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Highlights

- A Data Envelopment Analysis enabled Markovian model for decision making
- Radial and additive Markov-DEA models
- Tractable Markov manpower planning using Data Envelopment Analysis
- Converting stochastic to deterministic approaches through DEA and goal programming
- A decomposable measure of efficacy of alternative paths to a target outcomes vector

Using Data Envelopment Analysis in Markovian Decision Making

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Abstract

This paper introduces a modelling framework which combines Data Envelopment Analysis and Markov Chains into an integrated decision aid. Markov Chains are typically used in contexts where a system (e.g. staff profile in a large organisation) is at the start of the planning horizon in a given state, and the aim is to transform the system to a new state by the end of the horizon. The planning horizon can involve several steps and the system transits to a new state after each step. The transition probabilities from one step to the next are influenced by both organisational and external (non-organisational) factors. We develop our generic methodology using as a vehicle the homogeneous Markov manpower planning system. The paper recognizes a gap in existing Markovian manpower planning methods to handle stochasticity and optimization in a more tractable manner and puts forward an approach to harness the power of DEA to fill this gap. In this context, the Decision Maker (DM) can specify potential anticipated future outcomes (e.g. personnel flows) and then use DEA to identify additional feasible courses of action through convexity. These feasible strategies can be evaluated according to the 5 aΩă judgement over potential future states of nature and then employed to guide the organisation in making interventions that would affect transition probabilities to improve the probability of attaining the ultimate state desired for the system. The paper includes a numerical illustration of the suggested approach, including data from a manpower planning model previously addressed using classical Markov modelling.

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Keywords: Data Envelopment Analysis, Markov Manpower Planning, Markov processes, Efficiency, Goal Programming

1. Introduction and literature review

This paper integrates Markov Chains (MCs) and Data Envelopment Analysis (DEA) into a synergistic decision aid methodology. The methodology developed exploits the aspects of DEA to generate alternative courses of action and to identify the best among them, in order to simplify an otherwise stochastic framework and enhance the usefulness from the information that a MCs approach can yield as a decision aid. As a vehicle for integrating the use of DEA into a MCs based approach we use the Markovian human resource planning context. The use of a specific decision context facilitates the communication of the approach without detracting from its generalisability as the approach can be readily adapted to other decision contexts as will be discussed later.

1.1 Data Envelopment Analysis

Data Envelopment Analysis (DEA), is a non-parametric mathematical programming approach, that has been used widely to evaluate the relative performance (relative efficiency) of homogeneous entities, that is, operating units that use similar resources (inputs) to produce comparable products or services (outputs). One key feature of DEA is that under minimal assumptions it can use the data from observed operating units to generate other feasible in principle operating units even if not observed in practice. Another key feature of DEA is that it is a *boundary method* in that out of the [illegible] unit level, maximum output levels for given resources or minimum input levels for given output levels. It is these two properties of DEA that we shall deploy in the context of a MCs methodology to [illegible] Since the publication of the seminal paper by Charnes, Cooper and Rhodes (1978) who took forward ideas from a paper by Farell (1957) on the measurement of efficiency, the literature on DEA has flourished. The original idea was to provide a methodology that would yield a measure of efficiency of a Decision Making Unit (DMU) relative to a set of comparable DMUs in not for profit organisations. However, since then, DEA has evolved into a modelling approach to efficiency and productivity analysis which yields much additional information for exploring the production space and managing performance. For instance, it identifies benchmark DMUs (peers) inefficient units may draw best practice from to improve performance, it identifies targets inefficient units may seek to attain, the nature of returns to scale prevailing at a point of the efficient frontier, productivity change over time and much more. Due to its strength, applicability and minimal a priori assumptions, DEA has seen rapid growth and widespread acceptance in the last forty years, both in theory and practice. For more details on DEA see Thanassoulis (2001), Cooper (2005), Cooper et al. (2007a), Thanassoulis et al. (2008), Liu et al. (2013), Kao (2014) and Portela and Thanassoulis (2014). A more recent survey on the practice and progress of the theory and applications of DEA can be found in Emrouznejad and Yang (2018).

1.2 The Markovian manpower models and their use

The variability inherent in all processes involving human interaction influences the ability to forecast and eradicate the risks within an organization. As mentioned in a recent study on a Markov model for manpower prediction (Hrustek et al. 2020), in order to reduce business failures, various mathematical and probabilistic instruments have been developed with the purpose of mitigating these risks. MC models are among those methods, with a broad range of applications including the investigation of the mobility of personnel and the management of human potential within organizations. Hrustek et al. (2020) analyse a case study of an ICT service and use an irreducible, ergodic MC to predict numbers of people in the main staff categories thus assisting the management to plan ahead for budget requirements. As pointed out in a paper about Markovian manpower planning in the armed forces, ~ƪdzfƨ Ŝű I·fɸ óнллуύΣ άűKŜ ăƪλffă ƴŜŜŘŜŘ to perform assigned tasks are acquired through experience and training, shortfall or surpluses of skilled staff might be costly and λƴŜŦŦλŎλŜƴűέɸ ¢ƻ ŎƻLJŜ ǿλűK ăűƻŎKI·ăűλŎ ŦfdzŎűdzI·űλƻƴă ƻŦ ŦdzűdzNJŜ ƴŜŜŘă, predictions and strategies need to be in place. Often estimates are based on previous experiences. Nevertheless, often experience alone does not suffice without the application of appropriate quantitative models and the associated analyses.

MCs have been used in many cases in the last fifty or so years in stochastic decision contexts. These include health care, corporate manpower planning, defence, educational establishments, civil services and, more recently, even in machine learning and artificial intelligence (AI). As McClean (1991) and Smith and Bartholomew (1988) mention, manpower planning can be traced back to 1779 when John Rowe used an actuarial model to plan careers in the Royal Marines. Pioneering in the I·NJŜI· I·NJŜ Ŏƻƴƴ̃ăλŘŜNJŜŘ űKŜ ăűdzŘŜƴűăΩ ŜƴNJƻfΥŜƴű ΥƻŘŜf ƻŦ DI·ƴλ όмφсούΣ the work of Young and Almond (1965) on predicting distributions of staff, the volumes edited by A.R Smith (1971, 1976 and 1980) on manpower planning systems, manpower planning in the civil services and corporate manpower planning, the classic book by S. Vajida (1978) on the mathematics of manpower planning and the seminal book by Bartholomew (1982). It is also not a coincidence at all that, in a series of papers, the pioneers of goal programming and DEA, A. Charnes and W.W. Cooper with their colleagues (1968, 1972, 1973, 1976) set the fundamentals for *predictive* and *normative* manpower modelling for civil services and/or corporate settings, using both probabilistic approaches and mathematical programming techniques, including multiple objective optimization. They even considered crucial issues that are still at the top of the the social agenda of ƵdzǊ ǘƛϒŜăƐ ădzŎK Iǡ ǡŜƴădzǊƛƴϡ ŜIJdzIǡ employment opportunity representation of social groups within the organization that matches their ǊŜLJǊŜăŜƴǘIǘƛƵƴ΄ƛƴ΄ǘKŜ ŜƴǾƛǊƵƴϒŜƴǘ ădzǊǊƵdzƴŘƛƴϡ ǘKŜ ƵǊϡIƴƛǍIǘƛƵƴǭΦ LƴΊIŘŘƛǘƛƵƴΣ I ƵLJǘƛƴă ómфулύ dzăŜŘ I Markov chain based model as an aid in setting realistic numerical goals for the employment of women and minority persons in a real university environment.

The key features of Markov chains as an instrument for investigating future distributions of populations across states is that the recruitment, the internal transitions and the attrition probabilities drive the whole process. Manpower in organisations of this type, consists of sub-groups who interact purposefully and are usually stratified into network structures. Typically, state vectors (also called stock vectors) are used to describe the distribution of people (numbers of employees) that reside in various states of the system (groups, grades etc.) based on miscellaneous attributes such as job positions, departments, length of stay or age, skills, job description etc. These sub-groups are sometimes hierarchical in the sense that the grades correspond to promotional opportunities. The state vectors change in time according to flows which are typically regulated by internal transfers, wastage due to attrition, and recruitment of newcomers. The flow probabilities reflect, in most decision contexts, stochastic outcomes of purposeful actions such as recruitment and retention policies in manpower planning and medical treatments and their impact on health outcomes of a population.

In a MCs approach to human resource planning staff levels always converge in some asymptotic form when internal probabilities and recruitment vectors are kept constant over time. However, in reality, organizations exercise control over their personnel strategies to guide the process towards desired staff levels. In this respect, one of the many problems that has been given attention to in the Markov manpower literature is the problem of attaining desired staff levels, and in particular the achievement of desired structural configurations of personnel, using control variables such as recruitment flows, internal transfers, retirement and at times redundancies. The Markovian modelling framework lends itself naturally to this kind of personnel planning since it encompasses staff flows reflected in transition probabilities. The target is to use Markovian models in various forms to describe personnel mobility (forecasting aspect) and to support the preconditions to create feasible, satisfying or even optimal policies for attaining or maintaining appropriate population structures (normative aspect).

Formally, MC based manpower models are divided into two main streams; *explorative and normative. The explorative* (*descriptive, predictive*) models are used to get insight into the way a system operates (descriptive) and how it might respond to various interventions (predictive). *Normative (optimization)* models are used to identify optimal decisions based on control variables such as recruitment, training, promotional rates or retention. Markovian human resource models fall in the exploratory category and are used for predicting the population levels of organizations, contingent on alternative scenarios of the future (Wang, 2005). The solution of such stochastic [illegible] personnel and can guide [illegible] structure. A good account of baseline modelling tools can be found in two influential texts: Bartholomew (1982) and Bartholomew et al. (1991). Other approaches of stochastic models in manpower planning using MCs can be found in Vassiliou (1982) on the nonhomogeneous Markov system, Vassiliou and Tsantas (1984) on one step maintainability, Vassiliou and Georgiou (1990) on asymptotic behaviour, McClean and Montgomery (1999) on semi-Markov models, Papadopoulou and Vassiliou (1994, 1999) on semi-Markov asymptotic theory, Nilakantan and Raghavendra (2005) on proportionality constraints of attainability, Ossai and Uche (2009) on departmentalized manpower structures, Guerry (2011) on the hidden heterogeneity in manpower systems, and more recently, Dimitriou and Georgiou (2020) who elaborated on departmental mobility in continuous time multivariate Markov settings. From the vast body of the relevant literature we distinguish two papers that are somewhat closer to our proposed approach, namely De Feyter and Guerry (2009) and Nilakantan (2015). Nilakantan (2015) established an effectiveness measure for the evaluation of staffing policies in Markov manpower systems, in relation to the career growth prospects afforded by the system to its members. Our paper has a slight similarity in that it evaluates policies specified up front. However, we measure the effectiveness of proposed manpower flows rather than the career prospects at employee level using management aspirations. In De Feyter and Guerry (2009) the authors propose an approach for the evaluation of recruitment strategies. We too address the issue of identifying efficacious recruitment strategies but unlike De Feyter and Guerry (2009) who use probabilities and fuzzy membership functions, we use DM judgements and preferences which are more user-friendly concepts. We compare later their approach to ours using an example from their paper.

A number of applications and case studies have emerged in the literature in tandem with the theoretical development of the field as outlined above. A frequent area of application of MCs is that of academic manpower planning, see Ledwith (2019) for a recent review. In earlier studies, such as Bleau (1981) and Hackett et al. (1999) we see that Markov approaches provide additional insight when planning for personnel in short or midterm horizons. In the academic setting the states of the Markov chain usually represent teaching and research staff by academic discipline and grade. The [illegible] recruitment, attrition and promotion. The stochasticity is of course also influenced by random events and factors not in the gift of the University, such as the external market for academic staff, Government policies on education etc.

Another major area of application of manpower planning is that of military forces. There exist several applications of both *forecasting* and *normative* MC methodologies for projecting career progressions and for Ύ2ŘŜſſʎУƎ ǈŜƝă2ỵỵŜſǾă ǀƎƎƝŜƎǀűŜ Ύ2ōʎſʎűè ōŜKǀØʎ2ǳƝ. For a good review see Wang (2005) and Ledwith (2019). Among the first to employ Markov chains to determine the structure of armed forces was Brothers (1974). In 1984, the RAND Corporation issued a report regarding the use of Markov chain models to predict the probability of officers staying in the Air Force (Gotz and McCall, 1984). Another well-known framework was put forward by Gass et al. (1988) and Gass (1991), who proposed Markov chain models and goal programming techniques to forecast flows and determine optimal policies using as personnel states combinations of ranks, skills, operational units and length of service. Hall (2009), built a Markov model to determine optimal policies ƝŜƎǀƝŘʎУƎ 2ŦŦʎÕŜƝăΩ ŘŜÕʎăʎ2ỵ Ŧ2Ɲ ƝŜűʎƝŜΎŜỵű óăŜe also Cashbaugh et al. 2007). ~kulj et al. (2008) presented a case study using Markovian manpower planning for the Slovenian armed forces to produce projections for several years ahead and to apply appropriate policies for achieving specific staff level goals. Van Utterbeeck et al. (2009), presented a combined simulation and Markov chain model for human resource management in the Belgian Defence Forces. Recently, Zais and Zhang (2016), developed a Markov chain model to predict individualized stay/leave decisions within the US Army and Ledwith (2019) presented a Markov model with absorbing states to forecast educational composition according to arrival and internal transition probabilities for a military academy.

Another field of application of MC manpower planning is that of healthcare organizations. Early applications of Markovian models in *health care* can be found in Shuman et al. (1971) and Smith et al. (1976) who developed mathematical programming models for investigating optimal staffing in health services. Trivedi et al. (1987) focused their attention on Markov models for health care manpower supply predictions and McClean et al. (2011) developed a modelling framework that combines Markov chains with simulation to investigate the whole care system of stroke patients for Belfast City Hospital. Josiah (2014) introduced a Markov forecasting model as a tool for Health Corps Administrators to forecast inventory levels across ranks and subspecialties. Also, Lagarde and Cairns (2012) used a Markov model to examine the dynamics of movements of health care workers in the professional labour market in South Africa. In the same direction, Srikanth (2015) modelled the progression of diabetic retinopathy estimating the time a patient spends on each one of five stages of the disease from mild retinopathy through to moderate, severe, PDR (Proliferative Diabetic Retinopathy) and ultimately blindness (single or double). Using new patient arrivals to each stage and transitions of existing patients consequent on any medical interventions, the number of patients at each stage of the disease at future points in time can be estimated. This information clearly has important implications both for prognosis at patient level but also for resource planning at health service provider level. In a recent review of the relevant literature, Bartelt‑Hofer et al. (2020), report that ǻ*Markov models using transition states were the most common type of modelling approach. Cost-utility models using a mid- to long-term time horizon with a national payer perspective were the most frequent type of economic evaluation identified*ǽ. A good account of patient flow modelling of healthcare delivery and performance analysis using various techniques such as queueing models, Markov chains, simulation modelling and statistical methods, can be found in Bhattacharjee and Ray (2014).

1.3 The research gap and the contribution in this paper

Although as noted above there is strong evidence that the HR community recognizes the need for strategic manpower planning, it seems that barriers such as preoccupation with short-term activities and the complexity of uncertainty hinder the wider adoption of MC manpower planning models. Taylor (2005) points out that when ōdzăλÿŜăă ŘŜŎλăλƧÿ Ɣ I· ƪŜǁă ǘǁè ǘƧ ǀǁŜŘλŎǘ I· ŎƧƔǀI·ÿèǁă λÿǘŜǁÿI·ƭ labour supply they lean towards empirical judgements and intuition and not on a thorough statistical analysis. ~ƪdzƭƨ et al. (2014) mention that other reasons raised by researchers in the field of management are hostility to statistical (quantitative) techniques, preference of intuitive judgment, ignorance, and short-term mentality. The reasons put forward in Taylor (2005) and ~ƪdzƭƨ et al. (2014) are clearly in line with the major findings of the report by Johnson and Brown (2004) attributed to the rapid change of the environment and high level of uncertainty in contemporary business environment. As Oczki (2014) emphasises, the doubts concerning the application of more formal (quantitative) methods in systematic labour force prediction and optimization, should not hinder the ŦI·Ŏǘ ǘKI·ǘ ǘKŜăŜ I·ǀǀǁƧI·ŎKŜăΣ λÿǘKŜ ƭƧÿƎ ǁdzÿΣ ŎI·ÿ ăλƎÿλŦλŎI·ÿǘƭè ŎƧÿǘǁλōdzǘŜ ǘƧ I·ÿ ƧǀǘλƔI·ƭ ǁŜăƻdzǁŎŜ ǳăŜΦ

In this respect, the method developed here aims to make the handling of uncertainty in an MC context more tractable. The majority of MCs approaches in manpower planning focus on examining the populationΩă (mostly) stochastic flowing mechanism and attempt to determine an optimal strategy (e.g. recruitment or promotion), that attains or maintains specific targets in terms of personnel. However, to the extent that these mobility patterns can be influenced by the actions of an agent (University in the case of employees, clinicians in the case of patients, the Department of Defence in the case of military personnel etc.) the issue arises how to practically identify alternative actions an agent can take to achieve desirable outcomes which are stochastic in nature. The approach needs to be tractable so as to be more accessible to non-specialist managers and practitioners. This paper recognizes the gap in existing Markovian manpower planning methods to

rather than above its specified value within the flow. Further, where necessary, the DM prefers target component values within N^* to be under rather than overshot.

It is noteworthy that with few exceptions the slacks and deviational variables take values in line with the weights intended to reflect DM expectations and preferences. Thus for R_1-R_4 the slacks s1m, s2m and s3m all have positive optimal values while the corresponding s(i)p values are zero, except s2p of R_4 which is positive. In other words, the optimal projections for R_1-R_4 incorporate overwhelmingly undershoots from the values specified in these flows. The practical implication of this is that the model uses projections of the flows R as close to expectation as possible. This in turn means the evaluation of the efficacy of the flows specified by the DM for getting to the target N^* is as close to the DM expectations of outturn values as possible.

Flow R_5 bucks the trend in that despite the DM expectations, the model produces a reflection with positive values for s1p and s3p suggesting it is preferable to plan for overshoots of these two components of this flow. The practical implication of this outcome for R_5 is that the DM should aim, to the extent it might be possible, to influence (for example through internal promotions and retentions) to overshoot these two components for a better chance of getting closer to the targets in N^*. It is perhaps worth noting that components 1 and 3 have very low values in R_5 and so overshooting them maybe desirable even if difficult or unlikely.

The ´$_s$ index values in Table 2 reflect how close the optimal projected values of each flow R are to its original specification. In the case of R_2 the value is 1 and so the optimal projection and the specified value coincide. At the other extreme the ´$_s$ value for R_5 at 1.368, is the highest suggesting this flow has a specified value the furthest from expectation.

As far as deviation values are concerned undershoot values d1m, d2m and d3m are the only ones to take positive values, in line with the weights used for deviational variables in model (18). The ´$_d$ index values are all close to 1 suggesting the projections of the original flows, if attained, would all get us close to the target N^*. Thus the key value of the model is in revealing projections of flows which are efficacious in reaching N^*. Of these the most efficacious is R_2 while R_1 and R_3 are the next best in terms of the ´$_s$ and indeed SBM overall values.

It is worth pointing out that these results are in line with the results in the DFG paper even though the modelling context is different. DFG find that in a deterministic framework of their approach R_2 is more efficacious while using a stochastic approach R_1 is preferred. In our approach, as noted above, both R_2 and R_1 are among the most preferred flows, notwithstanding the fact that our approach relies on a weighting structure in the objective function of model (18) to capture the stochastic information used in the DFG paper.

To further illustrate the working of the additive model we now focus on flow R_5 which in DFG, as well as in our model above, was ranked at the bottom of the preference list. Let us assume the probability of achieving at the end of the horizon overshoots of inflows of 8 and 7 in staff grades 1 and 3 respectively, is rather high and certainly higher than the probability of realisation of undershoots of these grades. This is not an unreasonable assumption if we assume that the low values of 8 and 7 do signal some confidence that these levels as a minimum have been secured. By a similar reasoning we can assume that overshooting of grade 2 is less likely than undershooting it, given the relatively high level specified for that component. Table 3 shows the weighting structure adopted to reflect these expectations. The relative values of the weights can be seen in parentheses next to the variable names. Thus the weight on s1m and s3m is 100 compared to 1 for s1p and s3p respectively. In contrast the weight is 100 for s2p and 1 for s1m. Regarding the achievement of targets in N^* we have assumed in this variation that over achievement of staff target numbers is preferred rather than under achievement and so we have used larger weights for all @̂.Ý

The solution of this model is presented in Table 3. As can be seen the model yields a solution where indeed the slacks s1m and s3m are zero and s1p and s3p are positive. In contrast, s2m is positive and s2p is zero. So the optimal slack values are in line with expectation. However, the resulting optimal reflection is quite far from the specified flow R_5 as ´$_s$ = 1.5146 in Table 3 compared to ´$_s$ =1.368 in Table 2 indicates. On the positive side, this reflection is marginally closer to the ideal values in N^* as the ´$_d$ = 1.0037 in Table 3 compared to ´$_d$ = 1.0044 in Table 2 shows. So the reflection of R_5 depicted in Table 3 is marginally better to aim for rather than that in Table 2. Note in this respect that the reflection of R_5 is in large measure R_3 I·ă ŘŜŘdzŐŜŘ ŦŊ2Ƴ <$_3$=0.9 in Table 3. This further signifies how the weights reflecting a combination of likelihood and preferences judgements by the DM alter the actions the DM should take in order to get as close as possible to the ultimate target N^*.

We continue our comparison of the additive deterministic DEA model with the approach in DFG by adding to the five staff flows above, a 6th flow R_6 = [14 25 10] which DFG present as one capable of attaining in one step the target N^* = [200 260 230] when starting from $N(0)$ = [200 275 225]. Apart from introducing this 6th ΨŘŜăŊI·ōfŜ Ŧf2ǿ ǿŜ I·fă2 incorporate differences in weights as might be the case when the DM is much more certain about the likely future value of one component in particular for the potential flows specified. This for example might be the case when for one potential staff category the DM may have a substantial degree of control, e.g. some level of current staff for which supply is plentiful. The weights we have used in this illustration appear next to the slacks s and deviational variables d in Table 4. The weights indicate that the DM is far more certain of an overshoot occurring in flow component 1 than is the case for undershoots or overshoots in

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components 2 and 3. Having said that, the DM is more certain of undershoots than overshoots in components 2 and 3, albeit not to the degree of certainty regarding component 1. The solution of model (18) modified to include R_6 and the foregoing weights can be found in Table 4.

Table 4 shows that in response all optimal reflections of the specified flows have at least the specified value of component 1 as can be seen from the zero G_3^2 values across all flows *R*. This leads to re-adjustments of the optimal projections for the other two components across all flows *R* and reflected in the fact that the ´$_s$ values in Table 4 are the same or lower than the corresponding values in Table 2. The improved ´$_s$ values in Table 4 compared to Table 2 suggest projected flows are on the whole closer to the originally specified flows which can be seen as a consequence of the DM having given the model a stronger indication of the likely outturn levels of staff going forward. It is noteworthy also that the overall SBM values in Table 4 are the same or closer to 1 than those in Table 2. This is as we might expect given the prevailing improvement in the ´$_s$ values from Table 2 to Table 4. It is also noteworthy that in Table 4 the preferred flow is R_6 just as in DFG. This is concluded ō2ǘK ŦŊ2Ƴ ǘKŜ Ŏf2ăŜăǘ 2ØŜŊŀff {. a ØŀfdzŜ 2Ŧ ŀfƳ2ăǘ м ʎƴ ¢ŀōfŜ п ŀƴŘ ǘKŜ Ŧŀ Ŏǘ ǘKŀǘ $<_6$ = 0.91 at the optimal solution to the instance of model (18) corresponding to R_6. This makes R_6 by far the most dominant influence on its optimal projection. In fact R_6 is also the dominant influence in the optimal projections of the flows R_1 and R_3 ŀă Ŏŀƴ ōŜ ŘŜŘdzŎŜŘ ŦŊ2Ƴ ǘKŜ ØŀfdzŜ 2Ŧ лΦуу Ŧ2Ŋ $<_6$ for these two flows in Table 4. Finally, a further interesting impact of the stronger information on likely outcome regarding staff category 1 compared to the rest can be seen when we compare the outcome for flow R_4 in Tables 2 and 4. Flow R_4 is one of the weaker flows in Table 2 with overall ´Γ1.2798. It has only ƳŀŊƎʎƴŀf ʎƳLJŀŎǘ 2ƴʎǘă 2LJǘʎƳŀf LJŊ2ƞŜŎǘʎ2ƴ ǿʎǘK $<_4$=0.10537. Yet in Table 4 its ´ ʎƴŘŜȄ drops to 1.0225 making it one of the stronger flows with its projection being flow R_4 ʎǘăŜfŦ ŀă ǿŜ Kŀ ØŜ $<_4$ = 1.

Clearly the results of the additive model are data dependent and contingent on the weighting structure used on slacks and deviational variables. Therein, however, also lies the strength of our approach. It offers the DM the means to explore, using sensitivity analysis, the decision space by bringing to bear a full range of potential courses of action, in combination with alternative levels of perceived certainty of likely outturn values all assessed relative to her underlying preferences over ultimate target outcomes.

Please place Tables 2, 3 and 4 near here

5.3 A schematic representation of the DEA-Markov approach

We conclude the illustration of our DEA ςMarkov decision aid with the schematic representation in Figure 1 of its main steps for the generic case.

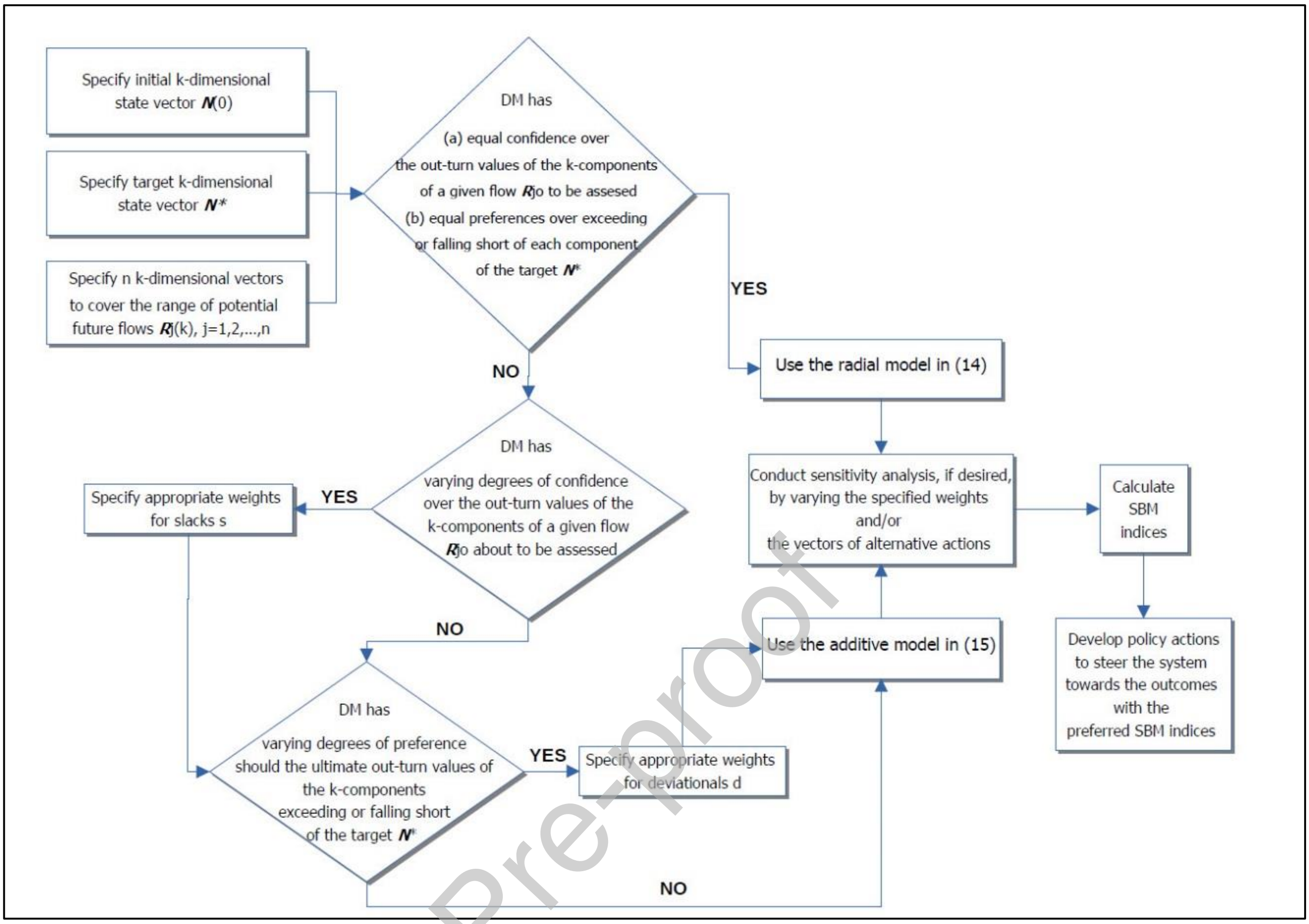

Figure 1. Schematic representation of the main steps

6. Conclusion

The main contribution of this paper is a DEA-enabled approach to aid decision making in contexts hitherto addressed using Markov Chains. The use of DEA makes it possible to circumvent complexities caused by the inherent stochastic nature of the problem addressed, through the use of a combination of user-specified potential alternatives and weights reflecting uncertainty and preferences over final outcomes. This approach will make easier the use of MC models in a variety of contexts.

To present our methodology, without loss of generality, we have used a classic Markov manpower planning model. The approach assumes an organisation which has a current personnel structure and wishes to attain an ideal structure within a given planning horizon. The method developed is used for generating and evaluating potential recruitment policies for reaching as close as possible the desired personnel structure. The approach begins with the Decision Maker specifying potential personnel flows over the planning horizon based on historical data and informed judgements of future recruitment and transition possibilities between personnel categories. The DEA assumption of convexity is then used to form an infinite set of feasible courses of action (recruitment flows). Then, still within the DEA framework, the efficacy of each initially specified course of action and of its optimal reflection within the infinite set of actions, is measured for attaining the ultimate state of the system under investigation. The most suitable of these virtual flows are identified as potential candidates that management can attempt to bring about, as far as possible, through instruments at its disposal within the organisation. Using weights in a goal programming objective function our hybrid DEA-Markov method can reflect the Decision Maker's subjective confidence about the realization of specific policies or staff flow levels as well as the desirability and importance of specific targets on staff categories. In essence, the proposed framework offers a means of converting an otherwise stochastic problem into a deterministic one, by offering the opportunity to the Decision Maker to express their attitudes on likelihoods and preferences in a tractable manner.

Our approach, presented here in the context of Markovian manpower planning, can be readily repurposed to aid other decision contexts. For example, a field where Markov chains are also used is that of health care delivery. Our approach can be used to identify the most efficacious way from the cost perspective of managing a chronic illness such as diabetes. In this case flows and transitions are affected by treatment protocols and healthcare interventions informed by medical research and expected patterns of mobility due to new drugs or healthcare policies (e.g. home treatment vs hospitalization). Models of the type we have proposed here, once developed to the more complex area where both clinical outcomes, alternative clinical interventions and quality of life are relevant, could aid health professionals in managing chronic diseases. Although Markov processes have long since been used to model mobility patterns in chronic diseases, we are not aware of any approaches comparing the efficiency of alternative policies using approaches like DEA.

We have used in this paper the classical DEA model in the context of Markov chains. However, further research is possible for additional contributions to Markov chain models that may be possible through extended DEA models. In particular, DEA models for estimating allocative efficiencies could be usefully explored. Allocative efficiencies take into account prices of the factors for delivering goods or services and reflect gains that could be made by aligning the mix of resources used with their prices. Another type of DEA model that could be explored are network models. In network DEA models, outcomes from one stage feed into subsequent stages before final outcomes are delivered. For example, we can expand the one step attainability problem we have considered into the multiple step attainability problem. For instance, in the manpower planning context the manpower structure resulting in a given step forms the starting manpower structure for the next stage and so on. This format is the type of problem handled in network DEA models. In addition, the possibility of using the transition probabilities as control variables could be explored further. This approach would give the decision maker an additional lever to drive the system towards desirable structures, e.g. manpower or health outcomes, as derived through the solution of the DEA models involved. This more direct approach is an important aspect, especially in manpower planning where it has been occasionally used in Markov modeling, with relative caution though, due its increased computational complexity.

In summary, our approach has opened an avenue for exploring how DEA models can render the handling of Markov chain models more tractable. This paper has used only the basic DEA models. Further research can explore benefits that may be available from using extended DEA models.

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Table 1: Example 1. Lambdas, slacks, deviational variables, radial ‘ and SBM indices for seven recruitment potentials ς radial model

	„1	„î	„ï	„4	„5	„6	„7	S1m (-0,0001)	S2m (-0,0001)	S3m (-0,0001)	S4m (-0,001)	d1m* (0,0001)	d1p (0,0001)	d2m (0,0001)	d2p (0,0001)	d3m (0,0001)	d3p (0,0001)	d4m (0,0001)	d4p (0,0001)	}	Œg	Œd	Œ = Œg + Œd
R1		1							100	75	150	50		50			115		45	1	1.40063	1.39881	1.96661
R2		1										50		50			115		45	1	1	1.39881	1.39881
R3			1																	1	1	1	1
R4		0.125	0.875						2.5		86.875	6.25		6.25			14.375		5.625	0.975	1.2214	1.0498	1.2821
R5					1							100		100			390		195	1	1	2.30015	2.30015
R6			1					150		290	195									1	1.579	1	1.579

																					2		2
R7		0.971143	0.02857					1.428			73	48.571		48.571			111.714		43.714	0.811	1.1231	1.3867	1.5574

* dim or dip represent the deviational variables @ and @ of the corresponding potential recruitment vector

Table 2: Example 2. Results for Flows R1-R5 - additive model in (18)

	„ í	„ î	„ 3	„ ð	„ ñ	S1m(1)	S1p(10)	S2m(1)	S2p(10)	S3m(1)	S3p(10)	d1m(1)	d1p(10)	d2m(1)	d2p(10)	d3m(1)	d3p(10)	Œg	Œd	Œ = Œg + Œd
R1	0.502888	0.465388		0.031733		0.275		1.65		0.370199						0.670199		1.03933	1.00010	1.04033
R2		1										0.725		1.35		1.3		1	1.00048	1.00048
R3	0.502888	0.465388		0.031733		0.275		2.65		0.370199						0.670199		1.05044	1.00010	1.05115
R4		0.896643		0.105337		6.275			4,48214	3.585571				1.86786		0.885571		1.27752	1.00037	1.27798
R5		0.887500			0.112500		4.4375	10.65			1.775	1.2875				1.52500		1.36880	1.00044	1.37440

Table 3: Example 2. Results for R_5 ς additive model an assortment of weights reflecting alternative likelihoods and desires

	„ 1	„ 2	„ 3	„ 4	„ 5	S1m (100)	S1p (1)	S2m (1)	S2p (100)	S3m (100)	S3p (1)	d1m (100)	d1p (1)	d2m (100)	d2p (1)	d3m (100)	d3p (1)	Œs	Œd	Œ Œs Œd
R5			0.9	0.1			6.6	8.9			3.3		0.875		1.75			1.5146	1.0037	1.5202

Table 4: Example 2. Lambdas, slacks, deviational variables, and SBM indices for six recruitment potentials ς additive model uncertainties differ across components

	„ 1	„ î	„ ï	„ 4	„ ñ	„ ò	S1m (100)	S1p (1)	S2m (1)	S2p (10)	S3m (1)	S3p (10)	d1m (1)	d1p (10)	d2m (1)	d2p (10)	d3m (1)	d3p (10)	Œs	Œd	Œ = Œs + Œd
R1			0,11667			0.88333			1,65					0,27500			0,30000		1.0204	1.0009	1.0213
R2		1											0.725		1.35		1.30000		1	1.0048	1.0048
R3			0,1166			0,8833			2,65					0,2750			0,3000		1.0	1.0	1.0

			7			3								0			0		315	009	325
R4				1										6,27500	6,35000			2,7	1	1.0225	1.0225
R5		0.88750			0.11250			4.4375	10.65			1.775	1.2875				1.52500		1.3680	1.0044	1.3740
R6		0.05441			0.03676	0.90882											0.46471		1	1.0007	1.0007