Breather Dynamics in Ultrafast Fibre Lasers and Their Intelligent Control

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Abstract: We review our recent work on the dynamics of breathing solitons in fibre lasers, including single breathers, breather molecular complexes, breather explosions and breather frequency locking at Farey fractions, and their control by genetic algorithms.

1. Introduction

Mode-locked fibre lasers that exploit nonlinearity in the pulse formation process are well-known to display a rich landscape of "dissipative soliton" dynamics, resulting from the interplay of the nonlinearity with dispersion and dissipation. Breathing solitons, manifesting themselves as localised temporal/spatial structures that exhibit periodic oscillatory behavior, are fundamental modes of many nonlinear physical systems and relate to a wide range of important nonlinear dynamics. First studied experimentally in fibre Kerr cavities and optical micro-resonators, optical breathers have also emerged as a ubiquitous ultra-short pulse regime of passively mode-locked fibre lasers thanks to the recent development of real-time measurements. In the present work, we review our recent results and advances in this fast-growing area, by reporting on the first observation of breathing solitons and breather molecular complexes (BMCs) in ultrafast lasers [1,2], breather explosions [3], and the fractal dynamics of breathers [4]. Our experimental findings are confirmed by numerical simulations of the laser model. We also report on the possibility of using genetic algorithms (GAs) to generate breather dynamics with controlled characteristics [4,5].

2. Breather laser dynamics

Breathing solitons in mode-locked lasers feature synchronous periodic variations of their spectrum and temporal intensity over cavity round trips. In [1], we have captured such a fast evolutionary behaviour in real time using time-stretch dispersive Fourier transform (DFT)-based single-shot spectral measurements and spatio-temporal intensity measurements. In the normal dispersion regime of the laser cavity breathers are excited in the laser under the pump threshold for stationary mode locking. Our work first establishes a general, deterministic route to induce soliton breathing in normal-dispersion fibre cavities in that the breather generation regime is accessible by solely changing the pump strength. One of the remarkable properties of dissipative solitons, which are mostly absent in integrable systems, is the ability to form stable multi-soliton bound states, also termed "soliton molecules", and showing similar dynamics to matter molecules. For the first time in experiments with mode-locked fibre lasers, breather-pair molecules have been also generated in the cavity [1]. While soliton pairs constitute the central soliton molecule case, soliton molecules can exist in various isomers, and a large population of solitons can self-assembly into macromolecules and soliton crystals. In [2], by tuning the cavity loss through small rotations of the polarisation controllers at fixed pump strength in an anomalous-dispersion laser cavity, we have demonstrated various types of BMCs. These include multi-breather molecules, and molecular complexes formed by the binding of two diatomic molecules, or a diatomic and a monoatomic molecule. The inter-molecular temporal separation of such complexes is more than an order of magnitude larger than that of their stationary soliton counterparts and is a signature of long-range interactions. We have also observed non-equilibrium dynamics of breathers, including collisions and annihilation of an elementary breather within a complex.
Soliton explosions are among the most striking nonlinear dissipative phenomena that can manifest in mode-locked lasers. In this regime, a dissipative soliton circulating in the laser cavity experiences an abrupt structural collapse, but within a few round trips returns to its original quasi-stable state. By pushing the analogy in dynamics between stationary and breathing solitons further, we have demonstrated the explosive dynamics of breathers in a mode-locked fibre laser [3]. Different laser regimes, including breather explosions, stable breathers, and continuous-wave mode locking, are accessible by varying the pump power in the normal-dispersion laser cavity. In contrast to soliton explosions that are observed above the pump level for generating stable solitons, breather explosions occur under the pump threshold for stable breather mode locking.
The excitation of breather oscillations in a laser naturally leads to a second characteristic frequency in the system, which therefore shows competition between the cavity repetition frequency and the breathing frequency. The theory of nonlinear systems with two competing frequencies predicts locking or resonances, in which the system locks into a resonant periodic response featuring a rational frequency ratio (winding number), and quasi-periodicity following

the hierarchy of the Farey tree and the structure of a devil's staircase [6]. In [4], we have established the link between breathing solitons and frequency locking by demonstrating for the first time a breather mode-locked fibre laser is a passive system showing frequency locking at Farey fractions. The frequency-locked states occur in the sequence they appear in the Farey tree and within a pump-power interval given by the width of the corresponding step in the devil's staircase (Fig. 1(a)). The fractal dimension of 0.906 determined from the measured staircase indicates the universal nature of this nonlinear system. The locked breathing frequencies feature narrow linewidth and high signal-to-noise ratio (SNR; Fig. 1(a)) and are robust against parameter (pump power and polarisation) variations.

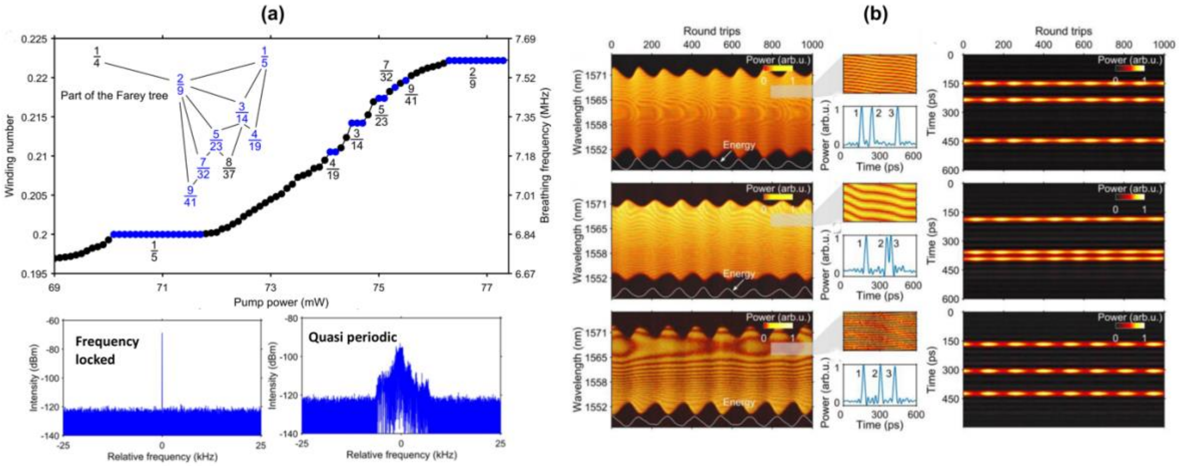

Fig. 1. (a) Top, Farey tree and devil's staircase of a breather laser: Measured breather frequency (winding number) as a function of the pump power. In the inset is shown the part of the Farey tree containing the observed Farey fractions. Bottom, Radiofrequency spectral measurements for frequency-locked and quasi-periodic breather operations of the laser. (b) Typical GA optimisation results for BMCs formed of three breathers: Dynamics of a (2+1) BMC, a (1+2) BMC, and a breather-triplet molecule. Left, DFT recording of single-shot spectra over consecutive cavity round trips (RTs). The white curve represents the energy evolution. Middle, Close-up view of the DFT recording and corresponding temporal intensity. Right, Temporal evolution of the intensity relative to the average RT time over consecutive RTs.

3. Intelligent control of breather dynamics generation

Reaching a specific mode-locked regime in a laser generally involves adjusting multiple control parameters, which is quite difficult to do manually. Machine-learning tools have recently shown promising for the design of smart lasers that can tune themselves to desired operating states, but these algorithms have been mainly designed to target regimes of parameter-invariant, stationary pulse generation. In [5], we have demonstrated, for the first time, the use of GAs for searching and optimising the breather regime in ultrafast lasers. We have implemented a GA in a laser mode-locked through a four-parameter nonlinear polarisation evolution, and we have designed merit functions relying on the characteristic features of the radiofrequency spectrum of the laser output, which are capable to locate various self-starting breather states in the laser, including single breathers with controllable breathing ratio and period, and BMCs with a controllable number of elementary constituents (Fig. 1(b)). The use of a GA with a merit function designed to exploit the distinguishing trait of frequency-locked breather states, i.e., a high SNR of the breathing frequency, has been the key to tailoring the frequency locking process in [4].

4. Conclusion

Our work further demonstrates that mode-locked fibre lasers are an ideal test bed for the study of complex nonlinear wave dynamics relevant to a large variety of physical systems. The transitions between subharmonic and non-subharmonic breather structures discussed here, which mirror the well-known commensurate-incommensurate phase transitions in condensed-matter physics, are not restricted to a single-breather laser emission regime, but we have observed alike transitions also when the laser operates in a diatomic or triatomic breather molecule generation regime. This further substantiates the universal nature of the frequency-locking phenomenon in nonlinear systems with two competing frequencies. Our recent advances in the intelligent control of non-stationary nonlinear wave dynamics also include the first demonstration of the possibility to use GAs to promote the emergence and control the intensity of rogue waves in nonlinear optical systems.

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