Operations Research Perspectives 8 (2021) 100184

Contents lists available at ScienceDirect

Operations Research Perspectives

journal homepage: www.elsevier.com/locate/orp

A novel best worst method robust data envelopment analysis: Incorporating decision makers' preferences in an uncertain environment

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ARTICLE INFO

Keywords:
Best Worst Method (BWM)
Robust Optimization
Data Envelopment Analysis (DEA)
Airline Efficiency

ABSTRACT

Data Envelopment Analysis (DEA) has been widely applied in measuring the efficiency of Decision-Making Units (DMUs). The conventional DEA has three major drawbacks: a) it does not consider Decision Makers' (DMs) preferences in the evaluation process, b) DMUs in this model are flexible in weighting the criteria to reach the maximum possible efficiency, and c) it ignores the uncertainty in data. However, in many real-world applications, data are uncertain as well as imprecise and managers want to impose their opinions in decision-making procedure. To address these problems, this paper develops a novel multi-objective Best Worst Method (BWM)-Robust DEA (RDEA) for incorporating DMs' preferences into DEA model in an uncertain environment. The proposed model tries to provide a new efficiency score which is more reliable and compatible with real problems by taking the advantages of the BWM to apply experts' opinions and RDEA to model the uncertainty This bi-objective BWM-RDEA model is solved utilizing amin-max technique and so as to illustrate its usefulness, this model is implemented for assessing Iranian airlines.

1. Introduction

In today's competitive marketplace,improving the performance of a system over time is of cardinal importance [29]. To this end, managers are required to continuously monitor the current status of the system and according to this, adopt appropriate approaches to achieve an acceptable level of efficiency. Indeed, without gaining knowledge about the system's progress towards reaching goals and without obtaining feedback on the impact of improvement actions in the system, continuous performance improvement will not be possible [43]. Since efficiency scores can provide managers with valuable information about the system's performance, this issue has received the attention of both academia and industrial experts in the wide range of fields. There are many real-world applications that performance evaluation plays a significant role in reaching the goals of systems. For example, in manufacturing industries, financial performance evaluation is crucial in increasing the revenue of companies [1]. Due to the indispensable role of healthcare systems in our lives, it is needed to evaluate the performance of hospitals to achieve desired and efficient conditions [35]. Performance of renewable energy resources is an issue which affects our surrounding environment and society [10]. The air transportation industry is another important field that regarding its impact on countries' economic growth, efficiency measurement of airlines is necessary [23]. Since uncertainty is the inherent feature of any business and activity, ignoring this item in the performance evaluation will lead to unreliable results; therefore, it is needed to consider uncertain parameters in the evaluation process.

Data Envelopment Analysis (DEA) is one of the most popular nonparametric methods for measuring the efficiency of Decision-Making Units (DMUs) applied in numerous industries [50]. DEA models have been used in diverse studies to calculate efficiency scores. The conventional type of this model allows DMUs to reach their maximum efficiency degree with the most favorable set of weights [18]. In fact, DMUs are flexible to choose the weights of inputs and outputs without any limitation until their efficiency became maximum. In some cases, the conventional DEA models assign zero or extreme values to some inputs or outputs weights in order to increase the efficiency degree of DMUs [34]. Besides, many DMUs may appear to be efficient (always more than one DMUs) due to the flexibility in the weights' selection for input and output variables. Indeed, each unit tries to achieve the maximum efficiency score by favorable weights, independently of other units. Hence, DEA is not capable to distinguish and compare the efficient DMUs [30].

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http://www.deazone.com

https://doi.org/10.1016/j.orp.2021.100184
Received 15 May 2020; Received in revised form 9 March 2021; Accepted 30 March 2021
Available online 15 April 2021

In order to solve these problems and incorporate Decision Makers' (DMs) preferences, researchers have proposed four approaches including common weights, weight restriction, Assurance Region (AR), and cone-ratio [33]. In the first model, weights are not flexible since the weights assigned to inputs and outputs for all DMUs are same.These models were first introduced by Roll et al. [46]. Then, Kao and Hung [26] developed their models and presented a nonlinear Common Weight DEA (CWDEA) model. Following this, in 2010, Zohrehbandian et al., improved the CWDEA model by introducing a linear goal programming approach for finding the common weights of inputs and outputs.The weight restriction approach was initially introduced by Dyson and Thanassoulis [15]. This approach considers DMs' priority and adds some extra constraints on the inputs and/or outputs weights in multiplier DEA models to reduce the flexibility of weights selection and distinguish efficient DMUs in DEA models [14]. However, this technique may lead to infeasible efficiency scores. The next one, AR approach, was first proposed by Thompson et al., [49]. This method imposes ratios between weights based on the DMs' preferences and limits the weights by assigning a lower and an upper bound for some selected sets of weights. The AR techniques are sensitive to the scaling of inputs and outputs. Finally, the cone-ratio approach, presented by Charnes et al., [12], is based on pre-selection of DMUs or favor inputs/outputs and the inputs and outputs weights are allowed taking even zero values.

On the other hand, conventional DEA models could not consider uncertainty in input and output data which is the inevitable feature of real-world applications. There are five extended DEA models to deal with this problem including Chance Constraint DEA (CCDEA), Imprecise DEA (IDEA), bootstrap DEA, Fuzzy DEA (FDEA), and Robust DEA (RDEA) models. In CCDEA, some or all inputs and outputs data are considered as random variables with certain probability distribution function,and the constraints of the DEA are considered as chance constraints. Quadratic programming technique is applied to solvethis version of the DEA models. However, finding a suitable distribution function for the data in real-world applications isdifficult, and by considering different probability distribution functions for data, different efficiency scores are achieved. In IDEA, different bounds are considered for data and the efficiency scores are estimated as an interval.In this case, the DEA model is changed to nonlinear programming which makes it difficult to reachan optimal solution [33]. In the next one, the bootstrap DEA which is used for re-sampling and re-producing the new units, a confidence interval is determined for efficiency scores. In this model, an infinite population of units corresponding to a data generating process is considered, and it is assumedthat the current set is a sample of the population [41]. FDEA models are used to consider some data as fuzzy numbers; therefore, the efficiency score of units with fuzzy inputs and outputs can be calculated. Readers can refer to Emrouznejad et al., [16] to get more details about the FDEA models. The term "robust optimization" includes approaches which protect DMs facing uncertainty [53]. The RDEA model tries to find out a robust solution that ensures the solution remains close to optimal by changing input data. This model can dealwith uncertain data and also data with an unknown probability distribution. Besides, using the RDEA model, the conventional DEA constraints will be immune against violation, and reliable efficiency scores will be achieved [39]. Regarding these advantages, the RDEA model will be developed in this paper. The robust optimization technique was originally presented by Soyster [42] and later developed by Ben-Tal and Nemirovski [4,5] and [6], Bertsimas and Sim [7] and [9], and Bertsimas et al., [8]. Sadjadi and Omrani [38] introduced a new RDEA model with assuming the existence of perturbation in data. Also, theydemonstrated that the complexity of solving RDEA based on the Bertsimas and Sim [8] approach is less than Ben-Tal and Nemirovski [6] approach. Sadjadi et al., [39] extended the conventional super-efficiency DEA model and presented robust super-efficiency DEA model to estimate DMUs' efficiency in an uncertain environment with assumingellipsoidal uncertainty set for data. Omrani [33]considered uncertain data in CWDEA model and presented Robust CWDEA model. He evaluated and ranked provincial gas companies in Iran using this-model. Salahi et al., [40] presented optimistic robust optimization for a common set of weights in DEA model with giving robust counterpart of the DEA-CRS model and calculated robust efficiency of DMUs. Arabmaldar et al., [3] investigated equivalent constraints in conventional DEA models with considering uncertainty. They presented a new RDEA model based on Bertsimas and Sim [8]. Also, they applied two linear robust super-efficiency DEA models for evaluating and ranking the DMUs. Zahedi-Seresht et al., [52] used RDEA model with considering different scenarios for evaluating decision units. Their model was based on the robust approach proposed by Mulvey et al., [31].Alizadeh and Omrani [2]addressedMulti Response Taguchi (MRT) problems with uncertainty in data based on Neural Network (NN)and RDEA model. They used NN technique to design experiments of the MRT method and applied RDEA to handle uncertainty in NN results.MardaniNajafabadi and Taki [32] used RDEA model to evaluate the energy flow and optimize the energy consumption for greenhouse cucumber production by considering different values of uncertainty and probability levels.

As can be seen, numerous studies have been attempted to cover the shortcomings of the conventional DEA models. However, an approach is still needed to calculate the efficiency score of DMUs based on the DEA model features and also the experts' opinions simultaneously under the uncertain circumstance. In some cases, DMs want to evaluate DMUs by considering different preferences for the criteria. This study aims to incorporate the experts' opinions about inputs and outputs weights into DEA models and simultaneously considering uncertainty in data. The proposed model reduces the flexibility of variables weight by incorporating DMs' preferences and opinions in the DEA model based on the Best Worst Method (BWM) which may lead to more compatible outcomes with practical applications. This method is one of the latest Multi-Criteria Decision-Making (MCDM) techniques applied to determine the weight of criteria based on the expert's judgments. In other words, the weights of criteria are determined according to the preference values (scales between 1 and 9) of the best criterion overall all criteria and all criteria over the worst criterion. Although the BWM is based on the pairwise comparisons like Analytic Hierarchy Process (AHP) and Analytic Network Process (ANP) methods, it requires less pairwise comparison and the results produced have higher consistency ratio and are more reliable [45]. Besides, in the conventional DEA models, the uncertainty of data which is an undeniable feature of real-world problems is ignored; hence the results will be unreliable. For this reason, RDEA model is developed in this study. The presented model empowers managers to evaluate the performance of DMUs based on the importance of criteria while considering the uncertainty of data. This approach can provide various sets of robust efficiency scores, and DMs can analyze the effect of one criterion over the performance of DMUs by considering various pairwise comparisons.To implement the proposed BWM-RDEA model, firstly, the best and the worst criteria are selected among the inputs and outputs and then, the objective function and constraints of BWM are added to DEA model. In the next step, it is assumed that there is uncertainty in inputs and outputs data. Therefore, the RDEA model is considered to handle the uncertainty in data. Unfortunately, there is an equality constraint in the conventional DEA model which does not allow considering uncertainty in input data. For solving this problem, according to Omrani [33], a specific version of the DEA model is used which has not any equality constraint in multiplier form. The final proposed BWM-RDEA model is bi-objective linear programming which is solved by the min-max technique. So as to illustrate the applicability of this model, it is applied to measure the efficiency of 14 airlines in Iran.

The rest of this paper is organized as follows: In Section 2, the DEA, BWM, and RDEA models are expressed. Section 3 describes the proposed bi-objective BWM-RDEA model. Section 4 introduces the case study of this paper and provides a brief review of studies carried out on airlines performance evaluation. Sections 5 and 6 discusses the results obtained based on the proposed model and presents some recommendation for policymakers, respectively. Finally, the conclusion of the paper is summarized in Section 7.

2. Methodology

In this section, the basic methods which are used in the proposed BWM-RDEA model are described.

2.1. Data envelopment analysis (DEA)

DEAis a nonparametric method used to measure the relative efficiency of DMUs with multiple inputs and outputs. In this method, efficiency is calculated as a ratio of a weighted sum of outputs to a weighted sum of inputs and whenever a DMU is placed on the frontier formed by data, is evaluated as an efficient unit. Since DEA models are solved by linear programming and this method is not sensitive to the unit of measurement, inputs and outputs can be based on different units.Each of these models may be output-oriented or input-oriented. The first one tries to maximize the outputs while the quantity of input is kept constant and the second one, conversely, tries to minimize the inputs required for a given level of outputs. The input-oriented DEA-CCR model is as follows [13]:

$$\begin{aligned}
&\theta_o^{CCR} = \max \sum_{i=m+1}^{m+s} w_i y_{io} \\
&st: \\
&\sum_{i=m+1}^{m+s} w_i y_{ij} - \sum_{i=1}^{m} w_i x_{ij} \le 0,\ j = 1,\dots,n \\
&\sum_{i=1}^{m} w_i x_{io} = 1 \\
&w_i \ge 0,\ i = 1,\dots,m+s
\end{aligned} \tag{1}$$

In the model (1), there are n DMUs (j= 1,…n) and eachDMU produces s outputs $y_{(m+1)j},\dots,y_{(m+s)j}$ using m inputs $x_{1j},\dots,x_{mj}$. Also, $\theta_o^{CCR}$and w_i denote the efficiency score of DMU under evaluation and the weight of ith criterion, respectively.

2.2. Best worst method (BWM)

The BWM is a novel MCDM method which has been initially proposed by Rezaei [44], to determine the weighs of criteria through pairwise comparisons like AHP and/or ANP methods. There are two main advantages in BWM in compared with these methods: First, it uses less pairwise comparison and second it has a higher consistency ratio. In BWM, preferences of the best criterion over other criteria and preferences of other criteria over the worst criterion are determined and assigned to a scale between 1 and 9 and therefore the weights of criteria are specified [20]. The steps of the BWM is as follows ([44] and [45]):

1. Determine a set of criteria as $\{c_1, c_2, \dots, c_n\}$.
2. Determine the best and the worst criteria by an expert or a team of experts.
3. Determine the preference vector of the best criterion over all criteria using numbers between 1-9 as: $A_B = (a_{B1}, a_{B2}, \dots, a_{Bn})$. Note that$a_{BB} = 1$.
4. Determine the preference vector of all criteria over the worst criterion using numbers between 1-9 as: $A_W = (a_{1W}, a_{2W}, \dots, a_{nW})^T$. Note that $a_{WW} = 1$.
5. Find the optimal weights $(w_1^*, w_2^*, \dots, w_n^*)$.

If the preferences are $a_{Bi}$and a_{iW}, the goal is to find the optimal weights which minimize the absolute maximum difference of the $|w_B/w_i - a_{Bi}|$,and $|w_i/w_W - a_{iW}|$. By assuming the sum of weights equal to one and non-negativity constraints, Rezaei [45] introduced the following linear BWM model:

$$\begin{aligned}
&\min \xi \\
&s.t: \\
&|w_i - a_{iW} w_W| \le \xi,\quad i = 1,\dots,n \\
&|w_B - a_{Bi} w_i| \le \xi,\quad i = 1,\dots,n \\
&\sum_{i=1}^{n} w_i = 1 \\
&w_i \ge 0,\qquad j = 1,\dots,n
\end{aligned} \tag{2}$$

In this paper, we intend to integrate the above linear BWM (model 2) in DEA to calculate the weights of criteria.

2.3. The BWM-DEA model

In this section, the BWM-DEA model is explained. As can be seen in the DEA model (1), there is a normalization equality constraint $\sum_{i=1}^{m} w_i x_{io} = 1$ which is different from the normalized equality constraint $\sum_{i=1}^{n} w_i = 1$ of the BWM model (2). Instead of DEA model (1), let us consider the DEA model (3) as follows:

$$\begin{aligned}
&\max \sum_{i=m+1}^{m+s} w_i y_{io} - \theta_o^{CCR} \sum_{i=1}^{m} w_i x_{io} \\
&s.t: \\
&\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} \le 0 \qquad j = 1,\dots,n \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&w_i \ge 0,\ i = 1,\dots,m+s
\end{aligned} \tag{3}$$

where$\theta_j^{CCR}$is the efficiency score estimated for jth DMU in model (1).In model (3), the normalization constraint $\sum_{i=1}^{m} w_i x_{io} = 1$ has been replaced by $\sum_{i=1}^{m+s} w_i = 1$. The normalization constraint $\sum_{i=1}^{m+s} w_i = 1$ is similar to the weight's normalization in BWM.

Zohrehbandian et al., [54] demonstrated that the models (1) and (3) have the same optimal solution. The proposed bi-objective BWM-DEA model can be expressed as follows:

$$\begin{aligned}
&\max f_1 = \sum_{i=m+1}^{m+s} w_i y_{io} - \theta_o^{CCR} \sum_{i=1}^{m} w_i x_{io} \\
&\max f_2 = -\xi \\
&s.t: \\
&\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} \le 0,\quad j = 1,\dots,n \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&|w_B - a_{Bi} w_i| \le \xi,\quad i = 1,\dots,m+s \\
&|w_i - a_{iW} w_W| \le \xi,\quad i = 1,\dots,m+s \\
&w_i \ge 0,\quad i = 1,\dots,m+s
\end{aligned} \tag{4}$$

In the above model, the first objective function and the first constraint belong to the DEA model (3). The second objective function and the third and fourth constraints belong to the BWM. The second constraint $\sum_{i=1}^{m+s} w_i = 1$ belongs to both DEA and BWM. The constraint $|w_B - a_{Bi} w_{Bi}| \le \xi$ can be transformed into two linear constraints $w_B - a_{Bi} w_{Bi} \le \xi$ and $a_{Bi} w_{Bi} - w_B \le \xi$ easily.

The model (4) is a bi-objective programming model and we solve it using a min-max technique and subsequently,obtain optimal weights $(w_1^*, \dots, w_m^*, w_{m+1}^*, \dots, w_{m+s}^*)$ for inputs and outputs. As the result, the efficiency score of jth DMU is calculated by Equation (5) as follows:

$$\theta_j^{BWM-DEA} = \frac{\sum_{i=m+1}^{m+s} w_i^* y_{ij}}{\sum_{i=1}^{m} w_i^* x_{ij}} \tag{5}$$

2.4. The robust DEA model (RDEA)

In many real-world applications, there is uncertainty in inputs and outputs data. Wang and Wei [51] demonstrated the DEA results are not credible when uncertainty is not considered. One of the approaches for handling the uncertainty of data is robust optimization. Based on the robust optimization approach, RDEA model was firstly introduced by Sadjadi and Omrani [38] and then extended by Omrani [33]. In RDEA model, it is assumed that there is a perturbation in inputs and outputs data. Indeed, in this case, the data are not stochastic or finding the probability distribution function for data is not obvious. Hence, the CCDEA models cannot be applied in such situations. For modeling the perturbation in data and estimating the efficiency scores in an uncertain environment, RDEA model has shown to be a useful tool. With existing perturbations in the data, the efficiency scores will not be reliable. In other words, the constraint $\sum_{i=m+1}^{m+s} w_i y_{ij} - \sum_{i=1}^{m} w_i x_{ij} \le 0$ of the DEA model might be violated and some of the DEA efficiency scores may be greater than one if the perturbation of data is considered. In this paper, we integrate the RDEA model of Omrani [33] to the BWM-DEA model presented previously. For this purpose, consider the RDEA model as follows [33]:

$$\begin{aligned}
&\max \sum_{i=m+1}^{m+s} w_i y_{io} - \theta_o^{CCR} \sum_{i=1}^{m} w_i x_{io} - p_o \Gamma_o - \sum_{o\in J_o} q_{io} \\
&s.t: \\
&\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} + p_j \Gamma_j + \sum_{j\in J_j} q_{ij} \le 0 \qquad j = 1,\ldots,n \qquad i = 1,\ldots,m+s \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_j + q_{ij} \ge e_{ij} y_{ij} z_i \qquad \forall i,j \qquad i = m+1,\ldots,m+s \\
&p_j + q_{ij} \ge e_{ij} x_{ij} z_i \qquad \forall i,j \qquad i = 1,\ldots,m \\
&-z_i \le w_i \le z_i \qquad i = 1,\ldots,m+s \\
&w_i \ge \varepsilon \qquad i = 1,\ldots,m+s \\
&p_j, q_{ij}, z_i \ge 0
\end{aligned} \tag{6}$$

where, e_{ij} is the value of the perturbation for ith data (input or output) in jth constraint. Let $J_o$and J_j be the set of coefficients that are subject to uncertainty in constraint jth. There is a parameter Γ_j for each j that not integer necessarily. Parameter Γ_j has the role of adjusting the robustness of the proposed method against the level of conservatism of the solution. As the value of $\Gamma_j$increase, the protection level of the model against violation increase, too Bertsimas and Sim [8]. For fully protection of the RDEA model against the perturbations, the $\Gamma_j$can be selected as the sum of the number of inputs and outputs [33,38].

3. The proposed BWM-RDEA model

In this section, a novel bio-objective BWM-RDEA model is explained. In order to incorporate the DMs' preferences into RDEA model, the objective function and constraints of BWM are added to RDEA model. In fact, the proposed BWM-RDEA model considers DMs' preferences and uncertainty in data, simultaneously. To put it in another way,the RDEA models consider no preferences of DMs in the evaluation process, while the results of BWM are only based on the subjectivity of DMs. This model takes the advantages of single BWM and RDEA models simultaneously and can cover some of the shortcomings of every single model. Regarding the symbols introduced in the previous section, this model is as follows:

$$\begin{aligned}
&\max f_1 = \sum_{i=m+1}^{m+s} w_i y_{io} - \theta_o^{CCR} \sum_{i=1}^{m} w_i x_{io} - p_o \Gamma_o - \sum_{o\in J_o} q_{io} \\
&\max f_2 = -\xi \\
&s.t: \\
&\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} + p_j \Gamma_j + \sum_{j\in J_j} q_{ij} \le 0 \qquad j = 1,\ldots,n \qquad i = 1,\ldots,m+s \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_j + q_{ij} \ge e_{ij} y_{ij} z_i \qquad \forall i,j \qquad i = m+1,\ldots,m+s \\
&p_j + q_{ij} \ge e_{ij} x_{ij} z_i \qquad \forall i,j \qquad i = 1,\ldots,m \\
&-z_i \le w_i \le z_i \qquad i = 1,\ldots,m+s \\
&w_i \ge \varepsilon \qquad i = 1,\ldots,m+s \\
&|w_B - a_{Bi} w_i| \le \xi \qquad i = 1,\ldots,m+s \\
&|w_i - a_{iW} w_W| \le \xi \qquad i = 1,\ldots,m+s \\
&p_j, q_{ij}, z_i \ge 0
\end{aligned} \tag{7}$$

where, the constraints such as $|w_B - a_{Bi} w_{Bi}| \le \xi$ which can easily be transformed to two linear constraints $w_B - a_{Bi} w_{Bi} \le \xi$ and $a_{Bi} w_{Bi} - w_B \le \xi$.

Since the model (7) have two distinct objectives, it should be converted to a single-objective problem. In fact, optimization problems with more than one objective function have a set of non-dominate optimal solutions; because the objective functions are often in conflict together and approaching to an optimal solution for one of which lead to deviation from the optimal solution of other objective functions [25]. Hence, in many studies, scholars tend to convert multi-objective problems to a single-objective problem. Although there are different methods to solve such model, in this study the min-max technique is used to solve the proposed model regarding its advantages compared with other methods. Firstly, the min-max method has lesser computing steps and higher speed. Besides, this technique guarantees to obtain a solution for non-convex multi-objective optimization problems [22]. The min-max technique initially introduced by Lightner and Director [27] and is based on minimizing the maximum deviations of the objective functions from their ideal value. The ideal value of each objective function is also obtained by solving each objective function individually with the existing constraints of the problem. Regarding these descriptions, the min-max technique is used as follows to solve the model (7):

$$\begin{aligned}
&\text{minmax}\left\{\left[\left(f_1^* - \sum_{i=m+1}^{m+s} w_i y_{io} - \theta_o^{CCR} \sum_{i=1}^{m} w_i x_{io} - p_o \Gamma_o - \sum_{o\in J_o} q_{io}\right)\right], [f_2^* - (-\xi)]\right\} \\
&s.t: \\
&\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} + p_j \Gamma_j + \sum_{j\in J_j} q_{ij} \le 0 \qquad j = 1,\ldots,n \qquad i = 1,\ldots,m+s \\
&\sum_{i=1}^{m+s} w_i = 1 \\
&p_j + q_{ij} \ge e_{ij} y_{ij} z_i \qquad \forall i,j \qquad i = m+1,\ldots,m+s \\
&p_j + q_{ij} \ge e_{ij} x_{ij} z_i \qquad \forall i, \qquad i = 1,\ldots,m \\
&-z_i \le w_i \le z_i \qquad i = 1,\ldots,m+s \\
&w_i \ge \varepsilon \qquad i = 1,\ldots,m+s \\
&|w_B - a_{Bi} w_i| \le \xi \qquad i = 1,\ldots,m+s \\
&|w_i - a_{iW} w_W| \le \xi \qquad i = 1,\ldots,m+s \\
&p_j, q_{ij}, z_i \ge 0
\end{aligned} \tag{8}$$

wheref_1^* and f_2^* are the ideal values of first and second objective functions, respectively. In order to calculate f_1^* and f_2^*, the objective functions f_1 and f_2 are optimized on the constraints of model (8), separately. In other words, for obtaining the ideal solution for f_1, the objective function f_2 is eliminated from the model (8) and the model is solved using the objective function f_1 . Also, the ideal value for f_2 is obtained by eliminating the objective function f_1 from the model (8). It is clear that the min-max model (8) is easily converted to a linear model (9) as follows:

$$
\begin{aligned}
& \min\ \alpha \\
& s.t: \\
& f_1^* - \left(\sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} + p_j \Gamma_j + \sum_{j \in J_j} q_{ij} \right) \le \alpha \\
& f_2^* - (-\xi) \le \alpha \\
& \sum_{i=m+1}^{m+s} w_i y_{ij} - \theta_j^{CCR} \sum_{i=1}^{m} w_i x_{ij} + p_j \Gamma_j + \sum_{j \in J_j} q_{ij} \le 0 \qquad j=1,\dots,n \quad i=1,\dots,m+s \\
& \sum_{i=1}^{m+s} w_i = 1 \qquad\qquad (9) \\
& p_j + q_{ij} \ge e_{ij} y_{ij} z_i \quad \forall i,j \quad i=m+1,\dots,m+s \\
& p_j + q_{ij} \ge e_{ij} x_{ij} z_i \quad \forall i,j \quad i=1,\dots,m \\
& -z_i \le w_i \le z_i \qquad i=1,\dots,m+s \\
& w_i \ge \varepsilon \qquad i=1,\dots,m+s \\
& |w_B - a_{Bi} w_i| \le \xi \qquad i=1,\dots,m+s \\
& |w_i - a_{iW} w_W| \le \xi \qquad i=1,\dots,m+s \\
& p_j, q_{ij}, z_i \ge 0
\end{aligned}
$$

Assume (w_1^*, ..., w_m^*, w_{m+1}^*, ..., w_{m+s}^*) is an optimal solution of model (9). Then, the new efficiency score of DMUj is calculated using Equation (5).

4. 4. A real application of assessing airlines

One of the important application fields of the DEA models is evaluating the performance of airlines due to the undeniable role of air transportation sector in the economic prosperity of countries. Indeed, to make sure that the quality service is provided in this sector, the efficiency of the airlines should be measured [17]. Several researchers have evaluated the performance of airlines using DEA models. Ha et al., [19] studied the relationship between airline market structure and airport productivity. They measured the efficiency of airports using DEA and Stochastic Frontier Analysis (SFA). Chang et al., [11] surveyed the impact of geographical characteristics and service strategies on the performance of airports in Chinese. They employed the DEA model to calculate the technical efficiency of 41 airports in 2008. Tavassoliet al., [47] offered a Slacks-Based Measure NDEA (SBM-NDEA) model to determine the airlines' technical efficiency and service effectiveness. They assisted the performance of 11 domestic airlines in Iran in 2010. Using DEA model, Jain and Natarajan [24] investigated the technical and scale efficiencies of airlines in India during 2006-2010. Omrani and Soltanzadeh [36] presented a dynamic NDEA model to evaluate airlines' efficiency in Iran. Hadi-Vencheh et al., [21] proposed a new slack-based non-radial DEA model in order to evaluate the sustainability of airlines considering CO_2 emissions and applied it in Chinese airline assessment. Losa et al., [28] extended slacks-based network DEA model to a dynamic framework in order to measure the overall efficiency of the world's largest airline groups so as to provides more accurate definition of overall and divisional efficiencies.Phung [37] proposed a smixed nNetwork DEA with Shared Resources andTavassoli et al., [48] proposed a new super-efficiencyDEA model to measure the efficiency of airlines with zero and stochastic data.

In this study, the proposed BWM-RDEA model is implemented to measure the efficiency of Iranian airlines. Civil Aviation Organization (CAO) of Iran is a governmental organization established in 1946 and works under the supervision of the Ministry of Roads and Urban Development. CAO is responsible to assess and controls the airport's performance as well as policy-making and implementing the rules in the air transportation industry. There are several local airlines in Iran such as Iranair,Aseman, Ata, Mahan, Taban, Kaspianetc, etc which are responsible to carry passenger and cargo. In this study, 14 airlines are evaluated based on the date reported by CAO[1] in 2016 and the criteria are selected from the set of criteria used in previous studies regarding the availability of data in this case study. However, financial criteria such as income, profits, operating costs, fuel costs, employee wages, cost of flight equipment, etc. are not considered due to the lack of data in the statistical yearbooks of the Iranian CAO. Therefore, we only focus on the operational efficiency of airlines as follows:

Inputs:

- The number of employees: Number of staff working in the airline such as engineering, attendant, pilot, flight, etc.
- Available seat-kilometer: Sum of the products obtained by multiplying the number of passenger seats available for sale on each flight by the flight distance.
- Available ton-kilometer: Sum of the products obtained by multiplying the number of tons available for the carriage of revenue load on each flight by the flight distance.
- Fleet seat: Number of available seats in the fleet.

Outputs:

- Number of flights: Number of performed flights.
- Passenger-kilometer performed: Sum of products obtained by multiplying the number of revenue passengers carried on each flight by the flight distance.
- Ton-kilometer performed: Multiplication of carried weight (ton) in every origin and destination of flight by the distance between the same origin and destination.

The schematic structure of the application and also all of the airlines and the value of the criteria for each of which have been show in Figure 1 and Table 1 respectively.

5. Results and discussions

In this section, the proposed BWM-RDEA model is applied to evaluate efficiency of the Iranian airlines. Results are discussed in five subsectionsof BWM, DEA, BWM-DEA, RDEA, and BWM-RDEA results, respectively.

5.1. Result of the BWM

In this part, the results from the proposed linear BWM are explained. To implement this weighting technique, firstly, the set of criteria used in this case are defined as follows: employees (c_1), available seat-km (c_2), available ton-km (c_3), number of fleet seat (c_4), number of flights (c_5), passenger-km performed (c_6), and ton-km performed (c_7). Based on the experts' preferences, c_6 and c_4 are determined as the best and the worse criteria, respectively. In the next step, vector of best criterion over all criteria is assigned as $A_6 = (a_{61}, a_{62}, \dots, a_{67})^T$= (6,7,6,8,5,1,3) and vector of all criteria over worst criterion is assigned as $A_4 = (a_{14}, a_{24}, \dots, a_{74})^T$= (4,3,2,1,5,8,7). Using the model (2), the optimal weights are obtained as w_1^* = 0.0883, w_2^* = 0.0757, w_3^* = 0.0883, w_4^* = 0.0401, w_5^* = 0.1059, w_6^* =0.4253, and w_7^* =0.1765. As can be seen, c_6

[1] http://www.cao.ir/statistical-yearbook

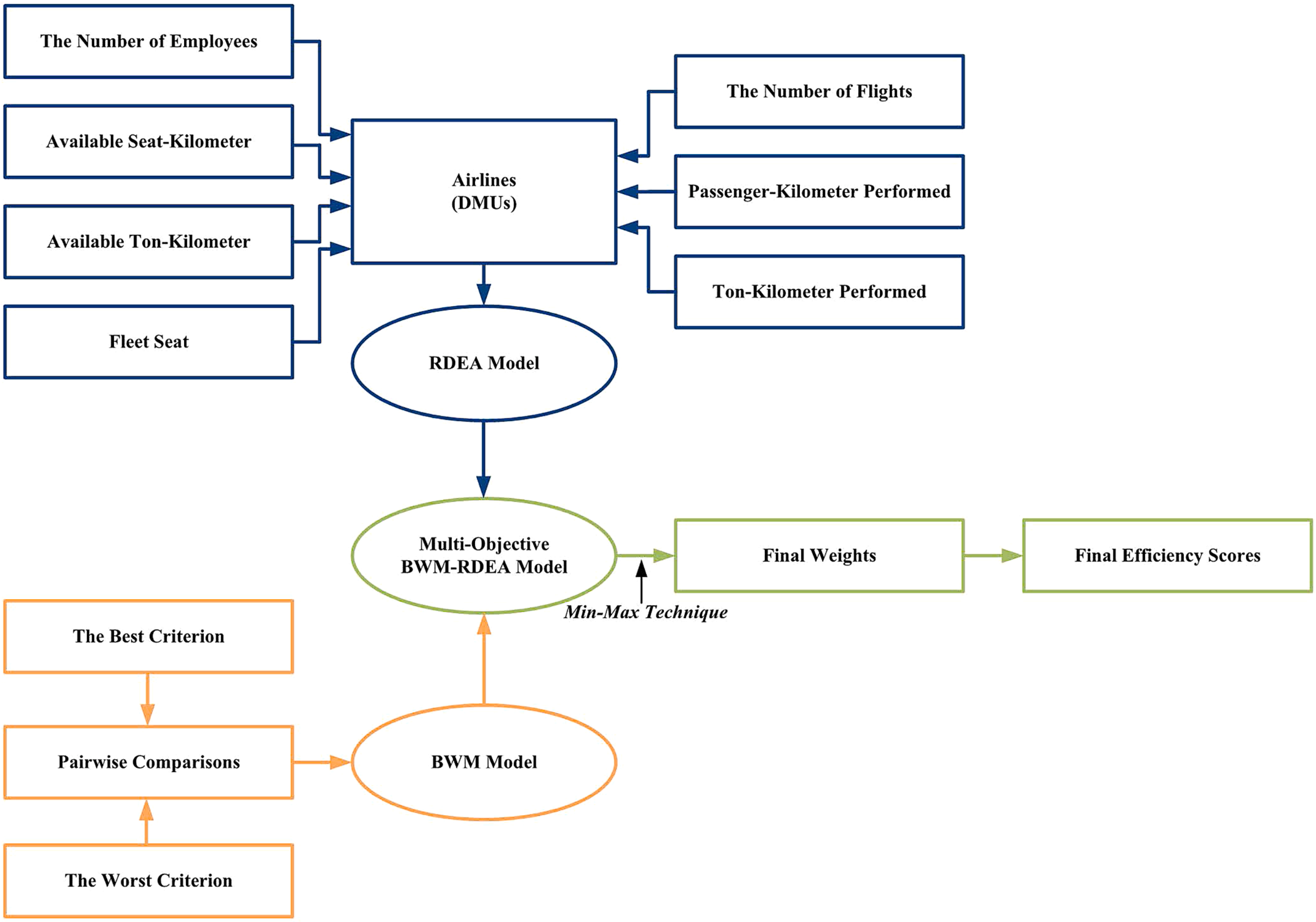

Fig. 1. The schematic structure of the application.

Table 1
The data of 14 Iranian airlines in 2016.

	Number of employees	Available seat- km	Available ton- km	Number of fleet seat	Number of flights	Passenger-km performed	Ton-km performed
Iranair	11118	1838923	210542	2694694	20212	1511100	155781
Iranairtour	683	1216026	109171	1471151	9439	1068524	96164
Ata	606	1371597	135969	2134435	13294	1183086	130268
Atrak	225	153794	15468	175835	1009	122514	13079
Aseman	3158	1925524	181954	2578760	25697	1682751	152382
Taban	832	1248996	124233	1535950	10225	1161512	113569
Zagros	684	2210951	221093	2598240	16239	1559286	141894
Pouyaair	103	11509	44190	21440	567	8860	17043
Gheshm	832	662626	56329	940416	9913	556859	47212
Kaspian	539	1070398	131057	1319386	8849	861441	89139
Kishair	826	1517847	166213	1843682	14005	1260049	126949
Mahan	4363	3295267	518048	3782430	20730	2591769	234907
Meraj	334	26319	30477	363640	2164	18679	23304
Naft	590	493163	48814	813830	10255	395334	34808

which is the most important for the experts, has achieved the highest weight. Also, due to the low importance of c_4, it has achieved the minimum weight. Hence, the arrangement of these criteria regarding the value of weights is $c_6 > c_7 > c_5 > c_1 = c_3 > c_2 > c_4$.

5.2. Result of the DEA model

In this section, the DEA model (1) has been solved to find out the optimal weights of the criteria. To this end, initially, the row data are normalized applying mean normalization. Table 2 shows the generated optimal weights for all airlines. Considering the mean of weights, the criteria can beranked as $c_4 > c_1 > c_7 > c_3 > c_5 > c_6 > c_2$. It can be seen that the importance of each criterion calculated by DEA is different with the one obtained from the BWM. Takepassenger-km performed (c_6) as an example. This criterion placed in the first rank in the BWM while, it is in sixth rank in the DEA model. In addition to this, c_4 which was the worst criterion based on the experts' preferences in the BWM, has the highest weight according to the DEA model. Besides, the flexibility in choosing weights by DEA has led to assigning the zero weight for several inputs and outputs. The efficiency scores generated by DEA model (1) have been shown in the last column of Table 2. It is clear that due to a large number of inputs and outputs in compared with the small number of

Table 2
Optimal weights and efficiency score of the DEA.

DMU	W1	W2	W3	W4	W5	W6	W7	Efficiency
Iranair	0.0000	0.6146	0.0058	0.0372	0.071	0.5237	0.0000	0.9159
Iranairtour	0.0000	0.4606	0.7043	0.0000	0.0189	0.9203	0.0000	1.0000
Ata	0.0000	0.5342	0.4169	0.0000	0.0972	0.395	0.3177	1.0000
Atrak	0.0000	0.0000	3.8532	5.261	0.0000	0.0000	7.3075	0.9721
Aseman	0.0000	0.0000	0.2394	0.4283	0.1601	0.0000	0.4167	1.0000
Taban	0.0000	0.0000	0.3846	0.6882	0.2572	0.0000	0.6697	1.0000
Zagros	0.5731	0.0000	0.1642	0.3212	0.2902	0.3806	0.0000	1.0000
Pouyaair	14.7375	0.0000	0.1436	7.5477	7.9894	1.324	3.4511	1.0000
Gheshm	0.186	0.0000	1.3037	0.6721	0.4538	0.0000	1.2758	1.0000
Kaspian	1.2031	0.1061	0.0000	0.6536	0.0795	0.6603	0.4025	0.9950
Kishair	0.8305	0.0732	0.0000	0.4512	0.0549	0.4558	0.2778	1.0000
Mahan	0.0000	0.0000	0.0000	0.4206	0.0000	0.3492	0.0000	0.9061
Meraj	0.0000	0.7705	2.2373	2.2072	0.0000	0.0000	4.2191	1.0000
Naft	0.0475	0.3374	0.8259	1.1034	0.5541	0.0000	1.4427	1.0000
Mean	1.2556	0.2069	0.7342	1.4137	0.7162	0.3578	1.1664	0.9849

DMUs, the DEA model (1) has failed to distinguish the airlines; hence, in this case, most airlines have the efficiency score equal to one expect Iranair, Atrak, Kaspian and Mahan.

5.3. Results of the BWM-DEA model

The bi-objective BWM-DEA model (4) is solved using the min-max technique. This model tries to find out optimal weights based on maximizing the efficiency of DMUs under evaluation and minimizing the deviations from DMs' preferences. The weights generated and efficiencies calculated have been reported in Table 3. As can be seen, the mean of weights is 0.1446, 0.1534, 0.1751, 0.1042, 0.0507, 0.3580, and 0.0140, respectively. In BWM-DEA model, passenger-km performed and ton-km performed have the highest and lowest weights, respectively. Accordingly, these criteria can bearranged in terms of weight as $c_6 > c_3 > c_2 > c_1 > c_4 > c_5 > c_7$. Indeed, the optimal solution of this model considers preferences form both BWM and DEA models. As it would be expected, the weights generated by BWM-DEA are different in compared with the weights of BWM and DEA and includes the DMs' preferences. For example, available seat-km (c_2) placed in the fifth rank in BWM and the last rank in DEA, is in the third rank in BWM-DEA model. The efficiency scores of most airlines have been reduced (see the last column in Table 3 in comparison with the previous model. As results show, the only efficient airline is Iranairtour while Poyaair is the least efficient airline with the score 0.3122. In fact, based on the mean of efficiency obtained by the BWM-DEA model which is less than the conventional DEA model, this method has a higher discrimination power.

5.4. Results of the RDEA model

In this section, the RDEA model is solved to calculate a set of reliable weights for airlines considering the existence of perturbation in data. Then, the efficiency score of airlines is calculated using the weights obtained from this model and Equation (5). It has been assumed the violation probability (e) is equal to 5% for all constraints. In other words, the constraints are violated with the maximum probability of 95%. Also, based on the experts' decisions, 10% perturbation has been considered in all inputs and outputs. For example, the value x is in interval [x-10%x, x+10%x].As mentioned, the role of the parameterΓ_j is to provide a balance between the conservatism level and the robustness of solution. With increasing its value, the value of the efficiency degree will decrease. Indeed, increasing Γ_j will worsen the objective function [33]. In this application, there are 7 inputs and outputs that all of which are subject to uncertainty. Hence, $|j_j|$ =7and assuming the maximum violation of 0.95, there is $0.95 = 1 - e^{-\frac{\Gamma_j^2}{2\times 7}}$ (see RDEA formulation in section 2). Then, the parameter Γhas been set to 6.476 for all constraints (i.e. $\Gamma = 6.476$ implies that all constraints are satisfied with probability at least 95%). The results of RDEA model (6) have been shown in Table 4.As shown in this Table, the mean of weights is 0.2578, 0.1250, 0.1470, 0.1284, 0.0546, 0.1697, and 0.1175. The number of employees and number of flights have the highest and the lowest weights, respectively. So, the criteria can bearranged in terms of weight as $c_1 > c_6 > c_3 > c_4 > c_2 > c_7 > c_5$.The weights generated by RDEA model are different from the weights of BWM, DEA and BWM-DEA models. For example, the number of fleet seat (c_4) which has been placed in the last rank in BWM, the first rank in DEA, and fifth rank in BWM-DEA, is in the fourth rank in RDEA. In the last column of Table 3, the RDEA efficiency scores for all airlines are shown. According to the results, Taban is in the first rank with the efficiency score 0.8204 while Mahan has the lowest score equal to 0.7364.

Table 3
Optimal weights and efficiency score of the BWM-DEA.

DMU	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Efficiency
Iranair	0.0259	0.1876	0.2189	0.1243	0.0000	0.4433	0.0000	0.6848
Iranairtour	0.1751	0.1501	0.1752	0.0918	0.0000	0.4078	0.0000	1.0000
Ata	0.1709	0.1465	0.1709	0.0952	0.0568	0.3597	0.0000	0.9552
Atrak	0.1674	0.1493	0.1741	0.0980	0.0000	0.3850	0.0262	0.7267
Aseman	0.1327	0.1524	0.1777	0.1050	0.1011	0.3311	0.0000	0.8945
Taban	0.1745	0.1496	0.1745	0.0931	0.0000	0.4017	0.0066	0.9951
Zagros	0.1747	0.1498	0.1458	0.1050	0.1114	0.3133	0.0000	0.8747
Pouyaair	0.1665	0.1427	0.1665	0.1123	0.0000	0.3132	0.0996	0.3122
Gheshm	0.1699	0.1457	0.1699	0.0959	0.0703	0.3483	0.0000	0.8991
Kaspian	0.1652	0.1416	0.1652	0.1029	0.0992	0.2985	0.0274	0.8686
Kishair	0.1680	0.1440	0.1680	0.0974	0.0962	0.3264	0.0000	0.9313
Mahan	0.0000	0.2017	0.2108	0.1374	0.0000	0.4501	0.0000	0.7125
Meraj	0.1637	0.1404	0.1637	0.1053	0.1102	0.2808	0.0359	0.3692
Naft	0.1704	0.1460	0.1704	0.0956	0.0645	0.3531	0.0000	0.8826
Mean	0.1446	0.1534	0.1751	0.1042	0.0507	0.3580	0.0140	0.7933

Table 4
Optimal weights and efficiency score of the RDEA.

DMU	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Efficiency
Iranair	0.0000	0.5496	0.0273	0.0000	0.0394	0.3837	0.0000	0.7455
Iranairtour	0.4881	0.0000	0.1445	0.0816	0.0000	0.2858	0.0000	0.8182
Ata	0.8194	0.0000	0.0000	0.0000	0.0596	0.0000	0.1210	0.8182
Atrak	0.0000	0.0000	0.2459	0.3543	0.0239	0.0000	0.3759	0.7909
Aseman	0.0000	0.0000	0.4623	0.1398	0.0700	0.0000	0.3279	0.8114
Taban	0.4123	0.0205	0.0241	0.2206	0.0239	0.1930	0.1056	0.8204
Zagros	0.8163	0.0000	0.0000	0.0000	0.1837	0.0000	0.0000	0.8182
Pouyaair	0.0000	0.5791	0.0000	0.0000	0.0120	0.4073	0.0016	0.8182
Gheshm	0.0000	0.0000	0.6175	0.0000	0.0098	0.1614	0.2113	0.8182
Kaspian	0.5639	0.0000	0.0000	0.1673	0.0000	0.1097	0.1592	0.7716
Kishair	0.4480	0.0028	0.0030	0.2282	0.1983	0.0358	0.0839	0.8185
Mahan	0.0000	0.0000	0.0000	0.5899	0.0390	0.3711	0.0000	0.7364
Meraj	0.0000	0.5788	0.0000	0.0000	0.0141	0.4071	0.0000	0.8182
Naft	0.0612	0.0196	0.5340	0.0156	0.0910	0.0201	0.2585	0.8199
Mean	0.2578	0.1250	0.1470	0.1284	0.0546	0.1697	0.1175	0.8017

5.5. Results of the proposed BWM-RDEA model

In this section, the results of implementing the BWM-RDEA model are presented. Similar to RDEA, the violation probability (e) has been considered to be 5% with 10% perturbation in all inputs and outputsand the value of Γhas been set equal to 6.476 for all constraints. In other words, all constraints are satisfied with the probability of at least 95%. The results have been shown in Table 5.As can be seen, the mean of weights is 0.1681, 0.1551, 0.1731, 0.1260, 0.0712, 0.2681, and 0.0384, respectively. The passenger-km performed and ton-km performed have the highest and the lowest weights, respectively and the criteria can bearranged in terms of weight as $c_6 > c_3 > c_1 > c_2 > c_4 > c_5 > c_7$. As can be seen, considering the perturbation in data and incorporating the experts' opinions simultaneously have made the weights of criteria different from the previous models. In the last column of Table 5, the efficiency scores of airlines have been reported. According to the results, Ata airline has the highest efficiency score which is equal to 0.8235 and received the first rank, while, Pouyaair is the most inefficient airline with a score of 0.3262.

Finally, Table 6 has summarized the efficiency scores and the rank of airlines based on the models discussed above. According to this, different efficiency scores have been obtained incorporating experts' opinions into DEA and RDEA models. It can be seen that the conventional DEA and RDEA models could not distinguish airlines and ranked 14 airlines in 5 and 9 places, respectively. In these models, DMUs were flexible to choose the weights of inputs and outputs without any limitation until their efficiency became maximum. Besides, according to Tables 2 and 4, it can be understood that the DEA and RDEA models have tried to increase the efficiency degree of DMUs by assigning zero values to several inputs and outputs weights. In these models, many DMUs appeared to be efficient because of this reason; hence, results cannot be reliable. On the other hand, although, the BWM-DEA model distinguished airlines, this model has not considered the uncertainty of data. However, in many real-world applications, data are uncertain as well as imprecise. The mean of efficiencies obtained from this model is higher than the BWM-RDEA model which can be attributed to ignoring the perturbation in data. Based on this model, the Iranairtour is the only efficient airline, while in the BWM-RDEA model this airline has been placed in the third rank. The proposed model of this study has tried to take the advantages of single BWM and RDEM model to cover some of the shortcomings of these models and produce more reliable results which are compatible with practical applications. To put it precisely, in real-world problems the uncertainty is an inevitable feature of data and without considering it, the results will be misleading. In addition to this, based on Table 6, the BWM-RDEA model has the highest discrimination power since the mean of efficiency in this method is lower than all the models discussed in this paper.

Figure 2 compares the efficiency scores generated by DEA, BWM-DEA, RDEA and BWM-RDEA models for all airlines schematically. As seen in this Figure, airlines such as Atrak, Poyaair and Meraj have a significant drop in their performance byincorporating BWM into DEA and considering uncertainty in data.

6. Remarks for policymakers

Efficiency measurement is a process through which useful information can be obtained on how to perform effectively by reinforcing positive behaviors and eliminating inappropriate and unnecessary behaviors in a system. In all organizations, increasing efficiency as well productivity has attracted the attention of policymakers and managers. DEA is a popular mathematical programming model to evaluate the performance of decision units in a wide range of industries. The biggest

Table 5
Optimal weights and efficiency score of the BWM-RDEA.

DMU	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Efficiency
Iranair	0.0289	0.1999	0.2332	0.1435	0.0000	0.3945	0.0000	0.5573
Iranairtour	0.1786	0.1531	0.1786	0.1154	0.0641	0.2871	0.0231	0.8205
Ata	0.1799	0.1542	0.1799	0.1065	0.0976	0.2330	0.0489	0.8235
Atrak	0.1816	0.1557	0.1816	0.1132	0.0259	0.3196	0.0224	0.5933
Aseman	0.1325	0.1653	0.1929	0.1229	0.0890	0.2974	0.0000	0.7406
Taban	0.1804	0.1546	0.1804	0.1147	0.0358	0.3068	0.0273	0.8211
Zagros	0.1991	0.0983	0.1790	0.1476	0.1087	0.2142	0.0531	0.7271
Pouyaair	0.1761	0.1509	0.1761	0.1317	0.0000	0.2500	0.1152	0.3262
Gheshm	0.1791	0.1535	0.1791	0.1125	0.0890	0.2868	0.0000	0.7743
Kaspian	0.1723	0.1476	0.1701	0.1274	0.1014	0.2115	0.0697	0.7268
Kishair	0.1721	0.1475	0.1721	0.1260	0.1008	0.2162	0.0653	0.7838
Mahan	0.2211	0.1895	0.0479	0.1623	0.1154	0.2276	0.0362	0.5466
Meraj	0.1711	0.1467	0.1711	0.1283	0.1022	0.2033	0.0773	0.4065
Naft	0.1808	0.1550	0.1808	0.1113	0.0662	0.3059	0.0000	0.7429
Mean	0.1681	0.1551	0.1731	0.1260	0.0712	0.2681	0.0384	0.6708

Table 6
The summary of the efficiency scores generated by the DEA, BWM-DEA, RDEA and BWM-RDEA models.

DMU	DEA		BWM-DEA		RDEA		BWM-RDEA	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank
Iranair	0.9159	4	0.6848	12	0.7455	8	0.5573	11
Iranairtour	1.0000	1	1.0000	1	0.8182	4	0.8205	3
Ata	1.0000	1	0.9552	3	0.8182	4	0.8235	1
Atrak	0.9721	3	0.7267	10	0.7909	6	0.5933	10
Aseman	1.0000	1	0.8945	6	0.8114	5	0.7406	7
Taban	1.0000	1	0.9951	2	0.8204	1	0.8211	2
Zagros	1.0000	1	0.8747	8	0.8182	4	0.7271	8
Pouyaair	1.0000	1	0.3122	14	0.8182	4	0.3262	14
Gheshm	1.0000	1	0.8991	5	0.8182	4	0.7743	5
Kaspian	0.9950	2	0.8686	9	0.7716	7	0.7268	9
Kishair	1.0000	1	0.9313	4	0.8185	3	0.7838	4
Mahan	0.9061	5	0.7125	11	0.7364	9	0.5466	12
Meraj	1.0000	1	0.3692	13	0.8182	4	0.4065	13
Naft	1.0000	1	0.8826	7	0.8199	2	0.7429	6
Mean	0.9849	-	0.7933	-	0.8017	-	0.6708	-

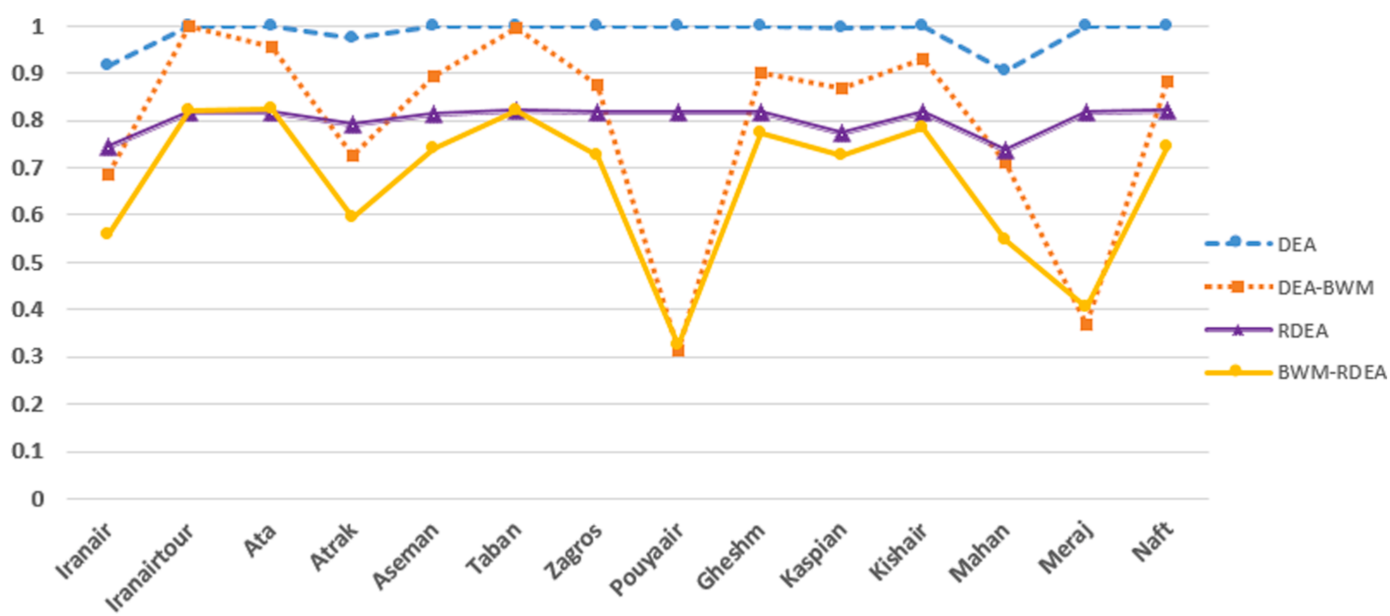

Fig. 2. Efficiency scores generated by DEA, BWM-DEA, RDEA and BWM-RDEA models.

advantage of this model is the ability to compare multiple DMUs based on multiple criteria. In fact, the DEA converts all of the criteria into a single criterion called efficiency by considering a number of which as input and some as output leading to providing a proper comparison and evaluation of DMUs' performance.

Air transportation which is considered in the service sector of countries is one of the essential components of the production and consumption cycle and play a key role in the truism industry. Airports and airline companies with an appropriate fleet will attract industrial investments. Therefore, it is necessary to make the right decisions to improve the airlines' efficiencies. The air transportation industry of Iran has fundamental and structural problems especially poor service quality and a high average of fleet life. Fleet exhaustion causes a lot of problems such as increasing delay of flights orcanceling flights, lacking fixed and regular flight schedules for all cities of the country, reducing the tourists' arrival to the country because of insecurity of flight lines, and finally, decreasing the use of airlines by people.

The proposed method of this study aimed to generate more reliable efficiency scores and provide DMs with helpful information so as to boost the performance of the airlines. In the evaluation of various industries such as air transportation, it is needed to apply the preferences of DMs about the criteria. However, the conventional DEA models consider all the criteria with the same importance. Besides, in real-world problems, it is almost impossible to provide accurate data for inputs and outputs in DEA applications. Hence, to reflect the uncertainty of data, provide robust solutions, cover some of the shortcomings of the DEA, and apply the experts' opinions about the criteria directly the score based on the BWM-RDEA model for efficiency measurement was recommended in this study. This model enables DMs or policymakers to evaluate and monitor the performance of DMUs based on different preferences of the criteria. Due to the mean of weights generated by the BWM-RDEA model, passenger-km performed and available ton-km are important criteria in the airlines' performance evaluation and making some changes in these criteria can create a big difference in the efficiency score of the airlines. Regarding the results of the proposed model, it can be seen that the airlines of the Meraj and Pouyaair have had a significant drop in their efficiency and it is needed to take serious actions to improve their performance. The policymakers of the airlines, especially the mentioned ones, are better to focus on improving the quality of their services in order to attract more passengers rather than increasing the capacity of cargos carriage. In other words, by increasing the number of passengers and decreasing the available capacity of cargos carriage on each flight, the efficiency score will be increased.

7. Conclusion and direction for future research

Transportationis an important issue in the country's economic development. Air transportation has a special role in the transportation industry due to the safety, convenience, speed and capacity, regularity of planning, and the ease of use. Policymakers and managers should evaluate the airlines' efficiency and make important decisions to improve the current situation. This paper proposed a DEA based method by incorporating experts' opinions about inputs and outputs weights and considering uncertainty in data,simultaneously. To this end, the objective function and the constraints of BWM were added to the RDEA model to calculate the efficiency of airlines. The proposed bi-objective BWM-RDEA model was solved through min-max technique and applied in a case study of Iranian airlines. This model had the highest discrimination power as compare with other models discussed in this paper regarding the lowest mean of efficiency achieved. One limitation of this study is that DEA might have multiple weight solutions. Future researchers could investigate the issue of alternative optimal solutions in their proposed model.

CRediT authorship contribution statement

Hashem Omrani: Conceptualization, Methodology, Supervision. **Mahsa Valipour:** Data curation, Software, Writing - original draft. **Ali Emrouznejad:** Supervision, Writing - review & editing.

Declaration of Competing Interest

All authors declare that there are no conflicts of interest.

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