{1p}^{-} y_{1p} \rightarrow (1 - \beta_{1p}^{-}) y_{1p} \geq 0 \rightarrow (1 - \beta_{1p}^{-}) \geq 0 \rightarrow \beta_{1p}^{-} \leq 1, \ \forall p \qquad (13)$$

Since variables are non-negative, thus

$0 \leq \beta_{1p}^{-} \leq 1, \ \forall p$ □ (14)

Theorem 2: For each *p*, the $\frac{\left[1 - \frac{1}{2}\left(\frac{1}{m}\sum_{i=1}^{m} \alpha_{ip} + \beta_{1p}^{-}\right)\right]}{\left[1 + \frac{1}{3}\left(\beta_{1p}^{+} + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s} \beta_{rp}\right)\right.}$ in model (7) is between zero and 1.

Proof: Consider any optimal solution of model (7). Given each *p*, we have

$\alpha_{ip} \leq 1$ and $\beta_{1p}^{-} \leq 1$ (15)

Therefore, according to (15), the following relations can be concluded.

$$\frac{1}{m}\sum_{i=1}^{m} \alpha_{ip} \leq 1, \quad \beta_{1p}^{-} \leq 1, \quad \forall p \qquad (16)$$

Therefore

$$\frac{1}{2}\left(\frac{1}{m}\sum_{i=1}^{m} \alpha_{ip} + \beta_{1p}^{-}\right) \leq 1 \rightarrow \ 1 - \frac{1}{2}\left(\frac{1}{m}\sum_{i=1}^{m} \alpha_{ip} + \beta_{1p}^{-}\right) \leq 1 \qquad (17)$$

It is clear that $1 - \frac{1}{2}\left(\frac{1}{m}\sum_{i=1}^{m} \alpha_{ip} + \beta_{1p}^{-}\right) \geq 0$. Simply, $(\beta_{1p}^{+} + \beta_{2p} + \sum_{r=3}^{s} \beta_{rp}) \geq 0$. Therefore, we have

$$\frac{1}{3}\left(\beta_{1p}^{+}+\beta_{2p}+\frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}\right)\geq 0\rightarrow 1+\frac{1}{3}(\beta_{1p}^{+}+\beta_{2p}+\frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp})\geq 1 \quad (18)$$

Consequently, for each p, we have

$$\frac{\left[1-\frac{1}{2}(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip}+\beta_{1p}^{-}\right]}{[1+\frac{1}{3}(\beta_{1p}^{+}+\beta_{2p}+\frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp})}\leq 1 \quad \square \quad (19)$$

Remark 2: After solving model (7), for each p, the following strategies can be introduced:

A) A strategy for reducing the undesirable output.

B) A strategy for increasing the undesirable output.

β_{1p}^{+} and β_{1p}^{-} cannot take positive values at the same time. For the first strategy, if $\beta_{1p}^{*-}>0$ and $\beta_{1p}^{*+}=0$, then according to constraint (20) of model (7), the undesirable output can be reduced.

$$\sum_{j=1}^{n}\lambda_{j}^{*p}y_{1j}=y_{1p}+(\beta_{1p}^{*+}-\beta_{1p}^{*-})y_{1p}=y_{1p}+(0-\beta_{1p}^{*-})y_{1p}=y_{1p}-\beta_{1p}^{*-}y_{1p},\quad \forall p \quad (20)$$

Note that the desirable and undesirable outputs are interdependent. Moreover, the undesirable output is considered a by-product of the desirable output. This point is taken into account in the model with respect to the following constraint:

$$\sum_{j=1}^{n}\lambda_{j}^{p}y_{1p}\leq F^{p}\left(\sum_{j=1}^{n}\lambda_{j}^{p}y_{2p}\right),\qquad p=1,\dots,n \quad (21)$$

Note that function "F" is determined by the decision-maker. Any change in the desirable and undesirable outputs is possible till constraint (21) is satisfied. According to relations (20) and (21), we have the following relations:

$$\sum_{j=1}^{n}\lambda_{j}^{p}y_{1j}=y_{1p}-\beta_{1p}^{-}y_{1p}\rightarrow\left(y_{1p}-\beta_{1p}^{-}y_{1p}\right)\leq F^{p}\left(\sum_{j=1}^{n}\lambda_{j}^{p}y_{2p}\right),p=1,\dots,n \quad (22)$$

$$\beta_{1p}^{-}y_{1p}\geq y_{1p}-F^{p}(\sum_{j=1}^{n}\lambda_{j}^{p}y_{2p})\rightarrow\beta_{1p}^{-}\geq\frac{y_{1p}-F^{p}(\sum_{j=1}^{n}\lambda_{j}^{p}y_{2p})}{y_{1p}},y_{1p}\neq 0 \quad (23)$$

Equation (23) defines the lower bound of β_{1p}^{-} by imposing relation (21) for decreasing the undesirable output. The β_{1p}^{-} takes positive values until relation (23) is held.

For the second strategy, if $\beta_{1p}^{*-}=0$ and $\beta_{1p}^{*+}>0$, then according to constraint (20), the undesirable output can be increased. According to relations (20) and (21), we have the following relations:

$$\sum_{j=1}^{n}\lambda_{j}^{p}y_{1j}=y_{1p}+\beta_{1p}^{+}y_{1p}\rightarrow\left(y_{1p}+\beta_{1p}^{+}y_{1p}\right)\leq F^{p}\left(\sum_{j=1}^{n}\lambda_{j}^{p}y_{2p}\right),p=1,\dots,n \quad (24)$$

$$\beta_{1p}^{+} y_{1p} \le F^{p}(\sum_{j=1}^{n} \lambda_j^p y_{2p}) - y_{1p} \;\rightarrow\; \beta_{1p}^{+} \le \frac{F^{p}(\sum_{j=1}^{n} \lambda_j^p y_{2p}) - y_{1p}}{y_{1p}}, y_{1p} \neq 0 \qquad (25)$$

Equation (25) defines the upper bound of β_{1p}^{+} by imposing relation (21) for increasing the undesirable output. The β_{1p}^{+} takes positive values until relation (25) is held. □

If the strategy of increasing the undesirable output is proposed for DMU_p, then the constraint $\sum_{j=1}^{n} \lambda_j^p y_{1p} \le F^p \left(\sum_{j=1}^{n} \lambda_j^p y_{2p}\right) (p = 1, \dots, n)$ is strictly unequal. Constraint $\sum_{j=1}^{n} \lambda_j^p y_{1p} \le F^p \left(\sum_{j=1}^{n} \lambda_j^p y_{2p}\right) (p = 1, \dots, n)$ shows the relation between the production of desirable and undesirable outputs for each p. Note that F^p describes the relation between the desirable and undesirable outputs for each DMU_p. It is clear that in most of the applications, the undesirable outputs are by-products of desirable outputs. Therefore, it is possible to increase the undesirable output to increase the desirable output. Hence, for DMU_p, increase in the undesirable output using β_{1p}^{+} continues until constraints of outputs and $\sum_{j=1}^{n} \lambda_j^p y_{1p} \le F^p \left(\sum_{j=1}^{n} \lambda_j^p y_{2p}\right) (p = 1, \dots, n)$ is satisfied for each p. If the strategy of decreasing the undesirable output is proposed, then the constraint $\sum_{j=1}^{n} \lambda_j^p y_{1p} \le F^p \left(\sum_{j=1}^{n} \lambda_j^p y_{2p}\right)$ is not satisfied. Thus, the undesirable output should be decreased. It is obvious that decreasing the undesirable output decreases the desirable output as well. Therefore, for DMU_p, decrease in the undesirable output using β_{1p}^{-} continues until constraints of outputs and $\sum_{j=1}^{n} \lambda_j^p y_{1p} \le F^p \left(\sum_{j=1}^{n} \lambda_j^p y_{2p}\right)$ are satisfied. Now, consider model (26).

$$Max\ \rho^3 = \sum_{p=1}^{n} \left(\frac{1}{m}\sum_{i=1}^{m} \alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s} \beta_{rp}\right)\right) \qquad (26)$$

$s.t.$

$$\sum_{j=1}^{n} \lambda_j^p x_{ij} = x_{ip} - \alpha_{ip} x_{ip}, \qquad i = 1, \dots, m;\ p = 1, \dots, n,$$

$$\sum_{j=1}^{n} \lambda_j^p y_{1j} = y_{1p} + (\beta_{1p}^{+} - \beta_{1p}^{-}) y_{1p}, \qquad r = 1; p = 1, \dots, n,$$

$$\sum_{j=1}^{n} \lambda_j^p y_{2j} = y_{2p} + \beta_{2p} y_{2p}, \qquad r = 2; p = 1, \dots, n,$$

$$\sum_{j=1}^{n} \lambda_j^p y_{rj} = y_{rp} + \beta_{rp} y_{rp}, \qquad r = 3, \dots, s; p = 1, \dots, n,$$

$$\sum_{p=1}^{n} \sum_{j=1}^{n} \lambda_j^p y_{1j} \le A$$

$$\sum_{j=1}^{n} \lambda_j^p y_{1p} \le F^p \left(\sum_{j=1}^{n} \lambda_j^p y_{2p}\right) \qquad p = 1, \dots, n$$

$$\alpha_{ip} \ge 0, \beta_{rp} \ge 0, \qquad i = 1, \dots, m; r = 2, \dots, s,$$

$$\beta_{1p}^{-} \ge 0, \beta_{1p}^{+} \ge 0, \qquad r = 1; p = 1, \dots, n,$$

$$\lambda_j^p \ge 0, \qquad j = 1, \dots, n;\ p = 1, \dots, n$$

The objective function of model (26) maximizes the sum of non-radial improvements. To calculate the efficiency and improved inputs and outputs of DMU_p, Expressions (27) should be considered.

$$E_p = \frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}\right) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}), \quad \forall p \tag{27}$$

The improved inputs and outputs of DMUs can be obtained by $(x_{ip} - \alpha_{ip}x_{ip},\ y_{1p} + (\beta_{1p}^{+} - \beta_{1p}^{-})y_{1p}, y_{2p} + \beta_{2p}y_{2p}, y_{rp} + \beta_{rp}y_{rp})$, for all i, r, and p.

Definition 1: Having optimal solution of Model (26), the efficiency score of DMU_p is obtained by the following relation.

$$\delta_p^{*} = \frac{\left[1-\frac{1}{2}(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip}{}^{*} + \beta_{1p}^{*-}\right]}{[1+\frac{1}{3}(\beta_{1p}^{*+} + \beta_{2p}{}^{*} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}{}^{*})} \tag{28}$$

In the objective function of model (26), the sum of non-radial changes of DMUs is maximized. Thus, the sum of input slacks and output surpluses of each DMU is not maximized separately. Therefore, there might be alternative optimal solutions for each DMU. Consequently, if we consider the constraint $\sum_{p=1}^{n}(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}((\beta_{1p}^{-} - \beta_{1p}^{+}) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}) = \rho^{*3}$ in model (26), we can introduce models (29) and (30). Note that ρ^{*3} represents the efficiency score of DMU_p. By adding the constraint $\sum_{p=1}^{n}(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}((\beta_{1p}^{-} - \beta_{1p}^{+}) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}) = \rho^{*3}$ to models (29) and (30), it is guaranteed that the optimal objective function of model (26) remains unchanged. Meanwhile, models (29) and (30) seek the maximum and minimum efficiency scores of DMU_p, respectively.

$$Max\ \rho^4 = \frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}\right) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp} \qquad (29)$$

$s.t.$

$\sum_{j=1}^{n}\lambda_j^p x_{ij} = x_{ip} - \alpha_{ip}x_{ip}, \quad i = 1,\dots,m;\ p = 1,\dots,n,$

$\sum_{j=1}^{n}\lambda_j^p y_{1j} = y_{1p} + (\beta_{1p}^{+} - \beta_{1p}^{-})y_{1p}, \quad r = 1; p = 1,\dots,n,$

$\sum_{j=1}^{n}\lambda_j^p y_{2j} = y_{2p} + \beta_{2p}y_{2p}, \quad r = 2; p = 1,\dots,n,$

$\sum_{j=1}^{n}\lambda_j^p y_{rj} = y_{rp} + \beta_{rp}y_{rp}, \quad r = 3,\dots,s; p = 1,\dots,n,$

$\sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_j^p y_{1j} \le A$

$\sum_{j=1}^{n}\lambda_j^p y_{1p} \le F^p\left(\sum_{j=1}^{n}\lambda_j^p y_{2p}\right) \quad p = 1,\dots,n,$

$\sum_{p=1}^{n}\left(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}\right) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}\right) = \rho^{*3},$

$\alpha_{ip} \ge 0, \beta_{rp} \ge 0, \quad i = 1,\dots,m; r = 2,\dots,s,$

$\beta_{1p}^{-} \ge 0, \beta_{1p}^{+} \ge 0, \quad r = 1; p = 1,\dots,n,$

$\lambda_j^p \ge 0, \quad j = 1,\dots,n;\ p = 1,\dots,n.$

Definition 2: For each DMU$_p$, the minimum efficiency score is obtained from $\delta_p^{*4} = \frac{1}{1+\rho^{*4}}$, where ρ^{*4} is the optimal objective value of model (29).

Now, consider model (30), which seeks the minimum non-radial improvements when the optimal objective function value of model (26) is kept unchanged, ρ^{*3}.

$$Min\ \rho^5 = \frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}\right) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp} \qquad (30)$$

$s.t.$

$\sum_{j=1}^{n}\lambda_j^p x_{ij} = x_{ip} - \alpha_{ip}x_{ip}, \quad i = 1,\dots,m;\ p = 1,\dots,n,$

$\sum_{j=1}^{n}\lambda_j^p y_{1j} = y_{1p} + (\beta_{1p}^{+} - \beta_{1p}^{-})y_{1p}, \quad r = 1; p = 1,\dots,n,$

$\sum_{j=1}^{n}\lambda_j^p y_{2j} = y_{2p} + \beta_{2p}y_{2p}, \quad r = 2; p = 1,\dots,n,$

$\sum_{j=1}^{n}\lambda_j^p y_{rj} = y_{rp} + \beta_{rp}y_{rp}, \quad r = 3,\dots,s; p = 1,\dots,n,$

$\sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_j^p y_{1j} \le A$

$\sum_{j=1}^{n}\lambda_j^p y_{1p} \le F^p\left(\sum_{j=1}^{n}\lambda_j^p y_{2p}\right) \quad p = 1,\dots,n,$

$\sum_{p=1}^{n}\left(\frac{1}{m}\sum_{i=1}^{m}\alpha_{ip} + \frac{1}{3}\left((\beta_{1p}^{-} - \beta_{1p}^{+}\right) + \beta_{2p} + \frac{1}{s-2}\sum_{r=3}^{s}\beta_{rp}\right) = \rho^{*3},$

$\alpha_{ip} \ge 0, \beta_{rp} \ge 0, \quad i = 1,\dots,m; r = 2,\dots,s,$

$\beta_{1p}^{-} \ge 0, \beta_{1p}^{+} \ge 0, \quad r = 1; p = 1,\dots,n,$

$\lambda_j^p \ge 0, \quad j = 1,\dots,n;\ p = 1,\dots,n.$

Definition 3: For each DMU_p, the maximum efficiency score is obtained from $\delta_p^{*5} = \frac{1}{1+\rho^{*5}}$, where ρ^{*5} is the optimal objective value of model (30).

Definition 4: The efficiency interval of DMU_p is defined as $[\delta_p^{*4}, \delta_p^{*5}]$.

Given different risks (α%); the technical efficiency of DMU_p is as follows:

$$EFFi_p = \alpha\, \delta_p^{*5} + (1-\alpha)\delta_p^{*4},\ 0 \le \alpha \le 1 \qquad (31)$$

Consider the following definitions:

1) Optimistic viewpoint (100% risk):

$$EFFi_p^{Opt} = \delta_p^{*5} \qquad (32)$$

2) Pessimistic viewpoint (0% risk):

$$EFFi_p^{Pes} = \delta_p^{*4} \qquad (33)$$

3) Neither optimistic nor pessimistic:

$$EFFi_p^{Risk} = \alpha\delta_p^{*5} + (1-\alpha)\delta_p^{*4},\ 0 \le \alpha \le 1 \qquad (34)$$

4. Case study

According to the Paris Agreement, countries were committed to submit their plans for climate actions known as nationally determined contributions (NDCs)[2]. Iran committed to reduce its greenhouse gas emissions by 4%[3]. Since the chemical industry in Iran is the main source of greenhouse gas emissions, the government has determined a cap for the companies working in this industry. Here, we assess Iranian resin manufacturers. The government has determined 24420.01 tons as a cap for the resin factories. Table 2 shows the dataset of twenty-one resin factories (DMUs). The dataset dates back to 2020 and it is obtained by looking at the documents and archives of the factories. The inputs include raw material costs, transportation costs, personnel (man-hour), and advertisement costs. The outputs include production volume, CO_2 emission, and profit. Table 3 shows the references on the inputs and outputs. Note that production volume and CO_2 emission are interdependent. In other words, CO_2 emission is an undesirable by-product of production volume. CO_2 emission is considered as a by-product as it is produced while the desirable outputs are produced. In other words, the undesirable outputs are produced along with desirable outputs. Therefore, any change in the desirable output may

[2] https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement
[3] Paris 2015: Tracking country climate pledges - Carbon Brief

change the undesirable output. The total undesirable output (CO_2 emission) should be unchanged or decreased. In the meantime, the modified undesirable output should be equal to or less than the cap.

Table 2. The dataset

Resin factories (DMUs)	Inputs				Outputs		
	Raw material costs (1000000 Rial)	Transportation costs (10000 Rial)	Personnel (Man-hour)	Advertisement costs (1000000 Rial)	Production volume (Ton)	CO_2 emission (Ton)	Profit (10000 Rial)
1	347,951	201,539	37,927	1,983	3,064	957	1,4215,956
2	593,275	314,383	57,634	3,029	5,248	1351	2,497,064
3	515,470	285,524	54,178	2,781	4,679	1234	2,193,751
4	396,158	216,103	39,414	1,725	3,564	1051	1,594,140
5	438,284	253,829	42.,737	1,583	3,992	1175	1,859,256
6	372,286	226,192	36,984	2,028	3,301	1053	1,601,868
7	534,180	299,735	56,937	3,382	4,901	1369	2,493,548
8	371,495	214,582	36,072	1,337	3,385	1027	1,594,140
9	454,693	282,193	43.,018	1,838	4,143	1295	2,184,193
10	401,749	220,638	41,183	2,325	3,637	1162	1,755,948
11	503,136	291,183	51,537	2,191	4,506	1147	1,964,434
12	383,739	235,573	37,194	2,138	3,493	993	1,284,825
13	579,153	383,146	56,438	3,584	5,305	1431	2,689,474
14	492,756	301,548	44.,184	2,382	4,459	1279	2,065,784
15	682,183	342,135	59,583	3,548	6,329	1457	2,385,183
16	319,753	167,194	34,547	1,493	2,793	931	1,028,847
17	543,569	290,943	54,937	2,781	4,864	1234	2,475,810
18	385,175	254,715	38,563	2,284	3,284	1075	1,754,573
19	642,274	382,433	59,937	4,027	6,647	1561	2,654,294
20	471,529	299,718	42.,545	1,784	4,439	1391	2,533,027
21	543,181	302,718	53,483	2,840	4,945	1249	2,265,734

Table 3. Inputs and outputs

Factors	Factor type	References
Raw material	Input	Rashidi et al. (2015)
Transportation costs	Input	Tajbakhsh and Hassini (2015)
Personnel (Man-hour)	Input	Tavassoli et al. (2021)
Advertisement costs	Input	Cook and Kress (1999)
Production volume	Output	Cavalheiro Francisco et al. (2012)
CO_2 emission	Output	Rashidi et al. (2015)
Profit	Output	Tavassoli et al. (2021)

As is shown in Table 4, DMUs 1, 5, 8, 14, 15, 19, and 20 are efficient. The inefficient DMUs can improve their efficiency by changing their inputs and outputs. Efficient DMUs can be benchmarks for inefficient DMUs.

Table 4. The efficiency scores using Expression (28)

DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency
1	1	8	1	15	1
2	0.45	9	0.86	16	0.23
3	0.18	10	0.18	17	0.24
4	0.37	11	0.19	18	0.6
5	1	12	0.11	19	1
6	0.12	13	0.4	20	1
7	0.18	14	1	21	0.19

As is seen in Table 5, there is no DMU with both positive β_1^{*+} and β_1^{*-}. However, there are DMUs with both zero β_1^{*+} and β_1^{*-}. As is seen in Table 5, for each DMU, the product of β_1^{*+} and β_1^{*-} is zero.

Table 5. The optimal β_1^{*+} and β_1^{*-}

DMUs	β_1^{*+}	β_1^{*-}	DMUs	β_1^{*+}	β_1^{*-}	DMUs	β_1^{*+}	β_1^{*-}
1	0	0	8	0	0	15	0	0
2	0	0.14	9	0	0.02	16	0	0.19
3	0.5	0	10	0	0.14	17	0.07	0
4	0	0.07	11	0.18	0	18	8.96	0
5	0	0	12	0.5	0	19	0	0
6	0	0.02	13	0	0.58	20	0	0
7	0	0.01	14	0	0	21	0.1	0

Table 6 depicts the improved inputs and outputs. Compared with the original inputs, some points can be discussed. For instance, DMUs 1 and 6 do not decrease their inputs. However, DMUs 9 and 11 should decrease their inputs. DMU 11 should decrease its three inputs. By calculating the sum of reduced inputs, we get 108254.4, 72839.79, 51852.86, and 920.25. Thus, the highest decrease in inputs corresponds to the third input, the second input, the first input, and the fourth input, respectively. Also, the changes for the three outputs can be calculated. Consider the original dataset in Table 2 and improved outputs in Table 6. For instance, DMU 3 has slightly increased its CO_2 emission from 1234 to 1295.55. Also, DMU 2 has decreased its CO_2 emission from 1351 to 1151.84. The CO_2 emission is changed in different DMUs until the sum is equal to or less than the cap. Here, the cap is considered 24420.01. Note that the sum of the original CO_2 emission is 25422. It means the total sum of CO_2 emission of DMUs should be reduced at least by 0.04%.

Increasing and decreasing CO_2 emissions can affect DMUs as the total CO_2 emissions of all DMUs should not exceed the cap. In Model (26), the relationship between desirable and undesirable outputs is considered. On the other hand, the undesirable output depends on the desirable output. The constraint introduced in Model (26) ensures that the sum of the undesirable outputs of all DMUs is less than 0.7 times the total output of the desired outputs of all DMUs. The sum of the undesirable and desirable outputs for all DMUs are 25422 and 90978, respectively. Also, the sum of the initial undesirable and the sum of the desirable output for all DMUs are equal to 24420.01 and 90894, respectively. The 0.7 is determined by the decision-maker.

Table 6. Improved inputs and outputs

DMUs	Improved inputs				Improved outputs		
	Raw material costs	Transportation costs	Personnel	Advertisement costs	Production volume	CO_2 emission	Profit
1	347951	201539	37927	1983	3064	957	14215956
2	577999.1	314383	48923.36	3029	5248	1151.84	5460756
3	515470	285524	47432.39	2781	4679	1295.55	11962311
4	390582.3	216103	16999.98	1725	3564	975.73	3444066
5	438284	253829	42.737	1583	3992	1175	1859256
6	372286	217191.5	35443.79	2028	3301	1022.09	13514803
7	534180	299735	53827.14	3021.67	4901	1354.91	13421319
8	371495	214582	36072	1337	3385	1027	1594140
9	446204.8	272982.2	41.63	1655.86	4143	1265.62	2184193
10	401749	220638	40107.99	2219.66	3637	1002.44	9907760
11	500796.3	290046.1	19991.1	2191	4506	1353.31	8829165
12	383739	225592.7	36300.45	2138	3493	1041.68	12074962
13	579153	369477.9	56438	3496.6	5305	605.86	2689474
14	492756	301548	44.18	2382	4459	1279	2065784
15	682183	342135	59583	3548	6329	1457	2385183
16	306826.6	167194	20693.26	1493	2793	754.41	4176379
17	536322.1	290943	45181.27	2781	4864	1311.29	9821249
18	385175	224872	38563	2134.96	3200	1061.65	14622042
19	642274	382433	59937	4027	6647	1561	2654294
20	471529	299718	42.454	1784	4439	1391	2533027
21	543181	302718	44874.24	2840	4945	1376.63	11364005

Considering a different production function between a desirable and an undesirable output can lead to different results. The production function should be introduced by the decision-maker. Here, the production function for DMU1 is set as $\sum_{j=1}^{n} \lambda_j^p y_{1j} \leq (0.75) \sum_{j=1}^{n} \lambda_j^p y_{2j}$. Assigning different values instead of 0.75 can lead to different solutions in model (26). After running Model (1), the results using Expression (28) are shown in Table 7.

Table 7. The efficiency scores using model (1)

DMUs	Efficiency scores	DMUs	Efficiency scores	DMUs	Efficiency scores
1	1	8	1	15	1
2	0.214	9	0.769	16	1
3	0.177	10	1	17	0.236
4	0.558	11	0.188	18	0.057
5	1	12	0.105	19	1
6	0.148	13	0.144	20	1
7	0.189	14	0.609	21	0.189

Now, consider the improved inputs and outputs obtained from Expression (27). By running model)1), the efficiency scores of improved DMUs obtained from Expression (27) are shown in Table 8.

Table 8. The efficiency score of improved DMUs

DMUs	Efficiency scores	DMUs	Efficiency scores	DMUs	Efficiency scores
1	1	8	1	15	1
2	0.451	9	1	16	1
3	1	10	1	17	1
4	1	11	1	18	1
5	1	12	1	19	1
6	1	13	0.121	20	1
7	1	14	0.602	21	1

Compared with the original efficiency scores, as is seen in Table 8, the efficiency scores of improved DMUs are increased. This implies that the strategy derived from model (26) and relation (28) improves the efficiency of DMUs. Now, according to models (29) and (30), the efficiency interval for each DMU_p $(p = 1, \dots, n)$ can be obtained. The result is reported in Table 9. Considering Table 9, the upper bounds of efficiency related to DMUs 2 and 14 are equal to 1. Thus, they are efficient in optimistic viewpoint. However, their lower bounds of efficiency are less than 1. Thus, in pessimistic viewpoint they are inefficient. Both optimistic and pessimistic viewpoints have their own merits and considering both of them in the analysis can lessen bias in the efficiency analysis.

Table 9. The efficiency intervals

DMUs	Lower bound	Upper bound	DMUs	Lower bound	Upper bound	DMUs	Lower bound	Upper bound
1	1	1	8	1	1	15	1	1
2	0.91	1	9	1	1	16	0.38	0.38
3	0.16	0.16	10	0.23	0.23	17	0.21	0.21
4	0.48	0.49	11	0.14	0.14	18	0.01	0.01
5	1	1	12	0.1	0.1	19	1	1
6	0.12	0.12	13	0.43	0.6	20	1	1
7	0.18	0.19	14	0.99	1	21	0.16	0.16

Note that the efficiency scores of DMUs, shown in Table 4, do not necessarily lie in the interval of Table 9 as the upper and lower bounds are obtained for each DMU. Model (27) maximizes the sum of non-radial improvements of DMUs. However, models (29) and (30), respectively, maximize and minimize the efficiency of DMUs provided that the objective function of model (26) remains unchanged. Assuming 30% risk, according to the efficiency intervals and using Expression (34), the efficiency scores are given in Table 10. As mentioned in Table 10, considering 30% risk, the efficiency scores of DMUs 2 and 14 are less than 1. This shows a bias in optimistic analysis that should be considered carefully by managers.

Table 10. The efficiency scores with 30% risk

DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency
1	1	8	1	15	1
2	0.937	9	1	16	0.38
3	0.16	10	0.23	17	0.21
4	0.483	11	0.14	18	0.01
5	1	12	0.1	19	1
6	0.12	13	0.43	20	1
7	0.183	14	0.99	21	0.16

Now, consider the proposed model of Momeni et al. (2019). Their proposed model is based on radial reductions and the radial efficiency of DMUs is calculated. By comparing Tables 4 and 11, it is seen that the efficiency scores obtained from our model are less than the efficiency scores obtained from the proposed model of Momeni et al. (2019). One of the main reasons is that our model takes into account all possible inefficiencies of inputs and outputs with slack variables as well as the production dependency of desirable and undesirable outpus. Moreover, in our proposed model, the relationship between the undesirable outputs of DMUs is considered.

Table 11. The results obtained from Momeni et al. (2019)

DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency
1	1	8	1	15	1
2	0.94	9	0.98	16	0.95
3	0.94	10	0.92	17	0.95
4	0.98	11	0.95	18	0.84
5	1	12	0.9	19	1
6	0.88	13	0.89	20	1
7	0.93	14	0.99	21	0.94

As is seen in Table 11, the efficiency scores are very close to each other and DMUs' ranking is tough. Moreover, it is not possible to consider different risks as done in Table 10.

4.2 Managerial insights

The proposed model can reallocate permitted CO_2 emissions among companies to increase their efficiencies concerning the market conditions and existing capacities. Using the cap-and-trade mechanism is a challenge for organizations as it faces with cap. However, it provides some advantages by applying market motivations. The results demonstrate that the proposed model can help managers to decide in a cap-and-trade context. As is shown in Fig. 1, the CCR radial efficiency scores are usually more than the efficiency scores obtained from model (27). The efficiency score with 30% risk, using Expression (34), is also shown in Fig. 1. The main reasons for the significant difference between the proposed model of Momeni et al. (2019) and model (27) are the non-radial improvements and dependency of desirable and undesirable outputs. In this way. The current model proposed in this study, considers all the possible inefficiencies with non-radial slacks into analysis. Therefore, model (27) considers all possible inputs and outputs' improvements, as well as the production dependency between the production of desirable and undesirable outputs.

As mentioned in Table 9, DMUs 1, 5, 8, 9, 15, 19, and 21 have the lower and upper bounds of unity. This shows that they are efficient in both pessimistic and optimistic viewpoints. Thus, they can be considered as benchmarks for inefficient DMUs. According to Table 9, DMUs 2 and 14 have the upper bounds of efficiency equal to 1 and the lower bounds less than 1. This shows that these DMUs are efficient in optimistic viewpoint but inefficient in pessimistic viewpoint. In Table 10, considering 30% risk, DMUs 1, 5, 8, 9, 15, 19, and 21 are efficient. However, DMUs 2 and 14 are inefficient. This clearly shows benchmarks to the managers of inefficient DMUs.

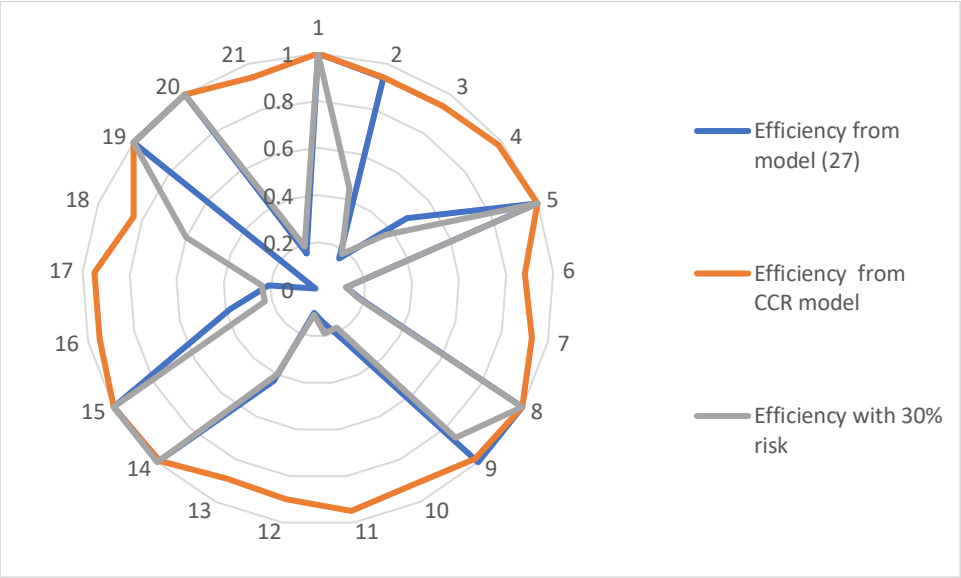

Fig. 1. The efficiency scores

5. Conclusions and future researches

Climate change has been the main concern for many governments around the globe. In this regard, greenhouse gases emissions have been identified as the main source of climate change. As such, the international community has tried to decrease greenhouse gases emissions. Market-based methods have been used for decreasing greenhouse gas emissions and improving the sustainability of organizations (Wang et al. 2018). Among market-based methods, cap-and-trade has been recognized as the most efficient solution to achieve sustainability goals (Momeni et al. 2019).

This paper proposed a new DEA model for reallocating CO_2 emissions among DMUs. We considered the dependency of production between the desirable and undesirable outputs in the cap-and-trade context. Therefore, it can be said that reducing the desired output to any extent is not the right way to deal with it. Because this work will directly affect the production of desired output, therefore, reducing the production of undesirable output should be done up to the calculated extent. Moreover, in the proposed model, the sum of efficiency scores of all DMUs was maximized. The efficiency intervals of DMUs were calculated given the alternative optimal solutions in the cap-and-trade models. A case study showed how well the proposed model can deal with the cap on greenhouse gas emissions. Presenting the optimistic and pessimistic approaches in the newly presented models can reveal more insights about the performance of DMUs. The DMUs that are efficient in both optimistic and pessimistic viewpoints, can be considered as benchmarks for inefficient DMUs.

The proposed model can be developed further in some directions. In this paper, we dealt with crisp data. However, there might be fuzzy data in reallocating greenhouse gas emissions in a cap-and-

trade context. To address such data, the proposed model can be redeveloped using fuzzy data. Furthermore, a new model can be developed in the presence of stochastic data.

Compliance with Ethical Standards

Conflict of Interest: Authors declare that they have no conflict of interest.
Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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