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The Emergence of the Noninteracting Channel in the Strongly Interacting 1D System

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A strongly interacting 1D system with many channels is studied. When single-electron interchannel backscattering processes become relevant, new fields are defined. Some fields are frozen (gapped) by these relevant perturbations, whereas others remain free. A mathematical procedure to separate gapped and conducting channels is proposed. The problem for a two-channel system with particular relevant perturbation is then solved exactly for arbitrary intra- and interchannel interactions, and effective Luttinger parameter and velocity for the free field are found. The parameters of the free field are independent of the interactions between right- and left-moving electrons in the same channel and between electrons moving in the same direction in different channels. Finally, if the interchannel interactions are weak, the free field becomes noninteracting (effective Luttinger parameter $K_{\text{eff}} = 1$) independently of how strong intra-channel interactions are.

1. Introduction

The physics of (quasi-) 1D systems is extremely rich and exhibits itself in a variety of phenomena, from edge states in quantum Hall effects[1] and topological insulators[2] to the most recent experimental results of the enhancement of superconductivity by disorder in quasi-1D materials[3] and zero-magnetic field fractional conductance in GaAs/AlGaAs heterostructures.[4] Strongly interacting electrons in 1D systems are described by Luttinger liquid (LL) theory.[5–8] Seminal renormalization group (RG) analysis[9] allowed to study the effect of a single impurity on the conductance of a one-channel LL based only on the Luttinger parameter K. It was later used for various two-channel LL systems[10–14] and was finally generalized for an arbitrary number of channels in LL by the introduction of the effective Luttinger matrix $\hat{K}$.[15] Generalized RG approach was used to study the effect of local impurity in a multichannel LL,[16] metal-insulator transition in sliding LL[17,18] and its instability,[19] and stability of edge states in topological insulators.[20,21] Recently two-terminal conductance with fractional values[22] and fractional short noise[23] were described theoretically in clean 1D systems with broken time-reversal symmetry. Experiments showed fractional transport in split-gate 1D constrictions made in germanium.[24]

The standard RG analysis studies possible perturbations evaluates the scaling dimension for each perturbation and defines perturbation as relevant when its scaling dimension becomes less than a physical dimension of the system. The initial phase is stable when all perturbations are irrelevant. The relevant perturbation opens a gap and transfers the system into a particular gapped phase. Relevant backscattering perturbations localize corresponding channels. The channel orthogonal to the localized one remains conducting.[25] Usually, the system analysis stops with the calculation of the remaining conductance.

In this letter, we go one step further. We start with a multichannel LL and suggest a mathematical procedure to separate gapped and conducting channels. Then, we solve the problem exactly for a two-channel system with particular relevant perturbation and study the properties of the remaining conducting channel.

2. Multichannel LL Model

The Lagrangian of the multichannel LL is

$$L_0 = \frac{1}{4\pi}\,\partial_t \boldsymbol{\theta}^{\mathrm{T}} \partial_x \boldsymbol{\phi} - \frac{1}{8\pi}\left[\partial_x \boldsymbol{\phi}^{\mathrm{T}} \hat{V}_{\phi}\, \partial_x \boldsymbol{\phi} + \partial_x \boldsymbol{\theta}^{\mathrm{T}} \hat{V}_{\theta}\, \partial_x \boldsymbol{\theta}\right] \tag{1}$$

where vectors $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$ are bosonic density and current fields correspondingly assigned to each channel. Matrices $\hat{V}_{\phi}$ and $\hat{V}_{\theta}$ describe intra- and interchannel density–density and current–current interactions. The most general perturbation can be written as[20]

$$L_{\text{pert}} = h(j, q)\, e^{i(j\boldsymbol{\phi} + q\boldsymbol{\theta})} \tag{2}$$

Luttinger matrix $\hat{K}$ can be obtained from the following equation[15]

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$$\hat{K}\hat{V}_{\phi}\hat{K} = \hat{V}_{\theta} \tag{3}$$

Its solution[26]

$$\hat{K} = \hat{V}_{\phi}^{-1}(\hat{V}_{\phi}\hat{V}_{\theta})^{\frac{1}{2}} \tag{4}$$

leads to the following scaling dimension of the perturbation

$$\Delta(j, q) = j\hat{K}j + q\hat{K}^{-1}q \tag{5}$$

If particular relevant perturbations freeze some combinations of the fields, it is natural to define a new basis containing these frozen combinations

$$\begin{pmatrix} \boldsymbol{\phi} \\ \boldsymbol{\theta} \end{pmatrix} = \hat{M}\begin{pmatrix} \boldsymbol{\phi}_{\text{new}} \\ \boldsymbol{\theta}_{\text{new}} \end{pmatrix} \tag{6}$$

The orthogonal matrix $\hat{M}$ has to preserve the form of the first term in Equation (1) (commutation relation)—we write it without derivatives for simplicity

$$\begin{aligned} \theta^{\mathrm{T}}\boldsymbol{\phi} &= \frac{1}{2}(\boldsymbol{\phi}^{\mathrm{T}}, \boldsymbol{\theta}^{\mathrm{T}})\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi} \\ \boldsymbol{\theta} \end{pmatrix} \\ &= \frac{1}{2}(\boldsymbol{\phi}_{\text{new}}^{\mathrm{T}}, \boldsymbol{\theta}_{\text{new}}^{\mathrm{T}})\hat{M}^{\mathrm{T}}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\hat{M}\begin{pmatrix} \boldsymbol{\phi}_{\text{new}} \\ \boldsymbol{\theta}_{\text{new}} \end{pmatrix} \\ &= \frac{1}{2}(\boldsymbol{\phi}_{\text{new}}^{\mathrm{T}}, \boldsymbol{\theta}_{\text{new}}^{\mathrm{T}})\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_{\text{new}} \\ \boldsymbol{\theta}_{\text{new}} \end{pmatrix} = \boldsymbol{\theta}_{\text{new}}^{\mathrm{T}}\boldsymbol{\phi}_{\text{new}} \end{aligned} \tag{7}$$

Therefore the additional condition for the orthogonal matrix $\hat{M}$ is

$$\hat{M}^{\mathrm{T}}\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\hat{M} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \tag{8}$$

3. Relevant Perturbation in a Two-Channel System

Below we consider a two-channel system and solve the problem exactly following the guidelines described above. Interaction matrices for two channels can be presented as

$$\begin{aligned} \hat{V}_{\phi} &= \begin{pmatrix} 1 + g_4 + g_2 & g_4' + g_2' \\ g_4' + g_2' & 1 + g_4 + g_2 \end{pmatrix} \\ \hat{V}_{\theta} &= \begin{pmatrix} 1 + g_4 - g_2 & g_4' - g_2' \\ g_4' - g_2' & 1 + g_4 - g_2 \end{pmatrix} \end{aligned} \tag{9}$$

where $g_{4(2)}$ is an interaction strength between electrons moving in the same (opposite) direction within the same channel, whereas couplings with prime have the corresponding meaning for the interchannel interactions. The solution for the Luttinger matrix $\hat{K}$ can be presented in the following form

$$\hat{K} = \frac{1}{2}\begin{pmatrix} K_{\parallel} + K_{\perp} & K_{\parallel} - K_{\perp} \\ K_{\parallel} - K_{\perp} & K_{\parallel} + K_{\perp} \end{pmatrix} \tag{10}$$

where we use notations introduced in ref. [20]

$$K_{\parallel} = K\sqrt{\frac{1 + (g_4' - g_2')/(1 + g_4 - g_2)}{1 + (g_4' + g_2')/(1 + g_4 + g_2)}} \tag{11}$$

$$K_{\perp} = K\sqrt{\frac{1 - (g_4' - g_2')/(1 + g_4 - g_2)}{1 - (g_4' + g_2')/(1 + g_4 + g_2)}} \tag{12}$$

K is a standard Luttinger parameter defined in the absence of interchannel interactions

$$K = \sqrt{\frac{1 + g_4 - g_2}{1 + g_4 + g_2}} \tag{13}$$

Next, we consider the following basic perturbation

$$L_{\text{rel}} \sim \exp\left[\frac{1}{2}i(\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2 + \boldsymbol{\theta}_1 - \boldsymbol{\theta}_2)\right] \tag{14}$$

describing interchannel counter-clockwise backscattering (see **Figure 1**). Its scaling dimension, according to Equation (5), is

$$\Delta = \frac{1}{2}\left(K_{\parallel} + \frac{1}{K_{\perp}}\right) \tag{15}$$

When, such disorder-induced perturbation is relevant ($\Delta < 3/2$), the corresponding channel $\boldsymbol{\phi}_{\text{g}} = (1/2)(\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2 + \boldsymbol{\theta}_1 - \boldsymbol{\theta}_2)$ becomes frozen (gapped). We want to stress that interchannel clockwise backscattering has the same scaling dimension, so for the system to remain nontrivial (not completely localized), we need to suppose that counter-clockwise backscattering is dominant over clockwise backscattering. This, for example, can be the case of a topological insulator in a magnetic field with backscattering caused by a particular harmonic of a disorder potential (suppressing clockwise backscattering[27]). In **Figure 2**, we present phase diagrams of the gapped channel $\boldsymbol{\phi}_{\text{g}}$ for equal intrachannel parameters $g_4 = g_2$, and two possible combinations of interchannel parameters: a) $g_2' = 0,\ g_4' \equiv g'$; b) $g_4' = g_2' \equiv g'$.

Now, we define a new basis including a gapped field $\boldsymbol{\phi}_{\text{g}}$. A field conjugate to $\boldsymbol{\phi}_{\text{g}}$ is

$$\theta_{\text{g}} = \frac{1}{2}(\boldsymbol{\phi}_1 - \boldsymbol{\phi}_2 + \boldsymbol{\theta}_1 + \boldsymbol{\theta}_2) \tag{16}$$

The field orthogonal to the gapped field remains free and together with its conjugate completes a new basis. In our case, these two fields are easily found

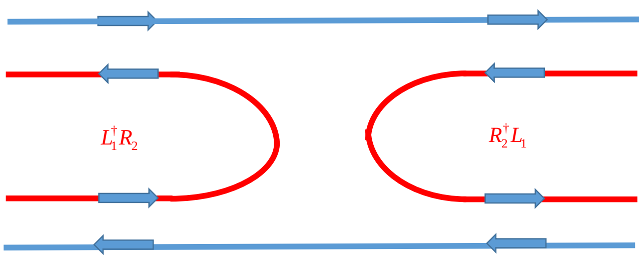

Figure 1. Relevant perturbation is a counter-clockwise interchannel backscattering.

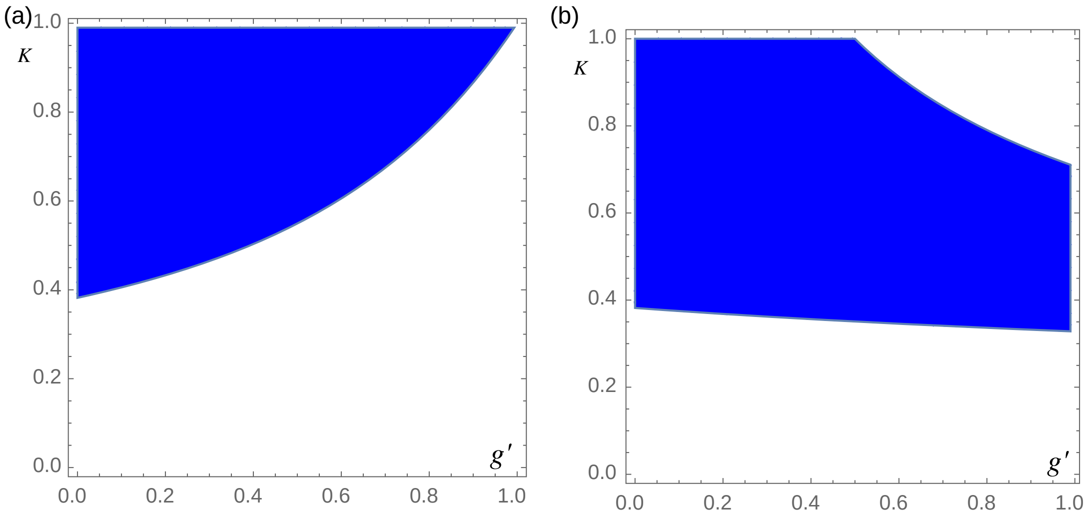

Figure 2. The blue regions represent regime with a gapped channel $\phi_g = (1/2)(\phi_1 + \phi_2 + \theta_1 - \theta_2)$ for the intrachannel parameters $g_4 = g_2$, and interchannel parameters: a) $g_2' = 0$, $g_4' \equiv g'$; b) $g_4' = g_2' \equiv g'$.

$$\boldsymbol{\phi}_f = \frac{1}{2}(\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2 - \theta_1 + \theta_2) \tag{17}$$

$$\boldsymbol{\theta}_f = \frac{1}{2}(-\boldsymbol{\phi}_1 + \boldsymbol{\phi}_2 + \theta_1 + \theta_2) \tag{18}$$

The orthogonal matrix $\hat{M}$ from Equation (6) is therefore

$$\hat{M} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix} \tag{19}$$

It is easy to check that it satisfies the commutation condition of Equation (8) to preserve the form of the kinetic term in the Lagrangian. For the particular relevant perturbation which we study in this Letter, the matrix $\hat{M}$ is also equal to its inverse $\hat{M} = \hat{M}^{-1} = \hat{M}^T$.

Now, we can write the Lagrangian of the system (see Equation (1)) in the new basis

$$\begin{aligned} L_0 = &\frac{1}{4\pi}(\partial_t \boldsymbol{\theta}_g \partial_x \boldsymbol{\phi}_g + \partial_t \boldsymbol{\theta}_f \partial_x \boldsymbol{\phi}_f) \\ &- \frac{1}{8\pi}\left[\partial_x \boldsymbol{\psi}^T \hat{M}^T \begin{pmatrix} \hat{V}_\Phi & 0 \\ 0 & 0 \end{pmatrix} \hat{M} \partial_x \boldsymbol{\psi} + \partial_x \boldsymbol{\psi}^T \hat{M}^T \begin{pmatrix} 0 & 0 \\ 0 & \hat{V}_\theta \end{pmatrix} \hat{M} \partial_x \boldsymbol{\psi}\right] \end{aligned} \tag{20}$$

where we introduced vector field $\boldsymbol{\psi}^T \equiv (\boldsymbol{\phi}_g, \boldsymbol{\phi}_f, \boldsymbol{\theta}_g, \boldsymbol{\theta}_f)$. We can now drop all the terms containing the gapped field $\boldsymbol{\phi}_g$ and its oscillating conjugate $\boldsymbol{\theta}_g$ (which self-averages to zero). Simple arithmetics provides the following Lagrangian for the single remaining conducting channel

$$\begin{aligned} L_f = &\frac{1}{4\pi}\partial_t \boldsymbol{\theta}_f \partial_x \boldsymbol{\phi}_f \\ &- \frac{1}{8\pi}[(1 + g_4 + g_2')(\partial_x \boldsymbol{\phi}_f)^2 + (1 + g_4 - g_2')(\partial_x \boldsymbol{\theta})^2] \end{aligned} \tag{21}$$

We immediately observe that the remaining conducting channel depends only on two (out of four) original Luttinger parameters. Its effective Luttinger parameter is then

$$K_f = \sqrt{\frac{1 + g_4 - g_2'}{1 + g_4 + g_2'}} \tag{22}$$

The effective velocity is correspondingly $v_f = \sqrt{(1 + g_4)^2 - (g_2')^2}$.

4. Conclusions

We have considered the effect of relevant perturbations on the quasi-1D system. When some channels become gapped, we suggest a mathematical procedure to describe the remaining conducting channels. For the case of relevant counter-clockwise backscattering in a two-channel system, we have shown that the remaining conducting channel depends only on two original Luttinger parameters, which is an unexpected result. Even more exciting result is that if there are no original interactions between particles moving in the different channels in opposite directions ($g_2' = 0$), then the remaining free channel becomes noninteracting ($K_{eff} = 1$) independently on the arbitrary intrachannel interactions (g_4 and g_2) as well as interactions between particles moving in the different channels in the same direction (g_4'). We speculate that the gapped channel screens effective interactions in the remaining conducting channel. We believe that the results we predict can be observed experimentally. More, the existing seminal experimental results (see our ref. [4]) present a variety of fractional conductances (indicating that the system is strongly interacting) but also the unit conductance, which can signal a noninteracting channel. To prove the absence of interactions conclusively, it will be essential to measure local conductivity inside the constriction and not two-terminal conductance, as is done in ref. [4].

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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