Coordination of public-private transport and sustainability measurement: A futuristic perspective in transport

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Abstract

Coordination in providing sustainable and effective transport services can be beneficial for both the public and private sectors. Although there is considerable research in each of public and private transport, the literature shows that research on public-private coordination is rare. Furthermore, performance measurement of public-private transport has been another less explored topic in transport systems. Based on these knowledge gaps, we make contributions to the transportation field by performance measurement and coordination of public-private transport within multimodal transport networks. In this paper, we model public-private transport networks by developing a novel network data envelopment analysis (NDEA) with some distinctive features. The model is built based on the directional distance function (DDF) and provides detailed insights into the sustainability and

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coordination of public-private transport networks operating in a supply chain. It also provides resource allocation among decision making units (DMUs).

Keywords: Data envelopment analysis (DEA); Public-private transport; Coordination; Performance measurement; Resource allocation.

1. Introduction

As transport plays a key role in strategic issues such as the economy, climate change, and healthcare, it has been considered a significant research direction in many areas. Transporting passengers in many megacities such as Tokyo, Hong Kong, Seoul, and London is typically based on public transport vehicles, including the metro and bus. These modes of transport are usually run either by the public sector or the private sector. Owing to the recent advances in digital technologies and analytical approaches, the coordination of public and private transport now is receiving considerable attention from decision-makers in transportation systems. This pushes public and private transport to improve performance by changing transport planning. This, in turn, requires novel methods for coordinating demands and available resources in public and private transport, simultaneously.

Sustainable transport has a considerable impact on societies and addresses ecological, social, and economic dimensions of transport. Based on formal statistics, roughly 25 percent of the produced CO2 in the world is stemmed from the transport sector (Zhao et al., 2020). A large proportion of green gas emissions in transport is due to consuming fossil fuels, which results in climate change (Boussauw and Vanoutrive, 2017). In addition, with the increasing population particularly in developing countries such as India and Indonesia, issues associated with transport are expected to be even much more complicated (Zope et al., 2019). As such, sustainable transport needs more attention in the transport sector by considering ecological, social, and economic dimensions. In this regard, sustainable transport aims to minimize ecological problems and maximize social and economic benefits for society (Stephenson et al., 2018). To do so, the focus of policymakers and managers of transport has been on adopting appropriate government policies, applying technology-based infrastructures, and collaborating with the public and private sectors (Soto et al., 2021).

In sustainable transport, the public and private sectors attempt to cooperate for achieving sustainability goals such as reduced costs, less pollution, and more satisfaction. To do so, optimal resource allocation and the right targets play significant roles. In this sense, allocation of the limited resources to the unlimited demands based on sustainability criteria in transport should be in a way that the most desirable outcome is obtained. Optimizing sustainable resource allocation in transport not only can prevent wasting resources but also increases the utility of passengers. Targets create directions for helping organizations to achieve objectives in line with their mission (Azadi et al.,

2014). By setting clear-cut and achievable targets, organizations can better conduct their settings and get more insights into their activities. Furthermore, setting such targets can assist organizations in improving performance.

To achieve sustainable urban mobility objectives, public transport service providers develop performance measurement frameworks and systems based on passengers' expectations (Georgiadis et al., 2020). Nonetheless, public transport performance depends on different factors such as demographic characteristics, transport system features, services planning, resource allocation, and network design (de Grange et al., 2013). In multimodal public transport networks, the incorporation of information flow, inter alia, and network planning play key roles (Roukouni et al., 2012; Rybnicek et al., 2020). From a performance perspective, a lack of appropriate coordination among public transport sub-systems can affect the overall sustainability of the system significantly.

Having said that, the question is how we can develop a new network data envelopment analysis (NDEA) model to increase the coordination and performance of public-private transport systems. Another research question is how we can allocate a budget for public transportation systems. Moreover, how we can model different types of data in network structures using data envelopment analysis (DEA) technique. The main objective of this paper is to investigate the coordination and performance of public-private transport systems. These systems deal with multimodal public transport networks consisting of road and rail sub-systems. We model public-private transport systems in a megacity using a novel NDEA for measuring the performance and investigating coordination within these systems. Our developed model also is applied to sustainable resource allocation in public-private transport systems. The proposed model in this study is built using the directional distance function (DDF) in the NDEA structure. Moreover, since there are different types of data, including integer data, negative data, and undesirable outputs in our case study, we address these types of data to get more reliable results and provide more realistic insights to managers and decision-makers of transport systems. The main contributions of this study are as follows:

- A DDF is developed for measuring the overall and stage sustainability.
- The proposed NDEA model is formulated in the presence of integer data.
- The proposed NDEA model can deal with undesirable outputs.
- The proposed NDEA model can handle negative data.
- A resource allocation approach using the NDEA model is developed.

The rest of this paper is organized as follows: The related works are discussed in Section 2. The proposed method is given in Section 3. In Section 4, we provide a case study. Conclusions and research directions are presented in Section 5.

2. Literature review

The term "collaboration" is used for establishing a close, deep, and long-run relationship among organizations for solving joint problems (Wankmüller and Reiner, 2020). The collaboration between the public sector and private sector provides organizations with improved efficiency and reduced risk (Iossa and Martimort, 2015). The concept of public-private collaboration was initially proposed in the transport sector (Delmon, 2011; Grimsey and Lewis, 2004). Public-private collaboration now is observed in many strategic areas such as healthcare and education (Spoann et al., 2019). Several factors such as service quality, reduced time, and costs are applied to assess public-private transport collaborations (Rybnicek et al., 2020).

Sustainability is of substantial importance in transport as it provides society with numerous benefits in all economic, ecological, and social dimensions (Wanke et al., 2020). Sustainable transport as a sustainable and effective strategy addresses today's key challenges, including air pollution, climate change, and road accidents that adversely affect human life and health even for future generations (Stefaniec et al., 2021). In sustainable transport, the focus of the economic dimension is on the cost reduction of carrying goods and individuals. The environmental dimension mostly concentrates on factors such as fuel consumption and CO2 emissions contributing to noise and air pollution and climate change. The social dimension also deals with individual quality of life and wellbeing in society (Shi et al., 2019). A sustainable transport system needs to meet several criteria (Gilbert et al., 2003). Firstly, it should be able to minimize energy consumption, noise pollution, carbon gas emissions, and the use of land. Secondly, individuals' needs for transportation to be met safely and efficiently. Thirdly, it should be able to provide individuals with different modes of transportation, be affordable, and support a dynamic economy. Performance evaluation and analysis of transportation systems considering sustainability criteria have received substantial attention over the last decade. Zito and Salvo (2011) presented some sustainable transport performance indicators for assessing the impacts of policy measures on urban transport. They integrated the indicators through equal weighting along with the Euclidean distance between cities based on the normalized transport sustainability index. Zhao et al. (2011) presented an assessment method named the downtown space reservation system for travel demand management strategy. To do so, they considered radial and slacks-based NDEA models in network structures. Haghshenas and Vaziri (2012) evaluated several cities considering the urban sustainable transport composite index. To do so, they selected sustainable transport metrics that encompass economic, environmental, and social dimensions. To allocate resources to rail and automotive systems, Vasco Correa (2012) proposed a DEA model. The results show that resource allocation in rail transport could be better compared with the road transport. Shiau and Liu (2013) presented an indicator system to assess and monitor

sustainable urban transport. First, they categorized several indicators into economic, environmental, and social dimensions. Then, they used fuzzy cognitive maps and the analytic hierarchy process for constructing the cause-effect relationships between main indicators and for assessing sustainable urban transport strategies. Azadi et al. (2014) evaluated public transport service providers and set some attainable and realistic targets for the service providers based on economic and ecological indicators using a two-stage NDEA. Mahdinia et al. (2018) evaluated transport sustainability in the US using an indicator-based algorithm. The algorithm combines composite indices in all sustainable transport aspects for performance evaluation. Mahmoudi et al. (2019) identified the assessment indicators to assess the sustainability of urban transportation networks and then proposed a Best-Worst approach for evaluating and prioritizing sustainability aspects and assessment indicators. They also identified potential strategies and appropriate policies for achieving sustainability goals in urban transportation. Ganji et al. (2023) developed a double-frontier cross-efficiency technique to evaluate Iranian airlines' performance. They incorporated prospect theory into the cross-efficiency technique. Ganji and Rassafi (2019) proposed a double-frontier Malmquist productivity index to assess Iranian road safety performance. Ganji et al. (2020) have also evaluated Iranian road safety using a novel cross-efficiency technique based on evidential reasoning approach and the ordered weighted averaging operator. Gupta et al. (2021) applied a DEA model to compute the performance of vehicles on a variety of routes of the transport networks. They also proposed a fuzzy model for getting preferred compromise transportation solutions aimed at sustainable development.

Resource allocation as a tool of strategic planning to achieve organizational objectives is a way of allocating limited resources among different units of an organization for producing goods and services (Yu and Chen, 2016). There is a rich literature on resource allocation in the transport sector with a focus on implementing policies and providing sustainable services. Melachrinoudis and Kozanidis (2002) proposed a method for allocating resources to improve highway safety. To do so, they used an integer model with linear multiple-choice constraints. Mishra and Khasnabis (2012) proposed an approach for resource allocation to implement safety improvement alternatives for urban transport over different time horizons. The approach maximizes the profit and decreases cost considering crashes of different severity categories, budget limits, and other constraints. Given several congested resources, Churchill and Lovell (2012) proposed a two-stage stochastic integer model to allocate resources for potential capacity outcomes in air transport. The model presented by Churchill and Lovell (2012) coordinates the air traffic system by addressing uncertainty. Yu and Chen (2016) presented an NDEA model for resource allocation in container shipping companies. They also considered shared inputs, desirable, and undesirable outputs in their proposed model for reallocating resources. Wang et al. (2017) examined the problem of dynamic resource allocation in intermodal freight transport considering network effects. To do so, they applied the Markov decision process model and developed some mathematical models such as the approximation model and booking control model. Wu et al. (2018) developed a target setting and resource allocation method to enhance the ecological efficiency of decision making units (DMUs) in regional highway transportation systems. They incorporated the common weight technique in their proposed model to reallocate discretionary inputs and set output targets for DMUs. Yan et al. (2019) used price signals for modifying resource allocation and developed a co-optimization model of resource allocation and fare rates of high-speed trains in rail transport. Zhao et al. (2022) developed an NDEA model to allocate CO2 emissions in transport systems. To do so, they combined two approaches with NDEA.

DEA as a nonparametric and applied technique is used for performance measurement and resource allocation of a set of peer DMUs (Amirteimoori et al., 2020; Charnes et al., 1978). The classic DEA models measure the performance of DMUs without considering the data type. One of these types of data is integer-valued data. There are some data in the performance evaluation of DMUs that are only integer-valued such as the number of staff in a hospital or the number of machines in a factory. Lozano and Villa (2006) embedded the integer data in DEA by presenting multi-integer linear programming. Their model was improved by Kazemi Matin and Kuosmanen (2009). Over the last decade, some scholars addressed integer-valued data in the DEA context, including Kazemi Matin and Emrouznejad (2011), Azadi and Farzipoor Saen (2014), and Fathi Ajirlo et al. (2019). Moreover, the traditional DEA models consider only non-negative data in performance evaluation and ignore negative data. By considering negative inputs as outputs and negative outputs as inputs, Scheel (2001) tackled the issue of negative data in DEA. A DDF model was developed for addressing this type of data by Portela et al. (2004). Kazemi Matin and Azizi (2011) and Tavana et al. (2018; 2021) also proposed models to deal with negative data in DEA. Undesirable output is considered another type of data that affects the obtained results by using DEA. CO₂ emissions produced by factories or cars are an example of undesirable output in real-world problems. Färe et al. (1989) were the first scholars to address undesirable outputs in DEA. Seiford and Zhu (2002) proposed a DEA model to reduce undesirable outputs. The issue of undesirable outputs has also been addressed over the last decade by several scholars such as Kalantary and Farzipoor Saen (2019); Mirhedayatian et al. (2014); Rashidi et al. (2015); Samavati et al. (2020), and Kazemi Matin et al. (2022).

The literature shows that performance measurement and coordination of public-private transport that operate in a supply chain have less been explored to date. Furthermore, sustainable resource allocation among DMUs in such a supply chain is another significant issue in the literature that needs to be explored by scholars. Moreover, the type of data, including integer data, undesirable data, and ratio data have not been considered in NDEA models by DDFs. The study in question makes several noteworthy contributions that advance the field of sustainability measurement and analysis. One of the main achievements of the study is the development of a DDF, which is a powerful tool for

assessing sustainability at both the overall and stage levels. This new framework is a significant improvement over existing methods, as it offers a more comprehensive and nuanced approach to evaluating sustainability across different stages of production or development. Another key finding of the study is the formulation of an NDEA model that can handle integer and negative data, as well as undesirable outputs. This is a critical development because many existing models are limited in their ability to account for these types of data, which can be common in sustainability research. By overcoming this limitation, the proposed NDEA model represents a significant step forward in the field of sustainability analysis. Furthermore, the study also introduces a resource allocation approach that utilizes the NDEA model. This approach has important implications for real-world sustainability applications, as it provides a practical tool for decision-makers to optimize resource allocation and management in a sustainable manner. Overall, the contributions of this study have the potential to significantly advance our understanding of sustainability and help us make more informed decisions about how to promote sustainable development and resource management. In the next section, we address these issues by modeling the public-private transport networks of a megacity.

3. Proposed models

The efficiency and sustainability are two key factors in designing and operating public-private transport systems. Efficiency refers to the ability of a system to perform a task with the least amount of resources. Sustainability, on the other hand, refers to the ability of a system to meet the needs of the present without compromising the ability of future generations to meet their own needs. In the context of public-private transport systems, efficiency and sustainability are interdependent. A transport system that is efficient in terms of resource utilization, travel time, and cost can contribute to sustainability by reducing energy consumption, emissions, and traffic congestion. For example, a well-designed public transit system that uses clean energy sources and provides seamless connectivity can be more efficient and sustainable than individual car use. Moreover, a sustainable transport system can contribute to efficiency by reducing the negative externalities associated with individual transport modes. These negative externalities include air and noise pollution, traffic congestion, and accidents. In contrast, a transport system that is unsustainable may be more expensive to operate, require more resources, and have negative impacts on the environment and social equity. Therefore, a balanced approach that considers both efficiency and sustainability is crucial for designing and operating public-private transport systems. This requires the integration of various modes of transportation, the use of advanced technologies, and the adoption of policies that incentivize sustainable transport behavior (Daimi and Rebai, 2022).

To measure efficiency (sustainability) and coordinate the public-private transport network, we model the multimodal transport network consisting of bus service providers and metro stations based on DDF in the NDEA context. Table 1 depicts the used notations in this paper.

Table 1. The notations

| Notations | Descriptions |
|-------------------------|---|
| x_{ij} | ith input of the DMU _j |
| $x_{ij}^{(k)}$ | <i>i</i> th external input of the DMU_j in the <i>k</i> th division |
| \mathcal{Y}_{rj} | rth output of the DMU _j |
| $y_{rj}^{(k)}$ | r th external output of the DMU $_j$ in the k th division |
| z_{dj} | dth intermediate product of the DMU _j |
| λ_j | Intensifier variable of the DMU _j |
| $\lambda_j^{(k)}$ | Intensifier variable of the DMU_j in the k th division |
| f_{io} | i th direction of the x_{io} |
| g_{ro} | r th direction of the y_{ro} |
| $oldsymbol{g}_{ro}^{U}$ | r th direction of the y_{ro} (undesirable output) |
| g_{ro}^D | r th direction of the y_{ro} (desirable output) |
| $f_{io}^{(k)}$ | <i>i</i> th direction of the x_{io} in the k th division |
| $g_{ro}^{(k)}$ | r th direction of the y_{ro} in the k th division |

For n DMUs with m inputs and s outputs, consider x_{ij} (i=1,...,m) and y_{rj} (r=1,...,s) represent the inputs and outputs (j=1,...,n), respectively. The input distance function fixes the outputs at the current level to find the smallest ratio of inputs that can generate the current level of outputs. However, the output distance function fixes the inputs at the current level to identify the largest expansion of outputs that can be generated at the current level of inputs. These approaches can only use two points as touchstones to measure the efficiency of the production function: one in the input direction and the other one in the output direction. A setback that these approaches may face is that the efficiency of DMUs outside the cone generated by the efficient frontier will be overestimated. There is no need to fix inputs or outputs for efficiency measurement. It is possible to select every dominant point on the production function as a touchstone for efficiency measurement

by defining a DDF. To measure the sustainability of DMUs, Chambers et al. (1996, 1998) defined DDF on the production possibility set (PPS) T as $\eta(\mathbf{x}, \mathbf{y}, \mathbf{f}, \mathbf{g}) = \max\{\eta \mid (\mathbf{x} - \eta \mathbf{f}, \mathbf{y} + \eta \mathbf{g}) \in T\}$, where \mathbf{f} and \mathbf{g} are predefined directions. The DDF that measures DMU_o (the DMU under evaluation) in the direction of $(-\mathbf{f}, \mathbf{g})$ under variable returns to scale in the presence of negative data is as below:

$$\max \eta_{o}$$
s.t.
$$\sum_{j=1}^{n} \lambda_{j} x_{ij} \leq x_{io} - \eta_{o} f_{i}, \quad i = 1, ..., m,$$

$$\sum_{j=1}^{n} \lambda_{j} y_{rj} \geq y_{ro} + \eta_{o} g_{r}, \quad r = 1, ..., s,$$

$$\sum_{j=1}^{n} \lambda_{j} = 1,$$

$$\lambda_{j} \geq 0, \quad j = 1, ..., n,$$

$$\eta_{o} \text{ unrestricted in sign.}$$
(1)

where $\lambda = (\lambda_1, ..., \lambda_n)$ is a non-negative vector in \mathbb{R}^n . Model (1) measures the distance of DMU_o from the efficient frontier in the direction of $(-\mathbf{f}, \mathbf{g})$. In the optimal scenario, an objective value $\eta_o^* = 0$ implies a sustainable DMU, whereas $\eta_o^* > 0$ implies an unsustainable DMU. Moreover, $(\sum_{j=1}^n \lambda_j^* x_{ij}, \sum_{j=1}^n \lambda_j^* y_{rj})$ denotes the DMU_o , which is a benchmark for unsustainable DMUs. Model (1) can identify sustainable and unsustainable DMUs; however, it cannot measure the extent of unsustainability as the upper bound of η_o is unknown. Portela et al. (2004) proposed a range directional measure in which the largest step that can be taken by a DMU is used in a way that η_o becomes equal to or smaller than one. Here, $1-\eta_o$ is defined as sustainability. In particular, a direction is defined as:

$$f_{io} = x_{io} - \min_{j=1,...,n} \{x_{ij}\}, \quad i = 1,...,m,$$

$$g_{ro} = \max_{j=1,...,n} \{y_{rj}\} - y_{ro}, \quad r = 1,...,s.$$
(2)

Thus, sustainability $1-\eta_o$ varies from 0 to 1. To measure sustainability, outputs are assumed to be disposable; i.e., they can be readily discarded. Although this assumption is reasonable for desirable outputs, the disposal of undesirable outputs may be costly. Färe et al. (1989) assumed that outputs are weakly disposable and developed a model to measure the efficiency in the presence of undesirable outputs. Assuming weak disposability of outputs, for b (undesirable outputs) and s-b (desirable outputs), the constraints are as follows:

$$\sum_{j=1}^{n} \lambda_{j} y_{rj} = y_{ro} - \eta_{o} g_{ro}^{U}, \quad r = 1, ..., b,$$

$$\sum_{j=1}^{n} \lambda_{j} y_{rj} \ge y_{ro} + \eta_{o} g_{ro}^{D}, \quad r = b + 1, ..., s.$$
(3)

where $\mathbf{g} = (g_r^U, g_r^D)$ is a direction vector. Assuming weak disposability of outputs, the sustainability of DMU_o can be obtained by incorporating constraints (3) into Model (1), which is as follows:

$$\max_{s.t.} \eta_{o}$$
s.t.
$$\sum_{j=1}^{n} \lambda_{j} x_{ij} \leq x_{io} - \eta_{o} f_{io}, \quad i = 1, ..., m,$$

$$\sum_{j=1}^{n} \lambda_{j} y_{rj} = y_{ro} - \eta_{o} g_{ro}^{U}, \quad r = 1, ..., b,$$

$$\sum_{j=1}^{n} \lambda_{j} y_{rj} \geq y_{ro} + \eta_{o} g_{ro}^{D}, \quad r = b + 1, ..., s,$$

$$\sum_{j=1}^{n} \lambda_{j} = 1,$$

$$\lambda_{j} \geq 0, \quad j = 1, ..., n,$$

$$\eta_{o} \text{ unrestricted in sign.}$$
(4)

Conventional DEA models assume that the inputs and outputs can only take real values. In many real-world applications, however, some inputs and/or outputs are only integer-valued. Assume that the inputs can be divided into $I = I^I \cup I^{NI} = \{1, ..., m\}$, and outputs can be divided into $O = O^I \cup O^{NI} = \{b+1, ..., s\}$, where I^I and O^I are the integer values and I^{NI} and O^{NI} are real values, respectively. Assuming weak disposable outputs and the presence of integer inputs and outputs, the sustainability of DMU_o is measured as below:

 $\begin{aligned} & \max \eta_o \\ & \text{s.t.} \\ & \sum_{j=1}^n \lambda_j x_{ij} \leq \widetilde{x}_{io}, \quad i \in I^I, \\ & \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io} - \eta_o f_{io}, \quad i \in I^{NI}, \\ & \sum_{j=1}^n \lambda_j y_{rj} = y_{ro} - \eta_o g_{ro}^U, \quad r = 1, \dots, b, \\ & \sum_{j=1}^n \lambda_j y_{rj} \geq \widetilde{y}_{ro}, \quad r \in O^I, \\ & \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro} + \eta_o g_{ro}^D, \quad r \in O^{NI}, \\ & x_{io} - \eta_o f_{io} = \widetilde{x}_{io}, \quad i \in I^I, \\ & y_{ro} + \eta_o g_{ro}^D = \widetilde{y}_{ro}, \quad r \in O^I, \\ & \widetilde{x}_{io}, \widetilde{y}_{ro} \in \mathbb{Z}_+, \quad i \in I^I, r \in O^I, \\ & \sum_{j=1}^n \lambda_j = 1, \\ & \lambda_j \geq 0, \quad j = 1, \dots, n, \\ & \eta_o \text{ unrestricted in sign.} \end{aligned}$

Figure 1 depicts a general two-division network. The first division uses $m^{(1)}$ external inputs $x_{ij}^{(1)}$ ($i=1,\ldots,m^{(1)}$) to produce $s^{(1)}$ external outputs $y_{rj}^{(1)}$ ($r=1,\ldots,s^{(1)}$), which are sent out of the network. Also, division 1 produces h intermediate products z_{dj} ($d=1,\ldots,h$), which are used by the second division. The second division uses $m-m^{(1)}$ external inputs $x_{ij}^{(2)}$ ($i=m^{(1)}+1,\ldots,m$) and h intermediate products z_{dj} ($d=1,\ldots,h$) to produce $s-s^{(1)}$ outputs $y_{rj}^{(2)}$ ($r=s^{(1)}+1,\ldots,s$).

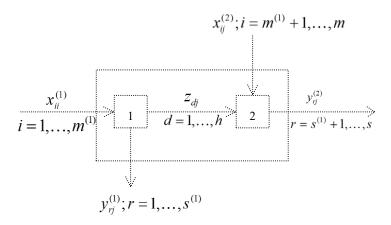


Figure 1. Structure of general two-division network

A variety of PPS has been defined in DEA literature for network systems. Färe and Grosskopf (2000) defined the PPS of a network system as an aggregate of PPSs of divisions. The PPS of the system shown in Figure 1 is defined as follows:

$$T^{\text{Network}} = \begin{cases} \left(\mathbf{x}, \mathbf{y}, \mathbf{z}\right)^{n} \lambda_{j}^{(1)} x_{ij}^{(1)} \leq x_{i}^{(1)}, i = 1, \dots, m^{(1)}, \sum_{j=1}^{n} \lambda_{j}^{(1)} y_{rj}^{(1)} \geq y_{r}^{(1)}, r = 1, \dots, s^{(1)}, \\ \sum_{j=1}^{n} \lambda_{j}^{(1)} z_{dj} \geq z_{d}, d = 1, \dots, h, \sum_{j=1}^{n} \lambda_{j}^{(2)} x_{ij}^{(2)} \leq x_{i}^{(2)}, i = m^{(1)} + 1, \dots, m, \\ \sum_{j=1}^{n} \lambda_{j}^{(2)} z_{dj} \leq z_{d}, d = 1, \dots, h, \sum_{j=1}^{n} \lambda_{j}^{(2)} y_{rj}^{(2)} \geq y_{r}^{(2)}, r = s^{(1)} + 1, \dots, s, \\ \sum_{j=1}^{n} \lambda_{j}^{(k)} = 1, k = 1, 2, \lambda_{j}^{(k)} \geq 0, k = 1, 2; j = 1, \dots, n \end{cases}$$

$$(6)$$

The intermediate products z_d (d=1,...,h) are produced and consumed within the system. For example, an intermediate product produced by Division 1 is expected to have maximum value so that Division 1 can be more sustainable. At the same time, it is expected to be as low as possible so that Division 2 can be more sustainable. Therefore, the supply and demand sides of intermediate products have contradictory objectives. Chen et al. (2016) used constraints $\sum_{j=1}^{n} \lambda_{j}^{(1)} z_{dj} = \hat{z}_{d}$ (d=1,...,h) and $\sum_{j=1}^{n} \lambda_{j}^{(2)} z_{dj} = \hat{z}_{d}$ (d=1,...,h) to find the shared objective \hat{z}_{d} for both divisions such that they would not be contradictory, where \hat{z}_{d} (d=1,...,h) is a decision variable. This study uses the approach proposed by Chen et al. (2016). Thus, assuming variable returns to scale, to measure the sustainability of DMU_o, an NDEA model is proposed as follows:

$$\max_{s.t.} \quad \eta_o^s
s.t. \quad \sum_{j=1}^n \lambda_j^{(1)} x_{ij}^{(1)} \le x_{io}^{(1)} - \eta_o^s f_{io}^{(1)}, \quad i = 1, ..., m^{(1)},
\qquad \sum_{j=1}^n \lambda_j^{(2)} x_{ij}^{(2)} \le x_{io}^{(2)} - \eta_o^s f_{io}^{(2)}, \quad i = m^{(1)} + 1, ..., m,
\qquad \sum_{j=1}^n \lambda_j^{(1)} z_{dj} = \hat{z}_{do}, \quad d = 1, ..., h,
\qquad \sum_{j=1}^n \lambda_j^{(2)} z_{dj} = \hat{z}_{do}, \quad d = 1, ..., h,
\qquad \sum_{j=1}^n \lambda_j^{(1)} y_{rj}^{(1)} \ge y_{ro}^{(1)} + \eta_o^s g_{ro}^{(1)}, \quad r = 1, ..., s^{(1)},
\qquad \sum_{j=1}^n \lambda_j^{(2)} y_{rj}^{(2)} \ge y_{ro}^{(2)} + \eta_o^s g_{ro}^{(2)}, \quad r = s^{(1)} + 1, ..., s,
\qquad \sum_{j=1}^n \lambda_j^{(k)} = 1, \quad k = 1, 2,
\qquad \lambda_j^{(k)} \ge 0, \quad k = 1, 2; \quad j = 1, ..., n.$$

$$(7)$$

For Model (7), the direction is defined as below:

$$\begin{cases}
f_{io}^{(1)} = x_{io}^{(1)} - \min_{j=1,\dots,n} \{x_{ij}^{(1)}\}, & i = 1,\dots,m^{(1)}, \\
f_{io}^{(2)} = x_{io}^{(2)} - \min_{j=1,\dots,n} \{x_{ij}^{(2)}\}, & i = m^{(1)} + 1,\dots,m, \\
g_{ro}^{(1)} = \max_{j=1,\dots,n} \{y_{rj}^{(1)}\} - y_{ro}^{(1)}, & r = 1,\dots,s^{(1)}, \\
g_{ro}^{(2)} = \max_{j=1,\dots,n} \{y_{rj}^{(2)}\} - y_{ro}^{(2)}, & r = s^{(1)} + 1,\dots,s.
\end{cases} \tag{8}$$

Then, sustainability $1-\eta_o^S$ varies from 0 to 1.

Theorem 1: Model (7) is always feasible.

Proof: Indeed,

$$\lambda_o^{(k)} = 1, \quad \lambda_j^{(k)} = 0, \quad j = 1, ..., n; j \neq o; k = 1, 2$$

$$\hat{z}_{do} = z_{do}, \quad d = 1, ..., h,$$

$$\eta_o^S = 0,$$

is a feasible solution to Model (7).

The overall network sustainability can be obtained using Model (7). The sustainability of Division 1 is obtained as below:

$$\max \quad \eta_o^{(1)}$$
s.t.
$$\sum_{j=1}^n \lambda_j^{(1)} x_{ij}^{(1)} \le x_{io}^{(1)} - \eta_o^{(1)} f_{io}^{(1)}, \quad i = 1, ..., m^{(1)},$$

$$\sum_{j=1}^n \lambda_j^{(1)} z_{dj} \ge z_{do}, \quad d = 1, ..., h,$$

$$\sum_{j=1}^n \lambda_j^{(1)} y_{rj}^{(1)} \ge y_{ro}^{(1)} + \eta_o^{(1)} g_{ro}^{(1)}, \quad r = 1, ..., s^{(1)},$$

$$\sum_{j=1}^n \lambda_j^{(1)} = 1,$$

$$\lambda_j^{(1)} \ge 0, \quad j = 1, ..., n.$$

$$(9)$$

Note that in Model (9), optimization is implemented on the indices of Division 1, and the remaining indices represent the interdependence between the two divisions. This enables the sustainability measurement of other divisions.

Theorem 2: Model (9) is always feasible.

Proof: Indeed,

$$\begin{split} & \lambda_o^{(1)} = 1, \quad \lambda_j^{(1)} = 0, \quad j = 1, \dots, n; j \neq o \\ & \eta_o^{(1)} = 0, \end{split}$$

is a feasible solution to Model (9).

Finally, the sustainability of Division 2 is measured as:

$$\max_{j=1} \eta_o^{(2)}$$
s.t.
$$\sum_{j=1}^n \lambda_j^{(2)} x_{ij}^{(2)} \le x_{io}^{(2)} - \eta_o^{(2)} f_{io}^{(2)}, \quad i = m^{(1)} + 1, ..., m,$$

$$\sum_{j=1}^n \lambda_j^{(2)} z_{dj} \le z_{do}, \quad d = 1, ..., h,$$

$$\sum_{j=1}^n \lambda_j^{(2)} y_{rj}^{(2)} \ge y_{ro}^{(2)} + \eta_o^{(2)} g_{ro}^{(2)}, \quad r = s^{(1)} + 1, ..., s,$$

$$\sum_{j=1}^n \lambda_j^{(2)} = 1,$$

$$\lambda_j^{(2)} \ge 0, \quad j = 1, ..., n.$$

$$(10)$$

Theorem 3: Model (10) is always feasible.

Proof: The proof is similar to the proofs of Theorems 1 and 2. \Box

Assume that $I^{(k)} = I_1^{(k)} \cup I_2^{(k)}$ (k = 1, 2), where $I_1^{(k)}$ (k = 1, 2) is the set of the indices of integer-valued variables, while $I_2^{(k)}$ (k = 1, 2) represents the indices of real-valued variables. Assume that $O^{(k)} = O_1^{(k)} \cup O_2^{(k)} \cup O_3^{(k)}$ (k = 1, 2), where $O_1^{(k)}$ (k = 1, 2) denotes the indices of integer-valued variables, $O_2^{(k)}$ (k = 1, 2) is the set of indices of real-valued variables, and $O_3^{(k)}$ (k = 1, 2) is the indices of undesirable outputs. Assume that $D = D_1 \cup D_2 = \{1, \dots, h\}$, where D_1 is the indices of integer-valued variables, while D_2 denotes the indices of real-valued variables. The proposed NDEA model is as follows:

$$\begin{aligned} & \max \quad \eta_o^S \\ & \text{s.t.} \quad \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} \leq \widetilde{x}_{i0}^{(i)}, \quad i \in I_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} \leq \widetilde{x}_{i0}^{(j)} - \eta_o^S f_{i0}^{(j)}, \quad i \in I_2^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} \leq \widetilde{x}_{io}^{(j)}, \quad i \in I_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} \leq \widetilde{x}_{io}^{(j)}, \quad i \in I_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} \leq \widetilde{x}_{io}^{(j)} - \eta_o^S f_{io}^{(j)}, \quad i \in I_2^{(2)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} = \widehat{x}_{io}, \quad d \in D_1, \\ & \sum_{j=1}^n \lambda_j^{(j)} x_{ij}^{(j)} \geq \widehat{y}_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)} + \eta_o^S g_{io}^{(j)}, \quad r \in O_2^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)} + \eta_o^S g_{io}^{(j)}, \quad r \in O_2^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(j)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)} + \eta_o^S g_{io}^{(j)}, \quad r \in O_2^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)} + \eta_o^S g_{io}^{(j)}, \quad r \in O_2^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)} + \eta_o^S g_{io}^{(j)}, \quad r \in O_2^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)} + \eta_o^S g_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(i)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)} \geq \widetilde{y}_{io}^{(i)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)} y_{ij}^{(j)}, \quad r \in O_1^{(i)}, \\ & \sum_{j=1}^n \lambda_j^{(i)}, \quad r$$

where $(-\mathbf{f},\mathbf{g}) = (-\mathbf{f}^{(1)},-\mathbf{f}^{(2)},\mathbf{g}^{(1)},\mathbf{g}^{(2)})$ is a direction vector. A DMU is efficient if $\eta_o^{S*} = 0$. The inefficient DMUs have positive values of η_o^{S*} , and DMUs with less efficiency have larger values of

 η_o^{S*} . The advantage of Model (11) is that different weights can be allocated to various inputs/outputs using different values of $(-\mathbf{f}^{(1)}, -\mathbf{f}^{(2)}, \mathbf{g}^{(1)}, \mathbf{g}^{(2)})$. The following theorem can be stated about Model (11):

Theorem 4: Model (11) is always feasible.

Proof: The proof is similar to the proofs of Theorems 1 and 2.

The sustainability of Division 1 is measured as follows:

$$\text{max} \quad \eta_{o}^{(1)}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_{j}^{(1)} x_{ij}^{(1)} \leq \widetilde{x}_{io}^{(1)}, \quad i \in I_{1}^{(1)},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} x_{ij}^{(1)} \leq x_{io}^{(1)} - \eta_{o}^{(1)} f_{io}^{(1)}, \quad i \in I_{2}^{(1)},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} x_{dj} \geq x_{do}, \quad d \in D_{1},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} y_{rj}^{(1)} \geq \widetilde{y}_{ro}^{(0)}, \quad r \in O_{1}^{(1)},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} y_{rj}^{(1)} \geq \widetilde{y}_{ro}^{(1)} + \eta_{o}^{(1)} g_{ro}^{(1)}, \quad r \in O_{2}^{(1)},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} y_{rj}^{(1)} = y_{ro}^{(1)} - \eta_{o}^{(1)} g_{ro}^{(1)}, \quad r \in O_{3}^{(1)},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} y_{rj}^{(1)} = y_{ro}^{(1)} - \eta_{o}^{(1)} g_{ro}^{(1)}, \quad r \in O_{3}^{(1)},$$

$$\sum_{j=1}^{n} \lambda_{j}^{(1)} y_{rj}^{(1)} = \widetilde{x}_{io}^{(1)}, \quad i \in I_{1}^{(1)},$$

$$y_{ro}^{(1)} + \eta_{o}^{(1)} g_{ro}^{(1)} = \widetilde{y}_{ro}^{(1)}, \quad r \in O_{1}^{(1)},$$

$$y_{ro}^{(1)} + \eta_{o}^{(1)} g_{ro}^{(1)} = \widetilde{y}_{ro}^{(1)}, \quad r \in O_{1}^{(1)},$$

$$\lambda_{i}^{(1)} > \widetilde{y}_{ro}^{(1)} \in Z_{+}, \quad i \in I_{1}^{(1)}, r \in O_{1}^{(1)},$$

$$\lambda_{i}^{(1)} \geq 0, \quad j = 1, \dots, n.$$

where $(-\mathbf{f}, \mathbf{g}) = (-\mathbf{f}^{(1)}, -\mathbf{f}^{(2)}, \mathbf{g}^{(1)}, \mathbf{g}^{(2)})$ is a direction vector. Furthermore, a DMU is efficient if $\eta_o^{(1)*} = 0$; otherwise, it is inefficient. The constraints on the intermediate products in Model (12) are treated similarly to the constraints on the non-discretionary variables in the conventional DEA. No optimization is implemented on this set of constraints. The following theorem can be stated about Model (12):

Theorem 5: Model (12) is always feasible.

Proof: The proof is similar to the proofs of Theorems 1 and 2. \Box

The sustainability of Division 2 is obtained as follows:

$$\max \quad \eta_o^{(2)}$$
s.t.
$$\sum_{j=1}^n \lambda_j^{(2)} x_{ij}^{(2)} \le \widetilde{x}_{io}^{(2)}, \quad i \in I_1^{(2)},$$

$$\sum_{j=1}^n \lambda_j^{(2)} x_{ij}^{(2)} \le x_{io}^{(2)} - \eta_o^{(2)} f_{io}^{(2)}, \quad i \in I_2^{(2)},$$

$$\sum_{j=1}^n \lambda_j^{(2)} z_{dj} \le z_{do}, \quad d \in D_1,$$

$$\sum_{j=1}^n \lambda_j^{(2)} z_{dj} \le z_{do}, \quad d \in D_2,$$

$$\sum_{j=1}^n \lambda_j^{(2)} y_{ij}^{(2)} \ge \widetilde{y}_{ro}^{(2)}, \quad r \in O_1^{(2)}$$

$$\sum_{j=1}^n \lambda_j^{(2)} y_{ij}^{(2)} \ge y_{ro}^{(2)} + \eta_o^{(2)} g_{ro}^{(2)}, \quad r \in O_2^{(2)}$$

$$\sum_{j=1}^n \lambda_j^{(2)} = 1,$$

$$x_{io}^{(2)} - \eta_o^{(2)} f_{io}^{(2)} = \widetilde{x}_{io}^{(2)}, \quad i \in I_1^{(2)},$$

$$y_{ro}^{(2)} + \eta_o^{(2)} g_{ro}^{(2)} = \widetilde{y}_{ro}^{(2)}, \quad r \in O_1^{(2)},$$

$$\widetilde{x}_{io}^{(2)}, \widetilde{y}_{ro}^{(2)} \in Z_+, \quad i \in I_1^{(2)}, r \in O_1^{(2)},$$

$$\lambda_i^{(2)} \ge 0, \quad j = 1, \dots, n.$$

$$(13)$$

where $(-\mathbf{f}, \mathbf{g}) = (-\mathbf{f}^{(2)}, \mathbf{g}^{(2)})$ is a direction vector. Moreover, a DMU is efficient if $\eta_o^{(2)*} = 0$; otherwise, it is inefficient. The following theorem can be stated about Model (13):

Theorem 6: Model (13) is always feasible.

Proof: The proof is similar to the proofs of Theorems 1 and 2.

3.1. A network DEA model for resource allocation

The fair allocation of resources and budget among a set of DMUs is very important for organizations (Amirteimoori et al., 2016). Cook and Zhu (2005) proposed an approach for resource allocation. However, their model cannot deal with DMUs with network structures. They assumed the budget as a non-discretionary input. To allocate budget R for n DMUs, resource allocation for Division 2 can be obtained by solving the following linear programming problem with an arbitrary objective function P, which is as follows:

min
$$P$$

s.t. $\sum_{j \in F} \lambda_j^{(2)t^*} r_j = r_t, \quad t \in N,$

$$\sum_{j=1}^n r_j = R$$
(14)

where F denotes the set of sustainable DMUs and N represents the set of unsustainable DMUs in Model (13). In addition, $\lambda_j^{(2)t^*}$ denotes the optimal values in Model (13) when evaluating the unsustainable DMUs ($t \in N$). The same approach can be applied to Division 1.

4. Case study

Tehran is the capital of Iran. The population of Tehran is roughly 9 million in the city and more than 15 million in the larger metropolitan area of Great Tehran. Tehran is the most populous city in the country. The transport system of Tehran is so complicated due to overpopulation, geographical characteristics, and size. It provides transport services to 9 million residents as well as more than 1 million people who commute daily to Tehran from the suburbs. The road network of Tehran covers more than 3500 km for passenger transportation. Due to population, unprecedented growth, and increased economic activities, air pollution and heavy traffic are major challenges in this megacity. Sustainable transport development has been considered the most reliable and effective way to tackle these challenges. The strategies and plans for the future of Tehran transport are based on a sustainable development approach. In Tehran megacity, buses and metro are the most important modes of public transport. Bus services are mostly provided by the private sector while metro services are provided by the governmental sector. In this regard, measuring sustainability and coordination of public-private transport and resource allocation have been significant matters for managers and decision-makers of the transport system of Tehran.

Here, to investigate the sustainability and coordination of public-private transport as well as resource allocation based on sustainability indicators, we deal with a supply chain consisting of bus service providers (stage 1) and metro stations (stage 2). In stage 1, there are four inputs, including the

number of seats, operating network, staff cost, and fuel cost. The outputs of stage 1 are CO2 emissions, profit, and number of arriving passengers. The number of passengers' complaints, the number of passengers, the average time spent on transferring, and the percentage of accurate data sharing are intermediate indicators between stage 1 and stage 2. In stage 2, the inputs include the number of passengers, maintenance cost, staff cost, and personnel training cost. The number of people traveled and profit are outputs of stage 2. Figure 2 shows the supply chain structure.

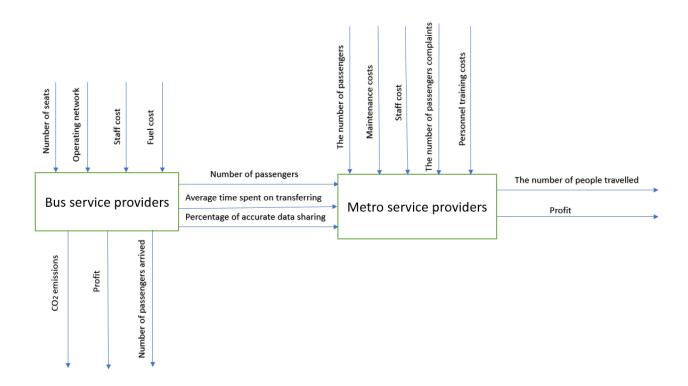


Fig. 2. The supply chain structure

Figure 2 provides an interesting structure for investigating the sustainability and coordination of public-private transport systems. The focus on a supply chain consisting of bus service providers and metro stations is particularly relevant given the increasing need for sustainable transport solutions in urban areas. One interesting aspect of Figure 2 is the inclusion of sustainability indicators as a basis for resource allocation. This highlights the importance of taking a holistic approach to transport system design and operation, which considers not only economic factors but also environmental and social factors. By integrating sustainability indicators into the structure, it becomes possible to evaluate the sustainability of different transport options and allocate resources accordingly. The use of intermediate indicators between stage 1 and stage 2 is also noteworthy. These indicators, such as the number of passengers' complaints, the average time spent on transferring, and the percentage of

accurate data sharing, can provide valuable insights into the effectiveness of the transport system and identify areas for improvement. By tracking these indicators, it becomes possible to identify unsustainabilities and make adjustments to improve the overall sustainability of the system. Overall, Figure 2 emphasizes the importance of taking a holistic approach to transport system and operation and highlights the value of sustainability indicators in resource allocation and system evaluation.

Table 2 presents the variables used in this case study. Also, Table 3 reports the dataset, which dates back to 2021 and was gathered from archives and documents of bus service providers and Tehran Urban & Suburban Railway Company.

Table 2. The used variables

| Stages | Variables | Variable type | Variable nature | Notation | Sustainability dimension | Measurement unit |
|--------|-----------|------------------|--------------------|----------|--------------------------|---------------------|
| | | | | | | |

| | Number of seats (Azadi et al., 2014) | Input | Integer | $x_1^{(1)}$ | Economic | Number |
|---------------------------------------|---|--------------|-------------|-------------|---------------|------------|
| | Operating network (Azadi et al., 2014) | Input | Real | $x_2^{(1)}$ | Economic | Km |
| | Staff cost (Mirhedayatian et al., 2014) | Input | Real | $x_3^{(1)}$ | Economic | Rial |
| | Fuel cost (Azadi et al., 2014) | Input | Real | $x_4^{(1)}$ | Economic | Rial |
| | CO ₂ emissions (Farzipoor Saen et al., 2022) | Output | Undesirable | $y_1^{(1)}$ | Environmental | Kg |
| Bus service providers (Stage 1) | Profit (Kazemi Matin et al., 2022) | Output | Real | $y_2^{(1)}$ | Economic | Rial |
| | The number of passengers (Azadi et al., 2014) | Output | Integer | $y_3^{(1)}$ | Economic | Number |
| | The number of passengers (Azadi et al., 2014) | Intermediate | Integer | z_1 | Economic | Number |
| | Average time spent on transferring (Espino and Román, 2020) | Intermediate | Real | z_2 | Economic | Second |
| | Percentage of accurate data sharing (Cottrill and Derrible, 2015) | Intermediate | Ratio | Z_3 | Economic | Percentage |
| | The number of passengers' complaints (Atalik, 2007) | Input | Integer | $x_1^{(2)}$ | Economic | Number |
| | The number of passengers (Azadi et al., 2014) | Input | Integer | $x_2^{(2)}$ | Economic | Number |
| | Maintenance costs (Azadi et al., 2014) | Input | Real | $x_3^{(2)}$ | Economic | Rial |
| Metro stations (Stage 2) | Staff costs (Izadikhah et al., 2021) | Input | Real | $x_4^{(2)}$ | Economic | Rial |
| | Personnel training costs (Izadikhah et al., 2021) | Input | Real | $x_5^{(2)}$ | Social | Rial |
| | The number of people (Azadi et al., 2014) | Output | Integer | $y_1^{(2)}$ | Economic | Number |
| | Profit (Kazemi Matin et al., 2022) | Output | Real | $y_2^{(2)}$ | Economic | Rial |

Table 3. The dataset

| | Bus services providers (Stage 1) | | | | | | | | | | | | Metro servic | es providers (St | age 2) | | |
|----------------------------|----------------------------------|------------------------------|------------------------------|------------------------------|---|---------------------------|---|--------------------------------|---|---|---|--------------------------------------|--------------------------------------|------------------------------------|--|---|-----------------------|
| | | Inp | uts | | | Outputs | | | Intermediates | | | | Inputs | | | Ou | tputs |
| Supply chains (DMUs) | Number of seats (Integer) | Operating network (Km) | Staff cost (1000 Rial) | Fuel cost (10000 Rial) | CO ₂ emission (10000 Kg) (Undesirable) | Profit (10000 Rial) | Number of passengers arrived (Integer) | Number of passengers (Integer) | Average time spent on transferring (Second) | Percentage of accurate data sharing (Ratio) | Number of passengers' complaints (Integer) | Number of passengers (Integer) | Maintenance costs (1,000 Rial) | Staff cost (1000,000) (Rial) | Personnel training costs (1000 Rial) | Number of people traveled (Integer) | Profit (1000 Rial) |
| 1 | 5419 | 117 | 25141650 | 1977935 | 512 | 17565378 | 2969025 | 1785459 | 241 | 96 | 217 | 2547846 | 10593493 | 21644 | 315700 | 4337305 | -15743000 |
| 2 | 7163 | 149 | 34854436 | 2413833 | 608 | 20837910 | 3617875 | 2607217 | 195 | 98 | 301 | 2862320 | 12835131 | 24121 | 391030 | 5469531 | -11494920 |
| 3 | 3831 | 92 | 18393192 | 1983712 | 372 | 10399013 | 1734910 | 1068484 | 217 | 93 | 174 | 1973018 | 8193201 | 15729 | 213580 | 3041502 | -7311539 |
| 4 | 5195 | 131 | 24975861 | 2115310 | 551 | 18972238 | 3191618 | 2291763 | 214 | 96 | 191 | 2102198 | 9948520 | 23938 | 371000 | 4393961 | -9643641 |
| 5 | 4727 | 112 | 21836870 | 1601839 | 503 | 16937631 | 2518209 | 1681915 | 191 | 95 | 214 | 2297103 | 11938978 | 18349 | 282210 | 3979018 | -12961390 |
| 6 | 6531 | 134 | 35118095 | 2393637 | 596 | 19175342 | 3325293 | 2064935 | 174 | 97 | 357 | 2753174 | 13965128 | 26865 | 412000 | 4818109 | -15937928 |
| 7 | 3428 | 84 | 15846291 | 1593198 | 271 | 8997018 | 1417275 | 1248936 | 197 | 99 | 127 | 1428965 | 6753746 | 16106 | 184400 | 2677901 | -9149793 |
| 8 | 5938 | 141 | 28753250 | 2338721 | 532 | 19396484 | 3294283 | 2193947 | 142 | 95 | 183 | 2302958 | 9853604 | 19328 | 445000 | 4496905 | -17868951 |
| 9 | 3193 | 81 | 13826875 | 1427839 | 235 | 8136928 | 1992195 | 1529193 | 178 | 94 | 219 | 1085932 | 7492041 | 15484 | 201460 | 2615125 | -10321927 |
| 10 | 4931 | 134 | 23291595 | 1824193 | 547 | 19382512 | 2916441 | 1413947 | 249 | 98 | 315 | 1912428 | 10124427 | 15138 | 217431 | 3326375 | -8139418 |
| 11 | 7541 | 162 | 38109531 | 3937592 | 685 | 27541485 | 4532164 | 3364395 | 213 | 95 | 364 | 2533175 | 10197426 | 32421 | 450164 | 5897570 | -9186563 |
| 12 | 3654 | 93 | 13928108 | 1695537 | 283 | 8583928 | 1638927 | 1134387 | 141 | 98 | 186 | 1043937 | 6028948 | 12937 | 219481 | 2178324 | 285753 |
| 13 | 5653 | 125 | 23644329 | 1853175 | 497 | 19472436 | 3232463 | 2243149 | 175 | 97 | 254 | 2321744 | 8643164 | 19875 | 273140 | 4564893 | -11748340 |
| 14 | 4265 | 118 | 20648145 | 1843015 | 519 | 12541256 | 2174828 | 1378064 | 246 | 98 | 196 | 1842912 | 9255173 | 12820 | 201538 | 3220976 | 237124 |
| 15 | 6173 | 121 | 31932816 | 2048391 | 482 | 15847927 | 2927302 | 1737138 | 201 | 95 | 293 | 1865937 | 10436429 | 21194 | 375185 | 3603075 | -8937928 |
| 16 | 3957 | 117 | 18674875 | 1798968 | 347 | 11073264 | 1953194 | 1246287 | 194 | 97 | 115 | 2285674 | 6786653 | 10187 | 145864 | 3527374 | 0 |
| 17 | 5751 | 135 | 26875956 | 2187377 | 548 | 18672932 | 3284283 | 2193982 | 228 | 95 | 253 | 2138289 | 8672623 | 19846 | 295970 | 4332271 | -7765493 |
| 18 | 6847 | 142 | 33827329 | 2374839 | 523 | 17849018 | 3492190 | 2045744 | 234 | 94 | 307 | 1598739 | 11847384 | 21637 | 312743 | 3644483 | -4675253 |
| 19 | 4593 | 96 | 19823018 | 1135307 | 412 | 12572073 | 2063153 | 1295482 | 197 | 97 | 252 | 1848327 | 7643932 | 16473 | 314931 | 3143809 | -15732850 |
| 20 | 7749 | 155 | 37645173 | 2864836 | 645 | 22947541 | 3986.193 | 2584270 | 239 | 94 | 329 | 2194583 | 14842017 | 22947 | 315924 | 4778853 | -14842927 |
| 21 | 3275 | 84 | 15865578 | 1697533 | 301 | 9137640 | 1586875 | 1478875 | 212 | 99 | 128 | 1783404 | 6475690 | 13972 | 157885 | 3262279 | -3942881 |
| 22 | 5176 | 110 | 23926521 | 1732026 | 473 | 16937721 | 2434183 | 1257937 | 193 | 97 | 171 | 2018963 | 6836027 | 17936 | 192614 | 3276900 | 764,95 |

| | l | | | | | | | | | | | | | | | | |
|----|------|-----|----------|---------|-----|----------|---------|---------|-----|----|-----|---------|----------|-------|------------------|---------|-----------|
| 23 | 4141 | 97 | 19932182 | 1384163 | 416 | 15267542 | 2042395 | 1398641 | 185 | 99 | 149 | 2595173 | 9175031 | 19105 | 353972 | 3993814 | -13523742 |
| 24 | 7382 | 159 | 35028191 | 2793145 | 632 | 19846741 | 3521938 | 2846173 | 209 | 96 | 282 | 3116186 | 9373928 | 21854 | 294119 217163 | 5962359 | 19105 |
| 25 | 4519 | 115 | 21103938 | 1523910 | 487 | 15919101 | 2849519 | 2084913 | 213 | 97 | 178 | 2917029 | 8143219 | 19047 | | 5001942 | 196853 |
| 26 | 3794 | 108 | 17937642 | 1739015 | 352 | 11948721 | 2014943 | 1439294 | 173 | 99 | 194 | 2392830 | 7939204 | 14862 | 196264 | 3832124 | -1964415 |
| 27 | 6937 | 141 | 36917617 | 2513178 | 617 | 21737527 | 3429936 | 2541191 | 185 | 95 | 312 | 3193548 | 13027539 | 24194 | 453932 | 5734739 | -863043 |
| 28 | 5218 | 123 | 23917140 | 2018825 | 545 | 17927019 | 3196268 | 2018917 | 214 | 97 | 210 | 2719163 | 8961528 | 20917 | 261189 | 4738080 | -9864421 |
| 29 | 5520 | 115 | 25085159 | 1796153 | 471 | 19017562 | 3392183 | 2429191 | 221 | 95 | 149 | 2819186 | 8018716 | 19528 | 301197 | 5248377 | -10743963 |
| 30 | 3179 | 83 | 15983210 | 1694028 | 310 | 8942329 | 1429193 | 1284104 | 141 | 98 | 174 | 1837054 | 6917109 | 15935 | 132018 | 3121158 | -5843311 |
| 31 | 7372 | 132 | 31294193 | 2018735 | 567 | 21654926 | 3736129 | 2472190 | 182 | 97 | 319 | 2582573 | 9385391 | 22917 | 321632 | 5054763 | -8836102 |
| 32 | 4268 | 101 | 19018684 | 1302572 | 432 | 15183729 | 2321503 | 1494823 | 205 | 97 | 237 | 1943793 | 12834793 | 19762 | 329793 | 3438616 | -13927945 |
| 33 | 6274 | 126 | 35591735 | 2193187 | 605 | 18928739 | 3422593 | 2321830 | 186 | 99 | 396 | 2941893 | 15938748 | 26937 | 496176 | 5263723 | -16563732 |
| 34 | 5839 | 131 | 27836264 | 2286927 | 539 | 17057827 | 2749295 | 1937436 | 211 | 95 | 343 | 2193835 | 11926186 | 19031 | 294935 | 4131271 | -16865095 |
| 35 | 3683 | 115 | 19836072 | 2191436 | 391 | 9846753 | 1583927 | 1138937 | 231 | 95 | 284 | 1638919 | 10937131 | 14648 | 247918 | 2777856 | -9864742 |
| 36 | 6112 | 141 | 33383037 | 2124711 | 585 | 18938262 | 3186028 | 1938022 | 154 | 98 | 312 | 2582771 | 11549263 | 24194 | 398226 | 4520793 | -12938658 |
| 37 | 4427 | 108 | 19083836 | 1393725 | 467 | 15654718 | 2381366 | 1382165 | 178 | 97 | 193 | 1856285 | 9745286 | 16438 | 214944 | 3238450 | -5419452 |
| 38 | 4915 | 131 | 23837594 | 1864542 | 516 | 18745625 | 3093864 | 2194871 | 213 | 98 | 208 | 2492861 | 11083337 | 16936 | 312947 | 4687732 | -7586315 |
| 39 | 3593 | 85 | 14864279 | 1586428 | 328 | 9643718 | 1386426 | 943745 | 196 | 96 | 159 | 1486439 | 5975427 | 12874 | 176973 | 3041502 | -7311539 |
| 40 | 5837 | 122 | 26938910 | 2284718 | 541 | 16947632 | 3193836 | 1884736 | 251 | 98 | 193 | 2387592 | 11848374 | 22193 | 321945 | 4272328 | -16948535 |
| 41 | 4382 | 98 | 19642783 | 1278842 | 473 | 14936665 | 2287524 | 1493826 | 218 | 97 | 237 | 2082837 | 9837252 | 16937 | 279176 | 3576663 | -10847351 |
| 42 | 7365 | 161 | 36928761 | 2794673 | 645 | 21747453 | 3828365 | 2973634 | 201 | 97 | 294 | 2684563 | 10873623 | 25928 | 427393 | 5658197 | -13876127 |
| 43 | 5137 | 125 | 24948759 | 2287371 | 539 | 18934726 | 3193746 | 1973327 | 217 | 95 | 231 | 2736252 | 7936262 | 18847 | 312920 | 4709579 | -9374634 |
| 44 | 6245 | 142 | 36928632 | 2593836 | 601 | 20184746 | 3492842 | 2382947 | 193 | 98 | 329 | 2947272 | 15193845 | 25937 | 419273 | 5330219 | -11865295 |
| 45 | 4538 | 107 | 20818387 | 1493922 | 458 | 15938232 | 2385479 | 1864292 | 175 | 97 | 212 | 2649473 | 7943627 | 16937 | 251984 | 4513765 | 17994 |
| 46 | 5072 | 113 | 24937363 | 1863537 | 499 | 17284635 | 2784829 | 2284639 | 209 | 98 | 237 | 2791634 | 9635284 | 20272 | 301833 | 5076273 | -4856459 |
| 47 | 3329 | 87 | 15937308 | 1693643 | 317 | 9638365 | 1693735 | 967252 | 193 | 96 | 184 | 2193639 | 7826341 | 14991 | 196390 | 3160891 | -1864284 |
| 48 | 5837 | 132 | 27465362 | 2257938 | 542 | 18837635 | 3402384 | 1938746 | 274 | 98 | 196 | 2602746 | 8937353 | 20187 | 293473 | 4541492 | -8654301 |
| | | | | | | | | | | | | | | | | | |
| 49 | 5175 | 105 | 21939441 | 1784617 | 395 | 13958465 | 2392392 | 1973629 | 219 | 95 | 201 | 2492193 | 10793826 | 18764 | 301831 | 4465822 | -11884914 |
| 50 | 3463 | 91 | 16793735 | 1783720 | 317 | 9534289 | 1580263 | 1194374 | 193 | 96 | 189 | 1975239 | 7177384 | 13936 | 189310 | 3169613 | -5765462 |
| 51 | 7782 | 153 | 35018310 | 2691813 | 619 | 21937354 | 3918367 | 3193792 | 218 | 96 | 352 | 2692027 | 13937359 | 26452 | 427395 | 5885819 | -9635289 |

4.1 Results and discussions

The results show the sustainability, coordination of public-private transport, and resource allocation based on sustainability indicators. In the first stage, there are bus service providers that are responsible for transporting people to different spots. In the second stage, metro stations play a key role in transporting people by public transport. Both stages in the supply chain of transport deal with all aspects of sustainability. Assuming variable returns to scale and using Model (11), we measure the sustainability of 51 supply chains of public-private transport. Table 4 depicts the results. As is seen, 35 out of 51 supply chains are sustainable. Supply chain 34 has the lowest sustainability score (0.7982). Also, this supply chain is unsustainable in stage 1 and stage 2 with sustainability scores of 0.8117 and 0.8991, respectively. In stage 1, the inputs' targets for supply chain 34 are 5793 (the number of seats), 120.9123 (operating network), 25009808.3246 (staff cost), and 2054582.6140 (fuel cost). The targets for intermediate variables, including the number of passengers, average time spent on transferring, and percentage of accurate data sharing are 2057658, 202.9012, and 96.6935, respectively.

Supply chain 34 also needs to improve output values in both stages. In stage 1, this supply chain should reduce CO2 emissions to 509.5439. Also, profit and the number of arriving passengers should increase to 19172950.9825 and 2749341, respectively. Moreover, in stage 2, the number of people travelled should increase to 4131317 and profit should increase to 13404836.193. Columns 3 to 5 of Table 4 represent projection points of intermediate products. For sustainable supply chains, the projection point corresponding to the intermediate variables of supply chain 5 differs from its original data (intermediate products). This is acceptable as NDEA models do not directly optimize intermediate products. Columns 6-10 of Table 4 provide projection points for the integer inputs and outputs.

Table 4. The sustainability scores and projection points of DMUs using Model (11)

| | Ovrama114-1. 1 '1' | | | \hat{z}_{2} . | $\widetilde{\mathbf{r}}^{(1)}$ $\widetilde{\mathbf{r}}^{(2)}$ | | | | |
|------------------|---|----------------|----------------|-----------------|---|----------------------------|----------------------------|--------------------------------------|----------------------------|
| DMU _j | Overall sustainability $(1\!-\!\boldsymbol{\eta}_j^{S*})$ | \hat{z}_{1j} | \hat{z}_{2j} | \hat{z}_{3j} | $\widetilde{x}_{1j}^{(1)}$ | $\widetilde{x}_{1j}^{(2)}$ | $\widetilde{x}_{2j}^{(2)}$ | $\widetilde{\mathcal{Y}}_{3j}^{(1)}$ | $\widetilde{y}_{1j}^{(2)}$ |
| 1 | 0.9216 | 2139473 | 215.5475 | 95.9702 | 5411 | 209 | 2547838 | 2969033 | 4337313 |
| 2 | 0.9624 | 2604926 | 195.1970 | 97.3366 | 7156 | 294 | 2862313 | 3617882 | 5469538 |
| 3 | 0.8983 | 1512967 | 184.9483 | 96.3868 | 3825 | 168 | 1973012 | 1734916 | 3041508 |
| 4 | 1.0000 | 2291763 | 214 | 96 | 5195 | 191 | 2102198 | 3191618 | 4393961 |
| 5 | 1.0000 | 1592700 | 227.0576 | 97.1108 | 4727 | 214 | 2297103 | 2518209 | 3979018 |
| 6 | 0.8884 | 2373878 | 194.8930 | 97.6058 | 6504 | 330 | 2753147 | 3325320 | 4818136 |
| 7 | 1.0000 | 1248936 | 197 | 99 | 3428 | 127 | 1428965 | 1417275 | 2677901 |
| 8 | 0.9559 | 2109643 | 211.6921 | 95.8792 | 5935 | 180 | 2302955 | 3294286 | 4496908 |
| 9 | 1.0000 | 1529193 | 178 | 94 | 3193 | 219 | 1085932 | 1992195 | 2615125 |
| 10 | 1.0000 | 1413947 | 249 | 98 | 4931 | 315 | 1912428 | 2916441 | 3326375 |
| 11 | 1.0000 | 3364395 | 213 | 95 | 7541 | 364 | 2533175 | 4532164 | 5897570 |
| 12 | 1.0000 | 1134387 | 141 | 98 | 3654 | 186 | 1043937 | 1638927 | 2178324 |
| 13 | 1.0000 | 2243149 | 175 | 97 | 5653 | 254 | 2321744 | 3232463 | 4564893 |
| 14 | 1.0000 | 1378064 | 246 | 98 | 4265 | 196 | 1842912 | 2174828 | 3220976 |
| 15 | 0.8483 | 2122680 | 208.8660 | 95.6099 | 6146 | 266 | 1865910 | 2927329 | 3603102 |
| 16 | 1.0000 | 1246287 | 194 | 97 | 3957 | 115 | 2285674 | 1953194 | 3527374 |
| 17 | 0.9058 | 2096328 | 204.4297 | 96.3566 | 5738 | 240 | 2138276 | 3284296 | 4332284 |
| 18 | 1.0000 | 2045744 | 234 | 94 | 6847 | 307 | 1598739 | 3492190 | 3644483 |
| 19 | 1.0000 | 1295482 | 197 | 97 | 4593 | 252 | 1848327 | 2063153 | 3143809 |
| 20 | 1.0000 | 2584270 | 239 | 94 | 7749 | 329 | 2194583 | 3986193 | 4778853 |
| 21 | 1.0000 | 1478875 | 212 | 99 | 3275 | 128 | 1783404 | 1586875 | 3262279 |
| 22 | 1.0000 | 1257937 | 193 | 97 | 5176 | 171 | 2018963 | 2434183 | 3276900 |
| 23 | 1.0000 | 1398641 | 185 | 99 | 4141 | 149 | 2595173 | 2042395 | 3993814 |
| 24 | 1.0000 | 2846173 | 209 | 96 | 7382 | 282 | 3116186 | 3521938 | 5962359 |
| 25 | 1.0000 | 2084913 | 213 | 97 | 4519 | 178 | 2917029 | 2849519 | 5001942 |
| 26 | 0.9747 | 1488838 | 187.7678 | 96.7158 | 3792 | 192 | 2392828 | 2014945 | 3832126 |
| 27 | 1.0000 | 2541191 | 185 | 95 | 6937 | 312 | 3193548 | 3429936 | 5734739 |
| 28 | 1.0000 | 2018917 | 214 | 97 | 5218 | 210 | 2719163 | 3196268 | 4738080 |
| 29 | 1.0000 | 2429191 | 221 | 95 | 5520 | 149 | 2819186 | 3392183 | 5248377 |
| 30 | 1.0000 | 1284104 | 141 | 98 | 3179 | 174 | 1837054 | 1429193 | 3121158 |
| 31 | 1.0000 | 2472190 | 182 | 97 | 7372 | 319 | 2582573 | 3736129 | 5054763 |
| 32 | 1.0000 | 1494823 | 205 | 97 | 4268 | 237 | 1943793 | 2321503 | 3438616 |
| 33 | 1.0000 | 2321830 | 186 | 99 | 6274 | 396 | 2941893 | 3422593 | 5263723 |

| 34 | 0.7982 | 2057658 | 202.9012 | 96.6935 | 5793 | 297 | 2193789 | 2749341 | 4131317 |
|----|--------|---------|----------|---------|------|-----|---------|---------|---------|
| 35 | 0.8817 | 1364416 | 169.9480 | 97.8576 | 3663 | 264 | 1638899 | 1583947 | 2777876 |
| 36 | 0.8985 | 2041782 | 208.0387 | 98.0631 | 6092 | 292 | 2582751 | 3186048 | 4520813 |
| 37 | 1.0000 | 1382165 | 178 | 97 | 4427 | 193 | 1856285 | 2381366 | 3238450 |
| 38 | 1.0000 | 2194871 | 213 | 98 | 4915 | 208 | 2492861 | 3093864 | 4687732 |
| 39 | 1.0000 | 943745 | 196 | 96 | 3593 | 159 | 1486439 | 1386426 | 3041502 |
| 40 | 0.9359 | 2134009 | 210.6691 | 96.8878 | 5832 | 188 | 2387587 | 3193841 | 4272333 |
| 41 | 1.0000 | 1493826 | 218 | 97 | 4382 | 237 | 2082837 | 2287524 | 3576663 |
| 42 | 1.0000 | 2973634 | 201 | 97 | 7365 | 294 | 2684563 | 3828365 | 5658197 |
| 43 | 1.0000 | 1973327 | 217 | 95 | 5137 | 231 | 2736252 | 3193746 | 4709579 |
| 44 | 0.8832 | 2358944 | 204.0391 | 96.8952 | 6220 | 304 | 2947247 | 3492867 | 5330244 |
| 45 | 1.0000 | 1864292 | 175 | 97 | 4538 | 212 | 2649473 | 2385479 | 4513765 |
| 46 | 1.0000 | 2284639 | 209 | 98 | 5072 | 237 | 2791634 | 2784829 | 5076273 |
| 47 | 1.0000 | 967252 | 193 | 96 | 3329 | 184 | 2193639 | 1693735 | 3160891 |
| 48 | 0.8889 | 2318299 | 214.2658 | 96.0621 | 5828 | 187 | 2602737 | 3402393 | 4541501 |
| 49 | 0.9535 | 2087923 | 203.4774 | 94.6887 | 5171 | 197 | 2492189 | 2392396 | 4465826 |
| 50 | 0.9595 | 1439417 | 186.2427 | 96.4444 | 3460 | 186 | 1975236 | 1580266 | 3169616 |
| 51 | 1.0000 | 3193792 | 218 | 96 | 7782 | 352 | 2692027 | 3918367 | 5885819 |

Table 5 represents the results obtained from Models (12)-(14). Column 2 reports the sustainability scores of the supply chains obtained from Model (12). According to Table 5, in stage 1, 42 out of 51 supply chains are sustainable. Supply chain 34 has the lowest sustainability score. Columns 3 and 4 of Table 5 list the projection points obtained from Model (12) for integer inputs and outputs.

Table 5. The results obtained from Models (12)-(14)

| DMIT | IU . Stage 1 | | | Stage 2 | | | | | |
|--------------------|-----------------------|----------------------------|----------------------------|---------------------|----------------------------|----------------------------|----------------------------|--------------|--|
| DMU_{j} | Sustainability | $\widetilde{x}_{1j}^{(1)}$ | $\widetilde{y}_{3j}^{(1)}$ | Sustainability | $\widetilde{x}_{1j}^{(2)}$ | $\widetilde{x}_{2j}^{(2)}$ | $\widetilde{y}_{1j}^{(2)}$ | r | |
| | $(1-\eta_{j}^{(1)*})$ | x_{1j} | y_{3j} | $(1-\eta_j^{(2)*})$ | x_{1j} | x_{2j} | y_{1j} | r_{j} | |
| | | 5.410 | 20.60025 | | 217 | 25.470.47 | 4227205 | 17000000000 | |
| 1 | 1.0000 | 5419 | 2969025 | 1.0000 | 217 | 2547846 | 4337305 | 170000000000 | |
| 2 | 1.0000 | 7163 | 3617875 | 1.0000 | 301 | 2862320 | 5469531 | 137514422166 | |
| 3 | 0.9540 | 3801 | 1863618 | 1.0000 | 174 | 1973018 | 3041502 | 0.0000 | |
| 4 | 1.0000 | 5195 | 3191618 | 1.0000 | 191 | 2102198 | 4393961 | 0.0000 | |
| 5 | 1.0000 | 4727 | 2518209 | 1.0000 | 214 | 2297103 | 3979018 | 0.0000 | |
| 6 | 0.9338 | 6309 | 3405223 | 1.0000 | 357 | 2753174 | 4818109 | 0.0000 | |
| 7 | 1.0000 | 3428 | 1417275 | 1.0000 | 127 | 1428965 | 2677901 | 0.0000 | |
| 8 | 0.9308 | 5747 | 3379979 | 1.0000 | 183 | 2302958 | 4496905 | 0.0000 | |
| 9 | 1.0000 | 3193 | 1992195 | 1.0000 | 219 | 1085932 | 2615125 | 0.0000 | |
| 10 | 1.0000 | 4931 | 2916441 | 0.8750 | 290 | 1912403 | 3655873 | 12152498751 | |
| 11 | 1.0000 | 7541 | 4532164 | 1.0000 | 364 | 2533175 | 5897570 | 101404487876 | |
| 12 | 1.0000 | 3654 | 1638927 | 1.0000 | 186 | 1043937 | 2178324 | 0.0000 | |
| 13 | 1.0000 | 5653 | 3232463 | 1.0000 | 254 | 2321744 | 4564893 | 0.0000 | |
| 14 | 1.0000 | 4265 | 2174828 | 1.0000 | 196 | 1842912 | 3220976 | 0.0000 | |
| 15 | 0.8333 | 5674 | 3194779 | 0.9157 | 278 | 1865922 | 3801891 | 97272514954 | |
| 16 | 0.9602 | 3926 | 2055955 | 1.0000 | 115 | 2285674 | 3527374 | 56934700587 | |
| 17 | 1.0000 | 5751 | 3284283 | 1.0000 | 253 | 2138289 | 4332271 | 0.0000 | |
| 18 | 1.0000 | 6847 | 3492190 | 1.0000 | 307 | 1598739 | 3644483 | 0.0000 | |
| 19 | 1.0000 | 4593 | 2063153 | 0.8540 | 232 | 1848307 | 3555276 | 130491091355 | |
| 20 | 1.0000 | 7749 | 3986193 | 1.0000 | 329 | 2194583 | 4778853 | 0.0000 | |
| 21 | 1.0000 | 3275 | 1586875 | 1.0000 | 128 | 1783404 | 3262279 | 0.0000 | |
| 22 | 0.9584 | 5093 | 2521380 | 1.0000 | 171 | 2018963 | 3276900 | 0.0000 | |
| 23 | 1.0000 | 4141 | 2042395 | 1.0000 | 149 | 2595173 | 3993814 | 0.0000 | |
| 24 | 1.0000 | 7382 | 3521938 | 1.0000 | 282 | 3116186 | 5962359 | 0.0000 | |
| 25 | 1.0000 | 4519 | 2849519 | 1.0000 | 178 | 2917029 | 5001942 | 0.0000 | |
| 26 | 1.0000 | 3794 | 2014943 | 1.0000 | 194 | 2392830 | 3832124 | 0.0000 | |
| 27 | 1.0000 | 6937 | 3429936 | 1.0000 | 312 | 3193548 | 5734739 | 0.0000 | |
| 28 | 1.0000 | 5218 | 3196268 | 0.9263 | 203 | 2719156 | 4828290 | 29850262242 | |
| 29 | 1.0000 | 5520 | 3392183 | 1.0000 | 149 | 2819186 | 5248377 | 0.0000 | |
| 30 | 1.0000 | 3179 | 1429193 | 1.0000 | 174 | 1837054 | 3121158 | 0.0000 | |
| 31 | 1.0000 | 7372 | 3736129 | 0.9951 | 318 | 2582572 | 5059212 | 17000000000 | |
| 32 | 1.0000 | 4268 | 2321503 | 0.8607 | 220 | 1943776 | 3790285 | 123282830393 | |
| 33 | 1.0000 | 6274 | 3422593 | 1.0000 | 396 | 2941893 | 5263723 | 0.0000 | |
| 34 | 0.8117 | 5338 | 3085091 | 0.8991 | 320 | 2193812 | 4315986 | 51373360508 | |
| 35 | 1.0000 | 3683 | 1583927 | 0.9408 | 274 | 1638909 | 2966288 | 112573262537 | |
| 36 | 0.9550 | 5980 | 3246611 | 1.0000 | 312 | 2582771 | 4520793 | 0.0000 | |
| 37 | 1.0000 | 4427 | 2381366 | 0.8974 | 185 | 1856277 | 3517825 | 77795431907 | |
| 38 | 1.0000 | 4915 | 3093864 | 1.0000 | 208 | 2492861 | 4687732 | 0.0000 | |
| 39 | 1.0000 | | 1386426 | 1.0000 | 159 | | | 170000000000 | |
| 40 | 1.0000 | 5837 | 3193836 | 0.9615 | 190 | 2387589 | 4337329 | 38697345578 | |
| 41 | 1.0000 | 4382 | 2287524 | 0.8607 | 220 | 2082820 | 3909096 | 103033981477 | |
| 42 | 1.0000 | 7365 | 3828365 | 1.0000 | 294 | 2684563 | 5658197 | 0.0000 | |
| 43 | 1.0000 | 5137 | 3193746 | 1.0000 | 231 | 2736252 | 4709579 | 0.0000 | |
| 44 | 1.0000 | 6245 | 3492842 | 0.9393 | 316 | 2947259 | 5368620 | 17000000000 | |
| 45 | 1.0000 | 4538 | 2385479 | 1.0000 | 212 | 2649473 | 4513765 | 0.0000 | |
| 46 | 1.0000 | 5072 | 2784829 | 0.9098 | 226 | 2791623 | 5156166 | 26257615602 | |
| 47 | 1.0000 | 3329 | 1693735 | 1.0000 | 184 | 2193639 | 3160891 | 0.0000 | |
| 48 | 1.0000 | 5837 | 3402384 | 0.9753 | 194 | 2602744 | 4576575 | 38431665427 | |
| 49 | 1.0000 | 5175 | 2392392 | 1.0000 | 201 | 2492193 | 4465822 | 0.0000 | |
| 50 | 0.9824 | 3458 | 1632233 | 0.9324 | 184 | 1975234 | 3358312 | 79562039873 | |
| 51 | 1.0000 | 7782 | 3918367 | 1.0000 | 352 | 2692027 | 5885819 | 0.0000 | |

As reported in Table 5 and assuming variable returns to scale, Model (13) is implemented for the second stage. According to Table 5 (column 5), in stage 2, 36 out of 51 supply chains are sustainable. Among the unsustainable supply chains, supply chain 19 has the lowest sustainability

score. Columns 6-8 of Table 5 show the projection points obtained from Model (13) for the integer inputs and outputs. In stages 1 and 2, 30 out of 51 supply chains are sustainable.

Regarding the specifications of the proposed models, the problem is explained concerning DMU₁₅. The projection points obtained from Model (11) are $\widetilde{x}_{1,15}^{(1)} = 6146$, $\overline{x}_{2,15}^{(1)} = 114.9326$ $\overline{x}_{3,15}^{(1)} = 29186409.2191$, $\overline{x}_{4,15}^{(1)} = 1909889.4944$, $\widetilde{x}_{1,15}^{(2)} = 266$, $\widetilde{x}_{2,15}^{(2)} = 1865910$, $\overline{x}_{3,15}^{(2)} = 9759760.1573$ $\overline{y}_{15}^{(1)} = 19524.3989$, $\overline{x}_{5,15}^{(2)} = 338300.1180$, $\overline{y}_{1,15}^{(1)} = 451.2079$, $\overline{y}_{2,15}^{(1)} = 17621668.9438$, $\overline{y}_{3,15}^{(1)} = 2927329$ $\widetilde{y}_{\text{\tiny 1,15}}^{(2)} = 3603102 \; , \quad \overline{y}_{\text{\tiny 2,15}}^{(2)} = -7538830.3202 \; , \quad \widehat{z}_{\text{\tiny 1,15}} = 2122680 \; , \quad \widehat{z}_{\text{\tiny 2,15}} = 208.8660 \; , \quad \text{and} \quad \widehat{z}_{\text{\tiny 3,15}} = 95.6099 \; , \quad \widehat{z}_{\text{\tiny 1,15}} = 2122680 \; , \quad \widehat{z}_{\text{\tiny 2,15}} = 208.8660 \; , \quad \widehat{z}_{\text{\tiny 3,15}} = 2122680 \; , \quad \widehat{z}_{\text{\tiny 3,15}} = 21226800 \; , \quad \widehat{z}_{\text{\tiny 3,15}} = 21226800 \; , \quad \widehat{z}_{\text{\tiny 3,15}} = 21226800 \; , \quad$ Model (11) reports the projection points of integer-valued inputs and outputs as integer values. Moreover, the projection point for the undesirable output of this DMU (i.e., 482) is reduced to 451.2079, something which is always true in this model. The factors of intermediate products may have decreasing or increasing values (as the output of Step 1 and the input of Step 2). The decreasing values mean that the relevant factors are dominated by the inputs of Step 2, whereas the increasing values mean that the relevant factors are dominated by the outputs of Step 1. Given the resultant projection points of this DMU, all three intermediate products have increasing values. In addition, the projection point of the first intermediate product is an integer value. Finally, the projection point of the second output of Step 2 is -7538830.3202, which is increased by 1399097.6798 from its initial value (i.e., -8937928). The projection points obtained from Model (12) are $\bar{x}_{3,15}^{(1)} = 28915159.1667$, $\bar{x}_{4,15}^{(1)} = 1896210.3333$, $\bar{y}_{1,15}^{(1)} = 448.1667$ $\overline{y}_{2,15}^{(1)} = 17796853.3333$, and $\widetilde{y}_{3,15}^{(1)} = 3194779$. Furthermore, the projection points obtained from Model 13) are $\widetilde{x}_{1,15}^{(2)} = 278$, $\widetilde{x}_{2,15}^{(2)} = 1865922$, $\overline{x}_{3,15}^{(2)} = 10060501.9926$, $\overline{x}_{4,15}^{(2)} = 20266.4441$, $\overline{x}_{5,15}^{(2)} = 354693.4058$ $\widetilde{y}_{1,15}^{(2)} = 3801891$, and $\overline{y}_{2,15}^{(2)} = -8160651.7748$. According to their projection points, both Models (12) and (13) can detect the integer-valued projection points.

Resource allocation is the art of distributing resources for different purposes. To run the daily affairs of metro stations, the government wishes to allocate 1.7 trillion IRR to metro stations. Due to the scarcity of resources, a maximum of 10% can be allocated to each metro station ($r_j \le 0.1 \times R$). Using Model (14), the last column of Table 5 shows the resources allocated to each supply chain. As is seen in Table 5, resources are allocated to only 20 supply chains. Metro stations 1 and 39 receive maximum resources. Resource leveling is performed for unsustainable metro stations. A lower bound of 1% is assumed for unsustainable DMUs ($r_i \ge 0.01 \times R$). The last column of Table 5 depicts that all unsustainable metro stations have been allocated a resource.

Models (7), (9), and (10) were employed to determine the overall sustainability and the sustainability of each division. The results can be seen in Table 6. In addition, the projection points of intermediate products can be seen in Table 6. The last column of Table 6 presents the resource allocation values obtained from Model (14). As is seen in Table 6, the overall sustainability and the sustainability of each division are smaller than or equal to the overall sustainability and the sustainability of each division obtained from Models (11)-(13). Hence, the results of Models (7), (9), and (10) can be confusing. Moreover, the values of resource allocation in Tables 5 and 6 are very different. Thus, considerable efforts should be made for correct resource allocation.

Table 6. The sustainability scores and projection points of DMUs using Models (7), (9), (10), and (14)

| DMUs | Overall sustainability $(1-\boldsymbol{\eta}_{j}^{S*})$ | Sustainability $(1-\eta_j^{(1)*})$ | Sustainability $(1-\eta_j^{(2)*})$ | \hat{z}_{1j} | \hat{z}_{2j} | \hat{z}_{3j} | r_j (10), and (14) |
|------|---|------------------------------------|------------------------------------|----------------|----------------|----------------|----------------------|
| 1 | 0.9208 | 0.9973 | 0.9158 | 2139473.4812 | 215.5331 | 95.9648 | 29444678608.9007 |
| 2 | 0.9579 | 1.0000 | 0.9036 | 2602962.9057 | 193.7766 | 97.3880 | 91663625897.139 |
| 3 | 0.8907 | 0.9427 | 1.0000 | 1512966.9428 | 183.8087 | 96.5410 | 170000000000 |
| 4 | 1.0000 | 1.0000 | 1.0000 | 2291763.0000 | 214.0000 | 96.0000 | 170000000000 |
| 5 | 0.9960 | 1.0000 | 0.9550 | 1592700.1038 | 223.0702 | 97.2698 | 89431055474.7062 |
| 6 | 0.8853 | 0.9019 | 0.9528 | 2369201.8015 | 194.1113 | 97.6371 | 28777444409.7212 |
| 7 | 1.0000 | 1.0000 | 1.0000 | 1248936.0000 | 197.0000 | 99.0000 | 0.0000 |
| 8 | 0.9422 | 0.8963 | 1.0000 | 2137503.5358 | 212.0327 | 95.7054 | 0.0000 |
| 9 | 1.0000 | 1.0000 | 1.0000 | 1529193.0000 | 178.0000 | 94.0000 | 0.0000 |
| 10 | 1.0000 | 1.0000 | 0.8571 | 1413947.0000 | 249.0000 | 98.0000 | 48933968809.4889 |
| 11 | 1.0000 | 1.0000 | 1.0000 | 3364395.0000 | 213.0000 | 95.0000 | 0.0000 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1134387.0000 | 141.0000 | 98.0000 | 0.0000 |
| 13 | 1.0000 | 1.0000 | 1.0000 | 2243149.0000 | 175.0000 | 97.0000 | 0.0000 |
| 14 | 1.0000 | 1.0000 | 1.0000 | 1378064.0000 | 246.0000 | 98.0000 | 0.0000 |
| 15 | 0.8428 | 0.8262 | 0.8903 | 2115851.1047 | 207.9009 | 96.0837 | 42871565510.8498 |
| 16 | 1.0000 | 0.9486 | 1.0000 | 1246287.0000 | 194.0000 | 97.0000 | 0.0000 |
| 17 | 0.8997 | 0.9455 | 0.9639 | 2101191.0010 | 201.8082 | 96.2355 | 52583743187.3269 |
| 18 | 1.0000 | 0.9580 | 1.0000 | 2045744.0000 | 234.0000 | 94.0000 | 0.0000 |
| 19 | 1.0000 | 1.0000 | 0.8294 | 1295482.0000 | 197.0000 | 97.0000 | 42622838198.4639 |
| 20 | 1.0000 | 1.0000 | 1.0000 | 2584270.0000 | 239.0000 | 94.0000 | 0.0000 |
| 21 | 1.0000 | 1.0000 | 1.0000 | 1478875.0000 | 212.0000 | 99.0000 | 0.0000 |
| 22 | 1.0000 | 0.9465 | 1.0000 | 1257937.0000 | 193.0000 | 97.0000 | 0.0000 |
| 23 | 1.0000 | 1.0000 | 1.0000 | 1398641.0000 | 185.0000 | 99.0000 | 0.0000 |

| 24 | 1.0000 | 0.9886 | 1.0000 | 2846173.0000 | 209.0000 | 96.0000 | 170000000000 |
|----|--------|--------|--------|--------------|----------|---------|------------------|
| 25 | 1.0000 | 1.0000 | 1.0000 | 2084913.0000 | 213.0000 | 97.0000 | 0.0000 |
| 26 | 0.9703 | 1.0000 | 1.0000 | 1488838.4317 | 183.3108 | 96.7687 | 0.0000 |
| 27 | 1.0000 | 1.0000 | 1.0000 | 2541191.0000 | 185.0000 | 95.0000 | 59325146302.129 |
| 28 | 1.0000 | 1.0000 | 0.9121 | 2018917.0000 | 214.0000 | 97.0000 | 30907533212.563 |
| 29 | 1.0000 | 1.0000 | 1.0000 | 2429191.0000 | 221.0000 | 95.0000 | 0.0000 |
| 30 | 1.0000 | 1.0000 | 1.0000 | 1284104.0000 | 141.0000 | 98.0000 | 0.0000 |
| 31 | 1.0000 | 1.0000 | 0.9925 | 2472190.0000 | 182.0000 | 97.0000 | 57282502376.3178 |
| 32 | 1.0000 | 1.0000 | 0.8270 | 1494823.0000 | 205.0000 | 97.0000 | 69845449837.9639 |
| 33 | 1.0000 | 1.0000 | 0.8802 | 2321830.0000 | 186.0000 | 99.0000 | 54465614865.1203 |
| 34 | 0.7946 | 0.7986 | 0.8973 | 2052473.1931 | 199.7469 | 97.2111 | 17000000000 |
| 35 | 0.8771 | 1.0000 | 0.8944 | 1368279.9985 | 167.0815 | 98.0629 | 58176813432.0112 |
| 36 | 0.8940 | 0.9277 | 1.0000 | 2039215.5904 | 207.5150 | 98.0561 | 0.0000 |
| 37 | 1.0000 | 1.0000 | 0.8970 | 1382165.0000 | 178.0000 | 97.0000 | 31673873697.9744 |
| 38 | 1.0000 | 1.0000 | 1.0000 | 2194871.0000 | 213.0000 | 98.0000 | 0.0000 |
| 39 | 1.0000 | 1.0000 | 1.0000 | 943745.0000 | 196.0000 | 96.0000 | 33195612582.0795 |
| 40 | 0.9263 | 1.0000 | 0.8634 | 2124286.2521 | 210.8730 | 96.8781 | 43851487806.0806 |
| 41 | 1.0000 | 1.0000 | 0.8440 | 1493826.0000 | 218.0000 | 97.0000 | 40752335864.1636 |
| 42 | 1.0000 | 1.0000 | 1.0000 | 2973634.0000 | 201.0000 | 97.0000 | 0.0000 |
| 43 | 1.0000 | 1.0000 | 1.0000 | 1973327.0000 | 217.0000 | 95.0000 | 0.0000 |
| 44 | 0.8816 | 0.9680 | 0.8712 | 2358943.8258 | 204.0012 | 96.8976 | 77012202703.2097 |
| 45 | 1.0000 | 1.0000 | 1.0000 | 1864292.0000 | 175.0000 | 97.0000 | 0.0000 |
| 46 | 1.0000 | 1.0000 | 0.9027 | 2284639.0000 | 209.0000 | 98.0000 | 90533419210.2263 |
| 47 | 1.0000 | 1.0000 | 1.0000 | 967252.0000 | 193.0000 | 96.0000 | 0.0000 |
| 48 | 0.8820 | 1.0000 | 0.8978 | 2314798.6863 | 211.1251 | 96.1466 | 32929850838.9266 |
| 49 | 0.9425 | 1.0000 | 0.9204 | 2073559.3870 | 204.1763 | 94.6323 | 26596039642.771 |
| 50 | 0.9568 | 0.9623 | 0.9216 | 1439416.5936 | 175.4240 | 96.2232 | 40123197531.8669 |
| 51 | 1.0000 | 1.0000 | 1.0000 | 3193792.0000 | 218.0000 | 96.0000 | 0.0000 |

4.2 Managerial implications

Sustainable transport is becoming increasingly important in addressing the ecological, social, and economic dimensions of transport in megacities. The coordination of public and private transport is receiving significant attention from decision-makers and requires novel methods for allocating limited resources to unlimited demands based on sustainability criteria. To achieve sustainable urban

mobility objectives, public transport service providers are developing performance measurement frameworks and systems based on passengers' expectations. The main objective of this study is to investigate the coordination and performance of public-private transport systems in a megacity using a novel NDEA model. The proposed model can handle different types of data, including integer data, negative data, and undesirable outputs, and can be used for sustainable resource allocation in public-private transport systems.

Coordination and sustainability measurement have been lees addressed in public-private transport and they are considered key challenges by managers and decision-makers of transportation systems. Allocating a limited budget for transport systems is another challenge for managers and decision-makers of transport systems. These challenges become more complex when it comes to the supply chain. Also, the type of data in sustainability measurement of public-private transport plays a key role in the results. In the case study, we dealt with negative and integer data. As well, the undesirable outputs were taken into account. In this paper, we measured the sustainability of public-private transport. Furthermore, projection points (sustainability goals) were determined for unsustainable DMUs. Also, the budget was allocated to different metro stations. The results showed the budget that should be allocated to each metro station. By allocating the budget, managers can improve the sustainability of transport.

This paper provides a novel method for measuring the performance and coordination of public-private transport systems in megacities. The study develops an NDEA model using DDF to measure the overall and stage sustainability of these systems. This model can also be used for sustainable resource allocation in public-private transport systems. The proposed NDEA model can handle different types of data, including integer data, negative data, and undesirable outputs, which allows for more reliable results and provides more realistic insights for managers and decision-makers of transport systems. In terms of sustainable transport, the study highlights the importance of optimal resource allocation and setting clear-cut, achievable targets to achieve sustainability goals such as reduced costs, less pollution, and increased passenger satisfaction. By adopting appropriate government policies, utilizing technology-based infrastructures, and collaborating with the public and private sectors, policymakers and managers of transport can work towards sustainable urban mobility objectives. Furthermore, the study suggests that public transport performance depends on various factors such as demographic characteristics, transport system features, services planning, resource allocation, and network design. In multimodal public transport networks, the incorporation of information flow, inter alia, and network planning play key roles. A lack of appropriate coordination among public transport sub-systems can affect the overall sustainability of the system significantly. Overall, this study provides a valuable tool for managers and decision-makers of public-private

transport systems in megacities to improve performance and coordination and achieve sustainable urban mobility objectives.

The model proposed in this study for the performance measurement and coordination of public-private transport within multimodal transport networks is a significant step forward for the transportation industry. Its potential advantages are numerous, including the ability to provide detailed insights into sustainability and coordination, efficient resource allocation, and the handling of complex data sets. One of the most significant advantages of the proposed model is its ability to evaluate the sustainability and coordination of public-private transport networks operating within a supply chain. This is a critical aspect of the transportation industry, as it has the potential to enhance both the efficiency and effectiveness of these networks. By using a DDF, the model can provide detailed insights into the performance of these networks, including areas where improvements can be made to enhance sustainability and coordination. Another important advantage of the proposed model is its ability to allocate resources efficiently among DMUs. This is achieved through the use of NDEA, which allows for the optimization of resource allocation and management in a sustainable manner. The ability to allocate resources efficiently is essential for both the public and private sectors, as it can lead to cost savings and improved overall performance. The proposed model is also highly advantageous due to its ability to handle complex data sets. Many existing models are limited in their ability to account for various types of inputs and outputs, including those that are integer, negative, and undesirable. By overcoming this limitation, the proposed model can provide a more comprehensive approach to performance measurement and coordination of public-private transport networks. Finally, the proposed model has the potential to significantly advance the field of transportation by providing decision-makers with valuable insights into the sustainability and coordination of public-private transport networks within multimodal transport systems. This information can help decision-makers make more informed decisions about resource allocation and management, leading to improved overall performance and sustainability. In conclusion, the proposed model for performance measurement and coordination of public-private transport within multimodal transport networks has numerous advantages that make it a valuable resource for the transportation industry. Its ability to evaluate sustainability and coordination, allocate resources efficiently, and handle complex data sets has the potential to significantly enhance the efficiency and effectiveness of public-private transport networks within multimodal transport systems.

The proposed model has several managerial implications for decision-makers in the transportation industry. Firstly, decision-makers can use the model to gain detailed insights into the performance of public-private transport networks within a supply chain. This information can help decision-makers identify areas where improvements can be made to enhance sustainability and

coordination, leading to improved overall performance and efficiency. Secondly, decision-makers can use the model to allocate resources efficiently among DMUs. This can help decision-makers sustainably optimize resource allocation and management, leading to cost savings and improved overall performance. Thirdly, decision-makers can use the model to handle complex data sets, including those that are integer, negative, and undesirable. This can provide a more comprehensive approach to performance measurement and coordination of public-private transport networks, which can help decision-makers make more informed decisions about resource allocation and management. Finally, decision-makers can use the proposed model to enhance the coordination between the public and private sectors in transport systems. By evaluating the performance of public-private transport networks, decision-makers can identify areas where coordination can be improved, leading to enhanced efficiency of transport systems. Overall, the proposed model has several managerial implications for decision-makers in the transportation industry. By using the model, decision-makers can gain valuable insights into the sustainability and coordination of public-private transport networks within multimodal transport systems, allocate resources efficiently, handle complex data sets, and enhance coordination between the public and private sectors in transport systems. These implications can help decision-makers make more informed decisions about resource allocation and management, leading to improved overall performance and sustainability of transport systems.

5. Conclusions and future research

Owing to population considerable growth and increased economic activities, air pollution, and heavy traffic have been major challenges in megacities in the last few decades. As such, public and private transport is of significant importance in megacities to transport a large number of passengers. Sustainability in transport has also received considerable attention because of its ecological, social, and economic impacts on society. The objective of a sustainable transport system is to minimize environmental problems and maximize social and economic benefits for society. In sustainable transport, the public and private sectors try to collaborate to attain sustainability goals. Thus, coordination of public-private transport and sustainability measurement as well as resource optimal allocation need to be carefully addressed and studied. In this study, we developed a model for measuring sustainability in public-private transport networks consisting of bus service providers and metro stations. The model was built based on the DDF approach in the NDEA structure and can address a variety of data types to obtain more accurate sustainability results. We also presented a resource allocation model among DMUs in public transport concerning budget limitations.

The study in question presented several significant contributions that propel the field of sustainability measurement and analysis forward. One of the study's primary accomplishments is the creation of a DDF, which is a potent tool for evaluating sustainability at both the overall and stage levels. This novel framework represented a considerable improvement over existing methods, as it provides a more comprehensive and nuanced approach to sustainability assessment across different production or development stages. Another crucial finding of the study was the formulation of an NDEA model that can handle integer and negative data and undesirable outputs. This development is critical since many current models are limited in their capacity to account for these data types, which are frequently encountered in sustainability research. By overcoming this limitation, the proposed NDEA model is a significant leap forward in the field of sustainability analysis. Additionally, the study introduces a resource allocation approach that employs the NDEA model. This approach has important implications for practical sustainability applications since it offers decision-makers a practical tool for optimizing resource allocation and sustainably. In conclusion, this study's contributions have the potential to considerably enhance our comprehension of sustainability and assist us in making better-informed decisions about how to encourage sustainable development and resource management. The development of the DDF and NDEA model, which can handle complex data sets, can significantly improve the accuracy and reliability of sustainability assessments. The resource allocation approach utilizing the NDEA model can provide practical and efficient solutions for sustainability management. These contributions have significant implications for decision-makers and practitioners in sustainability and can enable them to make more informed decisions that align with sustainability goals.

There were two major limitations in the current study. Firstly, owing to limitations in data gathering from public and private transportation units, we had to select 51 DMUs. In addition, almost all the data pertains to the economic dimension, with only one data point from the social dimension and one from the environmental. This could result in a solution that is predominantly focused on economic considerations, rather than a sustainable solution that takes into account all three dimensions. Due to the limited access to more environmental and social aspects, we do to run the model with existing data. As such a research direction could be developing a new model considering more environmental and social factors with different nature in the type of data. Another limitation in this study is that we assumed that all variables are controllable. However, in a transport system, there might be uncontrollable variables such as the size of buses or trains. As such, a research direction could be developing a novel NDEA model in the presence of uncontrollable variables. Moreover, due to a limitation in data gathering we evaluated the public-private system and allocated a budget between metro stations for only one period. However, there are conditions that the evaluation and

NDEA model based on our developed model can be another interesting research avenue. In this paper, we addressed the coordination and performance measurement in a series structure. However, there might be transportation systems with both parallel and series structures. Developing a NDEA model to address these sorts of structures can be an interesting research topic. Furthermore, developing other NDEA models such as Russel enhanced network model and bound-adjusted measure for evaluating transportation systems is another research avenue.

Compliance with Ethical Standards:

Conflict of Interest: Authors declare that they have no conflict of interest.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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