

Looking into surface plasmon polaritons guided by the acoustic metamaterials

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Abstract: Acoustic metamaterials are introduced as the structures with the alternating elements possessing effective properties that can be tuned seeking for the dramatic control on wave propagation. Homogenization of the structure under consideration is needed aiming to calculate permittivity of metamaterial. We present theoretical outcomes studying an acoustic composite possessing negative effective parameters in the acoustic frequency range. An acoustic metamaterial with an the alternating nanowires arranged in a building block and embedded in a host material was investigated. Propagation of surface plasmon polaritons at the metamaterial interface was predicted.

Keywords: surface plasmon polaritons; acoustic; metamaterial

1. Introduction

Acoustic metamaterials give rise to novel properties of the propagation of mechanical waves. The former are impossible in case of conventional materials. Recently, acoustic artificial structured materials possessing properties not found previously and opening the wide avenues for the diverse potential applications with remarkable functionalities [1-4] have been widely studied. Aiming to have a deeper insight into the properties of the structure under investigation the geometric parameters along with the effective permittivity can be tailored according to the elastic features of the scatterers. These might include either the medium preference [5, 6], or an some external factors, i.e. either electric field [7] or temperature [8].

Aiming to achieve propagation of acoustic waves in metamaterials, both density and stiffness should be negative. Recently, scientists made several suggestions and experiments aiming to attain negative effective parameters for acoustic metamaterials. To do so, different procedures, for instance, implanting soft inclusions in fluids [9], using Helmholtz resonators [10] and pipe-membrane compounds [11] have been employed.

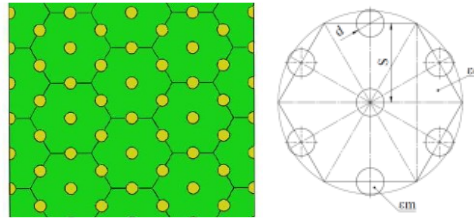
The goal of this study is to demonstrate possibilities to construct a metamaterial exhibiting negative effective parameters and show that acoustic waves can be characterized by unusual behavior. Aiming to achieve this goal we have chosen the structures that are composites of a hexagonal array of metal nanowires in a dielectric medium. An indefinite medium with a metallic nanowire array embedded in a dielectric matrix is not affected by the magnetic resonance and operates over a broad range of frequency with much lower material loss [12]. Such an anisotropic material possesses a negative electric permittivity along the nanowires and a positive permittivity perpendicular to the wires, i.e. indefinite permittivity, resulting in a hyperbolic dispersion. A great number of existing effective-medium theories [13] are limited to the optical response of nanowires that are isotropically distributed in the host material. The predicted response of these systems is not influenced by nanowire distribution and is characterized by the nanowire concentration only. These existing techniques are therefore not applicable for practical composites where the geometry is anisotropic due to fabrication process or as a result of a controlled mechanical deformation. Herein, we deal with a practical homogenized acoustic metamaterial for surface plasmon polariton guiding.

48 It should be mentioned, that the hyperbolic structures under investigation have already attracted
 49 interest before [14, 15]. Similar approaches have already been conducted for the optical frequency
 50 ranges [16-18]. However, to the best of our knowledge the enhanced structure based on the
 51 hexagonal distribution of the nanowires has not been used for acoustic SPP propagation before.

52 2. Homogenization of periodic phononic crystals

53 2.1 Mathematical model

54 A metamaterial structure under investigation is comprised of the building blocks composed of
 55 periodically arranged metallic cylinders made either of silver or gold implanted in a dielectric. These
 56 metals are chosen as the metallic nanowire materials for their lowest loss at the investigated
 57 frequency range. Figure 1 is as schematical illustration of the nanowire metamaterial. Figure 1
 58 presents the anticipated geometrical drawing of the nanowire structures. Nanowires are implanted
 59 in a host material.



60 Fig. 1. Schematic illustration of the nanowire composite (a); metamaterial unit cell (b)

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Nanowire filling fraction is described as

$$64 \quad f = \frac{\text{nanowire area}}{\text{unit cell area}} \quad (1)$$

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The nanowire filling ratio is calculated on the basis of the evaluation of the nanowire diameter
 (d) and spacing (S). It is assumed that the structure under consideration possesses a perfect
 hexagonal distribution. Doing so, we employ the equation as follows [18]:

$$68 \quad f = \frac{\pi d^2}{2\sqrt{3}S^2} \quad (2)$$

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By proceeding further with this assumption one may obtain a dispersion equation to
 characterize the modes propagating at the acoustic metamaterial interface.

72 2.2 Effective constitutive parameters

73 Herein, we study a composite containing rods with a perfect hexagonal distribution embedded
 74 in a fluid host. Nanowires are characterized by a round cylindrical shape and are made of elastic
 75 materials. Host material supports propagation of acoustic waves. It is worthwhile mentioning, that
 76 in this case wave number is perpendicular to the generatrix of the rods. A composite under
 77 consideration is quasi-isotropic. Doing so, geometrical dimensions of the inclusions needed to form
 78 the building blocks [19] are much smaller than the wavelength. We have made an assumption that
 79 all cylinders are identical. Moreover, they are distributed in a random order. It is worthwhile
 80 mentioning that distance between them is approximately equal. Our goal is to calculate the values of
 81 the effective parameters of a composite at a frequency range under consideration. Dispersion of the
 82 supported acoustic wave is also of particular interest.

83 Sonic or phononic crystals have provided a fertile ground for dealing with the composites for
 84 guiding acoustic or elastic waves. It is worthwhile noting, that in the low-frequency limit, an
 85 anisotropic compound exhibits behavior of a homogeneous composite described by outstanding
 86 effective features. The plane wave expansion method stands for as an alternative approach to
 87 multiple scattering aiming to perform homogenization of highly anisotropic medium consisting of
 88 periodic building blocks. The former methodology was for the first time considered by Krokhin [20]
 89 for sonic crystals and generalized for non-local phononic crystals in [21].

90 The system can be efficiently treated as a homogeneous uniaxial anisotropic material with a
 91 density parallel to wires (ρ_{\parallel}) and a density vertical to wires (ρ_{\perp}) [22, 23] if the wavelength of
 92 supported wave is much longer than the period of the array containing distributed nanorods. The
 93 derived expressions can be used in the low-frequency limit to get the effective parameters as follows

$$94 \quad \rho_{\parallel} = (1-f)\rho_d + f\rho_m \quad (3)$$

$$95 \quad \rho_{\perp} = \rho_d \left[\frac{\rho_m(1+f) + \rho_d(1-f)}{\rho_m(1-f) + \rho_d(1+f)} \right] \quad (4)$$

96 In a conventional case of a two-dimensional distribution of nanowire enclosures (parameters
 97 labeled with “m”) in a fluid background (parameters labeled “d”), f is the filling ratio of the crystal
 98 calculated by dividing area of the enclosures by the area of the unit cells. Aiming to modify the
 99 frequency range under consideration, one may apply the effective-medium theory designed for the
 100 specific frequency range [24].

101 It is possible to drastically alter the parameters under investigation by the dispersive
 102 corrections. For instance, the resonant Lorentz-type dispersion of the density [25] might be
 103 considered:

$$104 \quad \rho(\omega) = \rho_0 \omega_0^2 / (\omega_0^2 - \omega^2) \quad (5)$$

105 Herein, $\rho_0 > 0$ is a constant, and ω_0 is the resonant frequency of microresonators in the
 106 compound.

107 Some important relationship between the permittivity and the bulk density of the air-particle
 108 mixture is needed if the dielectric features, of granular or powdered solid materials are taken into
 109 consideration. Fundamentally, linearly dependent functions of the real and imaginary parts of the
 110 complex permittivity characterizing the specific materials such as pulverized coal, wheat, and
 111 whole-wheat flour and their bulk densities has already been found before [26, 27]. The former
 112 formalism is based on earlier studies. The linearity of $\sqrt{\varepsilon'}$ was observed by Klein in [28]. Quadratic
 113 nature of ε' and ε'' was found by Kent in [29] as follows:

$$114 \quad \varepsilon' = a\rho^2 + b\rho + 1 \quad (6)$$

$$115 \quad \varepsilon'' = c\rho^2 + d\rho \quad (7)$$

116 Herein, ρ characterizes the density of the air-particle mixture, a , b , c and d are constant values
 117 for a given particular material. It is worthwhile mentioning, that ε' and ε'' have values of 1 and 0,
 118 respectively, for air alone ($\rho = 0$).

119 3. Surface acoustic wave

120 It is worthwhile mentioning, that surface waves can not propagate at the boundary between
 121 two ideal fluids. The main reason lies in the fact of impossibility to satisfy boundary conditions in
 122 this case. Boundary states can be excited [30] if one of the materials possesses a parameter with
 123 negative values. The former provides a fertile ground for a direct analogy to surface plasmon states
 124 in plasma with $\varepsilon < 0$ [31].

125 Herein, we investigate a wave supported at the boundary separating a pure host fluid and a
 126 fluid containing nanowires implanted in a host material. The dispersion relation aiming to have a
 127 deeper insight into the properties of a surface wave is calculated as follows:

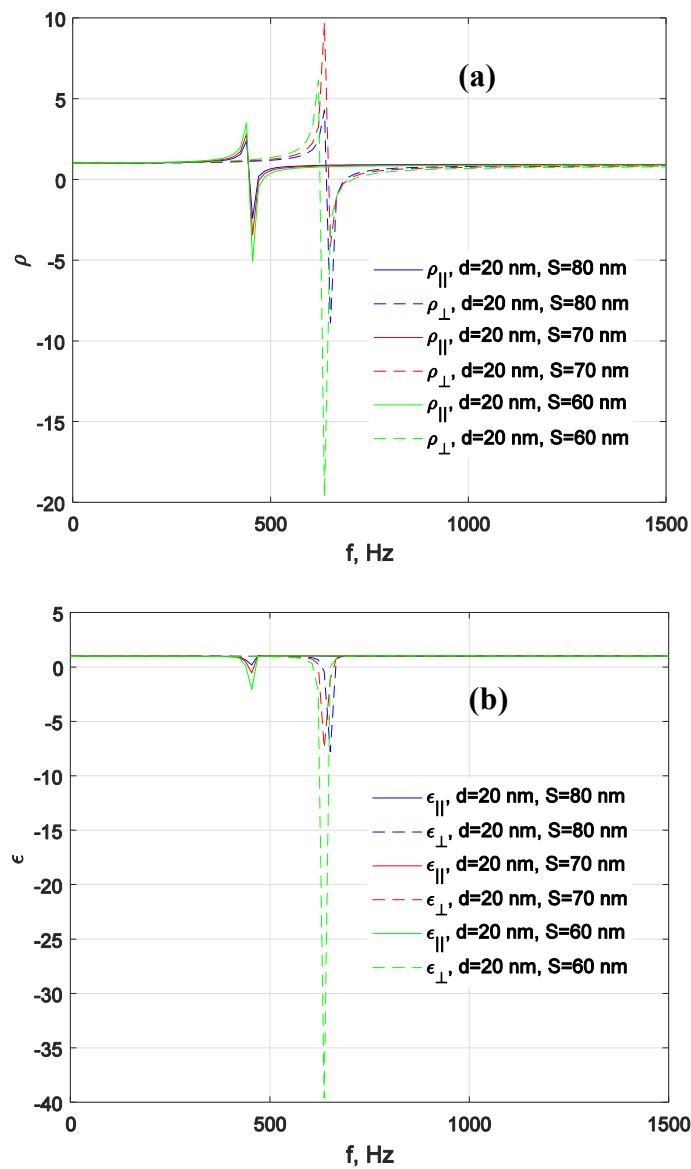
$$128 \quad \beta = k \left(\frac{(1 - \varepsilon_{\parallel}) \varepsilon_{\perp}}{1 - \varepsilon_{\perp} \varepsilon_{\parallel}} \right)^{1/2}, \quad (8)$$

129 where k – is the wavenumber.

130 **4. Results**

131 In the model employed for calculations the nanowires are considered to be conventional
 132 cylinders. Host material is air. Calculations in case of a bulk wave in the composite are presented in
 133 Figure 2. At low frequencies effective constitutive parameters have a tendency to approach “classic”
 134 values for composites [32]. However, at frequencies close to resonances inside the inclusions the
 135 values of dynamical effective parameters are significantly altered.

136 Fig. 2(a) demonstrates frequency dependence of the dynamical effective density. Because of the
 137 complex shear and longitudinal field distribution inside inclusion, resonances of different nature
 138 related to shear waves and longitudinal waves might be observed. Thus, there is a significant
 139 amount of the frequency ranges where the real part of the dynamical effective density is negative.
 140 In Fig. 2(b) permittivity versus frequency is depicted. It is worthwhile mentioning, that dimensions of
 141 the nanowires along with the distances between them have been chosen in order to meet the
 142 manufacturing requirements [33].

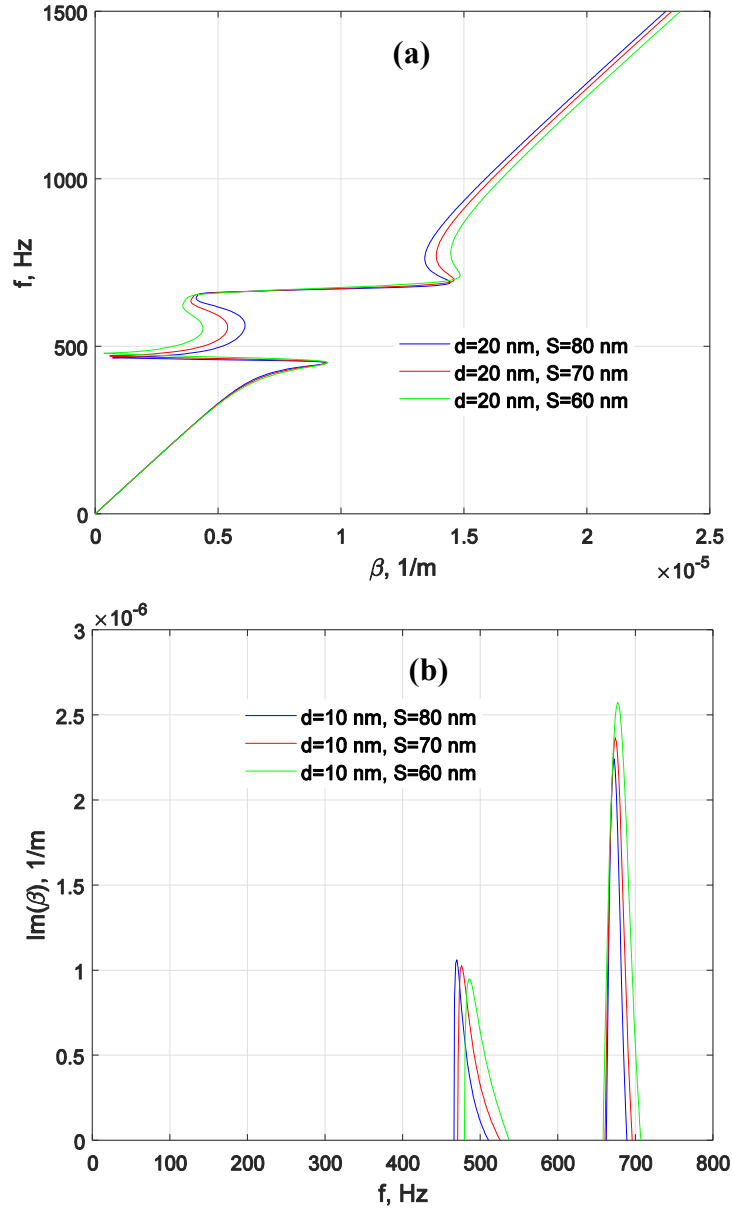


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Fig. 2. Effective density (a), permittivity (b) versus frequency for the composite under study.

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Fig. 3. The dispersion pattern of modes of surface plasmon polaritons supported by a structure under consideration demonstrated in Fig. 1: (a) $\text{Re}(\beta)$; (b) $\text{Im}(\beta)$.

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In what follows (Fig. 3), we have studied acoustic surface waves of the Rayleigh type, i. e. surface waves supported by the surface of a semi-infinite elastic metamaterial in vacuum. It is worthwhile mentioning, that the frequency for SPP propagation is determined by negative permittivity. As expected, there exists a common surface waves gap; in addition, when approaching asymptotic frequency from the low frequency direction, all the dispersion curves become very flat and asymptotically reach infinity, exhibiting behavior very similar to that of EM surface plasmon polaritons.

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5. Conclusions

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Herein, effective dynamic density of composite consisting of nanowires embedded in a host material in terms of coherent potential approximations is calculated. It is demonstrated that there are frequency regions in which dynamic constitutive parameters are instantaneously negative. In this relation, the wave under consideration becomes backward.

Dispersion pattern of the acoustic wave propagating at the boundary separating metamaterial and conventional medium is studied. It is demonstrated that there are frequency ranges in which the

164 surface states are bounded to the interface. Moreover, the exotic behavior of surface plasmon
165 polaritons has been investigated.

166 In summary, we presented modelling of the acoustic metamaterial. The wide spectral width of
167 the present research for acoustic double negative metamaterials is expected. Moreover, anticipated
168 applications including acoustic superlensing and cloaking [34-37] might be possible.

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173 **Conflicts of Interest:** The authors declare no conflict of interest.

174 **Availability of data and material:** The data that support the findings of this study are available from
175 the corresponding author upon reasonable request.

176 **Code availability:** All simulation parameters of this study are included in this manuscript.

177 **Authors' Contributions:** Conceptualization, data curation, and methodology, T.I.; investigation,
178 formal analysis, software, visualization, writing—original draft preparation, writing—review and
179 editing, T.I.; T.G.; E.R.; supervision and revision of the manuscript, validation, T.G.; E.R. All authors
180 have read and agreed to the published version of the manuscript.

181 References

- 182 1. Pennec Y., Vasseur J. O., Djafari-Rouhani B., Dobrzyński L., Deymier P. A. Two-dimensional phononic
183 crystals: examples and applications. *Surf. Sci Rep.* **2010**, 65, 229–91.
- 184 2. Ge H., Yang M., Ma C., et al. Breaking the barriers: advances in acoustic functional materials. *Nat Sci*
185 *Review* **2017**, 5, 159–82.
- 186 3. Khelif A., Adibi A. *Phononic crystals: fundamentals and applications*. Berlin, Springer, 2015.
- 187 4. Ma G., Sheng P. Acoustic metamaterials: from local resonances to broad horizons. *Sci Adv.* **2016**, 2,
188 e1501595.
- 189 5. Lin S. -C. S., Huang T. J., Sun J. -H., Wu T. -T. Gradient-index phononic crystals. *Phys. Rev. B* **2009**, 79,
190 094302.
- 191 6. Titovich A. S., Norris A. N., Haberman M. R. A high transmission broadband gradient index lens using
192 elastic shell acoustic metamaterial elements. *J. Acoust. Soc. Am.* **2016**, 139, 3357–64.
- 193 7. Yi K., Collet M., Ichchou M., Li L. Flexural waves focusing through shunted piezoelectric patches. *Smart*
194 *Mater. Struct.* **2016**, 25, 075007.
- 195 8. Yong G., Hong-Xiang S., Chen L., et al. Acoustic focusing by an array of heat sources in air. *Appl. Phys.*
196 *Express* **2016**, 9, 066701.
- 197 9. Li J., Chan C.T. Double-negative acoustic metamaterial. *Phys. Rev. E* **2004**, 70, 055602.
- 198 10. Fang N., Xi D., Xu J., Ambati M., Srituravanich W., Sun C., Zhang X. Ultrasonic metamaterials with
199 negative modulus. *Nature Materials* **2006**, 5, 452-456.
- 200 11. Lee S. H., Park C. M., Seo Y. M., Wang Z. G., Kim C. K. Composite acoustic medium with simultaneously
201 negative density and modulus. *Phys. Rev. Lett.* **2010**, 104, 054301.
- 202 12. Liu Y., Bartal G., Zhang X. All-angle negative refraction and imaging in a bulk medium made of metallic
203 nanowires in the visible region. *Opt. Exp.* **2008**, 16(15), 439–448.
- 204 13. G. W. Milton, *The Theory of Composites*, Cambridge University Press, Cambridge, 2002.
- 205 14. Wong Z. J., Wang Y., O'Brien K., Rho J., Yin X., Zhang S., Fang N., Yen T.-J. and Zhang X. Optical and
206 acoustic metamaterials: superlens, negative refractive index and invisibility cloak. *J. Opt.* **2017**, 19, 084007.
- 207 15. Fan B., Filonov D., Ginzburg P., and Podolskiy V. A. Low-frequency nonlocal and hyperbolic modes in
208 corrugated wire metamaterials. *Opt. Express* **2018**, 26, 17541-17548.
- 209 16. Ioannidis T., Gric T., Gorodetsky A., Trofimov A., Rafailov E. Enhancing the properties of plasmonic
210 nanowires. *Materials research express* **2019**, 6, 1-8.
- 211 17. Ioannidis T., Gric T., Rafailov E. Surface plasmon polariton waves propagation at the boundary of
212 graphene based metamaterial and corrugated metal in THz range. *Optical and quantum electronics* **2020**, 52,
213 1-12.

- 214 18. Starko-Bowes R., Atkinson J., Newman W., Hu H., Kallos T., Palikaras G., Fedosejevs R., Pramanik S. and
215 Jacob Z. Optical characterization of epsilon near zero, epsilon near pole and hyperbolic response in
216 nanowire metamaterials. *J. Opt. Soc. Am. B* **2015**, 32, 2074–80.
- 217 19. Caloz C., Itoh T. *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*.
218 Wiley-IEEE Press, Hoboken/New Jersey, 2005.
- 219 20. Krokhin A. A., Arriaga J., Gumen L. N. Speed of sound in periodic elastic composites. *Phys. Rev. Lett.* **2003**,
220 91, 264302.
- 221 21. Norris A. N., Shuvalov A. L., Kutsenko A. A. Analytical formulation of three-dimensional dynamic
222 homogenization for periodic elastic systems. *Proc. Roy Soc. A-Math Phy Eng. Sci* **2012**, 468, 1629–51.
- 223 22. Liu Y., Bartal G., Zhang X. All-angle negative refraction and imaging in a bulk medium made of metallic
224 nanowires in the visible region. *Opt. Exp.* **2008**, 16, 15439–15448.
- 225 23. Podolskiy V. A., Narimanov E. E. Strongly anisotropic waveguide as a nonmagnetic left-handed system.
226 *Phys. Rev. B* **2005**, 71, 201101.
- 227 24. Gric T., Hess O. Surface plasmon polaritons at the interface of two nanowire metamaterials. *J. Opt.* **2017**,
228 19, 085101.
- 229 25. Ambati M., Fang N., Sun C., and Zhang X. Surface resonant states and superlensing in acoustic
230 metamaterials. *Phys. Rev. B* **2007**, 75, 195447.
- 231 26. Nelson S. O. Observations on the density dependence of the dielectric properties of particulate materials.
232 *Journal of Microwave Power* **1983**, 18(2), 143-152.
- 233 27. Nelson S. O. Density dependence of the dielectric properties of wheat and whole-wheat flour. *Journal of*
234 *Microwave Power* **1984**, 19(1), 55-64.
- 235 28. Klein A. Microwave determination of moisture in coal -- comparison of attenuation and phase
236 measurement. *Journal of Microwave Power* **1981**, 16(3&4), 289-304.
- 237 29. Kent M. Complex permittivity of fish meal: a general discussion of temperature, density, and moisture
238 dependence. *Journal of Microwave Power* **1977**, 12(4), 341-345.
- 239 30. Ambati M., Fang N., Sun C., Zhang X. Surface resonant states and superlensing in acoustic metamaterials.
240 *Phys. Rev. B* **2007**, 75, 195447.
- 241 31. Caloz C., Itoh T. *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*.
242 Wiley-IEEE Press, Hoboken/New Jersey, 2005.
- 243 32. Berryman G. Long-wavelength propagation in composite elastic media I. Spherical inclusions. *J. Acoust.*
244 *Soc. Am.* **1980**, 68, 1809.
- 245 33. Yao B. J., Wang Y., Tsai K.-T., Liu Z., Yin X., Bartal G., Stacy A. M., Wang Y.-L. and Zhang X. Design,
246 fabrication and characterization of indefinite metamaterials of nanowires. *Phil. Trans. R. Soc. A* **2011**, 369,
247 3434–3446.
- 248 34. Ambati M., Fang N., Sun C. and Zhang X. Surface resonant states and superlensing in acoustic
249 metamaterials. *Phys. Rev. B* **2007**, 75, 195447.
- 250 35. Guenneau S., Movchan A., P`etursson G. and Ramakrishna S. A. Acoustic metamaterials for sound
251 focusing and confinement. *New J. Phys.* **2007**, 9, 399.
- 252 36. Chen H. and Chan C. T. Acoustic cloaking in three dimensions using acoustic metamaterials. *Appl. Phys.*
253 *Lett.* **2007**, 91, 183518.
- 254 37. Torrent D. and S`anchez-Dehesa J. Acoustic cloaking in two dimensions: a feasible approach. *New J. Phys.*
255 **2008**, 10, 063015.