Farey-Fraction Frequency Locking of a Breather Ultrafast Fibre Laser

X. Wu[1], Y. Zhang[1], J. Peng[1,2,*], S. Boscolo[3,*], C. Finot[4], and H. Zeng[1,2,*]

[1] *State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China*
[2] *Chongqing Institute of East China Normal University, Chongqing 401120, China*
[3] *Aston Institute of Photonic Technologies, Aston University, Birmingham B4 7ET, United Kingdom*
[4] *Laboratoire ICB, UMR 6303 CNRS - Université de Bourgogne Franche-Comté, F-21078 Dijon Cedex, France*
[*] *jspeng@lps.ecnu.edu.cn; s.a.boscolo@aston.ac.uk; hpzeng@phy.ecnu.edu.cn*

Abstract: We report frequency locking at Farey fractions of a breather mode-locked fibre laser. The ratios of the breathing frequency to the cavity-repetition frequency show the Farey-tree's hierarchy and devil-staircase's structure with a 0.906 fractal dimension. © 2022 The Author(s)

1. Introduction

The theoretical model describing nonlinear systems with two competing frequencies [1] predicts frequency locking, in which the system locks into a resonant periodic response featuring a rational frequency ratio, and quasi-periodicity following the hierarchy of the Farey tree and the structure of the devil's staircase [2]. In lasers, the two interacting frequencies can be the repetition rate of the cavity and a frequency that is externally applied to the system such as in modulated semiconductor lasers [3, 4]. Conversely, breathing solitons, which have recently emerged as a ubiquitous mode-locked regime of ultrafast fibre lasers [5–7], naturally trigger a second characteristic frequency in the nonlinear system, which therefore shows competition between the cavity repetition rate and the breathing frequency.

To date, the link between breathers and frequency locking in fibre lasers is largely missing, arguably because tuning the breathing frequency is a laborious task when done manually, requiring precise control of multiple laser parameters. Here we circumvent this difficulty by a machine-learning approach based on the use of an evolutionary algorithm (EA) and demonstrate, for the first time, frequency locking at Farey fractions of a breather mode-locked fibre laser. The frequency-locked states occur in the sequence they appear in the Farey tree and within a pump-power interval given by the width of the corresponding step in the devil's staircase. The breather laser may therefore serve as a simple model system to explore universal synchronisation dynamics of nonlinear systems. Further, frequency-locked breathers can give rise to wide and dense frequency combs, thereby providing an attractive alternative to the use of ultra-long unstable cavities in many practical applications such as, for instance, in high-resolution spectroscopy.

2. Breather Fibre Laser, Farey Tree and Devil's Staircase

The laser was an erbium-doped fibre ring cavity with normal dispersion, in which the transfer function of the nonlinear-polarisation evolution-based mode locking was controlled by three wave plates based on liquid crystal (LC) phase retarders working together with a polarisation beam splitter. The repetition rate of the laser was $f_r = 34.2\,\text{MHz}$. The laser output was split into two ports: a fraction was directly detected by a fast photodiode plugged to a real-time oscilloscope, while the remaining part was sent through a time-stretch dispersive Fourier transform (DFT) setup for spectral measurements. The oscilloscope was connected to a computer that ran the EA and controlled the polarisation state through the voltages applied on the LCs. The radio-frequency (RF) spectral properties of the laser output were further characterised by an electrical spectrum analyser (ESA).

Breathing solitons can be excited in a laser cavity by tuning the gain (pump strength) and the cavity loss (polarisation controllers) [5]. Panels (a1) and (a2) of Fig. 1 show two examples of breather operations of the laser recorded at different system parameters. The train of output pulses in Fig. 1(a1) shows periodic variations in intensity occurring across a well-defined period of 5 cavity roundtrips. The corresponding spatio-spectral representation of the laser regime (not shown) evidenced a periodic compression and stretching of the optical spectrum over cavity roundtrips, accompanied by synchronous periodic changes in pulse energy, which is a distinctive feature of breathing solitons. Whilst the period of oscillation seems to be unchanged for the pulse train in Fig. 1(a2), the quality of the periodic behaviour is clearly degraded in comparison with the previous case. The RF spectra of the laser emission taken from the ESA reveal the major difference between the two breather states. The breathing frequency of the unstable (quasi-periodic) breather state exhibits a noisy and broad structure (Fig. 1(a4)). By contrast, the stable breather state features a neat breathing frequency with narrow line-width (0.5 Hz) and high signal-to-noise ratio (SNR; Fig. 1(a3)). The breathing frequency of the stable breather state is $f_b = 6.84\,\text{MHz}$ exactly equalling one fifth of the fundamental repetition frequency, hence corresponding to a rational winding number of $f_b/f_r = 1/5$.

In [8], we introduced an approach based on an EA for the search and optimisation of the breather mode-locking regime in ultrafast fibre lasers, which relied on specific features of the RF spectrum of the breather laser output. Here, we have further developed our approach to directly pinpoint frequency-locked breathers so that the EA tunes the laser to these states only. Hence, benefiting from this reliable and efficient EA-based optimisation procedure, we have explored the transitions between the different breather states of the laser that can be accessed by varying the pump power starting from the range corresponding to a 1/5 frequency-locked state. Figure 1(b1) shows an example of a plot of the breathing frequency as a function of the pump power, revealing the presence of various plateaux (steps). The spectral measurements carried out with the ESA allow us to unambiguously relate the breathing frequencies associated with the plateaux to rational winding numbers: as shown in panels (b2-b4), when the laser operates in a frequency-locked state, the RF spectrum features a finite number n of spectral lines below the cavity repetition frequency f_r and equally spaced by f_r/n. For example, in panel (b3) the frequency-locked breather regime brings about the excitation of a RF comb that is 41 times denser than that obtained when the laser operates in the usual single-pulse stationary regime. The most intense line in the spectrum is the breathing frequency f_b, and if this is the mth line from the short-frequency side, then the corresponding winding number is given by m/n.

Importantly, in Fig. 1(b1) the winding numbers appear from left to right in the order predicted by the Farey tree, as shown in the inset of the figure, and the width of the step associated with a m/n frequency-locked state depends on the level where m/n appears in the Farey tree's hierarchy. The gaps (in pump power) between the stairs refer to quasi-periodic breather oscillations similar to the example shown in Fig. 1(a2, a4). The fractal dimension D of the set of gaps can be extracted from the width of the steps, and is calculated to be $D = 0.906 \pm 0.025$, which is close to the value of 0.87 expected from a complete devil's staircase [1]. Setting the laser to a slightly different initial polarisation state, Farey fractions belonging to other two parts of the Farey tree could be identified through the RF spectra while tuning the pump power. In both cases, the calculated dimension of the set complementary to the stairs approached that of a complete devil's staircase.

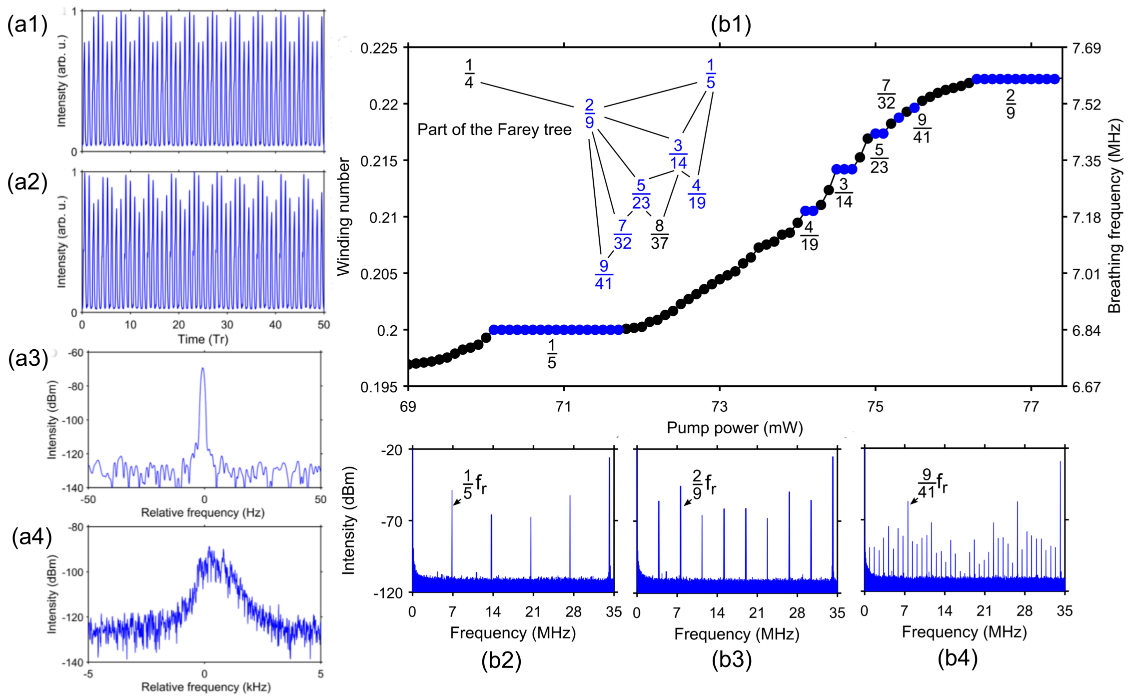

Fig. 1. (a) Characteristics of frequency-locked ((a1, a3)) and quasi-periodic ((a2, a4)) breather operations of the laser. (a1, a2): Photo-detected DFT output signals observed over 50 cavity roundtrips (T_r is the roundtrip time). (a3, a4): RF spectral measurements taken over spans of 100 Hz and 10 kHz, respectively. The reference frequency is one fifth of the fundamental repetition frequency. (b) Farey tree, devil's staircase and RF spectra. (b1): Measured breather frequency (winding number) as a function of the pump power. In the inset is shown the part of the Farey tree containing the observed Farey fractions. (b2-b4): RF spectra measured with the ESA for the frequency-locked states corresponding to the winding numbers 1/5, 2/9 and 9/41, respectively.

3. Conclusion

We have demonstrated that a fibre laser working in the breathing-soliton generation regime is a passive system showing frequency locking at Farey fractions. The frequency-locked breather states of the laser are characterised by robustness against parameter (pump power and polarisation) variations and a breathing frequency with narrow line-width and high SNR. The dimension of 0.906 determined from the measured devil's staircase indicates the universal nature of this nonlinear system. We have further demonstrated that frequency-locked breather lasers generate wide RF combs with a line spacing that is not constrained by the length of the laser cavity and can reach the sub-MHz range.

References

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