Disorder-induced phase transitions in a spinful one-dimensional system

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Abstract

We analyse a modified set of renormalisation group equations for disordered spinful fermions described by the Luttinger liquid model. The modification is necessary to take special care of the factitious admixture of the disorder to the interaction coupling constants undergoing renormalisation. Only properly separated amplitudes of elastic and inelastic processes allow the identification of true phases and the construction of the phase diagram (a similar procedure has been earlier implemented for the spinless case). In the spinful case, these modified equations enable us to demonstrate that in some region of the bare parameters values the phase diagram contains two massive phases, charge (CDW) and spin (SDW) density waves, which are separated by an insulating phase. These gapped phases are achieved at finite critical temperatures that vanish at the phase boundaries indicating the presence of a disorder-induced quantum phase transition. The critical temperatures as a function of disorder are reasonably well fit by a stretch exponential with the universal stretching critical exponent $\nu = 1/3$. A quantum phase transition between CDW and SDW phases driven by disorder strength has not been predicted before and this observation must be taken into account when analysing recent multiple experiments on phase transitions in quasi-one-dimensional structures.

Keywords: Quantum Phase Transitions, Disorder Induced, One-Dimensional

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1. Introduction

The phase transitions that occur in high-dimensional homogeneous systems are those between states with different symmetries and can be described by an order parameter which emerges below the transition temperature. The continuous phase transitions are described by an order parameter and characterised by non-analytic behaviour of susceptibilities, correlation length etc. The non-analyticity of various observables are related to each other. Those relations define critical exponents that are universal within each class of systems. The universality class is uniquely related to the set of critical exponents [1, 2, 3] and, therefore the knowledge of the critical exponents as a function of the parameters of a system must, in principle, allow a construction of a complete phase diagram (e.g. [4, 5, 6]).

Not all transitions break symmetry and not all systems obey power law non-analyticity. Whenever a mean field description breaks down, one may expect a different singularity to emerge. This typically happens in low-dimensional translation invariant or disordered systems. For example, the two-dimensional Kosterlitz-Thouless transition is of infinite order and does not break a symmetry, and the Ising model in two dimensions has a logarithmic divergence.

In one-dimensional disordered systems fluctuations are so strong that no long-range order (leading to a formation of an order parameter) can exist. These phases are not defined by the order parameter but by the most relevant (slowest decaying) correlations. The transition between phases are rather crossovers but they happen around characteristic temperatures which are loosely called critical. On the other hand, the one-dimensional quantum phase transitions (at zero temperature) are sharp transitions with boundaries that can be located by the analysis of scaling dimensions of the most relevant operators. The theory of quantum phase transitions [7, 8] is a rich area of physics due to the fascinating phenomena that can be extracted, particularly in the low dimensional regime [9].

In low-dimensional systems, the most relevant (in the renormalisation group (RG) sense) interaction occurs if a gap opens, which defines a formation of the phase. Interactions between spinful electrons [10, 11] may lead to formation of spin density wave (SDW) or charge density wave (CDW) phases. Both spin gapped phases occurs due to repulsion (SDW) or attraction (CDW) between opposite spins. For a SDW [13] phase, the densities of up- and down-spin electrons are shifted creating spacial modulation of the spin density. In the CDW [12] two spin densities compensate each other. These spin gapped phases can drastically change how a system responses to external perturbations, so determining and controlling the emergence of these phases is of practical interest for low-dimensional technologies.

One of the most disruptive effects on the phases and transitions between them are caused by disorder. Its effect is drastically enhanced in low dimension, resulting in disorder being used as a controllable mechanism to determine the modification of quantum effects [14]. Subsequently, disorder induced phase transitions are an interesting scenario to probe, particularly within the framework of superconductivity [15, 16].

The standard approach to critical low-dimensional systems is based on analysis of the RG equations written for coupling constants describing different interaction types present in the system. The most relevant in RG sense coupling constant (the one with the lowest scaling dimension) defines tendency towards formation of a particular phase. If the corresponding perturbation is relevant, i.e. its scaling dimension is below the physical dimension, the new phase is formed at a low temperature (infrared limit) when a small dimensionless bare coupling constant renormalises and becomes of the order of one.

Strongly correlated one-dimensional electrons are described by the Luttinger liquid model which is exactly solvable for arbitrary interaction strength. The Luttinger liquid affected by a disordered potential is not solvable but is known to be described by the set of the RG equations derived long ago [18]. These equations have been analysed in the original paper [18] as well as in the textbook [9] and many followed research papers. One subtlety of these equations which was recognised in the original paper [18] is that during the RG procedure it was unavoidable to introduce artificial inelastic process which must be removed at the end of the calculation. The authors [18] provided the recipe and verified its consistency with a weak coupling limit. Later this procedure separating inelastic admixture to elastic processes has been implemented in [19] to analyse phase diagram of spinless electrons with arbitrary strength of forward scattering electron-electron interactions. This modification was necessary to identify the true phase diagram of the disordered Luttinger liquid because only values of true amplitudes of various processes define genuine phase boundaries.

In a real system, under the presence of the spin degree of freedom, the phase diagram even of a clean Luttinger liquid becomes quite rich. The effect of disorder complicates the phase diagram by introducing one extra phase - an insulator brought by localisation associated with imperfections. Theoretical analysis of the original RG equations actually claimed the reduction of possible phases - clean Luttinger liquid, insulator and CDW [18]. Multiple recent experiments on quasi-one-dimensional strongly correlated structures suggest that the situation looks more complicated and the spin degrees of freedom play an important role. To analyse the phase diagram of a disordered spinful Luttinger liquid, there is a necessity to modify the original equations, in a spirit of [19], to separate the admixture of an inelastic process from the elastic and identify phases by renormalising the true coupling constants responsible for different mechanisms.

In this paper, we perform the procedure described above and numerically analyse the complete set of the RG equations for the true coupling constants. In particular, we will scan various domains of multi-dimensional parametric space of all values of bare parameters, to find whether disorder may induce phase transitions between distinct conducting phases as it was observed in [15, 16]. We will find that in some region of parameters a clean Luttinger liquid phase is unstable, and two gapped conducting phases, SDW and CDW, emerge with their boundaries being extremely sensitive to the disorder. This result, not obtained earlier within the original RG equations, demonstrates the phase transitions in a spinful one-dimensional system in the presence of interactions induced by disorder. We find the universal critical exponent $\nu = 1/3$, which describes the dependence of different transition temperatures on the disorder. We study how the state of the system depends on temperature, disorder, spin and charge interactions, and construct corresponding phase diagrams. These results unambiguously show disorder-driven phase transitions at finite temperatures and allow us to speculate also on quantum phase transitions, when the phases change entirely due to disorder, at zero temperature implying the phase transition is of a quantum nature.

2. Theoretical Framework

The Tomonaga-Luttinger liquid (TLL) describes physics in one-dimension (1D) [9]. The beauty of one-dimensional physics lies in the analytical framework with which it can be treated. Furthermore, the one-dimensional world is becoming readily accessible experimentally, and the relevance in condensed matter physics is only increasing. This physics depends strongly on both temperature and disorder. This work is motivated by incredible experimental results [16], demonstrating the emergence of disorder-enhanced superconductivity in quasi-one-dimensional superconductor. We will leave the treatment of superconductivity for the future research, and study a phase diagram of a one-dimensional system, depending on disorder and various interactions.

We consider 1D fermions with a spin, which plays a crucial role in the phenomenon we study. We follow a renormalisation group (RG) approach developed in Refs. [18, 19, 20]. These equations are written for six parameters, one of them is the disorder strength, D, and the other five describe the Luttinger liquid: two Luttinger parameters, $\tilde{K}_\rho$ and $\tilde{K}_\sigma$, accounts for the strengths of forward-scattering electron-electron interactions in two channels, charge and spin, with another two being velocities, $\tilde{u}_\rho$ and $\tilde{u}_\sigma$, of plasmon-like excitations in those two channels. The interaction strength $\tilde{y}$ is the amplitude of the interaction-induced backscattering accompanied by a flip of the particle spins. We present the set of equations only for the convenience of the reader:

$$\frac{d\tilde{K}_\rho}{dl} = -\frac{\tilde{u}_\rho}{2\tilde{u}_\sigma}\tilde{K}_\rho^2 D \tag{1}$$

$$\frac{d\tilde{K}_\sigma}{dl} = -\frac{1}{2}\tilde{K}_\sigma^2\tilde{y}^2 - \frac{1}{2}\tilde{K}_\sigma^2 D \tag{2}$$

$$\frac{d\tilde{y}}{dl} = 2(1-\tilde{K}_\sigma)\tilde{y} - D \tag{3}$$

$$\frac{dD}{dl} = -\left(\tilde{K}_\rho + \tilde{K}_\sigma + \tilde{y} - 3\right) D \tag{4}$$

$$\frac{d\tilde{u}_\rho}{dl} = -\frac{\tilde{u}_\rho^2}{2\tilde{u}_\sigma}\tilde{K}_\rho D \tag{5}$$

$$\frac{d\tilde{u}_\sigma}{dl} = -\frac{\tilde{u}_\sigma\tilde{K}_\sigma}{2}D\,, \tag{6}$$

Here all energy parameters are dimensionless since they are normalised by an ultraviolet cutoff and the dimensionless running ultraviolet cutoff is parametrised as $\Lambda = e^{-l}$. The ratio $\tilde{u}_\rho/\tilde{K}_\rho$ is not renormalised and one can substitute $\tilde{u}_\rho = (\tilde{u}_\rho^{(0)}/\tilde{K}_\rho^{(0)})\tilde{K}_\rho$. According to Ref. [18], the renormalisation group equations are written for 'tilded parameters' that account for the admixture of disorder to the running Luttinger parameters Eqs. (1-6). The tilded parameters are related to the true Luttinger parameters and in the linear in disorder approximation they are:

$$\tilde{K}_\rho = K_\rho - \frac{K_\rho^2+1}{4}D\gamma\,, \qquad \tilde{K}_\sigma = K_\sigma - \frac{K_\sigma^2+1}{4}D\gamma \tag{7}$$

$$\tilde{y} = y - D\gamma\,, \qquad \tilde{u}_\sigma = u_\sigma + \frac{K_\sigma^2-1}{4K_\sigma}u_\sigma D\gamma\,, \tag{8}$$

where the function γ is given by

$$\gamma = \left(\frac{u_\rho^{(0)}}{K_\rho^{(0)}} \frac{K_\rho}{u_\sigma} \right)^{K_\rho} . \tag{9}$$

Introducing a scaled spin velocity $u = u_\sigma / u_\rho^{(0)}$ and plugging Eq. (8) into Eq. (6), we can then rewrite the RG equations for the original parameters keeping only linear terms in the disorder in the RHS of the equations:

$$\frac{dK_\rho}{dl} = -\left(\frac{1}{2} \frac{K_\rho^3}{u K_\rho^{(0)}} + \frac{1}{4}(K_\rho^2+1)(K_\rho+K_\sigma+y-3)\gamma \right) D \tag{10}$$

$$\frac{dK_\sigma}{dl} = -\frac{1}{2}K_\sigma^2 y^2 - \left(\left[\frac{1}{4}(K_\sigma^2+1)(K_\rho+K_\sigma+y-3) - yK_\sigma^2 \right] \gamma + \frac{1}{2}K_\sigma^2 \right) D \tag{11}$$

$$\frac{dy}{dl} = 2(1-K_\sigma)y + \left(\left[1 + K_\sigma - K_\rho + \frac{y}{2}(K_\sigma^2-1) \right] \gamma - 1 \right) D \tag{12}$$

$$\frac{dD}{dl} = -\left(K_\rho + K_\sigma + y - 3\right) D \tag{13}$$

$$\frac{du}{dl} = \left(\frac{K_\sigma^2-1}{4K_\sigma}(K_\rho+K_\sigma+y-3)\gamma - \frac{K_\sigma}{2} \right) uD \, . \tag{14}$$

It is not difficult to check that these equations respect the rule "disorder does not produce interactions". In the absence of disorder only K_σ and y do vary. Their solutions provide RG trajectories

$$y^2 = 8\left(\frac{1}{K_\sigma} + \ln K_\sigma - C \right) \tag{15}$$

with a separatrix given by $C = 1$ as shown in Fig. 1.

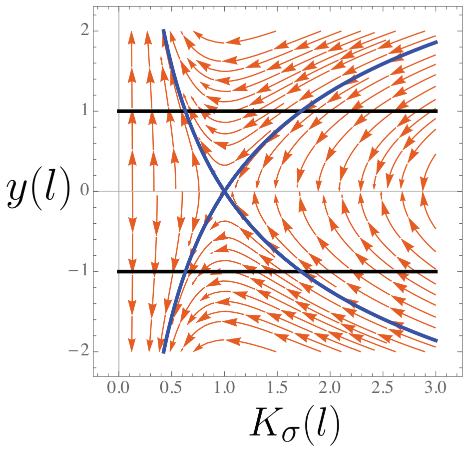

Figure 1: Phase portrait of a system without disorder. The red arrows show the trajectories of the RG flow. The black lines indicate where the spin gapped phase transition will occur. If the intial point is at $y(l) > 0$ and $K_\sigma(l) < 1$, then the RG flow will end up in the SDW phase. For $y(l) < 0$ and $K_\sigma(l) < 1$, the flow will end up in a CDW phase. Subsequently, if disorder is able to change the point at which the flow goes from positive to negative, this could result in a disorder induced QPT.

Solutions of the RG equations depend on the initial values of five parameters: $K_\sigma^{(0)}, y_0, K_\rho^{(0)}, D_0, u^{(0)}$. In order to choose these values properly to describe a real physical system we address the experimental data of Ref. [16]. Our initial point (at temperature $T_0 = 1$ must correspond to linear dependence of resistivity on temperature, observed in the experiment. In a Luttinger liquid [9, 18], resistivity is proportional to the product of disorder and temperature $\rho \sim DT$. It follows then from Eq. (14) that initial values of parameters must satisfy $K_\rho^{(0)} + K_\sigma^{(0)} + y_0 \approx 3$. More information for the proper choice of the initial values comes from Eq. (13). We will show below that there existence of qualitatively different RG trajectories (with increasing and decreasing y) is a necessary condition for the appearance of two qualitatively different phases at lower temperatures. We find that it is provided by a competition between the first and the third terms in Eq. (13), so we need to choose the first term to be positive, i.e. $K_\sigma^{(0)} < 1$.

3. Critical disorder.

Before presenting numerical results, let us discuss the expected behaviour of $y(l)$. In a clean system, any trajectory starting at $K_\sigma^{(0)} < 1$, $y_0 > 0$ is attracted to the branch of the separatrix with decreasing $K_\sigma(l)$ and $y \to +\infty$ (see Fig. 1). To simplify the qualitative analysis of a system with disorder, we choose $u(0) \gg 1$, which allows us to neglect the term containing the function γ in all equations. We then conclude that $K_\sigma(l)$ is a decreasing function for disorder. If the initial disorder D_0 is small enough to allow a positive RHS in Eq. (13) (derivative of $y(l)$) at $l = 0$), one can easily check that the derivative of the RHS in Eq.(13) (second derivative of $y(l)$) at $l = 0$) is also positive. This allows us to speculate that if $D_0 < 2(1 - K_\sigma^{(0)})y_0$ then $y(l)$ increases to $+\infty$. If, on the other hand, the initial disorder is big enough to invert the sign of this inequality, then $y(l)$ is decreasing, meaning that the RHS of Eq. (13) is always negative. We then conclude that if $D_0 > 2(1 - K_\sigma^{(0)})y_0$ then $y(l)$ decreases to $-\infty$. Obviously, the term we have neglected, as well as exact functions of the Luttinger parameters, add some corrections to the inequality, but they do not affect our qualitative conclusion: there exists a quantum phase transition at the critical disorder $D_{cr} \approx 2(1 - K_\sigma^{(0)})y_0$, which separates two distinct phases with spin gaps at $y \to +\infty$ and $y \to -\infty$. Our numerical results in the next Section confirm this fact. We will discuss the details of the spin-gapped phases below.

In order to describe this phase transition we suggest the following procedure. Taking into account that for initial disorder $D_0 < D_{cr}$ ($D_0 > D_{cr}$) Luttinger parameter y increases (decreases) to positive (negative) infinity, it is natural to expect that very close to the critical disorder it changes very slowly (if at all). We then define the temperature at which $|y| = 1$ as the transition temperature from the original gapless phase to a particular spin-gapped phase. The value $|y| = 1$ is reasonable, since the RG equations are perturbative, and define a perturbation to become relevant when its dimensionless amplitude reaches a value of 1. It is clear that for a system with disorder very close to the critical one this value of $|y| = 1$ is reached slowly at very large l_y (at very low temperature T_y). The quantum phase transition means that l_y diverges as the initial disorder D_0 approaches the critical initial disorder D_{cr}, so for a transition temperature ($T_y = \exp[l_y]$) we can write

$$T_y = \exp\Big[-a\,|D_0 - D_{cr}|^{-\nu}\Big]\,, \tag{16}$$

with the universal critical exponent $\nu > 0$ and a fitting parameter a.

This result can be understood from the following phenomenological consideration. We will use an analogy with magnetic systems where a magnetisation is caused by an applied magnetic field. The role of magnetisation (which is the order parameter) is played by the amplitude of spin (SDW) or charge (CDW) oscillations. Instead of a magnetic field, playing the role of a source field, we imply the disorder strength. The critical behaviour of a system at the phase transition $T = T_y$ is characterised then by the critical exponent δ that relates the order parameter m (absolute value of the oscillation amplitude in our case), and the source field which is $D_0 - D_{cr}$ for our problem:

$$|D_0 - D_{cr}| = m^{\delta}\,. \tag{17}$$

The 'magnetisation' is proportional to the spin-flip backscattering amplitude, y, in the regime of weak coupling where our perturbative RG analysis is valid. For a weak disorder and close to the separatrix of the clean system, the running parameter $y(l) \sim 1/l$ [9]. The 'magnetisation' scales as $|m(l)| = m_0/l$. Exactly at the transition, temperature $l \to l_y = -\ln T_y$, and 'magnetisation' $|m(l = l_y)| = m_0/(-\ln T_y)$. Under these assumptions, Eq. (17) leads to the following relation:

$$|D_0 - D_{cr}| = \left[-\frac{m_0}{\ln T_y}\right]^{\delta}\,. \tag{18}$$

The mean field universality class [1] corresponds to the critical exponent $\delta = 3$ and this result explains the numerically observed value $\nu = 1/\delta$ close to 1/3, as it is discussed in detail in the next section. Resolving Eq. (17) for the critical temperature T_y we reproduce our numerically observed behaviour described by Eq. (16) with $m_0 = 1/a$.

4. Numerical results for phase transition temperature.

Following the discussions in the previous section, we have chosen two sets of the initial values of Luttinger parameters:

$$K_\sigma^{(0)} = 0.8,\; y_0 = 0.25,\; K_\rho^{(0)} = 1.95,\; u^{(0)} = 8 \tag{19}$$

$$K_\sigma^{(0)} = 0.6,\; y_0 = 0.2,\; K_\rho^{(0)} = 2.2,\; u^{(0)} = 8\,. \tag{20}$$

We have run the RG equations for a wide spectrum of the initial disorder values D_0. In Fig. 2 we show two sets of typical trajectories, corresponding to a weak and strong initial disorder values. As we have described above, these trajectories run to zero value of K_σ and positive and negative infinite values of y correspondingly. The abrupt change in the qualitative behaviour of a function $y(l)$ takes place at critical disorder, which is very close to our rough estimates $D_{cr} \approx 2(1 - K_\sigma^{(0)})y_0$. The corrsponding values are 0.1 in Fig. 2a, and 0.16 in Fig. 2b.

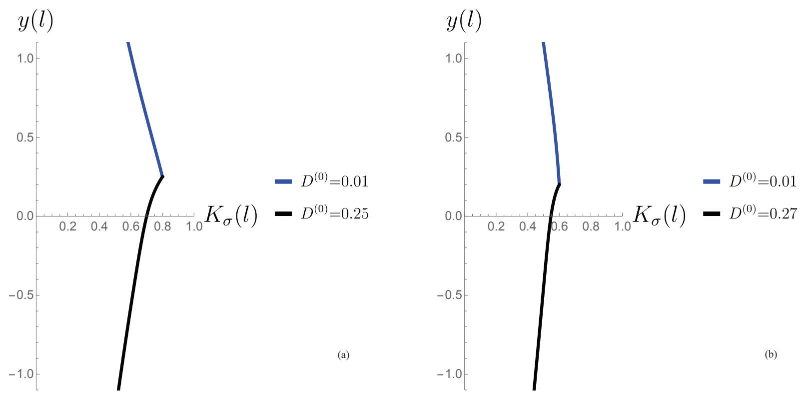

Figure 2: RG trajectories for weak and strong initial disorders. It is clear in both figures the trajectories have been changed due to disorder. The initial starting point is in the region $y(l) > 0$ and $K_\sigma(l) < 1$, but for strong disorder, the trajectory is changed such that the CDW phase is induced by disorder. To see how disorder has changed the system, compare Fig. 2 with Fig. 1.

Temperatures T_y (for both sets of the initial values) at which the absolute value of parameter $|y| = 1$ are presented in Figs. 3a and b. We find that these temperatures scale in almost perfect agreement with predictions of the phase transition Eq.(16). The most convincing proof of the second order phase transition is the universal critical exponent $\nu = 1/3$ in both graphs for both transitions (to SDW and CDW phases). The values of all three parameters ν, D_{cr}, and a have been found numerically by the least squares method. Amazingly, the critical disorder value is very close to our rough estimate in Fig. 3a (0.12 value is found whereas 0.1 is estimated) and both values simply coincide (0.16) in Fig. 3b.

We have also studied a very different ($K_\sigma > 1$) set of the initial values

$$K_\sigma^{(0)} = 1.1,\ y_0 = 0.5,\ K_\rho^{(0)} = 1.5,\ u^{(0)} = 1 \tag{21}$$

$$K_\sigma^{(0)} = 1.1,\ y_0 = -0.5,\ K_\rho^{(0)} = 1.5,\ u^{(0)} = 1\,. \tag{22}$$

For these parameters only single SDW (or CDW) to insulator transitions are observed (as we discuss below). In Fig. 3c we present T_y for a transition into SDW phase for a set from Eq. (21), which belongs to the same universality class with the critical exponent $\nu = 1/3$. The temperatures T_D of transitions into insulating phase (discussed in details in the next Section) are also shown in Fig. 3c. We notice the resemblance between Figs. 3a,b and Fig. 3c: SDW and CDW phases meet only at zero temperature in Figs. 3a,b, whereas SDW and insulating phases do the same in Fig. 3c; and correspondingly T_D and T_y scale with the same values of $\nu = 1/3$ and $D_{cr} = 0.1$. On the other hand, T_y for a transition into CDW phase for a set from Eq. (22) does not belong to the universality class discussed above, and disorder-driven transitions from a CDW to an insulating phase occur at the critical disorder in a wide interval of temperatures in Fig. 3d.

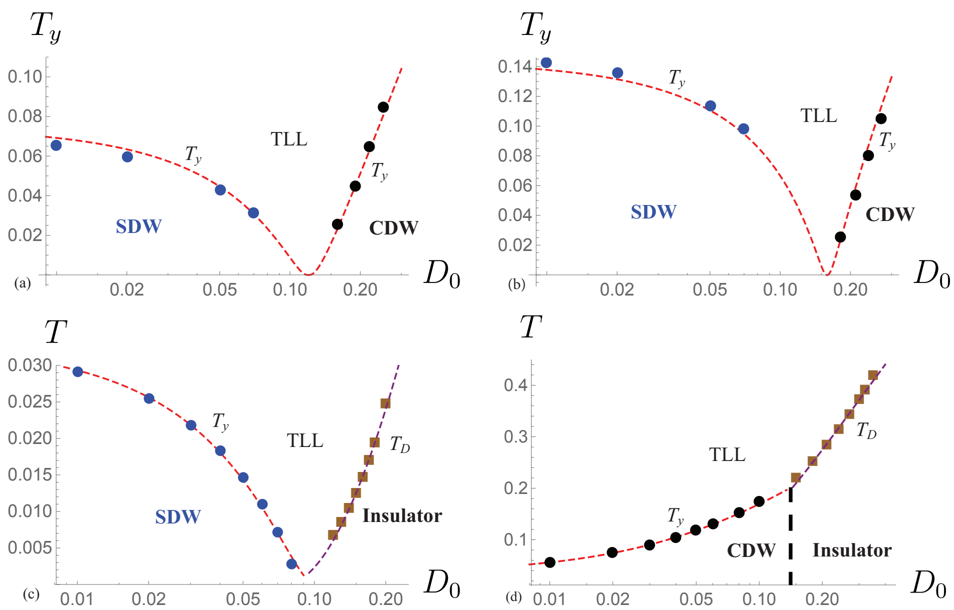

Figure 3: Temperatures of transition to spin-gapped phases as function of disorder. Blue circles correspond to $y = +1$, black circles correspond to $y = -1$, and brown squares correspond to $D = 1$. Temperatures T_y scale around critical disorder D_{cr} as $T_y = \exp\left[-a\,|D_0 - D_{cr}|^{-1/3}\right]$ with (a) $D_{cr} = 0.12$, $a = 1.27$ for initial values of Luttinger parameters from Eq. (19); (b) $D_{cr} = 0.16$, $a = 1.04$ for initial values of Luttinger parameters from Eq. (20); (c) $D_{cr} = 0.1$, $a = 1.6$ for initial values of Luttinger parameters from Eq. (21). Temperature T_D scales similar to T_y with $a = 1.9$. (d) Temperatures T_y ($y = -1$) and T_D for initial values of Luttinger parameters from Eq. (22)

5. Phase diagram in $y_0 - D_0$ plane.

To reconstruct the phase diagram containing three phases, the insulator if D renormalises to unity while $|y|$ is still less than one, and the SDW/CDW when renormalised coupling y reaches $+1/(-1)$ correspondingly while disorder is still weak ($D < 1$), it suffices to analyse the projection of the RG flows onto (y, D)-plane keeping track of trajectories within rectangle $0 \leq D \leq 1$ and $-1 \leq y \leq 1$ defined by the limits of applicability of the RG equations. The starting point (y_0, D_0) of the trajectory defines bare values characterising a material. Trajectory evolves in accordance with the RG equations and hits one of the boundaries of the rectangle. This event means that system enters into a new phase and the phase is uniquely defined by the side of the rectangle.

Phase diagrams presented in Fig. 4 clearly indicate the existence of three distinct phases (insulator, CDW and SDW) and a triple critical point at $(y = 0, D = 0)$. This point indicates that the direct transition between CDW and SDW phases is possible only at zero temperature (quantum phase transition).

At any nonzero temperature, there exists an insulating phase between two spin-gapped phases. It is important to stress that Figs. 4a,b clearly show that for some set of Luttinger parameters the change of disorder leads to a transition between SDW and CDW phases. This transition will pass through the insulating phase and can be qualified as two consecutive disorder-induced phase transitions. Transitions presented in Fig. 3a pass through a very narrow insulating phase (as can be seen in Fig. 4a) which allowed us to fit both branches of the temperature dependence on the initial disorder with the same value of the critical disorder (the same explanation applies to the transitions in Fig. 3b). When the gap opens wider (e.g. $y_0 = 0.25$ in Fig. 4b) there will be two slightly different critical disorder values for SDW and CDW branches. On the other hand, there is a big set of initial conditions which produce a finite gap that allow only a single SDW (or CDW) to insulator transition as can be seen in Fig. 4c. Transition temperatures into SDW phase belong to the same universality class with the critical exponent $\nu = 1/3$ and corresponding (with the same value of a parameter y) transitions into insulating phase exhibit similar scaling (Fig. 3c). Transitions into CDW and insulating phases (for large negative values of a parameter y) do not belong to the same universality class as we have stressed in the previous Section.

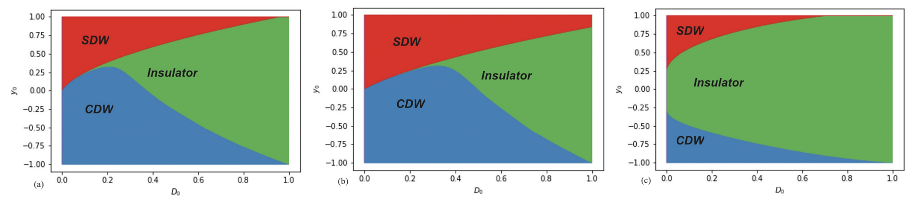

Figure 4: Phase diagram in $y_0 - D_0$ plane. Initial opposite spins' interaction y_0 and initial disorder D_0 define the phase of the system at low temperature. (a) $K_\sigma = 0.8\,, K_\rho = 1.95\,, u = 8\,$, (b)$K_\sigma = 0.6\,, K_\rho = 2.2\,, u = 8\,$, and (c) $K_\sigma = 1.1\,, K_\rho = 1.5\,, u = 1\,$,

6. Spin-gapped phases.

As we decrease temperature the amplitude of interactions between opposite spins y increases in absolute value. Due to its positive initial value $y_0 > 0$ it remains positive and increases (see Fig. 2) for a weak initial disorder. When it becomes large enough a spin gap opens, which freezes the spin density field at such a value that the average value of the disorder term goes to zero [9]. The positive parameter y describes opposite spins repulsion which favours antiferromagnetism and this phase is a SDW. The crossover between a gapless mode and a SDW phase is driven by temperature and is presented by the l.h.s. branches on both graphs in Fig. 3. On the other hand, when the initial disorder is strong enough, y decreases and becomes negative (see Fig. 2). When $y = -1$, a spin gap opens, which freezes the spin density field at such a value that does not annihilate disorder. The negative parameter y describes opposite spins attraction, which leads to a charge density modulation and zero spin density modulation. This phase is a CDW. The crossover between a gapless mode and a CDW phase is driven by temperature and is presented by the r.h.s. branches on both graphs in Fig. 3. The transition between SDW and CDW phases driven by disorder at zero temperature (horizontal axis in Fig. 3) represents a quantum phase transition. One can define a spin gap as an order parameter, which vanishes at the transition point D_{cr} as well as at temperatures T_y, but is non-zero in both spin-gapped phases. We can also speculate that our $T_y - D_0$ phase diagram looks similar to the phase diagrams containing "strange" metal [21]. This resemblance may be non-trivial, concerning the fact that the most peculiar feature of the "strange" metal is a linear dependence of the resistivity on the temperature down to low temperatures, which corresponds exactly to the transition temperature T_y reaching a zero value for the critical initial disorder.

7. Discussion

The results of this paper clearly show second order phase transitions and disorder-induced phase transitions in a one-dimensional system. Since different sets of initial conditions have shown similar behaviour, particularly the transition from the SDW to the CDW phase as disorder is increased, an intrinsic link between disorder and spin gapped phases seems obvious. The analysis suggests that this system has the universal critical exponent of $\nu = 1/3$ that relates critical disorder and transition temperature. Together with the linear-T dependence of resistivity, these results suggest similarity between the system under consideration and the so-called strange metals [21]. This presents scope for future research to establish whether these two models belong to the same universality class.

Another direction of future work would be extending the dimensionality by layering the model and formulating multi-wire coupled Tomonaga-Luttinger sliding liquid. Our preliminary results suggest that superconductivity can be enhanced by disorder in some region of Luttinger parameters. Non-perturbative analysis of interplay between disorder and superconductivity [22] may explain the experimentally observed results [16].

8. Conclusions

A disorder-induced phase transition (crossover) has been proposed and analysed in a one-dimensional system of spinful strongly correlated electrons. The analysis has been performed on the basis of renormalisation group equations with accurate separation of disorder and interaction initially mixed in the running Luttinger parameters. While the subtlety of having admixture of inelastic processes into an elastic effect within the original RG has always been appreciated, this separation for spinful electrons with arbitrary interactions has not been realised. The construction of a phase diagram required such a modification of the RG equations because only divergence (hitting applicability region) of true renormalised amplitudes defines the phase boundaries.

For the model of a spinful disordered Luttinger liquid, a critical exponent has been found numerically, allowing classification for the model. This critical exponent $\nu = 1/3$ has been found at the transition and we believe it is related to the critical exponent $\delta = 3$ in the notations standard for the theory of critical phenomena.The phase diagram shows a tricritical point between insulator, spin and charge density waves. Vanishing transition temperatures at the boundaries between phases implies a quantum phase transition driven by disorder strength.

Declaration of Competing Interests

There are no conflicting interests.

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