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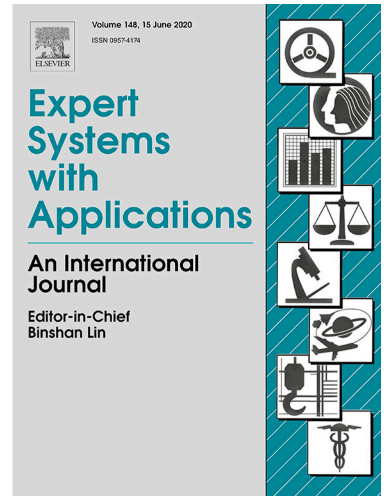

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A Robust Credibility DEA Model with Fuzzy Perturbation Degree: An Application to Hospitals Performance

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Abstract

Performance evaluation enables decision makers (DMs) to have a better view about the weaknesses and strengths of leading units to improve efficiencies as a crucial goal. Data envelopment analysis (DEA) is the most popular technique to measure performance efficiency of decision making units (DMUs). However, conventional DEA is unable to consider uncertainty of input and output data in the evaluations. In this study, in order to address uncertainty in data, a robust credibility DEA (RCDEA) model has been introduced. First, a fuzzy credibility approach is used to construct fuzzy data. Then, a robust optimization approach is applied to consider uncertainty in constructing fuzzy sets. Moreover, perturbation level is considered as exact and fuzzy values. To illustrate the capability of the proposed model, 28 hospitals are evaluated in northwestern region of Iran and results are analyzed. According to the results, as perturbation degree increases, DMUs get normalized lower efficiencies and vise-versa.

Keywords: Data envelopment analysis; Robust optimization; Fuzzy sets; Efficiency

1. Introduction

Decision makers (DMs) and managers consider "efficiency evaluation" as a useful tool for determining the weaknesses, strengths and performance of units. Indeed, "efficiency evaluation" gives a better picture of their leading units' performances and lead to make optimal and reliable decisions by Decision Makers (DMs) to improve efficiency.

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Improving efficiency is a crucial goal for units in almost all sectors such as health, education, industry, economic, environment and etc. (Wei et al. 2011).

Health sector plays a vital role in societies prosperities and progress. In the other words, health is precedent for any progress in the societies and is one of the main criteria in measuring development degree of societies. Therefore, performance evaluation of healthcare systems has a great importance for DMs to set policies and consequently improve service quality. Hospitals as the main components of healthcare system, are composed of human resources, medical equipment, materials, technologies, information and buildings which aim to provide primary, secondary and tertiary healthcare services (Al-Refaie et al. 2014). In other words, hospitals are considered as a huge part of health care systems that consume the health care resources (Omrani et al. 2018). Hospitals are very important organizations whose existence is directly related to the general health of the societies. In recent decades, countries, especially developing countries, have substantially invested on equipping hospitals and constructing new ones to improve the health situation in their societies. Hence, hospitals performance evaluation has devoted of a great attention in terms of increasing expenditures and competition among both practitioners and academics (Otay et al. 2017). Data Envelopment Analysis (DEA) is widely used technique to evaluate the performance of decision making units (DMUs). Conventional DEA is unable to consider the uncertainty in data whereas a small perturbation in data could make a big change in feasibility, ranking and evaluation of DEA approach. This problem has led researchers to extend optimization approaches which are immune against uncertainty. In this study, we have addressed the shortcoming of conventional DEA in considering uncertainty in data, using fuzzy credibility approach. Moreover, uncertainty of DMs preferences in constructing fuzzy upper, lower and medium values modelled using robust optimization. Finally, the perturbation degree of robust optimization technique has been considered as fuzzy sets.

The rest of paper is organized as follow: In section 2 literature related to the DEA, fuzzy DEA and robust DEA have been reviewed. Methodology of the paper has been explained in section 3. In section 4 proposed model is applied to evaluate efficiency of 28 hospitals in northwestern region of Iran. Section 5 presents the results of the model and finally, conclusion is made in section 6.

2. Literature review

In this section the methods have been used in this paper are reviewed. First, DEA as the most popular technique in evaluating the performance of DMUs has been reviewed. Then the weakness of conventional DEA in considering uncertainty has been addressed using fuzzy set. Furthermore, uncertainty in constructing fuzzy sets has been addressed with robust optimization. We proceeded one more level in dealing with uncertainty and considered the perturbation level in the robust optimization as the fuzzy sets.

2.1. Data Envelopment Analysis

Data envelopment analysis (DEA) is one of the most popular and widely used methods in measuring performance efficiency. DEA is a non-parametric model to measure the relative efficiency of a set of homogenous DMUs with multiple inputs and multiple outputs (Charnes et al. 1978). This model is based on a series of optimizations using linear programming techniques. In fact, DEA specifies an efficient frontier curve by applying a linear programming method. This popular method has been applied to measure performance efficiency of hospitals in many researches. For instance, Khushalani and Ozcan (2017) evaluated efficiency of producing quality of hospitals between 2009 and 2013 using dynamic network DEA. Dynamic network DEA was used to compute efficiency scores for hospital sub-divisions i.e. medical/surgical care (patient visits, surgeries and discharges) and quality. Chowdhury and Zelenyuk (2016) examined the production performance of hospital services in Ontario for the years of 2003 and 2006 using DEA with truncated regression approach. Omrani et al. (2018) in order to rank 288 hospitals in Iran applied integrated fuzzy clustering cooperative game DEA approach. They used a fuzzy C-means technique to cluster the DMUs. Then applied DEA combined with the game theory where each DMU is considered as a player using Core and Shapley value approaches within each cluster. The results show that the Core and Shapley values are suitable for fully ranking of efficient hospitals in the healthcare systems. Gandhi and Sharma (2018) used three methods of DEA, Malmquist productivity index (MPI) and Tobit regression to measure the efficiency of 37 Indian hospitals. As it can be seen, DEA is widely used method to measure hospitals performance efficiency in literatures, however, a vital assumption in linear DEA programming is that this model uses deterministic amount of inputs to product deterministic amount of outputs. Indeed, input and output data are deterministic. On the other hand, in real world applications, data are sometimes contaminated with uncertainty and uncertainty in data may lead to unreliable, imprecise and even infeasibility of solutions (Ben-Tal and Nemirovski, 2000; Alizadeh and Omrani, 2019). In literatures, various methods of fuzzy sets, stochastic programming and robust optimization have been applied to deal with uncertainty in DEA data. In this paper, we have addressed uncertainty in data using fuzzy sets and robust optimization simultaneously. Since in engineering and management practices, it is difficult to collect data with knowing probability distribution function, so stochastic programming is unavailable in such cases (Yin et al 2018). In the following a review of literatures on fuzzy sets and robust optimization is summarized.

2.2. Fuzzy Data Envelopment Analysis

The first steps of incorporating fuzzy set theory into DEA models was taken by Sengupta (1992). Sengupta (1992) introduced a DEA model with fuzzy objective and constraints by defining a tolerance level on DEA constraint violations and analyzed the model using Zimmermann (1978) approach. Later, Triantis and Girod (1998) applied a novel three stage DEA model to measure the technical efficiency in a fuzzy environment. In the first stage, for input and output variables a membership function was defined, so that imprecise input and output variables are expressed in terms of their risk-free and impossible bounds. In the second stage, conventional DEA-VRS and DEA-CRS were formulated in terms of the risk-free and impossible bounds and the membership function for each of the fuzzy input and output variables. Finally, technical efficiencies were measured according to the different membership function values. Their proposed three stage DEA model is categorized in the most popular fuzzy α-level based approach. This method enables decision maker (DM) to observe the impact of modifying input and output values between risk-free and impossible bounds on the efficiency scores. In the other words, the main purpose of the α-level based approach is to provide a pair of parametric programs which measures the upper and lower bounds of the α-level based membership function of efficiency scores (Hatami-Marbini et al. 2011). The α-level based approach was later extended and applied in many literatures. For instance, Liu (2008) in order to evaluate the upper and lower bound performance of flexible manufacturing systems (FMS) alternatives when the input and output data are represented as crisp and fuzzy data, transformed a two-level mathematical programming into a one-level DEA assurance region model. Soltanzadeh and Omrani (2018) to deal with fluctuations in data which can be represented by fuzzy numbers, extended the dynamic network DEA model in a fuzzy framework. Their proposed method provided more information for management, since data was presented by membership function. Hatami- Marbini and Saati (2018) proposed a common-weight DEA method to evaluate system efficiency and the component process efficiencies in fuzzy environment. Dotoli et al. (2015) presented a fuzzy DEA to evaluate healthcare systems performance in a region of Southern Italy under uncertainty. Moreover, Dotoli et al. (2015) developed the fuzzy cross efficiency DEA model for increasing discrimination power of the traditional fuzzy DEA model.

Another approach to deal with fuzziness in DEA model is the fuzzy ranking which was first developed by Guo and Tanaka (2001). In the fuzzy ranking approach, DM define a possibility level and the model converts to a linear programming model with crisp constraints and ranking occurs using the comparison rule for fuzzy numbers. Like α-level based approach, fuzzy ranking method has been widely applied in literatures. For instance, Guo (2009) proposed a fuzzy DEA for evaluating objects with fuzzy inputs and outputs and applied his method to analyze a case study involving a restaurant location problem in detail.

Finally, Lertworasikul et al. (2002a; 2002b) based on the Zadeh's (1978) fundamental principles of possibility theory for fuzzy sets, proposed "possibility" and "credibility" approaches to overcome ranking problem in DEA-CRS model. Later, Lertworasikul et al. (2003) extended the possibility approach to fuzzy DEA-VRS model. Their model converted the fuzzy DEA model to linear model which can be solved using linear programming solvers. Although "possibility" and "credibility" approaches are strong tool to deal with fuzzy and uncertain data, "credibility" mathematical programming is complex and difficult to solve (Amini et al. 2019). However, development of computer and appearance of algorithms have facilitated the use of these approaches in optimization problems (Liu and Liu, 2002). Unlike possibility approach, in credibility approach, expected credits are replaced with fuzzy data to deal with fuzzy constraints and fuzzy objectives and there is no need to define any parameter to rank fuzzy efficiency by DM. Generally, credibility approach has been applied in a narrow of literatures. For example, Fasanghari et al. (2015) applied fuzzy credibility constrained programming and P-robust approaches simultaneously for analyzing enterprise architecture scenarios. The catered DEA model was linear, flexible, robust and successful in discrimination power improvement. Also, Amini et al. (2019) to estimate the road safety efficiency of provinces in Iran used a credibility DEA based on road safety (DEA-RS) model. In fact, the constraints of DEA-RS model are considered as credibility constraints and a counterpart credibility DEA-RS (CreDEA-RS) model was proposed. Based on the literatures, credibility DEA has not been addressed widely yet and more research are needed for identifying the capability of the credibility approach in considering the uncertainty and its application in real world problems. Hence, in this manuscript, credibility DEA method has been examined to model uncertainty in the input and output data. For a comprehensive review on literatures about fuzzy DEA readers can refer to Hatami-Marbini et al. (2011).

2.3. Robust optimization

As mentioned before, fuzzy sets generally constructed based on the DM opinions. In real world applications, due to the dynamic and continues change in preferences of stakeholders and DMs, considering exact values for DMs' preferences may lead to the unrealistic and reliable evaluations (Omrani et al. 2018). Therefore, considering perturbation and noise for DMs' assigned values, in order to produce more realistic and precise results, is inevitable. One of the most effective approaches to incorporate uncertainty and immunizes model against uncertainty in linear programming is robust optimization (RO) which extended by Ben-Tal and Nemirovski (2000) and Bersimas and Sim (2004). According to the Bertsimas and Sim (2004) it is unlikely that all parameters get their worst case values, therefore, Bertsimas and Sim (2004) for each constraint *i* introduced a new parameter, Γ_i , to make a trade-off between the degree of conservatism of the solution and the protection level of the constraint *i*. Robust optimization has been used in diverse management and engineering practices such as supply chain management (Omrani et al. 2017), waste management (Saeidi-Mobarakeh et al. 2020), assessment of urban agriculture on the localized food system (Castello et al. 2021), project selection (Lee et al. 2020), and portfolio optimization (Toloo and Mensah, 2019). Also, robust optimization has been applied widely on the conventional DEA as a well-known linear programming model.

In this paper, for developing the credibility DEA model, the robust optimization approach (Bertsimas and Sim, 2004) is used to handle uncertainties in constructing fuzzy values. In fact, since data are considered as fuzzy sets, a small perturbation in DM preferences in specifying upper, medium and lower bounds may lead to unreal and imprecise results. In addition, perturbation degree (percentage of uncertainty) of the robust optimization model is considered as exact values which intensifies the impreciseness of the results. Therefore, in this study, perturbation level is handled using fuzzy sets. The main motivation of this study is to propose a novel model to handle uncertainty in data and DMs' judgement comprehensively using the state- of- the- art techniques. As mentioned before, due to the complexity, few research has addressed uncertainty using credibility approach for evaluating the efficiency of DMUs. So, the gap of applying credibility DEA approach in evaluating was obvious. Although, mixed robust fuzzy DEA method has been applied in a few research, however, to the best of the authors knowledge, no study has investigated the robust credibility DEA approach with uncertain perturbation level. For instance, Amirkhan et al. (2018) applied scenario-based robust Fuzzy DEA models under different return to scale conditions to evaluate Small and Medium-sized Enterprises (SMEs) units. Peykani et al. (2018) proposed several robust fuzzy DEA models by using of different fuzzy measures including possibility, necessity and credibility measures. Wardana et al. (2020) used P-robust fuzzy DEA to evaluate the welding process efficiency. As it can be seen, robust credibility with fuzzy perturbation degree has not investigated in the literature.

In summary, the main contributions of this study are as followings:

- Considering uncertainty in inputs and outputs using fuzzy credibility theory
- Considering uncertainty in constructing fuzzy sets using robust optimization approach
- Considering perturbation degree of data as a fuzzy number

3. Methodology

In this section, the proposed RCDEA model has been illustrated. In summary, first a fuzzy credibility DEA model is introduced. Then, considering uncertainty in constructing fuzzy sets, a robust credibility DEA (RCDEA) model is used. Finally, uncertainty in perturbation level modeled using fuzzy sets. The preliminaries of fuzzy set theory have been shown in Appendix A. Also, the parameters used in the equations of the methodology section have been defined in Table (1).

---[Table 1 about here] ---

In the following, methods applied into the proposed model have been defined separately.

3.1. Credibility DEA (CDEA) model

In this section, the credibility DEA (CDEA) model is described. For developing DEA using fuzzy credibility theory, first following lemma is proven.

Lemma: Let $\tilde{\lambda}_1=(m_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{\lambda}_2=(m_2,\alpha_2,\beta_2)_{LR}$ be two L-R fuzzy numbers with continuous membership functions. For a given confidence level $\gamma\in[0,1]$ it is proven that (Tavana et al. 2012):

I) If $\gamma\le 0.5$, then $Cr(\tilde{\lambda}_1\ge\tilde{\lambda}_2)\ge\gamma \Leftrightarrow m_1+\beta_1 R^{-1}(2\gamma)\ge m_2-\alpha_2 R^{-1}(2\gamma)$

II) If $\gamma>0.5$, then $Cr(\tilde{\lambda}_1\ge\tilde{\lambda}_2)\ge\gamma \Leftrightarrow m_1-\alpha_1 L^{-1}(2(1-\gamma))\ge m_2+\beta_2 L^{-1}(2(1-\gamma))$

Proof. Suppose that

$$\tilde{\lambda}=\tilde{\lambda}_1-\tilde{\lambda}_2=(m_1,\alpha_1,\beta_1)_{LR}\oplus(-m_2,\beta_2,\alpha_2)_{LR}=(m_1-m_2,\alpha_1+\beta_2,\alpha_2+\beta_1)_{LR}=(\overline{m},\overline{\alpha},\overline{\beta})_{LR} \quad \textbf{(8)}$$

According to definition 9, we have:

$$Cr(\tilde{\lambda}_1\geq\tilde{\lambda}_2)=Cr(\tilde{\lambda}_1-\tilde{\lambda}_2\geq 0)=Cr(\tilde{\lambda}\geq 0)=\frac{1}{2}\left[Pos(\tilde{\lambda}\geq 0)+Nec(\tilde{\lambda}\geq 0)\right]$$
$$=\frac{1}{2}\left[Pos(\tilde{\lambda}\geq 0)+1-Pos(\tilde{\lambda}\geq 0)\right]=\frac{1}{2}\left[\sup_{t\geq 0}\mu(t)+1-\sup_{t<0}\mu(t)\right] \quad \textbf{(9)}$$

It is clear that equation (9) can be expressed as follows:

$$Cr(\tilde{\lambda}\geq 0)=\begin{cases} 1 & 0\leq\overline{m}-\overline{\alpha} \\ \frac{1}{2}\left[1+1-L(\frac{\overline{m}}{\overline{\alpha}})\right]=1-\frac{1}{2}L(\frac{\overline{m}}{\overline{\alpha}}) & \overline{m}-\overline{\alpha}\leq 0\leq\overline{m} \\ \frac{1}{2}\left[R(-\frac{\overline{m}}{\overline{\beta}})+1-1\right]=\frac{1}{2}R(\frac{\overline{m}}{\overline{\beta}}) & \overline{m}\leq 0\leq\overline{m}+\overline{\beta} \\ 0 & \overline{m}+\overline{\beta}<0 \end{cases} \quad \textbf{(10)}$$

If $\gamma\leq 0.5$, then

$$C_r(\tilde{\lambda}\geq 0)\geq\gamma\Leftrightarrow\frac{1}{2}R(\frac{-\overline{m}}{\overline{\beta}})\geq\gamma\Leftrightarrow R(\frac{-\overline{m}}{\overline{\beta}})\geq 2\gamma\Leftrightarrow\frac{-\overline{m}}{\overline{\beta}}\leq R^{-1}(2\gamma)\Leftrightarrow-\frac{m_1-m_2}{\alpha_2+\beta_1}\leq R^{-1}(2\gamma)$$
$$\Leftrightarrow m_1-m_2\leq(\alpha_2+\beta_1)R^{-1}(2\gamma)\Leftrightarrow m_1+\beta_2R^{-1}(2\gamma)\geq m_2-\alpha_2R^{-1}(2\gamma)$$

If $\gamma>0.5$, then

$$C_r(\tilde{\lambda}\geq 0)\geq\gamma\Leftrightarrow 1-\frac{1}{2}L(\frac{\overline{m}}{\overline{\alpha}})\geq\gamma\Leftrightarrow\frac{1}{2}L(\frac{\overline{m}}{\overline{\alpha}})\leq 1-\gamma\Leftrightarrow L(\frac{\overline{m}}{\overline{\alpha}})\leq 2(1-\gamma)$$
$$\Leftrightarrow\frac{\overline{m}}{\overline{\alpha}}\geq L^{-1}(2(1-\gamma))\Leftrightarrow\frac{m_1-m_2}{\alpha_1+\beta_2}\geq L^{-1}(2(1-\gamma))\Leftrightarrow m_1-m_2\geq(\alpha_1+\beta_2)L^{-1}(2(1-\gamma))$$
$$\Leftrightarrow m_1-\alpha_1L^{-1}(2(1-\gamma))\geq m_2+\beta_2L^{-1}(2(1-\gamma))$$

The credibility counterpart of dual of model (7) can be expressed as follows:

$$
\begin{aligned}
&\max z_o \\
&s.t: \\
&Cr(\sum_{i=t+1}^{t+s} w_i \tilde{y}_{io} \geq \theta_o^{CCR} \sum_{i=1}^{t} w_i \tilde{x}_{io} + z_o) \geq \gamma_o \\
&Cr(\sum_{i=t+1}^{t+s} w_i \tilde{y}_{ij} \leq \theta_j^{CCR} \sum_{i=1}^{t} w_i \tilde{x}_{ij}) \geq \gamma_j,\ j=1,...,n \\
&\sum_{i=1}^{t+s} w_i = 1 \\
&w_i \geq 0,\ i=1,...,t+s \\
&z_o\ free
\end{aligned}
\tag{11}
$$

where $\tilde{x}_{ij} = (x_{ij}^m, x_{ij}^\alpha, x_{ij}^\beta)_{LR}, i=1,...,t$ and $\tilde{y}_{ij} = (y_{ij}^m, y_{ij}^\alpha, y_{ij}^\beta)_{LR}, i=t+1,...,t+s$ are the *L-R* fuzzy numbers.

According to definition 3, the membership functions of inputs and outputs $j=1,...,n$ can be expressed as follows, respectively:

$$
\mu_{\tilde{x}_{ij}}(r) = \begin{cases} L(\frac{x_{ij}^m - r}{x_{ij}^\alpha}) & r \leq x_{ij}^m, i=1,...,t \\ R(\frac{r - x_{ij}^m}{x_{ij}^\beta}) & r \geq x_{ij}^m, i=1,...,t \end{cases} \tag{12}
$$

$$
\mu_{\tilde{y}_{ij}}(r) = \begin{cases} L(\frac{y_{ij}^m - r}{y_{ij}^\alpha}) & r \leq y_{ij}^m, i=t+1,...,t+s \\ R(\frac{r - y_{ij}^m}{y_{ij}^\beta}) & r \geq y_{ij}^m, i=t+1,...,t+s \end{cases} \tag{13}
$$

According to Zadeh's extension principle, the membership functions of the constraints of model (11) can be expressed as follows:

$$
\mu_{\sum_{i=1}^{t} w_i \tilde{x}_{ij}}(r) = \begin{cases} L(\frac{\sum_{i=1}^{t} w_i x_{ij}^m - r}{\sum_{i=1}^{t} w_i x_{ij}^\alpha}) & r \leq \sum_{i=1}^{t} w_i x_{ij}^m, j=1,...,n \\ R(\frac{r - \sum_{i=1}^{t} w_i x_{ij}^m}{\sum_{i=1}^{t} w_i x_{ij}^\beta}) & r \geq \sum_{i=1}^{t} w_i x_{ij}^m, j=1,...,n \end{cases} \tag{14}
$$

$$\mu_{\sum_{i=t+1}^{t+s} w_i \tilde{y}_{ij}}(r) = \begin{cases} L\left(\dfrac{\sum_{i=t+1}^{t+s} w_i y_{ij}^m - r}{\sum_{i=t+1}^{t+s} w_i y_{ij}^\alpha}\right) & r \le \sum_{i=t+1}^{t+s} w_i y_{ij}^m, j = 1,\dots,n \\ R\left(\dfrac{r - \sum_{i=t+1}^{t+s} w_i y_{ij}^m}{\sum_{i=t+1}^{t+s} w_i y_{ij}^\beta}\right) & r \ge \sum_{i=t+1}^{t+s} w_i y_{ij}^m, j = 1,\dots,n \end{cases} \tag{15}$$

Based on the membership functions (14) to (15), the fuzzy numbers $\sum_{i=t+1}^{t+s} w_i \tilde{y}_{io}$, $\sum_{i=t+1}^{t+s} w_i \tilde{y}_{ij}$, $\theta_j^{CCR} \sum_{i=1}^{t} w_i \tilde{x}_{ij}$ and $\theta_o^{CCR} \sum_{i=1}^{t} w_i \tilde{x}_{io} + z_o$ are shown as below *L*-*R* fuzzy numbers:

$$\sum_{i=t+1}^{t+s} w_i \tilde{y}_{io} = \left(\sum_{i=t+1}^{t+s} w_i y_{io}^m, \sum_{i=t+1}^{t+s} w_i y_{io}^\alpha, \sum_{i=t+1}^{t+s} w_i y_{io}^\beta\right)_{LR} \tag{16}$$

$$\sum_{i=t+1}^{t+s} w_i \tilde{y}_{ij} = \left(\sum_{i=t+1}^{t+s} w_i y_{ij}^m, \sum_{i=t+1}^{t+s} w_i y_{ij}^\alpha, \sum_{i=t+1}^{t+s} w_i y_{ij}^\beta\right)_{LR}, j = 1,\dots,n \tag{17}$$

$$\theta_j^{CCR} \sum_{i=1}^{t} w_i \tilde{x}_{ij} = \left(\theta_j^{CCR} \sum_{i=1}^{t} w_i x_{ij}^m, \theta_j^{CCR} \sum_{i=1}^{t} w_i x_{ij}^\alpha, \theta_j^{CCR} \sum_{i=1}^{t} w_i x_{ij}^\beta\right)_{LR}, j = 1,\dots,n \tag{18}$$

$$\theta_o^{CCR} \sum_{i=1}^{t} w_i \tilde{x}_{io} + z_o = \left(\theta_o^{CCR} \sum_{i=1}^{t} w_i x_{io}^m + z_o, \theta_o^{CCR} \sum_{i=1}^{t} w_i x_{io}^\alpha, \theta_o^{CCR} \sum_{i=1}^{t} w_i x_{io}^\beta\right)_{LR} \tag{19}$$

In this study, the data are considered as triangular fuzzy numbers. Hence, according to *definition 4*, we have:

$$L(x) = R(x) = L^{-1}(x) = R^{-1}(x) = 1 - x \tag{20}$$

According to above lemma, for $\gamma_o \le 0.5$, the first constraint of model (11) is expressed as follows:

$$\begin{aligned} &\sum_{i=t+1}^{t+s} w_i y_{io}^m + R^{-1}(2\gamma_o) \sum_{i=t+1}^{t+s} w_i y_{io}^\beta \ge \theta_o^{CCR} \sum_{i=1}^{t} w_i x_{io}^m + z_o - R^{-1}(2\gamma_o) \theta_o^{CCR} \sum_{i=1}^{t} w_i x_{io}^\alpha \\ &\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{io}^m + R^{-1}(2\gamma_o) y_{io}^\beta) \ge \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^m - R^{-1}(2\gamma_o) x_{io}^\alpha) + z_o \\ &\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{io}^m + (1-2\gamma_o) y_{io}^\beta) \ge \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^m - (1-2\gamma_o) x_{io}^\alpha) + z_o \\ &\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{io}^m + (1-2\gamma_o) y_{io}^\beta) - \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^m - (1-2\gamma_o) x_{io}^\alpha) \ge z_o \end{aligned} \tag{21}$$

In addition, the second constraint of model (11) is converted to a linear constraint as follows:

$$\begin{aligned}
&\sum_{i=t+1}^{t+s} w_i y_{ij}^m - R^{-1}(2\gamma_j)\sum_{i=t+1}^{t+s} w_i y_{ij}^{\alpha} \le \theta_j^{CCR}\sum_{i=1}^{t} w_i x_{ij}^m + R^{-1}(2\gamma_j)\theta_j^{CCR}\sum_{i=1}^{t} w_i x_{ij}^{\beta} \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{ij}^m - R^{-1}(2\gamma_j) y_{ij}^{\alpha}) \le \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m + R^{-1}(2\gamma_j) x_{ij}^{\beta}) \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{ij}^m - (1-2\gamma_j) y_{ij}^{\alpha}) \le \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m + (1-2\gamma_j) x_{ij}^{\beta}) \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{ij}^m - (1-2\gamma_j) y_{ij}^{\alpha}) - \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m + (1-2\gamma_j) x_{ij}^{\beta}) \le 0
\end{aligned} \tag{22}$$

By considering the constraints (21) and (22), the final CDEA model for $\gamma_o, \gamma_j \le 0.5$ is expressed follows:

$$\begin{aligned}
&\max z_o \\
&s.t: \\
&\sum_{i=t+1}^{t+s} w_i (y_{io}^m + (1-2\gamma_o) y_{io}^{\beta}) - \theta_o^{CCR}\sum_{i=1}^{t} w_i (x_{io}^m - (1-2\gamma_o) x_{io}^{\alpha}) \ge z_o \\
&\sum_{i=t+1}^{t+s} w_i (y_{ij}^m - (1-2\gamma_j) y_{ij}^{\alpha}) - \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m + (1-2\gamma_j) x_{ij}^{\beta}) \le 0,\ j=1,\dots,n \\
&\sum_{i=1}^{t+s} w_i = 1 \\
&w_i \ge 0,\ i=1,\dots,\mathrm{t}+s \\
&z_o\ \mathit{free}
\end{aligned} \tag{23}$$

Similar to above way and according to above lemma, the first constraint of model (11) for $\gamma_o > 0.5$ is expressed as follows:

$$\begin{aligned}
&\sum_{i=t+1}^{t+s} w_i y_{io}^m - L^{-1}(2(1-\gamma_o))\sum_{i=t+1}^{t+s} w_i y_{io}^{\alpha} \ge \theta_o^{CCR}\sum_{i=1}^{t} w_i x_{io}^m + z_o + L^{-1}(2(1-\gamma_o))\theta_o^{CCR}\sum_{i=1}^{t} w_i x_{io}^{\beta} \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{io}^m - L^{-1}(2(1-\gamma_o)) y_{io}^{\alpha}) \ge \theta_o^{CCR}\sum_{i=1}^{t} w_i (x_{io}^m + L^{-1}(2(1-\gamma_o)) x_{io}^{\beta}) + z_o \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{io}^m - (2\gamma_o - 1) y_{io}^{\alpha}) \ge \theta_o^{CCR}\sum_{i=1}^{t} w_i (x_{io}^m + (2\gamma_o - 1) x_{io}^{\beta}) + z_o \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{io}^m - (2\gamma_o - 1) y_{io}^{\alpha}) - \theta_o^{CCR}\sum_{i=1}^{t} w_i (x_{io}^m + (2\gamma_o - 1) x_{io}^{\beta}) \ge z_o
\end{aligned} \tag{24}$$

Also, the second constraint of model (11) is written as follows:

$$\begin{aligned}
&\sum_{i=t+1}^{t+s} w_i y_{ij}^m + L^{-1}(2(1-\gamma_j))\sum_{i=t+1}^{t+s} w_i y_{ij}^\beta \le \theta_j^{CCR}\sum_{i=1}^{t} w_i x_{ij}^m - L^{-1}(2(1-\gamma_j))\theta_j^{CCR}\sum_{i=1}^{t} w_i x_{ij}^\alpha \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{ij}^m + L^{-1}(2(1-\gamma_j))y_{ij}^\beta) \le \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m - L^{-1}(2(1-\gamma_j))x_{ij}^\alpha) \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{ij}^m + (2\gamma_j - 1)y_{ij}^\beta) \le \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m - (2\gamma_j - 1)x_{ij}^\alpha) \\
&\Rightarrow \sum_{i=t+1}^{t+s} w_i (y_{ij}^m + (2\gamma_j - 1)y_{ij}^\beta) - \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m - (2\gamma_j - 1)x_{ij}^\alpha) \le 0
\end{aligned} \tag{25}$$

Finally, the CDEA model for $\gamma_o, \gamma_j > 0.5$ is shown as model (26).

$$\begin{aligned}
&\max z_o \\
&s.t: \\
&\sum_{i=t+1}^{t+s} w_i (y_{io}^m - (2\gamma_o - 1)y_{io}^\alpha) - \theta_o^{CCR}\sum_{i=1}^{t} w_i (x_{io}^m + (2\gamma_o - 1)x_{io}^\beta) \ge z_o \\
&\sum_{i=t+1}^{t+s} w_i (y_{ij}^m + (2\gamma_j - 1)y_{ij}^\beta) - \theta_j^{CCR}\sum_{i=1}^{t} w_i (x_{ij}^m - (2\gamma_j - 1)x_{ij}^\alpha) \le 0,\ j = 1,\dots,n \\
&\sum_{i=1}^{t+s} w_i = 1 \\
&w_i \ge 0,\ i = 1,\dots,\mathrm{t}+s \\
&z_o\ free
\end{aligned} \tag{26}$$

3.2 Robust optimization

Robust optimization is a preeminent approach in dealing with uncertainty of data (Alem and Morabito, 2012) since robust optimization assumes no probability distribution for uncertain data as well as the capability in modeling problems with large number of uncertain parameters. In this study we have investigated the robust optimization model with data uncertainty of U (Bental and Nemirovski, 2000).

Although first steps of robust optimization with data uncertainty of U was taken by Soyester (1973) and then developed by Ben-Tal and Nemirovski (2000) and Bertsimas and Sim (2004), however, the advantages of Bertsimas and Sim (2004) in converting models into simple linear models and generating less conservatism solutions led to wide application of this approach in real world problems.

To illustrate the robust optimization model (Bertsimas and Sim, 2004), consider the following optimization model:

$$\begin{aligned} & max\, c'x \\ & s.t: \\ & \sum_j a'_{ij} x_j \le b_i, \forall i \\ & l_j \le x_j \le u_j, \forall j \end{aligned} \tag{27}$$

In the optimization model (27), J_i is a set of uncertain coefficients in particular row *i* of matrix *A*. Each independent uncertain parameter $\tilde{a}_{ij}$ symmetrically distributed in a bounded interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ which centered at point a_{ij}. Also $\hat{a}_{ij} = e_{ij} \times a_{ij}$ where e_{ij} indicates percentage of perturbation in the nominal value a_{ij}. Moreover, Bertsimas and Sim (2004) introduced a scaled deviation $z_{ij} = (\hat{a}_{ij} - a_{ij}) / \hat{a}_{ij}$ which take values in interval $[-1, 1]$. Bertsimas and Sim (2004) believed that in real world problems, it is almost implausible that, all parameters take their worst case values simultaneously, which leads to the conservatism solution. Therefore, for each constraint *i*, Bertsimas and Sim (2004) introduced a new parameter of Γ_i, as a budget of uncertainty, which takes value in interval $[0, |J_i|]$. Indeed Γ_i values make a trade-off between protection level of constraint and conservatism level of solutions. When $\Gamma_i = 0$, the constraints are vulnerable in dealing with uncertainty while in $\Gamma_i = |J_i|$ constraints are fully protected, and solutions are too conservatism. Considering all aforementioned details, Bertsimas and Sim (2004) introduced the robust linear counterpart of model (27) as follows: (For more details, readers can refer to Bertsimas and sim (2004) and Alem and Morabito (2012))

$$
\begin{aligned}
& max\, c^{'} x \\
& \text{s}.t: \\
& \sum_j a'_{ij} x + \Gamma_i P_i + \sum_{j \in J_i} q_{rj} \le b_i, \quad \forall i \\
& P_j + q_{rj} \ge \hat{a}_{ij} y_j, \quad \forall j \\
& -y_j \le x_j \le y_j, \quad \forall j \\
& I_j \le x_j \le u_j, \quad \forall j \\
& q_{ij} \ge 0, \quad \forall i, j \in J_i \\
& y_j \ge 0, \quad \forall j \\
& p_i \ge 0, \quad \forall i
\end{aligned} \tag{28}
$$

3.3. Robust credibility DEA (RCDEA) model

In this section, the counterpart robust CDEA (RCDEA) models are proposed. The proposed RCDEA models are:

- RCDEA model with exact perturbation in fuzzy inputs and fuzzy outputs

- RCDEA model with fuzzy perturbation in fuzzy inputs and fuzzy outputs

Let $\tilde{x}_{ij} = (x_{ij}^m, x_{ij}^\alpha, x_{ij}^\beta)_{LR}, i = 1,...,t$ and $\tilde{y}_{ij} = (\mathrm{y}_{ij}^m, \mathrm{y}_{ij}^\alpha, \mathrm{y}_{ij}^\beta)_{LR}, i = t+1,...,t+s$ be the *L-R* fuzzy numbers. Let x_{ij}^m and y_{ij}^m are symmetrically distributed in intervals $[x_{ij}^m - \widehat{x}_{ij}^m, x_{ij}^m + \widehat{x}_{ij}^m]$ and $[y_{ij}^m - \widehat{y}_{ij}^m, y_{ij}^m + \widehat{y}_{ij}^m]$ where $\widehat{x}_{ij}^m = e_{ij} \times x_{ij}^m$ and $\widehat{y}_{ij}^m = e_{ij} \times y_{ij}^m$. Since x_{ij}^m and y_{ij}^m are contaminated with perturbation, hence, x_{ij}^α, x_{ij}^β, y_{ij}^α and y_{ij}^β are symmetrically distributed in determined intervals, too.

3.3.1. RCDEA model with exact perturbation in fuzzy inputs and fuzzy outputs

To show the RCDEA model with exact perturbation in fuzzy inputs and fuzzy outputs, credibility DEA models (23) and (26) are converted to the counterpart robust CDEA models. According to approach proposed by Bertsimas and Sim (2004), the robust counterpart of CDEA models for $\gamma \le 0.5$ and $\gamma > 0.5$ are as (29) and (30) respectively:

$$
\begin{aligned}
&\theta_o^{RCDEA} = \max_{\gamma_o,\gamma_i \le 0.5} \sum_{i=t+1}^{t+s} w_i (y_{io}^m + (1-2\gamma_o) y_{io}^{\beta}) - \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^m - (1-2\gamma_o) x_{io}^{\alpha}) - \Gamma_o p_o - \sum_{i=t+1}^{t+s} q_{io} - \sum_{i=1}^{t} q_{io} \\
&s.t: \\
&\sum_{i=t+1}^{t+s} w_i (y_{ij}^m - (1-2\gamma_j) y_{ij}^{\alpha}) - \theta_j^{CCR} \sum_{i=1}^{t} w_i (x_{ij}^m + (1-2\gamma_j) x_{ij}^{\beta}) + \Gamma_j p_j + \sum_{i=t+1}^{t+s} q_{ij} + \sum_{i=1}^{t} q_{ij} \le 0, \mathrm{j}=1,\ldots,\mathrm{n} \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_o + q_{io} \ge e_{io} (y_{io}^m + (1-2\gamma_o) y_{io}^{\beta}) z_i, \quad \forall i = t+1,..t+s \\
&p_o + q_{io} \ge e_{io} (x_{io}^m - (1-2\gamma_o) x_{io}^{\alpha}) z_i, \quad \forall i = 1,..t \\
&p_j + q_{ij} \ge e_{ij} (y_{ij}^m - (1-2\gamma_j) y_{ij}^{\alpha}) z_i, \quad \forall i = t+1,..t+s, \forall j = 1,\ldots,n \\
&p_j + q_{ij} \ge e_{ij} (x_{ij}^m + (1-2\gamma_j) x_{ij}^{\beta}) z_i, \quad \forall i = 1,..t, \forall j = 1,\ldots,n \\
&-z_i \le w_i \le z_i, \quad \forall i = 1,2,\ldots,t+s \\
&p_j, q_{ij} \ge 0, \quad \forall i, j \\
&w_i \ge 0, i = 1,\ldots,t+s
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&\theta_o^{RCDEA} = \max_{\gamma_o,\gamma_i > 0.5} \sum_{i=t+1}^{t+s} w_i (y_{io}^m - (2\gamma_o - 1) y_{io}^{\alpha}) - \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^m + (2\gamma_o - 1) x_{io}^{\beta}) - \Gamma_o p_o - \sum_{i=t+1}^{t+s} q_{io} - \sum_{i=1}^{t} q_{io} \\
&s.t: \\
&\sum_{i=t+1}^{t+s} w_i (y_{ij}^m + (2\gamma_j - 1) y_{ij}^{\beta}) - \theta_j^{CCR} \sum_{i=1}^{t} w_i (x_{ij}^m - (2\gamma_j - 1) x_{ij}^{\alpha}) + \Gamma_j p_j + \sum_{i=t+1}^{t+s} q_{ij} - \sum_{i=1}^{t} q_{ij} \le 0, \mathrm{j}=1,\ldots,\mathrm{n} \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_o + q_{io} \ge e_{io} (y_{io}^m - (2\gamma_o - 1) y_{io}^{\alpha}) z_i, \quad \forall i = t+1,\ldots t+s \\
&p_o + q_{io} \ge e_{io} (x_{io}^m + (2\gamma_o - 1) x_{io}^{\beta}) z_i, \quad \forall i = 1,\ldots t \\
&p_j + q_{ij} \ge e_{ij} (y_{ij}^m + (2\gamma_j - 1) y_{ij}^{\beta}) z_i, \quad \forall i = t+1,\ldots t+s, \forall j = 1,\ldots,n \\
&p_j + q_{ij} \ge e_{ij} (x_{ij}^m - (2\gamma_j - 1) x_{ij}^{\alpha}) z_i, \quad \forall i = 1,\ldots t, \forall j = 1,\ldots,n \\
&-z_i \le w_i \le z_i, \quad \forall i = 1,2,\ldots,t+s \\
&p_j, q_{ij} \ge 0, \quad \forall i, j \\
&w_i \ge 0, i = 1,\ldots,t+s
\end{aligned}
\tag{30}
$$

where the free variable z_o is removed and the first constraint of models (23) and (26) are moved to objective functions.

3.3.2. RCDEA model with fuzzy perturbation in fuzzy inputs and fuzzy outputs

In real world application, the level of uncertainty is unknown and considering certain values for uncertainty level of data (the value *e*) may be contributed to imprecise results. In this section, the percentage of perturbation is considered as *L-R* fuzzy number $\tilde{e}_{ij} = (e_{ij}^{m}, e_{ij}^{\alpha}, e_{ij}^{\beta})_{LR}$. Consider the fifth constraint of RCDEA model (29). In fuzzy status, this constraint is written as $Cre(p_j + q_{ij} \geq \tilde{e}_{ij}(y_{ij}^{m} + (2\gamma_j - 1)y_{ij}^{\beta})z_i) \geq \gamma_{ij}$. According to above lemma, the later fuzzy constraint is converted to $p_j + q_{ij} \geq [e_{ij}^{m} - (1-2\gamma_{ij})e_{ij}^{\alpha}](y_{ij}^{m} + (2\gamma_j - 1)y_{ij}^{\beta})z_i$. In fact, in order to obtain a RCDEA model with fuzzy perturbation, it is enough to replace e_{ij} with $[e_{ij}^{m} - (1-2\gamma_{ij})e_{ij}^{\alpha}]$ for $\gamma_{ij} \leq 0.5$ and $[e_{ij}^{m} + (2\gamma_{ij} - 1)e_{ij}^{\beta}]$ $\gamma_{ij} > 0.5$. Therefore, models (29) and (30) are converted to (31) and (32), respectively:

$$
\begin{aligned}
&\theta_o^{RCDEA} = \max_{\gamma_o, \gamma_i \leq 0.5} \sum_{i=t+1}^{t+s} w_i (y_{io}^{m} + (1-2\gamma_o) y_{io}^{\beta}) - \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^{m} - (1-2\gamma_o) x_{io}^{\alpha}) - \Gamma_o p_o - \sum_{i=t+1}^{t+s} q_{io} - \sum_{i=1}^{t} q_{io} \\
&s.t: \\
&\sum_{i=t+1}^{t+s} w_i (y_{ij}^{m} - (1-2\gamma_j) y_{ij}^{\alpha}) - \theta_j^{CCR} \sum_{i=1}^{t} w_i (x_{ij}^{m} + (1-2\gamma_j) x_{ij}^{\beta}) + \Gamma_j p_j + \sum_{i=t+1}^{t+s} q_{ij} + \sum_{i=1}^{t} q_{ij} \leq 0, \mathrm{j} = 1,\ldots,\mathrm{n} \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_o + q_{io} \geq [e_{io}^{m} - (1-2\gamma_{io}) e_{io}^{\alpha}](y_{io}^{m} + (1-2\gamma_o) y_{io}^{\beta}) z_i, \quad \forall i = t+1,..t+s \\
&p_o + q_{io} \geq [e_{io}^{m} - (1-2\gamma_{io}) e_{io}^{\alpha}](x_{io}^{m} - (1-2\gamma_o) x_{io}^{\alpha}) z_i, \quad \forall i = 1,..t \\
&p_j + q_{ij} \geq [e_{ij}^{m} - (1-2\gamma_{ij}) e_{ij}^{\alpha}](y_{ij}^{m} - (1-2\gamma_j) y_{ij}^{\alpha}) z_i, \quad \forall i = t+1,..t+s, \forall j = 1,\ldots,n \\
&p_j + q_{ij} \geq [e_{ij}^{m} - (1-2\gamma_{ij}) e_{ij}^{\alpha}](x_{ij}^{m} + (1-2\gamma_j) x_{ij}^{\beta}) z_i, \quad \forall i = 1,..t, \forall j = 1,\ldots,n \\
&-z_i \leq w_i \leq z_i, \quad \forall i = 1,2,\ldots,t+s \\
&p_j, q_{ij} \geq 0, \quad \forall i, j \\
&w_i \geq 0, i = 1,\ldots,t+s
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&\theta_o^{RCDEA} = \max_{\gamma_o,\gamma_i > 0.5} \sum_{i=t+1}^{t+s} w_i (y_{io}^m - (2\gamma_o - 1) y_{io}^\alpha) - \theta_o^{CCR} \sum_{i=1}^{t} w_i (x_{io}^m + (2\gamma_o - 1) x_{io}^\beta) - \Gamma_o p_o - \sum_{i=t+1}^{t+s} q_{io} - \sum_{i=1}^{t} q_{io} \\
&s.t: \\
&\sum_{i=t+1}^{t+s} w_i (y_{ij}^m + (2\gamma_j - 1) y_{ij}^\beta) - \theta_j^{CCR} \sum_{i=1}^{t} w_i (x_{ij}^m - (2\gamma_j - 1) x_{ij}^\alpha) + \Gamma_j p_j + \sum_{i=t+1}^{t+s} q_{ij} + \sum_{i=1}^{t} q_{ij} \le 0, \mathrm{j} = 1,\dots,\mathrm{n} \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_o + q_{io} \ge [e_{io}^m + (2\gamma_{io} - 1) e_{io}^\beta](y_{io}^m - (2\gamma_o - 1) y_{io}^\alpha) z_i, \quad \forall i = t+1,..t+s \qquad \textbf{(32)} \\
&p_o + q_{io} \ge [e_{io}^m + (2\gamma_{io} - 1) e_{io}^\beta](x_{io}^m + (2\gamma_o - 1) x_{io}^\beta) z_i, \quad \forall i = 1,..t \\
&p_j + q_{ij} \ge [e_{ij}^m + (2\gamma_{ij} - 1) e_{ij}^\beta](y_{ij}^m + (2\gamma_j - 1) y_{ij}^\beta) z_i, \quad \forall i = t+1,..t+s, \forall j = 1,\dots,n \\
&p_j + q_{ij} \ge [e_{ij}^m + (2\gamma_{ij} - 1) e_{ij}^\beta](x_{ij}^m - (2\gamma_j - 1) x_{ij}^\alpha) z_i, \quad \forall i = 1,..t, \forall j = 1,\dots,n \\
&-z_i \le w_i \le z_i, \quad \forall i = 1,2,\dots,t+s \\
&p_j, q_{ij} \ge 0, \quad \forall i,j \\
&w_i \ge 0, i = 1,\dots,t+s
\end{aligned}
$$

Model (32) maximizes the efficiency scores of the DMUs. The first constraint keeps the model immune against the uncertainty in fuzzy data. Also, in the first constraint, the minimum confidence level of feasibility has been set. The second constraint is normalizing constraint and tries to keep the sum of the weights equal to 1. The third to the sixth constraints are dual constraints in robust counterpart model and aims to consider uncertainty to some parameters of the model. Finally, the last two constraints are non-negativity constraints.

4. Data

In this section, data of 28 hospitals located in the northwestern region of Iran, including three provinces of Ardebil, East Azerbaijan and West Azerbaijan are collected. These provinces are almost homogenous in terms of cultural, technological, economic, climate and geographical structures. Thereby, evaluation of these provinces hospitals as a northwestern cluster gives an insight to DMs to improve healthcare service quality in that region. In this regard, 28 hospitals have been evaluated in this paper. The first step in evaluating hospitals efficiency is selecting suitable input and output variables. In literatures, different variables have been applied for inputs and outputs. For instance, Li et al. (2019) in order to evaluate 37 non homogenous hospitals in Hong Kong used two inputs of the number of Full-time Equivalent (FTE) staff and the number of beds, and six outputs of total inpatient length of stay, total Accident & Emergency attendances, total Specialist Outpatient attendances, family medicine specialist clinic attendances, total allied health outpatient attendance, and general outpatient attendances. Omrani et al. (2018) used three variables of personnel, active beds and equipment and four variables of inpatients, outpatients, bed-day and special patients as inputs and outputs respectively to evaluate 288 hospitals in 31 provinces of Iran. van Ineveld et al. (2016) used four inputs of number of beds, operational expenses, FTE physicians and FTE non-physicians and three outputs of number of (inpatient) admissions, number of primary outpatient visits and day care treatment to measure productivity of Dutch hospitals. Chowdhury and Zelenyuk (2016) applied six input variables of administrative staff hours, nursing hours, staffed beds, medical-surgical supplies costs, non-medical supplies costs, and equipment expenses and two outputs of ambulatory visits and case-mix weighted inpatient days in their hospitals evaluation. Mitropoulos et al. (2015) adopted four inputs of doctors, other personnel, beds and operating costs and two outputs of inpatients admissions and outpatients for evaluating Greek public hospitals. In the study of Kazley and Ozcan (2009) the inputs include non-physician full time equivalent employees (FTEs), beds set up and staffed, capital assets, and non-labor operating expenses and outputs include case mix adjusted admissions and outpatient visits. For a comprehensive review of papers use DEA with various input and output variables to evaluate hospitals readers can refer to Kahl et al. (2019).

As it can be seen in literatures, researchers have used different variables as inputs and outputs. This study considers the input variables as the total number of personnel, number of medical equipment in each hospital and number of active beds that means the beds which are available for use. The personnel are staffs, permanent staffs, contract workers and other staffs. The selected outputs are the number of inpatients, outpatients and special patients separately and the fourth output is bed-days. Table (2) presents the data of 28 hospitals. Data is retrieved from Iran ministry of health and medical education for year 2016.

---[Table 2 about here] ---

5. Results and discussion

In this section, the result of applying the proposed RCDEA model has been presented. We evaluated 28 hospitals in northwestern region of Iran including three provinces of Ardebil, East Azerbaijan and West Azerbaijan in 2 phases. In the first phase, a DEA-CCR model has been applied to estimate the efficiency scores of hospitals. Then, in the second phase, considering data as fuzzy sets, and considering perturbation in constructing fuzzy sets, RCDEA model with and without fuzzy perturbation in fuzzy inputs and fuzzy outputs results are calculated. The results of implementing each phase of the proposed model are discussed below.

5.1. DEA results

In the first phase, to calculate the hospitals performances, a DEA-CCR model has been applied and results are provided in the Table (3). According to the results, 12 hospitals with the efficiency score of one have the best performance in converting inputs to outputs. Indeed, these 12 hospitals are technically efficient and form the efficient frontier. On the other hand, 18 hospitals with efficiency scores less than 1 are inefficient. The inefficient units' scores are lay in interval [0.4424, 0.9468]. The weakest performance is related to hospital number 12 which is belong to East Azerbaijan province. Average efficiency score of each province indicates that, in overall, Ardebil has better performance in terms of hospitals efficiency and West Azerbaijan has the worst performance. Also, results indicate that the conventional DEA is unable to distinguish among DMUs and low distinguish power of DEA is the weakness of this popular model to evaluate efficiency scores. Moreover, in calculating efficiency scores, uncertainty in data has not considered and we assumed that data are exact which may lead to unreliable and imprecise results. Finally, the mean of efficiency scores of hospitals is 0.8772 which is more than both RCDEA results.

---[Table 3 about here] ---

5.2. RCDEA with exact perturbation level

In order to evaluate hospitals with considering data as fuzzy sets, first, some parameters have to be set. First without losing any generality, it is assumed that $\gamma = \gamma_i = \gamma_r$. So, for different γ ($\gamma \leq 0.5$ and $\gamma > 0.5$) either model (23) or (26) can be applied. In this paper, proposed model is implemented for $\gamma = 0.4$ and $\gamma = 0.8$ respectively. Then in constructing fuzzy sets, we assumed that upper and lower bound are 0.05 more and less than medial number respectively (data considered as triangular fuzzy numbers). However, we assumed that in determining medial values, there is 0.03 perturbation and this uncertainty degree is exact. Furthermore, to ensure full protection of data, budget of uncertainty parameter, Γ, is set equal 7 for all constraints. Indeed, model is robust if 100% of uncertain parameters take their worst-case value.

The normalized scores for $\gamma = 0.4$ and $\gamma = 0.8$ has been presented in the Table (3). These results have been calculated using RCDEA models (29) and (30), respectively. According to the results, for $\gamma = 0.4$, 12 hospitals with the efficiency scores of 0.9996 has the best performance among all 28 units. Again, hospital number 12, which is belong to East Azerbaijan province has the weakest performance with the score of 0.4164. Results indicate that all hospitals' efficiency scores get lower values in comparison with DEA results. Since we have considered the same level of fuzziness and perturbation for all units and this assumption led to almost the same ranking comparing with DEA model. For example, all units with the score of 1 get the same score of 0.9796 when we included uncertainty in the evaluations. Finally, as expected, the mean of all scores is 0.8546 which is less than DEA model mean score.

In this section, also RCDEA model (30) has been implemented for $\gamma = 0.8$ and results are presented in the Table (3). Results in general are lower than both DEA and RCDEA for $\gamma = 0.4$. The mean efficiency of scores is 0.7251 which implies that scores get lower values, however, ranking has not changed and is almost the same in comparison with both DEA and RCDEA for $\gamma = 0.4$. Like RCDEA for $\gamma = 0.4$ scores get lower values to keep feasibility. Moreover, hospitals' scores get lower with the same proportion, since we consider the same value of fuzziness and perturbation. Figure (1) illustrates the comparison of results between applied DEA, and RCDEA with exact perturbation level for $\gamma = 0.4$ and $\gamma = 0.8$.

--[Figure 1 about here] --

5.3. RCDEA with fuzzy perturbation in fuzzy inputs and outputs

In this section, to observe the impact of uncertainty on the perturbation level, we have considered perturbation degree is a fuzzy value and re-evaluated hospitals' efficiency scores. Results are presented in the Table (4).

--[Table 4 about here] ---

According to the RCDEA model (31) for $\gamma = 0.4$, scores get higher values in comparison with RCDEA model (29) with exact perturbation value. Also, the results are closer to the DEA model scores. Since in the model (31) we replace e_{ij} with $[e_{ij}^m - (1-2\gamma_{ij})e_{ij}^\alpha]$ and as a result perturbation level decreased. Hospitals with better efficiencies obtained the score of 0.9996 and the worst performance hospital, hospital number 12, get the value of 0.4308. The mean of all hospitals' efficiency is 0.8738 which is more than RCDEA model for $\gamma = 0.4$ with exact perturbation level.

In contrast, considering fuzzy sets for perturbation value for $\gamma > 0.5$, resulted in decreasing efficiency scores and consequently decrease in the mean of scores to 0.6602. Indeed, model to keep constraints against increase in uncertainty level releases optimal solutions. It should be noted that since uncertainty considered the same for all inputs, outputs and perturbation values, rankings remained almost unchanged in models. However, the efficiency of hospitals number 25 and 16, has decreased tangibly and consequently, ranking has been changed. Figure (2) presents the DMUs efficiency clearly. Also Figure (3) compares the results of all proposed models.

--[Figure 2 about here] --

--[Figure 3 about here] --

Sensitivity analysis can be performed on parameters such as γ, Γ and e. However it takes a lot of calculations due to the different combinations of parameters. Generally, by increasing perturbation degree (e) efficiency scores get lower, by decreasing the budget of uncertain parameters in each constraints (Γ), efficiencies get higher scores and by increasing value for γ in $\gamma \leq 0.5$ model (31), for example $\gamma = 0.5$, model converts to the conventional robust DEA model and scores take lower than RCDEA model for $\gamma = 0.4$. Finally, by decreasing values for γ in $\gamma > 0.5$ model (32), for example $\gamma = 0.6$, scores take higher values than RCDEA model for $\gamma = 0.8$.

Finally, to illustrate the capability of the proposed model in providing reliable results, we compared our ranking with the fuzzy DEA and fuzzy cross efficiency DEA models introduced by Dotoli et al. (2015). The results are presented in the Figure 4.

---[Figure 4 about here] --

As it can be seen, in fuzzy DEA, like RCDEA model, 12 hospitals have best performance with the efficiency scores of 0.5. On the other hand, DMUs 12 and 19 with the efficiency scores of 0.2207 have the least performance among all hospitals. The results approves that the fuzzy DEA and RCDEA provide approximately the same ranking. However, both models are still suffering from discrimination power. In other words, both models are unable to fully rank the DMUs. Hence, the comparison between RCDEA and fuzzy cross efficiency DEA models have been conducted and results are indicated in Figure 4. According to the fuzzy cross efficiency DEA results, hospital 15 with the efficiency score of 0.9291 has the best performance and is followed by hospitals 5 and 3 with the efficiency scores of 0.7686 and 0.7490 respectively. At the bottom of the ranking, hospital 25 with the efficiency score of the 0.1071 has been located. The results indicates that the fuzzy cross efficiency DEA can fully rank all hospitals.

For investigating the relation between the RCDEA, Fuzzy DEA and fuzzy cross efficiency DEA models rankings, the Spearman correlation test is applied. The results are shown in Table (5). As it can be seen, the correlation between ranks generated by different models is significant at the 0.01 level. Indeed, the proposed RCDEA model provides reliable ranking with considering perturbation degree in the robust concept as a fuzzy number.

---[Table 5 about here] ---

6. Conclusion and directions for future research

In real world applications, improving efficiency of units is the substantial goal of managers in all sectors such as healthcare systems. Hospitals as the main components of healthcare systems, play a major role in delivering health services to societies and increasing hospitals efficiency is very important for DMs in the healthcare sector. In this study, to measure the efficiency scores of 28 hospitals in northwestern region of Iran including three provinces of Ardebil, East Azerbaijan and West Azerbaijan we proposed a RCDEA model. First, data for 28 hospitals with three inputs of personnel, equipment and active beds and four outputs of inpatients, outpatients, special patients and Bed-day was collected. Then considering uncertainty in data fuzzy credibility approach was applied. Moreover, robust optimization was used to consider uncertainty in constructing fuzzy sets. Indeed, the proposed model addresses uncertainty by mixed robust credibility approaches. Therefore, results are more reliable and precise than DEA model. The results indicates that, hospitals located in the East Azerbaijan province such as hospitals 1,2,3,4,5,6, 8, 13 and 14 have mostly better performance in converting inputs to the outputs. Results was expectable, since among these three provinces, East Azerbaijan has better GDP per capita and more population than the other two provinces. Hospitals in West Azerbaijan and Ardebil provinces should learn from the best performance hospitals in the East Azerbaijan, by decreasing the number of personnel and active bed and increasing the number of inpatient and outpatients. In the other words, hospitals should provide treatment for more patients with a smaller number of personnel. Likewise, hospitals in order to obtain higher efficiencies are better to decrease number of active beds and provide more space for the incoming emergency patients. The results also provide better understanding of hospitals performance within each province and policy makers and hospitals managers can set policies based on the best performance hospitals in the province. In terms of methodology, the results indicated that since a same level of uncertainty considered for data, ranking has not changed tangible. Also, by increasing uncertainty in constructing fuzzy data, scores got lower values to keep model feasible. One of the limitations of this study is that, although it provides better ranking than conventional DEA, however, in comparison with the other mature DEA approaches such fuzzy cross DEA, has weaker performance in discriminating DMUs (Ghasemi, et al. 2014). Furthermore, setting perturbation degree has limitation, since it is based on the DMs suggestion and having access to the DMs has difficulty in real world problems. Finally, for the case study, providing extra input and output variables data is not easily achievable.

For future studies, it can be considered some issues such as:

- Applying cross version for the fuzzy credibility DEA approach to provide fully ranking of DMUs could be investigate.
- Robust credibility approach could be applied for other models of DEA such as super efficiency DEA, network DEA and slack based DEA models.
- Judgements of DMs can be included in the evaluations to obtain more reliable and precise results.
- Robust optimization with fuzzy perturbation could be applied in possibility and necessity DEA models and results to be compared with RCDEA model.

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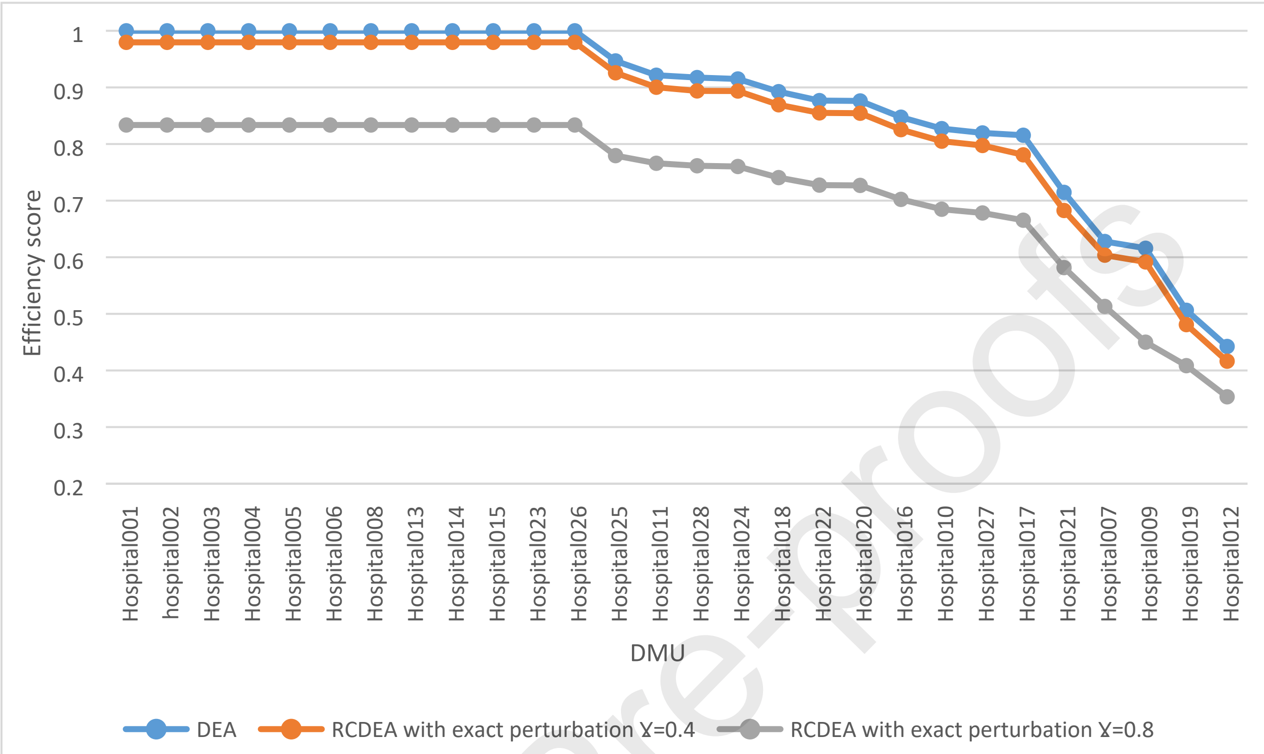

Fig.1. Efficiency scores of hospitals using RCDEA model with exact perturbation value

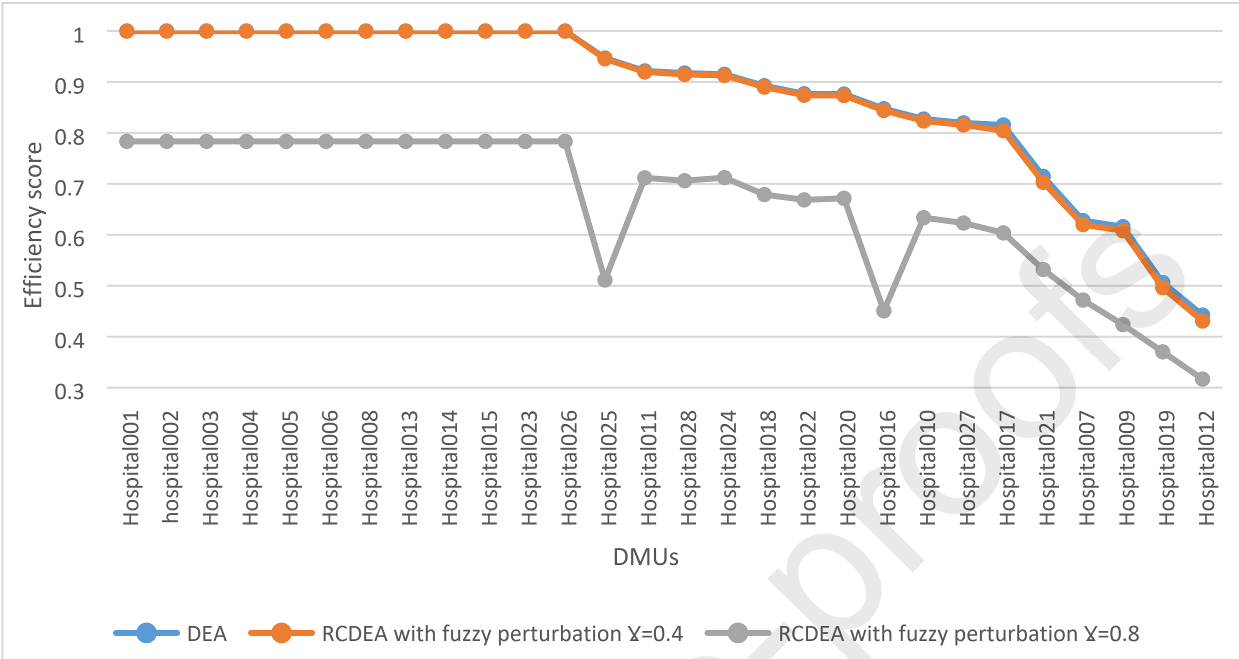

Fig.2. Efficiency scores of hospitals using RCDEA model with fuzzy perturbation value

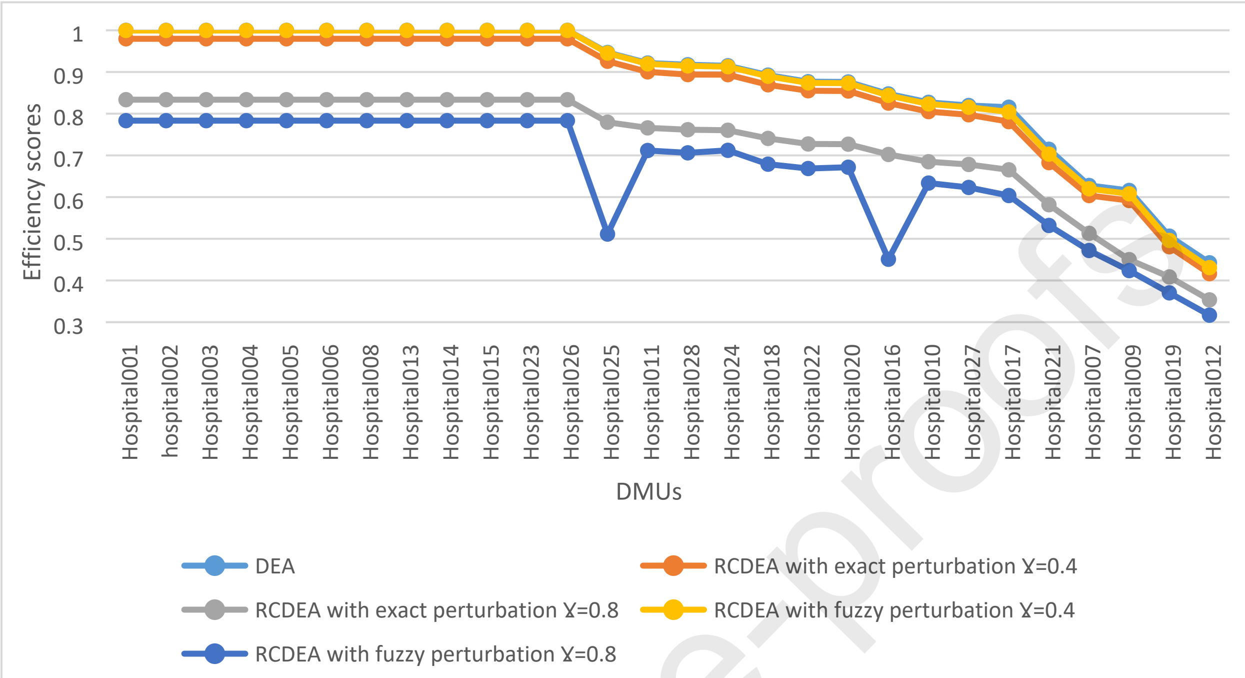

Fig.3. Efficiency scores of hospitals using RCDEA model

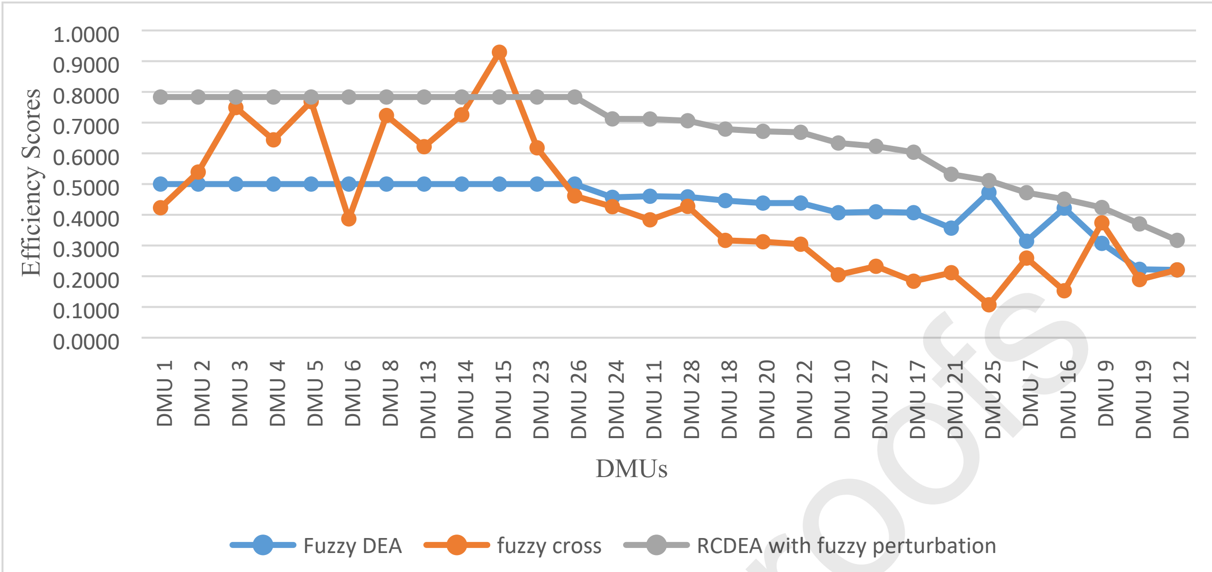

Fig.4. Efficiency scores of DMU in RCDEA, Fuzzy DEA and Fuzzy cross DEA model

Table 1. Parameters of the equations

Parameter	Definition
w_i	Weight of input and output variables
$(x_{ij}^{m}, x_{ij}^{\alpha}, x_{ij}^{\beta}), (y_{rj}^{m}, y_{rj}^{\alpha}, y_{rj}^{\beta})$	i^{th} fuzzy input and r^{th} fuzzy output for j^{th} DMU, respectively
Γ_0 , Γ_j	Budget of uncertainty for objective function and j^{th} constraint respectively
p_0 , p_j	Dual variable for objective function and constraints in robust counterpart model
q_{ij}	Dual variables for constructing robust model in Bertsimas and Sim (2004) approach
z_i	Linearization variable in Bertsimas and Sim (2004) approach
$(e_{ij}^{m}, e_{ij}^{\alpha}, e_{ij}^{\beta})$	Fuzzy perturbation percentage
γ	The confidence level for credibility theory $\gamma \in [0,1]$

Table 2. Data for 28 hospitals in northwestern region of Iran

Provinces	DMU	Inputs			Outputs			
		Personnel	Equipment	Active-bed	Inpatient	Outpatient	Special patients	Bed-day
East Azerbaijan	Hospital001	450	310	84	82840	606647	34149	25351
	hospital002	330	436	160	157514	168310	16480	51903
	Hospital003	687	678	581	389946	412736	118180	163089
	Hospital004	128	77	41	22460	533071	29541	9995
	Hospital005	152	19	78	19142	162417	58186	17923
	Hospital006	459	211	591	85547	56781	6756	199714
	Hospital007	417	271	206	49647	54077	36715	42228
	Hospital008	348	115	260	85405	130086	109904	66874
	Hospital009	314	36	119	34197	19798	22250	22767
	Hospital010	271	468	98	30894	112228	44872	25418
	Hospital011	952	1223	290	286882	637243	42198	84393
	Hospital012	160	194	67	22929	159602	8651	8766
	Hospital013	810	1258	258	490876	72982	52045	74451
	Hospital014	444	14	203	88563	213608	37101	66587
West Azerbaijan	Hospital015	722	249	244	299582	915744	46340	69522
	Hospital016	157	47	93	6112	52779	6	26343
	Hospital017	528	867	156	80796	43755	25053	41768
	Hospital018	598	572	220	119731	316633	27082	64428
	Hospital019	233	144	64	20973	23277	9075	10576
	Hospital020	444	606	142	115110	169227	14249	40337
	Hospital021	243	87	36	17577	30502	8127	8314
	Hospital022	645	732	247	136250	409164	28897	70783
	Hospital023	1154	397	439	281903	782631	116882	143668
Ardebil	Hospital024	681	968	288	206173	161174	147371	80515
	Hospital025	158	64	128	2097	1732	27	40957
	Hospital026	341	359	129	56579	47770	142534	37150
	Hospital027	528	402	190	58702	202569	28837	51456
	Hospital028	485	281	160	107568	140740	24841	48030

Table 3. Results of RCDEA with exact perturbation

Provinces	DMU	DEA	RCDEA with exact perturbation	
			$\gamma = 0.4$	$\gamma = 0.8$
East Azerbaijan	Hospital001	1	0.9796	0.8336
	hospital002	1	0.9796	0.8336
	Hospital003	1	0.9796	0.8336
	Hospital004	1	0.9796	0.8336
	Hospital005	1	0.9796	0.8336
	Hospital006	1	0.9796	0.8336
	Hospital007	0.6278	0.6037	0.5131
	Hospital008	1	0.9796	0.8336
	Hospital009	0.6159	0.5917	0.45
	Hospital010	0.8272	0.805	0.6848
	Hospital011	0.9215	0.9003	0.766
	Hospital012	0.4424	0.4164	0.3535
	Hospital013	1	0.9796	0.8336
	Hospital014	1	0.9796	0.8336
West Azerbaijan	Hospital015	1	0.9796	0.8336
	Hospital016	0.8474	0.8254	0.7022
	Hospital017	0.8155	0.7809	0.6654
	Hospital018	0.8925	0.8693	0.7407
	Hospital019	0.5063	0.481	0.4085
	Hospital020	0.8761	0.8544	0.7269
	Hospital021	0.7146	0.6826	0.5817
	Hospital022	0.8767	0.855	0.7274
	Hospital023	1	0.9796	0.8336
Ardebil	Hospital024	0.9149	0.8936	0.7603
	Hospital025	0.9468	0.9258	0.7795
	Hospital026	1	0.9796	0.8336
	Hospital027	0.8196	0.7974	0.6782
	Hospital028	0.9174	0.8938	0.7616
Mean		0.8772	0.8546	0.7251

Table 4. Results of RCDEA with fuzzy perturbation

Provinces	DMU	DEA	RCDEA with fuzzy perturbation	
			$\gamma = 0.4$	$\gamma = 0.8$
East Azerbaijan	Hospital001	1	0.9996	0.7833
	hospital002	1	0.9996	0.7833
	Hospital003	1	0.9996	0.7833
	Hospital004	1	0.9996	0.7833
	Hospital005	1	0.9996	0.7833
	Hospital006	1	0.9996	0.7833
	Hospital007	0.6278	0.6199	0.4719
	Hospital008	1	0.9996	0.7833
	Hospital009	0.6159	0.6078	0.4237
	Hospital010	0.8272	0.8233	0.6336
	Hospital011	0.9215	0.9195	0.7117
	Hospital012	0.4424	0.4308	0.3168
	Hospital013	1	0.9996	0.7833
	Hospital014	1	0.9996	0.7833
West Azerbaijan	Hospital015	1	0.9996	0.7833
	Hospital016	0.8474	0.8439	0.4509
	Hospital017	0.8155	0.8042	0.6037
	Hospital018	0.8925	0.8898	0.6789
	Hospital019	0.5063	0.496	0.3703
	Hospital020	0.8761	0.8732	0.6716
	Hospital021	0.7146	0.7029	0.5319
	Hospital022	0.8767	0.8738	0.6685
	Hospital023	1	0.9996	0.7833
Ardebil	Hospital024	0.9149	0.9127	0.7121
	Hospital025	0.9468	0.9453	0.5113
	Hospital026	1	0.9996	0.7833
	Hospital027	0.8196	0.8155	0.6231
	Hospital028	0.9174	0.9147	0.7061
Mean		0.8772	0.8738	0.6602

Table 5. Spearman correlation among different models

		Fuzzy DEA	Fuzzy Cross DEA	RCDEA
	Fuzzy DEA	1.000	0.806**	0.952**
	Fuzzy Cross DEA	0.806**	1.000	0.873**
	RCDEA	.952**	0.873**	1.000

**. Correlation is significant at the 0.01 level.

Appendix A

Preliminaries

Definition 1: Let U be a universe set. A fuzzy set $\tilde{A}$ of U is defined by a membership function $\mu_{\tilde{A}}(x) \rightarrow [0,1], \forall \mathrm{x} \in U$.

Definition 2: The *α-cut* of fuzzy set $\tilde{A}$, $\tilde{A}_\alpha$, is the crisp set $\tilde{A}_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 3: A fuzzy number *L-R* type is expressed as $\tilde{A} = (m, \alpha, \beta)_{LR}$ with below membership function:

$$\mu_{\tilde{A}}(r) = \begin{cases} L(\frac{m-r}{\alpha}) & r \leq m \\ R(\frac{r-m}{\beta}) & r \geq m \end{cases} \tag{A1}$$

where *L* and *R* are the left and right functions, respectively, and α and β are the (non-negative) left and right spreads, respectively.

Definition 4: An *L-R* fuzzy number, $\tilde{A} = (m, \alpha, \beta)_{LR}$ is a triangular fuzzy number if

$$L(x) = R(x) = \begin{cases} 1-x & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases} \tag{A2}$$

Definition 5: Let $\tilde{A} = (\mathrm{m}, \alpha, \beta)_{LR}$ and $\tilde{B} = (\overline{\mathrm{m}}, \overline{\alpha}, \overline{\beta})_{LR}$ be two positive triangular fuzzy numbers. The addition, subtraction and multiplication of $\tilde{A}$ and $\tilde{B}$ are as follows:

Addition: $\tilde{A} + \tilde{B} = (m, \alpha, \beta)_{LR} + (\overline{m}, \overline{\alpha}, \overline{\beta})_{LR} = (m + \overline{m}, \alpha + \overline{\alpha}, \beta + \overline{\beta})_{LR}$

Subtraction: $\tilde{A} - \tilde{B} = (m, \alpha, \beta)_{LR} - (\overline{m}, \overline{\alpha}, \overline{\beta})_{LR} = (m - \overline{m}, \alpha + \overline{\beta}, \beta + \overline{\alpha})_{LR}$

Multiplication (approximation):

$$\tilde{A} \otimes \tilde{B} = (m, \alpha, \beta)_{LR} \otimes (\overline{m}, \overline{\alpha}, \overline{\beta})_{LR} = (m\overline{m}, m\overline{\alpha} + \overline{m}\alpha - \alpha\overline{\alpha}, m\overline{\beta} + \overline{m}\beta + \beta\overline{\beta})_{LR}$$

Definition 6: A possibility space is defined as $(\Theta, P(\Theta), Pos)$ where Θ is nonempty set, $P(\Theta)$ is the power set of Θ and Pos is the possibility measure. The possibility measure satisfies the below axioms:

1. $Pos(\varnothing) = 0, Pos(X) = 1$;

2. $\forall\ A, B \in P(\Theta),\ if\ A \subseteq B \rightarrow Pos(A) \leq \mathrm{Pos(B)}$;

3. $Pos(A_1 \cup A_2 ... \cup A_k) = Sup Pos_j(A_j)$

where X is the universe set.

Definition 7: The necessity measure is defined as $Nec(A) = 1 - Pos(A^c)$ where A^c is the complementary set of A set. The necessity measure satisfies the below axioms:

1. $Nec(\varnothing) = 0, Nec(X) = 1;$

2. $\forall\ A, B \in P(\Theta),\ if\ A \subseteq B \rightarrow Nec(A) \leq Nec(B);$

3. $Nec(A_1 \cap A_2 ... \cap A_k) = Inf\ Nec_j(A_j)$

Definition 8: The credibility measure is defined as $Cre(A) = \frac{1}{2}\{Pos(A) + Nec(A)\}$. The credibility measure satisfies the below axioms:

1. $Cre(\varnothing) = 0, Cre(X) = 1;$

2. $\forall\ A, B \in P(\Theta),\ if\ A \subseteq B \rightarrow Cre(A) \leq Cre(B);$

3. $Cre(A) + Cre(A^c) = 1, \forall A \subseteq P(X)$

Definition 9: Let λ be a fuzzy variable. The possibility, necessity and credibility of the fuzzy event $(\lambda \geq r)$ are defined as:

$$Pos(\lambda \geq r) = \sup_{t \geq r} \mu_\lambda(t) \tag{A3}$$

$$Nec(\lambda \geq r) = 1 - Pos(\lambda < r) = 1 - \sup_{\lambda < r} \mu_\lambda(t) \tag{A4}$$

$$Cre(\lambda \geq r) = \frac{1}{2}\{Pos(\lambda \geq r) + Nec(\lambda \geq r)\} \tag{A5}$$

Credit Authorship Contribution Statement

List of Roles	Hashem Omrani	Arash Alizadeh	Ali Emrouznejad	Tamara Teplova
Conceptualization	√			
Data curation		√		
Formal analysis			√	√
Investigation				√
Methodology	√	√	√	

Software		√		
Supervision	√		√	
Validation	√			√
Writing - original draft		√		
Writing - review & editing			√	√

Hashem Omrani: Conceptualization, Methodology, Supervision, Validation

Arash Alizadeh: Data curation, Methodology, Software, Writing - original draft

Ali Emrouznejad: Formal analysis, Methodology, Supervision, Writing - review & editing

Tamara Teplova: Formal analysis, Investigation, Validation, Writing - review & editing

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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Highlights

- Efficiency of operational processes under uncertainty
- Developing a robust credibility DEA model with fuzzy perturbation degree
- Robust optimization approach is applied to consider uncertainty in constructing fuzzy sets
- An assessment of hospitals under uncertainty