FREQUENCY LOCKING OF A BREATHER FIBRE LASER, FAREY TREE AND DEVIL'S STAIRCASE

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Breathing solitons exhibiting periodic oscillatory behaviour form an important part of many different classes of nonlinear wave systems. Recently, thanks to the development of real-time detection techniques, they have also emerged as a ubiquitous mode-locked regime of ultrafast fibre lasers [1,2]. The excitation of breather oscillations in a laser naturally triggers a second characteristic frequency in the system, which therefore shows competition between the cavity repetition frequency and the breathing frequency. The theoretical model describing nonlinear systems with two competing frequencies predicts frequency locking, in which the system locks into a resonant periodic response featuring a rational frequency ratio, and quasi-periodicity following the hierarchy of the Farey tree and the structure of the devil's staircase [3]. Whilst frequency-locking phenomena have been extensively studied theoretically and experimentally in many physical systems including semiconductor lasers in the field of optics, all the investigations so far relate to systems where an external, accurately controllable modulation adds a new characteristic frequency to the system. Conversely, the link between breathers and frequency locking in fibre lasers is largely missing, arguably because tuning the breathing frequency is a laborious task when done manually, requiring precise control of multiple laser parameters. Here we circumvent this difficulty by a machine-learning approach based on the use of an evolutionary algorithm for the optimisation of the intra-cavity nonlinear transfer function steered by electronically driven polarisation control [4], and we demonstrate that a breather mode-locked fibre laser is a passive system showing frequency locking at Farey fractions [5]. The frequency-locked states, characterised by robustness against parameter (pump power and polarisation) variation and a high signal-to-noise ratio of the breathing frequency, occur in the sequence they appear in the Farey tree and within a pump-power interval given by the width of the corresponding step in the devil's staircase. The breather laser may therefore serve as a simple model system to explore universal synchronisation dynamics of nonlinear systems. Furthermore, frequency-locked breather lasers can generate wide radiofrequency combs with a line spacing that is not constrained by the length of the laser cavity and can reach the sub-megahertz range, thus representing an attractive alternative to long, unstable fibre cavities for many applications such as in high-resolution spectroscopy.

References:

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