Chapter 5

VECTOR AUTOREGRESSIVE MODELS[1,2]

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Abstract

This chapter reviews vector autoregressive (VAR) modelling and its applications in tourism demand research. Since the 1980s, VAR models have been a popular tool in macroeconomic analysis, in which endogeneity is of particular concern. This chapter revisits the classic VAR model. Then it introduces a recent advancement called the global VAR (GVAR) model, which is well suited to modelling large high-dimensional systems with multiple cross-sections. In addition, this chapter touches on the Bayesian approaches to VAR modelling. In the context of tourism demand research, VAR models can be used to capture the interrelations between tourism variables and economic variables and to simulate impulse responses to economic shocks. Using global tourism demand data for 24 major economies, this chapter demonstrates the applications of the classic VAR, GVAR, Bayesian VAR (BVAR) and Bayesian GVAR (BGVAR) models and compares their forecast accuracy with the accuracy of commonly used univariate time series models.

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5.1 Introduction

Macroeconomists are often tasked with describing and summarising macroeconomic data, making macroeconomic forecasts, quantifying what we do or do not know about the true structure of the economy and advising macroeconomic policymakers (Stock & Watson, 2001). In macroeconomics, a threshold concept that describes the complex interrelations among economic variables is *cumulative causation*, which refers to a self-reinforcing process whereby an initial shock triggers subsequent changes in other variables in the economic system through feedback loops. Vector autoregressive (VAR) models hold the promise of capturing the rich dynamics and interconnected nature of multiple economic variables by providing a coherent and credible approach to data description, forecasting, structural inference and policy analysis (Stock & Watson, 2001).

VAR models have been applied in tourism demand research since the early 2000s; for example, see Song, Romilly and Liu (2000). These studies are not limited to unrestricted VAR but also incorporate cointegration analysis in the form of vector error correction models (VECM) (e.g., Lim & McAleer, 2001) and structural analysis in the form of structural VAR (e.g., De Mello & Nell, 2005).

Empirically, each tourism destination country can be perceived as a single system within which interactions take place between tourism demand and other economic variables that characterise the local economy. In a global setting, which is characterised by the interconnectedness of destination countries and tourist-originating countries, cumulative causation can be readily transmitted across borders. The transmission can take place through various channels. As summarised by Chudik and Pesaran (2016), these include sharing scarce resources (such as oil and other commodities), political and technological developments, the movements of labour and capital across countries, and cross-border trade in financial assets as well as trade in goods and services. In addition, there might be residual interdependencies due to unobserved interactions and spillover effects not properly captured by the common interaction channels listed above.

To model the interdependencies among multiple countries, researchers need to handle the *curse of dimensionality* in the estimation of a vast number of coefficients. Initially proposed by Pesaran et al. (2004), the global VAR (GVAR) approach offers a relatively simple yet effective method for modelling complex high-dimensional systems such as the global economy. In the context of tourism, this approach enables research into the global tourism demand system (e.g., Cao, Li, & Song, 2017; Gunter & Zekan, 2021), which encompasses a host of tourism demand variables and macroeconomic determinants for multiple destination countries and treats all variables as *endogenous*.

Furthermore, a recent development in VAR models is the use of Bayesian statistics. Many of the macroeconomic databases used in modelling practice, including tourism statistics, provide relatively short sample periods, rendering VAR estimations less precise. To enhance forecast accuracy, many researchers impose additional restrictions on their estimations, which may help reduce the variance of unrestricted least squares (LS) estimators. As Kilian and Lütkepohl (2017) note, the Bayesian approach provides a formal framework for incorporating such extraneous information (i.e., priors) into estimations and inferences. It also reduces the need for estimating a large number of coefficients. The Bayesian approach to VAR modelling has long attracted the attention of tourism researchers; for example, see Wong, Song, and Chon (2006) and Assaf et al. (2019).

This chapter explores how VAR models are applied in tourism demand modelling and forecasting. Section 5.2 reviews a range of VAR models, from the classic VAR to the recently developed GVAR model and their Bayesian counterparts (i.e., Bayesian VAR and Bayesian GVAR). Section 5.3 demonstrates an application of VAR models using global tourism demand data and evaluates the models' forecasting performance. Section 5.4 reflects on VAR modelling and proposes future research directions.

5.2 Methods

VAR models relax the assumption of exogeneity imposed on independent variables in many econometric models. Developed by Sims (1980), a VAR model is a *k*-equation, *k*-variable linear model in which each variable is explained by its own lagged values plus the current and past values of the remaining *k* - 1 variables, whilst allowing for deterministic terms such as intercepts, trends and dummy variables (Song & Witt, 2006; Stock & Watson, 2001).

5.2.1 The VAR model

A classic VAR(*p*) model, where *p* is the lag order, can be written as

$$\boldsymbol{Y}_t = \sum_{l=1}^{p} \boldsymbol{A}_l \boldsymbol{Y}_{t-l} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t + \boldsymbol{U}_t \tag{5.1}$$

$$\boldsymbol{U}_t \sim IID(\boldsymbol{0}, \boldsymbol{\Sigma}) \tag{5.2}$$

where $\boldsymbol{Y}_t$ is a $k \times 1$ vector of *k* endogenous variables, $\boldsymbol{A}_l$ is a $k \times k$ matrix of coefficients to be estimated, $\boldsymbol{C}_0$ is a $k \times 1$ vector of intercepts, $\boldsymbol{C}_1$ is a $k \times 1$ vector of coefficients on the trend terms, and $\boldsymbol{U}_t$ is a $k \times 1$ vector of innovations or shocks. Moreover, dummy variables can be added to Equation (5.1) in the same manner as the trend term. The variables in $\boldsymbol{Y}_t$ are endogenous variables (either justified by theory or simply due to lack of evidence of exogeneity) in a system, such that changes in one variable can cumulatively cause changes to the other variables. The lag order *p* can be selected based on, among other criteria, the Akaike information criterion (AIC), the Schwarz Bayesian information criterion (BIC) or the Hannan–Quinn criterion (HQ) (Song, Witt, & Li, 2008, p. 42). In total, there are k equations to be estimated. Assuming $\boldsymbol{U}_t$ to be contemporaneously correlated but not autocorrelated, each equation in the system can be individually estimated with the ordinary least squares (OLS) estimator or the seemingly unrelated regression (SUR) estimator (Song & Witt, 2006).

More generally, VAR models can be extended to include exogenous variables, as follows:

$$\boldsymbol{Y}_t = \sum_{l=1}^{p} \boldsymbol{A}_l \boldsymbol{Y}_{t-l} + \boldsymbol{B}\boldsymbol{Z}_t + \boldsymbol{C}_0 + \boldsymbol{C}_1 t + \boldsymbol{U}_t \quad (5.3)$$

where $\boldsymbol{Z}_t$ is a $d \times 1$ vector of d exogenous variables, and $\boldsymbol{B}$ is a $k \times d$ matrix of coefficients to be estimated. Equation (5.3) is called a VAR with exogenous variables (VARX) model. Unlike Equation (5.1), where the k equations explain the causal relationships among k endogenous variables, Equation (5.3) uses $k + d$ variables to explain the k relationships.

Cointegration analysis can be incorporated into the above VAR models, which are then specified as VECM:

$$\Delta\boldsymbol{Y}_t = \sum_{l=1}^{p-1} \boldsymbol{A}_l \Delta\boldsymbol{Y}_{t-l} + \boldsymbol{\Pi}\boldsymbol{Y}_{t-1} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t + \boldsymbol{U}_t \quad (5.4)$$

where $\boldsymbol{\Pi}\boldsymbol{Y}_{t-1}$ is the error correction vector. If the elements of $\boldsymbol{Y}_t$ are $I(0)$, $\boldsymbol{\Pi}$ will be a full rank $k \times k$ matrix. If the elements of $\boldsymbol{Y}_t$ are $I(1)$ and not cointegrated, then $\boldsymbol{\Pi} = \boldsymbol{0}$ and a VAR model in first differences will be a more appropriate specification than a VECM. If the elements of $\boldsymbol{Y}_t$ are $I(1)$ and cointegrated with rank($\boldsymbol{\Pi}$) = r ($0 < r < k$), then $\boldsymbol{\Pi}$ can be expressed as $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$. Both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $k \times r$ full column rank matrices, and there are r cointegrating relations, i.e., $\boldsymbol{\xi}_t = \boldsymbol{\beta}'\boldsymbol{Y}_t$, which are $I(0)$. $\boldsymbol{\xi}_t$ captures the deviations from equilibrium, and $\boldsymbol{\alpha}$ is the matrix of adjustment or feedback coefficients, which measure how strongly the deviations from equilibrium feed back into the system (Garratt, Lee, Pesaran, & Shin, 2012, pp. 117-118). As in the classic VAR model, Equation (5.4) can also be extended to include exogenous variables, as described in Pesaran, Shin, and Smith (2000).

Whilst $\mathbf{\Pi}$ can be estimated unrestrictedly, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are not necessarily unique. It is possible to choose any non-singular $r \times r$ matrix, $\boldsymbol{Q}$, and write $\mathbf{\Pi} = \boldsymbol{\alpha\beta'} = (\boldsymbol{\alpha Q'^{-1}})(\boldsymbol{Q'\beta'}) = \boldsymbol{\alpha}_*\boldsymbol{\beta}'_*$, such that $\boldsymbol{\alpha}_* = \boldsymbol{\alpha Q'^{-1}}$ and $\boldsymbol{\beta}_* = \boldsymbol{\beta Q}$ constitute observationally equivalent alternative structures. To identify $\boldsymbol{\beta}$, at least r^2 restrictions are needed, formed from r restrictions on each of the r cointegrating relations (Garratt, Lee, Pesaran, & Shin, 2012, p. 36). One often-used restriction is a normalisation scheme of imposing an $r \times r$ identity matrix on $\boldsymbol{\beta}'$. Other subjective identification schemes are also plausible. In fact, the restrictions can be drawn from economic theories and other *a priori* information, which add economic interpretability to the cointegrating relations (Garratt, Lee, Pesaran, & Shin, 2012, pp. 36-37; Juselius, 2006, p. 120).

To determine the number of cointegrating relations, *r*, the Johansen Maximum Likelihood (JML) procedure can be adopted. This procedure is based on estimating the significance of the characteristic roots of matrix $\mathbf{\Pi}$ (Song, Witt, & Li, 2009, p. 129). The rank of a matrix is the same as the number of characteristic roots that are different from zero. In the JML procedure, two statistics can be calculated:

$$\lambda_{trace} = -T \sum_{i=r+1}^{m} \ln(1 - \hat{\lambda}_i) \tag{5.5}$$

$$\lambda_{max} = -T \ln(1 - \hat{\lambda}_{r+1}) \tag{5.6}$$

where $\hat{\lambda}_i$ are the estimated values of the characteristic roots or eigenvalues from matrix $\mathbf{\Pi}$ in Equation (5.4), and *T* is the total number of observations. The first test statistic λ_{trace} is the trace test. The null hypothesis is that there are at most *r* cointegrating relations, i.e., $\text{rank}(\mathbf{\Pi}) \leq r$, whereas the alternative hypothesis is there are more than *r* cointegrating relations, i.e., rank($\mathbf{\Pi}$) $> r$. The second test statistic λ_{max} is known as the maximal eigenvalue test. Its null hypothesis is that the rank of $\mathbf{\Pi}$ is *r*, and the alternative hypothesis is that the rank is *r* + 1. In practice, λ_{trace} and λ_{max} very often suggest different numbers of cointegrating relations. It is then at the discretion of the researchers to decide what value *r* should take, after considering both the test statistics and economic interpretability.

In the cointegrating relations, $\boldsymbol{\beta}'\boldsymbol{Y}_t$, which is *I*(0) if the variables of $\boldsymbol{Y}_t$ are cointegrated, it is possible to add deterministic terms such as intercept $\boldsymbol{C}_0$ and trend $\boldsymbol{C}_1 t$. Let

$$\boldsymbol{C}_0 = \boldsymbol{\alpha}\boldsymbol{\beta}_0 + \boldsymbol{\gamma}_0 \tag{5.7}$$

$$\boldsymbol{C}_1 = \boldsymbol{\alpha}\boldsymbol{\beta}_1 + \boldsymbol{\gamma}_1 \tag{5.8}$$

where $\boldsymbol{\alpha}$ has the same meaning as in $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$. The derivations of $\boldsymbol{\beta}_0$, $\boldsymbol{\beta}_1$, $\boldsymbol{\gamma}_0$ and $\boldsymbol{\gamma}_1$ are explained in Juselius (2006, pp. 95-99).

There are five cases under which Equations (5.7) and (5.8) can be incorporated into Equation (5.4) (Juselius, 2006, p. 100; Song, Witt, & Li, 2009, pp. 129-130).

Case I: $\boldsymbol{Y}_t$ does not have deterministic trends and the cointegration equations do not have intercepts, i.e., $\boldsymbol{C}_0 = 0$ and $\boldsymbol{C}_1 = 0$. Hence, $\mathbf{\Pi}\boldsymbol{Y}_{t-1} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t = \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{Y}_{t-1}$.

Case II: $\boldsymbol{Y}_t$ does not have deterministic trends but the cointegration equations have intercepts, i.e., $\boldsymbol{\beta}_0 \neq 0, \boldsymbol{\gamma}_0 = 0$ and $\boldsymbol{C}_1 = 0$. Hence, $\mathbf{\Pi}\boldsymbol{Y}_{t-1} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t = \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{Y}_{t-1} + \boldsymbol{\beta}_0)$.

Case III: $\boldsymbol{Y}_t$ has deterministic trends but the cointegration equations only have intercepts, i.e., $\boldsymbol{\beta}_0 \neq 0, \boldsymbol{\gamma}_0 \neq 0$ and $\boldsymbol{C}_1 = 0$. Hence, $\mathbf{\Pi}\boldsymbol{Y}_{t-1} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t = \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{Y}_{t-1} + \boldsymbol{\beta}_0) + \boldsymbol{\gamma}_0$.

Case IV: $\boldsymbol{Y}_t$ has deterministic trends and the cointegration equations have intercepts and deterministic trends, i.e., $\boldsymbol{\beta}_0 \neq 0, \boldsymbol{\gamma}_0 \neq 0, \boldsymbol{\beta}_1 \neq 0$ and $\boldsymbol{\gamma}_1 = 0$. Hence, $\boldsymbol{\Pi Y}_{t-1} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t = \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{Y}_{t-1} + \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 t) + \boldsymbol{\gamma}_0$.

Case V: $\boldsymbol{Y}_t$ has quadratic trends and the cointegration equations have intercepts and deterministic trends, i.e., $\boldsymbol{\beta}_0 \neq 0, \boldsymbol{\gamma}_0 \neq 0, \boldsymbol{\beta}_1 \neq 0$ and $\boldsymbol{\gamma}_1 \neq 0$. Hence, $\boldsymbol{\Pi Y}_{t-1} + \boldsymbol{C}_0 + \boldsymbol{C}_1 t = \boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{Y}_{t-1} + \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 t) + \boldsymbol{\gamma}_0 + \boldsymbol{\gamma}_1 t$.

Of the above five cases, *Case III* and *Case IV* are commonly used in economics.

One major usage of VAR modelling is to simulate the impulse responses of endogenous variables in the face of a shock to the i^{th} variable. For simplicity, consider a VAR(*1*) model:

$$\boldsymbol{Y}_t = \boldsymbol{A}_1 \boldsymbol{Y}_{t-1} + \boldsymbol{U}_t \tag{5.9}$$

By iterative substitution for *n* times, Equation (5.9) can be rearranged as follows:

$$\boldsymbol{Y}_t = \sum_{i=0}^{n} \boldsymbol{A}_1^i \boldsymbol{U}_{t-i} + \boldsymbol{A}_1^{n+1} \boldsymbol{Y}_{t-n+1} \tag{5.10}$$

If the time series data are stationary, i.e., $0 < |\boldsymbol{A}_1| < 1$, then $\lim_{n\to\infty} \boldsymbol{A}_1^n = 0$, and Equation (5.10) can be written as follows:

$$\boldsymbol{Y}_t = \sum_{i=0}^{\infty} \boldsymbol{A}_1^i \boldsymbol{U}_{t-i} \tag{5.11}$$

Equation (5.11) is called a vector moving average (VMA) form, where the vector of dependent variables is represented by an infinite sum of lagged random errors weighted by an exponentially diminishing coefficient. Hence, the endogenous variables of $\boldsymbol{Y}_t$ can be expressed by sequences of shocks to the VAR system. Equation (5.11) can capture the impacts of unitary changes in the error terms (i.e., shocks) on the endogenous variables.

5.2.2 The GVAR model

An issue with the classic VAR model is that whilst the number of coefficients to be estimated grows exponentially when more endogenous variables are included, as each variable is represented in *k* equations, the number of observations tends to be limited relative to the number of coefficients. The reduction in the degrees of freedom makes it difficult to estimate a high-dimensional VAR model. In response to this issue, Pesaran, Schuermann, and Weiner (2004) propose the GVAR model, the global version of the VAR, , which is suitable for a large multiple cross-sectional setting. Dees, Mauro, Pesaran, and Smith (2007) further develop the approach within a global common factor framework.

The GVAR approach entails a two-stage procedure: (1) treating each cross-section in a global system as an individual VAR model and estimating the coefficients for each model; and (2) stacking the individual VAR models to form the GVAR system.

Suppose that in a global system, there are *N* countries (i.e., cross sections). In the first stage, each country-specific model is specified as a VARX*(p_i, q_i) model:

$$\boldsymbol{\Phi}_i(L, p_i)\boldsymbol{x}_{it} = \boldsymbol{a}_{i0} + \boldsymbol{a}_{i1}t + \boldsymbol{\Lambda}_i(L, q_i)\boldsymbol{x}_{it}^{*} + \boldsymbol{\Upsilon}_i(L, q_i)\boldsymbol{d}_t + \boldsymbol{u}_{it} \quad (5.12)$$

Let $\boldsymbol{x}_{it}$ denote a $k_i \times 1$ vector of endogenous variables belonging to country $i \in \{1, \dots, N\}$. $\boldsymbol{x}_{it}$ is also called *domestic variables* in relation to country *i*. $\boldsymbol{x}_{it}^{*}$ is a $k_i^{*} \times 1$ vector of *foreign variables* specific to country *i*, which is supposed to capture the influence of country *i*'s trade partners. $\boldsymbol{x}_{it}^{*}$ is calculated as cross-sectional averages of the foreign counterparts of country *i*'s domestic variables:

$$\boldsymbol{x}_{it}^{*} = \sum_{j=1}^{N} w_{ij}\boldsymbol{x}_{jt} \quad (5.13)$$

where $\boldsymbol{x}_{jt}$ represents the domestic variables of country $j \in \{1, \dots, N\}$, and w_{ij} is the weight for country *j* (e.g., the share of country *j*'s trade with country *i* among country *i*'s total trade with the global system). In tourism demand research, the weight can be the share of tourist arrivals or the share of tourism expenditures. Preferably the weights are non-random (i.e., pre-determined) and granular (i.e., compared with the global system, each country's weight is small, and no country dominates the system) (Bussière, Chudik, & Sestieri, 2009). w_{ij} satisfies $w_{ii} = 0, \forall i = 1, \dots, N$, and $\sum_{j=1}^{N} w_{ij} = 1, \forall i, j = 1, \dots, N$. Equation (5.13) is a data shrinkage method that addresses the dimensionality problem. $\boldsymbol{d}_t$ is a $k_d \times 1$ vector of *observable global common factors*, which apply to all of country-specific VARX* models. As an aside, Equation (5.12) can incorporate cointegration analysis and be specified in its error correction representation VECMX* (see Dees, Mauro, Pesaran, & Smith, 2007 for technical details).

Domestic variables ($\boldsymbol{x}_{it}$), *foreign variables* ($\boldsymbol{x}_{it}^{*}$) and *observable global common variables* ($\boldsymbol{d}_t$) capture the different channels through which the cross-border transmission of business cycles takes place. Broadly, the transmission can be caused by observable global common shocks (e.g., changes in oil prices); it can arise as a result of unobserved global common factors (e.g., diffusion of technological progress and political developments); or it can be due to specific national or sectoral shocks. In a factor model framework, Dees, Mauro, Pesaran, and Smith (2007) show that *foreign variables* are proxies for unobserved global common factors.

In Equation (5.12), $\boldsymbol{\Phi}_i(L, p_i) = \boldsymbol{I}_{k_i} - \sum_{l=1}^{p_i} \boldsymbol{\Phi}_l L^l$ is a $k_i \times k_i$ matrix of unknown coefficients on *domestic variables*; $\boldsymbol{\Lambda}_i(L, q_i) = \sum_{l=0}^{q_i} \boldsymbol{\Lambda}_l L^l$ is a $k_i \times k_i^*$ matrix of unknown coefficients on *foreign variables*; and $\boldsymbol{\Upsilon}_i(L, q_i) = \sum_{l=0}^{q_i} \boldsymbol{\Upsilon}_l L^l$ is a $k_i \times k_d$ matrix of unknown coefficients on *global common variables*. L is the lag operator, and p_i and q_i respectively denote the lag order of *domestic variables* and the lag order of *foreign* and *global common variables*. p_i and q_i can be different. $\boldsymbol{\Phi}_i(L, p_i)$, $\boldsymbol{\Lambda}_i(L, q_i)$ and $\boldsymbol{\Upsilon}_i(L, q_i)$, together with $\boldsymbol{a}_{i0}$ and $\boldsymbol{a}_{i1}$, are the coefficients to be estimated in the first stage. Last, $\boldsymbol{u}_{it}$ is a $k_i \times 1$ vector of idiosyncratic country-specific shocks, and is assumed to be serially uncorrelated with a zero mean and a non-singular covariance matrix $\sum_{ii} = (\sigma_{ii,ls})$, where $\sigma_{ii,ls} = cov(u_{ilt}, u_{ist})$ with *l* and *s* denoting the l^{th} and s^{th} variables, respectively. More compactly, $\boldsymbol{u}_{it} \sim i.i.d.(0, \sum_{ii})$.

As noted by Dees, Mauro, Pesaran, and Smith (2007), in the first stage's country-specific VARX* models, $\boldsymbol{x}_{it}$ (*domestic variables*) is treated as *endogenous*, whereas $\boldsymbol{x}_{it}^*$ (*foreign variables*) is assumed to be *weakly exogenous*, which means that $\boldsymbol{x}_{it}^*$ is long-run forcing for $\boldsymbol{x}_{it}$ but there is no long-run feedback from $\boldsymbol{x}_{it}$ to $\boldsymbol{x}_{it}^*$. Lagged short-run feedback between the two sets of variables is, however, allowed. This weak exogeneity is in line with the assumption that most countries in the global system are perceived as small open economies, in the sense that they are operating under the influence of a global economic environment, which in turn is not subject to a particular country's influence. $\boldsymbol{d}_t$ (*Observable global common variables*) is also assumed to be *weakly exogenous* and treated in a similar manner to $\boldsymbol{x}_{it}^*$ (*foreign variables*).

Once $\boldsymbol{\Phi}_i(L, p_i)$, $\boldsymbol{\Lambda}_i(L, q_i)$, $\boldsymbol{\Upsilon}_i(L, q_i)$, $\boldsymbol{a}_{i0}$ and $\boldsymbol{a}_{i1}$ are estimated for all country-specific VARX* models, the second stage can proceed. Let $\boldsymbol{x}_t = (\boldsymbol{x}_{1t}', \boldsymbol{x}_{2t}', \dots, \boldsymbol{x}_{Nt}')'$, which is a $k \times 1$ vector that collects all of the domestic variables across *N* countries, with $k = \sum_{i=1}^{N} k_i$ denoting the total number of variables. Given that $\boldsymbol{x}_{it}^* = \sum_{j=1}^{N} w_{ij}\boldsymbol{x}_{jt}$, apparently $\boldsymbol{x}_t$ contains all of the variables that are used to construct each country-specific VARX* model. In the second stage, all of the elements in $\boldsymbol{x}_t$ are treated as *endogenously* determined from the standpoint of the global VAR system.

The second stage involves re-arranging Equation (5.12) as follows:

$$\mathbf{B}_i(L, p_i, q_i)\boldsymbol{x}_t = \boldsymbol{a}_{i0} + \boldsymbol{a}_{i1}t + \boldsymbol{\Upsilon}_i(L, q_i)\boldsymbol{d}_t + \boldsymbol{u}_{it} \tag{5.14}$$

where $\mathbf{B}_i(L, p_i, q_i) = [\boldsymbol{\Phi}_i(L, p_i)\boldsymbol{E}_i', -\boldsymbol{\Lambda}_i(L, q_i)\boldsymbol{W}_i']$. $\boldsymbol{E}_i$ is a $k \times k_i$ selection matrix that selects vector $\boldsymbol{x}_{it}$, namely $\boldsymbol{x}_{it} = \boldsymbol{E}_i'\boldsymbol{x}_t$. $\boldsymbol{W}_i$ is merely a $k \times k_i^*$ matrix that collects the weights w_{ij} used in calculating the *foreign variables*, such that $\boldsymbol{x}_{it}^* = \boldsymbol{W}_i'\boldsymbol{x}_t$.

Let $p = max\{max\ p_i, max\ q_i\}$, $\forall i = 1, \ldots, N$, and construct $\mathbf{B}_i(L, p)$ from $\mathbf{B}_i(L, p_i, q_i)$ by augmenting $p - p_i$ or $p - q_i$ additional terms in powers of L by zeros; similarly, construct $\boldsymbol{\Upsilon}_i(L, p)$. Then Equation (5.14) becomes

$$\mathbf{B}_i(L, p)\boldsymbol{x}_t = \boldsymbol{a}_{i0} + \boldsymbol{a}_{i1}t + \boldsymbol{\Upsilon}_i(L, p)\boldsymbol{d}_t + \boldsymbol{u}_{it} \tag{5.15}$$

The next step is to stack Equation (5.15) for all $i = 1, \ldots, N$, such that

$$\boldsymbol{G}(L, p)\boldsymbol{x}_t = \boldsymbol{a}_0 + \boldsymbol{a}_1 t + \boldsymbol{\Upsilon}(L, p)\boldsymbol{d}_t + \boldsymbol{u}_t \tag{5.16}$$

where $\boldsymbol{u}_t = (\boldsymbol{u}_{1t}', \ldots, \boldsymbol{u}_{Nt}')'$, $\boldsymbol{a}_0 = (\boldsymbol{a}_{10}', \ldots, \boldsymbol{a}_{N0}')'$, $\boldsymbol{a}_1 = (\boldsymbol{a}_{11}', \ldots, \boldsymbol{a}_{N1}')'$, $\boldsymbol{\Upsilon}(L, p) = \begin{pmatrix} \boldsymbol{\Upsilon}_1(L, p) \\ \vdots \\ \boldsymbol{\Upsilon}_N(L, p) \end{pmatrix}$

and $\boldsymbol{G}(L, p) = \begin{pmatrix} \mathbf{B}_1(L, p) \\ \vdots \\ \mathbf{B}_N(L, p) \end{pmatrix}$.

Eqution (5.16) is the GVAR model that explains the causal relationships among all $k = \sum_{i=1}^{N} k_i$ variables in the global system.

To obtain the reduced form of the GVAR model, Equation (5.16) can be further transformed into

$$\boldsymbol{G}(L, p)\boldsymbol{x}_t = \boldsymbol{G}_0\boldsymbol{x}_t - \sum_{j=1}^{p} \boldsymbol{G}_j\boldsymbol{x}_{t-j} = \boldsymbol{a}_0 + \boldsymbol{a}_1 t + \sum_{j=0}^{p} \boldsymbol{\Upsilon}_j\boldsymbol{d}_{t-j} + \boldsymbol{u}_t \tag{5.17}$$

Moving $\sum_{j=1}^{p} \boldsymbol{G}_j \boldsymbol{x}_{t-j}$ to the right-hand side and then pre-multiplying both sides of the above equation by $\boldsymbol{G}_0^{-1}$, which is a non-singular matrix, gives

$$\boldsymbol{x}_t = \boldsymbol{G}_0^{-1}\boldsymbol{a}_0 + \boldsymbol{G}_0^{-1}\boldsymbol{a}_1 t + \sum_{j=1}^{p} \boldsymbol{G}_0^{-1}\boldsymbol{G}_j \boldsymbol{x}_{t-j} + \sum_{j=0}^{p} \boldsymbol{G}_0^{-1}\boldsymbol{\Upsilon}_j \boldsymbol{d}_{t-j} + \boldsymbol{G}_0^{-1}\boldsymbol{u}_t$$

$$= \boldsymbol{b}_0 + \boldsymbol{b}_1 t + \sum_{j=1}^{p} \boldsymbol{F}_j \boldsymbol{x}_{t-j} + \sum_{j=0}^{p} \boldsymbol{\Gamma}_j \boldsymbol{d}_{t-j} + \boldsymbol{\varepsilon}_t \tag{5.18}$$

where $\boldsymbol{b}_0 = \boldsymbol{G}_0^{-1}\boldsymbol{a}_0$, $\boldsymbol{b}_1 = \boldsymbol{G}_0^{-1}\boldsymbol{a}_1$, $\boldsymbol{F}_j = \boldsymbol{G}_0^{-1}\boldsymbol{G}_j$, $\boldsymbol{\Gamma}_j = \boldsymbol{G}_0^{-1}\boldsymbol{\Upsilon}_j$ for j = 1, 2, …, p and $\boldsymbol{\varepsilon}_t = \boldsymbol{G}_0^{-1}\boldsymbol{u}_t$.

For impulse response analysis, Pesaran, Schuermann, and Weiner (2004) propose using the generalised impulse response (GIR) functions, instead of the more common orthogonalised impulse response (OIR) functions. Consider the reduced form GVAR model in Equation (5.18), and for simplicity, let the lag orders be 1:

$$\boldsymbol{x}_t = \boldsymbol{b}_0 + \boldsymbol{b}_1 t + \boldsymbol{F}\boldsymbol{x}_{t-1} + \boldsymbol{\Gamma}_0 \boldsymbol{d}_t + \boldsymbol{\Gamma}_1 \boldsymbol{d}_{t-1} + \boldsymbol{\varepsilon}_t \tag{5.19}$$

For predetermined values of $\boldsymbol{d}_t$ ($t = T + 1$, $T + 2$ …), it can be solved as follows:

$$\boldsymbol{x}_{T+n} = \boldsymbol{F}^n \boldsymbol{x}_T + \sum_{\tau=0}^{n-1} \boldsymbol{F}^\tau [\boldsymbol{b}_0 + \boldsymbol{b}_1 (T + n - \tau)]$$

$$+ \sum_{\tau=0}^{n-1} \boldsymbol{F}^\tau [\boldsymbol{\Gamma}_0 \boldsymbol{d}_{T+n-\tau} + \boldsymbol{\Gamma}_1 \boldsymbol{d}_{T+n-\tau-1}]$$

$$+ \sum_{\tau=0}^{n-1} \boldsymbol{F}^\tau \boldsymbol{\varepsilon}_{T+n-\tau} \tag{5.20}$$

The point forecasts of $\boldsymbol{x}_{T+n}$, conditional on the initial state of the system and the exogenous global variables, are given by

$$\boldsymbol{x}_{T+n}^f = E(\boldsymbol{x}_{T+n} | \boldsymbol{x}_T, \cup_{\tau=1}^{n} \boldsymbol{d}_{T+\tau})$$

$$= \boldsymbol{F}^n \boldsymbol{x}_T + \sum_{\tau=0}^{n-1} \boldsymbol{F}^\tau [\boldsymbol{b}_0 + \boldsymbol{b}_1 (T + n - \tau)]$$

$$+ \sum_{\tau=0}^{n-1} \boldsymbol{F}^\tau [\boldsymbol{\Gamma}_0 \boldsymbol{d}_{T+n-\tau} + \boldsymbol{\Gamma}_1 \boldsymbol{d}_{T+n-\tau-1}] \tag{5.21}$$

Under the assumption that $\boldsymbol{u}_t$ is normally distributed,

$$\boldsymbol{x}_{T+n}|\boldsymbol{x}_T, \bigcup_{\tau=1}^{n} \boldsymbol{d}_{T+\tau} \backsim N(\boldsymbol{x}_{T+n}^{f}, \boldsymbol{\Omega}_n) \qquad (5.22)$$

where $\boldsymbol{\Omega}_n = \sum_{\tau=0}^{n-1} \boldsymbol{F}^{\tau}\boldsymbol{G}_0^{-1}\boldsymbol{\Sigma}\boldsymbol{G}_0'^{-1}\boldsymbol{F}'^{\tau}$ and $\boldsymbol{\Sigma}$ is the $k \times k$ variance-covariance matrix of shocks $\boldsymbol{u}_t$. $\boldsymbol{\Sigma}_{ij}$, which measures the dependence of shocks in country *i* on shocks in country *j*, is defined as $\boldsymbol{\Sigma}_{ij} = cov(\boldsymbol{u}_{it}, \boldsymbol{u}_{jt})$. A typical element of $\boldsymbol{\Sigma}_{ij}$ is denoted by $\sigma_{ij,ls} = cov(u_{ilt}, u_{jst})$, which is the covariance of the l^{th} variable in country *i* with the s^{th} variable in country *j*.

According to Pesaran, Schuermann, and Weiner (2004), the GIR function that denotes the j^{th} shock in $\boldsymbol{u}_t$ (corresponding to the l^{th} variable in the i^{th} country) is given by

$$\boldsymbol{GI}_{x:u_{il}}\left(n, \sqrt{\sigma_{ii,ll}}, \mathcal{I}_{t-1}\right) = E\left(\boldsymbol{x}_{t+n}|u_{ilt} = \sqrt{\sigma_{ii,ll}}, \mathcal{I}_{t-1}\right) - E(\boldsymbol{x}_{t+n}|\mathcal{I}_{t-1}) \qquad (5.23)$$

where $\boldsymbol{\mathcal{I}}_t = (\boldsymbol{x}_t, \boldsymbol{x}_{t-1}, \ldots)$ is the information set at time *t*–1, and $\boldsymbol{d}_t$ is assumed to be given exogenously. On the assumption that $\boldsymbol{u}_t$ has a multivariate normal distribution, and using Eqution (5.20), it can be derived that

$$\boldsymbol{\psi}_j^{g}(n) = \frac{1}{\sqrt{\sigma_{ii,ll}}}\boldsymbol{F}^{n}\boldsymbol{G}_0^{-1}\boldsymbol{\Sigma}\boldsymbol{\zeta}_j \qquad (5.24)$$

where $\boldsymbol{\zeta}_j$ is a $k \times 1$ selection vector with unity as its j^{th} element (corresponding to a particular shock in a particular country), and zeros0 otherwise. Equation (5.24) measures the effect of one standard error shock to the j^{th} equation (corresponding to the l^{th} variable in the i^{th} country) at time *t* on the expected values of **x** at time *t* + *n*.

5.2.3 Bayesian approaches

As discussed above, when the number of coefficients in a VAR model is large relative to the number of observations, asymptotic theory will typically be an unreliable guide for the finite sample estimation properties (George, Sun, & Ni, 2008). Bayesian approaches address this issue by imposing priors on the VAR model to reduce the number of coefficients to be estimated whilst improving the significant uncertainty about the future paths projected by the model.

In a primer of Bayesian statistics, van de Schoot et al. (2021) describe the typical Bayesian workflow, which consists of three main steps: (1) capturing available knowledge about a given parameter in a statistical model via the *prior distribution*, expressed as a probability density function (PDF); (2) determining the likelihood function using parameter information available in the observed data, expressed as the conditional probability of the data given the model parameters; and (3) combining both the prior distribution and the likelihood function using Bayes' theorem in the form of the *posterior distribution*, which is the probability of the model parameters conditional on the observed data. The posterior distribution reflects one's updated knowledge, balancing prior knowledge with observed data, and is used to conduct inferences.

For the general VAR model in Equation (5.1), Bayesian VAR (BVAR) estimation centres around forming prior distributions for the matrices of lag coefficients $\boldsymbol{A}_l$ (l = 1, 2, …, p) and the covariance matrix $\boldsymbol{\Sigma}$. A popular prior is the Minnesota prior introduced by Litterman (1979). In the tourism demand literature, this prior is used by Wong, Song and Chon (2006), Gunter and Önder (2015), Ampountolas (2019) and others.

The Minnesota prior assumes that each variable follows an independent random walk process, possibly with drift:

$$\boldsymbol{Y}_t = \boldsymbol{C}_0 + \boldsymbol{Y}_{t-1} + \boldsymbol{U}_t \tag{5.25}$$

which represents the macroeconomic behaviour of an economic variable. As described by Giannone, Lenza, and Primiceri (2015) and Kuschnig and Vashold (2021a), the prior is imposed by setting the following moments for the prior distribution of the coefficients:

$$\mathbb{E}\left[(\boldsymbol{A}_s)_{ij}|\boldsymbol{\Sigma}\right] = \begin{cases} 1, & i = j \; and \; s = 1 \\ 0, & otherwise \end{cases} \tag{5.26}$$

$$cov\left[(\boldsymbol{A}_s)_{ij}, (\boldsymbol{A}_r)_{gh}|\boldsymbol{\Sigma}\right] = \begin{cases} \lambda^2 \frac{1}{s^\alpha} \frac{\boldsymbol{\Sigma}_{ig}}{\boldsymbol{\psi}_j/(d-k-1)}, & h = j \; and \; r = s \\ 0, & otherwise \end{cases} \tag{5.27}$$

where $\boldsymbol{A}_s$ and $\boldsymbol{A}_r$ are the coefficient matrices of the s^{th} and r^{th} lags, k is the number of endogenous variables given in Equation (5.1), d is the prior degrees of freedom and $\boldsymbol{\Psi}$ is a diagonal matrix of hyperparameters. The hyperparameter λ controls the tightness of the prior, i.e., it weighs the relative importance of the prior and data. For $\lambda \rightarrow 0$, the prior outweighs any information in the data, and the posterior approaches the prior. As $\lambda \rightarrow \infty$, the posterior distribution mirrors the sample. Governing the variance decay with increasing lag order, α controls the degree of shrinkage for more distant observations. Finally, ψ_j, the j^{th} variable of $\boldsymbol{\Psi}$, controls the prior's standard deviation on the lags of all of the variables other than the dependent variable.

Another popular prior is stochastic search variable selection (SSVS). As described by George, Sun, and Ni (2008), SSVS treats each possible set of restrictions as a distinct sub-model and then uses priors to describe the uncertainty across all of the sub-models. This can be done by assigning commonly used prior variances to the parameters that should be included in a model and prior variances close to zero to irrelevant parameters. As a result of this process, relevant parameters are estimated in the usual way, and posterior draws of irrelevant variables are close to zero and thus have no significant effect on forecasts or impulse responses. In modelling practice, a hierarchical prior can be added, where the relevance of a variable is assessed in each step of the sampling algorithm. Under such a setup, the posterior distribution will increase the prior weights on the restrictions that are best supported by the observed data. Although it is typically not feasible to exhaustively calculate all of the sub-model posterior probabilities, many of the higher posterior probability sub-models can be found through a stochastic search using Markov chain Monte Carlo (MCMC) algorithms.

In the context of GVAR modelling, Bayesian GVAR (BGVAR) is adopted by researchers in economics and tourism. Cuaresma, Feldkircher, and Huber (2016) implement the Minnesota prior. It is assumed that the endogenous variables *a priori* follow random walk processes at the country-specific (i.e., cross-sectional) level, and the prior mean is set to one for the first own lag of the endogenous variables in the level, and zero for contemporaneous and lagged foreign variables and higher lag orders of the endogenous variables. The prior variance of the coefficients is set according to whether the variable is endogenous, weakly exogenous (i.e., foreign variable at the country-specific level) or deterministic. Cuaresma, Feldkircher, and Huber (2016) also implement SSVS. Unlike the Minnesota prior, which applies a symmetric and equal degree of shrinkage across equations, SSVS allows for more flexibility in the specification of the prior variance-covariance matrix on the coefficients. As an alternative to the Minnesota prior and SSVS, Assaf et al. (2019) follow another strand of literature and adopt the Bayesian variant of the least absolute shrinkage and selection operator (LASSO) as priors for a BGVAR model. They find that BGVAR unequivocally outperforms other VAR models.

5.3 Application

To illustrate how VAR models can be applied to tourism research, we construct classic VAR, BVAR, GVAR and BGVAR models to forecast the tourism exports of Thailand, a major tourist destination. Methodologically, we treat Thailand as a single VAR system and use VAR and BVAR models in the forecasting exercise. In a global setting, we use GVAR and BGVAR models to account for the influence of the external economic environment on Thailand’s tourism exports. We then evaluate the forecast accuracy of these models against three benchmark models: no-change naïve, SARIMA and ETS.

5.3.1 A global tourism demand system

Our data set corresponds to a global system consisting of 24 major economies around the world, as listed in Table 5.1. From the perspective of GVAR modelling, each country is an individual VAR system, and the 24 countries altogether constitute a GVAR system. The selection of countries is based on data availability and their use in Dees, Mauro, Pesaran, and Smith (2007).

<Insert Table 5.1 about here>

Following Cao, Li, and Song (2017), we use real tourism exports (*rtex*), real GDP index (*y*) and exchange rate-adjusted consumer price index (CPI) (*p*) as endogenous variables. As indicated in Table 5.2, the raw data are collected from the following international macroeconomic databases: Balance of Payments Statistics of the IMF and International Financial Statistics of the IMF. In addition, for GVAR and BGVAR modelling, a weight matrix is constructed using bilateral trade flows between the 24 countries. The data source for these bilateral trade flows is the IMF's Direction of Trade Statistics database. The sample period is 2000Q1–2019Q4. All of the variables are log-transformed before the models are estimated, and the forecasts are then rid of the logarithms before being evaluated.

<Insert Table 5.2 about here>

In the GVAR and BGVAR settings, for each country the variables are arranged as *domestic variables* (i.e., $\boldsymbol{x}_{it}$) and *foreign variables* (i.e., $\boldsymbol{x}^{*}_{it}$) in the following manner: $\boldsymbol{x}_{it} = (rtex_{it}, y_{it}, p_{it})$ and $\boldsymbol{x}^{*}_{it} = (rtex^{*}_{it}, y^{*}_{it}, p^{*}_{it})$, where $rtex^{*}_{it} = \sum_{j=1}^{N} w_{ij} rtex$, $y^{*}_{it} = \sum_{j=1}^{N} w_{ij} y_{jt}$ and $p^{*}_{it} = \sum_{j=1}^{N} w_{ij} p_{jt}$; N is the number of countries in the global system; w_{ij} is the bilateral trade weight that country j accounts for among all of country i's trade partners; and $w_{ij} = 0$ where $i = j$. For the classic VAR and BVAR, only $\boldsymbol{x}_{it}$ for Thailand is used and treated as endogenous variables.

Descriptively, Figure 5.1 shows the developments of real tourism exports in each of the sampled countries. The tourism market of many countries see a marked growth after 2010, a sign of healthy recovery from the global financial crisis of 2008. For Thailand, its real tourism exports (our variable of interest) grow steadily throughout the sample period. It is worth noting that the evolution of real tourism exports is susceptible to not only volatility in tourism demand *per se* but also volatility in prices and exchange rates. As a result, the fluctuations in real tourism exports, as depicted in Figure 5.1, are occasionally dramatic.

<Insert Figure 5.1 about here>

5.3.2 Forecast evaluation

In executing the forecasts, we divide the sample data into a training set (2000Q1–2016Q4) and a test set (2017Q1–2019Q4). A rolling window scheme (also known as the "evaluation on a rolling forecasting origin", see Hyndman & Athanasopoulos, 2018) is adopted. We evaluate seven competing models (no-change naïve, SARIMA, ETS, VAR, GVAR, BVAR and BGVAR) in terms of ex post out-of-sample predictive means of Thailand's real tourism exports over horizons of 1, 2, 3, 4 and 8 quarters. The evaluation is based on forecasts from 2017Q1 to 2019Q4.

The modelling and forecasting exercises are implemented in R, using prewritten packages. Specifically, classic VAR models are constructed using the *vars* package (https://cran.r-project.org/package=vars), developed by Pfaff (2008). GVAR is implemented using the *GVARX* package (https://cran.r-project.org/package=GVARX), developed by Ho (2020). It should be noted that to carry out GVAR forecasting, one may also use the open-source toolbox (https://sites.google.com/site/gvarmodelling/) written by Smith and Galesi (2014). The toolbox is accessible and easy to use, with a comprehensive range of built-in analytics. For Bayesian analysis, BVAR is implemented using the *BVAR* package (https://cran.r-project.org/package=BVAR), developed by Kuschnig and Vashold (2021b). BGVAR is implemented using the *BGVAR* package (https://cran.r-project.org/package=BGVAR), developed by Böck, Feldkircher, and Huber (2020). Both the *BVAR* and *BGVAR* packages allow the Minnesota prior to be imposed, but the *BGVAR* package also provides the option of an SSVS prior. For illustration purposes, we use the Minnesota prior. The benchmark models are implemented using the *forecast* package (https://cran.r-project.org/web/packages/forecast/index.html), developed by Hyndman et al. (2022) and Hyndman and Khandakar (2008).

The results of the forecast evaluation are reported in Table 5.3. Overall, the univariate time series benchmark models perform fairly well, especially over short-term horizons. According to MAPE and MASE, the SNAIVE model performs the best in 2-step-ahead and 3-step-ahead forecasts. RMSE suggests that the SNAIVE model is better than the other models in 1-step ahead and 2-

step ahead forecasts. The performance of SARIMA and ETS is similar to that of the SNAIVE model, although they both have slightly higher forecast errors.

<Insert Table 5.3 about here>

Among the VAR models, BVAR and BGVAR consistently outperform their frequentist counterparts. Their forecast accuracy is close to that of the time series benchmark models. According to MAPE and MASE, BVAR is better than all of its rival models in 1-step-ahead and 8-step-ahead forecasts, and BGVAR is the best in 4-step-ahead forecasts. This finding is corroborated by RMSE, although this measure suggests that BVAR performs the best in 8-step-ahead forecasts only and that BGVAR outperforms the other models in both 3-step-ahead and 4-step-ahead forecasts. In comparison, the forecasting performance of the classic VAR and GVAR is less accurate, as they generate much higher forecast errors.

The pattern shown in Table 5.3 corroborates the findings of previous studies such as Wong, Song, and Chon (2006), Song, Smeral, Li, and Chen (2008) and Gunter and Zekan (2021). These studies also observe that frequentist VAR models are less accurate than univariate time series models, although VAR models can considerably improve their performance by imposing Bayesian priors. Among our seven competing models, GVAR is typically the least accurate. This is in line with Gunter and Zekan (2021). They also observe that GVAR ranks behind VAR, naïve and ARIMA, especially when the forecast horizon stretches beyond three quarters.

It is important to note that the evaluation in Table 5.3 is based on forecasts for Thailand only, so the results could be different for forecasts made for other countries. As Wong, Song, and Chon

(2006) note, there is no indication that univariate time series models always outperform econometric models such as VAR in all forecasting situations. Econometric models are more accurate in forecasting directional changes in tourism demand than simple time series models. From the perspective of economic intuition, Gunter and Zekan (2021) comment that time series models such as ARIMA are hardly economically interpretable and cannot answer questions about what drives tourism demand in a certain direction. In contrast, VAR models are well suited to adding economic interpretability to the modelling process.

5.4 Conclusion and future directions

This chapter reviews a popular class of macroeconometric models called VAR models, which are designed to capture the interrelations between economic variables. In a VAR model, each variable is treated as endogenous and is explained by its own lagged values as well as the remaining variables in the model. In recent years, VAR models have undergone rapid development and their capability has expanded. Notably, the GVAR approach is well suited to model a large high-dimensional system with multiple cross-sections. Moreover, the incorporation of Bayesian techniques substantially improves the forecasting performance of VAR models.

This chapter covers not only frequentist classic VAR and GVAR models but also their Bayesian counterparts (i.e., BVAR and BGVAR). As an illustration, this chapter uses both frequentist and Bayesian VAR models to forecast tourism demand for Thailand, and then evaluates their forecasting performance against three commonly used univariate time series models.

Unsurprisingly, we find that the classic VAR and GVAR models are not as accurate as the benchmarks, which is consistent with the tourism demand literature. However, by imposing Bayesian priors, BVAR and BGVAR greatly improve the forecast accuracy of VAR and GVAR. The performance of BVAR and BGVAR is closely comparable to the benchmark models in almost all horizons. In particular, BVAR outperforms all of the competing models in 1-step-ahead and 8-step-ahead forecasts, whilst BGVAR outperforms its rivals in 4-step-ahead forecasts. Although VAR models may not always outperform time series models, in tourism demand research they deserve more attention because they offer an analytical framework that can embed economics theories in the modelling process. With respect to the choice between frequentist and Bayesian methods, there is not a one-size-fits-all answer. As Kilian and Lütkepohl (2017) comment, researchers should rely on their own preferences and the convenience of implementation.

Future research in tourism demand could take advantage of the vast range of advanced VAR models available. Closely related to GVAR, spatial VAR methods (e.g., Ramajo, Márquez, & Hewings, 2017) are also able to model multi-regional/multi-country interrelations, with a weight matrix based on geographical criteria. Another development in VAR models is panel VAR (PVAR), which has garnered attention in recent years. It allows for individual heterogeneity by introducing fixed effects in VAR models (Xu & Reed, 2019). Focusing on temporal patterns, regime-switching VAR (RS-VAR) models (e.g., Huarng, Yu, & Solé Parellada, 2011; Yamaka, Pastpipatkul, & Sriboonchitta, 2015) are well suited to capture the cyclical asymmetry of the different phases of business cycles. They have already been further extended to the GVAR setting as RS-GVAR models (e.g., Binder & Gross, 2013). Other advanced developments include mixed-frequency VAR models (e.g., Ghysels, 2016) and threshold VAR models (e.g., Afonso, Baxa, & Slavík, 2018). All of these methods enrich the capability of VAR models and render VAR modelling a valuable tool in tourism demand research.

Self-study questions

(1) For a classic VAR model, what are the issues if an increasing number of lagged terms are included in the model?

(2) In a GVAR model, foreign variables are treated as weakly exogenous to country-specific VARX* systems. How do we test for the weak exogeneity of foreign variables?

(3) What are the advantages of Bayesian approaches to VAR modelling?

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Appendix: R code

```
rm(list=ls())
graphics.off()

library(lubridate)  # for use in dealing with date-time data
```

Chunk 1. Read data and create data frames

```
#  "GVAR": country-specific endogenous variables
#         rtex: real tourism exports ($ million)
#         y: real GDP index (2010 = 100)
#         pi: CPI (adjusted by exchange rate against USD) (2010 = 100)
#  "Weight": weight matrix
#           elements in each row are trade shares of partner countries
#           each row sums to unity (i.e., 1)

load("Chapter5_Data.RData")
lnGVAR <- GVAR
lnGVAR[,c(3:5)] <- log(lnGVAR[,c(3:5)])  # take logarithm
```

Chunk 2. Preparation

```
#  Create a template data frame to record each model's forecasts
df_sheets <- data.frame(matrix(nrow = 20, ncol = 6))
colnames(df_sheets ) <- c("Time", "1-step ahead",
                          "2-step ahead", "3-step ahead",
                          "4-step ahead", "8-step ahead")
df_sheets[,1] <- as.yearqtr("2015-01-01") + 0:19/4

#  Extract Thailand data for benchmarks, VAR and BVAR models
df_thai <- lnGVAR[lnGVAR$country == "TH",]
df_thai <- ts(df_thai[,c(3:5)], start = c(2000,1), end = c(2019, 4), freq = 4)

#  Forecast horizons
horizon <- c(1, 2, 3, 4, 8)
```

Chunk 3. Benchmark models: (1) No-change naive; (2) SARIMA; (3) ETS

```
library(forecast)
```

Chunk 3.1 No-change naive

```
tmp_f_naive <- df_sheets  # temporary sheet

for (i in 1:20) {
  # i loop over rolling windows
  train <- window(df_thai[,"rtex"],
                  start = 2000.00 + (i-1)/4, end = 2014.75 + (i-1)/4)

  for (j in 1:length(horizon)) {
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_naive <- naive(train, h = h)
      tmp_f_naive[i+h-1,j+1] <- fcst_naive[["mean"]][[h]]
    } else {
      next
    }
  }
}

tmp_f_naive[,2:6] <- exp(tmp_f_naive[,2:6])  # un-log forecasts
fcst_naive <- tmp_f_naive  # forecasts
rm(tmp_f_naive, train)

print(fcst_naive)  # print forecasts
```

Chunk 3.2 SARIMA

```
tmp_f_sarima <- df_sheets  # temporary sheet

fit_sarima <- auto.arima(window(df_thai[,"rtex"], end = c(2014,4), freq = 4),
                         ic = "aic",
                         seasonal = TRUE,
                         stepwise = FALSE,
                         approximation = FALSE)
order <- arimaorder(fit_sarima)

for (i in 1:20) {
  # i loop over rolling windows
  train <- window(df_thai[,"rtex"],
                  start = 2000.00 + (i-1)/4, end = 2014.75 + (i-1)/4)
  model_sarima <- arima(train, order = order[1:3], seasonal = order[4:6])

  for (j in 1:length(horizon)) {
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_sarima <- forecast(model_sarima, h = h)
      tmp_f_sarima[i+h-1,j+1] <- fcst_sarima[["mean"]][[h]]
    } else {
      next
    }
  }
}

tmp_f_sarima[,2:6] <- exp(tmp_f_sarima[,2:6])  # un-log forecasts
fcst_sarima <- tmp_f_sarima  # forecasts
rm(tmp_f_sarima, train)

print(fcst_sarima)  # print forecasts
```

Chunk 3.3 ETS

```
tmp_f_ets <- df_sheets  # temporary sheet

for (i in 1:20) {
  # i loop over rolling windows
  train <- window(df_thai[,"rtex"],
                  start = 2000.00 + (i-1)/4, end = 2014.75 + (i-1)/4)
  model_ets <- ets(train,
                   model = "ZZZ",
                   ic = "aic",
                   use.initial.values = TRUE)

  for (j in 1:length(horizon)) {
```

```
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_ets <- forecast(model_ets, h = h, PI = FALSE)
      tmp_f_ets[i+h-1,j+1] <- fcst_ets[["mean"]][[h]]
    } else {
      next
    }
  }
}

tmp_f_ets[,2:6] <- exp(tmp_f_ets[,2:6])  # un-log forecasts
fcst_ets <- tmp_f_ets  # forecasts
rm(tmp_f_ets, train)

print(fcst_ets)  # print forecasts
```

Chunk 4. VAR modelling: (1) classic VAR; (2) BVAR; (3) GVAR; (4) BGVAR

Chunk 4.1 Classic vector autoregression (VAR)

```
library(vars)

tmp_f_var <- df_sheets  # temporary sheet

p <- VARselect(window(df_thai[,c(1:3)], end = c(2014,4), freq = 4),
               lag.max = 4,
               type = "both",
               season = 4)[["selection"]][["SC(n)"]]  # select order of lags

for (i in 1:20) {
  # i loop over rolling windows
  train <- window(df_thai[,c(1:3)],
                  start = 2000.00 + (i-1)/4, end = 2014.75 + (i-1)/4)
  model_var <- VAR(train,
                   p = p,
                   type = c("both"),
                   season = 4)

  for (j in 1:length(horizon)) {
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_var <- predict(model_var, n.ahead = h, ci = 0.95)
      tmp_f_var[i+h-1,j+1] <- fcst_var[["fcst"]][["rtex"]][h,1]
    } else {
      next
```

```
    }
  }
}

tmp_f_var[,2:6] <- exp(tmp_f_var[,2:6])  # un-log forecasts
fcst_var <- tmp_f_var  # forecasts
rm(tmp_f_var, train)

print(fcst_var)  # print forecasts
```

Chunk 4.2 Bayesian vector autoregression (BVAR)

```
library(BVAR)

tmp_f_bvar <- df_sheets  # temporary sheet
set.seed(1234)

#  Setting priors
minnesota <- bv_mn(lambda = bv_lambda(mode = 0.5),
                   alpha = bv_alpha(mode = 1))
priors <- bv_priors(hyper = "auto", mn = minnesota)

#  Estimation and forecast
for (i in 1:20) {
  # i loop over rolling windows
  train <- window(df_thai,
                  start = 2000.00 + (i-1)/4, end = 2014.75 + (i-1)/4)
  model_bvar <- bvar(train,
                     lags = 4,
                     n_draw = 10000L,
                     n_burn = 5000L,
                     n_thin = 2L,
                     priors = priors,
                     verbose = FALSE)

  for (j in 1:length(horizon)) {
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_bvar <- predict(model_bvar, horizon = h)
      fcst_result <- as.matrix(fcst_bvar[["quants"]][,,1])
      tmp_f_bvar[i+h-1,j+1] <- fcst_result[2,h]
    } else {
      next
    }
  }
}
```

```
tmp_f_bvar[,2:6] <- exp(tmp_f_bvar[,2:6])  # un-log forecasts
fcst_bvar <- tmp_f_bvar  # forecasts
rm(tmp_f_bvar, train)

print(fcst_bvar)  # print forecasts
```

Chunk 4.3 Global vector autoregression (GVAR)

The forecast in this section is based on the first stage of GVAR modelling. An alternative, tractable way of implementing GVAR forecast is to use the GVAR Toolbox developed by Smith and Galesi (2014).

```
library(GVARX)

tmp_f_gvar <- df_sheets  # temporary sheet

tmp <- lnGVAR
names(tmp)[1:2] <- c("ID", "Time")  # set names as required by the package
tmp$ID <- as.character(tmp$ID)
tmp$Time <- as.character(as.Date(tmp$Time, format = "%y Q%q"))

#  Time-variant weight matrix 2000-2019
WeightALL <- split(Weight[,3:26], f = Weight$Time)

#  Foreign variables for Thailand, used as exogenous variables in forecast
exo <- GVAR_Ft(data = tmp,weight.matrix = WeightALL)[[21]]
exo <- ts(exo, start = c(2000,1), end = c(2019, 4), freq = 4)
exo <- cbind(exo, stats::lag(exo, k = -1))
exo <- window(exo, start = 2015.00, end = 2019.75)
colnames(exo) <- c("F.rtex.Lag0", "F.y.Lag0", "F.pi.Lag0",
                   "F.rtex.Lag1","F.y.Lag1", "F.pi.Lag1")
exo <- as.data.frame(exo)

#  Estimation and forecast
for (i in 1:20) {
  # i loop over rolling windows
  train_idx <- as.yearqtr(tmp$Time, format = "%Y-%m-%d") >= 2000.00 + (i-1)/4 & as.yearqtr(t
mp$Time, format = "%Y-%m-%d") <= 2014.75 + (i-1)/4
  train <- tmp[which(train_idx),]

  p <- 2
  FLag <- 2
  lag.max <- 4
  type <- "const"
  ic <- "SC"
```

```
  w_yrs <- as.character(unique(year(train$Time)))
  weight.matrix <- WeightALL[w_yrs]

  model_gvar <- GVARest(data = train,p,FLag,lag.max,type,ic,weight.matrix)

  # Extract VARX* model for Thailand, on which the forecast is based
  thai_gvar <- model_gvar[["gvar"]][[21]]

  for (j in 1:length(horizon)) {
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_gvar <- predict(thai_gvar,
                          dumvar = exo[i:(i+h-1),],
                          n.ahead = h,
                          ci = 0.95)
      tmp_f_gvar[i+h-1,j+1] <- fcst_gvar[["fcst"]][["TH.rtex"]][h,1]
    } else {
      next
    }
  }
}

tmp_f_gvar[,2:6] <- exp(tmp_f_gvar[,2:6])  # un-log forecasts
fcst_gvar <- tmp_f_gvar  # forecasts
rm(tmp_f_gvar, train)

print(fcst_gvar)  # print forecasts
```

Chunk 4.4 Bayesian global vector autoregression (BGVAR)

```
library(BGVAR)

tmp_f_bgvar <- df_sheets  # temporary sheet
set.seed(1234)

#  Weight matrix 2014
matrname <- colnames(Weight)[3:26]
Weight2014 <- as.matrix(Weight[Weight$Time == "2014",c(3:26)])
rownames(Weight2014) <- matrname

#  Data input should be a list object
#  Convert the data set (data frame) into a list
df2list <- split(lnGVAR[,c(2:5)], f = lnGVAR$country)
df2list <- sapply(names(df2list),
                  function(x) xts(df2list[[x]][,c(2:4)],
                                  order.by = as.yearqtr(df2list[[x]][[1]]),
```

```
                              "%Y/Q%q"),
                simplify=FALSE)
df2list <- df2list[c(matrname)]

#  Estimation and forecast
for (i in 1:20) {
  # i loop over rolling windows
  train <- sapply(names(df2list),
                function(x) window(df2list[[x]],
                                  index = index(df2list[[x]]),
                                  start = as.yearqtr(2000.00 + (i-1)/4),
                                  end = as.yearqtr(2014.75 + (i-1)/4)),
                simplify = FALSE)
  model_bgvar <- bgvar(Data = train,
                    W = Weight2014,
                    plag = 2,
                    draws = 5000,
                    burnin = 2500,
                    prior = "MN",
                    SV = TRUE,
                    trend = FALSE,
                    thin = 2)

  for (j in 1:length(horizon)) {
    # j loop over five horizons
    h <- horizon[j]   # h-step ahead
    if (i+h-1 <= 20) {
      fcst_bgvar <- predict(model_bgvar, n.ahead = h)
      fcst_result <- as.matrix(fcst_bgvar[["fcast"]][,,4])
      tmp_f_bgvar[i+h-1,j+1] <- fcst_result["TH.rtex",h]
    } else {
      next
    }
  }
}

tmp_f_bgvar[,2:6] <- exp(tmp_f_bgvar[,2:6])  # un-log forecasts
fcst_bgvar <- tmp_f_bgvar  # forecasts
rm(tmp_f_bgvar, train)

print(fcst_bgvar)  # print forecasts
```

Table 5.1 Sampled countries in the global tourism demand system

Australia	Austria	Brazil	Canada	China	France
Germany	India	Indonesia	Italy	Japan	Korea
Malaysia	Mexico	Netherlands	New Zealand	Philippines	Singapore
Spain	Sweden	Thailand	Turkey	United Kingdom	United States

Table 5.2 Description of endogenous variables

Variable	Definition	Source	Mean	Median	Maximum	Minimum	Std Dev.	Observations
rtex	Tourism exports in constant price and constant US dollars (base year 2010), in millions of US dollars, seasonally adjusted	Balance of Payments Statistics, IMF	7,061.14	4,661.44	44,596.96	509.27	7,100.00	1,920
y	Real GDP index, base year 2010 = 100	International Financial Statistics, IMF	100.92	100.21	206.69	35.72	21.87	1,920
p	Consumer price index (CPI) (adjusted by exchange rate against US dollar), base year 2010 = 100	International Financial Statistics, IMF	90.06	92.64	129.39	30.67	19.21	1,920

Note: *rtex* is measured using nominal travel credits (in millions of US dollars) deflated by exchange rate-adjusted CPI. Missing values are imputed in R using the *imputeTS* package. Seasonal adjustment is carried out using the *seasonal* package.

Table 5.3 Forecasting performance evaluation

	Forecasting horizons				
	1	2	3	4	5
	MAPE (%)				
SNAIVE	2.189	3.360	4.272	5.087	10.923
SARIMA	2.388	3.634	4.730	6.472	14.287
ETS	2.166	3.892	5.335	6.775	10.110
VAR	3.003	5.352	6.875	8.637	15.313
GVAR	2.946	5.700	7.587	9.813	22.407
BVAR	2.015	3.761	5.092	6.562	9.573
BGVAR	2.492	3.655	4.334	4.146	11.285
	MASE				
SNAIVE	0.504	0.781	0.994	1.186	2.523
SARIMA	0.550	0.844	1.098	1.504	3.305
ETS	0.501	0.903	1.244	1.577	2.367
VAR	0.700	1.248	1.604	2.007	3.581
GVAR	0.686	1.331	1.775	2.282	5.183
BVAR	0.465	0.872	1.186	1.524	2.240
BGVAR	0.578	0.851	1.011	0.972	2.546
	RMSE				
SNAIVE	329	530	697	837	1,617
SARIMA	345	587	824	1,045	2,039
ETS	357	591	790	1,003	1,543
VAR	425	737	988	1,265	2,148
GVAR	432	789	1,057	1,357	3,114
BVAR	335	563	766	991	1,497
BGVAR	391	532	650	608	2,310

Note: RMSE is scale dependent, whereas MAPE and MASE are not. The forecasts are for Thailand's real tourism exports, which are in the region of 10,000 million US dollars. Hence, the values of RMSE are quite large compared with the other two accuracy measures.

Figure 5.1 Evolution of real tourism exports (seasonally adjusted) of major economies, 2000–2019