Third-derivative filters predict edge locations in spatial vision

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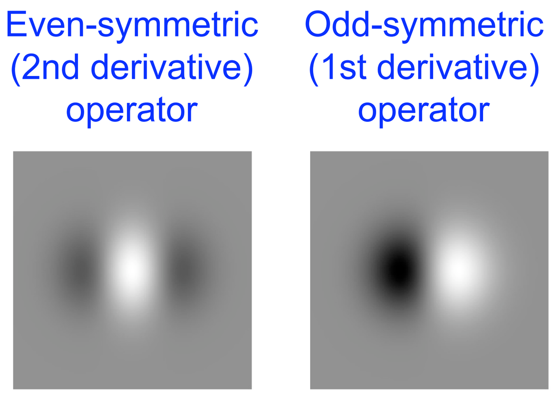
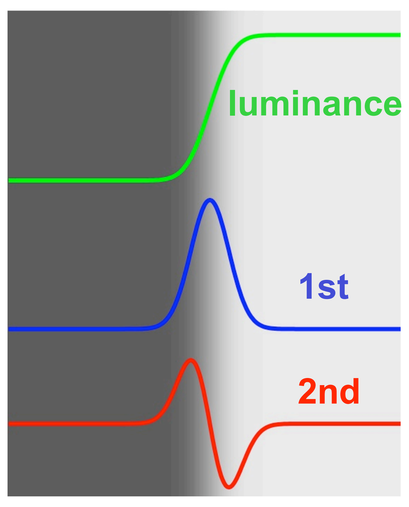
1. Edge detection in vision

Edges are key points of information in visual scenes. But, how are they extracted from the eye's neural output? It is widely accepted that the retinal image is filtered by even- and odd-symmetric spatial operators of various scales, early in the visual pathway [1]. But how the operators are used remains an open question.



A blurred edge with its 1st and 2nd derivatives



One important class of models supposes that edges correspond to the steepest parts of the luminance profile, implying that they can be found as peaks and troughs in the response of a gradient (first derivative) filter [2], or as zero-crossings (ZCs) in the second derivative [3]. An alternative approach uses peaks in the local contrast energy [4]. Both approaches have experimental support, but neither is entirely successful [5].

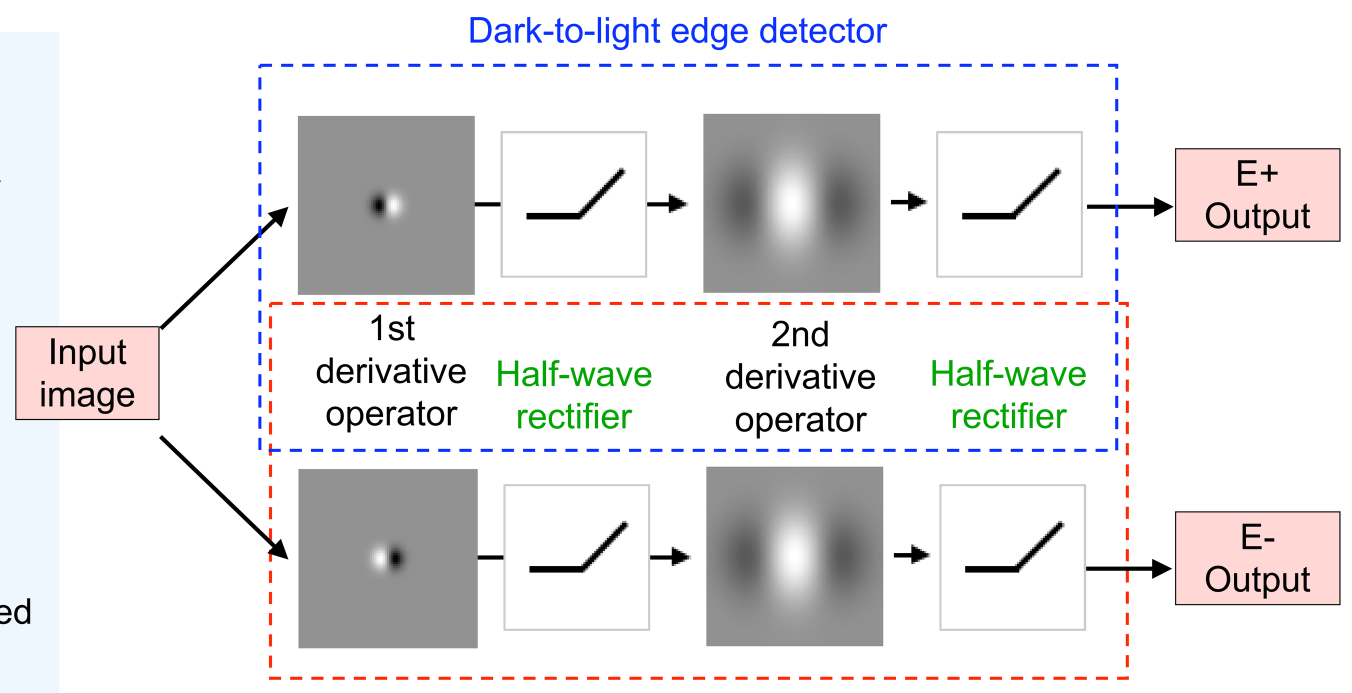
Our experiments test a new model (box 2) against the 1st & 2nd derivative approaches. This model accurately predicted perceived location and blur of edges [6]. Here we predict & test a new phenomenon: 'Mach Edges'. These are visible edges with no peak in the 1st derivative and no zero-crossing in the 2nd derivative (see box 3). They are analogous to Mach Bands, which are light (and dark) bars where there is no peak (or trough) in the luminance profile.

2. Model: Edges as peaks in third derivative

Five steps:
- 1st derivative operator
- Half-wave rectifier
- 2nd derivative operator
- Half-wave rectifier
- Find response peaks

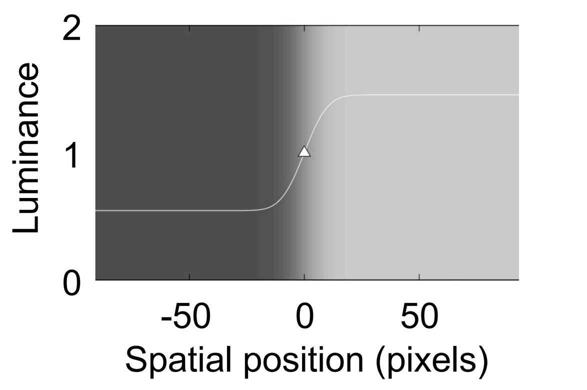
This recovers edges of one polarity (E+). A second channel is needed for the opposite polarity (E-).

The image is differentiated 3 times, so this is a '3rd derivative' model.

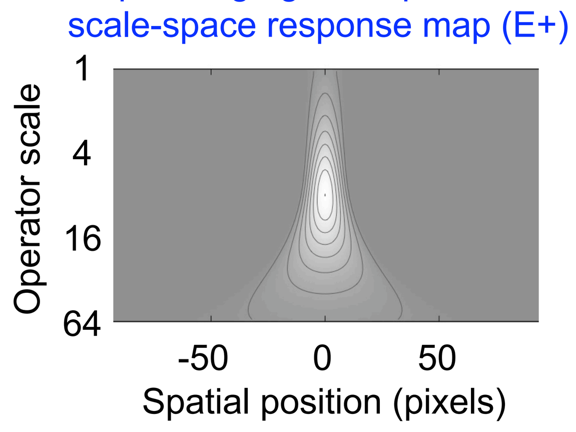


This nonlinear filtering is repeated at many scales. The outputs form a scale-space response map. A peak on the map (far right), indicates the position and blur (scale) of the edge [6].

Input image: A single blurred edge



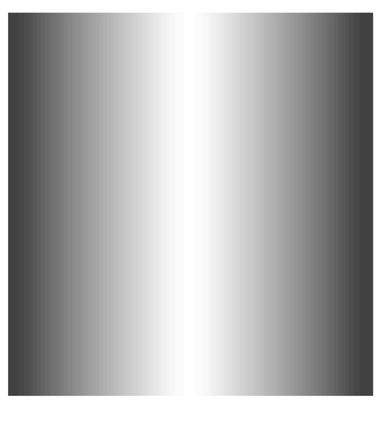
Output: Edge gives a peak in the scale-space response map (E+)



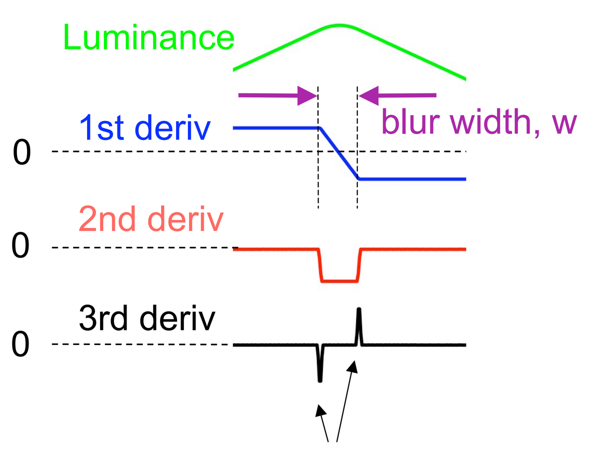
3. Experiment 1: Mach Edges ?

- Peak in the 1st or 3rd derivative might be the cue for edge location.
- We devised a stimulus to test the 3rd derivative model.
- Its 1st derivative is like a Mach ramp, with no peaks.
- Its 2nd derivative has no zero-crossings.
- Its 3rd derivative has peaks & troughs that should signal edges.

Blurred triangle wave



Stimulus generation
- A single period of a triangle wave was blurred with a box function (1 - 64 pixels blur width).
- Its 1st derivative profile is a trapezoidal wave
- Image size: 256 by 256 pixels (4 degrees)
- Surround: full-screen mid-grey
- Contrast: 0.2 or 0.4
- Waveforms also inverted for opposite polarity images.

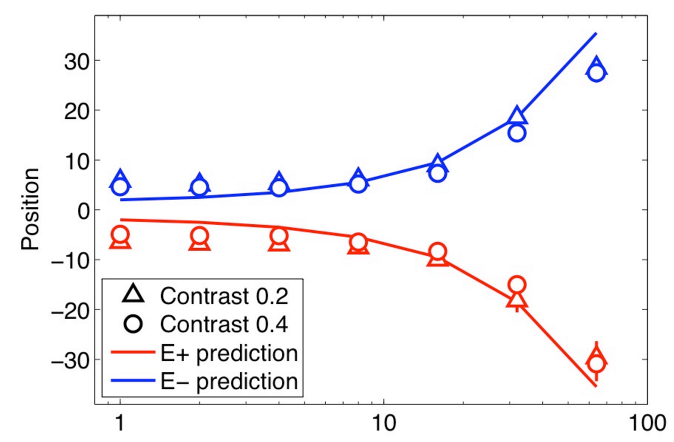


Procedure
- Flashing presentation (0.3s on, 0.6s off)
- 7 blur widths x 2 contrasts x 2 polarities = 28 conditions
- 3 subjects
- Task: mark the position & polarity of all edges
- Marker was two black dots (:), 1 deg apart
- Subjects told to fixate midway between the dots

4. Results

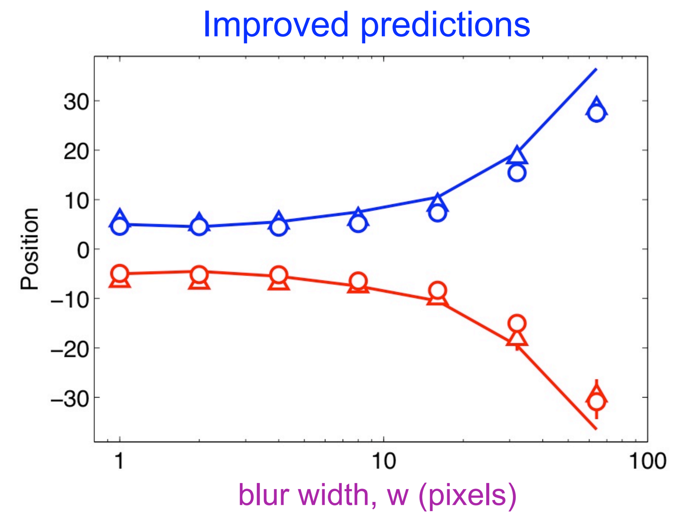
- **Subjects do see Mach edges**
- Data were similar for all 3 subjects; means are shown.
- Perceived edge positions (symbols) are close to model predictions (curves)
- Data are reliable. Error bars (± 1 se) are very small.
- Results were similar for opposite polarity (not shown)

Perceived edge positions: data & model



Model predictions improved with a small amount of Gaussian smoothing (σ=2 pixels) of the model stimulus. This could represent optical and/or neural blur in the observer. Fit improved markedly at smaller blur widths, w.

Improved predictions



Conclusion
THE 3RD DERIVATIVE MODEL CORRECTLY PREDICTS EXISTENCE & POSITION OF MACH EDGES.
Simple 1st derivative model predicts no Mach edges. At a coarse scale, it predicts edges at ± 60 pixels for all blur widths - not marked by observers.

EPSRC

5. Critical test: add a luminance ramp

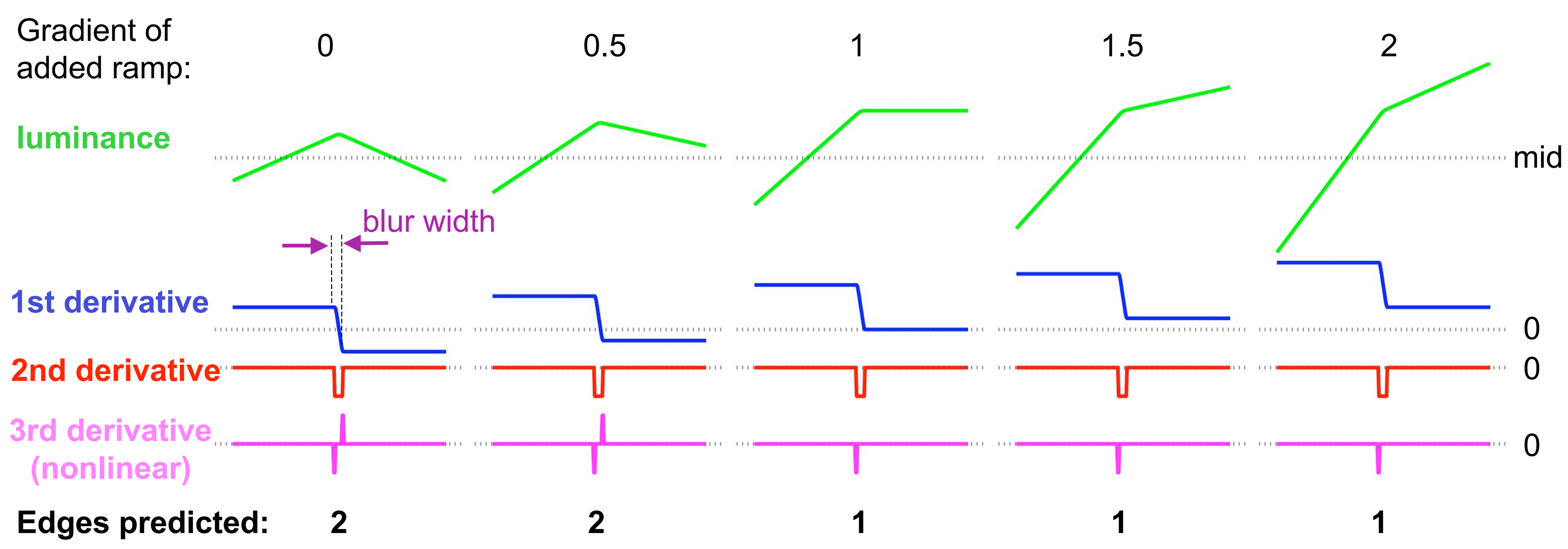
As a critical test of the model, we added a linear luminance ramp to the blurred triangle waves used previously. This ramp has no effect on the second or higher derivatives. But the nonlinear 3rd derivative model predicts
- TWO EDGES, E+, E-, for ramp gradients of less than ±1 (below, 2 left plots).
- ONE EDGE, for steeper added ramp gradients (below, 3 right plots).

Why ? As the added ramp is increased, the 1st derivative becomes entirely positive (blue curves below). Hence the E- channel is silenced, because its first rectifier makes it responsive only to negative gradients.

Stimulus generation
- Basic stimulus was one period of a triangle-wave (green, below, left). It was blurred by a rectangular function 2 or 8 pixels wide. The gradients of its rise and fall are defined as +1,-1.
- Linear luminance ramp was added to form the stimulus luminance profile. Its gradient was -2 to +2.
- 2 blur widths x 9 ramp gradients x 2 polarities = 36 conditions.

Model used the nonlinear channels (box 2) at a single operator scale, σ = 3 pixels

Stimulus luminance profiles and their derivatives (only positive added gradients shown here)

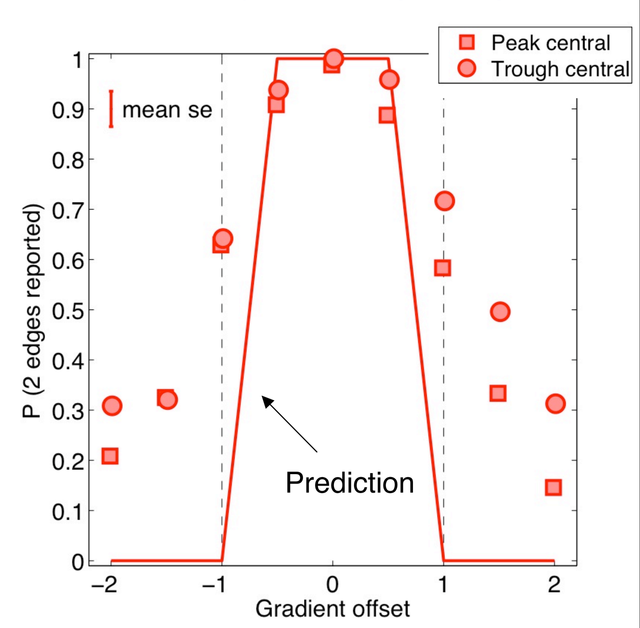


6. Experiment 2: Yes-No method

Procedure
- Images presented once per trial for 0.3s. Inter-trial interval > 1s
- Blur width, w = 2 or 8 pixels
- Task: were 1 or 2 edges seen near the centre of the image?
- 9 added ramps x 2 blur widths x 2 polarities
- 4 subjects, 60 trials per condition per subject

Results
- Data broadly similar for all 4 subjects; averages shown for w = 8.
- Results similar for blur width 2 pixels (not shown).
- 2 edges seen reliably with added gradients up to ±0.5. This matched model predictions (solid line).
- With steeper added ramps, reports of 2 edges fell away as model predicts, but more gradually than predicted.
- Model cannot predict 2 edges for offsets ±1 to ±2, unlike the data.

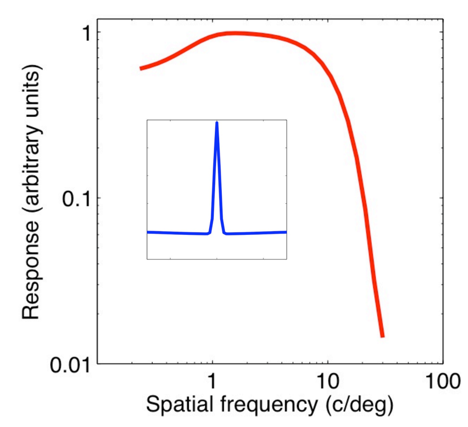
Probability of seeing 2 edges



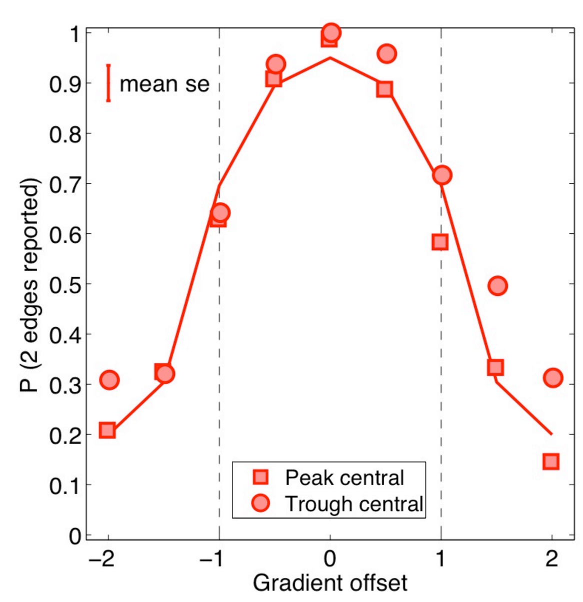
7. Model refinement

Rationale
Two Mach edges were reliably seen in a triangle-wave, but one disappeared when a steep linear ramp was added. This general trend was predicted by the nonlinear 3rd derivative model, but the transition from 2 to 1 edge was less rapid than the model predicted (box 6). We suggest that mild, bandpass filtering (e.g. by the retina) can explain this.

MTF and RF profile of the pre-filter



Data (box 6) with revised predictions

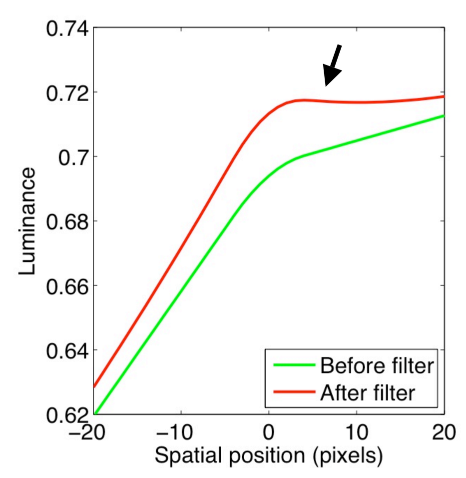


Model refinement - add a retina
A linear, even-symmetric, pre-filter was added to the model. It has a Difference-of-Gaussians receptive field profile, inspired by ganglion cell physiology [8].

It accentuates points of high curvature. For our images with purely positive gradient (offset 1 or 1.5) this filter introduces a weak, central negative gradient (arrowed). That leads to a peak in the E- channel, alongside the positive edge (E+), so predicting 2 edges instead of 1.

We assumed a noisy decision process, with independent decisions about each edge, and a false-positive rate of 0.20. Revised predictions fit well (left).

Filtered waveform, Gdt offset = +1.5

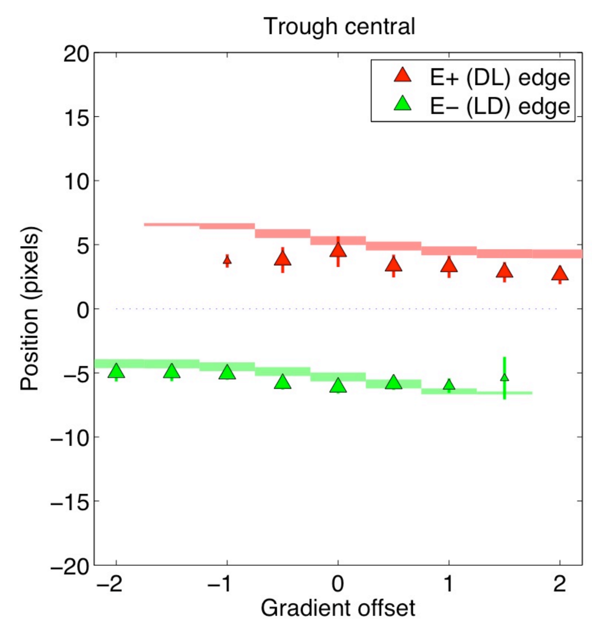


8. Experiment 3: Feature-marking

Procedure
- Stimuli were the same as experiment 2
- Images were flashed repeatedly (0.3s on, 0.6s off).
- Task: mark the position and polarity of all edges seen.
- 1 blur width (8 pixels) x 9 ramp gradients x 2 phases = 18 conditions.
- 4 subjects; 24 trials per condition per subject.

Results
- Data were broadly similar for all 4 subjects, averaged here.
- Results similar for opposite polarity images (not shown).
- All subjects marked a central bar with 2 edges for offsets up to ±0.5.
- For gradient offsets at & beyond ±1, perception of 2 edges gave way to 1 edge (smaller symbols show reduced no. of markings).
- Position and polarity of marked edges was well predicted by the revised 3rd derivative model, with pre-filter.

Perceived edge positions: data & model



9. Conclusions

The revised 3rd derivative model, with retinal pre-filter, correctly predicts Mach Edges and the way they are affected by added luminance ramps.
However, we find that the retinal pre-filter introduces 1st derivative peaks & troughs that match the 3rd derivative predictions (not shown). Mach Edges thus support the 3rd derivative model but, with the pre-filter, the 1st derivative approach also remains viable.

References
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