Conical refraction with generalized Bessel-Gaussian beams and their application to conical refraction cavities

Valentin Yu. Mylnikov[1], Edik U. Rafailov[2], Grigorii S. Sokolovskii[1]

1. Ioffe Institute, St. Petersburg, 194021, Russia

2. Optoelectronics and Biomedical Photonics Group, AIPT, Aston University, Aston Triangle, Birmingham B4 7ET, UK

It is well known that the light beam transforms into a hollow light cylinder after passing along one of the optical axes of a biaxial crystal. This optical phenomenon is known as conical refraction (CR) and fascinates researchers for almost two centuries. The characteristic features of CR find a number of practical applications in optical manipulation, polarization metrology, and singular optics to name a few [1]. However, these seen to be hindered by the lack of intuitive understanding and prediction of the CR transverse intensity pattern. This problem was partially solved, by expressing the CR field as a superposition of the generalized Bessel-Gaussian (BG) beams [2].

In this paper, we theoretically investigate the CR of a linearly polarized generalized BG beam. One can write the input electric field amplitude E at its focal plane in the form: $E(R)=\exp(-½(R/w_0)^2)J_0((\gamma+ir_0)R/w_0)$, where J_0 is the Bessel function of the first kind of zeroth order, w_0 is a beam width in a focal plane, γ is a normalized semi-aperture of the wave vectors cone and r_0 is a normalized input beam radius. We start with the "Gaussian" case of $\gamma=r_0=0$ (Fig. 1 (a)), which yields the classical two-ring Lloyd's distribution in the focal plane (Fig. 1 (b)). The radius of Lloyd's ring distribution R_0 is determined by the parameters of the CR crystal. Then semi-aperture γ can be increased, and the Bessel function starts to superimpose on the Gaussian distribution (Fig. 1 (c)). This Bessel function modulation leads to a transition from a double- to a multi-ring intensity pattern of CR in the focal plane, as shown in Fig. 1 (d). The next step is to increase the input radius of the generalized BG beam r_0 (Fig. 1 (e)), which causes the spatial separation of the CR cones [3] (Fig. 1 (f)). The radius of each cone is changed by the input beam radius. With a further increase in the input radius r_0 (Fig. 1 (g)), the inner cone passes through the center, and internal intensity distribution rotates 180° (Fig. 1(h)). The last is consistent with experiment in which the CR from the axicon was observed [4]. We also show that the CR beam can be described in terms of generalized BG beams in the limit of large values of the semi-aperture γ. This observation essentially simplifies the physical interpretation of the CR spatial evolution and allows to avoid the complicated integral expressions.

Summary, we investigate the features of the CR intensity pattern for the different shapes of the input generalized BG beams. We also show that the CR beam can be described through the generalized BG beams. This leads to the conclusion that the generalized BG beams are modes of a laser cavity formed with spherical and conical mirrors, where the CR crystal acts as an active element. We believe that the mode of this resonator has a CR polarization distribution and a fractional orbital angular momentum.

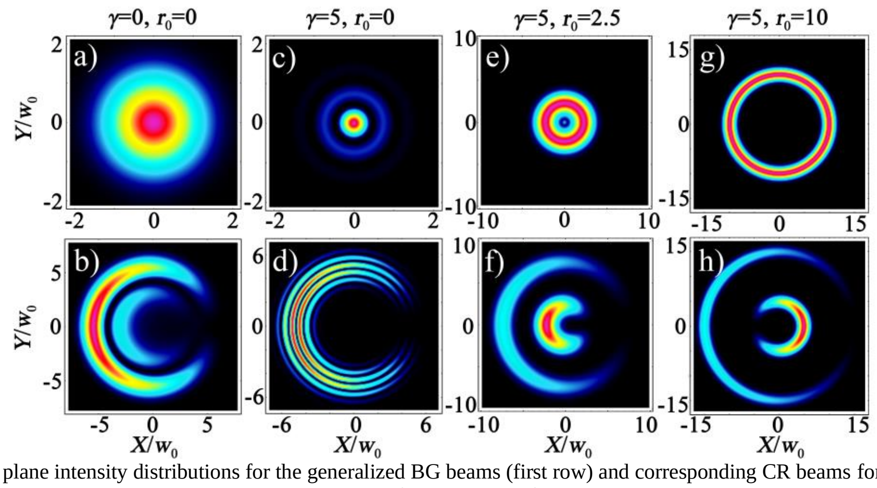

Fig. 1 Focal plane intensity distributions for the generalized BG beams (first row) and corresponding CR beams for $R_0=5w_0$ (second row). The numerical calculations were carried out for the values of the parameters $\gamma=0$, $r_0=0$ (a, b), $\gamma=5$, $r_0=0$ (c, d), $\gamma=5$, $r_0=2.5$ (e, f), and $\gamma=5$, $r_0=10$ (g, h).

References

[1] A. Turpin, Y. V. Loiko, T. K. Kalkandjiev, and J. Mompart, "Conical refraction: fundamentals and applications," Laser Photonics Rev. **10**, 750-771 (2016).

[2] V. Yu. Mylnikov, E. U. Rafailov, and G. S. Sokolovskii, "Close relationship between Bessel–Gaussian and conical refraction beams," Opt. Express **28**, 33900-33910 (2020).

[3] G. S. Sokolovskii, D. J. Carnegie, T. K. Kalkandjiev, and E. U. Rafailov, "Conical Refraction: New observations and a dual cone model," Opt. Express **21**, 11125-11131 (2013)

[4] A. Turpin, Yu. V. Loiko, T. K. Kalkandjiev, H. Tomizawa, and J. Mompart, "Wave-vector and polarization dependence of conical refraction," Opt. Express **21**, 4503-4511 (2013).