Mach Edges: A key role for 3rd derivative filters in spatial vision

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1. Introduction to edge detection:

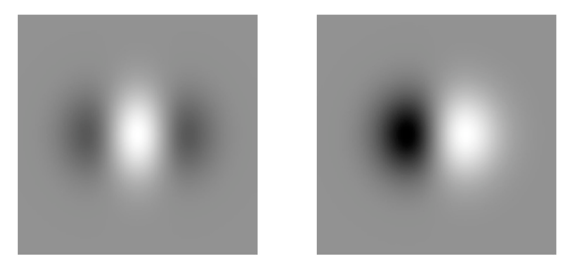
Edges are key points of information in visual scenes. But, how are they extracted from the eye's neural output?

It is widely accepted that the retinal image is filtered by even- and odd-symmetric spatial operators of various scales, early in the visual pathway [1].

Early psychophysical work proposed an edge-detector role for odd-symmetric operators and an edge-detector role for odd-symmetric operators [2]. However, such a simple interpretation is incomplete, as edge-detectors also respond to bars, and bar-detectors also respond to edges, placed as shown here.

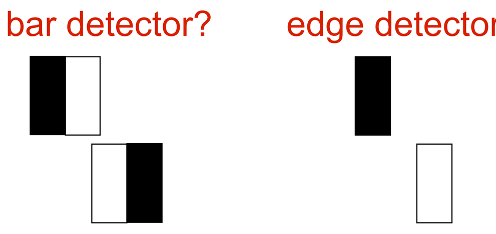
Even-symmetric (2nd derivative) operator

Odd-symmetric (1st derivative) operator



bar detector?

edge detector?

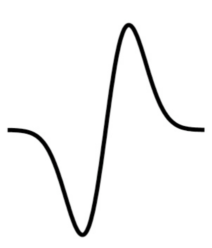


responds to edges

responds to bars

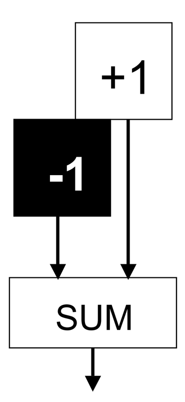
Another view of these operators is to consider that their role is to compute the spatial gradients and higher derivatives of the image [1]. For the odd-symmetric operator shown here, this process can be illustrated as follows….

Odd-symmetric operator profile



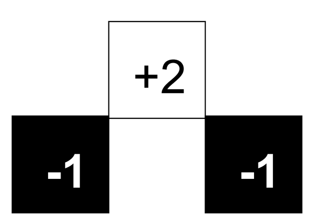
A horizontal section through the odd-symmetric profile shown above produces the profile shown here (left). This can be simplified to two adjacent regions of positive and negative response. If the output of these two regions is summed, this is equivalent to obtaining the difference in luminance between these two regions.

If this operator is 'swept along' (convolved with) a 1-D image so that it makes this comparison between every point and its neighbour then the output at every point is proportional to the rate of change of luminance, i.e. its 1st derivative.

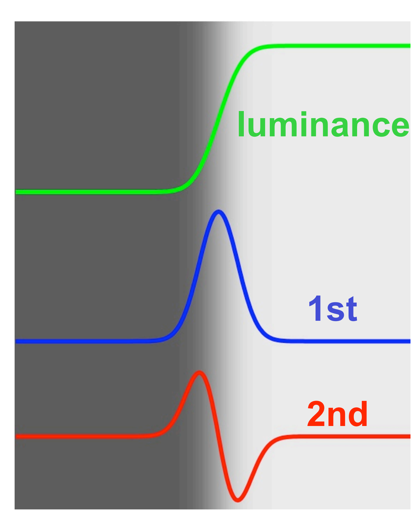


A similar argument applies to obtain operators that compute the 2nd, 3rd or any higher derivative, but with different weights applied to different regions.

Simplified 2nd derivative operator



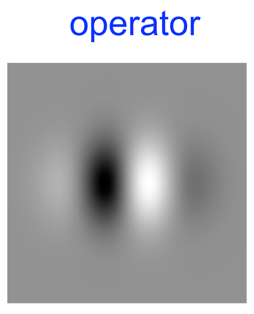
A blurred edge and its 1st and 2nd derivatives



One important class of models supposes that edges correspond to the steepest parts of the luminance profile, implying that they can be found as peaks and troughs in the response of a gradient (first derivative) filter [3], or as zero-crossings (ZCs) in the second derivative [4]. A variety of multi-scale models are based on this idea [5 - 8].

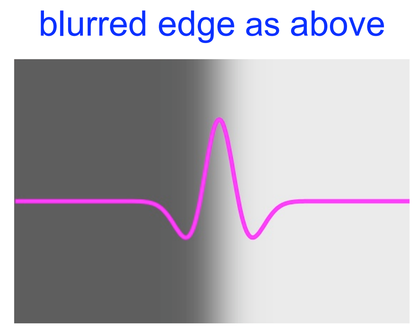
2. Our approach: the 3rd derivative

3rd derivative operator



A new approach is to consider the 3rd derivative of the luminance profile. This operator also produces a peak or trough at each edge. We have devised stimuli that have peaks in the 3rd derivative but no corresponding peaks in the 1st derivative or ZCs in the 2nd derivative.

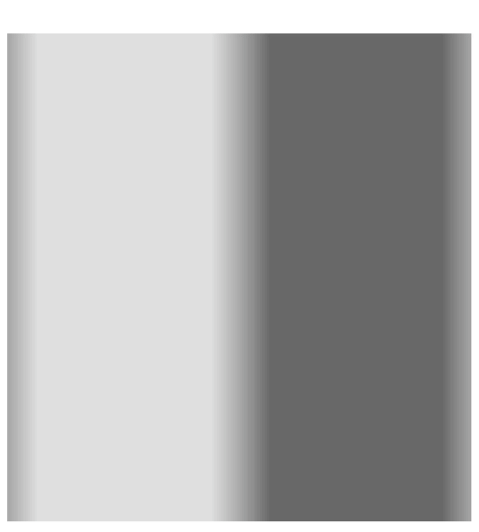
3rd derivative of same blurred edge as above



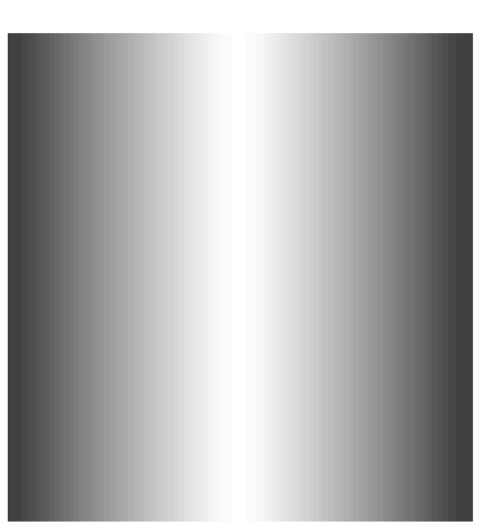
Question: Will subjects see edges at 3rd derivative peaks ?

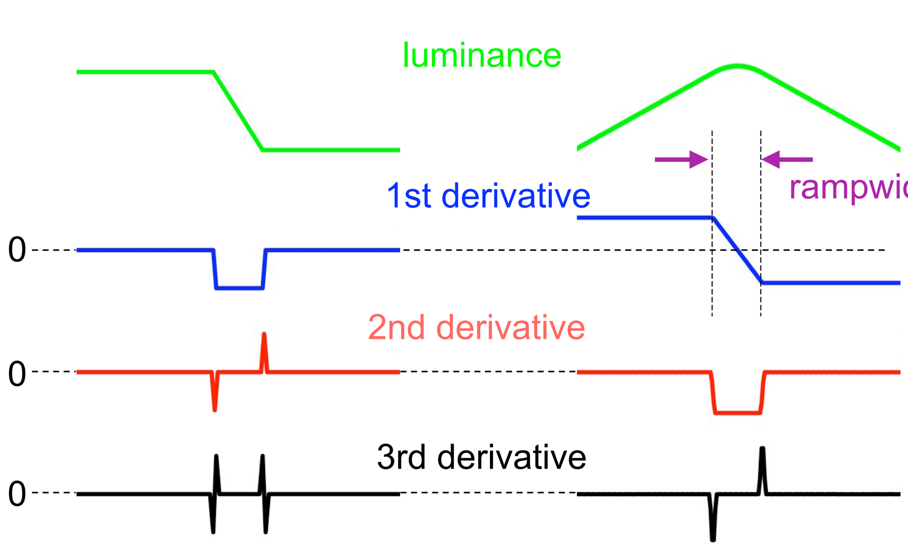
3. Stimuli design:

Mach band image



Experimental image





The stimuli had no local peaks of gradient and no ZCs, at any scale. Our stimulus profile is analogous to the classic Mach band stimulus, but it is the luminance **gradient** (not the absolute luminance) that increases as a linear ramp between two plateaux.

Stimuli generation:

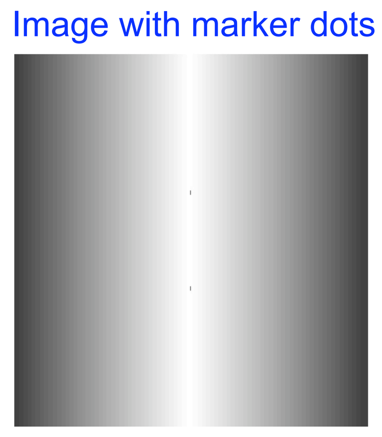
- The starting point was a single period of a trapezoidal wave whose walls were 1, 2, 4, 8, 16, 32 or 64 pixels wide (rampwidth), shown here in the **1st derivative** profile. This defined the gradient profile of the stimulus.
- The resulting waveform was then integrated to form the **luminance profile.**
- Note that the waveform now has peaks in the 3rd derivative but none in the 1st, and no zero-crossings in the 2nd derivative.
- Image size was 256 by 256 pixels (4 degrees).
- Surrounded by a full-screen mid grey.
- Viewed at contrasts of 0.2 or 0.4.
- Waveforms were also inverted to obtain opposite polarity images.

4. Procedure:

Procedure:

- Flashing presentation (duration 0.3s, isi 0.6s)
- 7 rampwidths x 2 contrast levels x 2 polarities = 28 conditions
- 3 subjects
- The task was to mark the position and polarity of all edges
- The marker comprised two black dots, each of 1 x 3 pixels, vertically arranged, each 32 pixels (1/2 degree) from image midline
- Subjects were instructed to fixate midway between the dots

Image with marker dots



5.Results:

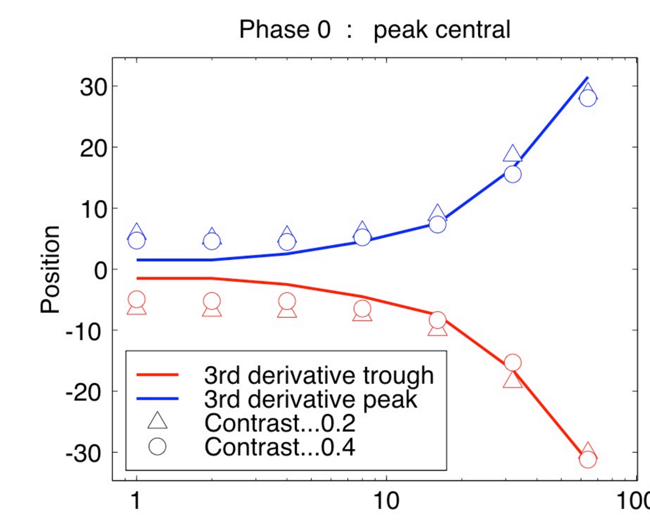
Results were similar for all 3 subjects so are shown averaged.

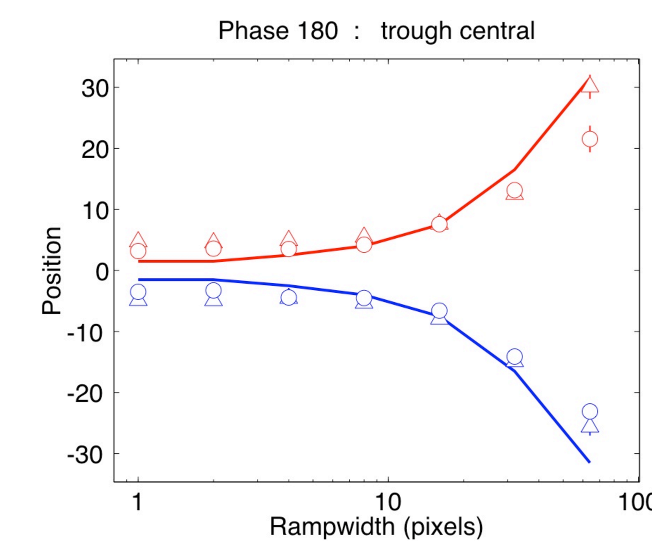
Main Results

- Subjects reliably see edges flanking the luminance peaks
- Their polarity is as predicted
- Their positions (data points) are close to 3rd derivative peaks and troughs (solid lines)
- Data are reliable ~ Error bars are ± 1 se and are plotted behind data points

Conclusion

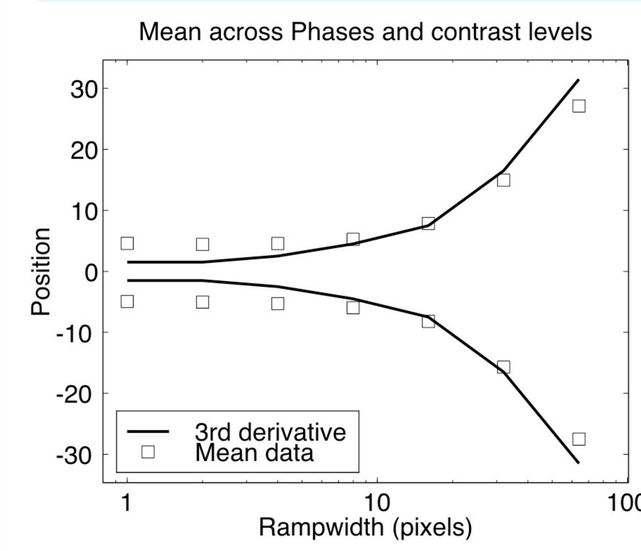
The use of the 3rd derivative is strongly supported.





Secondary Result

Edges appeared slightly more separated for the 0 polarity conditions (upper right) than the 180 polarity conditions (right).



Conclusion

This appears to be an example of the Helmholtz irradiation effect [9], which is neutralised by averaging across phase (left).

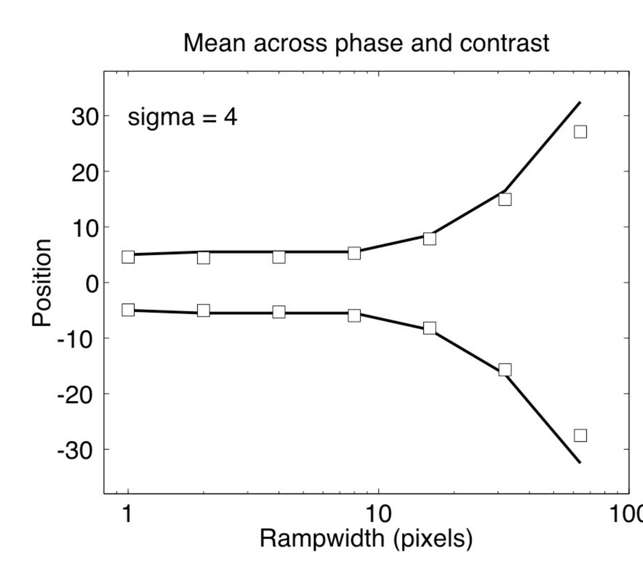
Note that 1st derivative peaks predict no edges here. They predict edges at ± 60 pixels for all rampwidths - not marked by subjects.

6. Add optical and neural blur:

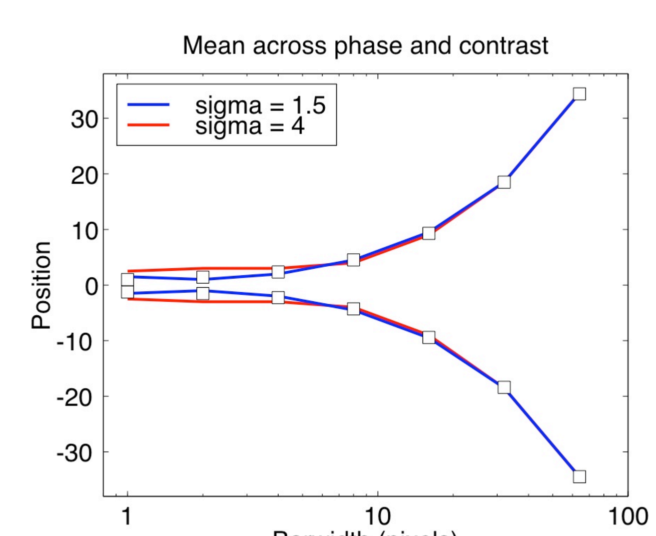
The shift of the data away from the 3rd derivative prediction at low rampwidths can be explained by optical and neural blur. This should have maximum effect at small scales.

A small amount of Gaussian blur (σ =4 min arc) was applied to the luminance waveforms before derivatives were obtained. This could represent the combination of optical blur and the scale of the filter used by the subject.

The 3rd derivative now fits the data better at low rampwidths.



7. Control experiment ~ step edges:



Question: Can 3rd derivatives *with blur* still resolve edges that are close together?

To test this, we applied the same feature-marking method to images containing sharp-edged bars whose widths were comparable to experiment 1.

Sharp-edged bar

Results

Fit was better with σ =1.5 min arc than with σ =4 min arc.

Conclusion

Subjects apply smaller scale operators (less neural blur) for sharp-edges than for more blurred edges.

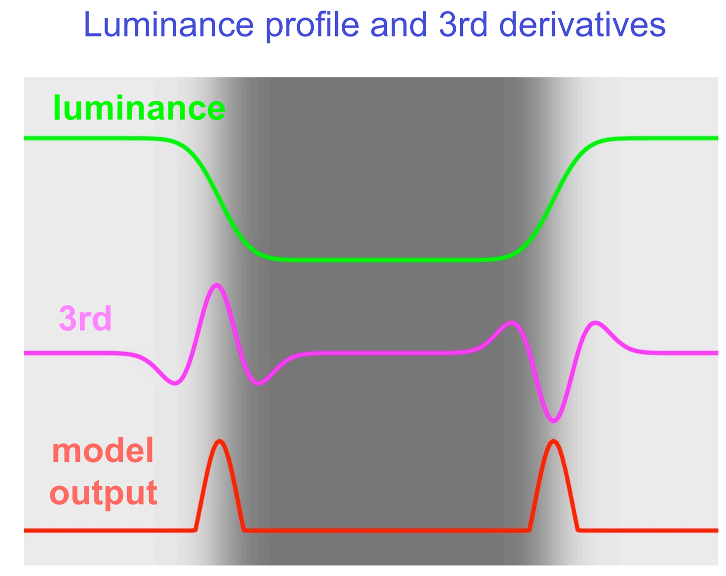
8. Discussion:

Edges are 3rd derivative peaks

The edges seen here are a new phenomenon - Mach Edges - analogous to the classic Mach Bands. There is no peak in the gradient profile that would indicate their existence.

Mach Edges strongly support the use of the 3rd derivative in edge-finding, but defeat 1st & 2nd derivative models.

Luminance profile and 3rd derivatives



Problem of spurious edges

Not all peaks and troughs in the 3rd derivatives signify edges, as the central peak or trough is spanned by two others of opposite sign. This suggests that some form of rectification is needed. A single half-wave rectifier will not suffice, as it would remove the trough that indicates a dark-to-light edge (see right edge shown here). However, this problem of spurious peaks is solved by a **new model** that uses a nonlinear 3rd derivative, incorporating 2 stages of rectification [12]. This returns the lower plot shown here.

References:

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