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RESEARCH ARTICLE

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The Skewness-Kurtosis plane for cryptocurrencies' universe

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Abstract

Cryptocurrency returns diverge excessively from normality, with the interrelationship of *Skewness* and *Kurtosis* being accordant with a parabolic form, yet this connection is scantly documented. We begin by demonstrating diagrammatically the attributes of the *S-K plane* for cryptocurrencies. Moreover, by taking advantage of the panel structure of the data, we estimate a quadratic model for the S-K plane. Then we investigate whether the type and the infrastructure of the cryptocurrency, as well as the period under examination, alter the architecture of the plane. We find that the squared *Skewness* of tokens substantially lowers the slope of Kurtosis, while the same applies to the earlier era of the market.

KEYWORDS

bitcoin, cryptocurrencies, higher moments, kurtosis, skewness

1 | INTRODUCTION

It is broadly discussed in the literature, that returns of several financial asset classes exhibit deviations from Gaussianity (i.e., Campbell et al. (1997)). *Skewness* captures the distortion, in comparison to a normal distribution, separately for each tail, demonstrating the possibility for extreme events occurrence and whether it's a positive or negative one. *Kurtosis* on the other hand measures the extreme values in either tail, describing the shape of a distribution's tails in contrast to its overall shape. What is less known however, especially within finance, is that *Kurtosis* and *Skewness* are correlated. Pearson (1916) and Klaassen et al. (2000) proved that their interrelationship obeys well-defined rules, yet not exact.

The launch of the Bitcoin (Nakamoto (2008))—the first cryptocurrency—was followed by a surge in the creation of other altcoins[1] resulting in the explosive expansion of the cryptocurrency market over the past decade, both in terms of its intensive and extensive margin (Ballis and Drakos (2021)). What this essentially means is that this expansion consisted not only of an immense increase in the number of traded cryptocurrencies, but also by a rather significant inflow of funds in the newly born cryptocurrency market. Today (as of June 2021), the total cryptocurrency market capitalization stands well above 1.5 trillion dollars.

Over the past few years, this erratic market and this new type of asset have managed to capture the attention of academia, resulting into a growing academic literature. In a systematic review regarding the empirical academic literature, Corbet et al. (2019) indicate that in the topic of cryptocurrencies both quantitative and non-quantitative approaches coexist. The latter puts mainly under scrutiny topics like cybercriminality and regulation (Vandezande

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(2017); Hendrickson and Luther (2017)). In terms of quantitative research Urquhart and Zhang (2019) examined the market's hedging capabilities in their study, whereas Urquhart (2016) and Wei (2018) had market efficiency under their spotlight. Cheah and Fry (2015) and Corbet et al. (2018) deal with asset pricing bubbles. Furthermore, other topics like the herding behaviour exhibited at the market (Bouri et al. (2019); Ballis and Drakos (2020)) and its high volatility (Feng et al. (2018); Katsiampa (2017)) have at the spotlight of the research community. Finally, another strand of literature that has drawn increasing volume of academic interest and attention is price dynamics (Phillip et al. (2018)).

Our research aims at contributing to the fast expanding cryptocurrency literature by investigating an often-neglected issue, which is how the higher moments are interrelated. To do so, we utilize all available cryptocurrencies, over a 5-year period. The novelty of this research is twofold. First and foremost, to the best of our knowledge, and despite the rapidly growing academic literature on cryptocurrency issues, this is the first study that puts under scrutiny the interrelationship of *Skewness* and *Kurtosis* within the cryptocurrencies market. Second, this study takes under consideration parameters pertinent to the functioning of cryptocurrencies, such as the *Type* of the asset and its *Mineability*.

The rest of this paper is organized as follows. The next section offers a literature review on the issue at hand. Section 3 describes the dataset and the construction of the variables used in the analysis. Section 4 provides a review of the empirical methodology deployed in the analysis. Section 5 presents and discusses the empirical findings. Finally, Section 6 presents the conclusions of this study.

2 | LITERATURE REVIEW

Among the most typical historical anomalies existent on 'unformed' markets, momentum stands out. Within cryptocurrencies research, Caporale and Plastun (2020), investigate whether a momentum impact is present in the cryptocurrency market, whereas Liu et al. (2020) identify three common risk factors in the returns on cryptocurrencies. In their study Chu et al. (2020) indicate that the momentum method has the potential to be utilized successfully for Bitcoin trading at a high frequency, and Jia et al. (2022) introduce and test a three-factor pricing model including market, size, and momentum factors, outperforming relevant models suggested in the literature.

As the literature has established (Wilkins (1944); Groeneveld and Meeden (1984); MacGillivray and Balanda (1988)), in chaotic systems that deviate from normality, the *S-K* plane is in general compatible with a parabolic form, but the precise structure is contingent on various factors and almost certainly includes case-specific components (Alberghi et al. (2002)). Vargo et al. (2017) depicted the *Skewness-Kurtosis* relationship, for 37 different distributions and offered a roadmap for the appropriate selection of a distribution based on empirical data. This interrelationship has been investigated in various physical phenomena (Schopflocher and Sullivan (2005), Sattin et al. (2009), Cristelli et al. (2012)) and in a very restricted set of asset classes (ibid). In such unstable systems this interrelationship gives rise to a *S-K* plane that conveys rich information about the joint realization of *Kurtosis* and *Skewness* and the subsequent obedience or deviation from normality of the underlying returns' generation process.

In other fields, for instance for healthcare costs, the choice of the appropriate distribution has been a subject with wide coverage (Jones et al. (2014); Jones et al. (2015); Mauler and McDonald (2015)). McDonald et al. (2013) utilized the observed *Skewness* & *Kurtosis* on *GB1* and *GB2* distributions. Afterward, income data were fitted and a broad comparison was attempted between *Weibull*, *gamma*, *Pareto* distributions; with *GB2* accommodating the greatest levels of *Kurtosis* for positive *Skewness* (fat tails with positive returns).

Building upon the aforementioned research, we are exploiting parts of the methodology, while expanding and applying it in the cryptocurrency market. Undoubtedly cryptocurrencies, due to their structure and nature, are exposed to excess skewness and kurtosis and the risks deriving from them. Jia et al. (2021) analysing the cross-sectional return predictability of the higher moments of 84 cryptocurrencies showcase a positive relationship between kurtosis and volatility related to future returns, while the predictability of returns for skewness is found to be negative. Karagiorgis and Drakos (2022) explored how skewness and kurtosis of hedge funds' returns are interrelated and what are the differences among the various investment strategies. Therefore, we structure our analysis based on the aforementioned research.

Regarding the interrelationship between the higher moments, Pearson (1916) established as the lower bound Equation (1):

$$K \geq S^2 + 1, \tag{1}$$

While Schopflocher and Sullivan (2005) and later Sattin et al. (2009) concluded to a more general form of the *Skewness-Kurtosis* taking the form of a quadratic Equation (2).

$$K = A \times S^2 + B, \tag{2}$$

Moreover, Klaassen et al. (2000) approximated the lower bound of the *Skewness-Kurtosis* relationship, in a more specific form with the formulation of Equation (3).

$$K \geq S^2 + \frac{189}{125} \quad (3)$$

In a relatively recent work, Cristelli et al. (2012) experimented by comparing physical phenomena, such as earthquakes, with financial markets, taking as an example the S&P 500. The main goal was to diagnose if their respective higher moments have similarities and whether a universal power-law could be established. The equation of the proposed power-law was the following:

$$K = N^{\frac{1}{3}} \times S^{\frac{4}{3}} \quad (4)$$

Moreover, a validity factor Δ was created as described in 5 in order to test the legitimacy of the power law regime as described by Cristelli et al. (2012). Variable r represents the return of the respective cryptocurrency by week, while r_{max} is the maximum return of the given period *N*. The proposed check levels are $\Delta < 1$, $1 < \Delta < 10$, $\Delta > 10$. $\Delta < 1$ indicates a normal distribution, $1 < \Delta < 10$ demonstrates an intermediate state, while if Δ takes a value greater than 10, suggests that the distribution is vastly dominated by the extreme events.

$$\Delta = \frac{r_{max}^4}{\sum_{i=1}^{N-1} r^4}. \quad (5)$$

3 | DATA ISSUES AND VARIABLES CONSTRUCTION

In this section, we describe the process followed to collect the data and then we analyse the variables constructed. Then, we provide the descriptive statistics of the core variables, while we exhibit some introductory graphs to set the basis of the methodology in the following section.

3.1 | Data issues

Data on daily closing prices of the cryptocurrencies were collected for 4142 distinct cryptocurrencies (coinmarketcap.com[2]) spanning the period from January 1st 2016 to November 2nd 2020, resulting into 2,223,794 observations. Both active and inactive cryptocurrencies are selected in order to avoid potential survivorship bias. Besides the daily prices, we collect data for two additional cryptocurrency-specific variables. The first is the type, which denotes whether the cryptocurrency is a *Coin* or a *Token*, while the second denotes whether the cryptocurrency can be *Mined* or not.

A mineable cryptocurrency is initially created and acquired through the process of cryptocurrency mining, mainly as a" reward" for verifying the transactions of a block in a particular blockchain. On the other hand, non-mineable cryptocurrencies are cryptocurrencies that cannot be mined by individuals utilizing their computational power. This category includes cryptocurrencies whose maximum supply has been already achieved or cases, that depending on the fundamental design of each particular cryptocurrency, the total amount of currency units is not yet in circulation. Furthermore, it is important to comprehend the significant difference between *Coins* and *Tokens*. Even though both these words are quite often used interchangeably, they denote two quite different concepts in the cryptocurrency market. *Coins* refer to cryptocurrencies that are built on their independent blockchains, like *Bitcoin (BTC)*, *Ethereum (ETH) and Litecoin (LTC)* among others. These independent blockchains may have different characteristics (size of chain, procedures, mining process, performance, etc.). On the other hand, the term token refers to cryptocurrencies that do not operate on an independent blockchain and are built on another blockchain, with the *Ethereum* blockchain and its smart contracts technology being one of the most popular choices.

3.2 | Variables construction

We proceed by calculating the daily returns using Equation (6), where *i* denotes the cryptocurrency, *t* the time period and *P* the closing prices.

$$R_{i.t} = ln\frac{P_{i,t}}{P_{i,t-1}}. \quad (6)$$

Skewness and *Kurtosis*[3] which are the core variables under consideration, are calculated on a *weekly*[4] basis, as shown in Equations (7) and (8), respectively. *N* stands for the total number of observations, μ is the sample mean, while σ is the standard deviation and *r* the cryptocurrency returns:

$$S = \frac{1}{N}\sum_{i=1}^{N}\frac{(r_i - \mu)^3}{\sigma^3}, \quad (7)$$

$$K = \frac{1}{N}\sum_{i=1}^{N}\frac{(r_i - \mu)^4}{\sigma^4}. \quad (8)$$

A set of dummy variables are constructed to facilitate our analysis of the cryptocurrencies *S-K* plane. *Crypto type*, takes the value 0 if the cryptocurrency is a coin,

TABLE 1 Descriptive statistics of higher moments by category

		Skewness					Kurtosis				
	Obs	Mean	StDev	p25	p50	p99	Mean	StDev	p25	p50	p99
Tokens	178,652	0.168	0.834	−0.362	0.164	2.007	2.593	0.923	1.878	2.395	5.166
Coins	140,204	0.216	0.835	−0.307	0.214	2.015	2.614	0.948	1.879	2.403	5.165
Mineable	87,424	0.231	0.844	−0.290	0.232	2.023	2.631	0.961	1.881	2.418	5.166
Unmineable	231,432	0.173	0.830	−0.354	0.170	2.006	2.591	0.923	1.877	2.391	5.165
Maturity period	281,904	0.173	0.843	−0.359	0.172	2.017	2.610	0.938	1.881	2.406	5.166
Early period	36,952	0.312	0.759	−0.186	0.291	1.950	2.546	0.901	1.860	2.340	5.032
All	318,856	0.189	0.835	−0.338	0.186	2.011	2.602	0.934	1.878	2.398	5.166

Note: Obs is Observations, StDev is Standard Deviation, while p denotes the respective percentiles.

while it takes value 1 when it is a token. In the same manner, the variable *Mineable* takes the value 1 if the cryptocurrency can be mined, while it takes value 0 when it cannot. For the purposes of this research, we follow the analysis of Ballis and Drakos (2021) regarding the timeline of the expansion of the cryptocurrency market, and we estimate two distinct periods of the cryptocurrency market.

The variable *Era* separates the dataset between two periods; the early phase up until the end of 2017 and a maturity phase from 2018 until the end of the sample period. Finally, based on Equation (5) factor Δ is created. However, we utilize the *ln* version for illustration purposes.

3.3 | Univariate properties of variables

In Table 1 we provide the summary statistics of *Skewness* and *Kurtosis* for the variables in question. It is evident that the cryptocurrency market is almost split between *Tokens* and *Coins*. The latter has higher mean for both higher moments, while the median of the observations indicates a greater distance from *Normality* for *Coins' Skewness* and *Token's Kurtosis*, since the Gaussian *Normality* is the zero *Skewness* and three *Kurtosis*. Moreover, the *Unmineable* assets are the vast majority of the dataset and display relatively lower *Skewness* and *Kurtosis* than the *Mineable* assets. Additionally, the *Earlier* period of the market appears to deviate substantially more from *Normality* for both higher moments in comparison to the *Maturity* phase after 2017. *Unminable Tokens* dominate the rest of the categories in terms of observations and display the lowest mean *Skewness* out of the four categories. Finally, the sector as a whole exhibits a mean *Kurtosis* below *Normality*, while the majority of the observations are positively *Skewed*.

3.4 | Graphical illustration of the interrelationship between Skewness & Kurtosis

Utilizing a series of appropriate graphical illustrations, we aim to investigate if the theoretical parabolic relationship of the two higher moments is also present within the cryptocurrencies sector. In Figure 1 we show the relationship between *Skewness* and *Kurtosis*, essentially the *S-K* plane, for the whole set of the cryptocurrencies returns. It is clear, that while a portion of observations falls near the Gaussian values (*Kurtosis* = 3 and *Skewness* = 0), the vast majority of observations are located either at the tails of the plane with spikes at certain thresholds, or at higher *Kurtosis* levels with near-normal *Skewness*.

Figure 2 depicts the plane for the two different eras of cryptocurrencies, the early stages and the mature phase. While they appear to be similar, there are some distinct differences. The highest level of *Kurtosis* for the lowest level of *Skewness* appears to be lower for the *Early* (a cluster around K = 5, S = −2) period in comparison to the *Maturity* period (a cluster around K = 6, S = −2). Translating to fatter tails in the *Maturity* period with higher probability of extreme negative returns. On the other hand, the highest levels of *Kurtosis* for the positively skewed returns are about the same for both periods. Moreover, the tails appear to be fatter (*Kurtosis*) for the Maturity period, with a similar possibility of being on either tail (*Skewness*). The spikes at the various thresholds as previously seen are attributed to the later period.

4 | METHODOLOGY

Having the graphical evidence as a springboard, we proceed by enhancing the aforementioned mentioned

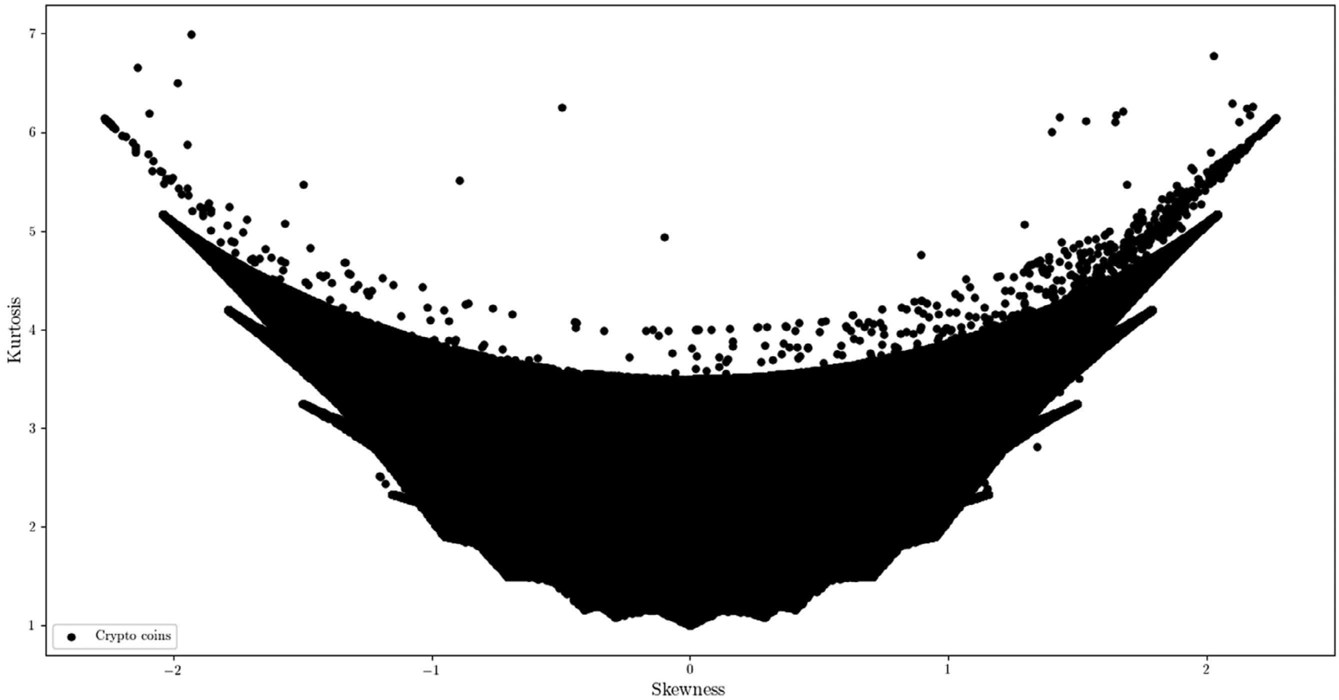

FIGURE 1 Skewness-Kurtosis relationship for crypto universe

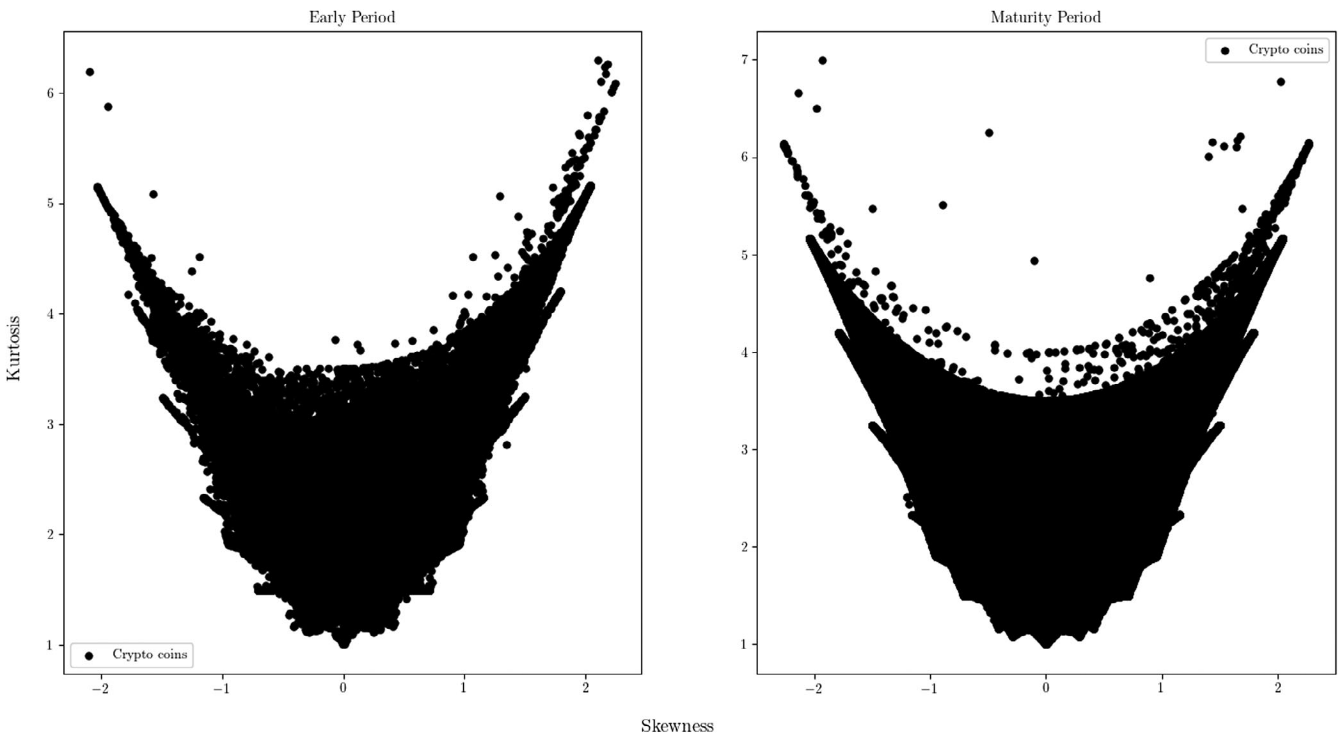

FIGURE 2 Skewness-Kurtosis relationship for crypto universe separated for Early period (−2017) and Maturity period (2018-)

Equations (1) and (2), that theoretically describe the Skewness-Kurtosis relationship. By fitting two versions, one only with the quadratic relationship and one also including a linear term of Skewness, as shown below in Equations (9) and (10). We expect these models to describe adequately the interrelationship of the cryptocurrency higher moments, given the fundamental nature of the relationship and the graphical evidence already acquired.

$$K_{i,t} = \beta_0 S_{i,t}^2 + \varepsilon_{i,t}, \tag{9}$$

$$K_{i,t} = \beta_0 S_{i,t} + \beta_1 S_{i,t}^2 + \varepsilon_{i,t}, \tag{10}$$

In order to further gauge this relationship, we augment the above models with cryptocurrency-specific variables and test relevant hypotheses about which characteristics of each asset affect the *K-S plane*. Therefore, we proceed by utilizing the dummy variables which we introduced earlier, initially as an interaction effect with $Skewness^2$ and later also standalone. Commencing with Equations (11) and (12), we include the cryptocurrency *Type* as *CT*.

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 CT_i + \varepsilon_{i,t}, \tag{11}$$

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 CT_i + \beta_2 CT_i + \varepsilon_{i,t}. \tag{12}$$

In the same spirit, Equations (13) and (14) include the dummy variable *Mineable* as *M*, which captures the cryptocurrency's fundamental infrastructure.

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 M_i + \varepsilon_{i,t}, \tag{13}$$

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 M_i + \beta_2 M_i + \varepsilon_{i,t}. \tag{14}$$

Equation (15) considers jointly the two interaction terms used so far. Furthermore, Equation (16) adds the actual dummy variables on top of the interaction terms to explore their direct effect on *Kurtosis*.

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 CT_i + \beta_2 S_{i,t}^2 M_i + \varepsilon_{i,t}, \tag{15}$$

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 CT_i + \beta_2 S_{i,t}^2 M_i + \beta_3 CT_i + \beta_4 M_i + \varepsilon_{i,t}. \tag{16}$$

Then, we introduce the dummy variable *Era* as *E* on Equations (17) and (18). With that particular variable, we are able to compare the early cryptocurrency market period to the more recent era, investigating whether there are any discernible changes.

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 E_i + \varepsilon_{i,t}, \tag{17}$$

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 E_i + \beta_2 E_i + \varepsilon_{i,t}. \tag{18}$$

Consequently, estimation 19 includes all the interaction terms and 20 also incorporates the direct impacts of the standalone dummy variables:

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 E_i + \beta_2 S_{i,t}^2 CT_i + \beta_3 S_{i,t}^2 M_i + \varepsilon_{i,t}, \tag{19}$$

$$K_{i,t} = \beta_0 S_{i,t}^2 + \beta_1 S_{i,t}^2 E_i + \beta_2 S_{i,t}^2 CT_i + \beta_3 S_{i,t}^2 M_i + \beta_4 E_i + \beta_5 CT_i + \beta_6 M_i + \varepsilon_{i,t}. \tag{20}$$

For each equation, from 11 to 20, we test the hypothesis for zero interaction effect and for zero direct and total effects where applicable.

5 | EMPIRICAL RESULTS

In Figure 3, we visually assess the fitness of the three equations discussed in the literature, for cryptocurrencies. Among the competing representations of the *S-K* plane, the one proposed by Klaassen et al. (2000) ($K = S^2 + 189/125$), seems to have the best fit for the totality of the sector. The Pearson (1916) formula, encompasses a wider area of the plane with the lower constant factor. Although the proposed power law of Cristelli et al. (2012) can successfully fit the flanks of the *S-K* plane, it lacks to provide an adequate representation for the rest of the relationship.

Furthermore, Figure 4 depicts the same formulae for the two different periods that we have proposed. As anticipated, the equation by Klaassen et al. (2000) emerges as the best fitting for the *S-K* plane for both subperiods.

With Figures 5 and 6 we depict the literature formulae on the various cryptocurrencyspecific categories. Figure 5, demonstrates the *Type* of the asset, while Figure 6 the *Infrastructure*. Undoubtedly, *Tokens* show higher levels of *Kurtosis* when compared to *Coins*, yet again the equation of Klaassen et al. (2000) fits both equally and superiorly to the other two formulae.

When segregated for *Infrastructure*, it appears *Unmineable* assets produce slightly fatter tails and resemble the *Tokens* behaviour, which is justified since the vast majority of the *Tokens* are *Unmineable*. As expected, Klaassen et al. (2000) appears to have a greater fit on the cryptocurrency market.

Moreover, Figure 7 is an ambitious effort to provide an extensive view of the interrelationship between *Skewness-Kurtosis*. It depicts how the higher moments behave through time, with the addition of factor Δ as proposed by Cristelli et al. (2012). For demonstration purposes we are using $ln(\Delta)$. Having set 1 (0 for *ln*) as the initial threshold which would represent the normal distribution, it accounts for somewhere below 25% of the returns. The intermediate state ($ln(\Delta) < 2.3$) represents about half of the dataset's observations, while those dominated by the extreme events demonstrate similar spikes at certain

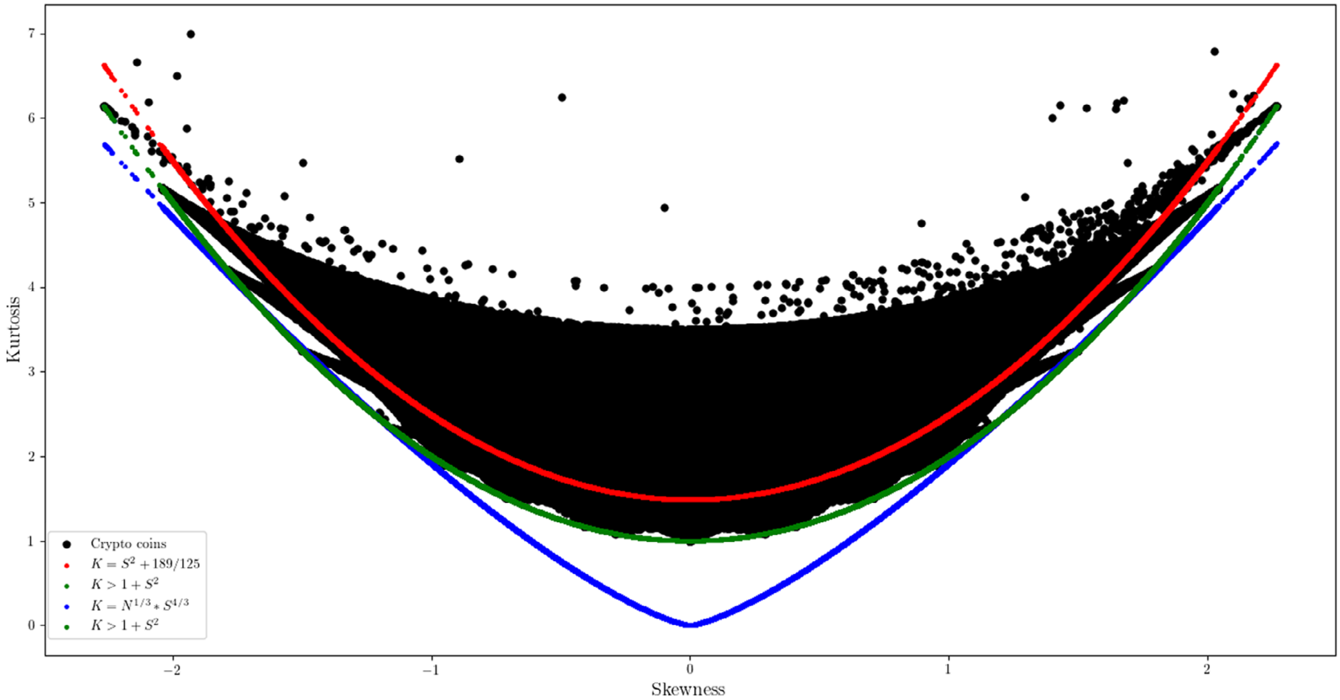

FIGURE 3 Skewness-Kurtosis relationship for crypto universe with fitted equations [Colour figure can be viewed at wileyonlinelibrary.com]

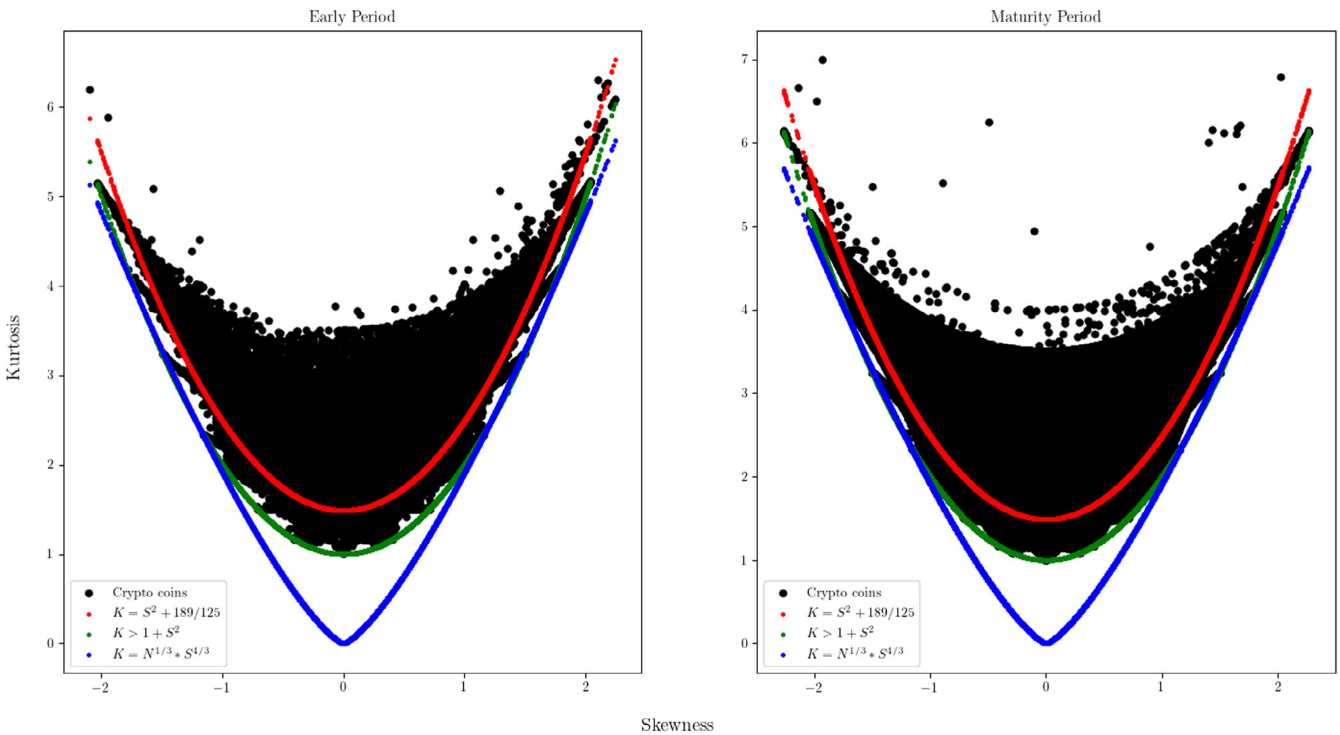

FIGURE 4 Skewness-Kurtosis relationship for crypto universe separated for Early period (−2017) and Maturity period (2018-) with fitted equations [Colour figure can be viewed at wileyonlinelibrary.com]

thresholds of higher levels of *Kurtosis*. High Δ observations can be traced across the plane, but the most extreme values are mainly located on the highest and lowest levels of *Skewness* as anticipated. Apparently, Δ increases gradually when *Kurtosis* reaches values above 4, while a significant cluster also exists at relatively low

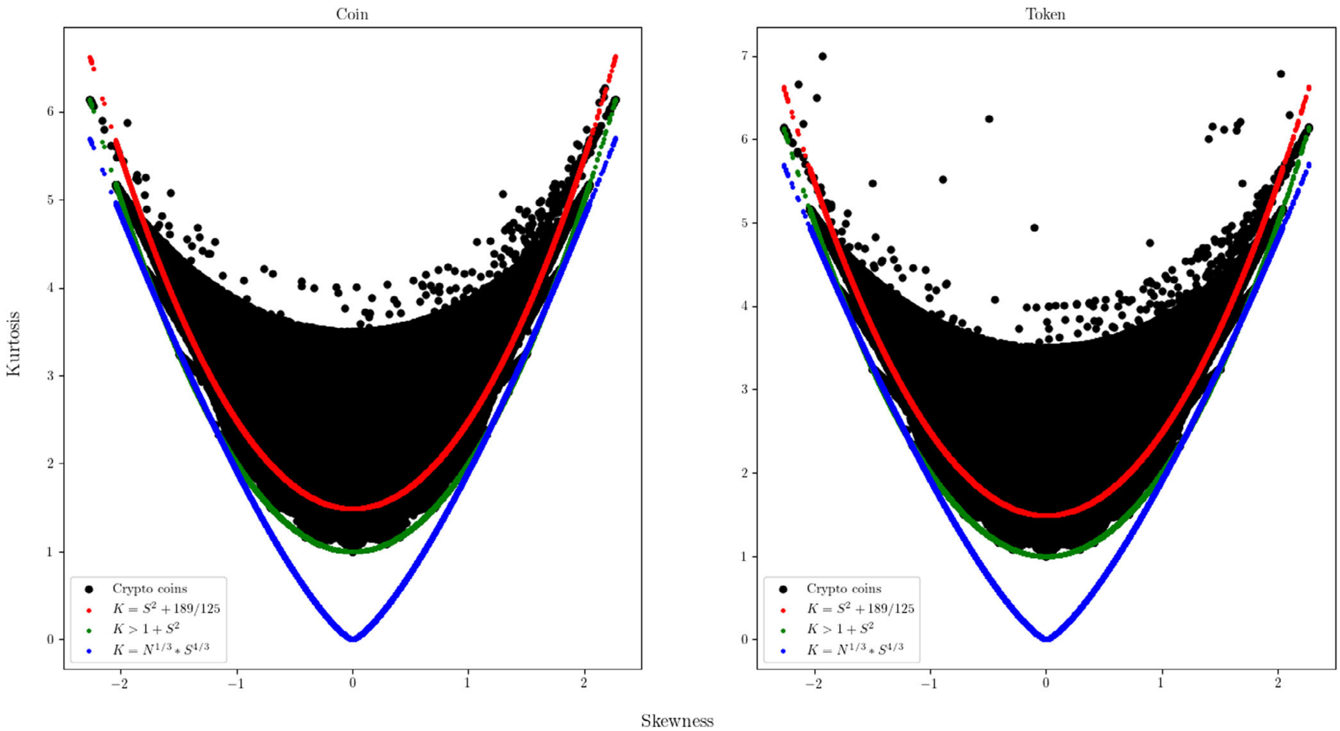

FIGURE 5 Skewness-Kurtosis relationship for crypto universe separated for instrument type with fitted equations [Colour figure can be viewed at wileyonlinelibrary.com]

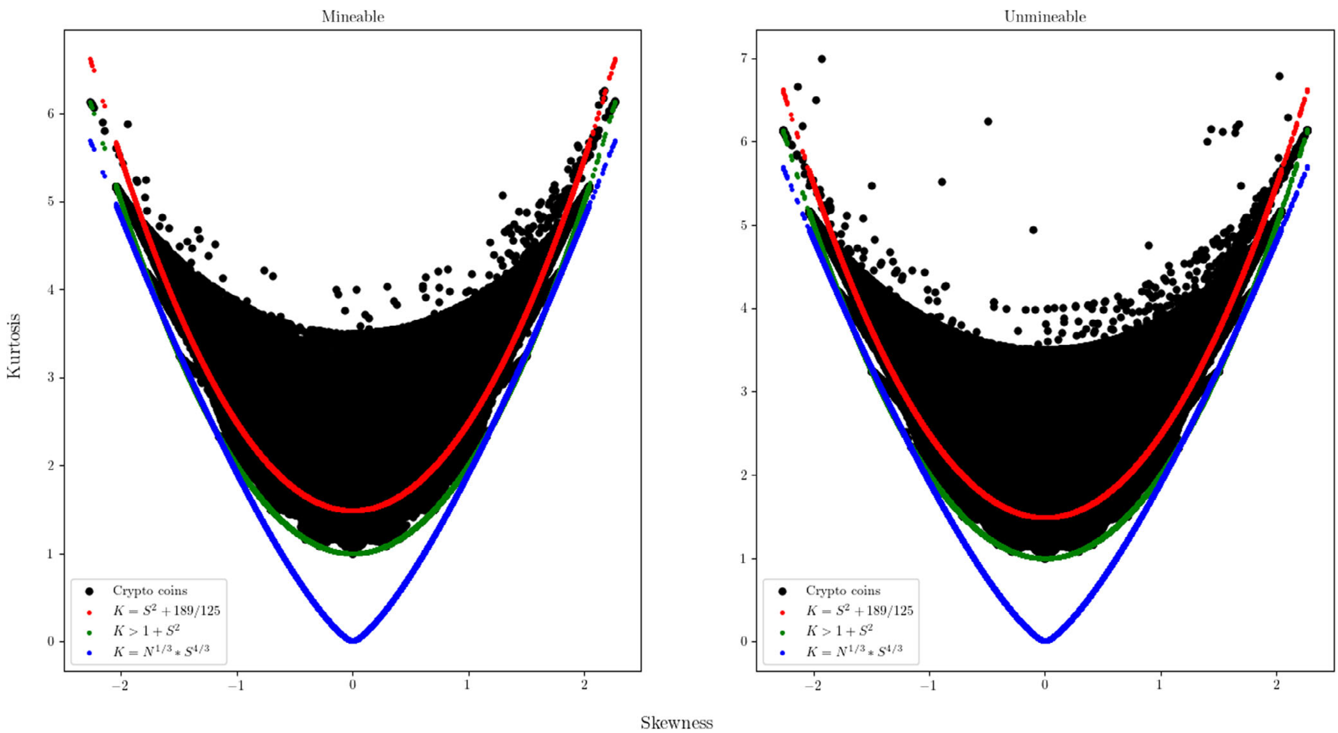

FIGURE 6 Skewness-Kurtosis relationship for crypto universe separated for mineability with fitted equations [Colour figure can be viewed at wileyonlinelibrary.com]

Kurtosis while *Skewness* is around the *Normality* region. It's also visible that $ln(\Delta)$ escalates to values, well above the 2.3 threshold, demonstrating the erratic behaviour of cryptocurrencies. As for the time dimension, it is evident

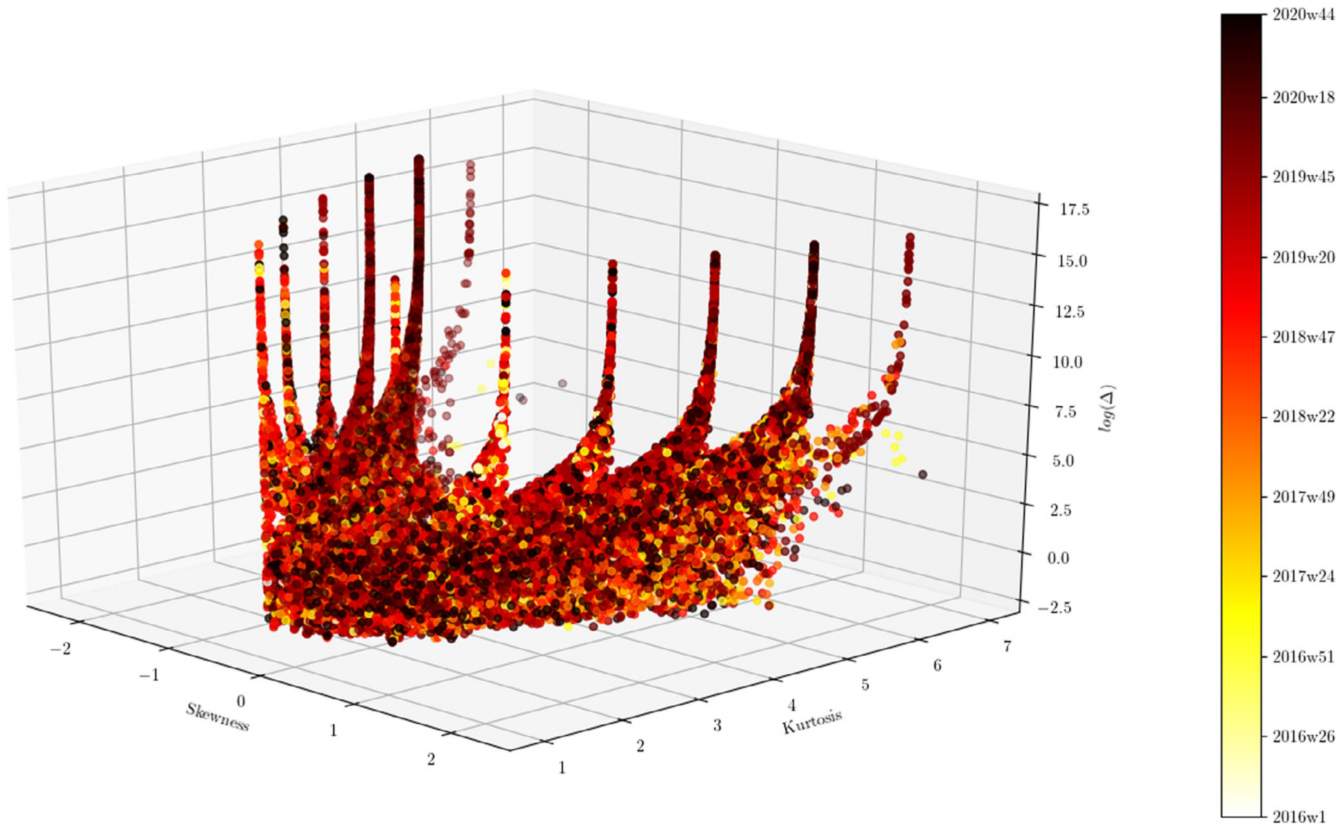

FIGURE 7 Skewness-Kurtosis-time-Δ relationship [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 2 Quadratic model

Kurtosis		(9)	(10)
Skewness^2		0.838***	0.833***
		(0.000825)	(0.000846)
Skewness			0.0237***
			(0.000974)
Constant		1.980***	1.979***
		(0.00210)	(0.00205)
Observations		318,856	318,856
R-squared	Within	0.7624	0.7629
	between	0.8022	0.8044
	Overall	0.7703	0.7707

Note: Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, and *$p < 0.1$.

that darker colours, hence during the COVID-19 era, dominate the extreme areas.

We proceed by examining in a formal econometric way the exact shape of the *S-K* plane, building on top of the visual evidence we accumulated. Table 2 displays the regression results for the quadratic equation, without (equation 9) and with (Equation (10)) the linear term; both are estimated with random effects. On both occasions, *Skewness*^{2} appears to have a positive and statistically significant coefficient at 1%, while the estimated coefficients resemble the findings reported by Karagiorgis and Drakos (2022) for hedge funds' returns, yet somehow on lower levels. Additionally, plasma physics literature, being the sector with similar performed researches, also exhibits similar numbers. On the other hand, *Skewness* also has a minor positive coefficient with the same, highest, statistical significance. R-squared is around 80% supporting the hypothesis that the quadratic equation has a robust explanatory ability.

In order to compare the performance of Equations (3) and (4) along with the estimation of 10 on the cryptocurrency data, we produced a series of metrics. Initially, the difference between the actual Kurtosis and those generated by Klaassen et al. (2000) and Cristelli et al. (2012) equations was created, as a residual generation. Then, the Mean Squared Error (MSE), the Root-MSE for Equations (3) and (4) and the Standard Error of Regression (SER) for all three. Table 3 demonstrates the aforementioned metrics, from which we deduce that Klaassen et al. (2000) equation has the minimum SER between the models, but the basic quadratic relationship takes similar values.

In order to test for time variance of the quantified interrelationship of the cryptocurrency sector ($K = 0.838 \times S^2 + 1.980$ (95% *CI* [0.8363, 0.8396], [1.9757, 1.9839])), we perform a Chow stability test. As demonstrated in Table 4 we segregated the dataset in accordance to the *Era* variable we have already created. It appears that there is a slight variance among the periods, a conclusion backed by the Chow test, which takes the value 897, therefore rejecting the null hypothesis for stable coefficients between the periods and validating the hypothesis behind the prior analysis.

TABLE 3 Fitting of equations

	Root-MSE	MSE	SER
Quadratic	0.670	0.447	0.823
Klaasen	0.595	0.354	0.559
Cristelli	1.451	2.105	0.617

Note: Error and SER the standard error of regression.
Abbreviation: MSE, mean squared.

TABLE 4 Stability estimations

Kurtosis		2016–2017	2018–2020
Skewness^2		0.881***	0.833***
		(0.0026)	(0.0009)
Constant		1.953***	1.992***
		(0.0028)	(0.0010)
Observations		36,952	281,904
R-squared	Within	0.760	0.762
	Between	0.746	0.807
	Overall	0.758	0.772

Note: Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, and *$p < 0.1$.

The various cryptocurrency-specific variables will provide an additional framework to better comprehend what affects the interrelationship between the two higher moments. Table 5 commences with equation 11, where the interaction of *Skewness*^{2} with *Tokens* has a negative and highly statistically significant effect on the slope of *Kurtosis*. When including the direct effect of *Tokens* in comparison to *Coins* (Equation (12)), the interaction term holding its statistical significance albeit with a lower negative effect to the slope of *Kurtosis*; *Tokens* also reduce the level of *Kurtosis* in comparison to *Coins*. Meaning that for positive *Skewness*, *Coins* are the preferable asset for an investor due to higher *Kurtosis*. The three hypotheses for zero interaction/direct/total effect are all rejected. On the estimations of 13 and 14, we incorporate the mineability of each cryptocurrency. It appears that the interaction effect is zero, while the direct effect is positive but significant only at 10%.

TABLE 5 Quadratic model: controlling for crypto type and mineability

Kurtosis		(11)	(12)	(13)	(14)	(15)	(16)
Skewness^2		0.843***	0.842***	0.837***	0.837***	0.848***	0.847***
		(0.00119)	(0.00122)	(0.000974)	(0.000984)	(0.00197)	(0.00205)
Skewness^2 × Crypto type		−0.00871***	−0.00732***			−0.0139***	−0.0123***
		(0.00159)	(0.00166)			(0.00222)	(0.00233)
Crypto type			−0.0125***				−0.0129**
			(0.00429)				(0.00571)
Skewness^2 × Mineable				0.00256	0.00172	−0.00812***	−0.00773***
				(0.00174)	(0.00180)	(0.00244)	(0.00253)
Mineable					0.00917*		−0.00153
					(0.00521)		(0.00685)
Constant		1.981***	1.989***	1.980***	1.978***	1.980***	1.989***
		(0.00208)	(0.00344)	(0.00209)	(0.00233)	(0.00208)	(0.00511)
Observations		318,856	318,856	318,856	318,856	318,856	318,856
R-squared	Within	0.7624	0.7624	0.7624	0.7624	0.7624	0.7624
	Between	0.8030	0.8046	0.8023	0.8029	0.8032	0.8046
	Overall	0.7703	0.7703	0.7703	0.7703	0.7703	0.7703
Joint test for zero interaction effects		H 30.17***	1 19.53***	2.17	0.91	4 41.26*** 282	28.90***
Joint test for zero direct effects		–	8.51**	–	3.09*	–	7.88**
Joint test for overall zero direct effects		–	38.8***	–	5.27*	–	49.32***

Note: Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, and *$p < 0.1$.

TABLE 6 Quadratic model: controlling for era differentiation

Variables		(17)	(18)	(19)	(20)
Skewness^2		0.867***	0.878***	0.874***	0.884***
		(0.00222)	(0.00259)	(0.00280)	(0.00313)
Skewness^2 × Era		−0.0314***	−0.0444***	−0.0304***	−0.0446***
		(0.00228)	(0.00273)	(0.00234)	(0.00283)
Skewness^2 × Crypto type				−0.0102***	−0.00624***
				(0.00224)	(0.00236)
Skewness^2 × Mineable				−0.00916***	−0.00931***
				(0.00244)	(0.00254)
Era			0.0292***		0.0305***
			(0.00340)		(0.00346)
Crypto type					−0.0173*** (0.00572)
Mineable					−0.000249 (0.00685)
Constant		1.980***	1.954***	1.981***	1.964***
		(0.00209)	(0.00376)	(0.00208)	(0.00593)
Observations		318,856	318,856	318,856	318,856
R-squared	Within	0.7626	0.7626	0.7626	0.7626
	Between	0.8025	0.8022	0.8030	0.8047
	Overall	0.7704	0.7704	0.7704	0.7705
Joint test for zero interaction effects		189.68***	2122222ss 263.47***	210.86***	277.03***
Joint test for zero direct effects		–	73.91***	–	86.59***
Joint test for overall zero direct effects		–	263.63***	–	297.46***

Note: Standard errors in parentheses. ***$p < 0.01$, **$p < 0.05$, and *$p < 0.1$.

As a next step, we estimate a model (15) with both interactions with *Skewness*^{2} utilized so far. While *Tokens* from crypto *Type* remains significant, its effect is magnified in comparison to the results from model 11. Moreover, the interaction term of the assets that are *Mineable* turns statistically significant at 1% and decreases the rate of change of cryptocurrencies *Kurtosis* in contrast to the *Unmineable* assets. The hypothesis for zero interaction effects is rejected. The final estimation shown in Table 5, adds the two dummy variables with the interaction terms retaining their behaviour as before, while *Mineable* cryptocurrencies displaying no effect on the level of *Kurtosis*. Again, all three hypotheses are rejected.

Undoubtedly the cryptocurrency market has reached a relatively mature phase in comparison to its earlier days. Hence, it is deemed as appropriate to induct a variable to capture any effects between the earlier phases of the market. As demonstrated in Table 6, *Skewness*^{2} of the latter period has a soothing effect to the slope of *Kurtosis* which is significant at 1% (Equation (17)), while the direct effect of *Era* on *Kurtosis* (18) is positive. Again, all the hypotheses for zero effects on *Kurtosis* are rejected. The third estimation (Equation (19)) constitutes of all the interaction terms encountered so far; all retain their significance as anticipated. It is worth noting, that the interaction of variable *Mineable* is insignificant when used as the sole independent variable but becomes significant at 1% when additional interaction terms are incorporated into the estimation. In the final estimation of the Table, it is demonstrated that all variables affect *Kurtosis*, yet at a subdued level in comparison to the separate estimations. All the hypotheses, for zero interaction/direct/total effects are rejected.

6 | CONCLUSIONS

The literature investigating the interrelation between *Skewness* and *Kurtosis* is barely sufficient, especially for financial assets returns. The cryptocurrency market is a relatively new asset class with distinctive traits and as such is drawing increasing volumes of novel research in an attempt to grasp and establish its fundamental properties. Our study contributes to the literature by exploring the shape of the *S-K* plane with data from the cryptocurrency market. We offer diagrammatic representations of the *S-K* plane for the whole market, as well as for two different eras of the market, the *Type* of the asset and its *Mineability*. To the best of our knowledge, this is the first time that such dimensions are taken under consideration in the cryptocurrency literature. Moreover, we address the ability, of the already proposed in the literature alternative models, to outline this plane. We confirmed that the cryptocurrencies' higher moments exhibit a parabolic form, in accordance with the literature.

Furthermore, based on a formal econometric framework, we estimate a quadratic model for the *S-K* plane. We find that the relationship for the cryptocurrencies takes the form of $K = 0.838 \times S^2 + 1.980$. After establishing the ability of the quadratic model to superiorly describe the *S-K* plane, we further investigate whether the structure of the plane exhibits any significant variations whose source can be traced to the market's *Maturity*, or cryptocurrency-specific characteristics and mainly their *Type* and *Infrastructure*. We confirmed that there are variations among the effects of the *Era* and cryptocurrency characteristics on the *S-K* plane. The combination of the two cryptocurrency characteristics though did not seem to offer a valuable.

With Figure 7 we aspired to exhibit the interrelationship of the higher moments along with factor Δ (Cristelli et al. (2012)) which would display whether the distribution is dominated by the extremes and time. Volatile periods, such as the Covid-19 era are located in the flanks with excess *Kurtosis* and high Δ values, validating the hypothesis of the extreme events domination.

To the best of our knowledge, this research is the first to examine the interrelationship of *Skewness* and *Kurtosis* within the cryptocurrencies market. Consequently, the fitness of the known formulas of the literature on such a dataset is also attempted for the first time, producing comparable results to the other scientific fields. Besides academia, the findings of this study provide useful information for investors. For market participants, the results of this research could be valuable for evaluating risk, especially in a market that exhibits constant and enormous price movements, within short periods of time.

Future directions of research could include testing the same methodology on different financial datasets or as a case study in particular volatile periods, such as Covid-19, which could produce valuable outcomes for the behaviour of the higher moments.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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ENDNOTES

[1] The term Altcoin is a portmanteau of" alternative" and" coin". It refers ultimately to all the cryptocurrencies other than Bitcoin. Altcoins use similar cryptography technology but are based on different algorithmic structures.

[2] http://coinmarketcap.com

[3] 252 dummy variables were created, one per week of the dataset, in order to test each higher moment for time variance. For *Skewness* 236/251 weeks and for *Kurtosis* 233/251, demonstrate statistically significant variance. For the sake of brevity we do not report the table, but results are available upon request.

[4] We have conducted the same analysis with *Skewness* and *Kurtosis* calculated on a *monthly* basis, with the results being qualitatively similar and available upon request.

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