Michali, M., A. Emrouznejad A. Dehnokhalaji, and B. Clegg (2021) Noise-pollution efficiency analysis of European railways: A Network DEA model, Transportation Research Part D, 98: 102980. https://doi.org/10.1016/j.trd.2021.102980

Noise-pollution efficiency analysis of European railways: A Network DEA model

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Abstract

One of the most important effects that railways have on the environment is noise pollution, notably in Europe. The purpose of this study is to evaluate the environmental efficiency of railways in 22 European countries, considering two factors; a country's response in retrofitting their wagon fleet with more silent braking technology and the number of people affected by railway noise. The railway transport process efficiency is decomposed into assets and service efficiency. The additive decomposition network Data Envelopment Analysis (NDEA) approach is customised to account for intermediate and undesirable outputs. Results suggest that Estonia, Germany and Poland are overall environmentally efficient and that except for Finland, asset efficient countries are also service efficient; the inverse does not hold. Sensitivity analysis revealed that efficiency rankings are robust to alterations in the decomposition weight restrictions. This is the first study that uses DEA to incorporate the noise-pollution problem in railway efficiency measurement.

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Keywords: Network DEA, Undesirable output, Efficiency decomposition, Railways, Noise pollution

1 Introduction

An efficient transport system is critical for attaining economic growth; it allows for the movement of people, goods and resources, provides access to services and facilities and enhances the quality of life. Governments need to invest in transport infrastructure while being considerate of sustainable development. During the last decades, the important share of transport in energy consumption and air pollution increased concerns about the impacts it will have on the natural ecosystem and climate change. Since railways are the most eco-friendly means of transport while demonstrating the lowest traffic congestion levels and high safety performance, they are becoming increasingly popular, and plans regarding their improvement and broader adoption are included in many governmental agendas.

However, railway noise generated by the wheel-rail contact, as well as aerodynamic noise were proved to create a major environmental problem. Specifically in the European Union (EU), railways are considered to be the second-highest source of noise pollution after road, both inside and outside urban areas (EEA Report 22/2019, 2019). Prolonged noise exposure is linked to well-being and health problems, such as sleep disturbances, annoyance and higher risk of cardiovascular diseases (EEA Report 10/2014, 2014). Railway noise is also related to economic costs, such as the depreciation of houses close to rail lines and productivity decrease.

Efficiency evaluation of railways is very important in order to identify its sources of inefficiency and further improve its operation. It is crucial for the society and the economy to keep this sustainable mode of transport competitive and manage to form the modal split in its favour. Data envelopment analysis (DEA) is a linear programming technique, which is broadly used to measure the technical efficiency of Decision Making Units (DMUs) relatively to an empirically constructed production frontier. Its great advantage lies in its non-parametric nature which allows for the inclusion of multiple inputs and outputs in the production model. Therefore, there is a large number of studies using DEA models to assess the efficiency of railways in different geographical areas.

The purpose of this study is to evaluate the efficiency of railways in the European countries during 2016-2017, considering the noise pollution generated and its impact on humans. For this reason, a network DEA (NDEA) model with intermediate and undesirable outputs is formulated upon the assumption of variable returns to scale (VRS). As distinct from the conventional one-stage DEA approaches, there is a quite limited number of studies in the DEA literature which consider the inner structure of railways' operation. In this study, we suggest that the final output of the railway transport process, i.e. passenger and freight movement, is a result of a two-stage process, the first one of which is related to asset management and the second one to the service offering. Ignoring the role of one of the two stages may result in misleading conclusions for the efficiency level of a country's railway system. The number of people exposed to high levels of railway noise is considered as an undesirable output. Despite the extended DEA literature, to the best of our knowledge, this is the first study that uses DEA to incorporate noise effects on the operation of railway transport. Using the additive efficiency decomposition approach, it is possible to define the source of inefficiency for each country. Furthermore, a sensitivity analysis is performed to investigate how the different choices of stage weights affect the efficiency scores and rankings of the countries.

The rest of this paper is structured as follows. Section 2 is a review of the relevant DEA and NDEA literature on railway transport. In Section 3, DEA and efficiency decomposition methodologies are explained. In Section 4, a two-stage NDEA model with undesirable output is formulated. In Section 5, the properties of the decomposition weights of the two stages, under the VRS assumption, are examined. In Section 6, the empirical study is discussed. Finally, in the last section, conclusions, main contributions of this study and some future research directions are provided.

2 Literature Review

Many studies have aimed their attention at measuring the performance of the railway transport industry. The first studies that used DEA in this direction were in the 1990s, from Moesen (1994) and Oum and Yu (1994). Since then, many studies have assessed railway performance in different regions. Graham (2008) assessed the efficiency and productivity of 200 urban railways globally, non-parametrically, using VRS and CRS DEA, and parametrically, by decomposing the total factor productivity (TFP) change. Yu (2008) used directional distance functions and NDEA to measure the efficiency of 40 global railways. Yu and Lin (2008) measured passenger and freight services' efficiency and effectiveness of 20 railway companies using a multi-activity NDEA. Kutlar et al. (2013) evaluated the technical and allocative efficiency of 31 railway companies using CCR and BCC DEA models and used a second stage Tobit regression to test the impact of outputs on the efficiency scores. Chapin and Schmidt (1999) measured the performance of class I railroads using the CCR and BCC DEA models on panel data and Shi et al. (2011) using sequential DEA and Malmquist index (MI). Marchetti and Wanke (2017) used CCR and BCC DEA models to assess the efficiency of rail concessionaires in Brazil and a second stage bootstrap truncated regression to measure the impact of exogenous variables on the efficiency scores. Li et al. (2018) used CCR DEA and generalised DEA to measure the efficiency of Chinese railway administrations and Kuang (2018) applied BCC and super cross-efficiency DEA to assess the efficiency of China Railway Bureau. Jitsuzumi and Nakamura (2010) used BCC DEA to identify the sources of inefficiency in Japanese railways and calculate the optimal levels of subsidies. Mapapa (2004) applied CCR and BCC DEA and MI to evaluate the performance of sub-Saharan African railways. Mohajeri and Amin (2010) combined DEA and analytical hierarchy process (AHP) for the selection of the optimal railway station location in Mashhad. Rayeni and Saljooghi (2014) assessed and compared the efficiencies of Iranian railways over a 30-year period using cross-efficiency DEA. Azadeh et al. (2018) assessed the performance of Tehran-Karaj railway electrification system using BCC DEA and considering resilience engineering (RE). George and Rangaraj (2008) applied CCR and cross efficiency DEA to measure the performance of Indian railways. Bhanot and Singh (2014) used CCR and BCC DEA to compare indicators of business performance of Indian Railway container transport. Sharma et al. (2016) assessed the performance of 16 railway zones in India in terms of the services they provide applying BCC DEA and Malmquist Index. Kim et al. (2011) studied the modal shift to railways in Korea, as a more environmental means of transport. They measured railway freight transport efficiency applying CCR and BCC DEA models and made suggestions about how to expand the use of railways in freight transportation. Reorganisation, incorporation or privatization as well as passenger services, freight carriage, safety and energy consumption of railways are some common research topics in DEA literature. Mahmoudi et al. (2020) provided an extended review of DEA applications on the transportation and railway industry.

In the late 1980s, the need for increasing railways' eroded modal market share and coping with the new demands arising from globalisation sparked a series of reforms in European railways. That stimulated many studies to assess the performance of the railway system in Europe before and after such transformations, to extract useful conclusions towards its efficiency improvement. Oum and Yu (1994) assessed the efficiency of railways which were mainly focused on passenger services in 18 European countries and Japan, during the time period from 1978 to 1989. They suggested that managerial autonomy and less dependence on subsidies have a positive effect on the efficiency of a railway system. UK, Ireland, Netherlands, Spain, and Sweden had the most efficient railway systems during that period. According to Cantos et al. (1999), in the years that followed - from 1985 to 1995 - the financial and managerial autonomy continued to have a positive impact on the efficiency of railways and that reforms that took place, resulted in increasing productivity. During the same time period, smaller European railway companies seemed to have higher technical efficiency (De Jorge-Moreno and Garcia-Cebrian, 1999).

Coelli and Perelman (1999) investigated the efficiency of 17 European railway firms from 1988 to 1993, using distance functions. De Jorge and Suarez (2003) used quadratic functions to observe the efficiency of 19 railway firms in Europe for a long time period, from 1965 to 1998. They concluded that the separation of railway operations from railway infrastructure management - introduced in 1991 - and the reductions in personnel, affected the efficiency of firms. Hilmola (2007) studied the productivity and efficiency of freight railways in 31 European countries, from 1980 to 2003. During the 1990s there was an efficiency downfall of all the previously best-performing countries. Also, a high level of divergence in freight transport among the European countries was observed (Hilmola, 2007; 2008; Salvolainen and Hilmola, 2009). Countries in the Baltic region, and notably Estonia and Latvia, were performing better in freight transport. However, in passenger transport, Netherlands, UK, Spain and Denmark were more efficient, while Eastern European countries were showing low performance (Hilmola, 2008). Salvolainen and Hilmola (2009) suggested that an associated development of railway and airline passenger transport would probably increase the efficiency. Growitsch and Wetzel (2009) used a DEA super-efficiency model with bootstrapping on 54 railway European firms during 2000-2004 and found that vertical integration favours the performance improvement in the majority of the railways included in the study. Cantos et al. (2010) studied the vertical and horizontal separation of railways in 16 European countries for the time period 1985-2005.

In more recent years, Sozen et al. (2012) and Sozen and Cipil (2018) compared Turkish railways to the railways of 23 EU member countries. Rotoli et al. (2015) considered accessibility among the European countries and suggested it could be improved by giving importance to the increase of railway speed. Rotoli et al. (2018) ranked the efficiency of Italian rail segments, from the standpoint of three different stakeholders; rail regulator, rail operator and the infrastructure manager. Khadem Sameni et al. (2016) were the first to implement DEA to assess the efficiency of 96 railway stations in Great Britain in terms of how well they manage train stops considering the existing station capacity.

Although railways are one of the safest means of transport, reduction of existing safety risks such as train collisions, derailments, level crossings or exposure of railway stuff to moving trains and electricity of high voltage can further improve its sustainability. Concerning railway safety in Europe, Noroozzadeh and Sadjadi (2013) measured the efficiency of 25 European passenger railways in 2008. Djordjević et al. (2018) used a non-radial DEA model to assess the efficiency of European railways regarding their level of safety in railway level crossings, during 2010-2012 and 2014. Roets et al. (2018) measured the efficiency of railway traffic control centres in Belgium in 2015, using cost allocation restrictions and a metafrontier approach. In such studies, the number of accidents, number of victims, surveillance staff, number of safety and non-safety interventions are some of the variables used to measure railway safety.

Within the global movement towards decreasing greenhouse gas emissions, some studies used DEA models to assess the energy-environmental efficiency of transport considering CO_2 emissions as an undesirable output. The majority of those studies refers to China (Chang et al., 2013; Cui and Li, 2014; Zhou et al., 2014). Concerning railway transport in China, Liu et al. (2016) used a non-radial DEA model, window analysis and a second stage Tobit regression. Song et al. (2016) combined a non-radial DEA model with a second stage panel beta regression, and Liu et al. (2017) applied a SBM DEA model with parallel structure. Ha et al. (2011) measured the environmental inefficiency of railway companies in Japan, considering CO_2 emissions produced both during the train operation and the infrastructure construction. The environmental efficiency of railways in the EU countries during 2014-2016 was studied by Djordjević and Krmac (2019) using a non-radial DEA model.

In most of the studies in the DEA transport literature, the production process is considered as a 'black box', where inputs are directly transformed to outputs. However, some studies model the production process considering its inner structure. Yu (2008) measured the efficiency and effectiveness of 40 railways globally, during 2002, using a NDEA model with two sub-processes - production and consumption process - with shared, intermediate and exogenous inputs. Yu and Lin (2008) assessed the efficiency and effectiveness of 20 selected railway companies during the same year. In this study, the production stage was divided into two parallel processes - passenger and freight subprocesses - with stage-specific and shared inputs. Mallikarjun et al. (2014) used a non-oriented four-stage series NDEA model to study the performance of public railway transport in the US. Zhou and Hu (2017) used an additive two-stage NDEA model to measure the performance of railways in China, considering dust as an undesirable output of the second stage. Similarly, the two stages are production and service related respectively. Wanke et al. (2018) applied directional distance functions in a multi-stage NDEA model combining series and parallel structure, with an undesirable output - number of accidents - to evaluate the efficiency of Asian railways.

3 The DEA framework

Suppose there is a set of N Decision Making Units (DMUs), each consuming P inputs to produce S outputs. Let $x_j = (x_{1j}, ..., x_{Pj}) \in \mathbb{R}_+^P$ and $y_j = (y_{1j}, ..., y_{Sj}) \in \mathbb{R}_+^S$ denote the non-negative input and output vectors of DMU$_j$, $j = 1, 2, ..., N$. The production possibility set (PPS) is defined as

$$P = \{(x, y) \in \mathbb{R}_+^{P+S} | x \text{ can produce } y\},$$

and it includes all the feasible input-output correspondences.

Definition 3.1 *Let $(x, y) \in P$ be the activity vector of DMU_j, $j = 1, 2, ..., N$. DMU_j is strongly (weakly) efficient if and only if there is no other feasible activity $(x', y') \neq (x, y)$, such that $x' \leq x$ $(x' < x)$ and $y' \geq y$ $(y' > y)$.*

For a weakly efficient DMU, it is possible to further improve its activity, without worsening any other of its inputs or outputs. In this paper, both strongly and weakly efficient DMUs are regarded as efficient.

In DEA, the boundary of the PPS is defined by the observed set of DMUs and some assumptions that are made about the technology under which the DMUs operate. Let P^{CRS} and P^{VRS} denote the PPSs under the assumptions of constant returns to scale (CRS) and variable returns to scale (VRS), respectively. Then,

$$P^{CRS} = \Big\{(x, y) \in \mathbb{R}_+^{P+S} | \sum_{j=1}^{N} \lambda_j x_j \leq x, \sum_{j=1}^{N} \lambda_j y_j \geq y, \lambda \in \mathbb{R}_+^N \Big\}, \tag{1}$$

$$P^{VRS} = \Big\{(x, y) \in \mathbb{R}_+^{P+S} | \sum_{j=1}^{N} \lambda_j x_j \leq x, \sum_{j=1}^{N} \lambda_j y_j \geq y, \sum_{j=1}^{N} \lambda_j = 1, \lambda \in \mathbb{R}_+^N \Big\}. \tag{2}$$

The basic DEA model, suggested by Charnes et al. (1978), is used to measure the relative efficiency of a homogeneous set of DMUs, under the CRS. Banker et al. (1984) extended their model to the VRS case. The feasible set of the CRS and VRS envelopment models corresponds to P^{CRS} and P^{VRS}, respectively. Models (3) and (4) are the dual forms of models (1) and (2) or the multiplier models, used to measure the input-oriented relative efficiency of DMU_{j_0}, under the CRS and VRS assumptions, respectively, where $j_0 \in \{1, ..., N\}$ denotes the index of the DMU under evaluation.

$$
\begin{aligned}
\theta_{j_0}^{CRS*} = \max \; & \sum_{s=1}^{S} u_s y_{sj_0} \\
\text{s.t.} \; & \sum_{p=1}^{P} v_p x_{pj_0} = 1 \\
& \sum_{s=1}^{S} u_s y_{sj} - \sum_{p=1}^{P} v_p x_{pj} \leq 0, \\
& \qquad j = 1, ..., N, \\
& u_s, v_p \geq 0, s = 1, ..., S, p = 1, ..., P
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\theta_{j_0}^{VRS*} = \max \; & \sum_{s=1}^{S} u_s y_{sj_0} + u^{j_0} \\
\text{s.t.} \; & \sum_{p=1}^{P} v_p x_{pj_0} = 1 \\
& \sum_{s=1}^{S} u_s y_{sj} - \sum_{p=1}^{P} v_p x_{pj} + u^{j_0} \leq 0, \\
& \qquad j = 1, ..., N, \\
& u_s, v_p \geq 0, s = 1, ..., S, p = 1, ..., P, \\
& u^{j_0} \text{ free in sign.}
\end{aligned}
\tag{4}
$$

The optimal solutions (u_p^*, u_s^*) and (u_p^*, u_s^*, u^{j_0*}) for models (3) and (4), respectively, satisfy the constraints and maximise the corresponding objective functions. The optimal objective values $\theta_{j_0}^{CRS*}$ and $\theta_{j_0}^{VRS*}$ are the CRS and VRS-efficiency scores of DMU_{j_0}.

Definition 3.2 *A DMU_{j_0} is CRS strongly efficient if the optimal solution $(\theta_{j_0}^{CRS*}, u_p^*, u_s^*)$ satisfies $\theta_{j_0}^{CRS*} = 1$ and all slacks are zero. It is weakly efficient if $\theta_{j_0}^{CRS*} = 1$ and has non-zero slacks. Otherwise, it is CRS-inefficient. DMU_{j_0} is considered as VRS strongly efficient if the optimal solution $(\theta_{j_0}^{VRS*}, u_p^*, u_s^*, u^{j_0*})$ satisfies $\theta_{j_0}^{VRS*} = 1$ and all slacks are zero. It is weakly efficient if $\theta_{j_0}^{VRS*} = 1$ and has non-zero slacks. In any other case, it is VRS-inefficient.*

In conventional DEA, the transformation of inputs to outputs is considered to occur in one stage. Network DEA (NDEA) models deal with more complex production processes, where some inner structure needs to be taken into account. Consider the simple two-stage production process illustrated in Figure 1. In the first stage, each DMU$_j$, $j = 1, 2, ..., N$ uses P inputs $x_j = (x_{1j}, ..., x_{Pj}) \in \mathbb{R}^P_+$ to produce Q outputs $z_j = (z_{1j}, ..., z_{Qj}) \in \mathbb{R}^Q_+$, which are referred to as intermediate products. In the second stage, intermediate products obtained from the first stage, are inserted as inputs to the second stage, to produce S final outputs $y_j = (y_{1j}, ..., y_{Sj}) \in \mathbb{R}^S_+$. Let θ^0_j be the overall efficiency, θ^1_j be the efficiency of the first stage and θ^2_j be the efficiency of the second stage for DMU$_j$, for $j = 1, 2, .., N$.

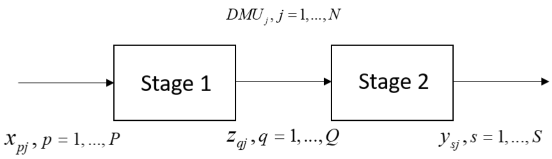

Figure 1: Two-stage process

There are two main decomposition approaches in two-stage NDEA literature used to derive the overall efficiency of a DMU; the additive approach (Kao and Huang, 2008) and the multiplicative approach (Chen et al., 2009).

Under the CRS, the first and second stage efficiency scores for DMU$_{j_0}$ in the input orientation, can be calculated independently by solving the following mathematical models, respectively:

$$\begin{aligned} \max\ \theta^1_{j_0} &= \frac{\sum_{q=1}^{Q} \gamma^A_q z_{qj_0}}{\sum_{p=1}^{P} v_p x_{pj_0}} \\ \text{s.t.}\ \frac{\sum_{q=1}^{Q} \gamma^A_q z_{qj}}{\sum_{p=1}^{P} v_p x_{pj}} &\leq 1 \\ v_p, \gamma^A_q > 0, &\quad j = 1, ..., N, \end{aligned} \tag{5a}$$

$$\begin{aligned} \max\ \theta^2_{j_0} &= \frac{\sum_{s=1}^{S} e_s y_{sj_0}}{\sum_{q=1}^{Q} \gamma^B_q z_{qj_0}} \\ \text{s.t.}\ \frac{\sum_{s=1}^{S} e_s y_{sj}}{\sum_{q=1}^{Q} \gamma^B_q z_{qj}} &\leq 1 \\ e_s, \gamma^B_q > 0, &\quad j = 1, ..., N. \end{aligned} \tag{5b}$$

In order to take into consideration the series relationship between the two stages, Kao and Huang (2008) assumed that the multipliers related to the intermediate products z_{qj}, are the same in both stages, i.e. $\gamma^A_q = \gamma^B_q$. That means that the optimal aggregated outputs from the first stage become inputs to the second stage.

Kao and Huang (2008) defined the overall efficiency θ^0_j, $j = 1, 2, ..., N$ as the product of stage efficiencies, i.e.

$$\theta_j^0 = \frac{\sum_{s=1}^S e_s y_{sj}}{\sum_{p=1}^P v_p x_{pj}} = \frac{\sum_{q=1}^Q \gamma_q z_{qj}}{\sum_{p=1}^P v_p x_{pj}} \cdot \frac{\sum_{s=1}^S e_s y_{sj}}{\sum_{q=1}^Q \gamma_q z_{qj}} = \theta_j^1 \theta_j^2. \tag{6}$$

In this case, the overall efficiency of DMU_{j_0} is given by the following model:

$$\begin{aligned} \theta_{j_0}^{0*} = \max\ \theta_{j_0}^1 \theta_{j_0}^2 &= \frac{\sum_{s=1}^S e_s y_{sj_0}}{\sum_{p=1}^P v_p x_{pj_0}} \\ \text{s.t. } \theta_j^1 &= \frac{\sum_{q=1}^Q \gamma_q z_{qj}}{\sum_{p=1}^P v_p x_{pj}} \leq 1 \\ \theta_j^2 &= \frac{\sum_{s=1}^S e_s y_{sj}}{\sum_{q=1}^Q \gamma_q z_{qj}} \leq 1 \\ & e_s, v_p, \gamma_q > 0, \quad j = 1, ..., N. \end{aligned} \tag{7}$$

From the constraints $\frac{\sum_{q=1}^Q \gamma_q z_{qj}}{\sum_{p=1}^P v_p x_{pj}} \leq 1$ and $\frac{\sum_{s=1}^S e_s y_{sj}}{\sum_{q=1}^Q \gamma_q z_{qj}} \leq 1$, it is implied that $\frac{\sum_{s=1}^S e_s y_{sj}}{\sum_{p=1}^P v_p x_{pj}} \leq 1$. Therefore, this last constraint is superfluous and is not included in model (7).

Applying the Charnes-Cooper transformation (Charnes and Cooper, 1962), model (7) can be converted into a linear one. However, under the VRS assumption, the product of stage efficiencies would be

$$\frac{\sum_{q=1}^Q \gamma_q z_{qj} + u^A}{\sum_{p=1}^P v_p x_{pj}} \cdot \frac{\sum_{s=1}^S e_s y_{sj} + u^B}{\sum_{q=1}^Q \gamma_q z_{qj}}. \tag{8}$$

The above quantity cannot be linearised. Therefore, as Chen et al. (2009) noted, the multiplicative approach is not applicable under the VRS.

If intermediate products are treated as outputs and inputs simultaneously, the overall efficiency of DMU_j is defined as

$$\theta_j^0 = \frac{\sum_{s=1}^S e_s y_{sj} + \sum_{q=1}^Q \gamma_q z_{qj}}{\sum_{p=1}^P v_p x_{pj} + \sum_{q=1}^Q \gamma_q z_{qj}}. \tag{9}$$

In the additive decomposition approach introduced by Chen et al. (2009), the overall efficiency (9), can be decomposed into a weighted average of the stage efficiencies:

$$\frac{\sum_{s=1}^{S} e_s y_{sj} + \sum_{q=1}^{Q} \gamma_q z_{qj}}{\sum_{p=1}^{P} v_p x_{pj} + \sum_{q=1}^{Q} \gamma_q z_{qj}} = w_{1j} \frac{\sum_{q=1}^{Q} \gamma_q z_{qj}}{\sum_{p=1}^{P} v_p x_{pj}} + w_{2j} \frac{\sum_{s=1}^{S} e_s y_{sj}}{\sum_{q=1}^{Q} \gamma_q z_{qj}} \tag{10}$$

$$\text{and} \quad w_{1j} + w_{2j} = 1. \tag{11}$$

The decomposition weights w_{1j} and w_{2j} can be defined endogenously by solving the system of equations (10) and (11). Then, for DMU$_{j_0}$ under evaluation we have:

$$w_{1j_0} = \frac{\sum_{p=1}^{P} v_p x_{pj}}{\sum_{p=1}^{P} v_p x_{pj} + \sum_{q=1}^{Q} \gamma_q z_{qj}}, \quad \text{and} \quad w_{2j} = \frac{\sum_{q=1}^{Q} \gamma_q z_{qj}}{\sum_{p=1}^{P} v_p x_{pj} + \sum_{q=1}^{Q} \gamma_q z_{qj}}. \tag{12}$$

Intuitively, the decomposition weights w_{1j} and w_{2j} represent the relative contribution of each stage to the overall efficiency. Therefore, in the input orientation, they are defined as the ratio of the stage-specific inputs to the overall process inputs. In the output orientation, endogenous decomposition weights would be defined similarly, as the proportion of outputs consumed by each stage.

Then, the overall efficiency of DMU$_{j_0}$ can be derived by solving the following fractional programme:

$$\begin{aligned} \theta_{j_0}^{0*} = \max\ [w_{1j_0}\theta_{j_0}^{1} + w_{2j_0}\theta_{j_0}^{2}] &= \frac{\sum_{s=1}^{S} e_s y_{sj_0} + \sum_{q=1}^{Q} \gamma_q z_{qj_0}}{\sum_{p=1}^{P} v_p x_{pj_0} + \sum_{q=1}^{Q} \gamma_q z_{qj_0}} \\ \text{s.t.}\ \theta_j^1 &= \frac{\sum_{q=1}^{Q} \gamma_q z_{qj}}{\sum_{p=1}^{P} v_p x_{pj}} \leq 1 \\ \theta_j^2 &= \frac{\sum_{s=1}^{S} e_s y_{sj}}{\sum_{q=1}^{Q} \gamma_q z_{qj}} \leq 1 \\ v_p, \gamma_q, e_s &> 0, \quad j = 1, ..., N. \end{aligned} \tag{13}$$

Instead of defining the decomposition weights endogenously, fixed weights could be assigned to each stage. However, that would imply that for all DMUs each stage would be of the same relative importance. However, by defining the decomposition weights endogenously, their optimum values for each DMU can be obtained through linear programming, avoiding any arbitrary decisions. Furthermore, endogenous decomposition weights allow the linearization of the fractional problem; using the Charnes-Cooper transformation, model (13) can be transformed into a linear one.

Let $\theta_{j_0}^{0*}$ be the overall efficiency score of DMU_{j_0}, derived from the optimal solution $(v_p^*, \gamma_q^*, e_s^*)$ of model (13). The efficiency of the stage that is given pre-emptive priority is calculated while preserving the optimal overall efficiency level $\theta_{j_0}^{0*}$. Let θ_j^{kp*}, $k = 1, 2$ denote the optimal efficiency score of the priority stage. Suppose stage one is the priority stage, then, we solve the following model to calculate the efficiency score of the first stage:

$$
\begin{aligned}
\max\ & \theta_{j_0}^1 \\
\text{s.t. } & \theta_j^1 \leq 1 \\
& \theta_j^2 \leq 1 \\
& \frac{\sum_{s=1}^{S} e_s y_{sj_0} + \sum_{q=1}^{Q} \gamma_q z_{qj_0}}{\sum_{p=1}^{P} v_p x_{pj_0} + \sum_{q=1}^{Q} \gamma_q z_{qj_0}} = \theta_{j_0}^{0*} \\
& v_p, \gamma_q, e_s > 0, \quad j = 1, ..., N.
\end{aligned}
\tag{14}
$$

Replacing the optimal weights $(v_p^*, \gamma_q^*, e_s^*)$ obtained from model (13) to equations (12), optimal decomposition weights for DMU_j are derived. Then, from equation (10), the efficiency of the second stage is calculated as

$$
\theta_{j_0}^{2*} = \frac{\theta_{j_0}^{0*} - w_{1j_0}^* \theta_{j_0}^{1p*}}{w_{2j_0}^*}. \tag{15}
$$

Definition 3.3 *DMU_{j_0} is considered to be overall efficient if and only if $\theta_{j_0}^{0*} = 1$ and $\theta_{j_0}^{k*} = 1,\ k = 1, 2$. It is stage-k efficient if $\theta_{j_0}^{k*} = 1,\ k = 1, 2$.*

4 A two stage Network DEA model with undesirable outputs

The great advantage of the additive decomposition approach against the multiplicative is that the first one can be used under VRS as in that case, the resulting models can be transformed into linear ones. Chen et al. (2009) introduced the additive decomposition method for a closed two-stage process with no external intermediate inputs or outputs. In this section, the additive approach is going to be applied to an open two-stage production process, with external intermediate outputs and undesirable outputs. The assumption introduced by Kao and Huang (2008), that the optimal level of outputs resulting from the first stage is introduced unchanged to the second stage is also adopted. The formulation of the models is done in the input orientation, and under the assumption of VRS.

Consider a production process consisted of two serially connected stages (see Figure 2). Suppose there are N DMUs. Each DMU$_j$, $j = 1, ..., N$ consumes P inputs x_{pj}, $p = 1, ..., P$ in the first stage to produce L final outputs $(z_f)_{lj}$, $l = 1, ..., L$ and Q intermediate products z_{qj}, $q = 1, ..., Q$, which are then used as inputs in the second stage. From the second stage S good outputs y_{sj}, $s = 1, ..., S$ and D bad outputs $(y_b)_{dj}$, $d = 1, ..., D$ are produced.

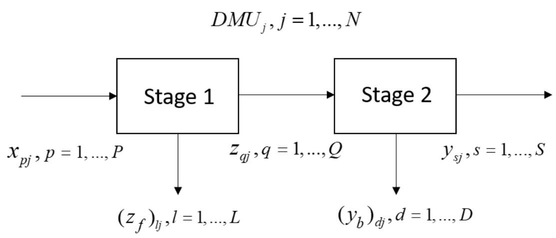

Figure 2: Two-stage process with undesirable outputs

If undesirable outputs are treated as normal outputs, then a DMU with lower undesirable products would be falsely considered as less efficient. In this approach, bad outputs produced from the second stage are treated as normal inputs to this stage, and thus, through the optimisation process, the aim is to proportionally decrease inputs to the second stage and undesirable outputs simultaneously. The first and second stage input-oriented efficiency scores of the DMU$_{j0}$, under the VRS assumption, can be calculated independently one from another as

$$\begin{aligned}
\max\ \theta^1_{j0} &= \frac{\sum_{q=1}^{Q} \gamma_q^A z_{qj0} + \sum_{l=1}^{L} m_l (z_f)_{lj0} + u^A}{\sum_{p=1}^{P} v_p x_{pj0}} \\
\text{s.t. } & \frac{\sum_{q=1}^{Q} \gamma_q^A z_{qj0} + \sum_{l=1}^{L} m_l (z_f)_{lj0} + u^A}{\sum_{p=1}^{P} v_p x_{pj0}} \leq 1 \qquad (16) \\
& v_p, \gamma_q^A, m_l > 0, \quad j = 1, ..., N \\
& u^A \text{ free in sign}
\end{aligned}$$

and

$$
\begin{aligned}
\max\ \theta^2_{j_0} &= \frac{\sum_{s=1}^{S} e_s y_{sj_0} + u^B}{\sum_{q=1}^{Q} \gamma_q^B z_{qj_0} + \sum_{d=1}^{D} k_d (y_b)_{dj_0}} \\
\text{s.t.}\ & \frac{\sum_{s=1}^{S} e_s y_{sj_0} + u^B}{\sum_{q=1}^{Q} \gamma_q^B z_{qj_0} + \sum_{d=1}^{D} k_d (y_b)_{dj_0}} \leq 1 \\
& e_s, \gamma_q^B, k_d > 0, \quad j = 1, ..., N \\
& u^B \text{ free in sign.}
\end{aligned} \tag{17}
$$

In order to link the two stages, it is assumed that for the multipliers of the intermediate products, $\gamma_q^A = \gamma_q^B = \gamma_q$. Treating intermediate products as outputs and inputs at the same time, the overall efficiency of DMU_j under VRS, is defined and additively decomposed as

$$
\theta_j^0 = \frac{\sum_{q=1}^{Q} \gamma_q^A z_{qj} + \sum_{l=1}^{L} m_l (z_f)_{lj} + u^A + \sum_{s=1}^{S} w_s y_{sj} + u^B}{\sum_{p=1}^{P} v_p x_{pj} + \sum_{q=1}^{Q} \gamma_q z_{qj} + \sum_{d=1}^{D} k_d (y_b)_{dj}} \tag{18}
$$

$$
= w_{1j} \frac{\sum_{q=1}^{Q} \gamma_q z_{qj} + \sum_{l=1}^{L} m_l (z_f)_{lj} + u^A}{\sum_{p=1}^{P} v_p x_{pj}} + w_{2j} \frac{\sum_{s=1}^{S} e_s y_{sj} + u^B}{\sum_{q=1}^{Q} \gamma_q z_{qj} + \sum_{d=1}^{D} k_d (y_b)_{dj}} \tag{19}
$$

$$
= w_{1j}\theta_j^1 + w_{2j}\theta_j^2, \tag{20}
$$

where for the decomposition weights, it holds that $w_{1j} + w_{2j} = 1$, $j = 1, 2, ..., N$.

Then for a DMU_j, the decomposition weights w_{1j}, w_{2j} can be defined as the proportion of inputs consumed by each stage, as

$$
w_{1j} = \frac{\sum_{p=1}^{P} v_p x_{pj}}{\sum_{p=1}^{P} v_p x_{pj} + \sum_{q=1}^{Q} \gamma_q z_{qj} + \sum_{d=1}^{D} k_d (y_b)_{dj}}, \tag{21}
$$

$$
w_{2j} = \frac{\sum_{q=1}^{Q} \gamma_q z_{qj} + \sum_{d=1}^{D} k_d (y_b)_{dj}}{\sum_{p=1}^{P} v_p x_{pj} + \sum_{q=1}^{Q} \gamma_q z_{qj} + \sum_{d=1}^{D} k_d (y_b)_{dj}}. \tag{22}
$$

The overall VRS efficiency of DMU_{j_0} is given by solving the following fractional programme:

$$
\begin{aligned}
\theta_{j_0}^{0*} = \ & \max \frac{\sum_{q=1}^{Q}\gamma_q^A z_{qj_0} + \sum_{l=1}^{L} m_l (z_f)_{lj_0} + u^A + \sum_{s=1}^{S} w_s y_{sj_0} + u^B}{\sum_{p=1}^{P} v_p x_{pj_0} + \sum_{q=1}^{Q}\gamma_q z_{qj_0} + \sum_{d=1}^{D} k_d (y_b)_{dj_0}} \\
& \text{s.t.} \ \frac{\sum_{q=1}^{Q}\gamma_q z_{qj} + \sum_{l=1}^{L} m_l (z_f)_{lj} + u^A}{\sum_{p=1}^{P} v_p x_{pj}} \le 1 \\
& \frac{\sum_{s=1}^{S} w_s y_{sj} + u^B}{\sum_{q=1}^{Q}\gamma_q z_{qj} + \sum_{d=1}^{D} k_d (y_b)_{dj}} \le 1 \\
& v_p, w_s, \gamma_q, m_l > 0, \quad j = 1, ..., N \\
& u^A, u^B \text{ free in sign.}
\end{aligned} \tag{23}
$$

The constraint $\frac{\sum_{q=1}^{Q}\gamma_q^A z_{qj_0} + \sum_{l=1}^{L} m_l (z_f)_{lj_0} + u^A + \sum_{s=1}^{S} w_s y_{sj_0} + u^B}{\sum_{p=1}^{P} v_p x_{pj_0} + \sum_{q=1}^{Q}\gamma_q z_{qj_0} + \sum_{d=1}^{D} k_d (y_b)_{dj_0}} \le 1$ is omitted, as it is implied by the two constraints included in model (23).

Applying the Charnes-Cooper transformation, the fractional model (23) can be transformed into a linear one:

$$
\begin{aligned}
\theta_{j_0}^{0*} = \ & \max \sum_{q=1}^{Q} \eta_q z_{qj_0} + \sum_{l=1}^{L} \mu_l (z_f)_{lj_0} + u^1 + \sum_{s=1}^{S} \xi_s y_{sj_0} + u^2 \\
& \text{s.t.} \ \sum_{p=1}^{P} \pi_p x_{pj_0} + \sum_{q=1}^{Q} \eta_q z_{qj_0} + \sum_{d=1}^{D} c_d (y_b)_{dj_0} = 1 \\
& \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{l=1}^{L} \mu_l (z_f)_{lj} - \sum_{p=1}^{P} \pi_p x_{pj} + u^1 \le 0 \\
& \sum_{s=1}^{S} \xi_s y_{sj} - \sum_{q=1}^{Q} \eta_q z_{qj} - \sum_{d=1}^{D} c_d (y_b)_{dj} + u^2 \le 0 \\
& \pi_p, \mu_l, \eta_q, \xi_s, c_d > 0, \quad j = 1, ..., N \\
& u^1, u^2 \text{ free in sign.}
\end{aligned} \tag{24}
$$

Let $(\theta_{j_0}^{0*}, \pi_p^*, \mu_l^*, \eta_q^*, \xi_s^*, c_d^*)$ be the optimal solution to model (24). The optimal decomposition weights are

$$
w_{1j_0}^* = \frac{\sum_{p=1}^{P} \pi_p^* x_{pj_0}}{\sum_{p=1}^{P} \pi_p^* x_{pj_0} + \sum_{q=1}^{Q} \eta_q^* z_{qj_0} + \sum_{d=1}^{D} c_d^* (y_b)_{dj_0}}, \tag{25}
$$

$$w_{2j_0}^{*}=\frac{\sum_{q=1}^{Q}\eta_q^{*}z_{qj_0}+\sum_{d=1}^{D}c_d^{*}(y_b)_{dj_0}}{\sum_{p=1}^{P}\pi_p^{*}x_{pj_0}+\sum_{q=1}^{Q}\eta_q^{*}z_{qj_0}+\sum_{d=1}^{D}c_d^{*}(y_b)_{dj_0}}. \tag{26}$$

If stage one is considered as the priority stage, then, the first stage efficiency of DMU_{j_0} is calculated by maximising $\theta_{j_0}^{1p}$, while maintaining optimal overall efficiency $\theta_{j_0}^{0*}$, as follows:

$$\begin{aligned}
\theta_{j_0}^{1*} = \max\ & \frac{\sum_{q=1}^{Q}\eta_q z_{qj_0}+\sum_{l=1}^{L}\mu_l(z_f)_{lj_0}+u^1}{\sum_{p=1}^{P}\pi_p x_{pj_0}} \\
\text{s.t.}\ & \frac{\sum_{q=1}^{Q}\eta_q z_{qj_0}+\sum_{l=1}^{L}\mu_l(z_f)_{lj_0}+u^1}{\sum_{p=1}^{P}\pi_p x_{pj_0}}\leq 1 \\
& \frac{\sum_{s=1}^{S}\xi_s y_{sj_0}+u^2}{\sum_{q=1}^{Q}\eta_q z_{qj_0}+\sum_{d=1}^{D}c_d(y_b)_{dj_0}}\leq 1 \\
& \frac{\sum_{q=1}^{Q}\eta_q z_{qj_0}+\sum_{l=1}^{L}\mu_l(z_f)_{lj_0}+u^1+\sum_{s=1}^{S}\xi_s y_{sj_0}+u^2}{\sum_{p=1}^{P}\pi_p x_{pj_0}+\sum_{q=1}^{Q}\eta_q z_{qj_0}+\sum_{d=1}^{D}c_d(y_b)_{dj_0}}=\theta_{j_0}^{0*} \\
& \pi_p,\mu_l,\eta_q,\xi_s,c_d>0, \quad j=1,...,N \\
& u^1,u^2 \text{ free in sign.}
\end{aligned} \tag{27}$$

The equivalent linear model is

$$\begin{aligned}
\theta_{j_0}^{1p*} = \ \max\ & \sum_{q=1}^{Q}\eta_q z_{qj_0}+\sum_{l=1}^{L}\mu_l(z_f)_{lj_0}+u^1 \\
\text{s.t.}\ & \sum_{p=1}^{P}\pi_p x_{pj_0}=1 \\
& \sum_{q=1}^{Q}\eta_q z_{qj}+\sum_{l=1}^{L}\mu_l(z_f)_{lj}+u^1-\sum_{p=1}^{P}\pi_p x_{pj}\leq 0 \\
& \sum_{s=1}^{S}\xi_s y_{sj}+u^2-\sum_{q=1}^{Q}\eta_q z_{qj}-\sum_{d=1}^{D}c_d(y_b)_{dj}\leq 0 \\
& (1-\theta_{j_0}^{0*})\sum_{q=1}^{Q}\eta_q z_{qj_0}-\theta_{j_0}^{0*}\sum_{d=1}^{D}c_d(y_b)_{dj_0}+\sum_{l=1}^{L}\mu_l(z_f)_{lj_0}+\sum_{s=1}^{S}\xi_s y_{sj_0}+u^1+u^2=\theta_{j_0}^{0*} \\
& \pi_p,\mu_l,\eta_q,\xi_s,c_d>0, \quad j=1,...,N \\
& u^1,u^2 \text{ free in sign.}
\end{aligned} \tag{28}$$

The second stage efficiency score of DMU_{j_0} is obtained from equation (15), where $w^*_{1j_0}$ and $w^*_{2j_0}$ are calculated from formulas (25) and (26) respectively.

Similarly, if stage two is considered as the priority stage, then

$$
\begin{aligned}
\theta^{2p*}_{j_0} = \ & \max \sum_{s=1}^{S} \xi_s y_{sj_0} + u^2 \\
& \text{s.t.} \sum_{q=1}^{Q} \eta_q z_{qj_0} + \sum_{d=1}^{D} c_d (y_b)_{dj_0} = 1 \\
& \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{l=1}^{L} \mu_l (z_f)_{lj} + u^1 - \sum_{p=1}^{P} \pi_p x_{pj} \leq 0 \\
& \sum_{s=1}^{S} \xi_s y_{sj} + u^2 - \sum_{q=1}^{Q} \eta_q z_{qj} - \sum_{d=1}^{D} c_d (y_b)_{dj} \leq 0 \\
& \sum_{q=1}^{Q} \eta_q z_{qj_0} + \sum_{l=1}^{L} \mu_l (z_f)_{lj_0} + u^1 + \sum_{s=1}^{S} \xi_s y_{sj_0} + u^2 - \theta^{0*}_{j_0} \sum_{p=1}^{P} \pi_p x_{pj_0} = \theta^{0*}_{j_0} \\
& \pi_p, \mu_l, \eta_q, \xi_s, c_d > 0, \quad j = 1, ..., N \\
& u^1, u^2 \text{ free in sign.}
\end{aligned}
\tag{29}
$$

Then, using the optimal decomposition weights given by (25) and (26), the efficiency of the first stage is calculated as

$$
\theta^{1*}_{j_0} = \frac{\theta^{0*}_{j_0} - w^*_{2j_0} \theta^{2p*}_{j_0}}{w^*_{1j_0}}. \tag{30}
$$

Remark 4.1 *If $w^*_{1j} = 0$ or $w^*_{2j} = 0$ are obtained for some DMU_j, then, some restrictions can be imposed on the decomposition weights, i.e $w_{1j} \geq \kappa$ and $w_{2j} \geq \kappa$, where $\kappa \in (0, 0.5]$, for $j = 1, 2, ..., N$,. Therefore, for DMU_{j_0} under evaluation, the following two constraints need to be added to model (24):*

$$
\begin{aligned}
& (\kappa - 1) \sum_{p=1}^{P} \pi_p x_{pj_0} + \kappa \sum_{q=1}^{Q} \eta_q z_{qj_0} + \kappa \sum_{d=1}^{D} c_d (y_b)_{dj_0} \leq 0 \\
& \kappa \sum_{p=1}^{P} \pi_p x_{pj_0} + (\kappa - 1) \sum_{q=1}^{Q} \eta_q z_{qj_0} + (\kappa - 1) \sum_{d=1}^{D} c_d (y_b)_{dj_0} \leq 0 \\
& \kappa \in (0, 0.5], \quad j = 1, ..., N.
\end{aligned}
$$

Since by definition $w_{1j} + w_{2j} = 1$, when both decomposition weights are restricted to take values greater than a value κ, κ can only take values in the interval $(0, 0.5]$.

Concerning the relationship between the decomposition weights in this network structure, by making use of the first inequality constraint of the linearised model (24), the following inequality holds true:

$$
\begin{aligned}
w_{1j} - w_{2j} &= \frac{\sum_{p=1}^{P} \pi_p x_{pj} - \sum_{q=1}^{Q} \eta_q z_{qj} - \sum_{d=1}^{D} c_d (y_b)_{dj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{d=1}^{D} c_d (y_b)_{dj}} \\
&\geq \frac{\sum_{l=1}^{L} \mu_l (z_f)_{lj} + u^1 - \sum_{d=1}^{D} c_d (y_b)_{dj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{d=1}^{D} c_d (y_b)_{dj}}.
\end{aligned}
\tag{31}
$$

If using model's (24) constraints resulted in the last fraction of inequality (31) being greater (less) than 0, then it would hold that $w_{1j} \geq w_{2j}$ ($w_{1j} \leq w_{2j}$), for all $j = 1, ..., N$. That would imply that the relative importance of the stages in the efficiency evaluation of DMU$_j$ would be biased, in favour of the first (second) stage. Nevertheless, the sign of the last fraction in inequality (31) varies among the different DMUs. Therefore, in this case, no stage is favoured against the other by definition. That means that endogenous decomposition weights can be used without incorporating arbitrary restrictions into the production process.

5 Decomposition weight properties in the two-stage process, under the VRS

Ang and Chen (2016) and Despotis et al. (2016) showed that under the CRS, the endogenous definition of the decomposition weights results in a non-increasing relationship between them, i.e. $w_{1j} \geq w_{2j}$, for $j = 1, ..., N$, for some series network structures. This results in giving higher priority to the first stage in the efficiency decomposition. To overcome this problem, Ang and Chen (2016) suggested the use of constant decomposition weights, while Despotis et al. (2016) introduced a novel overall efficiency composition approach. In the previous section, it was shown that for the network structure elaborated in this paper (see Figure 2), under the VRS, the non-increasing relationship between the endogenous decomposition weights ($w_{1j} \geq w_{2j}, j = 1, ..., N$) cannot be established. In this section, we are going to show that the same result holds for all the four types of two-stage series network structures under the VRS.

The four types of two-stage series structures of a DMU_j, $j = 1, ..., N$, are depicted in Figure 3. Let $x_{pj}, p = 1, ..., P$ be the inputs to the first stage, $(z_f)_{lj}, l = 1, ..., L$ the final outputs resulting from the first stage, $z_{qj}, q = 1, ..., Q$ the intermediate products, used as inputs to the second stage, $x_{dj}, d = 1, ..., D$ the external intermediate inputs to the second stage, and $y_{sj}, s = 1, ..., S$ the final outputs of the second stage.

Under the CRS, for the decomposition weights of structures of type 1 and 3, in the input-oriented, additive decomposition model, Ang and Chen (2016) showed that $w_{1j} \geq w_{2j}$, for all $j = 1, ..., N$. The relation between the decomposition weights for the four types of two stage structures, under the VRS, is investigated below.

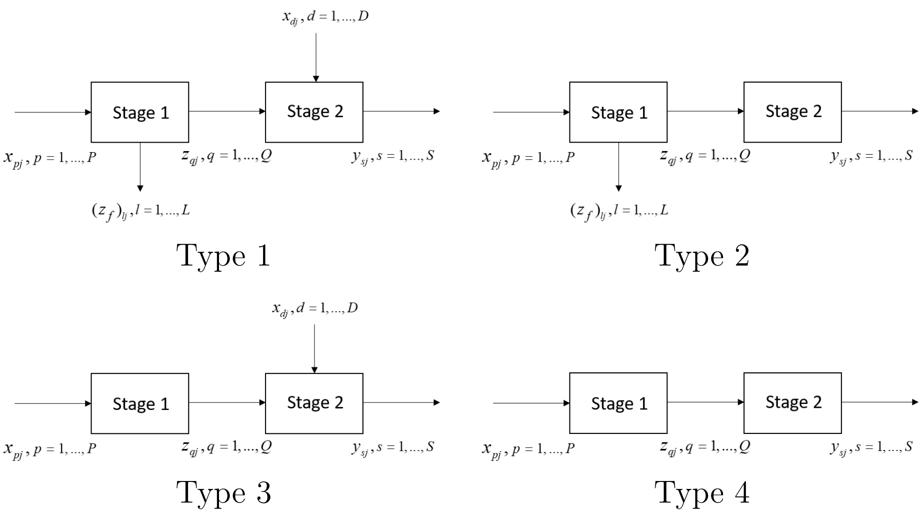

Figure 3: The four types of two-stage series structures

Type 1 structure:

Type 1 structure is the same as the one presented in the previous section (Figure 2), as in that case, undesirable outputs were treated as external inputs to the second stage. Therefore, it holds that

$$w_{1j} - w_{2j} \geq \frac{\sum_{l=1}^{L} \mu_l (z_f)_{lj} + u^1 - \sum_{d=1}^{D} c_d x_{dj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{d=1}^{D} c_d x_{dj}}. \tag{32}$$

Type 2 structure:

$$w_{1j} - w_{2j} = \frac{\sum_{p=1}^{P} \pi_p x_{pj} - \sum_{q=1}^{Q} \eta_q z_{qj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj}} \geq \frac{\sum_{l=1}^{L} \mu_l (z_f)_{lj} + u^1}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj}}. \tag{33}$$

Type 3 structure:

$$\begin{aligned} w_{1j} - w_{2j} &= \frac{\sum_{p=1}^{P} \pi_p x_{pj} - \sum_{q=1}^{Q} \eta_q z_{qj} - \sum_{d=1}^{D} c_d x_{dj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{d=1}^{D} c_d x_{dj}} \\ &\geq \frac{u^1 - \sum_{d=1}^{D} c_d x_{dj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj} + \sum_{d=1}^{D} c_d x_{dj}}. \end{aligned} \tag{34}$$

Type 4 structure:

$$w_{1j} - w_{2j} = \frac{\sum_{p=1}^{P} \pi_p x_{pj} - \sum_{q=1}^{Q} \eta_q z_{qj}}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj}} \geq \frac{u^1}{\sum_{p=1}^{P} \pi_p x_{pj} + \sum_{q=1}^{Q} \eta_q z_{qj}}. \tag{35}$$

In all four cases, the denominator of the final fraction is positive, whereas the sign of the nominator varies, depending on the values of the optimal weights and the value and sign of the scalar u^1.

Remark 5.1 *In the input-oriented additive decomposition model, under the VRS assumption, the order relation between the endogenously defined decomposition weights is not fixed, but it depends on the optimal input output mix and the first stage scalar (u^1) of each DMU_j, $j = 1, ..., N$.*

Hence, unlike the CRS case, under the VRS assumption, the decomposition weights can be defined endogenously without introducing any bias in the production process.

6 Empirical Study

6.1 Railway noise pollution in Europe

The most serious problem that railways cause to the environment is noise pollution. Notably in Europe, after road traffic noise, noise generated from railways is the second highest environmental health problem. According to 2017 estimations, about 22 million people were exposed to high levels of railway noise inside and outside urban areas (EEA

Report 22/2019, 2019).

Railway noise mainly comes from the wheel and rail vibrations, which are generated by the contact of the rolling wheel with the rail (Kitagawa, 2009, pg. 1). Bad rolling conditions originating from the poor maintenance of the rail lines or wheel flats, result in augmented noise levels. The braking technology that is used, also plays an important role; cast iron brake blocks corrugate the wheel surface resulting in higher rolling noise levels, while composite and sinter material blocks cause low roughness to the wheel, and thus, produce less noise (Pyrgidis, 2016, pg. 428-429). Until the mid 2000s, cast iron brakes was the only brake technology used in freight wagons, due to their lower cost. On the other hand, the disc brake technology that is used in passenger trains, which are usually high-speed trains, generates lower rolling noise levels (Thomson and Gautier, 2006, pg. 400). In this type of trains, aerodynamic noise seems to be the major problem (Thomson et al., 2015).

In contrast to passenger trains which are mainly operating during the day, the majority of freight wagons operate during the night hours, and are therefore considered, under the current brake technology, as the main source of noise pollution in Europe (ERA 006REC1072 Impact Assessment, 2018). In 2006, the technical specifications for interoperability (TSI) which were introduced by the European Commission (EC), set noise emission limits for new wagons and implicitly prohibited the use of cast iron blocks (Commission Decision 2006/66/EC, 2006). However, since the lifetime of wagons can be over 40 years, the renewal procedure would be very slow. Therefore, in 2008, the Commission announced new measures for noise emissions reduction, which suggested the retrofitting of the existing wagon fleet with composite brake blocks (Council Directive 2008/57/EC, 2008). This would result in up to 10dB noise reduction.

Depending on its duration and intensity, noise can affect human health, causing from mild problems such as annoyance, sleeping disturbances or stress to the body to more serious problems such as increased blood pressure, insomnia and risk for cardiovascular diseases. Environmental noise in the classroom - coming from the road, rail and air traffic - is also related to children cognitive impairment (Clark and Stansfeld, 2007; EEA, 2010).

The Environmental Noise Directive (END) (Council Directive 2002/49/EC, 2002) defined L_{den} indicator to be used as a threshold against which human exposure to environmental noise is monitored. L_{den} is defined as the yearly average sound pressure level during all days, evenings and nights, where evening sound pressure value has a penalty of 5dB and night value has a penalty of 10 dB, where dB is considered as an A-weighting scale, used to measure loudness corresponding to the frequencies that human ear can perceive. L_{den} is calculated by the following formula:

$$L_{den} = 10 \cdot log\frac{1}{24}\left((\text{day hours}) \cdot 10^{\frac{L_{day}}{10}} + (\text{evening hours}) \cdot 10^{\frac{L_{evening}+5}{10}} + (\text{night hours}) \cdot 10^{\frac{L_{night}+10}{10}}\right),$$

where L_{day}, $L_{evening}$, and L_{night} are the yearly average sound pressure levels over day, evening and night hours, respectively.

According to the END, EU member states should keep environmental noise at levels where $L_{den} \leq 55$ dB and $L_{night} \leq 50$ dB. According to the World Health Organization (WHO) guidelines, noise should not exceed 40 dB during night (WHO, 2009, pg. 108-109).

6.2 Railway model considering the impact of environmental noise

Railways is a capital-intensive industry that relies a lot on investments to maintain, improve and expand its assets, rolling stock and infrastructure, aiming to provide passenger and freight services of high quality and continue to be competitive in the modal market share. Therefore, from the operational perspective, the railway transport process is divided into two stages; the asset related and the services related. The first stage is related to the acquirement of rolling stock that satisfies EC standards and the development of a rail network with adequate line length. In the second stage, the quality of the services offered is evaluated by measuring the passenger and freight carriage as well as the impact that noise generated from the moving trains had on the population.

In this study, the infrastructure investment costs and the operating and maintenance costs are considered as inputs, and the length of operating lines, the number of total rail wagons and the number of wagons that are compliant with the noise standards specified in TSI, are regarded as outputs of the first stage. The length of the operating lines and the total number of wagons are then introduced as inputs to the second stage, to produce two desirable outputs, million-tonne freight-kilometres (MT-km) and million passenger-kilometres (M-km), and one undesirable output, the total number of people exposed to high levels of railway noise ($L_{den} \geq 55$ dB) inside and outside urban areas (see Figure 4). The passenger-km and tonne freight-km are calculated as the total distance travelled divided by the number of passengers or tonnes of freight carried and represent the transport of one passenger or one tonne of freight, respectively, over one kilometre.

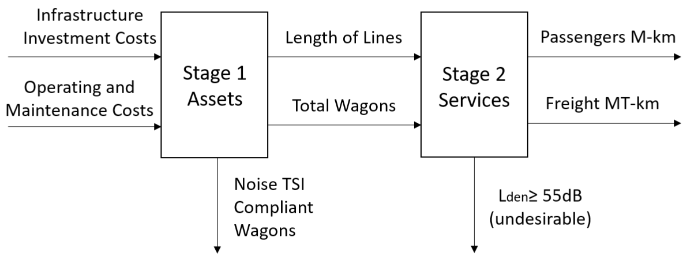

Figure 4: Railways model structure

This study assesses the environmental efficiency of railway systems in 22 European countries in 2016-2017. During that time, 20 of the countries under investigation were members of the European Union (EU) - United Kingdom left the EU on 31st January 2020. Switzerland and Norway are also included in the dataset, as they belong to the Schengen area. Concerning the rest of the EU members not included in this study, some of them have missing data and some others, such as Malta and Cyprus, have no railway system.

Data provided in Table 1, were collected from various sources. Infrastructure investment and operational and maintenance costs were extracted from the 2019 European Commission report (van Essen et al., 2019, pg. 153) and are both measured in billion euros. The number of new and retrofitted wagons which are compliant with the TSI, as well as the total number of wagons in each country, were found in the 2018 report of the European Union Agency for the Railways (ERA) on the noise TSI (ERA 006REC1072 Impact Assessment, 2018, pg. 23). The length of operating lines, passenger M-km and freight MT-km were extracted from the Eurostat (2016) database. The number of people exposed to noise levels higher than those established by the END as acceptable was found in Noise Country Fact Sheets 2019, in the European Environment Agency (EEA) webpage (EEA, 2020). It should be noted that until 1st January 2019 - when common noise assessment methods (CNOSSOS-EU) started to be applied by all the EU member states - each country was using its own methods for noise pollution measurement, which involved the use of different parameters to capture meteorological conditions, ground absorption or population assignment to buildings (EEA Report 22/2019, 2019). All variables refer to 2016 measurements, except for the last one, which refers to 2017, since noise pollution impact measurements took place in 2007, 2012 and 2017.

The non-parametric Spearman correlation analysis (Table 2) indicates that there is a positive relationship between the input and output variables. This can be interpreted as that an increase in the amount of inputs consumed results in a certain increase in the amount of outputs produced.

Because the efficiency scores calculated with the DEA methodology are relative and not absolute measures of the performance of a DMU, as the number of inputs and outputs increases, the discrimination power of the model diminishes and a number of inefficient DMUs may be falsely rated as efficient. There are several rules of thump in the DEA literature for relating the number of inputs and outputs to the sample size (Charles et al., 2019). Following the threshold $N \geq max\{p \times q \times s, 3(p+q+s)\}$, where N is the number of observations, p is the number of inputs, q the number of the first stage outputs, and s the number of second stage outputs, and given that in this case, due to data unavailability, the number of observations is not possible to be increased, there is a need to reduce the model dimensions. Here, the aggregation of the infrastructure investment costs and operation and maintenance costs by simple addition is applied, since these two variables are both measuring different types of costs and are highly correlated (Podinovski and Thanassoulis, 2007).

According to Peterson (1996), among the western European railway companies included in the study, the smallest operators showed increasing returns to scale (IRS), the medium sized operators showed CRS and the largest operators showed decreasing returns to scale (DRS). Since there are CRS, IRS, and VRS operators, the VRS DEA model was adopted in this study to provide a more equitable efficiency analysis, regardless of the size of railway operators[1].

Table 1: Data Set

	DMU	Invest. Costs (bn €)	O&M Costs (bn €)	TSI Wagons	Total Wagons	Length of Lines (km)	Freight MT-km	Pass. M-km	$L_{den} \geq$ 55 dB
1	Austria	2.61	1.66	6511	23345	5491	21361	12497	1081900
2	Belgium	1.78	0.38	2312	12013	3607	0	10025	324400
3	Bulgaria	0.30	0.25	568	16915	4029	3434	1455	42300
4	Croatia	0.19	0.30	383	2274	2604	2160	827	26400
5	Czech Rep.	1.35	1.36	8000	42199	9564	15619	8738	268500
6	Denmark	0.39	0.13	225	366	2045	2616	6332	84300
7	Estonia	0.06	0.14	0	20849	1161	2340	316	6100
8	Finland	0.41	0.18	200	9942	5926	9456	3868	119400
9	France	5.09	3.67	8558	77678	28364	32569	90612	3780000
10	Germany	7.74	3.92	59626	165653	38623	126686	95465	6390500
11	Ireland	0.16	0.21	100	254	1931	101	1991	42600
12	Latvia	0.11	0.17	0	11888	1860	15873	584	40600
13	Lithuania	0.22	0.31	0	14828	1911	13790	280	11600
14	Netherlands	2.73	1.02	9000	21226	3058	6641	17483	312500
15	Poland	3.50	0.69	2750	83500	19132	50650	19067	419700
16	Portugal	0.71	0.26	3123	3313	2546	2774	4266	137100
17	Slovenia	0.23	0.18	226	3230	1209	4360	611	47600
18	Spain	5.23	0.73	6781	20833	16167	10550	26646	69300
19	Sweden	1.07	0.45	931	11000	10882	21406	12800	549400
20	UK	6.46	3.45	15467	18246	16253	17053	68010	1709400
21	Norway	0.52	0.48	516	1623	3895	3312	3695	123400
22	Switzerland	2.50	1.58	19236	21200	3650	12447	20657	482400

Table 2: Non-parametric Spearman correlation matrix

	Invest. Costs	O&M Costs	TSI Wagons	Total Wagons	Length Lines	Freight MT-km	Pass. M-km	$L_{den} \geq$ 55 dB
Invest. Costs	1.000							
O&M Costs	0.864	1.000						
TSI Wagons	0.903	0.882	1.000					
Total Wagons	0.657	0.707	0.645	1.000				
Length Lines	0.840	0.766	0.715	0.586	1.000			
Fr. MT-km	0.597	0.637	0.479	0.689	0.674	1.000		
Pass. M-Km	0.951	0.810	0.888	0.591	0.831	0.561	1.000	
$L_{den} \geq$ 55dB	0.862	0.801	0.822	0.536	0.764	0.616	0.886	1.000

[1]For a review of the different models used to assess the efficiency of railways under different returns to scale assumptions see Mahmoudi et al. (2020).

6.3 Results

In the model described above, the first stage measures the performance of European countries in building and maintaining their railway infrastructure and rolling stock, while the second stage efficiency measures their performance in providing passenger and freight services considering the less possible environmental noise impact on humans. The efficiency of the whole process and the two sub-processes is evaluated using the additive decomposition methodology elaborated in the previous section while assuming that railways operate under the VRS technology.

Through the optimisation process, for five countries, namely France, Lithuania, Poland, Spain and the UK, the optimal decomposition weights take values $w_1^* = 0$ and $w_2^* = 1$, whereas for Portugal and Switzerland it results that $w_1^* = 1$ and $w_2^* = 0$. That means that for these countries, one of the two stages' contribution to the overall process is ignored. Therefore, the decomposition weight restrictions given in remark 4.1 are incorporated into model (24).

Sensitivity analysis of the overall efficiency scores was performed for different given values of the lowest allowed level κ of decomposition weights, i.e. $w_{ij}, w_{2j} \geq \kappa$, $\kappa \in S$, $S = \{0.01, 0.02, 0.03, ..., 0.48, 0.49, 0.5\}$. Results reveal that although for some countries overall efficiency has a very slight downward tendency as κ increases, for the majority of countries, overall efficiency scores and optimal decomposition weights are generally stable. Furthermore, for all countries, stage efficiency scores are stable.

Efficiency scores start to be more sensitive to changes of κ, as κ exceeds some threshold, and they are completely destabilised when $\kappa = 0.5$. For $\kappa = 0.5$, Austria and the Netherlands show infeasibility in the stage efficiency models.

Rankings based on the overall efficiency score seem not to be significantly affected for most of the countries, even for large values of κ. Portugal, Switzerland, Latvia, UK, Bulgaria and Finland are the most sensitive to weight restrictions. Estonia and Germany are overall efficient for all $\kappa \in S$, and Poland is overall efficient for all $\kappa \in S \backslash \{0.47, 0.48, 0.49, 0.5\}$. For space-saving, rankings of the countries for half of the κ values - with κ step change being 0.02 - are given in Table 3.

Table 3: Overall efficiency rankings for different decomposition weight restrictions

κ \ DMU	Austria	Belgium	Bulgaria	Croatia	Czech Rep.	Denmark	Estonia	Finland	France	Germany	Ireland	Latvia	Lithuania	Netherl.	Poland	Portugal	Slovenia	Spain	Sweden	UK	Norway	Switzer.
0.02	21	22	15	17	16	12	1	14	4	1	13	8	6	19	1	10	18	5	11	7	20	9
0.04	21	22	15	17	16	12	1	14	4	1	13	7	6	19	1	10	18	5	11	8	20	9
0.06	21	22	15	17	16	12	1	14	4	1	13	5	7	19	1	11	18	6	10	8	20	9
0.08	21	22	15	17	16	12	1	14	4	1	13	5	7	19	1	11	18	6	10	8	20	9
0.10	21	22	15	17	16	12	1	14	4	1	13	5	7	19	1	11	18	6	10	8	20	9
0.12	21	22	15	17	16	12	1	14	4	1	13	5	7	19	1	11	18	6	10	8	20	9
0.14	21	22	15	17	16	12	1	14	4	1	13	5	7	19	1	11	18	6	10	8	20	9
0.16	21	22	15	17	16	12	1	14	4	1	13	5	7	19	1	11	18	6	10	8	20	9
0.18	21	22	15	17	16	11	1	14	4	1	13	5	7	19	1	12	18	6	10	8	20	9
0.20	21	22	15	17	16	11	1	14	5	1	13	4	7	19	1	12	18	6	10	8	20	9
0.22	21	22	15	17	16	11	1	14	5	1	13	4	7	19	1	12	18	6	10	8	20	9
0.24	21	22	15	17	16	11	1	14	5	1	13	4	7	19	1	12	18	6	9	8	20	10
0.26	21	22	15	17	16	11	1	14	5	1	12	4	7	19	1	13	18	6	9	8	20	10
0.28	21	22	15	17	16	11	1	14	5	1	12	4	7	19	1	13	18	6	9	8	20	10
0.30	21	22	15	17	16	10	1	14	5	1	12	4	7	19	1	13	18	6	9	8	20	11
0.32	21	22	14	17	16	10	1	13	5	1	12	4	7	19	1	15	18	6	8	9	20	11
0.34	21	22	14	17	16	10	1	13	5	1	11	4	7	19	1	15	18	6	8	9	20	12
0.36	21	22	13	17	16	10	1	12	5	1	11	4	7	19	1	15	18	6	8	9	20	14
0.38	21	22	13	17	16	10	1	12	5	1	11	4	7	19	1	15	18	6	8	9	20	14
0.39	21	22	13	17	16	9	1	12	5	1	11	4	8	19	1	15	18	6	7	10	20	14
0.40	21	22	13	17	16	9	1	12	5	1	11	4	8	19	1	15	18	6	7	10	20	14
0.42	21	22	13	16	14	9	1	12	5	1	11	4	8	19	1	17	18	6	7	10	20	15
0.44	21	22	13	15	14	9	1	12	5	1	11	4	8	19	1	17	18	6	7	10	20	16
0.46	21	22	13	15	14	9	1	12	5	1	11	4	8	19	1	17	18	6	7	10	20	16
0.48	21	22	12	15	14	9	1	10	5	1	13	4	8	19	3	17	18	6	7	11	20	16
0.50	21	22	11	15	14	9	1	10	5	1	13	3	7	19	4	17	18	6	8	12	20	16

Portugal and Switzerland are the only countries in the set, for which, for all the different weight restrictions, efficiency decomposition is not unique and changing the priority stage yields different stage efficiency scores. As κ increases and restrictions on the decomposition weights become more severe, the optimisation process is forced to assign greater optimal values to some decomposition weights. Therefore, the optimal values of the decomposition weights tend to coincide with the values of κ and $1-\kappa$ for a growing number of countries. In this analysis, this upturn starts happening for $\kappa \geq 0.14$. Therefore, above that threshold, for some countries the relative contribution of each stage to the overall process is forced to change. However, for most countries, this does not affect their rankings significantly. Nevertheless, as κ increases, the number of countries for which it is not possible to have a unique efficiency decomposition raises. For example, four countries do not have unique efficiency decomposition for $\kappa = 0.2$, and ten countries for κ=0.5.

For the cases when decomposition weight restrictions are needed, there is no rule for choosing a value for κ and the choice depends on what the managerial preferences are.

Large values of κ may be too restrictive, impacting on the efficiency scores and sometimes resulting in infeasibility problems. Therefore, there is a range of smaller κ values for which efficiency scores and optimal decomposition weight values are not significantly affected. That means that for this range of κ values, efficiency scores show low volatility.

In order to deduce the sensitivity threshold, a volatility measure of the overall efficiency scores was evaluated as follows[2]:

1. We calculate the overall efficiency scores of each DMU$_j$, $j = 1, ..., N$, for all $\kappa \in S$
2. For a small $r \in \mathbb{Z}^+$ we calculate the volatility index as the sum of the standard deviations of the overall efficiency scores of each DMU, i.e. $V_\kappa = \sum_{j=1}^{N} sd\{\theta^{0*}_{j,\kappa-r}, ..., \theta^{0*}_{j,\kappa+r}\}$
3. We choose the range of κ values which minimise V_κ.

Here, the above algorithm is repeated for $r = 1, 2, 3, 4$. Lower values of the volatility index indicate greater stability of the efficiency scores. The resulting volatility indices are presented in Table 5 in the Appendix. As it is shown in Table 5, the volatility index is low and stable for all $\kappa \leq 0.15$ for all $r = 1, 2, 3, 4$. Volatility increases for $\kappa \geq 0.19$, $\kappa \geq 0.18$, $\kappa \geq 0.17$ and $\kappa \geq 0.16$ for $r = 1, 2, 3, 4$ respectively. Therefore, $\kappa = 0.15$ is deduced as an overall sensitivity threshold in this analysis.

However, if κ is too small, for some DMUs one stage will be assigned a very low contribution to the overall process. There is a range of κ values which are not too restrictive, but also ensure that no stage will be ignored. Table 4 shows the overall and stage efficiency scores, as well as the decomposition weights for the case when $\kappa = 0.1$, where the exponent p indicates the priority stage. By imposing $w_{1j}, w_{2j} \geq 0.1$, we prevent one of the two stages to undertake the weight of the whole process, and secondly, $\kappa = 0.1$ lies below the defined sensitivity threshold.

According to the results, four countries, Estonia, Finland, Germany and Poland are first-stage efficient. These countries are also efficient in the second stage, except for Finland which shows a relatively low performance in the second stage. In total, 11

[2]This algorithm is based on the algorithm suggested by Politis et al. (2001) for the selection of subsample size when applying subsampling bootstrap.

out of 22 countries are efficient in the second stage - without including Switzerland. These countries constitute half of the sample, which seems to be a great difference to the number of first-stage efficient countries. However, performing Wilcoxon signed-rank test for the efficiency scores of the two stages, we fail to reject the null hypothesis that the scores of the two stages do not differ significantly, for any level of significance. Also, the Spearman correlation between the stage efficiency scores is zero, indicating that an increase (decrease) in one stage's efficiency score does not imply an increase (decrease) in the other stage's score.

Table 4: Efficiency scores and optimal decomposition weights, when $w_{1j}, w_{2j} \geq 0.1$

	DMU	θ^{0*}	w_{1j}^*	w_{2j}^*	θ^{1p*}	θ^{2*}	θ^{1*}	θ^{2p*}
1	Austria	0.4336	0.6219	0.3781	0.2748	0.6949	0.2748	0.6949
2	Belgium	0.3974	0.6892	0.3108	0.3332	0.5397	0.3332	0.5397
3	Bulgaria	0.7038	0.6206	0.3794	0.9088	0.3685	0.9088	0.3685
4	Croatia	0.6653	0.7025	0.2975	0.7270	0.5195	0.7270	0.5195
5	Czech Rep.	0.6673	0.6950	0.3050	0.7356	0.5116	0.7356	0.5116
6	Denmark	0.7629	0.5592	0.4408	0.5760	1	0.5760	1
7	Estonia	1	0.4310	0.5690	1	1	1	1
8	Finland	0.7051	0.5351	0.4649	1	0.3657	1	0.3657
9	France	0.9882	0.1000	0.9000	0.8822	1	0.8822	1
10	Germany	1	0.8531	0.1469	1	1	1	1
11	Ireland	0.7313	0.6844	0.3156	0.7373	0.7181	0.7373	0.7181
12	Latvia	0.9773	0.2785	0.7215	0.9186	1	0.9186	1
13	Lithuania	0.9493	0.1000	0.9000	0.4932	1	0.4932	1
14	Netherlands	0.5480	0.8053	0.1947	0.4387	1	0.4387	1
15	Poland	1	0.4198	0.5802	1	1	1	1
16	Portugal	0.7873	0.9000	0.1000	0.8250	0.4480	0.7995	0.6769
17	Slovenia	0.6600	0.8027	0.1973	0.5764	1	0.5764	1
18	Spain	0.9559	0.1000	0.9000	0.5594	1	0.5594	1
19	Sweden	0.8058	0.3903	0.6097	0.8628	0.7692	0.8628	0.7692
20	UK	0.9380	0.1000	0.9000	0.3804	1	0.3804	1
21	Norway	0.4676	0.7176	0.2824	0.4818	0.4317	0.4818	0.4317
22	Switzerland	0.8936	0.9000	0.1000	0.9552	0.3398	0.8818	1

Bulgaria, Croatia, Czech Republic, Finland, Portugal, Sweden and Switzerland - considering the first stage as priority stage - have significantly lower second stage efficiency score. To investigate whether their lower second stage efficiency is due to the number of people affected by noise, L_{den} variable is excluded from the model. In this case, Switzerland and the Netherlands show infeasibility when the first stage is considered as the priority stage. According to the results (see Table 6 in the Appendix for the case when

$w_{1j}, w_{2j} \geq 0.1$), overall efficiency scores are lower for all countries except for Estonia and Germany, which remain overall efficient. However, it is not possible to extract a safe conclusion about whether this happens because countries perform relatively well in terms of the number of people affected by railway noise or because the reduction of the model dimensions results in increasing its discrimination power. Similarly, for the majority of the countries, the second stage efficiency scores are the same or lower than those when L_{den} is included in the model. Austria and Belgium are the only countries whose second stage efficiency increases when L_{den} variable is omitted.

6.4 Policy Implications

Based on the optimal decomposition weights obtained, it is possible to specify which stage is of the highest relative importance for each country. In other words, the optimal decomposition weights can be used by the countries included in the data set as guidance about defining the optimal portion of inputs that they should devote to each stage.

According to the results, 13 out of 22 countries included in the study, namely Portugal, Switzerland, Germany, the Netherlands, Slovenia, Norway, Croatia, Czech Republic, Belgium, Ireland, Austria and Bulgaria, should give more importance to their assets investment, operation and maintenance to improve their efficiency, since, for these countries, the contribution of the first stage to the overall process is higher.

On the other hand, railway industries in France, Spain, the UK and Lithuania, should focus their operation management almost completely on the services they provide, aiming to optimise their freight and passenger carriage, while reducing its noise effects on the environment. Railway operation in Latvia and Sweden should also be more services-related, while Denmark, Finland, Estonia and Poland should give approximately the same balance in their assets and services operation.

Considering the interoperability framework in which European railways operate, to limit the railway noise pollution problem and improve environmental efficiency, changes and measures should be planned and adopted in a cross-country context. Abatement of the railway noise sources in a single country would not resolve the problem and could even harm the competitiveness of railways against other means of transport. Therefore, the common standards set by the European Commission through the Directives can help in this direction. Cooperation and exchange of expertise among the European countries could further foster efficiency improvement of the railway sector.

Furthermore, in reducing railway noise, countries should also focus both on the good maintenance of rail tracks and the increase of the number of wagons that are compliant with the EC standards to achieve the maximum possible noise reduction.

The multiplier model, which was formulated in previous sections, is used to calculate the efficiency scores of DMUs. In NDEA, it is also possible to provide targets for the input/output variables of each DMU by solving the envelopment form of the model, which is based on the PPS. However, this study focuses on the efficiency evaluation of European railways, and the formulation of the envelopment model is beyond its scope.

7 Conclusion

Railways have unarguably many advantages, such as higher safety, less energy consumption, less pollution and less traffic congestion, compared to other means of transport. While recognising that the development and maintenance of railways should be given priority, it is vital to take into consideration the impact that railways have on the environment in order to be able to mitigate it. Acknowledging that noise pollution is a major environmental problem caused by railways, this paper focused on incorporating it in the efficiency evaluation of the railway transport process.

The railway industry is capital-intensive, and its purpose is to optimise its passenger and freight services. For this reason, the railway transport process was divided into two stages, assets and services. The problem of noise pollution is linked to both stages. In the asset stage, good maintenance of the rail lines and retrofitting of the rail wagons with more silent, composite brake technology can mitigate the noise generation. On the other hand, high-quality railway services should entail the minimisation of the number of people affected by railway noise. Therefore, both these factors were taken into account when building the model.

We extended the NDEA additive decomposition approach to account for intermediate and undesirable outputs. This allowed us to have a better insight into the railways' operation, detect which part of the production process is the main source of inefficiency, and which stage has the highest relative importance for each country.

The performance of railways in 22 European countries during 2016-2017 was studied since the railways' pollution problem seemed to be more significant in this area. The asset, services and overall efficiency scores obtained, revealed that there was no significant difference in the performance of European railways in total, between the two stages. An interesting result is also that, except for Finland, countries which show efficient performance in the asset stage are also efficient in services provision. However, although many countries seemed to be efficient in the second stage, they got a low asset efficiency score, indicating that the inverse relationship did not hold.

The overall efficiency rankings were not significantly affected by imposing different constraints on the decomposition weights of each stage. Consequently, changing the relative importance of each stage, in general, did not affect its relative performance significantly.

A limitation of this study is that due to data unavailability, the collected variables refer to consecutive years, and this has probably affected the accuracy of the results reported. Furthermore, due to missing data, some European countries were not included in the data set. Since DEA provides relative efficiency measurement, the inclusion or omission of DMUs impacts the efficiency scores of the sample. Therefore, the obtained efficiency scores can only be indicative of the real noise-pollution picture in European railways, as the complete data set of European countries would be needed to have a more accurate efficiency measurement.

This research can be extended by using DEA models to study the railway noise pollution problem in different regions other than Europe. Furthermore, a future study could distinguish between the noise generated by the passenger high-speed trains, and freight wagons or between the impact that railway noise has inside and outside urban areas. Finally, another future research may consider the impact of railway noise on wildlife.

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Appendix

Table 5: Volatility indices for the overall efficiency scores

	V_κ			
κ	r=1	r=2	r=3	r=4
0.01	-	-	-	-
0.02	0.02677	-	-	-
0.03	0.02677	0,04233	-	-
0.04	0.02677	0.04233	0.05784	-
0.05	0.02677	0.04233	0.05784	0.07332
0.06	0.02677	0.04233	0.05784	0.07332
0.07	0.02677	0.04233	0.05784	0.07332
0.08	0.02677	0.04233	0.05784	0.07332
0.09	0.02677	0.04233	0.05784	0.07332
0.10	0.02677	0.04233	0.05784	0.07332
0.11	0.02677	0.04233	0.05784	0.07332
0.12	0.02677	0.04233	0.05784	0.07332
0.13	0.02677	0.04233	0.05784	0.07332
0.14	0.02677	0.04233	0.05784	0.07332
0.15	0.02677	0.04233	0.05784	0.07332
0.16	0.02677	0.04233	0.05784	0.07402
0.17	0.02677	0.04233	0.05863	0.07590
0.18	0.02677	0.04328	0.06073	0.07837
0.19	0.02799	0.04568	0.06343	0.08152
0.20	0.03076	0.04861	0.06679	0.08501
0.21	0.03313	0.05189	0.07025	0.08857
0.22	0.03466	0.05437	0.07335	0.09201
0.23	0.03547	0.05569	0.07564	0.09498
0.24	0.03565	0.05651	0.07696	0.09710
0.25	0.03606	0.05695	0.07780	0.09877
0.26	0.03636	0.05738	0.07875	0.10020
0.27	0.03647	0.05822	0.07986	0.10183
0.28	0.03727	0.05915	0.08139	0.10451
0.29	0.03817	0.06060	0.08401	0.10747
0.30	0.03923	0.06326	0.08690	0.11042
0.31	0.04167	0.06576	0.08964	0.11324
0.32	0.04344	0.06782	0.09186	0.11582
0.33	0.04381	0.06903	0.09364	0.11816
0.34	0.04381	0.06941	0.09507	0.12042
0.35	0.04404	0.07007	0.09624	0.12344
0.36	0.04493	0.07123	0.09882	0.12848
0.37	0.04602	0.07416	0.10432	0.13448
0.38	0.04895	0.08015	0.11065	0.14099
0.39	0.05523	0.08615	0.11691	0.14779
0.40	0.05894	0.09112	0.12280	0.15471
0.41	0.05940	0.09439	0.12823	0.16276
0.42	0.06059	0.09649	0.13406	0.17116
0.43	0.06270	0.10101	0.13990	0.18142
0.44	0.06707	0.10686	0.14910	0.19375
0.45	0.07175	0.11558	0.16147	0.20728
0.46	0.07844	0.12697	0.17421	0.22148
0.47	0.08775	0.13726	0.18658	-
0.48	0.09380	0.14618	-	-
0.49	0.09634	-	-	-
0.50	-	-	-	-

Table 6: Efficiency scores and optimal decomposition weights, when L_{den} is omitted from the model, and $w_{1j}, w_{2j} \geq 0.1$

	DMU	θ^{0*}	w_{1j}^*	w_{2j}^*	θ^{1p*}	θ^{2*}	θ^{1*}	θ^{2p*}
1	Austria	0.4204	0.7703	0.2297	0.2748	0.9089	0.2748	0.9089
2	Belgium	0.3655	0.8872	0.1128	0.3328	0.6226	0.3328	0.6226
3	Bulgaria	0.6985	0.6430	0.3570	0.9081	0.3210	0.9081	0.3210
4	Croatia	0.6650	0.7123	0.2877	0.7421	0.4742	0.7421	0.4742
5	Czech Rep.	0.6431	0.7890	0.2110	0.7356	0.2970	0.7356	0.2970
6	Denmark	0.6490	0.7700	0.2300	0.5735	0.9016	0.5735	0.9016
7	Estonia	1	0.9000	0.1000	1	1	1	1
8	Finland	0.7006	0.5678	0.4322	1	0.3072	1	0.3072
9	France	0.9474	0.4462	0.5538	0.8822	1	0.8822	1
10	Germany	1	0.9000	0.1000	1	1	1	1
11	Ireland	0.7261	0.7161	0.2839	0.7339	0.7065	0.7339	0.7065
12	Latvia	0.9473	0.6478	0.3522	0.9186	1	0.9186	1
13	Lithuania	0.5885	0.7721	0.2279	0.4932	0.9117	0.4932	0.9117
14	Netherlands	0.4454	0.9000	0.1000	0.5146	-0.1774	0.3838	1
15	Poland	0.8495	0.4829	0.5171	1	0.7089	1	0.7089
16	Portugal	0.7826	0.9000	0.1000	0.8250	0.4008	0.7943	0.6769
17	Slovenia	0.6537	0.8174	0.1826	0.5764	1	0.5764	1
18	Spain	0.4528	0.6516	0.3484	0.4851	0.3926	0.4851	0.3926
19	Sweden	0.7696	0.4717	0.5283	0.8628	0.6863	0.8628	0.6863
20	UK	0.5788	0.6567	0.3433	0.3586	1	0.3586	1
21	Norway	0.4620	0.7717	0.2283	0.4808	0.3985	0.4808	0.3985
22	Switzerland	0.7207	0.9000	0.1000	0.9552	-1.3899	0.6896	1