Machine learning control of complex nonlinear dynamics in fibre lasers

Sonia Boscolo[1,*], *Junsong* Peng[2], *Xiuqi* Wu[2], *Ying* Zhang[2], *Christophe* Finot[3], and *Heping* Zeng[2]

[1]Aston Institute of Photonic Technologies, Aston University, Birmingham B4 7ET, United Kingdom
[2]State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China
[3]Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR 6303 CNRS-Université de Bourgogne, 21078 Dijon Cedex, France

Abstract. We review our recent work on the use of genetic algorithms to control non-stationary nonlinear wave dynamics in ultrafast fibre lasers, including the generation of breathing-soliton dynamics with controlled characteristics, the disclosure of the fractal dynamics of breathers, and the generation of rogue waves with controlled intensity.

1 Introduction

Ultrafast fibre lasers represent interesting realisations of dissipative nonlinear systems with dynamics driven by the interplay among the effects of nonlinearity, dispersion and energy exchange, thus providing an ideal platform for the study of complex nonlinear wave phenomena relevant to a large variety of physical systems. Reaching a specific mode-locked regime in a laser generally involves adjusting multiple control parameters in a high-dimensional space, which is a laborious task if addressed with a trial and error procedure. Machine-learning tools, especially the use of evolutionary and genetic algorithms (GAs), have recently shown promising for the design of smart lasers that can tune themselves into desired operating states [1]. Yet, these algorithms have been mainly designed to target regimes of parameter-invariant, stationary pulse generation, while the intelligent excitation of evolving pulse patterns in a laser remains largely unexplored. In this paper, we review our recent progress in the control of dynamic, non-stationary operating regimes of ultrafast lasers by the use of GAs. These include repetitive patterns, such as breathing solitons and multi-breather complexes, and non-repetitive rare events. Numerical simulations of a lumped laser model confirm our experimental findings and provide insight into the observed dynamics.

2 Breather dynamics

Breathing solitons, manifesting themselves as localised temporal/spatial structures that exhibit periodic oscillatory behaviour, are fundamental modes of many nonlinear physical systems and relate to a wide range of important nonlinear dynamics. First studied experimentally in fibre Kerr cavities and optical micro-resonators, optical breathers have also emerged as a ubiquitous ultra-short pulse regime of passively mode-locked fibre lasers [2, 3] thanks to the recent development of real-time measurements. In [4], we have demonstrated, for the first time, the use of GAs for searching and optimising the breather regime in ultrafast lasers. The merit function is the key ingredient of any GA as it defines the optimisation target. To drive the optimisation of the four-parameter nonlinear polarisation evolution transfer function for the auto-setting of the laser onto the breather mode-locked regime, we have employed a merit function derived from the feature that the breathing frequency manifests itself as two symmetrical sidebands around the cavity repetition frequency in the radio-frequency (RF) spectrum of the laser emission. By including additional components in the definition of the merit function for breather mode locking we have also achieved advanced control of the characteristics of the breather state, such as tuning of the breathing period and breathing ratio (defined as the ratio of the largest to the narrowest width of the pulse spectrum within a period). Furthermore, using the merit function for breather mode locking when the pump power is set to a level that favours multi-pulse self-starting of the laser, and subsequently applying pulse count we have generated different types of breather molecular complexes with a controllable number of elementary constituents.

A breather laser show competition between the cavity repetition frequency and the breathing frequency. The theory of nonlinear systems with two competing frequencies predicts locking or resonances, in which the system locks into a resonant periodic response featuring a rational frequency ratio (winding number), and quasi-periodicity following the hierarchy of the Farey tree and the structure of a devil's staircase [5]. Whilst frequency-locking phenomena have been extensively studied in many physical systems, all the investigations so far relate to systems where an external, accurately controllable modulation adds a new characteristic frequency to the system. Conversely, in the case of breather oscillations in a laser, the second frequency is not externally controlled and is intrinsic to the nonlinear system. In [6], further development of our GA-based optimisation approach has been the key to manipulating the breathing frequency of the laser system, hence to establishing the link between breathing

*e-mail: s.a.boscolo@aston.ac.uk

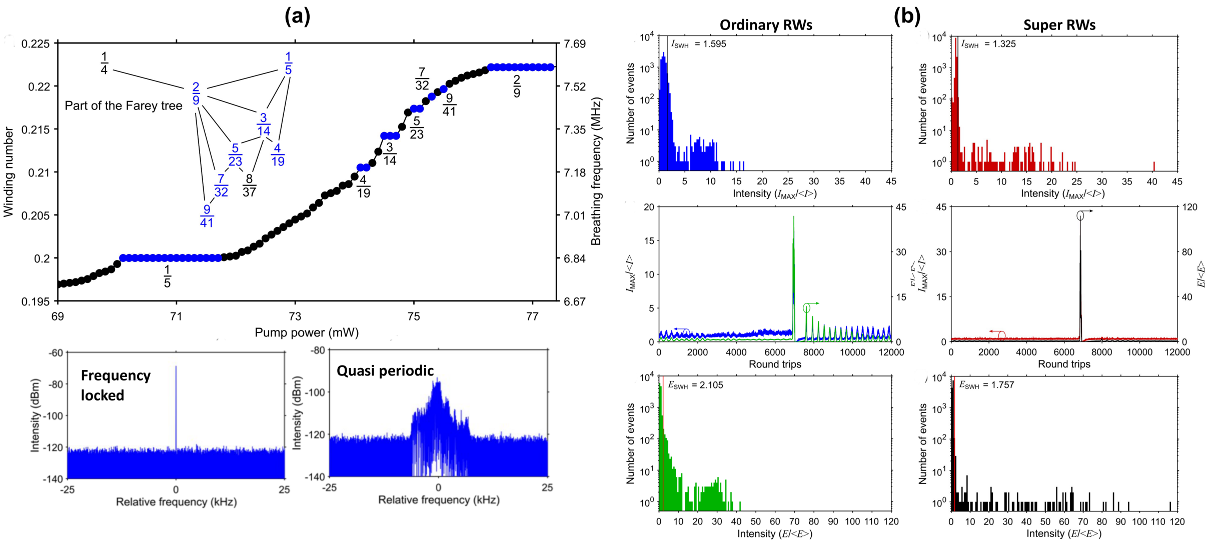

Figure 1. (a) Top, Farey tree and devil's staircase of a breather laser: Measured breather frequency (winding number) as a function of the pump power. In the inset is shown the part of the Farey tree containing the observed Farey fractions. Bottom, RF spectral measurements for frequency-locked and quasi-periodic breather operations of the laser. (b) Examples of statistical analysis for the observed ordinary and super RW operations of the laser. Top, histograms showing the distributions of the spectral intensity maxima for 12,000 successive cavity round trips. The black lines denote the associated significant wave heights. Middle, evolutions of the spectral intensity maxima (blue and red curves) and pulse energies (green and black curves) over 12,000 successive roundtrips. Bottom, corresponding histograms of the energy of the pulses. The red lines denote the associated significant wave heights.

solitons and frequency locking. We have demonstrated for the first time that a breather mode-locked fibre laser is a passive system showing frequency locking at Farey fractions. The frequency-locked breather states occur in the sequence they appear in the Farey tree and within a pump-power interval given by the width of the corresponding step in the devil's staircase. The fractal dimension of 0.906 determined from the measured staircase indicates the universal nature of this nonlinear system. For the GA to directly pinpoint frequency-locked states, we have defined a merit function which takes into account the distinguishing trait of these states, namely, a high signal-to-noise ratio of the breathing frequency.

3 Extreme events

First introduced in the context of oceanic waves, the concept of extreme events or RWs, i.e., statistically-rare giant-amplitude waves, has been transferred to other natural environments such as the atmosphere, as well as to the solid grounds of research laboratories [7]. As RWs appear from nowhere and disappear without a trace, their emergence is unpredictable and non-repetitive, which make them particularly challenging to control. In [8], we have extended the use of GAs to the active control of extreme events in a fibre laser cavity. Feeding real-time spectral measurements into a GA that uses merit functions based on the statistical defining characteristics of RWs to optimise the cavity parameters, we have been able to trigger the generation of both ordinary and super RWs with tailored intensity in the cavity. These extreme spectral events correlate with extreme variations of the pulse energy, thus qualifying as temporal RWs as well. Quite remarkably, significant frequency up- or down-shifting of the optical spectrum is also associated with the emergence of these waves, which suggests a new physical scenario for their emergence and disappearance: an initially coherent but asymmetric pulse circulating in the cavity causes one edge of the spectrum to grow and eventually evolve into a RW through the effect of self-phase modulation, while a concomitant intra-pulse shock wave develops in the time domain. This shock wave ultimately leads to pulse collapse and the emergence of noise-like broadband structures.

4 Conclusion

We have overviewed recent examples of the promise of machine learning for the control and study of highly dynamic, non-stationary operating regimes of ultrafast fibre lasers. It is reasonable to expect the machine-learning approach used in this work to be applicable to the control of alike dynamics in other nonlinear systems. As demonstrated here in the particular case of a laser system, the use of control algorithms can make complex dynamics and instabilities easily accessible so that laborious manual tuning of the system's parameters is no longer required, where this facilitates the exploration of the rich underlying physics.

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