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A Scenario-based Model for Resource Allocation with Price Information

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Abstract In this paper, we consider the problem of allocating resources among Decision Making Units (DMUs). Regarding the concept of overall (cost) efficiency, we consider three different scenarios and formulate three Resource Allocation (RA) models correspondingly. In the first scenario, we assume that overall efficiency of each unit remains unchanged. The second scenario is related to the case where none of overall efficiency scores is deteriorated. We improve the overall efficiencies by a pre-determined percentage in the last scenario. We formulate Linear Programming problems to allocate resources in all scenarios. All three scenarios are illustrated through numerical and empirical examples.

Keywords: Data Envelopment Analysis; Resource Allocation; Overall Efficiency.

1. Introduction

1.1. Literature review and motivation

Data envelopment analysis is a mathematical programming technique for measuring the efficiency of a group of Decision Making Units (DMUs) initially proposed by [5]. Later on, [4] generalized their model to a Variable Returns to Scale case later. All DMUs are evaluated based on their observed input/output values, and an efficiency score is obtained for each one.

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One interesting application of DEA is in the problem of Resource Allocation (RA) in a production plan that is well-known in operations research and management sciences literature. Recent developments exhibit an additional planning orientation for the resource allocation problem. The use of DEA provides an alternative to the RA problem because it makes it possible to consider feasible production plans and trade-offs between inputs/outputs of different units based on the empirical characterization of the Production Possibility Set (PPS). [9] utilized the additive DEA model and introduced an RA model in this framework. Besides efficiency, they also consider effectiveness and equality in their RA model. [3] and [2] applied goal programming models in order to propose a two-phase RA model in a general DEA framework.

In many RA applications like emission, banks, police stations, hospitals, schools, etc., there is a top manager who is interested in maximizing the efficiency of each individual unit. Simultaneously, (s)he might minimize the total input consumption or maximize the total output production. This sort of situation occurs when a DM is managing and providing resources for all units. [11] considered the corresponding RA problems as the centralized RA problems. They suggested centralized DEA models to reduce the total value of resources consumed by all units in an organization, rather than considering the consumption of individual units separately. [12] dealt with the RA problem in a centralized framework in the presence of undesirable outputs. They took into account the emission permit reallocation in their RA model based on DEA. developed a number of non-radial, output-oriented, centralized DEA models to determine individual and collective output targets and apply the proposed approach to the Spanish Port Agency. [10] developed a multiple-objective linear programming approach for resource allocation with the aim to maximize the total amount of outputs of all units simultaneously. They assumed that a central unit controls all the units simultaneously and proposed RA models by taking into account the total amount of the outputs of all units to be maximized. Their RA model keep the technical efficiency of all DMUs unchanged.

[7] proposed two RA models based on the cost efficiency concept. Their first RA model guarantees both unchanged technical efficiency and unchanged cost efficiency level. Their second model guarantees that the cost efficiency of none of DMUs deteriorates after allocating resources. Defining the overall performance of the whole system as the ratio of the efficiencies before and after the RA, [13] formulated a linear programming RA model to improve the performance of the system. [18] provided a review of DEA-based resource and cost allocation approaches by classifying them by industry and by model formulation. [19] dealt with the scale economies in the RA problem based on DEA approach. [8] dealt with the emission issues in the process of resource allocation and proposed an emission-based RA model-based DEA and environmental efficiency measures. [14] dealt with the centralized RA and developed two generalized models based on DEA for the centralized RA problem. They extended the centralized RA model of [11] for the possibility of reallocation of resources. [6] incorporated the cross-efficiency measures in the centralized RA problem. [15] considered the target setting line in the RA problem and concentrated on the profit improvement of targeted units. [17] studied the centralized RA models based DEA from an axiomatic perspective. They described the envelopment form and multiplier form of associated DEA models. In the current paper, we propose a scenario-based RA model that is more generalized and more flexible compared with exisiting models in the literature. We take

the overall efficiency into consideration and provide different setings that help decision-maker in the process of allocating resources.

1.2. Primary comparison with relative work in the literature and contribution of the current work

In this paper, we propose three RA models based on the concept of overall (cost) efficiency. The first model preserves the overall efficiency scores of all DMUs. Our first RA model in this study is more flexible compared to the first RA model in [7]. Particularly, our first RA model does not force input-output changes to be proportional, while the first RA model in [7] was formulated based on that assumption. Our second RA model in this paper guarantees that the overall efficiency of none of DMUs deteriorates after allocating resources. Basically, the main structure of the second RA model imposes no restrictions on allocation of extra resources and improvements of overall efficiency scores, but the managerial and resource limitations that usually occur in real-life applications are incorporated into the second model. The third RA model improves the overall efficiency score of all units by a predetermined percentage desired for the DM. This RA model can be utilized for the short, mid, and long-run planning based on the DM's wishes and limitations. All proposed RA models in this paper are linear, and there is no concern regarding to solving and finding their optimal solutions.

This paper unfolds as follows. Section 2 provides basic preliminaries necessary for our results. Section 3 investigates three scenarios and RA models are formulated and illustrated by numerical examples correspondingly. Section 4 provides an empirical example using the real data to show the strength and applicability of proposed RA models. Concluding results are provided in the last section.

2. Preliminaries

Suppose that there are n decision making units DMU_j, consuming the input vector $X_j = (X_{1j}, \dots, X_{mj})$ to produce the output vector $Y_j = (Y_{1j}, \dots, X_{pj})$, for $j=1,\dots n$. All input and output components are assumed to be positive. The famous Production Possibility Set (PPS) can be defined as

$$T = \left\{(X,Y) \middle| \sum_{j=1}^{n} \lambda_j X_j \leq X, \sum_{j=1}^{n} \lambda_j Y_j \geq Y, Y \geq 0, \lambda \in \Lambda\right\}, \quad (1)$$

where

- $\Lambda = \mathbb{R}_+^n$ stands for the PPS with the constant returns to scale (CRS) technology,
- $\Lambda = \left\{\lambda \in \mathbb{R}_+^n \middle| \sum_{j=1}^{n} \lambda_j = 1\right\}$ stands for the PPS with variable returns to scale (VRS) technology,
- $\Lambda = \left\{\lambda \in \mathbb{R}_+^n \middle| \sum_{j=1}^{n} \lambda_j \leq 1\right\}$ stands for the PPS with the non-increasing returns to scale (NIRS) technology, and
- $\Lambda = \left\{\lambda \in \mathbb{R}_+^n \middle| \sum_{j=1}^{n} \lambda_j \geq 1\right\}$ stands for the PPS with the non-decreasing returns to scale (NDRS) model.

Correspondingly, we denote the PPS by T_{CRS}, T_{VRS}, T_{NIRS}, and T_{NDRS}, in the four above-mentioned cases.

The fundamental DEA model to measure the technical efficiency of DMU_k, for $k = 1,2,\dots,n$, can be written as follows:

$$
\begin{aligned}
TE_k = Min\ \ \theta \qquad & \\
s.t.\ \sum_{j=1}^{n} \lambda_j X_{ij} \le \theta X_{ik}, & \quad i = 1,\dots,m \\
\sum_{j=1}^{n} \lambda_j Y_{rj} \ge Y_{rk}, & \quad r = 1,\dots,p \\
\lambda \in \Lambda. &
\end{aligned}
\tag{2}
$$

Model (2) is said to be an input-oriented CCR (BCC) model in envelopment form assuming the CRS (VRS) technology for the PPS (See [5] and [4] for more details.).

Definition 1. Let (θ^*, λ^*) be the optimal solution of problem (2). The value of θ^* is said to be the technical efficiency score for DMU_k. If $\theta^* = 1$, we say DMU_k is technically efficient. Otherwise, it is technically inefficient.

In case that the prices of inputs are is available, the concept of overall (cost) efficiency can be obtained for each unit. Assume that c_i is the price of one unit of i-th input, for $i = 1,\dots,m$. The following model provides the minimum cost for producing output vector Y_k:

$$
\begin{aligned}
C^*(Y_k) = Min\ \sum_{i=1}^{m} c_i X^i \qquad & \\
s.t.\ \sum_{j=1}^{n} \lambda_j X_{ij} \le X^i, & \quad i = 1,\dots,m \\
\sum_{j=1}^{n} \lambda_j Y_{rj} \ge Y_{rk}, & \quad r = 1,\dots,p \\
\lambda \in \Lambda. &
\end{aligned}
\tag{3}
$$

Definition 2. Let (X^*, λ^*) be the optimal solution for model (3). The overall efficiency of DMU_k is defined as the ratio of the minimum cost to the actual cost of producing Y_k

$$CE_k = \frac{C^*(Y_k)}{C(Y_k)} = \frac{\sum_{i=1}^{m} c_i x^{i^*}}{\sum_{i=1}^{m} c_i x_{ik}}.$$

If $CE_k = 1$, DMU_k is said to be overall efficient.

On this definition of cost efficiency, refer to Cooper et al, (pp 236-237). It can be shown that the overall efficiency is a sufficient condition for technical efficiency. In other words, if a unit is overall efficient, it is also technically efficient, but the converse does not necessarily hold true. Technical efficiency does not provide a full picture of causes of inefficiency (See [7]). On the other hand, the number of efficient units are fewer if the performance of the system is evaluated by means of overall efficiency instead of technical efficiency (See [16]). Furthermore, overall efficiency can be regarded as a special case of effectiveness when multipliers must reflect realistic values or prices (See [1] for more details about effectiveness). All these facts motivate us to consider overall efficiency instead of technical efficiency analysis in the context of resource allocation.

3. Overall Efficiency Analysis in the context of Resource Allocation

Suppose that an expectation level is available for each DMU_k in the production plan. Let $\bar{Y}_{rk}$ be the value of expected r-th output for DMU_k for $r = 1, \dots, p$ and $k=1,\dots,n$. The question is how much input is required for producing the expected output levels. Bearing the concept of overall efficiency in mind, this aim can be met considering three different goals:

I. Overall efficiency of all units stays unchanged.
II. Overall efficiency of none of DMUs deteriorates.
III. Overall efficiency of individual units improves by a pre-determined percentage desirable for the DM.

We consider three different scenarios based on the above-mentioned goals. The first scenario defines the situation where the DM is interested in keeping the overall efficiency scores unchanged. Such a scenario is useful in the short-term where there are some resources available and the DM is interested in devoting them to all decision making units and keeping the efficiencies unchanged in a short period seems reasonable. The second scenario is related to the situation where improvements in efficiencies are expected. We model a linear programming RA problem and prove that overall efficiencies are not deteriorated after solving it. The third scenario relates to the case that the exact amount of improvements in overall efficiencies is pre-determined. The RA model formulated in this case guarantees the achievement of pre-determined values for overall efficiencies.

3.1 First Scenario: Unchanged Overall Efficiencies

Improving the performance of DMUs may not be an easy task in the short run, and so we should not expect a revolution in the performance of DMUs via the RA process. This motivates us to construct the first scenario of RA assuming unchanged overall efficiency scores of all units in the process of RA.

Assume that $\bar{Y}_{rk}$ is the expected value for output r of DMU_k. We formulate and solve the following linear programming model for DMU_k:

$$Min \quad \sum_{i=1}^{m} c_i X^i \tag{4}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j X_{ij} \leq X^i, \qquad i = 1, \dots, m \tag{4.1}$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq \bar{Y}_{rk}, \qquad r = 1, \dots, p \tag{4.2}$$

$$\lambda \in \Lambda.$$

Denote the optimal solution of model (4) as $(\tilde{\lambda}, \widetilde{X}_k)$. Assume that $(X_k + \Delta X_k,\ Y_k + \Delta Y_k)$ is the activity level of DMU_k after RA. The optimal value for input/output changes for units after resource allocation can be obtained from solving the following RA model:

$$Min \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_i \, \Delta X_{ij} \tag{5}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_{kj} X_{ij} \leq (X_{ik} + \Delta X_{ik}), \quad i = 1, \dots, m, k = 1, \dots, n \tag{5.1}$$

$$\sum_{j=1}^{n} \lambda_{kj} Y_{rj} \geq Y_{rk} + \Delta Y_{rk}, \qquad r = 1, \dots, p, \ \ k = 1, \dots, n \tag{5.2}$$

$$Y_{rk} + \Delta Y_{rk} \geq \bar{Y}_{rk}, \qquad r = 1, \dots, p, k = 1, \dots, n \tag{5.3}$$

$$C^t \ (X_k + \Delta X_k) = \frac{C^t \, \tilde{X}_k}{CE_k}, \qquad k = 1, \dots, n \tag{5.4}$$

$$\lambda_k \ \in \ \Lambda_k, \qquad k = 1, \dots, n. \tag{5.5}$$

Although model (5) may look similar to model (7) proposed by Dehnokhalaji et al. (2017), there are differences between them. While the latter is a multi-objective linear programming model, model (5) is a linear programming model and easier to solve. Also, the constraint (5.4) is new here to guarantee that the overall efficiency score remains unchanged after allocation. The following theorem proves our claim.

Theorem 1. The overall efficiency of DMU_k remains unchanged after the resource allocation obtained from model (5).

Proof. Let $(X_k + \Delta X_k,\ Y_k + \Delta Y_k)$ be the input/output vector for DMU_k after solving model (5). We solve the following problem:

$$C^*(Y_k + \Delta Y_k) = \ Min \quad \sum_{i=1}^{m} c_i X^i \tag{6}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j X_{ij} \leq X^i, \qquad i = 1, \dots, m \tag{6.1}$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{rk} + \Delta Y_{rk}, \qquad r = 1, \dots, p \tag{6.2}$$

$$\lambda \in \Lambda.$$

Let (λ', X') be the optimal solution of problem (6). We prove that

$$\frac{\sum_{i=1}^{m} c_i X^{i^*}}{\sum_{i=1}^{m} c_i X_{ik}} = \frac{\sum_{i=1}^{m} c_i X^{i'}}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik})}. \quad (7)$$

From (5.4) we have

$$\frac{\sum_{i=1}^{m} c_i X^{i^*}}{\sum_{i=1}^{m} c_i X_{ik}} = CE_k = \frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik})}. \quad (8)$$

From (5.3), we have $Y_{rk} + \Delta Y_{rk} \geq \bar{Y}_{rk}$ for all r, k. Therefore, any feasible solution for problem (6) is also a feasible solution for problem (4), and since constraints (4.1) and (6.1) hold as equality at optimality, we can conclude that the following equation holds at the optimality of problems (4) and (6):

$$\sum_{i=1}^{m} c_i \tilde{X}^i = \sum_{i=1}^{m} c_i X^{i'}. \quad (9)$$

Now from (8) and (9), we can conclude that (7) holds true. Thus the overall efficiency score of all units remains unchanged after solving RA model (5). ■

In contrast with the first RA model of [7], which also guarantees unchanged overall efficiency scores, model (5) is more flexible and general. Input/output changes are assumed to be proportional in RA model proposed by [7] while we do not impose such a constraint in RA model (5). Regardless of input-output changes, in RA model (5) we only impose constraints that guarantee unchanged overall efficiency scores for all units.

Numerical Example 1. Consider five DMUs using two inputs to produce one single output. We consider C= (2,5). The input prices are imaginary here, and our approach works, no matter how we select prices. The data and overall efficiency scores are reported in Table 1.

Table 1: Input-output data and overall efficiency scores for Numerical Example 1.

DMUs	Input 1	Input 2	Output	Overall Efficiency (OE)
A	5	8	3	0.1457
B	3	4	7	0.6538
C	9	5	5	0.2823
D	4	6	6	0.3834
E	6	1	7	1.0000

Using the first scenario and assuming the increment of 2 and 0.5 units for the output of units A and B, respectively, we obtain new input/output values by solving RA model (5) reported in Table 2. It can be seen from the last column of Table 2 that the overall efficiency scores of all units remain unchanged in the first scenario.

Table 2: Result of the first scenario for Numerical Example 1.

DMUs	Input 1	Input 2	Output	OE	OE NEW
A	2.1429	15.8095	5.0000	0.1457	0.1457
B	3.2143	4.2857	7.5000	0.6538	0.6538
C	2.1429	7.7429	5.0000	0.2824	0.2824
D	2.5714	6.5714	6.0000	0.3835	0.3835
E	6.0000	1.0000	7.0000	1.0000	1.0000

3.2 Second Scenario: Overall Efficiencies are improved or remain unchanged

The second scenario assumes more freedom for the overall efficiency of DMUs; that is, it assumes the possibility of improving overall efficiency scores for all units. Consequently, the second scenario is more flexible comparing to the first one.

The second RA model that guarantees overall efficiency of none of DMUs deteriorates can be formulated as the following linear programming problem:

$$Min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_i\,\Delta X_{ij} \tag{10}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_{kj} X_{ij} \le (X_{ik} + \Delta X_{ik}), \quad i = 1,\dots,m,\ k = 1,\dots,n \tag{10.1}$$

$$\sum_{j=1}^{n} \lambda_{kj} Y_{rj} \ge Y_{rk} + \Delta Y_{rk}, \quad r = 1,\dots,p,\ \ k = 1,\dots,n \tag{10.2}$$

$$Y_{rk} + \Delta Y_{rk} \ge \bar{Y}_{rk}, \quad r = 1,\dots,p, k = 1,\dots,n \tag{10.3}$$

$$C^t\ (X_k + \Delta X_k) \le \frac{C^t\,\tilde{X}_k}{CE_k}, \quad k = 1,\dots,n \tag{10.4}$$

$$\lambda_k \in \Lambda_k, \quad k = 1,\dots,n. \tag{10.5}$$

Although model (10) may look similar to model (13) proposed by Dehnokhalaji et al. (2017), there are differences between them. While the latter is a multi-objective linear programming model, model (5) has a linear programming formulation, and hence it is easier to solve. Also, the constraint (10.4) is new here to guarantee that the overall efficiency score is not deteriorated after allocation. Theorem 2 provides the evidence for this claim.

Theorem 2. The overall efficiency of *DMUk* is not deteriorated after resource allocation.

Proof. Let $(X_k + \Delta X_k,\ Y_k + \Delta Y_k)$ be the input/output vector for *DMUk* after solving model (10). We first solve the following model:

$$C^*(Y_k + \Delta Y_k) = \quad Min \quad \sum_{i=1}^{m} c_i X^i \tag{11}$$

$$s.t. \quad \textstyle\sum_{j=1}^{n} \lambda_j X_{ij} \leq X^i, \qquad i = 1, \dots, m \tag{11.1}$$

$$\textstyle\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{rk} + \Delta Y_{rk}, \qquad r = 1, \dots, p \tag{11.2}$$

$$\lambda \in \Lambda.$$

Let (λ', X') be the optimal solution of problem (11). We prove that

$$\frac{\sum_{i=1}^{m} c_i X^{i^*}}{\sum_{i=1}^{m} c_i X_{ik}} \leq \frac{\sum_{i=1}^{m} c_i X^{i'}}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik})}. \tag{12}$$

From (10.4) we have

$$\frac{\sum_{i=1}^{m} c_i X^{i^*}}{\sum_{i=1}^{m} c_i X_{ik}} = CE_k \leq \frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik})}. \tag{13}$$

From (10.3), we get $Y_{rk} + \Delta Y_{rk} \geq \bar{Y}_{rk}$ for all *r, k*. Therefore, any feasible solution for problem (11) is also feasible for problem (4), and since constraints (4.1) and (11.1) hold as equality at optimality, we can conclude that the following equation holds at the optimality of problems (4) and (11):

$$\textstyle\sum_{i=1}^{m} c_i \tilde{X}^i = \sum_{i=1}^{m} c_i X^{i'} \tag{14}$$

Now from (13) and (14), we can conclude that (12) holds true. Thus the overall efficiency of DMU_k is not deteriorated for all *k* after solving RA model (10). ■

The next theorem compares our second RA model with the second RA model proposed by [7], which is as follows

$$Min \quad \textstyle\sum_{i=1}^{m} \sum_{j=1}^{n} c_i \, \Delta X_{ij} \tag{15}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_{kj} X_{ij} \leq X_{ik} + \Delta X_{ik}, \qquad i = 1, \dots, m, \qquad k = 1, \dots, n$$

$$\sum_{j=1}^{n} \lambda_{kj} Y_{rj} \geq Y_{rk} + \Delta Y_{rk}, \qquad r = 1, \dots, p, \qquad k = 1, \dots, n$$

$$\sum_{i=1}^{m} c_i \Delta X_{ik} \leq 0 \qquad k = 1, \dots, n$$

$$\lambda_k \in \Lambda_k, \qquad k = 1, \dots, n.$$

Theorem 3. Considering the same managerial assumptions and expecting the same output level of $Y_{rk} + \Delta Y_{rk}$, the total cost of RA model (10) is not greater than the total cost of model (15).

Proof. Let $(\lambda^*, \Delta X^*, \Delta Y^*)$ be the optimal solution of RA model (15). Thus we have

$$\sum_{j=1}^{n} \lambda_{kj} X_{ij} \leq X_{ik} + \Delta X_{ik}^*, \qquad i = 1, \dots, m, \quad k = 1, \dots, n$$

$$\sum_{j=1}^{n} \lambda_{kj} Y_{rj} \geq Y_{rk} + \Delta Y_{rk}^*, \qquad r = 1, \dots, p, \quad k = 1, \dots, n$$

$$\textstyle\sum_{i=1}^{m} c_i \Delta X_{ik}^* \leq 0 \qquad k = 1, \dots, n$$

$$\lambda_k^* \in \Lambda_k, \qquad k = 1, \dots, n.$$

The input and output set constraints and intensity constraint of $\lambda_k \in \Lambda_k$ are the same in both RA models; thus the optimal solution of RA model (15) satisfies associated constraints in RA model (10). We just need to check the third constraint of this model. Please note that

$$\textstyle\sum_{i=1}^{m} c_i \Delta X_{ik}^* \leq 0 \quad k = 1, \dots, n.$$

Therefore,

$$\frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik}^*)} = \frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{\sum_{i=1}^{m} c_i X_{ik} + \sum_{i=1}^{m} c_i \Delta X_{ik}^*} \geq \frac{\sum_{i=1}^{m} c_i X_{ik}^*}{\sum_{i=1}^{m} c_i X_{ik}} = CE_k\ .$$

This implies that

$$\frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{CE_k} \geq \textstyle\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik}^*).$$

Equivalently,

$$\textstyle\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik}^*) \leq \frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{CE_k}.$$

This means that the optimal solution of the RA model (15) satisfies the constraint set (10.4) of the RA model (10). Therefore, $(\lambda^*, \Delta X^*, \Delta Y^*)$ satisfies all constraint of the RA model (10) that implies $\sum_{i=1}^{m} \sum_{j=1}^{n} c_i\, \Delta \tilde{X}_{ij} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} c_i\, \Delta X_{ij}^*$ and this ends the proof. ■

Although models (10) and (15) has the same structure, our RA model (10) provides better solutions in terms of the total cost comparing to model (5), the second RA model of [7], and both models guarantee that overall efficiency of none of units deteriorates after allocation.

Numerical Example 2. Consider the data of Example 1 and outputs changes reported in the fourth column of Table 3.

Table 3: Input-output data and overall efficiency scores for Numerical Example 2.

DMUs	Input 1	Input 2	Output	OE
A	5	8	3+2	0.1457
B	3	4	7+0.5	0.6538
C	9	5	5+1	0.2823
D	4	6	6+1.5	0.3834
E	6	1	7+0.75	1.0000

Following the second scenario, we find the following results for input changes reported in Table 4. As you can see, the overall efficiency scores of all units are improved after using the new input/output values.

Table 4: Result of the second scenario for Numerical Example 2.

DMUs	Input 1	Input 2	Output	OE	OE NEW
A	4.2857	0.7143	5.0000	0.1457	1.0000
B	6.4286	1.0714	7.500	0.6538	1.0000
C	5.1429	0.8571	6.0000	0.2824	1.0000
D	6.4286	1.0714	7.500	0.3835	1.0000
E	6.6429	1.1071	7.7500	1.0000	1.0000

Observe that the new overall efficiency scores of all DMUs become unity. This fact is not surprising when we do not impose any restriction for improving cost efficiency scores due to the existence of no limitation in changing (increasing or decreasing) the current input levels. However, there are some limitations for input changes in practice, and we are not allowed to freely increase or decrease input values. Numerical Example 3 illustrates the practical situation in scenario 2.

Numerical Example 3. Consider the same output changes as in numerical example 2, where we impose the constraint $-0.3X_{ik} < \Delta X_{ik} < 0.3 + X_{ik}$ as the restriction on input values. We solve RA model (5) and obtain the results reported in Table 5.

Table 5: Result of the *modified* second scenario.

DMUs	Input 1	Input 2	Output	OE	OE NEW
A	3.5000	5.6000	5.0000	0.1457	0.3469
B	3.9000	3.6000	7.5000	0.6538	0.7060
C	6.3000	3.5000	6.0000	0.2824	0.4841
D	3.3000	4.2000	7.5000	0.3835	0.6599
E	6.6429	1.1071	7.7500	1.0000	1.0000

Observe that the overall efficiency scores of all DMUs improve after allocation, but we find more rational results in the sense that input changes are restricted, and overall efficiency scores of all units are not equal to unity. Note that limitation on input changes can be considered in all RA models, including RA models in scenario 1 and scenario 3, if it is necessary.

3.3 Third Scenario: Improvement of Overall efficiencies by certain values

The third scenario might be more applicable for a long-run strategy and planning, where we assume a specific percentage of improvement for the efficiency score of each unit which is at the DM's discretion limitations, desires, goals, etc. can be considered in the planning process of the third RA scenario.

Assume that h_k is the percentage improvement for DMU_k.The third RA model can be formulated as the following linear programming problem:

$$Min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_i \,\Delta X_{ij} \tag{16}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_{kj} X_{ij} \le (X_{ik} + \Delta X_{ik}), \qquad i = 1, \dots, m,\ k = 1, \dots, n \tag{16.1}$$

$$\sum_{j=1}^{n} \lambda_{kj} Y_{rj} \ge Y_{rk} + \Delta Y_{rk}, \qquad r = 1, \dots, p,\ \ k = 1, \dots, n \tag{16.2}$$

$$Y_{rk} + \Delta Y_{rk} \ge \bar{Y}_{rk}, \qquad r = 1, \dots, p, \qquad k = 1, \dots, n \tag{16.3}$$

$$(1 + h_k)\, C^t(X_k + \Delta X_k) \geq \frac{C^t\, \tilde{X}_k}{CE_k}, \quad k = 1, \dots, n \qquad (16.4)$$

$$\lambda_k \in \Lambda_k, \qquad k = 1, \dots, n. \qquad (16.5)$$

Clearly, if no improvement is assumed, the RA model (16) will be similar to the RA model (5).

Theorem 4. The overall efficiency of *DMU*$_k$ increases h_k times at most for all *k* after solving the RA model (16).

Proof. Let $(X_k + \Delta X_k,\ Y_k + \Delta Y_k)$ be the input-output vector for *DMU*$_k$ after solving model (16). We solve the following model:

$$C^*(Y_k + \Delta Y_k) = \ Min \ \sum_{i=1}^{m} c_i X^i \qquad (17)$$

$$s.t. \ \ \sum_{j=1}^{n} \lambda_j X_{ij} \leq X^i, \qquad i = 1, \dots, m \qquad (17.1)$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{rk} + \Delta Y_{rk}, \qquad r = 1, \dots, p \qquad (17.2)$$

$$\lambda \in \Lambda.$$

Let (λ', X') be the optimal solution of problem (17). We prove that

$$\frac{\sum_{i=1}^{m} c_i X^{i'}}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik})} \leq (1 + h_k) \frac{\sum_{i=1}^{m} c_i X^{i^*}}{\sum_{i=1}^{m} c_i X_{ik}}. \qquad (18)$$

From (16.4) we have

$$\frac{\sum_{i=1}^{m} c_i X^{i^*}}{\sum_{i=1}^{m} c_i X_{ik}} = CE_k \geq \frac{\sum_{i=1}^{m} c_i \tilde{X}^i}{\sum_{i=1}^{m} c_i (X_{ik} + \Delta X_{ik})} \frac{1}{(1 + h_k)} \qquad (19)$$

From (16.3), we get $Y_{rk} + \Delta Y_{rk} \geq \bar{Y}_{rk}$ for all *r, k.* Therefore, any feasible solution for problem (17) is also feasible for problem (4), and since constraints (4.1) and (17.1) hold as equality at optimality, we can conclude that the following equation holds at the optimality of problems (4) and (17):

$$\sum_{i=1}^{m} c_i\, \tilde{X}^i = \sum_{i=1}^{m} c_i X^{i'} \qquad (20)$$

Now from (19) and (20), we can conclude that (18) holds true. Therefore the overall efficiency of *DMU*$_k$ increases h_k times at most for all *k* after solving the RA model (16).■

Numerical Example 4. With the same data, we consider new output values reported in the fourth column of Table 6. We also assume a specific improvement percentage for the overall efficiency of each unit reported in the last column of Table 6.

Table 6: Input-output data and overall efficiency scores for Numerical Example 3.

DMUs	Input 1	Input 2	Output	OE	Improvement (%)
A	5	8	3+2.00	0.1457	0.9
B	3	4	7+0.50	0.6538	0.3
C	9	5	5+1.00	0.2823	0.8
D	4	6	6+1.50	0.3834	0.7
E	6	1	7+0.75	1.0000	0.0

Using the aforementioned setting for new output levels and overall efficiency improvement percentages and using the third scenario, we find the following results reported in Table 7 by solving the RA model (16). As you see, the new input/output values guarantee to produce the expected output levels and overall efficiency improvement percentage.

Table 7: Result of the third scenario for Numerical Example 3.

DMUs	Input 1	Input 2	Output	Improvement (%)	OE	OE NEW
A	14.787	2.8571	5.00	0.9	0.1457	0.2769
B	5.3572	2.1428	7.50	0.3	0.6538	0.8500
C	5.7619	3.4286	6.00	0.8	0.2824	0.5083
D	3.2563	4.2857	7.50	0.7	0.3835	0.6519
E	6.6429	1.1071	7.75	0.0	1.0000	1.0000

4. Empirical illustration

In this section, we apply our models for a data set of 25 supermarkets in Finland taken from [10] reported in Table 8. Man-hours and size are considered as inputs (second and third columns), and sales and profit are considered as outputs (fourth and fifth columns). Man-hours refer to the labor force used within a certain period, and size is the total retail floor space of the supermarket. Assuming the input cost vector of C = (1.5, 2), the overall efficiency scores are reported at the sixth column of Table 8. Note that the input prices are imaginary here and we can obtain the results of our proposed RA model for any choice of input prices.

Table 8: Input-output data and overall efficiency of 25 Finnish supermarkets

SM	Man-hour	Size	Sales	Profit	OE
SM1	4.99	79.1	115.3	1.71	0.7986
SM2	3.30	60.1	75.2	1.81	0.6877
SM3	8.12	126.7	225.5	10.39	1.0000
SM4	6.70	153.9	185.6	10.42	0.7521
SM5	4.74	65.7	84.5	2.36	0.7032
SM6	4.08	76.8	103.3	4.35	0.7491
SM7	2.53	50.2	78.8	0.16	0.8642
SM8	2.47	44.8	59.3	1.30	0.7274
SM9	2.32	48.1	65.7	1.49	0.7527
SM10	4.91	89.7	163.2	6.26	1.0000
SM11	2.24	56.9	70.7	2.80	0.6900
SM12	5.42	112.6	142.6	2.75	0.6979
SM13	6.28	106.9	127.8	2.70	0.6561
SM14	3.14	54.9	62.4	1.42	0.6241
SM15	4.43	48.8	55.2	1.38	0.6131
SM16	3.98	59.2	95.9	0.74	0.8860
SM17	5.32	74.5	121.6	3.06	0.8913
SM18	3.69	94.6	107	2.98	0.6255
SM19	3.00	74.0	65.4	0.62	0.7621
SM20	3.87	54.6	71	0.01	0.7102
SM21	3.31	90.1	81.2	5.12	0.5991
SM22	4.25	95.2	128.3	3.89	0.7437
SM23	3.79	80.1	135	4.73	0.9290
SM24	2.99	68.7	98.9	1.86	0.7947
SM25	3.10	62.3	66.7	7.41	1.0000

Scenario 1

Consider the expected output values for units reported in 4th and 5th columns of Table 9, that is, 10 percentage increment for both outputs. We aim to find the required input levels that can produce the expected output value with the same overall efficiency scores. The input and outputchanges after allocation are reported in the 2nd and 3rd columns of Table 9.

As can be seen from Table 9, the changes in man-hour can be positive or negative for all supermarkets and the total changes required for the man-hours is equal to -5.24. On the other hand, the required changes in the size of supermarks are all positive, with the total value of changes equal to 201.8. Also changes in both sales and profit are positive as expected and the total sum of changes in Sales and profit is 258.6 and 8.19 units for all supermarkets altogether. The overall efficiency scores remain unchanged after the allocation as we expected according to Table 9. This allocation of resources to supermarkets require 395.7 extra costs for all supermarkets to be paid.

According to the results of the first scenario, if the top management team sets targets to increase the sales and profits for all supermarkets, and expects each local branch to have the same performance in the short-run, then it is expected to supply more resources for the second output and decrease the resources of the first ourput. If changing the size is impossible for supermarkets, then this input can be considered as a non-sicretionary one and we expect to get different results for our analysis. The model can be generalized to the case where there are non-discretionary inputs in the dataset.

The difference between our approach and Korhonen and Syrjanen's (2004) mthod is that we solve an LP and our assumprion is to keep Overal efficiencies unchanges while their model is an MOLP and they concentrate on the technical efficiencies remaining unchanged. The motivation of considering the cost efficiency instead of technical efficiency in resource allocation problems has been explained in the work of Dehnokhalaji and Ghiasi (2017).

Scenario 2

We keep the same setting as the first scenario, that is, 10 percentage increments of both outputs and our goal is finding the required input levels such that the overall efficiency score of none of the supermarkets deteriorates after allocation. However, input levels are allowed to be decreased or increased at least by 10 percentage. Table 10 reports the results of this run. The changes in input/output values after allocation for each supermarket is reported in Table 10. Both man-hour and size show positive and negative changes for different supermarkets. But the the sum of changes in both man-hour and size are both negative and equal to -11.73 and -281.9 respectively. This means that both resources need to be increased with the emphasize on the size in order to improve the overall efficiency scores. The overall efficiency scores remain unchanged only for the overall efficient units SM3, SM10 and SM25 and both inputs are increased after resource allocation for this units to keep their efficiency unchanged and increase their ourputs. The input/output changes are exactly the same as in the first scenario for these three units. For all other 22 supermarkets, overall efficiencies are improved between 0.071 as for SM23 and 0.4007 as for SM21. Man-hour and size change are both positive for SM23, while the size change for SM21 is equal to -31.2 which is the largest value of decrease in the size among all supermarkets. This can explain how the largest value of increase in efficiency scores happens for SM21.

This scenario is applicable when the top management aim is to improve the efficiency scores of supermarkets in long term and there are no extra resources available for supermarkets, and each local branch needs to increase the profit and sale, so the only way to achive this goal is to decrease staff members and the size of the supermarkets. In reality, this scenario may not work if the inputs are not easy to decrease, for instance in the long-run, it is not easy to decrease the human resources, or size in this specific example.

Scenario 3

In order to determine specific improvements for overall efficiency scores in resource allocation, we consider the third scenario and the corresponding RA model. The percentage of improvements of overall efficiency scores of all units are reported in Table 11. The assumed changes vary from one supermarket to another. Therefor the overall efficiency of the whole system is improved as well.

of the input/output changes after allocation are reported in Table 11 for each supermarket. As can be seen, the total man-hour increased by 1.31 unit while the total size decreased by 233.82. So this scenario concentrates on decreasing the second input again to allow the pre-determined improvement level for each supermarket. The overall efficiency scores remain unchanged only for the overall efficient units SM3, SM10 and SM25, as there was no room for more improvement for these units and the percentage of improvement was defined as 0 percent for these three brancesh. Both inputs are increased after resource allocation for this units to keep their efficiency unchanged and increase their ourputs. All three scenarios provide the same input changes values for us because in the all three models, the efficiency scores remain unchanged. Assuming the aforementioned overall efficiency improvement for the whole system, it is possible to save 432.65 units of costs in total. This scenario may be considered as a mid-run or a long-run plan. Scenario 1 concentrates on keeping the overall efficiencies unchanged, and so it is useful in short term. In order to increase the outputs, the management teams need to devote extra resources to the system. Both Scenario 2 and 3 concentrate on improving the overall performance of the system when there are no extra resources available. Hence, to achieve larger values of outputs, there is an essential need to decrease inputs in both scenarios. While scenario 2 does not impose any restriction on the level of improvement of efficiency scores for branches, scenario 3 sets some target for the level of improvement to control improvements and the process of allocation more. Scenario 3 may be preferred to scenario 1 because it is more systematic, but how to indicate the percentage of improvements for each unit is an issue, and needs to be considered by the managerial team. Overal efficient units show the same level of changes in all three scenarios. Scenario 1 asks for increasing resources whereas scenarios 2 and 3 show decreasing resources. All provide different changes for resources for inefficient units.

5. Concluding Remarks

In this paper, we developed a resource allocation model in the context of overall efficiency for three different scenarios. First, we consider the problem of allocating resources among decision making units in order to keep the overall efficiency scores unchanged. This scenario is useful in the short-run when the DM is interested in allocating available resources to units. The second scenario assumes that overall efficiencies are not deteriorated through resource allocation. Finally, we supposed a pre-determined improvement of the overall efficiency score of each decision making unit and allocated resources to DMUs to achieve this goal.

We formulated three linear programming models for all scenarios, which is the preference of this approach to other existing approaches like [7]. All three scenarios are illustrated by numerical and empirical examples, and they provided promising results.

All models can be generalized for the case that there are non-discretionary and discretionary inputs simultaneously, so we can keep the input values unchanged when changing them is out of the control of the decision maker.

Appendix

Table 9: Results of the first scenario for the Empirical Example.

SM	Man-hour Changes	Size Changes	Sales Changes	Profit Changes	OE	OE NEW
SM1	-1.17	9.2	11.5	0.17	0.7986	0.7964
SM2	-0.81	6.9	7.5	0.18	0.6877	0.6876
SM3	0.81	12.7	22.6	1.04	1.0000	1.0000
SM4	1.60	14.7	18.6	1.04	0.7521	0.7563
SM5	-1.67	8.2	8.5	0.24	0.7032	0.7002
SM6	1.01	7.2	10.3	0.44	0.7491	0.7507
SM7	0.08	5.2	7.9	0.02	0.8642	0.8655
SM8	-0.51	5.0	5.9	0.13	0.7274	0.7273
SM9	-0.15	5.1	6.6	0.15	0.7527	0.7543
SM10	0.49	9.0	16.3	0.63	1.0000	1.0000
SM11	1.34	4.9	7.1	0.28	0.6900	0.6938
SM12	-0.70	12.2	14.3	0.28	0.6979	0.6994
SM13	-2.05	12.7	12.8	0.27	0.6561	0.6552
SM14	-1.07	6.5	6.2	0.14	0.6241	0.6236
SM15	-2.60	7.2	5.5	0.14	0.6131	0.6060
SM16	-0.81	6.8	9.6	0.07	0.8860	0.8824
SM17	-1.30	8.8	12.2	0.31	0.8913	0.8865
SM18	-0.15	9.8	10.7	0.3	0.6255	0.6288
SM19	-0.84	5.6	6.5	0.06	0.7621	0.7598

SM20	-1.52	6.9	7.1	0	0.7102	0.7065
SM21	2.52	7.4	8.1	0.51	0.5991	0.6034
SM22	0.00	9.8	12.8	0.39	0.7437	0.7462
SM23	1.66	7.0	13.5	0.47	0.9290	0.9313
SM24	0.28	6.9	9.9	0.19	0.7947	0.7977
SM25	0.31	6.2	6.7	0.74	1.0000	1.0000
Sum	-5.24	201.8	258.6	8.19		

Table 10: Modified input/output values and results of the second scenario.

SM	Man-hour Change	Size Change	Sales Change	Profit Change	OE	OE NEW
SM1	-1.17	-9.4	11.5	0.17	0.7986	0.9734
SM2	-0.81	-14.6	7.5	0.18	0.6877	0.8405
SM3	0.81	12.7	22.6	1.04	1.0000	1.0000
SM4	0.98	-27.4	18.6	1.04	0.7521	0.9206
SM5	-1.67	-14.6	8.5	0.24	0.7032	0.8558
SM6	-0.33	-13.7	10.3	0.44	0.7491	0.9175
SM7	0.08	-2.6	7.9	0.02	0.8642	1.0000
SM8	-0.51	-8.9	5.9	0.13	0.7274	0.8889
SM9	-0.15	-8.4	6.6	0.15	0.7527	0.9219
SM10	0.49	9.0	16.3	0.63	1.0000	1.0000
SM11	0.17	-14.0	7.1	0.28	0.6900	0.8449
SM12	-0.70	-26.4	14.3	0.28	0.6979	0.8548
SM13	-2.05	-29.6	12.8	0.27	0.6561	0.8008
SM14	-1.07	-17.2	6.2	0.14	0.6241	0.7622
SM15	-2.60	-15.4	5.5	0.14	0.6131	0.7406
SM16	-0.81	-1.2	9.6	0.07	0.8860	0.9950
SM17	-1.30	-1.0	12.2	0.31	0.8913	0.9926
SM18	-0.15	-29.9	10.7	0.30	0.6255	0.7685
SM19	-0.84	-7.5	6.5	0.06	0.7621	0.9287
SM20	-1.52	-11.7	7.1	0.00	0.7102	0.8635
SM21	0.15	-31.2	8.1	0.51	0.5991	0.7366
SM22	0.00	-17.6	12.8	0.39	0.7437	0.9109

SM23	0.68	1.5	13.5	0.47	0.9290	0.9313
SM24	0.28	-8.9	9.9	0.19	0.7947	0.9694
SM25	0.31	6.2	6.7	0.74	1.0000	1.0000
Sum						

Table 11: New input-output data based on the third scenario.

SM	%	Man-hour Change	Size Change	Sales Change	Profit Change	OE	OE NEW
SM1	0.3	-1.30	-11.8	11.5	0.17	0.7986	1.0000
SM2	0.4	1.63	-14.6	7.5	0.18	0.6877	0.9627
SM3	0.0	0.81	12.7	22.6	1.04	1.0000	1.0000
SM4	0.3	0.61	-24.9	18.6	1.04	0.7521	0.9833
SM5	0.3	0.00	-10.6	8.5	0.24	0.7032	0.9103
SM6	0.3	-0.56	-11.9	10.3	0.44	0.7491	0.9759
SM7	0.2	-0.02	-4.3	7.9	0.02	0.8642	1.0000
SM8	0.3	0.00	-7.2	5.9	0.13	0.7274	0.9455
SM9	0.3	0.00	-7.7	6.6	0.15	0.7527	0.9806
SM10	0.0	0.49	9.0	16.3	0.63	1.0000	1.0000
SM11	0.3	0.37	-9.3	7.1	0.28	0.6900	0.9019
SM12	0.3	0.00	-17.9	14.3	0.28	0.6979	0.9092
SM13	0.4	0.00	-23.9	12.8	0.27	0.6561	0.9173
SM14	0.4	0.00	-12.3	6.2	0.14	0.6241	0.8731
SM15	0.4	0.00	-11.2	5.5	0.14	0.6131	0.8484
SM16	0.2	-0.98	-4.4	9.6	0.07	0.8860	1.000
SM17	0.2	-1.54	-5.4	12.2	0.31	0.8913	1.000
SM18	0.4	0.33	-21.1	10.7	0.30	0.6255	0.8803
SM19	0.3	-0.16	-7.5	6.5	0.06	0.7621	0.9878
SM20	0.3	0.00	-8.8	7.1	0.00	0.7102	0.9185
SM21	0.4	0.44	-20.2	8.1	0.51	0.5991	0.8448
SM22	0.3	0.13	-15.2	12.8	0.39	0.7437	0.9700
SM23	0.1	0.57	-0.4	13.5	0.47	0.9290	1.0000
SM24	0.3	0.17	-11.0	9.9	0.19	0.7947	1.0000
SM25	0.0	0.31	6.2	6.7	0.74	1.0000	1.0000
		1.31	-233.82	258.61	8.17		

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