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Evolution of Tail Risk during the Global Financial Crisis (2008)

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Doctor of Philosophy

ASTON UNIVERSITY

September 2019

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THESIS SUMMARY

The primary objective of this thesis is to understand the behaviour of extreme events taking place in the foreign exchange (FX) and stock exchange (SX) markets around the Global Financial Crisis using parametric and non-parametric methods. Specifically, it models the evolution of the extreme values and Black Swans measures of daily stock returns and volatility proxies from a set of more than thirty SX and FX markets and compares their statistical properties in the pre- and post-crisis periods.

My findings suggest that irrespective of the modelling approach followed, there are substantial differences in the tail-behaviour of the two periods that cannot be accounted for by the mainstream mean-variance model although the identification of latent Non-linearities in the underlying mean and/or variance dynamics does indeed lead to a significant reduction in tail asymmetry and kurtosis. Moreover, the mainstream parametric models prove insufficient in encompassing the volatility tails. Consequently, this study demonstrates that during times of extreme financial turbulence in financial markets the existing financial risk models are inadequate to provide guidance for the probability of extreme events taking place and subsequently underlines the need for additional research on parsimonious models and for the use of encyclopaedic exercise of judgment in the real-world risk management.

Keywords: Extreme Value theory, Black Swans, Latent Non-linearities, Financial market returns, Volatility of volatility.

I dedicate this PhD to my mother.

'The love of a mother is the veil of a softer light between the heart and heaven.'

– Adapted from Samuel Taylor Coleridge (1772 - 1834)

Acknowledgements

Writing this document has been a gruelling, beautiful and immensely gratifying journey. It has been one of reflection and has required monumental growth – academically, emotionally and mentally.

My first thanks is to my mother, who has been my fortress of strength while showering me with unconditional love. She taught me to be brave and persistent. I also want to thank my husband for being patient with me while I discovered and achieved my dreams – it hasn’t been easy. Next, I want to thank my best friend, Divya Jyoti, who was a kind listening ear to the woes of this journey - *Thank you for standing by me when I was lost but also letting me find my own way as I navigated this document.* Last but not the least, I also want to thank Ali, Monty, Elena and all my friends for being my family while I was so far away from home.

The biggest acknowledgment goes to my Supervisor, Dr. Michail Karoglou, without whom this document could not have been what it is today - *Thank you for being ever so patient but also stern where you had to be. Thank you, for expecting only excellence from me and having the confidence that I could achieve it. I hope I have done your belief in me justice*. I also want to thank Prof. Jun Du for her support.

Finally, I would also like to thank my examiners, Dr. Ioannis and Dr. Bissoondeeal for their valuable feedback on my work.

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Chapter 1: Introduction

The behaviour of financial markets when undergoing extreme duress or expecting a crisis has gained much popularity in recent financial, economic and risk literature. Much of this is due to the recent tidal waves of financial and economic crises affecting developing and emerging economies alike.

It is well known that daily returns of financial assets display stylized properties such as volatility clustering, excess kurtosis, significant negative asymmetry and long memory; the effects of these are ostensibly heightened in times of a crisis thereby augmenting tail risk.

The Global Financial Crisis of 2008 popularized the idea of classifying highly extreme events as *Black Swans* which in this context are defined as events that are outliers and hence lay beyond the realm of normal probability (Taleb, 2007b). Quite a few studies have focused on the so-called *de-blackening* of such Black Swans by proposing that uncertainty comes in several forms but the most popular versions are: known unknowns and unknown unknowns. The former stands for knowledge that we know we don't know and the latter for knowledge that we do not know we do not know.

Generally, there is a plethora of research that emphasizes the multiple causes of a Black Swan such as the bursting of the housing bubble, abrupt effects of deregulation of the financial services sector, incongruous subprime mortgage derivatives, and so on. However, schematically there appears to be two strands that emerge: management-inspired that concentrate on transparency in communication, and adaptive governance, and finance-inspired which deliberates over the adequacy of financial models to incorporate extreme values.

On the one hand, the management-inspired strand of literature highlights various methods by which Black Swans can become prospectively more detectable and/or moderated. For example: if they emerge from the lack of knowledge (an known unknown), an increase in knowledge of various possibilities would decrease their *blackness* (Aven, 2015)[1], and if they emerge from unknown unknowns then predictability can be achieved through joint cognitive processes that turn the implicit knowledge into collective practices (Lindaas and Pettersen, 2016). Other studies focus on adapting a holistic communication technique focused on adaptive governance techniques that allow firms to dynamically expand their scope when faced with extreme uncertainly (Wardman and Mythen, 2016).

On the other hand, the finance-inspired strand of literature focused more on studying the causes of such extreme volatility such as: the inappropriate use of derivatives in asset securitization transactions and deregulation of financial services (Catanach Jr and Ragatz, 2010, Chi, 2008, Crotty, 2009), the exclusion of dynamic conditional forecasts of risks that allow for rapidly changing "bursts" in the economy (Marsh and Pfleiderer, 2012), financial models that do not include modelling joint marginal tail behaviour, conditional heteroscedasticity and the extremal dependence structure (Hyung and De Vries, 2007, Beine et al., 2010, Hilal et al., 2011) and the omission of stress testing by assigning probabilities to extreme events and including them into formal risk models (Aragonés and Blanco, 2008).

While the empirical work on the causes of the Global Financial Crisis grows considerably, an emergent topic of interest in the area of tail risk is the presence of structural breaks in the underlying mean and/or variance dynamic of asset returns and the effect it has on the tail risk.

The study of structural breaks is essential to accurately modelling tail risks because it can lead to an misestimating of kurtosis, if neglected, thereby leading to a specious probability of the occurrence of an extreme event (Marsh and Pfleiderer, 2012, Skinner, 2010, Vodra, 2009, Cowen and Abuaf, 2010, Seaberg, 2009), skewness (Harvey and Siddique, 1999, León et al., 2005, Jondeau and Rockinger, 2003a) and spurious detection of long memory in asset returns (Charfeddine and

[1] Typically the author recommends an extension of the conceptualization of the domains of probability of extreme events, the development of risk assessment models that incorporate this extreme risk and finally resilience engineering in the decision making process of the management.

Guégan, 2012, Chen and Tiao, 1990, Diebold and Inoue, 2001, Granger and Hyung, 2004, Gourieroux and Jasiak, 2001, Liu, 2000).

Additionally, there are a multitude of papers that provide ample empirical evidence regarding the inadequacy of parametric risk assessments tools such as GARCH type processes in measuring tail risk (McNeil and Frey, 2000). With respect to the limited literature that covers tail risk during the Global Financial Crisis of 2008, a plethora of papers exist that find evidence regarding the superiority of using Extreme Value Theory (EVT) to model tail risk see for example Totić and Božović, 2016 for six stock markets from Eastern Europe; (Straetmans and Chaudhry, 2015) for evolution of tail and systematic risk for banks in US and the Eurozone; (Allen et al., 2013a) for application to the benchmark indices of US, UK along with their VIX indexes; Billio et al., 2016 for a Markov-regime switching approach[2] for hedge funds because it can adequately capture time and space dependent risk exposures.

However, these models assumed that the parameters of volatility such as the variance remained stable and linear. Furthermore, these papers failed to explicitly capture the evolution of tail risk in the presence of latent Non-linearities in the underlying mean and/or variance structure of the asset returns.

A study that focused on studying structural breaks during the Global Financial Crisis found that the inclusion of breaks in the mean and conditional variance reduced although did not fully eliminate long memory properties in the time series for the GCC countries (Aloui and Hamida, 2014).

The following thesis uses structural breaks combined with the EVT-GARCH methodology to capture the evolution of tail risk during the Global Financial Crisis because an essential assumption of the traditional regime switching models is that the number of states is finite and the causes are endogenous and recurrent while

[2] A popular nonlinear time series model that assumes that changes are frequency and random. These are captured by equations (structures) that include the complex dynamic characteristics of the time series at that point.

structural breaks impose no such restrictions allowing for the exogenous estimation of infinite states.

The findings of this thesis will contribute to the literature of Extreme Value theory as its ability to capture the evolution of tail risk with the inclusion of structural breaks has not been considered in the literature thus far. Consequently, the following thesis aims to build on this work by exploring the effect of the inclusion of structural breaks have on tail risk termed as Black Swans in recent literature, specifically the change in kurtosis, and tail asymmetry of returns. The thesis focuses on black swans instead of extreme values because extreme values will exist in every data set irrespective of a crisis (returns above/below 1 percentile on both tails) however, black swans are highly improbable and tend to appear during a crisis (returns that are three standard deviations away from the mean). Therefore, in order to study extreme tail risk during a crisis, black swans are the more appropriate measure.

Therefore, the estimation of tail risk and inclusion of latent Non-linearities in the mean and/or variance of underlying asset return process has important implications for asset pricing models, option pricing as well as portfolio diversification and hedging strategies.

Specifically, the first two empirical chapters (chapter 3 and 4) will focus on the effect of structural breaks on tail risk arising from the mean dynamics of the returns process in the equity and forex market. The next two chapters (chapter 5 and 6) will study tail risk arising from the variance dynamics by looking at the change in the frequency of extreme values in volatility proxies such as absolute returns, squared returns, log-squared returns and log range in the equity and forex markets.

Moreover, all four chapters briefly focus on the effect of the inclusion of breaks on the contagion effects/volatility spill-overs building on the work of Jung and

Maderitsch (2014)[3]. This was tested by implementing a model that combined extreme value theory with GARCH models on daily log returns (chapters 3 and 4) and first-order differenced volatility proxies (chapters 5 and 6) with and without the provision of breaks which were identified using the break-testing procedure of Karoglou (2010a). The results were then contextualized using the Global Financial Crisis of 2008 (identified as a major Black Swan (Taleb, 2007a) since the effect of time-varying conditional third and fourth moment of asset returns are heightened during a crisis).

The research philosophy of this thesis is primarily positivist in its ontological and epistemological assumptions. It will, therefore, be using a deductive approach of reasoning starting with Extreme value theory to rationalise the importance of information that sits in the tails of an assets return distribution. The thesis will then build a set of hypotheses in each chapter regarding the evolution of this tail risk during the Global Financial Crisis of 2008 i.e. is tail risk in equity and forex markets influenced by structural breaks in the mean and/or variance structure (chapters three and four) and if it is then is there a difference in tail risk from mean dynamics versus volatility dynamics (chapters five and six). In order to do this, I use a quantitative research design strategy which applies superior econometric modelling methods to cross-sequential data that combines cross-section and longitudinal financial time series data to test the hypothesis (I test for structural breaks in multiple countries over multiple time periods with possible structural breaks within the data set).

The empirical findings of chapters three and four are three-fold. First, that provision for breaks leads to fewer extreme events being identified. Second, the frequency of the resulting clusters of extreme events changes from one segment to the next in the majority of the cases in the data set thereby providing evidence that extreme events clusters are not homogenous in nature. Finally accounting for breaks reduces tail asymmetry and kurtosis overall making the asset returns distribution appear more normal.

[3] Having studied volatility spill over between the equity markets of Hong Kong, Europe and the United States during the Global Financial Crisis of 2008, provided evidence that structural breaks were the main cause of increased cross-market volatility

The empirical finding of chapters five and six is that black swans that arise from the returns processes are different to the ones that arise from the volatility processes. While there is a significant degree of overlap, disregarding extreme volatility arising from the innovations in variance could lead to a potential misspecification of the overall financial risk faced by market participants in these markets.

Chapter 2: Literature Review

The foundational financial theory measures risk in financial markets by the variability of returns around the mean (often with variants of the standard deviation or sigma i.e. σ) or by relative volatility (often with variants of the CAPM model beta i.e. β). However, their hard-wired assumption, that returns follows a normal distribution, has long been proved invalid. Several modelling methods have been devised to bridge the gap between theory and empirics but so far unsuccessfully at least in providing with a widely accepted mainstream answer.

The primary problem of the stylised non-normality is not around the mean of the distribution where the theory is actually quite successful in modelling the market returns but in the tails, when extreme events take place. This has paved the way for a supplementary view of risk measurement in financial returns: one that focuses primarily on the tails of the distribution where catastrophic events lie. The Extreme Value Theory (henceforth EVT) introduced by (Mandelbrot, 1963) has provided now a well-established strand of literature in finance. Interestingly, market practitioners are more familiar with a related notion borrowed from manufacturing processes: the notion of Black Swans as, typically three, standard deviations away from the mean. This explains the confusion surrounding the term Black Swan since some researchers use the same term to refer to any outstanding phenomenon and not something that can be quantified as strictly as that. In this thesis I use the concept of Black Swan for both these notions and clarify whenever it is important, whether I am referring to the quantitative and/or qualitative term. At this stage it should also be noted that the main difference between the notions of extreme values and Black Swans is that the former stems from the EVT theory and refers to a pre-specified number of observations that are characterised as extreme values, while the latter stems from the practitioners' approach and refers to a pre-specified interval outside which observations are characterised as Black Swans. However, both of these approaches capture the same thing i.e. extreme events.

The remainder of this section is organised in two parts. The first part overviews the theoretical framework of EVT and focuses on describing each of the two methods for identifying extreme values. The second part provides a flavour of the voluminous empirical finance literature that is tangent to the notion of Black Swans.

Theoretical Framework of EVT

In order to efficiently realize extreme financial risk which requires focus on the tails of a financial distribution, the EVT branch of statistics has provided two popular techniques: one is the Block Maxima method and the other is Peak Over Threshold.

Block Maxima (BM)

Being the more traditional of the two methods, BM fits a generalized extreme value distribution (GEV) to the maximum values within certain blocks of time derived from a set of identical and independently distributed returns. The GEV distribution can take three specific forms: Gumbel, Fréchet or Weibull (see (Embrechts et al., 1997); Kellezi & Gilli, 2000; McNeil, 1998 for detailed theoretical evidence).

Starting with the daily returns of the chosen index, R_i, assumed to be identically and independently distributed with an unknown underlying cumulative distribution function, F_R, the chosen time period for analysis is then broken into non-overlapping blocks of length *n*, from which the maximas in each consecutive block are chosen, X_i. These maximas M_n *where* $M_n=\max(X_1,X_2,\ldots,X_n)$ are then normalized and their cumulative distribution function can be given as:

$$P\,(M_n \leq x) = P(X_1 \leq x, \ldots\ldots X_n \leq x) = \prod_{i=1}^{n} F(x) = F^R(x) \qquad (1)$$

Theorem: Fisher and Tippett, 1928; Gnedenko, 1943

In order to achieve a non-degenerate distribution function Y_i, maximas, M_n, have to be standardized to achieve a non-degenerate behavior limit when $n \to \infty$ using:

$$Y_i = \frac{(X_i - \beta)}{\alpha} \qquad \alpha > 0 \qquad (2)$$

Where α and β are location and scale parameters respectively.

The Fisher –Tippet theorem states if Y_i converges to some nondegenerate distribution function, this must be a generalized extreme value (GEV) whose standard one parameter distribution is given by:

$$Y_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}} & if\ \xi \neq 0 \\ e^{-e^{-x}} & ,if\ \xi = 0 \end{cases} \tag{3}$$

Where ξ is known as the shape parameter of the GEV distribution which determines the tail behaviour and Y_ξ gives the type of distribution. GEV is a generalized representation which can take one of the following three forms:

ξ > 0 for Fréchet distribution in which the tail of F_r declines by a power function. The result will be fat tailed distributions like pareto, gauchy or student's t. it can be represented as:

$$\Phi_\alpha(x) = \begin{cases} 0, & x \leq 0 \\ e^{-x^{-\alpha}}, & x > 0 \end{cases} \tag{4}$$

ξ < 0 for the Weibull distribution in which the tail is finite which will result in distributions with bounded support like uniform or beta which can be represented as:

$$\Psi_\alpha(x) = \begin{cases} e^{-(-x)^\alpha}, & x \leq 0 \\ 1, & x > 0 \end{cases} \tag{5}$$

ξ = 0 for the Gumbel distribution in which the tail of F_r declines exponentially. The result will be a thin tailed distribution like normal, log-normal, exponential or gamma which can be represented as:

$$\Lambda(x) = e^{-e^{-x}} \tag{6}$$

It is important to remember that all three limiting distributions of the GEV are of normalized maxima's, however, in reality, the underlying distribution of the sample extremals are unknown. Therefore, it is important to calculate the parameters for the GEV distribution (the tail index, ξ, and constants $\alpha \text{ and } \beta$) using various statistical techniques, the most common one being the maximum likelihood estimation (MLE) using a three parameter specification.

Peak over Threshold (POT)

Peak over threshold approach is an alternative approach that fits the generalized pareto distribution (GPD) to independent data which exceed a given high threshold, μ. Unlike the block maxima method, POT is considered more efficient since time series data often exhibit volatility clustering and where BM method would ignore all values except the highest within the block, POT allows for all values above a predefined high threshold to be taken into consideration. The conditional excess distribution function for this method was introduced by Pickands (1975), Balkema and de Haan (1974) motivated by Fisher and Tippett (1928) who provided the types of limiting distribution that sample maxima can converge to given it has specific mathematical properties.

Theorem: Pickands (1975), Balkema and de Haan (1974)

Using the i.i.d. daily returns r_i in the financial time series arising from an underlying distribution F_r the returns that are above the threshold u follow a conditional excess distribution function $F_u\,(y)$ where, $y_i = r_i - u$. The behaviour of these exceedances y_i can be approximated by the following cdf:

$$F_u(y) = P(r_i - u \ \le y \mid r_i > u) = \frac{F_r\,(y-u) - \ F_r\,(u)}{1 - F_r(u)}, 0 \le y \le R_F - u \qquad (7)$$

Pickands (1975), Balkema and de Haan (1974) provided evidence that when the predefined threshold is high enough, i.e. that the Fisher-Tippet theorem is satisfied, the excess distribution, $F_u(y)$ of most distributions, F_r, can be approximated by the Generalized Pareto Distribution, GPD:

$$H_{\xi,\beta(u)}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\beta(u)}\right)^{-1/\xi} & \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\beta(u)}\right) & \xi = 0 \end{cases} \qquad (8)$$

When the shape parameter, $\xi\ > 0$ then $y\ \ge 0$ and $\beta(u)$ is a positive function where F belongs to the Fréchet family and $H_{\xi,\beta(u)}(y)$ is a Pareto Distribution. When the shape parameter, $\xi = 0, y\ and\ \beta(u)$ remain positive but F is in the Gumbell family and $H_{\xi,\beta(u)}(y)$ is an exponential distribution. When the shape parameter, $\xi\ < 0$ then y lies between 0 and ${}^{-\beta(u)}/_{\xi}$ and F is in the Weibull family and $H_{\xi,\beta(u)}(y)$ is a Pareto-type II distribution.

In order to estimate the tails of the distribution, Pickands-Blkema-de Haan theorem states that the following approximations can be used given that a sufficiently high threshold,u, is selected:

$$F_u(y) \approx H_{\xi,\beta(u)}(y) \qquad u \to \infty \qquad (9)$$

For $x - u \geq 0$, the excess distribution function can be written as:

$$F(x) = 1 - F(u))\, H_{\xi,\beta(u)}(y) + F(u) \tag{10}$$

The function $F(u)$ is estimated using historical simulation using the empirical cumulative distribution function:

$$\hat{F}(u) = \frac{n-k}{n} \tag{11}$$

Where n represents the total number of observations in the sample and k represents the number of exceedances over the threshold, u.

Using the method of maximum likelihood, the tail estimator can be obtained using:

$$\hat{F}(x) = 1 - \frac{k}{n}(1 + \hat{\xi}\,\frac{(x-u)}{\hat{\beta}})^{-\frac{1}{\hat{\xi}}} \tag{12}$$

Where, $\hat{\xi}$ and $\hat{\beta}$ are estimates of ξ and β , respectively.

Choosing the right threshold remains of key importance to the efficiency of the model, if the threshold is too low then it will violate the asymptotic property of the model and lead to higher bias; if the threshold selected is too high then the number of exceedances will reduce reducing the sample size for model estimation, this can lead to high variance. Therefore, selection of the threshold remains an issue to balancing bias and variance which will be looked into more detail in the Methodology section of this report.

Empirical Literature

While the Extreme Value theory was originally applied extensively to the fields of hydrology and engineering, by the late 1900's there was an increased interest in its applicability to financial markets and insurance (see (Embrechts et al., 1997, Leadbetter et al., Diebold et al., 2000, Danielsson and De Vries, 2000, Thomas and Reiss, 1997).

The remainder of this section is organized as follows: the first part covers the empirical literature of Extreme Value theory i.e. in equity markets, the various EVT models (conditional, unconditional and hybrid), the second section provides empirical work on tail asymmetry and the final section focuses on research done on Black Swans with respect to the Global Financial Crisis of 2008.

EVT and Equity Markets

The introduction of EVT and its validation in estimating a superior VaR at higher quantiles exists in abundance within the finance literature. For example: Bekiros and Georgoutsos (2005) applied EVT to the financial data from Dow Jones Index as well as the Cyprus stock exchange and found EVT based VaR to be more accurate when compared to conventional methods (see Gilli (2006) and more recently (Allen et al., 2013b) with respect to US, UK, German, French and the Australian Stock exchanges as well as volatility indices, (Daníelsson and Morimoto, 2000), for application to Japanese Stock Markets, (Wagner, 2003) for application to the German stock market, (Kabundi and Muteba, 2011) for application to the left tail of the US, UK, Japanese, South African, German and French equity markets, (Vilasuso and Katz, 2000) for application to seventeen developed markets, namely Australia, Austria, Canada, Denmark, France, Germany, Hong Kong, Italy, Japan, The Netherlands, Norway, Singapore, Spain, Sweden, Switzerland, the UK and the US).

Although empirical evidence supporting fatter tails of financial distributions as opposed to the previously assumed normal distribution of stock returns emerged relatively early (see (Mandelbrot, 1963, Fama, 1965) and more recently works by (Embrechts et al., 1999, Longin, 2000, Danielsson and De Vries, 2000)) yet research focusing on the application of EVT to emerging markets became more significant after studies by Harvey (1995) and Claessens et al. (1995) revealed that the deviation from normality was significant in emerging markets in comparison to developed markets (see (LeBaron and Samanta, 2005) for additional evidence). Empirical evidence has shown emerging markets to be characterized by size constraints, limited liquidity and higher volatility caused by lack of information transparency, added trading expenses and other market inefficiencies among others in comparison to industrialized economies (Domowitz et al., 1998). All these contribute to the significant difference in the performance of asset returns from emerging markets and therefore require differential treatment as well as testing of the applicability of EVT. Research focusing and validating the efficiency of EVT based VaR estimates in being able to capture tail risk in comparison to conventional risk measurement techniques such as variance-covariance method, HS method and ARCH type processes in emerging markets is adequate in literature (see (Assaf, 2009, Maghyereh and Al-Zoubi, 2006) for application to Middle Eastern and North African equity markets, (Gencay and

Selcuk, 2004) for application in Argentina, Brazil, Hong Kong, Indonesia, Korea, Mexico, Singapore, Taiwan and Turkey, (Jondeau and Rockinger, 2003b), for application to twenty countries covering industrialized markets, Asian markets, Eastern European and Latin American markets, (Ergen, 2014), (Cotter, 2004) for application to six Asian and Five European markets, (De Jesús and Ortiz, 2011), for application to the Brazilian and Mexican stock exchange, (Cerović and Karadžić, 2015, Cerović et al., 2015) for application to the Montenegro stock market; (Da Silva and de Melo Mendes, 2003) for ten Asian emerging markets).

Although evidence exists that Asian markets are more prone to experiencing market crashes and other extreme events as opposed to the American and European markets (Cotter, 2006, Cotter and Dowd, 2011) yet a study by Bao et al. (2006)studying five Asian stock markets found inconclusive evidence of the efficiency of unconditional EVT as opposed to conventional methods because neither method provided a superior VaR forecast. The same conclusion was also drawn by another study (Kittiakarasakun and Tse, 2011) which tested the performance of EVT-based VaR as opposed to ARCH-based VaR and tested it on developed Asian markets (Japan), advanced Asian markets (Hong Kong, Singapore, Taiwan, and Korea), developing Asian markets (Thailand, Indonesia, and Malaysia) and compared it to the US S&P stock market. They found evidence that although there was a substantial difference in the development between the Asian and American stock markets yet the stock markets behaved quite similarly. Their conclusion, based on the similarity in the shape parameter of the returns distributions, was that the difference was only a matter to scale and although the Asian markets were more volatile, both Asian and American stock markets shared the same probability when it came to the occurrence of extreme events. Another study (Dimitrakopoulos et al., 2010) focusing on a wider sample consisting of sixteen emerging and four developed markets across America, Asia, and Europe found similar evidence that the classification of the country had no significant difference in the performance of volatility models. In fact, they claim that majority of the previous research disregarded the importance of conditional efficiency of back testing the VaR estimates.

Hybrid EVT models

The conflicting results provided by literature thus far led to the development to hybrid EVT models because unconditional EVT model did not consider certain essential characteristics of the financial time series data such as time varying volatility and clustering. Some examples of hybrid models designed to capture these are the Hawkes-POT method proposed by Chavez-Demoulin et al. (2005), the Autoregressive conditional intensity POT, ACD-POT model suggested by Herrera and Schipp (2013) the duration-POT, D-POT model introduced by Araújo Santos and Fraga Alves (2013) and self-exciting marked point processes combined with POT, SEMPP-POT model, presented by Herrera and Schipp (2014) which theorized and proved that the intervals between extreme events are an essential consideration in forecasting the magnitude and intensity of extreme events in stock markets.

The most popular however remains the conditional EVT model which is a two-step process introduced by McNeil and Frey (2000) whereby a GARCH filter is used on the returns to obtain independent and identically distributed residuals to which the chosen EVT distribution is then fitted. The most significant contribution of this improvement is that this model captures the conditional heteroscedasticity that exists in financial time series via GARCH filtering along with the ability to model the tails of the distribution via EVT. Although Chavez-Demoulin et al. (2005) alleged that the two step procedure meant that the efficiency of the final results of the conditional EVT model would be based on the accuracy of the GARCH model fitting, this allegation was refuted by Furió and Climent (2013) who found their empirical results to be robust under multiple GARCH models in the US, UK, and Japanese markets.

Conditional EVT has been applied to international markets (see (Fernandez, 2005) for application in United States, Europe, Asia, and Latin America, (Cotter, 2007, Ghorbel and Trabelsi, 2008) for application to the Tunisian stock exchange, (Furió and Climent, 2013), for application to both tails of the S&P 500, FTSE 100 and NIKKEI 225 index returns distribution), emerging markets (see (Ozun et al., 2010) for application to the Turkish stock market, (Vee et al., 2014), for application to the stock markets of Mauritius, Tunisia, Sri Lanka, Pakistan, Croatia and Kazakhstan, (Totić and Božović, 2015) for application to the left hand tail of the returns distribution of six emerging markets of the Southeastern Europe, namely Serbia, Romania, Hungary, Croatia, Bulgaria and Slovenia) and to a combination of both industrialized and frontier stock markets (see (Chen et al., 2012), for application to Chinese, Hong King, Taiwan, Japan, Korea, India, Singapore, US, Canda, Mexico, France, Germany and UK stock markets, (Karmakar and Shukla, 2015) for application to three developing and three developed markets across US, Europe and Asia) and found to provide more accurate VAR estimates.

An exceptional benefit of a dynamic VAR estimate resulting from a conditional EVT model that adjusts to the current volatility in the market also leads to a robust margin system in the Indian stock market compared to the current ad hoc margins being imposed by the National Stock Exchange during periods of higher volatility (Bhattacharyya and Ritolia, 2008).

Studies have also focused on extending the conditional EVT model which was originally proposed to be applied to GPD distributions to GEV distributions as well (see (Byström, 2004) for its application to the Swedish and the US indices). Samuel (2007) proposed the improvement of the conditional EVT model by combining it with the Markov switching process in order to capture excess volatility being caused by the structural changes in the financial return series. The SWARCH-EVT model was then applied to three stock indices - S&P 500 Index (S&P), Hang Seng and Hang Seng China Enterprise Index (CEI). Their proposed model outperformed the GARCH-EVT model at 99.5% confidence levels in both in-sample and out-of-sample tests, however when yearly back-testing was done GARCH-EVT model produced the most accurate VaR estimates.

Research that focused on the comparison of conditional and unconditional EVT model during tranquil and volatile market conditions found the conditional model to be superior although both models performed well at confidence intervals of 99%. However, conditional EVT model was deemed to be superior at higher confidence intervals capturing time-varying volatility more efficiently (see (Byström, 2004) for application in Swedish and US stock markets,(Zikovic and Aktan, 2009) for application to the Turkish and Croatian stock markets before, during and after the Global Financial Crisis of 2008, (Seymour and Polakow, 2003) for application to nine stocks from the South African stock market). Another recent study by Hoque et al. (2013) which found empirical evidence of the superior performance of the conditional EVT model during the Global Financial Crisis by applying it to five emerging (Brazil, China, Mexico, South Africa and India) and five developed markets (France, Germany, Japan, UK, and US). However, they also found that the tail parameter of the GPD was quite different in pre-crisis and during crisis periods implying that extreme loss estimated of one period needed to be tested for another and might not be an automatic fit.

It is important to note that even though conditional EVT is highly superior to the unconditional EVT model the choice between selecting between the two models still exists. A distinctive study by Danielsson and De Vries (2000) provided evidence that for longer time horizons where a continuously modernizing VaR was not required, that an unconditional model was still the optimal choice as it provided optimal asymptotic results (this was also supported by (Christoffersen and Diebold, 2000), whose study found that volatility forecasting display a higher speed of decay as there is a shift from shorter to longer time horizons. More specifically if the time period of interest is more than ten or twenty days, given a specific asset class that volatility forecasts are less relevant). So a conditional EVT model would be most suitable for a risk manager understanding the financial losses that could arise for events in the next ten to fifteen days or daily risk factors. Therefore, the final decision of model choice depends largely on the time horizon of interest and if continual updating of parameters is essential.

Another facet of sound financial risk management is international diversification but due to the contagion effect and volatility spillovers, it became important to extend the understanding of financial risk caused by extreme events to a multivariate setting i.e. there has been an increasing focus on modeling extreme value dependencies between markets using EVT. This focus is necessary as the assumption of multivariate normality, which gives equal weight to all the values in the distribution, leads to a gross underestimation of the risks caused by co-crashes especially since evidence exists that joint extremal dependence increases during financial distress. For example, a study by Galbraith and Zernov (2009) studying extremal dependence between the NASDAQ and S&P found that although GARCH-type models produced similar second-moment dependence between the two markets, GEV combined with a extremal index revealed significantly highly extremal dependence in the NASDAQ returns.

Asymptotic dependence and independence are the two types of cross-section dependence which determine if the relatively large values from each variable can transpire collectively or not, respectively (see (Ledford and Tawn, 1996, Ledford and Tawn, 1997, Ledford and Tawn, 1998, Coles et al., 1999, Poon et al., 2003). Poon et al. (2003) studied five international markets, namely US, UK, Germany, France, and Japan and find evidence of stronger left tail dependence due to correlated conditional volatilities (a study (Bhatti and Nguyen, 2012), found similar empirical evidence between the Australian and US stock markets). Also, markets within Europe displayed increasing asymptotic dependence whereas extremal dependence between Europe, North America, and Asia was asymptotically independent. Another significant outcome of the same study was that where asymptotic dependence exists filtering conditional heteroscedasticity using GARCH models can reduce tail dependence substantially (see (Fernandez, 2005) for a more recent study validating this conclusion for US stock markets i.e. extremal dependence reduces drastically when controlling for conditional heteroscedasticity and serial correlation. See (Chen et al., 2012) whose study followed similar methodology testing for extremal dependence between the Chinese stock market with thirteen international and emerging stock markets from Asia, North America and Europe finding that although there exists a positive correlation between the movements of the Chinese and other stock markets yet they are asymptotically independent when compared pairwise).

Research has also focused on combining the conditional EVT model with multidimensional marginal distributions also known as Multivariate Extreme Value Theory (MEVT) to estimate linear and non-linear dependencies and found it to be an efficient parameter during periods of calm as well as crisis especially because they integrated dynamic conditional correlations (conditional EVT-Copula model) of the selected asset classes (see (Berger and Missong, 2014) application to German national stock exchange and foreign exchange market, see (de Melo Mendes and de Souza, 2004), for evidence on modelling of the joint dependence structure of bivariate financial distributions from the Brazilian and American stock market, (Bradley and Taqqu, 2004) for international markets, (Bhatti and Nguyen,

2012) for modelling tail dependence using time varying copulas along with conditional EVT between the Australian financial market and the US, UK, Hong Kong, Taiwan and Japanese stock markets, (Schich, 2004), for application to the European stock markets, namely Germany, UK, France, Italy and the Netherlands finding that dependencies during periods of extreme negative returns are higher than compared to extreme positive returns as well as that conditional probabilities between the markets are symmetric).

A study on fifteen emerging markets (Ergen, 2014) provided evidence of an inverse relationship between diversification benefits and extremal tail dependence in emerging markets. They also found that although pairs of emerging markets are asymptotically independent yet their dependence at even finite levels of extremes is underestimated by simple correlation measures (see, (Turgutlu and Ucer, 2010, Chollete et al., 2012) for joint extremal dependence of international and emerging markets leading to similar results). For deeper understanding of diversification benefits that can be gained with a global portfolio, (Bekiros and Georgoutsos, 2008a) found slightly contradictory results when they studied extremal dependence between stock indices of seven Asia-Pacific stock markets and the US using MEVT. For example, their first substantial conclusion was that extreme correlations were not statistically different from correlations achieved through multivariate GARCH models or unconditional models which means that amplified correlations during crisis periods are not prevalent. Their second key finding was that in order to achieve diversification benefits Asia Pacific markets would have to be identified as high correlation clusters to attract investors from the other two markets: US and Europe. However this was not the case between all the seven indices, i.e. Hong Kong, Singapore, Malaysia and Thailand show a highly integrated capital market, whereas Taiwan and Indonesian stock markets depict lower systematic correlation. Japan, on the other hand, exhibited fluctuating degrees of extremal dependence with the US stock markets

Chapter 3: Influence of Latent Non-linearities in Mean and/or Variance dynamics of Equity returns and their effect on the tail risk

3.1 Introduction

The use of methods based on econometric analysis has been the backbone of financial models – the success of which depends mostly on their ability to prepare their users for any foreseeable risks in the market. The Global Financial Crisis of 2008 followed by the European Debt crisis highlighted the importance of paying special importance to tail risks or extreme volatility of asset returns during times of heightened financial distress. The presence and scope of these tail risks have a significant impact on the continuance and perseverance of a crisis, making it harder to predict their severity and therefore incorporating the amplified risk into asset prices.

Neoclassical finance theories such as the modern portfolio theory, derivative pricing and financial forecasting models include financial models such as Capital Asset Pricing model, Modigliani-Miller Theorem of capital structure and the Black–Scholes–Merton approach to option pricing rely heavily on the Efficient Market hypothesis and therefore typically assume that market participants are rational, well-informed and asset prices in the market are informationally efficient. However, empirically it has long been shown that asset returns display properties of persistence, volatility clustering, asymmetry, and kurtosis and yet there exist few parametric financial and econometric models that can comprehensively allow for each property to be included in the modelling of asset returns (see for example (Harvey and Siddique, 1999, León et al., 2005, Jondeau and Rockinger, 2003a)[4] for the implications of higher moment time-varying dynamics on asset pricing and allocation).

While important, the literature that exists on the importance of encompassing time-varying conditional higher-moments remains emergent and the evidence

[4] These studies provide strong evidence that the presence of conditional skewness and kurtosis affects the persistence of conditional variance of asset returns, i.e. the inclusion of conditional third and fourth moment led to a significant decrease in asymmetric variance thereby improving model estimates and performance.

limited (see for example (Clark and Baccar, 2018, Gong et al., 2010, Appadoo et al., 2012, Flynn et al., 2005). The predominant paradigm is still based on the adoption of a mean-variance model such as GARCH or Stochastic Volatility model at best with some realised volatility measures.

A second strand of literature that has emerged in the past decade, states extreme risks specifically that arising from leptokurtosis have been synonymized with Black Swan events (Marsh and Pfleiderer, 2012, Skinner, 2010, Vodra, 2009, Cowen and Abuaf, 2010, Seaberg, 2009). Black Swans here are defined as events that are effectively treated as outliers and hence lay beyond the realm of normal probability (Taleb, 2007b). Quite a few studies have focused on the de-blackening of Black Swans by proposing that uncertainty comes in several forms but the most popular versions are: known unknowns and unknown unknowns. The former stands for knowledge that we know we don't know and the latter for knowledge that we do not know we do not know. The literature highlights various methods by which Black Swans become prospectively more detectable. For example: if Black Swans emerge from the lack of knowledge (an unknown unknown), an increase in knowledge of various possibilities would decrease the blackness of a Black Swan (Aven, 2015) [5], and if Black Swans emerge from known unknowns then predictability can be achieved through joint cognitive processes that turn the implicit knowledge into collective practices (Lindaas and Pettersen, 2016). Other studies focus on adapting a holistic communication technique focused on adaptive governance techniques that allow firms to dynamically expand their scope when faced with extreme uncertainly (Wardman and Mythen, 2016). This chapter effectively contributes to this literature by focusing on the de-blacking of Black Swans through the testing and, in anticipation of the results, proposal of adopting a simple form of time-variation in the fourth moment of asset returns.

A final strand of literature on volatility that is tangent to the research objectives of this chapter involves the evidence on the presence of structural breaks in the mean and/or variance dynamics of asset returns which can lead to higher estimates of kurtosis leading to an upward bias in the estimates of long memory in the variance of stock returns (Starica and Granger, 2005, Hillebrand, 2005,

[5] Typically the author recommends an extension of the conceptualization of the domains of probability of extreme events, the development of risk assessment models that incorporate this extreme risk and finally resilience engineering in the decision making process of the management.

Karoglou, 2010a, Lamoureux and Lastrapes, 1990). Studies show that incorporating structural breaks in the volatility along with time-varying volatility of returns leads to better estimates of Value At Risk using conditional and unconditional tests (Hood and Malik, 2018). However, these studies focus primarily on structural breaks and excess time-varying kurtosis generically and do not study its effect on tail risk. This chapter can be seen as a natural extension of this work but with an explicit focus on the evolution of tail risk, i.e. identification of extreme values and Black Swan clusters due to time-varying kurtosis.

Combining the three stands of literature reveals an essential gap and in turn my research question: could unaccounted for structural breaks in the mean and/or variance dynamic of asset returns be a cause of excess kurtosis and if so, how is this effect played out when markets undergo extreme distress namely the Global Financial Crisis? Consequently, this chapter endeavours to examine from an empirical point of view, the extreme variability of equity markets of 35 countries, with respect to the existence of Black Swan clusters. Specifically, it tests whether the weak existing empirical evidence of time-varying kurtosis and skewness could be viewed as the manifestation of structural breaks in the underlying asset return distribution that are not taken into account. Such breaks would effectively segment the returns into subsamples, and by extension lead to the clustering of Black Swans.

Additionally, by examining the homogeneity or heterogeneity of these clusters we would have effectively been informed as to whether time-varying kurtosis, as captured by the changing frequency of Black Swans and the threshold values of the extreme values, could provide a simple albeit overarching explanation for the tail risk behaviour of asset returns. Therefore, a comparison of how the statistical properties of these clusters differ before and after the Global Financial Crisis would inform us as to how the stock markets respond in this respect during extreme financial distress. In effect, the contribution of this chapter lies in recognizing the effect that latent Non-linearities can have on the variabilities in equity returns, the ignorance of which can lead to the misestimating risk during a crisis. It is not hard to infer that this would have a significant impact on a variety of financial tools, such as those dealing with risk measurement, efficient asset pricing, optimal portfolio selection, and option pricing.

The rest of this chapter is organized as follows. Section II sets the context for the research with a literature review; Section III provides the specific research questions this paper seeks to answer; Section IV describes the data; Section V presents my research methodology. The final segments of the chapter are section VI, which presents the empirical results on the frequency of Black Swan clusters and extreme returns that exist in the stock market, and section VII, which discusses the implications for key market participants. Section VIII concludes.

3.2 Literature review

While extreme value theory has remained highly popular in the literature (as evidenced by the plethora of studies summarized in chapter 2) for its ability to capture tail risk in equity markets thereby provide superior estimates in comparison to parametric models yet there is a component of tail risk that requires further study. This section will focus on important studies that are directly related to the research questions of this chapter: tail asymmetry, black swans and structural breaks. Tail risk is synonymized with excess kurtosis in the distribution of asset returns. Another term for tail risk coined by Taleb (2007b) are Black Swans.

Tail Asymmetry

There are multiple studies that have focused on the implications of time-varying skewness and kurtosis on the asset returns (see (Harvey and Siddique, 1999, León et al., 2005, Jondeau and Rockinger, 2003a). Therefore, tail asymmetry is known to have a direct and significant effect on asset allocation and portfolio management (see (Chunhachinda et al., 1997, Guidolin and Timmermann, 2008), as well as risk management (So and Wong, 2012)).

This has resulted in the formation of various models that have the ability to capture tail asymmetry considered to be superior in comparison to models that hold the third and fourth moment as constant because of their inability to explain ex-ante market risk premiums and cross-section variation in asset returns (see (Grigoletto and Lisi, 2009, Premaratne and Bera, 2000, Jondeau and Rockinger, 2003b).

However, the main challenge of modeling tail asymmetry is fitting a suitable distribution since traditional parametric distributions were not appropriate. This is resolved by Extreme value theory (EVT) that specifies the distribution of extreme values as Generalized Extreme Distributions (GED). McNeil and Frey (2000) then improved the existing EVT model by first filtering the returns using GARCH models and then fitting the GED distribution to the tails of the distribution. While this method implicitly allowed for the incorporation of tail asymmetry in asset returns by modeling the left and right tails separately, its efficiency was never tested explicitly.

So and Chan (2014) investigated the significance of tail asymmetry by proposing a threshold extreme value distribution combined with GARCH to calculate Value at Risk over multiple periods. Their VaR estimates outperformed most benchmark financial models of extreme quantiles. While the study conclusive confirmed the efficiency of using EVT with GARCH to study tail asymmetry and the significant effect it has on risk management in equity markets, it also found finding further evidence regarding the degree of tail asymmetry which increases significantly when markets are in turmoil.

Black Swan clusters

The term Black Swan was formally coined with respect to risk and financial markets by Taleb (2007a) who defines it from the viewpoint of its attributes: Black Swan clusters are rare, have extreme impact and are essentially unpredictable prospectively (Taleb, 2007a). Although this research accepts the first two factual attributes about the extreme nature of Black Swan clusters, it integrally stunts the third as it deals more with human psychology as a coping mechanism after extreme events rather than the capability of statistical models to capture them. There exist other definitions in the literature with regards to Black Swans: Aven (2013) describes Black Swan as extreme events that are surprising given the context of present knowledge; Aven and Krohn (2014) identify three types of Black Swans, namely: 1. as an unknown unknown (a completely unknown event beyond the realm of the scientific, professional and academic knowledge), 2. Unknown knowns (events who occurrence was known to some and unknown to others) and 3. Known events with such low probability that their occurrence is considered negligible.

There has been a growing strand of literature that focuses on the identification of Black Swans in the financial markets during the Global Financial Crisis of 2008. A few focus specifically on the changes in the interest rates spreads between the overnight federal funds (OIS) and the interbank offer rate (LIBOR) since it is widely accepted as one of the key measures of financial stress (Brunnermeier, 2009, Mizen, 2008, Taylor and Williams, 2009, Ji and In, 2010). While Taylor and Williams (2009) identified August 2007 as the occurrence of an extraordinary Black Swan using visual observation of the trend line exhibiting an exceptionally high and volatile spread between OIS and LIBOR, the Ji and In (2010) used impulse response analysis in a multivariate setting in conjunction with bias-corrected bootstrapping to discuss the impact of the crisis on cross currency linkage of the OIS-LIBOR spread. Both studies found in part that markets were depicting stress signals before the collapse of the Lehman Brothers in 2008 as markets started demanded excess liquidity.

However, none of these studies account for structural breaks.

Structural breaks

Recent research has found the existence of structural breaks in financial time series (see (Koedijk et al., 1990, Werner and Upper, 2004, Lin and Kao, 2008, Quintos et al., 2001) and confirmed that tail behaviour does not remain stable over time. Liu (2013) tested for structural breaks using a transformed Generalized Pareto Distribution in the foreign exchange market, specifically UK Pound, Japanese Yen, and New Taiwan Dollar, all versus US Dollar. Their study provided evidence that there are multiple structural breaks in the tail index at quantiles lower than 1% and 5% with a 95% confidence interval.

A recent paper by Olson et al. (2012) used the Bai and Perron (1998) technique to identify structural breaks and therefore possible Black Swan clusters with 95% confidence intervals for eight international LIBOR-OIS spreads and the global credit default swap index. Their findings remain similar to those previously mentioned, in the essence that during the Global Financial Crisis of 2008 global markets (mainly emerging economies) were sending financial stress signals well before September of 2008. Another interesting finding of their study was that the statistical significance of shocks to the spread became statistically insignificant after a 30-day time period for all countries except the US.

Marsh and Pfleiderer (2012) find that the frequency and intensity of Black Swan reduces dramatically when existing models efficiently include predictable shifts (structural breaks) in the markets by using stochastic volatility models. For example: using S&P returns over a 20-year period, the paper provided empirical evidence that when returns were scaled by the prior day VIX –typically including the EGARCH effect, the returns were less extreme and closer to a Gaussian Distribution. Another interesting finding of the paper is that most asset prices specifically the yield premiums on corporate bonds were already inclusive of the time-varying risk premium which becomes heightened during crisis periods (Bao et al., 2011, Lin et al., 2011).

All the definitions assume the nature of Black Swan clusters lies essentially in its immense impact (Bogle, 2008).

3.3 Research Question

Evidence exists that by identifying structural breaks in the mean/volatility dynamics, the assumption of normality becomes more plausible in comparison to conditional volatility models. However, a very limited literature exists that endeavours to use EVT methods to measure financial risk arising from the probability of an extreme returns/outlier events after incorporating conditional volatility models and the use of non-parametric approaches to identify multiple breaks in the mean and/or variance dynamics. This chapter aspires to contribute to this literature.

Specifically, by allowing EVT to set a threshold of identification of these outliers combined with a break detection procedure it becomes possible to identify, test the homogeneity of, and estimate the frequency of Black Swan clusters and in turn test the hypothesis that excess kurtosis in the extreme tails of the asset distribution could be a result of unaccounted latent non-linearity in the mean and/or variance structure of the asset returns. More tangibly, it seeks to answer:

- Could unaccounted structural breaks in the underlying conditional mean and/or conditional variance dynamics be responsible for tail asymmetry/excess kurtosis in the distribution of asset returns?
- Are clusters of Black Swans homogenous in terms of their frequency?
- How does this variability of Black Swans change ex-ante and ex-post the Global Financial Crisis?

The impact of these critical questions can be calibrated with the appropriate pricing of financial investments as well as the investment horizon when there exists a probability of occurrence of an extreme event. For example, to underestimate the frequency of Black Swan clusters before the occurrence of a crisis would mean that the asset is under-priced and not taking into account the higher risk. Therefore, the risk-adjusted returns are not reflecting the plausible variability around the mean of the stock market returns.

The use of traditional financial models used to make standardized investment decisions that average out the risk (assuming homogeneity in the mean and/or variance dynamic of asset returns i.e. constant change in conditional kurtosis and skewness) of those assets could be disastrous because financial markets are inter-linked and pro-cyclical (Commission, 2009). Consequently, the collective behaviour of underestimating the probability of a (negative) Black Swan and investing in assets that do not reflect it, would lead to a far greater crisis than otherwise. In other words, by categorising the frequency of Black Swan clusters using structural breaks, my research will contribute to the current literature by making it possible to ascertain the exogenous risks of financial assets which could then be properly assigned to market participants that have the capacity to absorb that risk.

3.4 Data

I use daily closing prices of the benchmark stock indices of 34 countries extracted from the OECD[6] database accessed through the Thomson Reuters DataStream which is a comprehensive financial and economic information platform covering the equity, bond, forex, commodities and derivatives markets for several countries. The primary reason for choosing the OECD countries was that this basket of data is varied and inclusive, allowing for the demonstration of market risk in stock markets that according to the FTSE Russell's unique 4-tiered country classification structure ranges from most advanced (Japan, U.S., U.K., Germany and Switzerland among others) to advanced emerging (Brazil, Mexico and Turkey among others) to secondary emerging (China, Pakistan and India among others) and finally frontier economies (Slovakia, Slovenia and Estonia) (Russell, 2016).

The sample period runs from as January of 1965 to May of 2016 overall; however, the time period varies for some countries depending on when their benchmark index was introduced. Table 1 contains this information.

[6] The Organization for Economic Cooperation and Development (OECD) was founded in 1960 and consists of 35 member countries from the North and South of America to Europe and the Asia-Pacific.

Market	Full Name	Symbol	Start Date	Source
Greece	Athex Composite	GRAGENL	30/09/1988	Athens Stock Exchange
Austria	ATX - Austrian Traded Index	ATXINDX	07/01/1986	Wiener Boerse
Belgium	Belgium 20	BGBEL20	02/01/1990	BEL Group
Hungary	Budapest (BUX)	BUXINDX	02/01/1991	Budapest Stock Exchange
Chile	Chile Santiago Stock Exchange General (IGPA)	IGPAGEN	02/01/1987	Santiago Stock Exchange
Germany	DAX 30 Performance	DAXINDX	31/12/1964	Deutsche Boerse
France	France CAC 40	FRCAC40	09/07/1987	Euronext Paris
Ireland	Ireland Stock Exchange Overall (Iseq)	ISEQUIT	05/01/1983	Irish Stock Exchange
Israel	Israel Ta 125	ISTA100	23/04/1987	Israel Stock Market
Luxembourg	Luxembourg Stock Exchange General	LUXGENI	04/01/1999	Luxembourg Stock Exchange
Mexico	Mexico IPC (Bolsa)	MXIPC35	04/01/1988	Mexican Stock Exchange
Japan	Nikkei 225 Stock Average	JAPDOWA	03/04/1950	NIKKEI
Denmark	OMX Copenhagen (OMXC20)	DKKFXIN	04/12/1989	Nasdaq OMX
Finland	OMX Helsinki (OMXH)	HEXINDX	02/01/1987	Nasdaq OMX
Czech Republic	Prague Stock Exchange PX	CZPXIDX	06/04/1994	Prague Stock Exchange
Australia	Standard and Poor's / Australian Stock Exchange 200	ASX200I	29/05/1992	S&P/ASX
Canada	Standard and Poor's / Toronto Stock Exchange Composite Index	TTOCOMP	31/01/1950	S&P/TSX
Netherlands	AEX Index (AEX)	AMSTEOE	03/01/1983	Euronext Amsterdam
Turkey	Bist National 100	TRKISTB	04/01/1988	-
Italy	FTSE MIB Index	FTSEMIB	31/12/1997	FTSE
Spain	IBEX 35	IBEX35I	05/01/1987	BME, Spanish Exchanges
South Korea	Korea Stock Exchange Composite (KOSPI)	KORCOMP	31/12/1974	Korea Stock Exchange
United States	NASDAQ Composite	NASCOMP	05/02/1971	NASDAQ Stock Market
Norway	Oslo Exchange All Share	OSLOASH	03/01/1983	Oslo Bors
Portugal	Portugal PSI-20	POPSI20	31/12/1992	Euronext Lisbon
New Zealand	Standard and Poor's / NZX 50	NZ50CAP	29/12/2000	New Zealand Exchange (NZX)
Slovakia	Slovakia SAX 16	SXSAX16	14/09/1993	Bratislava Stock Exchange
Slovenia	Slovenian Blue Chip (SBI Top)	SLOETOP	31/03/2006	Ljubljana Stock Exchange
Switzerland	Swiss Market (SMI)	SWISSMI	30/06/1988	SWX Swiss Exchange
Poland	Warsaw General Index	POLWIGI	16/04/1991	Warsaw Stock Exchange
Sweden	OMX Stockholm (OMXS)	SWSEALI	28/12/1979	Nasdaq OMX
United Kingdom	FTSE All Share	FTALLSH	10/04/1962	FTSE

Table 1: Countries along with the full name of their benchmark stock index, code, inauguration date, and source.

Descriptive Statistics

The values of the aforementioned stock market indices are first converted into log returns by taking the natural log-differences using the following equation:

$$R_i = log\left[\frac{P_{i+1}}{P_i}\right] = \log(P_{i+1}) - \log(P_i) \tag{13}$$

Appendix 1 presents the time-series graph of each series and Table 1 below provides a brief overview of their statistical properties.

Table 1 – Summary Statistics for Equity Market Daily Returns

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
No. of Observations	6255	7923	9493	8950	6883
Mean	0.02%	0.02%	0.03%	0.03%	0.06%
Standard Deviation	0.95%	1.33%	0.98%	1.04%	1.12%
Skewness	-0.44	-0.32	-0.22	-0.74	0.21
Kurtosis	5.66	7.57	10.38	12.68	6.75
	Finland	**France**	**Germany**	**Greece**	**Hungary**
No. of Observations	7665	7531	13406	7210	6622
Mean	0.03%	0.01%	0.02%	0.01%	0.05%
Standard Deviation	1.61%	1.38%	1.22%	1.86%	1.61%
Skewness	-0.38	-0.14	-0.25	-0.1	-0.51
Kurtosis	9	5.31	7.49	5.93	11.75
	Italy	**Japan**	**Korea**	**Luxembourg**	**Mexico**
No. of Observations	4797	13406	10798	4534	7404
Mean	-0.01%	0.02%	0.03%	-0.01%	0.08%
Standard Deviation	1.56%	1.24%	1.50%	1.68%	1.52%
Skewness	-0.09	-0.42	-0.31	0.22	0.07
Kurtosis	3.79	10.28	8.27	59.32	7.49
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**
No. of Observations	5764	6101	5918	2383	7402
Mean	0.01%	0.01%	0.02%	-0.01%	0.02%
Standard Deviation	1.80%	1.16%	1.44%	1.66%	1.38%
Skewness	-0.16	-0.37	1.49	-0.1	-0.1
Kurtosis	5.4	7.16	41.8	291.74	5.83
	UK	**USA**	**Estonia**	**Turkey**	**Norway**
No. of Observations	9731	13406	5209	7142	7665
Mean	0.03%	0.03%	0.04%	0.13%	0.03%
Standard Deviation	1.09%	1.01%	1.51%	2.60%	1.50%
Skewness	-0.53	-1.04	-1.01	-0.04	-0.97
Kurtosis	21.79	28.42	24.71	4.45	15.6
	Switzerland	**New Zealand**	**Ireland**	**Denmark**	**Czech**
No. of Observations	7014	4015	8707	6904	5772
Mean	0.03%	0.01%	0.03%	0.03%	0.00%
Standard Deviation	1.14%	0.69%	1.22%	1.17%	1.33%
Skewness	-0.39	-0.49	-0.6	-0.27	-0.44
Kurtosis	8.21	5.54	11.04	5.83	12.15

	Israel	Sweden	Netherlands	Iceland
No. of Observations	7586	7664	8709	6101
Mean	0.05%	0.04%	0.03%	0.02%
Standard Deviation	1.45%	1.43%	1.32%	1.71%
Skewness	-0.42	0.01	-0.27	-45.27
Kurtosis	6.24	4.59	8.27	2822.65

Table 1 – Descriptive Statistics for daily log returns of stock markets of OECD countries

It is evident from the descriptive statistics that the empirical distribution of stock returns deviates from normal where third and fourth moments are concerned, characterized by high kurtosis and some degree of negative asymmetry.

A visual test of normality is conducted using Q-Q plots (presented in Appendix 2) which provides evidence that the data is characterized by heavy tails indicating a higher probability of extreme returns than a normal distribution.

3.5 Methodology

The research question primarily revolves around three aspects: tail risk, breaks that may be the manifestation of structural changes or caused by latent Non-linearities in the mean and/or variance dynamic of equity returns, and possible volatility dynamics. The chapter will use Extreme value theory to identify the threshold for tail risk (log returns above a 1% threshold on the left and right tails), the Karoglou (2010a) break-test procedure to identify potential structural breaks in the data and finally a best fit ARMA-APARCH model to account for the effects of volatility clustering and leverage effects.

The first model will include daily log returns from the entire sample as well as segments of daily log returns sliced according to break dates identified by the break-test procedure. Once the two groups of returns have been recognised, tail risk will be measured using extreme values identified by taking returns that lie above the 1% threshold on the left and right rails of the respective asset distributions and Black swans which are returns that are three standard deviations from the mean. The focus, however, will remain on black swans as they measure tail risk during a crisis where as extreme values will exist in every segment irrespective of the existence of extreme volatility.

Finally, a Black Swan cluster is established when there is a change in the frequency of Black Swans from one segment to the next. By comparing these Black Swan clusters in segment to those of the full sample, a consensus can be ascertained regarding the nature of extreme tail risk. Specifically, if the frequencies are the same, then structural breaks do not account for excess kurtosis; however if there is a significant difference in the number of Black Swan clusters, then two inferences can be drawn: first, that Black Swans clusters are not homogenous in nature and second that excess kurtosis can be a result of unaccounted structural breaks in the underlying stochastic process of stock returns or latent Non-linearities in the mean and/or variance dynamics. Based on these, I can then extrapolate the impact of the Global Financial Crisis simply by comparing the respective measures in the before and after the crisis periods.

To ensure that the analysis is comprehensive, I also look at a second model namely one that aims to tests the same hypothesis but with standardized residuals from the best-fit ARMA-APARCH model to account for stylized properties of equity returns such as volatility clustering, leverage effects and excess kurtosis.

The following section starts with detailing the method of identifying structural breaks in the daily returns, trailed by two models listing their respective hypotheses for the testing of homogeneity of Black Swan clusters in the returns and residuals respectively. In both models, the extant underlying theme is related to extreme values, Black Swan clusters, and structural breaks.

Identification of Structural Breaks

By definition structural breaks in a time series indicate unforeseen swings in the series which have the potential of rescinding the results of financial econometric techniques such as: imprecise parameter estimates resulting from incorrect model specification, prejudiced forecasts, and unreliable interpretations drawn from econometric tests (Pesaran and Timmermann, 2004). Financial time series are characterized by significant leptokurtosis but it is routinely assumed that the tail behaviour remains constant over time. However, when structural breaks exist in the tail index it can pose a fundamental challenge in the implementation of econometric models and extrapolation of its conclusions especially when applying EVT that studies returns in extreme quantiles.

The structural breaks in this research have been recognized using the two stages of the Nominating-Awarding procedure of Karoglou (2010a). The reason this approach has been chosen is because it acknowledges different strains of break-tests that can be combined to identify structural changes in the mean/variance dynamics as well as latent non-linearity's in the data that might add bias to the model. Therefore, it fortitudes the selection of a particular break date.

The first stage of Nominating-Awarding procedure is the Nominating stage which identifies break dates involves using break tests such as:

1. The Inclan-Tiao test – which uses simple binary-division algorithm like the iterated Cumulative Sum of Squares (ICSS) algorithm to retrospectively detect changes in the unconditional variance of a stochastic process (Inclan and Tiao, 1994).
2. Sansȯ Aragȯ and Carrion test$_1$ which takes into account kurtosis properties of financial data and allows for conditional heteroscedasticity (Sansó et al., 2004)
3. Sansȯ Aragȯ and Carrion test$_2$ (Sansó et al., 2004) with the Bartlett kernel and the Quadratic Spectral kernel, implemented using the automatic procedure for bandwidth selection (Newey and West, 1994) as well as the Vector Autoregressive Heteroscedasticity and Auto-Correlation consistent (VARHAC) kernel (Den Haan and Levin, 1998) to bypass the bandwidth selection issues.
4. Kokoszaka and Leipus test (Kokoszka and Leipus, 2000) refined by Andreou and Ghysels (2002) including the Bartlett, Quadratic spectral and VARHAC kernel correspondingly.

While these test detect structural breaks in mean and/or volatility dynamics, they do not differentiate between the two (Karoglou, 2006b). Their efficiency, however, remains unchallenged in the literature (Andreou and Ghysels, 2002, Sansó et al., 2004, Karoglou, 2006b) which confirms that even for strongly dependent time series with the existence of outliers, the segments do not exhibit size distortions. This ingenious evidence is opportune to this analysis of Black Swan clusters and extreme outliers as distributional properties of the returns remain unscathed. This procedure also allows for segments differentiated by structural changes in the mean/variance dynamic to be identified while respecting their occurrence in the time series.

The second stage of the Nominating-Awarding procedure is that of eliminating breaks identified in the first stage and uniting continuous segments with homogenous first and second order moments. This process continues until one of the two below mentioned conditions are satisfied till there are no segments that can be united:

1. If the t-test (when variance of continuous segments are similar) and the Satterthwaite-Welch t-test (when variance of continuous segments are disparate) confirm that the means of neighbouring segments are statistically diverse;
2. If the standard F-test, the Siegel-Tukey test with continuity correction (Siegel and Tukey, 1960, Sheskin, 2003), the adjusted Bartlett test (Sokal and Rohlf, 1995, Judge et al., 1982), the Levene test (Levene, 1960) and the Brown-Forsythe test (Brown and Forsythe, 1974) confirm the homogeneity of variances of neighbouring segments.

The final outcome of this awarding break dates procedure are multiple segments that are homogenous in their mean and/or variance dynamics.

Model Specification

The model empirically tests the effect of unaccounted structural breaks on extreme volatility, i.e. could latent non-linearity in the underlying mean and/or variance dynamic of the asset return distribution be a potential cause of tail asymmetry? This is tested by looking at the homogeneity of Black Swan clusters in the financial series with and without a provision for structural breaks i.e. does allowing a provision for incorporating structural breaks in the mean and/or variance dynamics lead to a significant difference in the frequency of Black Swan clusters, thereby approving the claim that these breaks could be a potential cause of tail asymmetry in financial returns. For robustness, the hypothesis will be tested first on returns and then on residuals of returns obtained from the best fit ARMA-APARCH model in order to include the effect of volatility clustering, leverage and negative skewness which are stylized properties of financial returns.

The results of both models will be contextualized using the Global Financial Crisis of 2008, which effectively provides the underlying research question of this chapter, namely, how does the frequency of Black Swans change due to the Global Financial Crisis.

Schematically, structural breaks in the underlying asset return distribution can be presented as follows:

$$r_t = \begin{pmatrix} \mu_0 + \sigma_0 \cdot u_{o,t} & for\ 0 \le t \le \tau_1 & , \\ \mu_1 + \sigma_1 \cdot u_{1,t} & for\ \tau_1 \le t \le \tau_2 & , \\ \mu_n + \sigma_n \cdot u_{n,t} & for\ \tau_n \le t \le T & \end{pmatrix} \quad (14)$$

Where r_t stands for the daily log returns of the equity market, T is the sample size, n denotes the number of structural breaks which occur at dates point $\tau_1, \tau_2, \ldots, \tau_n$ when there is a change in the unconditional mean μ and or variance σ^2 of the underlying non-parametric process of equity returns. The standardized unexpected return is signified by u_t which have a mean of 0 and variance of 1. It is also important to note that minimalistic assumptions have been made regarding the underlying dynamics of the model.

In order to capture the heteroskedastic properties of the returns process as well as the leverage effects, the conditional variance of the returns process can be defined by the APARCH (m, n):

$$\varepsilon_t = z_t \sigma_t; \qquad z_t \sim N(0,1) \quad (15)$$

$$\sigma_t^\delta = \varphi_0 + \sum_{i=1}^{m} \psi_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{n} \omega_j \left(\sigma_{t-j}\right)^\delta \quad (16)$$

Section 3.6.1 empirically test the homogeneity of Black Swan clusters in the full sample and then compare it to the frequency of Black Swan clusters when there is a provision that recognizes structural breaks in the underlying data in daily returns. The overarching objective will be observing any reduction in tail asymmetry in segmented returns. Sections 3.6.2 will investigate the equivalent outcome in residuals of returns, thereby controlling for the effect of stylized properties of financial returns such as volatility clustering and negative kurtosis. The second model is critical from a robustness perspective to control the effect of conditional heteroscedasticity in the data which is known to be heightened during a crisis.

Both sections will also examine the change in the ratio of Black Swans to extreme values during the Global Financial Crisis to contextualize the outcomes.

3.6 Empirical Results

Using the Nominating-Awarding procedure has identified two to eight structural breaks in each data set. The empirical results in the following section summarizes the findings of the study with respect to the above-mentioned research questions.

3.6.1 Model 1 Outcomes

Summary of the frequency of the Black Swan clusters in the full sample and segments (with breaks) of daily stock market returns

The results in table 2 depict the ratio of Black Swan clusters in daily log returns segments with and without the provision of structural breaks. The full segment assumes homogeneity in first and second order moments and does not allow for a provision of structural breaks in the mean and/or variance dynamics. The segmented sample addresses these issues. The nominating-awarding procedure typically finds four to eight structural breaks in the daily log equity returns of these OECD countries during the data timeline which spans over two decades. More detailed results can be found in Appendix 5.

Table 2 - Differences in the frequency of Black Swan clusters between the full sample and the segmented sample

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	78.85%	3.92%	6.45%	22.74%	*20.29%*
Left tail	36.55%	14.84%	28.00%	26.73%	*20.29%*
	Czech	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	-6.90%	38.78%	8.92%	15.23%	55.96%
Left tail	7.85%	48.29%	19.11%	26.75%	37.04%
	Mexico	**Netherlands**	**Denmark**	**Estonia**	**Finland**
Right tail	13.58%	47.96%	*18.72%*	*14.17%*	*49.11%*
Left tail	12.41%	23.18%	*3.85%*	*13.06%*	*42.05%*
	France	**Germany**	**Greece**	**New Zealand**	**Norway**
Right tail	*7.41%*	*16.25%*	*3.64%*	28.77%	9.53%
Left tail	*20.25%*	*36.77%*	*21.22%*	37.16%	5.26%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Ireland**

Right tail	51.88%	-7.41%	-14.17%	0.00%	19.91%
Left tail	15.76%	25.59%	-25.78%	0.00%	18.03%
	Israel	**Switzerland**	**Hungary**	**Iceland**	**Spain**
Right tail	11.51%	19.11%	-8.34%	-378%	18.66%
Left tail	-3.39%	7.52%	13.88%	-186%	0.00%
	Sweden	**Turkey**	**UK**	**USA**	
Right tail	22.31%	46.36%	7.90%	16.13%	
Left tail	33.95%	20.63%	10.23%	13.04%	

Table 2 summarizes the difference in the frequency of Black Swan Clusters in the full sample versus the segmented sample in the equity markets of the OECD countries of the data set. By differentiating the black swan clusters on each tail, the result represents the skewness of the data.

The above table 2 presents the results that test the hypothesis of the homogeneity of Black Swan clusters when changes in mean and/variance volatility are accounted for as opposed to not. The evidence allows for the rejection of the null hypothesis as clearly accounting for structural breaks reduces the frequency of Black Swans by approximately 20% (with the exception of Iceland and Slovakia). Therefore, by incorporating structural breaks, the probability of unusual events reduces substantially along with reducing tail asymmetry in equity market returns. The countries with the highest reduction in total Black Swans are Australia (51.49%), Finland (45.52%) and Italy (44.63%).

The case of Iceland is extremely unusual as the table shows that full segments have eight Black Swan clusters whereas taking the change in mean and/or variance dynamics finds 89 Black Swan clusters within the segments. This can be attributed in part to the latent non-linearity's of a crisis (like the banking collapse of 2008) which can prejudice interpretations.

A closer look at table 2 in terms of a horizontal country comparison with respect to overall Black Swan clusters depicts that the countries with the highest frequency/percentage of trading days of Black Swans in the full sample and segmented samples are: USA (184/2.32%), Japan (197/2.49%) and Germany (184/2.32%); and Japan (170/2.15%), UK (159/2.01%) and Korea (154/1.94%). Not surprisingly, these are also the countries with the highest frequency/percentage of trading days that account for Black Swans in the full segment sample on the left tail and right tail respectively: Japan (115/1.45%), Germany (104/1.31%) and Korea (98/1.24%); and Japan (82/1.03%), Korea (92/1.16%), Germany (80/1.01%) and USA (94/1.19%).

In terms of the frequency of Black Swans in the left and right tail, the evidence suggests that given the total number of Black Swans in the full sample, on average, 56% are negative and 44% are positive. This can be translated as: a) there is always a slightly higher probability of events occurring that will affect stock returns negatively and/or b) markets overreact to negative news leading to a higher probability and emanation of bearish markets as compared to reactions to positive news leading to softer impact and culmination of bullish markets. The above trend is consistent across 31 of the 35 countries with the exception of UK, USA, Estonia, and Mexico where the proportion of Black Swans in the right tail exceeded those in the left tail by an average of 7%. For example, the countries with the highest Black Swans within the segmented samples of the left tail were: Japan (95/1.20%), Korea (75/0.95%), Ireland (81/1.02%) and USA (79/1.00%) and those with the highest Black Swan clusters in the segmented samples of the right tail were: Japan (75/0.95%), Korea (79/1.00%) and USA (80/1.01%).

A final observation is that in terms of frequency of Black Swans on the left and tail right, the segmented sample follow the orientation of full samples but with a much lower frequency, i.e. allowing for structural breaks reduces the frequency of Black Swan in both tails thereby reducing the skewness of the distribution. This result is consistent with the empirical results in the equity market returns as well discussed in chapter 4.

Summary of the ratio of Black Swans to extreme values in the full sample and segments (with breaks) of daily stock market returns pre and post the Global Financial Crisis (2007-2008)

The following section examines the variations to volatility in daily log returns using the ratio of Black Swans (as three standard deviations away from the mean) to extreme values. Given that the former measure of tail risk pre-specifies the interval of values that is considered ‘abnormal’ and the latter the number of observations that are considered ‘abnormal’, which should be the same for two different samples with exactly the same number of observations, changes in the ratio of the two would effectively indicate that the intervals of ‘abnormal’ uncertainty have also changed.

As before, I use two lenses: first, a preliminary analysis is done on how the ratio of Black Swans to extreme values vicissitudes when there exists a provision for structural breaks in the mean and/or variance dynamic as opposed to not. This contributes to the literature on the impact of structural change onto the tail risk. Second, I focus on how the ratio changes when the study epicentres on the Global Financial Crisis of 2008-2009, and seek to answer how the tail risk in equity markets evolve with respect to the occurrence of a crisis.

The threshold for extreme values has been set to the 1st and 99th percentile of the returns distribution and the table 3 illustrates the ratio of Black Swans to extreme values overall as well as in the left and right tails of the full sample and segmented samples of the daily log returns of the equity market (See Appendix 6 for more detailed results).

Table 3 - Difference in the ratio of Black Swans to extreme values between full sample and segmented samples

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	83.50%	6.39%	10.58%	27.09%	24.55%
Left tail	41.20%	17.31%	32.12%	31.08%	24.55%
	Czech	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	-5.19%	44.84%	10.39%	18.87%	62.28%
Left tail	9.56%	54.35%	20.58%	30.38%	43.36%
	Mexico	**Netherlands**	**Denmark**	**Estonia**	**Finland**
Right tail	20.03%	52.40%	18.72%	16.03%	52.93%
Left tail	18.86%	27.63%	3.85%	14.93%	45.87%
	France	**Germany**	**Greece**	**New Zealand**	**Norway**
Right tail	12.54%	17.72%	5.00%	35.83%	13.35%
Left tail	23.38%	38.24%	22.58%	44.22%	9.09%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Ireland**
Right tail	55.27%	-4.18%	-10.89%	8.00%	22.16%
Left tail	19.15%	28.82%	-22.50%	8.00%	20.27%
	Israel	**Switzerland**	**Hungary**	**Iceland**	**Spain**
Right tail	15.38%	23.24%	-6.86%	-374%	21.29%
Left tail	1.81%	11.66%	15.37%	-181%	2.63%
	Sweden	**Turkey**	**UK**	**USA**	
Right tail	27.38%	47.74%	11.90%	17.60%	
Left tail	39.02%	22.01%	14.23%	14.51%	

Left tail	-3.39%	7.52%	13.88%	-186%	0.00%
	Sweden	**Turkey**	**UK**	**USA**	
Right tail	22.31%	46.36%	7.90%	16.13%	
Left tail	33.95%	20.63%	10.23%	13.04%	

Table 3 summarizes the difference in the ratio of Black Swan Clusters to extreme values in the full sample versus the segmented sample in the equity markets of the OECD countries of the data set. By differentiating the black swan clusters on each tail, the result represents the skewness of the data.

There are two key observations about the trends in the ratio of Black Swans to extreme values that provide support to previously drawn conclusions based upon the frequency of Black Swans when a provision for structural breaks exists as opposed to not.

First, when provision of mean and/or variance breaks are accounted for the ratio of Black Swan clusters to extreme values declines considerably i.e. in the full segment of daily log returns which do have a provision of structural breaks, the model identifies 3 blacks swans for every 4 extreme values (approximately 76%) and this ratio falls to 2 Black Swans for every 4 extreme value when structural breaks are provided for (approximately 60%) for every country except Iceland and Slovakia.

The second observation is that in terms of the ratio of Black Swans to extreme values in the left tail and right tail, the former is heavier (83% of extreme values were Black Swans) than the latter (66% of extreme values were Black Swans). This trend fundamentally endures even when there is a provision for breaks but the ratio decreases substantially as was evidenced in section 1.6.1 as well i.e. in the left tail, of all the extreme values, 68% are classified as Black Swans and in the right tail 53% of extreme values are Black Swans.

In order to analyse the transformation in the frequency of Black Swans to extreme values before, during, and post the Global Financial Crisis, I have selected the segments defined by the three breaks that were closest to the collapse of the Lehman Brothers (15.09.2008). Table 4 reports the breaks and demonstrates that apart from a few cases there was typically a break detected close to the 15.09.2008.

Table 4 – Structural breaks identified for the Global Financial Crisis, ex-post and ex-ante segments

Country	**Australia**	**Austria**	**Belgium**	**Canada**
Pre-Crisis	29/10/2001	08/10/1992	26/07/2007	29/10/1997
Crisis	27/07/2007	30/07/2007	17/01/2008	21/08/2009
Post-Crisis	21/07/2009	09/11/2009	26/05/2009	06/01/2012
Country	**Chile**	**Czech**	**Denmark**	**Estonia**
Pre-Crisis	12/06/1998	07/04/1994	14/07/1997	04/06/1996

Crisis	18/05/2007	24/06/2006	10/08/2007	22/10/2008
Post-Crisis	05/12/2011	15/06/2010	02/07/2009	05/12/2011
Country	**France**	**Germany**	**Greece**	**Hungary**
Pre-Crisis	15/04/2003	19/06/2003	27/09/2001	09/07/1993
Crisis	17/01/2008	17/01/2008	25/06/2008	06/04/2005
Post-Crisis	18/05/2009	20/07/2009	16/10/2014	25/01/2012
Country	**Iceland**	**Ireland**	**Israel**	**Italy**
Pre-Crisis	26/08/2004	11/02/1988	05/04/1995	10/04/2003
Crisis	12/12/2008	26/07/2007	23/04/2005	08/09/2008
Post-Crisis	08/03/2011	13/07/2010	21/09/2009	26/05/2009
Country	**Korea**	**Luxembourg**	**Mexico**	**Netherlands**
Pre-Crisis	14/03/1986	28/03/2003	09/01/2001	10/07/2003
Crisis	02/05/2006	11/05/2007	19/10/2007	17/01/2008
Post-Crisis	23/07/2009	05/08/2010	28/07/2009	20/07/2009
Country	**New Zealand**	**Norway**	**Poland**	**Portugal**
Pre-Crisis	01/01/2001	17/12/1992	08/06/1995	28/02/2003
Crisis	09/01/2008	16/05/2006	08/02/2005	11/01/2008
Post-Crisis	27/08/2009	06/08/2009	31/05/2010	11/12/2008
Country	**Slovenia**	**Spain**	**Sweden**	**Switzerland**
Pre-Crisis	04/04/2007	08/01/1988	13/04/2004	28/06/2004
Crisis	20/11/2012	12/04/2006	28/07/2008	29/07/2008
Post-Crisis	20/06/2014	19/01/2009	08/07/2010	09/04/2010
Country	**Turkey**	**UK**	**USA**	**Finland**
Pre-Crisis	30/10/1998	13/02/1989	31/03/1998	22/07/2003
Crisis	19/04/2006	19/07/2005	21/10/2006	27/07/2007
Post-Crisis	26/05/2010	21/07/2010	21/07/2010	21/07/2009
Country	**Japan**	**Slovakia**		
Pre-Crisis	23/02/1990	12/02/2003		
Crisis	08/01/2008	11/09/2008		
Post-Crisis	22/05/2009	01/07/2010		

It is important to note in table 4 that while most of the break dates regarding the advent of the Global Financial Crisis are similar for the countries in the dataset yet there are a few that experienced the decline much later. A plausible explanation for this could be the lead-lag effect caused by the delay in information diffusion across global equity markets.

Table 5 builds on the above results to report the difference in the ratio of Black Swans to extreme value before and after the Global Financial Crisis.

Table 5 – Difference in the ratio of Black Swans to extreme value ex-post and ex-ante the GFC of 2008

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Pre-Crisis	56.67%	70.51%	70.37%	77.42%	50.00%
Crisis	41.67%	100.00%	0.00%	57.14%	56.45%
Difference	-30.75%	34.94%	N/A	-30.37%	12.14%

Post-Crisis	35.71%	56.25%	75.00%	31.25%	25.00%
Difference	-15.42%	-57.54%	N/A	-60.35%	-81.45%
	Czech	**Denmark**	**Estonia**	**Finland**	**Germany**
Pre-Crisis	95.45%	55.56%	84.29%	68.18%	37.50%
Crisis	65.63%	80.00%	50.00%	58.33%	100.00%
Difference	-37.47%	36.46%	-52.22%	-15.60%	98.08%
Post-Crisis	80.00%	53.33%	81.25%	43.75%	75.00%
Difference	19.81%	-40.55%	48.55%	-28.77%	-28.77%
	Greece	**Hungary**	**Iceland**	**Ireland**	**Israel**
Pre-Crisis	55.56%	106.67%	12.50%	80.39%	62.50%
Crisis	52.94%	58.82%	75.00%	68.75%	72.73%
Difference	-4.82%	-59.52%	179.18%	-15.64%	15.15%
Post-Crisis	66.67%	45.83%	45.00%	50.00%	57.14%
Difference	23.05%	-24.95%	-51.08%	-31.85%	-24.12%
	Italy	**Japan**	**Korea**	**Mexico**	**Netherlands**
Pre-Crisis	56.67%	60.64%	84.44%	44.44%	45.83%
Crisis	50.00%	75.00%	64.71%	66.67%	125.00%
Difference	-12.52%	21.26%	-26.62%	40.55%	100.33%
Post-Crisis	38.89%	35.29%	66.67%	97.62%	57.14%
Difference	-25.13%	-75.38%	2.99%	38.14%	-78.28%
	New Zealand	**Norway**	**Poland**	**Portugal**	**Slovakia**
Pre-Crisis	50.00%	81.43%	61.11%	42.31%	113.33%
Crisis	30.00%	88.89%	59.09%	66.67%	100.00%
Difference	-51.08%	8.77%	-3.36%	45.47%	-12.52%
Post-Crisis	45.45%	50.00%	58.33%	47.50%	94.44%
Difference	41.55%	-57.54%	-1.29%	-33.90%	-5.72%
	Slovenia	**Spain**	**Turkey**	**UK**	**Switzerland**
Pre-Crisis	40.00%	74.42%	66.67%	44.64%	68.18%
Crisis	20.00%	53.85%	53.13%	82.76%	80.00%
Difference	-69.31%	-32.36%	-22.71%	61.72%	15.98%
Post-Crisis	33.33%	61.54%	65.63%	42.86%	78.57%
Difference	51.08%	13.35%	21.13%	-65.81%	-1.80%
	USA	**France**	**Luxembourg**	**Sweden**	
Pre-Crisis	25.00%	53.85%	18.18%	58.33%	
Crisis	79.03%	87.50%	38.89%	41.67%	
Difference	115.10%	48.55%	76.03%	-33.65%	
Post-Crisis	71.43%	72.22%	58.33%	68.75%	
Difference	-10.12%	-19.19%	40.55%	50.08%	

Table 5 summarizes the difference in the frequency in the ratio of Black Swan Clusters to extreme values in segments identified using the break-testing procedure as mentioned in the methodology to show the evolution of black swan cluster before the crisis in comparison to after the crisis.

The dominant trend that is apparent in the ratio of Black Swans to extreme values in terms of pre and post-crisis periods is that the ratio seems to peak before a crisis and then continues to decline steadily suggesting that as the crisis becomes more apparent and economic measures are taken to stabilize the economy in the majority of the data set the probability of extreme events becomes more moderate.

A plausible assumption here is that stock markets were already exhibiting financial distress which culminated into a global crisis in the next period by which time investors had already become wary of the warning signals and were taking a more cautious investment position. However, when averages are taken there are 13% more Black Swans to extreme values during the segment identified as the Global Financial Crisis as opposed to the segment before and 20% less Black Swan clusters to extreme values in the segment after (identified during the Nominating-Awarding Procedure).

The countries with the highest ratio of Black Swan clusters to extreme values during the crisis period are: USA, Iceland, Germany and the Netherlands.

3.6.2 Model 2 Outcomes

Summary of the frequency of Black Swans in the residuals of the daily equity returns using the best fit ARMA-APARCH model

With regard to robustness, model 2 removes the effect of volatility clustering in the series by using the residuals of the best fit ARMA-APARCH model. The tail risk of these residuals is then studied closely for the effect of structural breaks on tail risk presented in table 6 (See Appendix 7 for detailed results).

Table 6 – Difference in the frequency of Black Swan clusters in residuals of the full sample versus the segmented sample (Equity Markets)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	51.08%	0.00%	11.78%	104.98%	8.00%
Left tail	24.69%	-1.98%	7.52%	37.04%	-13.40%
	Czech	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	17.19%	35.67%	-8.89%	4.88%	28.77%
Left tail	-5.88%	19.11%	16.30%	0.00%	0.00%
	Mexico	**Netherlands**	**Denmark**	**Estonia**	**Finland**
Right tail	8.00%	18.23%	0.00%	-19.57%	-57.54%
Left tail	0.00%	6.74%	-3.28%	0.00%	2.15%
	France	**Germany**	**Greece**	**New Zealand**	**Norway**
Right tail	-7.41%	-10.01%	-5.00%	0.00%	4.88%
Left tail	-9.24%	-7.28%	-19.57%	23.64%	5.56%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Ireland**

Right tail	14.31%	12.78%	-25.13%	-150.41%	10.54%
Left tail	29.85%	-8.00%	-3.08%	-36.77%	-14.66%
	Israel	**Switzerland**	**Hungary**	**Iceland**	**Spain**
Right tail	7.15%	19.42%	6.90%	-297.04%	26.83%
Left tail	14.76%	-2.82%	-2.35%	-184.58%	-15.82%
	Sweden	**Turkey**	**UK**	**USA**	
Right tail	-20.48%	0.00%	-42.61%	0.00%	
Left tail	-25.49%	4.65%	-63.13	0.00%	

Table 6 summarizes the frequency of black swan clusters in the residuals of daily log returns of the equity markets in the data set (full samples versus the segmented samples). The frequency of clusters on the left and right tail are representative of the skewness of the data.

The above table 6 suggests that when autoregressive and generalized autoregressive conditional heteroscedasticity models (in this case the best fit ARMA-APARCH model was applied) are used to sieve volatility clustering in the daily log returns to achieve standardized residuals, the null hypothesis of homogeneity of the frequency of Black Swan clusters is still rejected. By taking into account the structural breaks in the mean and/or variance dynamics, the frequency of Black Swans is reduced by 12% in segmented samples as compared to full samples for more than half of the countries in the sample.

However, it is important to note that the other half of the data set, depict a reversed trend in terms of the frequency of Black Swans in segmented samples *vis-à-vis* the full sample i.e. Black Swans in segmented samples exceed those in full samples by approximately 30% (including Iceland where the frequency of Black Swans in segmented samples are 226% more than those in the full sample). A possible explanation for this phenomenon could be that the presence of conditional heteroscedasticity exaggerated the tail risk. However, swarms of Black Swans reduced drastically once ARMA-APARCH residuals were extracted thereby removing any autocorrelation or dependency in the daily log returns distribution. This assumption is further validated by the fact that after these models are implemented the percentage of Black Swans in full samples constitute on average 1.49% of the trading days drops to 0.87%; and in segments with a provision for breaks the percentage of Black Swans that constitute on average 13.31% of the trading days shrinks to 0.90%.

A final development in residuals that is contrasting to earlier outcomes observed in returns is that the percentage of Black Swans in left tail (61%) is higher than the percentage of Black Swans in the right tail (39%) irrespective of the provision of breaks. This proves that while accounting for structural breaks reduces tail asymmetry overall, equity markets continue to display negative skewness.

Summary of the ratio of Black Swan clusters to extreme values in the residuals of daily equity returns using the best fit ARMA-APARCH model

For appropriateness and robustness of the model results, this section of the results examines the fluctuations in the ratio of Black Swans to extreme values in the residuals of the best-fit ARMA-APARCH model. Again, the study is two-fold: it begins by examining the change in the ratio of Black Swans to extreme values in residuals when compared to returns i.e. whether the Black Swans identified in section 3.6.1 are reduced when non-parametric models are fitted to the series. The second fragment of the model centres specifically on the Global Financial Crisis to study the evolution of the ratio when heteroscedasticity and structural breaks are accounted for. In particular, it seeks to answer the questions such as: how do equity markets behave before, during and after the Global Financial Crisis when volatility clusters are removed in samples that also exhibit the same non-parametric behaviour in terms of the first and second central moments.

Table 7 provides evidence to the first section of the model testing the change in the ratio of Black Swans to extreme values when a provision is incorporated for changes in the mean and/or variance dynamics. It also emphases on the left and right tail of the distribution to highlight any underlying skewness (see Appendix 8 for more detailed results).

Table 7 – Difference in the ratio of Black Swans to extreme values in the residuals of the full sample versus the segmented sample (Equity Markets)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	55.73%	2.47%	15.90%	109.33%	12.26%
Left tail	29.34%	0.49%	11.65%	41.39%	-9.10%
	Czech	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	18.89%	41.73%	-7.42%	8.52%	35.09%
Left tail	-4.17%	25.17%	17.72%	3.64%	6.32%
	Mexico	**Netherlands**	**Denmark**	**Estonia**	**Finland**
Right tail	14.46%	22.68%	0.00%	-17.71%	-53.71%
Left tail	6.45%	11.19%	-3.28%	1.87%	5.97%
	France	**Germany**	**Greece**	**New Zealand**	**Norway**
Right tail	-2.28%	-8.54%	-3.64%	7.06%	8.70%
Left tail	-4.11%	-5.81%	-18.21%	30.70%	9.38%

	Poland	Portugal	Slovakia	Slovenia	Ireland
Right tail	-27.98%	16.01%	-21.85%	-142.40%	12.78%
Left tail	-12.44%	-4.78%	0.20%	-28.77%	-12.41%
	Israel	**Switzerland**	**Hungary**	**Iceland**	**Spain**
Right tail	11.02%	23.55%	8.38%	-292.24%	29.46%
Left tail	18.64%	1.32%	-0.87%	-179.78%	-13.19%
	Sweden	**Turkey**	**UK**	**USA**	
Right tail	-15.42%	1.38%	-38.61%	1.47%	
Left tail	-20.42%	6.03%	-59.13%	1.47%	

Table 7 summarizes the difference in the ratio of Black Swan Clusters to extreme values in the full sample versus the segmented sample of the residuals of log returns in the equity markets of the OECD countries of the data set. By differentiating the black swan clusters on each tail, the result represents the skewness of the data.

Table 7 presents twofold evidence regarding the variation in the ratio of Black Swan clusters to extreme values in residuals. First, the ratio is reduced in residuals when compared to returns and second the trend of skewness towards the left is persistent in residuals as is in returns albeit at a smaller scale.

With regards to the first observation, preliminary comparisons indicate that there are fewer Black Swans to extreme values overall within residuals as opposed to returns, i.e. in full segments, 74% of extreme values are Black Swans whereas in residuals this ratio declines to 43%. In other words, there is a higher probability of an extreme value being a Black Swan in returns as opposed to residuals. This can be explained by the implementation of autoregressive heteroskedastic models that eliminate in-sample volatility clustering in the data, a piece of information that would not impossible to be known out-of-sample i.e. in real conditions.

In the second segment of the model, despite using the best fit ARMA-APARCH model, skewness towards the left persists even though it is at a modest degree. In the full segment of the residuals, there are 2 negative Black Swans for every 4 negative extreme values (53%) and on the right tail, there is only 1 Black Swan for every 4 positive extreme value (33%) as compared to returns where the ratio is 0.83 and 0.65 respectively. When there is a provision for breaks in the residuals, the ratio of Black Swans to extreme values in the left tail is 0.53 as opposed to 0.34 in the right tail and within the returns, these values are 0.69 and 0.54 respectively.

To further investigate if the modification of the ratio of Black Swans to extreme values emerges from a single swarm or a cluster, the model uses the structural segments of the residuals from the best-fit ARMA-APARCH model identified in section 3.6.2. The results revealed in table 8 illustrate that the increasing ratio of Black Swans to extreme values experienced from pre-crisis to during crisis periods in the returns were in fact clusters of Black Swans instead of a single occurrence.

Table 8: Difference in the ratio of Black Swans to extreme value ex-post and ex-ante the GFC (2008)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Pre-Crisis	30.00%	51.28%	37.04%	43.55%	33.33%
Crisis	8.33%	25.00%	0.00%	28.57%	38.71%
Difference	-128.1%	-71.85%	N/A	-42.15%	14.95%
Post-Crisis	7.14%	18.75%	25.00%	37.50%	N/A
Difference	-15.42%	-28.77%	N/A	27.19%	N/A
	Czech	**Denmark**	**Estonia**	**Finland**	**Germany**
Pre-Crisis	63.64%	40.63%	N/A	N/A	33.33%
Crisis	40.63%	40.00%	62.50%	N/A	25.00%
Difference	-44.88%	-1.55%	N/A	N/A	-28.77%
Post-Crisis	63.64%	43.33%	68.75%	18.75%	N/A
Difference	44.88%	8.00%	9.53%	N/A	N/A
	Greece	**Hungary**	**Iceland**	**Ireland**	**Israel**
Pre-Crisis	33.33%	90.00%	N/A	59.80%	56.25%
Crisis	35.29%	39.71%	N/A	31.25%	40.91%
Difference	5.72%	-81.83%	N/A	-64.91%	-31.85%
Post-Crisis	33.33%	29.17%	55.00%	40.00%	14.29%
Difference	-5.72%	-30.85%	N/A	24.69%	-105.21%
	Italy	**Japan**	**Korea**	**Mexico**	**Netherlands**
Pre-Crisis	40.00%	44.68%	36.67%	50.00%	25.00%
Crisis	25.00%	25.00%	32.35%	40.48%	25.00%
Difference	-47.00%	-58.07%	-12.52%	-21.13%	0.00%
Post-Crisis	16.67%	26.47%	27.78%	N/A	14.29%
Difference	-40.55%	5.72%	-15.25%	N/A	-55.96%
	New Zealand	**Norway**	**Poland**	**Portugal**	**Slovakia**
Pre-Crisis	39.47%	40.00%	N/A	38.46%	93.33%
Crisis	0.00%	5.56%	40.91%	16.67%	90.00%
Difference	N/A	-197.41%	N/A	-83.62%	-3.64%
Post-Crisis	22.73%	12.50%	N/A	27.50%	83.33%
Difference	N/A	81.09%	N/A	50.08%	-7.70%
	Slovenia	**Spain**	**Turkey**	**UK**	**Switzerland**
Pre-Crisis	N/A	29.07%	N/A	44.64%	22.73%
Crisis	20.00%	50.00%	N/A	25.86%	50.00%
Difference	N/A	54.23%	N/A	-54.59%	78.85%

Post-Crisis	50.00%	30.77%	40.63%	14.29%	28.57%
Difference	91.63%	-48.55%	N/A	-59.35%	-55.96%
	USA	**France**	**Luxembourg**	**Sweden**	
Pre-Crisis	0.00%	30.77%	50.00%	23.33%	
Crisis	30.65%	37.50%	16.67%	41.67%	
Difference	N/A	19.78%	-109.86%	57.98%	
Post-Crisis	28.57%	16.67%	25.00%	16.67%	
Difference	-7.01%	-81.09%	40.55%	-91.63%	

N/A's represent the zero Black Swan clusters being identified in the respective series.

With respect to variations in the ratio of Black Swan clusters to extreme values pre- and post-2007/2007 crisis, the residuals show a remarkable trend which challenges the trend in the ratio when returns were analysed i.e. on average, there is a significant reduction of 32% in the ratio of Black Swan clusters to extreme values from pre-crisis periods to the crisis period and a further 10% reduction from the crisis period to the post-crisis period. This is an important finding as it confirm two assumptions: a) markets were showing sign of distress before the collapse of Lehman Brothers in 2008, and b) recognition and inclusion of structural breaks along with ARMA-APARCH models reduces the deceivability of greater risk during crisis periods i.e. the potential upward bias of excess kurtosis. The countries with the highest reduction in the ratio of Black Swan clusters to extreme values during the crisis period as compared to the previous segment were Australia and Norway.

To add context to this circumstance,

- Australia had braced itself for a crisis from an economic point of view before the fall of the Lehman brothers (its markets had been experiencing a mining boom before the crisis hit), from the banking perspective (its banks were ranked in the top three according to the World Economic Forum in 2008; sub-prime loans account for only 1% of the outstanding housing loans in Australia); and from a macro-economic standpoint (the government budget released in May 2008 included measures that recognized the warning signs and incorporated guards to cushion against an emerging Global Financial Crisis; the government had built a $21.7 billion budget surplus; and the balance sheet was in a net asset position). The ratio of Black Swan clusters to extreme values continue to decline post-crisis periods because of the government's speedy and bold responses to stabilize the economy and reduce the impact of global financial contagion (the Reserve bank of Australia cut interest rates by 100 basis points and the Australian government announced that it would guarantee all bank deposits along with wholesale funding of national banks along with a $10.4 billion stimulus package in October of 2008) (Kennedy, 2009).

- In the case of Norway, a similar status quo existed where: a) the country built its wealth from being the world's third-largest gas exporter leading to a strong economic performance before the crisis hit, b) it had cushioned itself with the sovereign wealth fund called the Government Pension fund valued at $326billion at the time of the Lehman Brothers fall (the largest pension fund in Europe and the second-largest in the world, c) the government has a budget surplus of 11% of its GDP, and d) the housing market had nominal lending excesses (Economist, 2013, Thomas, 2009).

Overall, this analysis highlights that unaccounted structural breaks in the underlying mean and/or variance dynamic of asset returns could be a probable source of excess kurtosis in returns of equity markets which is heightened during crisis periods results. Extreme tail risk is also intensified due to persistence/volatility clustering and leverage effects in the returns which can be decreased with the application of the best fit ARMA-APARCH model.

It is the primary reason there is a contrasting trend in the movement of the Black Swan to extreme value ratio when comparing returns and residuals indicating that markets are much closer to a normal distribution than predicted when volatility clustering is accounted for.

The model was unable to recognize Black Swan clusters in some of the series and extreme values remained constant since they accounted for 1% of the distribution. This indicates that while there are extreme values within the distribution, there are no returns that are rare enough to be classified as Black Swans during that segment.

3.7 Discussion and Implications

In this chapter, the aim was to understand the effect of unaccounted structural breaks in the mean and/or variance dynamic of asset returns on the evolution of extreme tail risk i.e. tail asymmetry and excess kurtosis and use it to examine how the tail behaviour has changed around the Global Financial Crisis. This was tested by implementing a model that combined extreme value theory with GARCH models on daily log returns with and without the provision of breaks.

The empirical findings are three-fold. First, the inclusion of a provision for breaks leads to fewer Black Swans. Second, their frequency changes from one segment to the next in majority of the cases in the data set thereby providing evidence that Black Swans clusters are not homogenous in nature and therefore there is some form of time-varying behaviour of tails that needs to be encompassed in existing models. Finally, accounting for breaks reduces tail asymmetry overall making the equity returns distribution more normal.

With regards to the first result and research question, the acknowledgment of structural breaks, on average, reduces the frequency of Black Swans; there was a 20% reduction in the frequency of Black Swans in returns and a 12% reduction in standardized residuals from the best-fit ARMA-APARCH model. In other words, accounting for potential breaks in the mean and/or variance dynamic of asset returns dramatically reduces the probability of extreme events thereby reducing the possibility of an incorrect upward bias of excess kurtosis estimates in the tails.

The second major finding, which is the focus of the second research question and builds on the first finding, is that Black Swans are not homogenous in nature and their frequency changes from one segment to the next. This implies that the frequency of Black Swans is reflective of the events that are causing changes in an economy whether gradual or dramatic and assigns credibility to the use of break-testing procedures in financial modelling. Hence, structural breaks in the mean and/or variance dynamic of asset returns can be used as a reliable proxy to capture the evolution of risk in the market. This becomes even more essential in times of extreme financial tension as markets would display distress signals that can be projected by the break-testing procedure. Therefore, it is wise to incorporate a provision for changes in the mean and/or variance dynamics in financial models in order to efficiently assess the risk from extreme events.

The third and final major finding from the data, related to the third research hypothesis, is that when volatility clustering and leverage effects are removed using the best fit ARMA-APARCH model along with a provision of structural breaks, the equity markets exhibit less negative skewness i.e. the distribution becomes more normal when breaks are accounted for. Therefore while equity markets still remain highly reactive to the possibility of the occurrence of rare negative events in comparison to positive ones (Pagan and Schwert, 1990, Engle and Ng, 1993), this intensity is reduced by recognizing potential latent Non-linearities in the underlying asset dynamic i.e. the reduction in the frequency of negative Black Swans is higher than those in the right tail. Therefore, the contribution of my findings further the current literature by supporting the incorporation of structural breaks and conditional volatility models would significantly reduce the overall asymmetry in the tails as well the excess kurtosis.

From the perspective of risk premiums being inclusive of the underlying possibility of the negative or positive shocks, it is important to consider provisions that incorporate changes in the mean/variance dynamic of asset returns along with removing any effects of autocorrelation or long memory between the log returns to truly reflect the risk-adjusted returns on investments along with deciding their investment horizon. For the former support exists from existing research that shows optimal portfolio allocation is significantly different when the existence and identification of time-varying conditional kurtosis and skewness are considered (Chunhachinda et al., 1997, Jondeau and Rockinger, 2004). Furthermore, portfolio selection based on Betas that allow for the inclusion of systematic risks like Black Swans outperforms passive investment strategies (Estrada and Vargas, 2015) when studied over four decades for 47 countries including 57 industries.

For the latter there exists similar evidence (Bogle, 2008, Olson et al., 2012, Estrada, 2008) that there are extreme but infrequent changes that can influence daily returns, however, they become less significant with time. Therefore, the crisis becomes amplified when investors panic as it expounds augmented volatility clustering emerging from the behavior of the investors rather than the investments themselves. This crucial distinction between the influence of behavior of market participants and asset returns was identified as early as 1936 by the John Maynard Keynes (1883-1946) in his revolutionary book titled The General Theory of Employment, Interest, and Money. Using the example of the Great Depression of 1987, the drop in corporate earnings was a mere 2-sigma event within the 95 percent probability range as compared to a 20-sigma event in the market overall attributable to dominant ownership by individual investors lacking market knowledge leaving markets fuelling the domino effect of the crisis (Keynes, 2016).

3.8 Conclusion

The identification of structural breaks in the underlying stochastic process of equity returns could lead to better inferences about the performance and change in the market with respect to extreme tail risk. My findings show that the inclusion of a provision of structural breaks can reduce tail asymmetry as well as moderate the upward bias of excess kurtosis in markets overall and more so during a crisis.

Using extreme value theory to examine the evolution of extreme tail with and without a provision for breaks allows for a better understanding of the nature of Black Swans as well i.e. they exhibit clusters which are not homogenous in nature changing from one segment to the next and seem they are reflective of the modifications in the market.

In this chapter, using daily returns of stock indices from 35 countries, which can be considered as a dynamic portfolio proxy, it has been successfully recognized that Black Swans are not homogenous when breaks in the mean and/or variance dynamic are considered and can lead to a significant reduction in tail asymmetry. Within the returns and residuals, it is revealed that the frequency of Black Swans varies from one segment to the next and reduces dramatically when ARMA-APARCH volatility models are used to remove time-dependence structures in the underlying distribution. Therefore, the provision of structural breaks can possibly increase the efficiency of financial models as it reduces the inference bias that can result from the existence of latent nonlinearities in the daily returns. In other words, recognizing these structural breaks can lead to a significant decrease in the tail asymmetry of the asset return distribution.

Another important finding, with regards to the research questions, is that the ratio of Black Swans to extreme values depict reverse trends in returns and residuals during the Global Financial Crisis of 2008 i.e. the tail variability in the market that can be captured by identified clusters of Black Swans shows that although the probability of extreme events taking place during a financial turmoil increases, it is not long before it returns back to its normal level and in fact even lower probably to the enhanced regulation and market practices that are implemented exactly due to the crisis itself.

Chapter 4: Influence of Latent Non-linearities in Mean and/or Variance of asset returns and their effect on the tail risk in Foreign Exchange Markets

4.1 Introduction

Understanding the behaviour of equity markets during extreme financial distress is an essential aspect of broadly understanding asset pricing and hedging strategies, however, the global financial system continues to become more integrated and therefore there are now varied asset classes with different volatility classes that are available for trading globally. In order to add depth to the current evidence of this thesis that structural breaks in the mean and/or variance dynamic of asset returns can have an effect on extreme tail risk, this chapter will focus on Foreign Exchange rates (henceforth forex) returns.

While the equity markets have received considerable scholarly attention with regards to tail risk and managing volatility during the recent financial crisis, there has been growing interest in literature towards the behaviour of foreign exchange markets as well. There is a growing recognition of the vital link[7] between these markets that become more responsive to each other during a crisis and therefore potentially increasing the magnitude of a crisis. This also has important implications for risk management via portfolio diversification.

There have been various empirical studies looking at each of the markets during a crisis and its subsequent effect on the other. Some researchers argue that change in exchange rates lead to changes in the stock market (see (Mun, 2008, Aquino, 2005, Chung, 2005, Yau and Nieh, 2006)) while others claim that stock returns influence foreign exchange markets (see (Kanas, 2000, Yang and Doong, 2004, Choi et al., 2010, Aloui, 2007). There is a notable paucity of empirical research

[7] The extreme domino effect between the two markets is explained by macro-economic flow-oriented models DORNBUSCH, R. & FISCHER, S. 1980. Exchange rates and the current account. *The American Economic Review,* 70, 960-971. and the portfolio balance theories FRANKEL, J. 1983. Monetary and portfolio-balance models of exchange rate determination, in economic interdependence and flexible exchange rates. *MIT Press, Cambridge, MA.*.

that have focused on this relationship during a crisis while testing the presence of structural breaks (see (Kallberg et al., 2005) with respect to volatility in Indonesia, Malaysia, Philippines, ,South Korea, Taiwan, and Thailand during the recent Asian crisis found evidence of higher responsiveness of stock markets to volatility in corresponding domestic exchange rate ex-post a crisis).

Lahmiri et al. (2017) studied the short and long term dynamic development of the co-movement between the equity and forex market specifically during the Global Financial Crisis of 2008 finding evidence of hierarchical clustering, the presence of which could lead to higher contagion effects and global instability (for similar results see (Abid and Kaffel, 2018) for analysis on the co-movement between the different asset classes such as gold, oil, equity and forex in US markets with the implied volatility index during a crisis with regards to the time and frequency domain using wavelet analysis; (Inci and Lee, 2014) for US markets along with five European markets, Japan and Canada). This chapter aims to build on this work by considering the unique behaviour of foreign exchange rate returns during, ex-ante and ex-post the Global Financial Crisis of 2008 with a provision for latent non-linearity in the mean and/or variance with an added consideration for time-varying third and fourth order moments.

Therefore, while this chapter will focus mainly on the evolution of tail risk in the forex markets during the financial crisis, a complemental area of the discussion section of this chapter will be dedicated to the degree of co-movement between the equity and forex markets. In other words, the primary objective of this chapter is to understand the behaviour of extreme tail risk during a crisis in returns and residuals of the foreign exchange market of 35 countries with respect to the Global Financial Crisis of 2008 when there is a provision for structural breaks as opposed to without. Additionally, it provides a brief glimpse on the evolution of tail risk in forex markets in comparison to equity markets i.e. it will not study the co-movement or volatility spill over formally.

The rest of this chapter is organized as follows. Section II sets the context for the research followed by theoretical underpinnings and a literature review, section III provides the specific research questions the chapter seeks to answer, Section IV make available the description of the data and section V presents the research methodology used to obtain results. The final segments of the papers are section VI which presents the empirical results on the frequency of Black Swans and extreme returns that exist in the stock market, and section VII discusses the implications for key market participants. Section VIII concludes.

4.2 Literature Review

Foreign exchange markets are more concentrated than equity markets and have a significant effect on equity returns. There have been several studies covering various time periods and geographical areas providing evidence of its significance. For example: studies have focused on developed countries and found vigorous ties between stock market returns and exchange rates (see (Ajayi and Mougoue, 1996), who studied eight industrialized economies using co-integration analysis) as well as developing economies (see (Harvey, 1995) who provided evidence that conditional variation in the stock market returns of 16 OECD countries were typically explained by foreign exchange risk and expected returns on the global market portfolio).

The following section will start with the definition of a currency crisis, followed by the models used for empirical research on understanding extreme events in forex markets. The final section will summarize the literature on the co-movement of extreme volatility between the equity and forex market.

Definition of a Currency Crisis

An extreme event in a foreign exchange market would be defined as a currency crisis which would occur when there is a large movement in the nominal exchange rates of a country. For example, Frankel and Rose (1996) define a currency crisis as an event when the exchange rate of a country depreciates by 25% or more in the last fiscal year. A currency crisis is a financial crisis that could have a distressing effect on the economy and therefore research has focused on identifying the factors can cause this currency crisis. One of the common possible triggers of a currency attack is when a local currency is under selling pressure in the forex market and this is called a speculative attack. However not all speculative attacks turn into a currency crisis as relevant authorities intervene into the market by changing interest rates, using their foreign reserves and/or imposing capital control measures.

In order to deal with an extreme event like that it is important to distinguish successful, speculative attacks from a currency crisis. There is growing literature that focuses on differentiating these by looking at changes in other variables such as exchange rates, interest rates and international reserves. Consequently, this led to the development of the Exchange Market Pressure Index (EMPI). Some of the prominent contributors to the EMPI were Wyplosz (1996) and Kaminsky and Schmukler (1999) with the latter suggesting the exclusion of interest rates.

Empirical Framework

A recent study (Iglesias, 2012) which focused on The Majors EUR/ USD (Euro/US dollar), the USD/JPY (US Dollar/ Japanese Yen), the GBP/USD (British Pound/US dollar) and the USD/CHF (US Dollar/Swiss Franc and The Commodity Forex pairs (AUD/USD (Australian Dollar/Us Dollar) and NZD/USD (New Zealand Dollar/US Dollar) associated with gold commodities and the USD/CAD (US Dollar/Canadian Dollar) associated with oil commodities) to study the VAR using a hybrid conditional EVT model. They found that the Japanese Yen had the highest VAR followed by the Swiss Franc with the UK pound being the safest during times of extreme turmoil.

However, the competence of using EVT to measure the risk originating in the foreign exchange market remains questionable. For example, Bekiros and Georgoutsos (2008b) provide evidence with respect to the daily returns of the US Dollar/Cyprus Pound exchange rate. They found that for foreign exchange returns EVT provided efficient estimates only at significance levels higher than 98% (this conclusion is confirmed by Wang et al. (2010), who applied EVT to study the exchange rate risk of the Chinese Yuan with the US Dollar, British Pound, Japanese Yen, and Hong Kong Dollar). In all studies, EVT provided results similar to other conventional methods such as Historical Simulation and GARCH models and therefore was not proven to be more efficient overall. This was due to the absence of strong leptokurtosis in the foreign exchange returns even though the null hypothesis of normality was rejected. Another study that recommended using EVT to study currency crisis was by Karimi and Voia (2014) who provided evidence that EVT is only appropriate when using high-frequency data using daily returns from 20 OECD members and South Africa over a period of 28 years from 1970-1998.

4.3 Research Questions

The aims of this chapter are threefold: to understand the effect that latent non-linearity, in the mean and/variance dynamic of the returns distribution, have on the frequency of Black Swans in global currency markets; to empirically examine the evolution of tail risk ex-ante and ex-post the Global Financial Crisis of 2008 and finally, to examine the variation in tail asymmetry and kurtosis when structural breaks have been accounted for. Specifically, it is seeking to answer the following questions:

- Does the frequency of Black Swans in forex markets change once there is a provision that recognizes structural breaks in the mean and/or variance dynamic of the underlying distribution?
- Could tail asymmetry in returns be caused by unaccounted structural breaks?
- What was the impact of the Global Financial Crisis onto the frequency of Black Swans?

All of these objectives will be tested on two levels: that of log returns and that of the residuals from the best fit ARMA-APARCH model so to capture the possible effect of volatility clustering on the frequency of Black Swans.

4.4 Data

To understand the dynamic probability of extreme events aka Black Swans occurring in the foreign exchange market, 31 currencies were used. Daily exchange rates measured per USD were extracted using the DataStream software.

The countries chosen for extreme risk analysis in the foreign exchange market are varied and inclusive of various level of financial and economic development i.e. developed economies, developing economies and economies in transition. While the FTSE global equity indexes were used for country classifications when analyzing extreme risk in equity markets, the basis for forex market country classifications is the World Economic Situation and Prospects (WESP) annex which is published by the Development Policy and Analysis division of the Department of Economic and Social Affairs of the United Nations Secretariat. Consequently, the current basket includes Developed countries (such as Australia, Canada, UK, Germany, and USA) as well as Emerging countries, (such as Brazil, China, India, Chile, and Turkey) and transition economies (such as Russia).

Table 9 below summarizes the basket of countries and their respective currency along with the symbols and fractional units as well as the ISO code.

Table 9 – Data set of currencies along with symbols, fractional codes and ISO code

No.	State or territory	Currency Name	Currency Symbol	ISO code	Fractional unit
1	Argentina	Argentine peso	$	ARS	Centavo
2	Australia	Australian dollar	$	AUD	Cent
3	Brazil	Brazilian real	R$	BRL	Centavo
4	Canada	Canadian dollar	$	CAD	Cent
5	Chile	Chilean peso	$	CLP	Centavo
6	China	Chinese yuan	¥ or 元	CNY	Fen
7	Denmark	Danish krone	kr	DKK	Øre
8	Fiji	Fijian dollar	$	FJD	Cent
9	Germany	Euro	€	EUR	Cent
10	Hong Kong	Hong Kong dollar	$	HKD	Cent
11	Iceland	Icelandic króna	kr	ISK	Eyrir
12	India	Indian rupee	₹	INR	Paisa
13	Indonesia	Indonesian rupiah	Rp	IDR	Sen
14	Kenya	Kenyan shilling	Sh	KES	Cent
15	Korea, South	South Korean won	₩	KRW	Jeon
16	Malaysia	Malaysian ringgit	RM	MYR	Sen
17	Mexico	Mexican peso	$	MXN	Centavo
18	New Zealand	New Zealand dollar	$	NZD	Cent
19	Norway	Norwegian krone	kr	NOK	Øre
20	Pakistan	Pakistani rupee	₨	PKR	Paisa
21	Poland	Polish złoty	zł	PLN	Grosz
22	Russia	Russian ruble	₽	RUB	Kopek
23	Singapore	Singapore dollar	$	SGD	Cent
24	South Africa	South African Rand	R	ZAR	Cent
25	Sweden	Swedish krona	kr	SEK	Öre
26	Turkey	Turkish lira	₺	TRY	Kuruş

27	United Kingdom	British pound	£	GBP	Penny
28	Taiwan	New Taiwan dollar	$	TWD	Cent
29	Solomon Islands	Solomon Islands dollar	$	SBD	Cent
30	Nigeria	Nigerian naira	₦	NGN	Kobo
31	Papua New Guinea	Papua New Guinean kina	K	PGK	Toea

Table 9 – Introduction to foreign exchange markets chosen as well as currency symbols, ISO codes and Fractional units.

Descriptive Statistics

Initial analysis was conducted for every data set with respect to the four moments of the return's distribution, namely: mean, standard deviation, skewness, and kurtosis, finding a significant deviation from normality. The descriptive statistics of each forex market substantiating this are summarized in table 10 below.

Table 10 – Summary Statistics of Daily Forex returns

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**
No. of obs.	5840	5840	5711	5840
Mean	0.05%	0.00%	0.02%	0.00%
Standard Dev.	0.92%	0.77%	0.97%	0.53%
Skewness	20.40	0.69	0.37	-0.13
Kurtosis	719.36	12.51	17.70	6.52
	Chilean Peso	**Chinese Yuan**	**Danish Krone**	**Euro**
No. of obs.	5840	2829	5840	5840
Mean	0.01%	-0.01%	0.00%	0.00%
Standard Dev.	0.58%	0.12%	0.61%	0.60%
Skewness	0.51	-0.59	-0.20	-0.19
Kurtosis	7.08	60.15	2.44	2.47
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	4558	5840	4954	5840
Mean	0.00%	0.00%	0.01%	0.01%
Standard Dev.	0.59%	0.03%	0.85%	0.36%
Skewness	13.06	-2.49	-0.41	0.46
Kurtosis	502.02	62.54	57.31	10.86
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	5840	4954	5840	5840
Mean	0.03%	0.01%	0.01%	0.03%
Standard Dev.	1.35%	0.51%	0.73%	0.90%
Skewness	2.06	0.48	17.31	2.71
Kurtosis	84.01	23.87	1000.10	91.71

	New Guinea Kina	New Turkish Lira	New Zealand Dollar	Nigerian Naira
No. of obs.	5612	5840	5840	5455
Mean	0.02%	0.09%	0.00%	0.02%
Standard Dev.	0.68%	1.18%	0.79%	0.60%
Skewness	-3.89	6.86	0.33	1.78
Kurtosis	100.31	212.96	5.70	62.10
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
No. of obs.	5840	4743	5578	5272
Mean	0.00%	0.02%	0.01%	0.05%
Standard Dev.	0.72%	0.30%	0.80%	1.60%
Skewness	-0.04	0.92	0.17	5.33
Kurtosis	4.92	37.20	5.52	322.67
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**
No. of obs.	5840	2587	5840	5840
Mean	0.00%	0.00%	0.03%	0.01%
Standard Dev.	0.36%	0.76%	0.96%	0.86%
Skewness	-0.46	-0.13	0.27	-0.76
Kurtosis	10.38	15.54	6.97	106.18
	Swedish Krona	**Taiwan New Dollar**	**UK Sterling**	
No. of obs.	5840	5840	5840	
Mean	0.00%	0.00%	0.00%	
Standard Dev.	0.71%	0.27%	0.54%	
Skewness	-0.18	1.08	0.01	
Kurtosis	3.50	26.80	4.22	

Table 10: Descriptive statistics for the daily log returns of the forex market

While the daily mean is not significantly different from 0, the third and fourth moment show signs of substantial non-normality. For example, the data generally seems to be more positively skewed which is a contrast to the trend observed in equity markets. The currencies with the highest positive skewness are the Argentine Peso (20.4), Fijian Dollar (13.1) and Malaysian Ringgit (17.3) indicating a much longer right tail of the returns distribution. In terms of the Kurtosis, four out of every 5 countries displays significantly heavier tails i.e. leptokurtic properties than a normal distribution which translates as a higher probability of the occurrence of Black Swans in comparison to traditional financial models.

The data can also be visually inspected for non-normality and heavy tails using the Q-Q plots presented in Appendix 3. The plots for each currency display significant deviations in the tails in comparison to a Gaussian distribution thereby providing strong support for the rejection of the Null hypothesis of normality in forex returns and providing greater justification towards understanding the complexity of Black Swans, their nature and corresponding probabilities for efficient risk management during crisis periods.

4.5 Methodology

The research question primarily revolves around three aspects: tail risk, structural breaks caused by latent Non-linearities in the mean and/or volatility dynamic of forex returns and excess kurtosis. The chapter will use Extreme value theory to identify the threshold for tail risk (log returns above a 5% threshold on the left and right tails), the Karoglou (2010a) break-test procedure to identify potential structural breaks in the data and finally a best fit ARMA-APARCH model to account for the effects of volatility clustering and leverage effects.

The following section details the three hypotheses that are being empirically tested with respect to measuring volatilities in the daily foreign exchange market by taking into account the frequency of Black Swans and the ratio of Black Swans to extreme values in the full and segmented samples. The following section is divided into three segments: Black Swans, Structural breaks and finally model specification. Each segment begins with a brief literature review of seminal studies thereby providing a structural base and scaffold to the overarching research objectives of this chapter. The segments then detail the empirical methods used in the model to measure them.

Black Swans

Although extensive research has been carried out on the recognition and resolution of extreme events in equity markets, with a significant focus on Black Swans since the Global Financial Crisis, no single study exists within the current literature review that adequately focuses on the identification and decryption of Black Swans in foreign exchange markets at a global scale during the Global Financial Crisis of 2008.

A single study by Lleo and Ziemba (2015) focused on a single Black Swan i.e. the abandonment of the Euro peg of Swiss Franc by the Swiss national banks on 15th January 2015 and the effect it had on the economy, financial markets of Switzerland. However, the study is limited in that it failed to recognize the effect of volatility clustering as well as the effect of structural changes in the underlying distribution on the occurrence of Black Swans.

In this chapter, the threshold for the identification of Black Swans is set according to the routine practice of three standard deviations above or below the mean rule. This rule has been applied to full samples as well as segments.

Structural Breaks

The issue of recognizing structural breaks in order to draw accurate inferences regarding the persistence of the realized volatility has received considerable critical attention in the literature (Lamoureux and Lastrapes, 1990, Dieobold, 1986, Rydén et al., 1998, Mikosch and Starica, 1998, Liu, 2000, Zikovic and Aktan, 2009). The common conclusion drawn by this overwhelming number of seminal studies is that accounting for structural breaks dramatically reduces the degree of persistence in long memory volatility processes. For example, Han (2016) used the Adaptive-FIGARCH model to identify the effect of recognizing structural breaks on the long memory volatility property in the daily USD-GBP returns. The paper found evidence that long memory volatility property in exchange rates is greater due to the presence of frequent structural breaks.

Therefore, not accounting for them could lead to an upward bias and overstating of long memory persistence of the conditional variance (see (Granger and Hyung, 2004) for application to the S&P returns and long memory persistence; (Morana and Beltratti, 2004) studies the Deutsche mark/US dollar and Japanese yen/US dollar exchange rates finding evidence that inclusion of structural breaks lead to superior forecasts at longer horizons; (Martens et al., 2004) studied three exchange rates, namely DM/$, ¥/$ and ¥/DM to find that even though the leverage effect is less important in foreign exchange returns in comparison to equity markets, yet integrating latent nonlinearity's still produced more efficient in-sample fit and out-of-sample forecasts).

However, most of these studies modeled the structural breaks without identifying the actual location of breakpoints. Another stream on literature that focuses on the importance of incorporating structural breaks in variance includes pivotal papers by Javed (2011), van Dijk et al. (2005), Rodrigues and Rubia (2007), Koseoglu and Cevik (2013) which provide empirical evidence that undetected structural breaks can lead to biased and overestimated GARCH parameters leading to causality-in-variance test results that are inefficient due to severe size distortions. The most recent paper (Koseoglu and Cevik, 2013) found the existence of several structural breaks when testing for latent non-linearities in the variance in the foreign exchange returns series in the Czech Republic, the Hungarian and Turkish foreign exchange market, against the backdrop of the Global Financial Crisis of 2008 (see (Kočenda, 2005) for application to the foreign exchange markets of the Czech Republic, Hungary, Poland, Slovakia, Slovenia, Albania, Bulgaria, Romania, Estonia, Latvia, and Lithuania from January 1991 to December 2003).

A paper that tested both i.e. the long term persistence and distortion of the GARCH parameters due to structural breaks with respect to the currencies of Czech Republic, Hungary, Poland and Russia namely koruna, forint, zloty and ruble found inconclusive evidence for the former but strong evidence of the latter (Zivkov et al., 2015). However, these papers focused on eliminating the effect of structural breaks instead of studying them and this current chapter aims to fill that gap in the literature by studying its influence on the behavior of extreme variability in the foreign exchange returns of 35 countries.

The structural breaks in this research have been recognized using the two stages of the Nominating-Awarding procedure of Karoglou (2010a). The reason this approach has been chosen is, that, it acknowledges different strains of break-tests that can be combined to identify structural changes in the mean/variance dynamics as well as latent non-linearities in the data that might add bias to the model. Therefore, it fortitudes the selection of a particular break date. Detailed information is included in section 3.5 of chapter 3. The final outcome of this awarding-break dates procedure are multiple segments that are homogenous in their mean and/or variance dynamics.

Model Specification

Similar to the previous chapter, there are two focal notions underlying this work: The first notion is the identification of Black Swans with respect to non-parametric detection of structural changes in the underlying process. The second notion is their evolution in times of the Global Crisis. In order to test the research questions mentioned in section 3 of this chapter, two models have been designed to test the following hypothesis:

Model 1: There is a reduction in tail risk of financial returns in forex markets when structural breaks in the mean/variance properties are taken into account i.e. Black Swans clusters are not homogenous in returns and segmented returns.

Model 2: There is a reduction in the tail asymmetry of residuals of financial returns of forex markets obtained using the best-fit ARMA-APARCH models when there is a provision, accounting for latent non-linearities in the underlying distribution.

Schematically, and using the same notation as in the previous chapter, structural breaks in the underlying asset return distribution can be presented as follows:

$$r_t = \begin{cases} \mu_0 + \sigma_0 \cdot \mu_{o,t} & for\ 0 \leq t \leq \tau_1 \quad , \\ \mu_1 + \sigma_1 \cdot \mu_{1,t} & for\ \tau_1 \leq t \leq \tau_2 \quad , \\ u_n + \sigma_n \cdot \mu_{n,t} & for\ \tau_n \leq t \leq T \end{cases} \tag{17}$$

where, r_t stands now for the daily log returns of the forex market. And as before, in order to capture the heteroskedastic properties of the returns process as well as the leverage effects, the conditional variance of the returns process can be defined by the APARCH (m, n):

$$\varepsilon_t = z_t \sigma_t; \qquad z_t \sim N(0,1) \tag{18}$$

$$\sigma_t^\delta = \varphi_0 + \sum_{i=1}^{m} \psi_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{n} \omega_j \left(\sigma_{t-j}\right)^\delta \tag{19}$$

4.6 Empirical Results

The Nominating-Awarding procedure has identified two to seven structural breaks in each series. The empirical results in the following section summarizes the findings of the study with respect to the research questions mentioned in section 3.

4.6.1 Model 1 Outcomes

Summary of the frequency of the Black Swan clusters in the full sample and segments (with breaks) of daily forex returns

The table 11 below illustrates the number of Black Swans in foreign exchange returns when there is no provision for structural breaks in the mean and/or variance dynamic of the series and then compares it to the frequency of Black Swans when these latent non-linearities are accounted for to test the hypothesis of homogeneity of Black Swans. Furthermore, both tails of the distribution are also examined closely to test this hypothesis and to study the skewness of the distribution i.e. the frequency of negative Black Swans versus the frequency of positive Black Swans (See Appendix 9 for more detailed results).

Table 11 – Difference in the frequency of Black Swans in Forex returns full sample versus the segmented sample

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	13.35%	0%	23.26%	44.47%	2.35%
Left tail	-52.30%	54.36%	36.77%	5.72%	-3.39%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	40.55%	60.61%	49.64%	33.65%	28.09%
Left tail	-18%	3.08%	2.99%	-44%	9.53%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	29.73%	12.90%	-21.87%	1.71%	-63.10%
Left tail	7.41%	-15%	26.24%	5.94%	-62.20%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	0%	4.17%	-1.98%	4.45%	-8.70%
Left tail	-12.50%	-9.76%	-7.41%	27.87%	-18.60%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	49.47%	3.85%	18.23%	-80.70%	11.78%
Left tail	37.47%	-18.60%	25.95%	-94.10%	40.55%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-8.00%	24.89%	-27.63%	42.12%	14.46%
Left tail	-35.70%	32.16%	15.42%	6.45%	16.51%

	UK Sterling				
Right tail	29.73%				
Left tail	7.15%				

Table 11 - Summary of the frequency of the Black Swans in the full sample and segments (with breaks) of daily forex market returns. The black swan clusters on the left and right tail are representative of the skewness of the data.

An inspection of the results reveals two interesting results: the first being that Black Swans are not homogenous when structural breaks in the series are accounted for, i.e. on average there is slight decrease in the probability of a Black Swans occurring with breaks; and second that data is more symmetric when the existence of a provision for breaks are taken into account indicating that, in forex markets, which implies that latent Non-linearities may well be a factor causing at least partially the observed asymmetry in the tails.

For the first result, the table shows that the frequency of total Black Swans is not the same in full samples and segments for 29 of the 31 foreign exchange returns series. With the exception of Chilean Peso and Fijian Dollar, all currencies show a change in the frequency of Black Swans when there exists a provision for structural breaks in the series thus rejecting the null hypothesis of homogeneity in different clusters of Black Swans when deviations in the mean and/or variance dynamic are accounted for. However, looking further into the results, a counterintuitive trend emerges. Although on average, there is a 5% decrease in the number of total Black Swans (from representing 1.41% of the trading days to 1.36%) when latent nonlinearities are taken into consideration, some countries depict a reverse trend when taken individually. For example, 11 out of the 31 countries display a higher number of Black Swans in segments (20.69%) as opposed to the full sample, however because of a slightly high proportion of decline (20.90%) in the remaining 20 countries, the resulting average shows a negative trend. The highest increase in overall Black Swans is in the Russian Rouble (86.22%) and the highest decline in total Black Swans is for the Norwegian Krone (44.03%). Therefore, on average, the probability of extreme events is overstated in the daily returns of foreign exchange markets if a provision for structural breaks is not provided which is analogous to daily returns in the equity market which overstate this probability by 20% (excluding Iceland and Slovakia).

One notable difference between the equity markets, examined in the previous chapter, and the foreign exchange markets is the skewness of the distribution is reduced in the tails specifically when there exists a provision of breaks. While in equity market returns, 56% of overall Black Swans in full segments were negative and 46% were positive, the trend is reversed in foreign exchange market returns where on average 46% of the Black Swans occur in the left tail and 54% in the right. When there exists a provision for structural breaks the skewness remains but more moderate (52% positive Black Swans and 48% negative Black Swans). This can be translated as the possibility of unaccounted structural breaks causing partially tail asymmetry in the forex markets thereby reducing the probability to extremely extreme returns.

When it comes to comparing the pre- and post- crisis period, I first focus examining the ratio of Black Swans to extreme values in full sample where no provision for structural breaks exist and compare it to the ratio of Black Swans to extreme values in segments which have a provision for breaks; and then I examine the evolution of this ratio when the crisis occurs i.e. the pattern that emerges ex-ante and ex-post the Global Financial Crisis of 2008.

Summary of the ratio of the Black Swan clusters to extreme values in the full sample and segments (with breaks) of daily forex returns

The following table 12 summarizes the ratio of Black Swans to extreme values overall in the full sample as well as in the left and right tails and compares it to segments (See Appendix 10 for detailed results).

Table 12 – Difference in the ratio of Black Swans to extreme values in the Forex returns full sample and segmented sample

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	11.64%	4.96%	28.31%	47.80%	7.31%
Left tail	-50.64%	59.32%	41.82%	9.05%	1.57%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	40.55%	63.95%	52.98%	37.90%	33.05%
Left tail	-18.23%	6.41%	6.32%	-42.74%	16.09%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**

Right tail	33.65%	16.25%	-18.54%	5.63%	-59.79%
Left tail	11.33%	-13.33%	29.57%	9.86%	-58.84%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	4.96%	4.17%	-0.03%	11%	-5.13%
Left tail	-7.56%	-9.76%	-5.73%	34.43%	-16.77%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	56.03%	9.91%	60.84%	-76.94%	16.74%
Left tail	44.03%	-12.55%	29.46%	-92.44%	45.51%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-0.59%	29.85%	-21.07%	43.80%	17.79%
Left tail	-28.26	37.12%	21.97%	8.13%	19.84%
	UK Sterling				
Right tail	33.06%				
Left tail	10.48%				

Table 12 - Summary of the ratio of Black Swans to extreme values in the full sample and segments (with breaks) of daily forex returns. The black swan clusters in the left and right tail are representative of the skewness of the data.

The results in table 12 above display two interesting findings: one, that is similar to equity markets, is that providing for structural breaks decreases the average ratio of Black Swans to extreme values in segments when compared to the full sample irrespective of their nature and two, that is contrary to the trend in equity markets, is that the decrease in the ratio of Black Swans to extreme values in the left tail is much smaller than the decrease in the ratio on the right tail indicating a higher probability of market variability when forex markets are bullish as opposed to bearish.

With respect to the first outcome, there is on average a 8% decrease in the ratio of Black Swans to extreme values when there is a provision for structural breaks which translates that the probability of an extreme event being a Black Swan is lower when latent non-linearity in the mean and/or variance of the underlying distribution are taken into account. Taking a closer look at the tails of the distribution, the decrease in ratio of Black Swans to extreme values in the right tail is much higher than the left tail, i.e. the ratio of positive Black Swans to extreme values decreases by 13% whereas the ratio of negative Black Swans to extreme values decreases by a marginal extent of 3%.

Irrespective of the higher proportion of decrease in the probability of extreme events being Black Swans when there is provision for structural breaks, the data appear normalized with decreased skewness in segments as compared to the full sample i.e. in the right tail, without breaks 75% of all extreme values are Black Swans, and this reduces to 67% when structural breaks are taken into account; in the left tail, 64% of all extreme values are Black Swans when a provision for structural breaks does not exist and this reduces to 62% when structural breaks are taken into account.

Summarizing these results depicts that identification of structural breaks reduce the probability of extreme events being classified as Black Swans while reducing tail asymmetry in the distribution and that foreign exchange markets display contrary trends to the equity markets in terms of skewness where the probability of extreme events is concerned.

In order to analyse the transformation in the frequency of Black Swans to extreme values in foreign exchange markets before, during, and post the Global Financial Crisis, segments were chosen which were closest to the collapse of the Lehman Brothers (15.09.2008). These segments are given in the table 13 below:

Table 13 – Structural breaks identified for the Global Financial Crisis of 2008 along with ex-post and ex-ante segments

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-Crisis	18/07/2001	24/10/1997	09/06/2003	21/08/1998	06/07/2001
Crisis	08/07/2007	31/07/2007	08/09/2008	13/09/2007	10/01/2008
Post Crisis	21/12/2015	20/07/2009	29/06/2009	13/09/2010	13/05/2009
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-Crisis	NA	18/08/2004	18/08/2004	02/12/1998	24/09/2003
Crisis	20/07/2005	12/08/2008	12/08/2008	27/08/2006	21/11/2007
Post Crisis	20/02/2014	18/11/2011	18/11/2011	29/06/2009	12/01/2016
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-Crisis	27/05/1997	27/08/1998	26/10/1999	27/05/1997	22/07/2005
Crisis	26/03/2006	01/05/2008	21/10/2006	20/11/2007	10/03/2008
Post Crisis	11/06/2009	08/10/2013	23/06/2009	27/01/2009	02/12/2014
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-Crisis	22/12/1994	29/07/1998	26/02/2001	17/12/1997	12/07/2000

Crisis	06/12/2005	24/12/2002	26/11/2005	27/07/2007	05/12/2003
Post Crisis	26/05/2009		04/05/2009	20/07/2009	01/12/2008
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Pre-Crisis	26/08/1998	15/10/2001	26/05/1998	27/08/1998	01/12/1998
Crisis	18/01/2008	21/02/2008	07/08/2008	11/01/2006	14/04/2008
Post Crisis	24/08/2009	28/01/2009	09/08/2012	03/11/2014	21/03/2012
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-Crisis	23/06/2006	27/05/1998	11/08/1998	01/09/1998	20/10/1998
Crisis	06/11/2007	03/12/2001	17/03/2008	12/08/2008	02/03/2004
Post Crisis	08/07/2011	04/05/2009	19/05/2009	16/12/2011	15/11/2011
	UK Sterling				
Pre-Crisis	04/01/1994				
Crisis	28/06/2002				
Post Crisis	17/06/2010				

It is important to note that while most of the break dates regarding the advent of the Global Financial Crisis are similar for the countries in the dataset yet there are a few that experienced the decline much later. A plausible explanation for his could be the lead-lag effect caused by the delay in information diffusion across global equity markets.

Table 14 summarizes the change in the ratio of Black Swans to extreme values in the segment before the collapse of the Lehman Brothers in 2008 and compares to the segment within which the crisis came to the forefront of global financial markets. It then delves deeper into the probability of extreme events in the segment following the crisis.

Table 14 – Ratio of Black Swans to extreme values ex-post and ex-ante the GFC (2008)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-crisis	66.67%	46.15%	57.14%	43.75%	50.00%
Crisis	22.22%	91.67%	83.33%	56.25%	62.50%
Difference	-109.86%	68.62%	37.73%	25.13%	22.31%
Post-crisis	100.00%	35.71%	67.86%	50.00%	43.75%
Difference	150.41%	-94.26%	-20.54%	-11.78%	-35.67%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-crisis	NA	36.36%	36.36%	43.33%	50.00%

Crisis	56.52%	38.89%	38.89%	15.38%	96.88%
Difference	NA	6.71%	6.71%	-103.56%	66.14%
Post-crisis	83.33%	44.44%	44.44%	21.43%	0.00%
Difference	38.82%	13.35%	13.35%	33.14%	NA
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-crisis	55.00%	128.85%	75.00%	101.79%	132.61%
Crisis	65.91%	56.67%	63.89%	100.00%	55.56%
Difference	18.09%	-82.14%	-16.03%	-1.77%	-87.00%
Post-crisis	7.14%	64.29%	90.91%	125.00%	50.00%
Difference	-222.22%	12.62%	35.27%	22.31%	-10.54%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-crisis	100.00%	95.83%	75.00%	61.54%	94.44%
Crisis	72.22%	68.57%	75.00%	75.00%	119.23%
Difference	-32.54%	-33.47%	0.00%	19.78%	23.30%
Post-crisis	61.11%	NA	43.33%	28.57%	100.00%
Difference	-16.71%	NA	-54.86%	-96.51%	-17.59%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Pre-crisis	38.00%	88.24%	44.44%	150.00%	46.00%
Crisis	60.00%	83.33%	54.55%	94.05%	81.82%
Difference	45.68%	-5.72%	20.48%	-46.68%	57.59%
Post-crisis	14.29%	92.50%	35.71%	100.00%	68.75%
Difference	-143.51%	10.44%	-42.35%	6.14%	-17.40%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-crisis	100.00%	85.00%	76.92%	42.31%	67.86%
Crisis	80.00%	45.00%	37.50%	55.56%	64.29%
Difference	-22.31%	-63.60%	-71.85%	27.24%	-5.41%
Post-crisis	110.00%	47.22%	85.71%	37.50%	77.78%
Difference	31.85%	4.82%	82.67%	-39.30%	19.05%
	UK Sterling				
Pre-crisis	41.30%				
Crisis	64.29%				
Difference	44.24%				
Post-crisis	0.00%				
Difference	NA				

Table 14 – Summary of the ratio of Black Swans to extreme values in the full sample and segments (with breaks) of daily forex market returns pre and post the Global Financial Crisis (2007-2008) identified using the Awarding-Nominating procedure.

While the forex markets of the 31 countries on average display a declining ratio of Black Swans to extreme events as the Global Financial Crisis become apparent, the rate of decline increases in the succeeding segment suggesting that the rate of variability reduced substantially in the aftermath of the crisis i.e. there were 7% less Black Swans to extreme values in the segment preceding the Global Financial Crisis and 13% fewer in the succeeding segment. One plausible explanation of this behaviour, similar to the equity markets, is that markets had begun showing evidence of distress and investors were starting to expect the worst, thereby reducing the probability to a collapse. This means that the probability of the occurrence of a Black Swans is not constant with the expectation of increased volatility in markets by the participants, in fact, that leads to a reverse trend – a decrease in variability.

A closer look at the results, however, reveal that is there no dominant trend in the number of currencies that follow that trend. Half of the countries in the data set (such as Argentine Peso, Fijian Dollar, majority of the developing Asian currencies) experienced a smaller ratio of Black Swans to extreme values in preceding segments of the Global Financial Crisis and the other half experienced a reverse trend (such as Australian Dollar, Canadian Dollar, Euro, UK Sterling, Singapore dollar). This trend between global currencies remains consistent when comparing succeeding segment with the one that includes that Lehman Brothers fall.

An important implication of these results is that because returns in equity markets displayed more Black Swans to extreme values in preceding segments, a finding that is contradictory to the average trend found in foreign exchange markets, there are strong implications for the purposes of diversification. This reverse trend could be a possible strategy for risk divergence during periods of high volatility. Possible causes of this trend could be the heightened responsiveness of equity markets in comparison to forex markets and/or the reversal of the positive extremal dependence during a crisis (Walid et al., 2011, Diamandis and Drakos, 2011, Jorion, 1990)

4.6.2 Model 2 outcomes

Summary of the frequency of the Black Swan clusters in the full sample and segments (with breaks) of the residuals of daily forex returns

In order to assess the probability of extreme events when there is a provision of structural breaks while negating the effects of volatility persistence in the forex returns, the residuals of the best fit ARMA-APARCH model were tested for homogeneity of Black Swans. The following table 15 summarizes the frequency of Black Swans overall and in the left and right tail of the residuals (see Appendix 11 for detailed results).

Table 15 – Summary of the difference in the frequency of Black Swans in residuals of full sample and segmented samples (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	-125%	5.00%	-353%	-4.88%	0%
Left tail	-230%	-57.50%	N/A	4.65%	4.45%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	22.30%	-18.20%	-6.90%	-25.80%	-5.56%
Left tail	-62.90%	-3.39%	3.51%	-22.30%	-2.41%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	28.80%	4.17%	15.40%	1.98%	N/A
Left tail	-3.64%	-46.10%	0%	-41.70%	-309%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	-21.70%	-16.51%	-88.73%	6.32%	2.02%
Left tail	-14.30%	7.70%	-256.49%	-28.77%	-25.49%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	20.07%	8.00%	6.90%	14.46%	0%
Left tail	8.70%	-11.12%	4.26%	3.39%	-8.46%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-3.92%	7.60%	-2.15%	0%	25.42%
Left tail	32.85%	0%	80.44%	11.33%	9.31%
	UK Sterling				
Right tail	6.06%				
Left tail	-8.34%				

Table 15 - Summary of the frequency of Black Swans in the full sample and segments (with breaks) of the residuals of the best fit ARMA-APARCH model for forex returns

An inspection of the results reveals that with the exception of the Canadian Dollar and the Euro, all currencies display a different frequency once there is a provision for breaks in the mean and/or variance dynamics of the underlying distribution. While there is 35% increase in the number of Black Swans when there is a provision for breaks, upon closer examination, this result is analogous to all the currencies i.e. majority of the currencies (18 of 31) display, on average, 53% more Black Swans in the segments as opposed to the full sample whereas the remaining (11 of 31) display 11% fewer Black Swans when there is a provision of breaks. The currency with the highest number of Black Swans in segments, as opposed to the full sample, is Brazilian Real i.e. when there was no provision for breaks, Black Swans constituted 0.02% of trading days and once a provision was provided for, Black Swans constituted 0.84% of the trading days. The currency which exhibited an extreme decline in the number of Black Swans when there was a provision for structural breaks was South Korean Won which showed a decline of 27%.

The second outcome of this analysis is the reduction in the skewness of the distribution which normalises even further when there is a provision of breaks and without the effect of volatility clustering i.e. when there is no provision for breaks 57% of the Black Swans are positive and the remaining are negative and when there is a provision for breaks 55% of the Black Swans remain positive while the remaining are negative. This endorses the result in section 6.2 that the unaccounted structural breaks in the underlying mean/variance dynamic of the distribution could be a possible cause for skewness in forex markets even when autocorrelation in the returns have been weeded out.

Keeping this consistent skewness in mind, the final result of this analysis was to scrutinize the degree of change in the different tails of the distribution when there is a provision for structural breaks. Overall, there is a greater increase in negative Black Swans (32%) in comparison to positive Black Swans (26%) when there is a provision for breaks. This translates as even though structural breaks attempt to normalize the data and reduce skewness on the right tail that was resulting from volatility clustering in returns, forex markets pertinaciously remain more susceptible to positive shocks as opposed to negative ones.

Overall these results indicate that forex returns, even with the application of the most efficient parametric volatility models, understate the probability of extreme events. Application of structural breaks exposes the probability experiencing extreme events to be much higher than the initial examination. When comparing these trends to the equity stock markets, it is a distinctly antipode result as standardized residuals of equity market returns show a lower probability of experiencing negative extreme returns whereas standardized residuals of the daily forex markets depict a higher probability of experiencing positive extreme results.

Summary of the ratio of Black Swans to extreme values in the residuals of daily forex returns

This section will focus on the ratio of Black Swans to extreme values in segments and full samples once the auto-correlation in returns has been sieved using residuals from the best fit ARMA-APARCH model. The results presented in table 16 below summarize the extreme variability markets experience without the effect of volatility clustering (see Appendix 12 for detailed results).

Table 16 – Difference in the ratio of Black Swans to extreme values in residuals' full sample versus segmented samples (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	-121.94%	9.96%	-347.59%	-1.55%	4.96%
Left tail	-226.92%	-52.58%	N/A	7.99%	9.40%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	22.31%	-14.90%	-3.57%	-21.53%	1%
Left tail	-62.86%	-0.06%	6.84%	-18.06%	4.15%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	32.69%	7.50%	18.75%	5.90%	N/A
Left tail	0.29%	-42.80%	3.33%	-37.77%	-305.77%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	-16.75%	-16.51%	-87.05%	12.88%	7.33%
Left tail	-9.35%	7.70%	-254.81%	-22.21%	-20.18%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	26.63%	14.07%	10.41%	18.16%	4.96%
Left tail	15.26%	-5.06%	7.77%	7.09%	-3.50%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	3.49%	12.56%	4.41%	1.68%	28.76%
Left tail	40.26%	4.96%	87%	13.01%	12.64%
	UK Sterling				
Right tail	9.40%				
Left tail	-5%				

Table 16 - Summary of the ratio of Black Swans to extreme values in the residuals of the best fit ARMA-APARCH model. Clusters on the left and right tail are representative of the skewness of the data.

Table 16 results regarding the ratio of Black Swans to extreme values in the residuals provide fertile grounds for trend comparisons with two other important variables: the ratio of Black Swans to extreme values in the returns and persistence of skewness in the residuals.

With regard to the first observation, removing auto-correlation between returns using the best fit ARMA-APARCH model has substantially reduced the ratio of Black Swans to extreme values. In returns, section 1.6.3 depicted that 70% of all extreme values were Black Swans when structural breaks were not accounted for, in residuals this ratio is 48%. When structural breaks are accounted for, returns had exhibited that 64% of all extreme values could be identified as Black Swans, although in residuals, this ratio drops to 50%. It can be clearly deduced from these results that the high number of Black Swans identified in the returns were resulting from volatility clusters or 'Black Swans swarms' instead of a single Black Swan.

With respect to the second result of persistence, the data remains skewed to the right irrespective of dependence between the data as well as the existence of structural breaks. For example: in the right tail, with a provision for breaks, 55% of extreme values are Black Swans as compared to the left tail where 46% of the extreme values are Black Swans. Results in section 1.6.3, with respect to returns, display the same skewness, i.e. with a provision for structural breaks 67% of all extreme values were Black Swans in the right tail as compared to 62% on the left tail. These results can be interpreted as accounting for structural breaks and auto-correlation between data, forex markets, in general, and on average, continue to exhibit a persistent skewness to the right suggesting that markets remain more volatile during upswings rather than downturns.

A final finding from Table 16 results regarding the ratio of Black Swans to extreme values in the residuals, when structural breaks are taken into account, can be summarized that appears to be quite contrary to the results in the previous sections is the increase in the average number of extreme events that can be identified as Black Swans i.e. there is a marginal increase of 3% in extreme events that are identified as Black Swans in the segments as compared to the full sample. This result can be translated as an overestimation of the frequency of positive Black Swans within extreme values when structural breaks are not accounted.

To further investigate if the increased volatility in the forex market during the Global Financial Crisis of 2008 emerges from a single swarm or a cluster, the model distinguishes the ratio of Black Swans to extreme values within segments of the residuals from the best-fit ARMA-APARCH model identified in the previous section. The results revealed in table 17 illustrate that the increasing ratio of Black Swans to extreme values experienced from pre-crisis to during crisis periods in the returns were in fact clusters of Black Swans instead of a single occurrence.

Table 17 – Difference in the ratio of Black Swans to extreme values in residuals ex-post and ex-ante the GFC (2008)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-crisis	16.67%	44.23%	46.43%	22.92%	44.12%
Crisis	2.78%	41.67%	66.67%	31.25%	75.00%
Difference	-179.18%	-5.97%	36.18%	31.02%	53.06%
Post-crisis	50.00%	28.57%	50.00%	37.50%	37.50%
Difference	289.04%	-37.73%	-28.77%	18.23%	-69.31%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-crisis	NA	22.73%	22.73%	40.00%	50.00%
Crisis	45.65%	44.44%	38.89%	57.69%	79.69%
Difference	NA	67.07%	53.71%	36.62%	46.61%
Post-crisis	16.67%	55.56%	50.00%	28.57%	0.00%
Difference	-100.76%	22.31%	25.13%	-70.27%	NA
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-crisis	45.00%	80.77%	62.50%	85.71%	2.17%
Crisis	61.36%	36.67%	77.78%	37.50%	50.00%
Difference	31.02%	-78.97%	21.87%	-82.67%	313.55%
Post-crisis	7.14%	50.00%	40.91%	112.50%	50.00%
Difference	-215.07%	31.02%	-64.25%	109.86%	0.00%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-crisis	33.33%	87.50%	125.00%	55.77%	100.00%
Crisis	45.83%	52.86%	52.50%	41.67%	73.08%
Difference	31.85%	-50.40%	-86.75%	-29.15%	-31.37%
Post-crisis	55.56%	NA	40.00%	28.57%	100.00%
Difference	19.24%	NA	-27.19%	-37.73%	31.37%

	Norwegian Krone	Pakistan Rupee	Polish Zloty	Russian Rouble	Singapore Dollar
Pre-crisis	30.00%	73.53%	48.15%	100.00%	44.00%
Crisis	30.00%	83.33%	22.73%	65.48%	54.55%
Difference	0.00%	12.52%	-75.07%	-42.35%	21.48%
Post-crisis	7.14%	50.00%	42.86%	100.00%	68.75%
Difference	-143.51%	-51.08%	63.43%	42.35%	23.14%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-crisis	100.00%	55.00%	57.69%	34.62%	46.43%
Crisis	60.00%	40.00%	62.50%	22.22%	61.90%
Difference	-51.08%	-31.85%	8.00%	-44.32%	28.77%
Post-crisis	100.00%	19.44%	35.71%	50.00%	66.67%
Difference	51.08%	-72.13%	-55.96%	81.09%	7.41%
	UK Sterling				
Pre-crisis	47.83%				
Crisis	28.57%				
Difference	-51.52%				
Post-crisis	10.00%				
Difference	-104.98%				

Table 17 depicts the difference in the ratio of black swans to extreme values in the forex market segments before and after the Global Financial Crisis.

With respect to the alteration of the ratio of Black Swans to extreme values in residuals ex-post and ex-ante the Global Financial Crisis, table 17 results corroborate the findings of the previous section albeit with a reduced persistence i.e. the ratio of Black Swans to extreme values decreases by 2% during the crisis period (which was 6% in the returns) when compared to the preceding segments and reduces by a further 9% (which was 13% in the returns) in the succeeding segment. Although there appear to be no dominant trends in the data, the average decline in the ratio illustrate and reiterate that foreign exchange markets, like equity markets, had started to exhibit signs of financial stress and key market participants had recognized these signs early thereby reducing exposure by positioning themselves in risk-averse investment strategies. This is a prime example of a self-defeating prophecy where investors had started to fear the worst before the fall of the Lehman Brothers and because they expected the worst and took conservation investment strategies, they consequently reduced the probability of Black Swans occurring during that period.

4.7 Discussions and Implications

In this investigation, the central aim was to gain a better understanding about the evolution of tail risk, via a focus on the nature and frequency of Black Swans when there is a provision that recognizes structural breaks in the first and second moments of the underlying stochastic process in foreign exchange markets with an additional scaffolding of the Global Financial Crisis of 2008.

The findings clearly indicate that when the model identifies and embraces significant breaks in the mean and/or variance dynamic of the return's distribution, it affects the probability of Black Swans occurring. Empirical results of section 4.6.1 and 4.6.2 permit the evident rejection of the first null hypothesis regarding the homogeneity of the various clusters of Black Swans when structural breaks are included i.e. in returns, there is dominant trend of a 5% reduction in the occurrence of Black Swans in segments as opposed to full samples and in residuals, there is a 35% reduction in the manifestation of Black Swans. Overall, these results indicate that ignoring structural changes or latent non-linearities in the underlying distribution could lead to an erroneous estimation of financial risk via kurtosis estimates in foreign exchange markets i.e. accounting for structural breaks reduces the probability of the occurrence of extreme events.

The second corresponding finding is regarding the nature of Black Swans which allows deeper understanding about the existence of skewness in the distribution. Summarizing the skewness trends in section 6, foreign exchange markets consistently display a higher propensity of experiencing a marginally higher frequency of Black Swans during positive shocks as opposed to negative ones i.e. with a provision for structural breaks, the greater part of total Black Swans occur on the right tail - 52% and 57% in the returns and residuals respectively. Therefore, while accounting for structural breaks reduces tail asymmetry, the forex market remains skewed to the right unlike the equity market which skews to the left.

The third major finding of this chapter was regarding the second aspect of the research question about the evolution of tail risk ratio ex-ante, during and ex-post the Global Financial Crisis of 2008. It investigates the question: Were markets experiencing a steady increase in volatility up until the fall of the Lehman Brothers on 15th September 2008 and if so, how did the ratio advance when markets supposedly started to stabilize in the succeeding segment. Summarizing the particularly remarkable but uniform results in section 6.3 and 6.4, there is evidence of a consistent trend between the returns and residuals of the data i.e. with the inclusion of the effects of latent non-linearity in the mean and/or variance dynamic, majority of the 31 currencies illustrated a decreasing trend leading up to the fall of the Lehman brothers (returns exhibited a 7% lower ratio of Black Swans to extreme values in the preceding segments and residuals a 2% decline). This deteriorating volatility remains evident in the segment succeeding the fall of the Lehman Brothers i.e. returns demonstrate 13% fewer Black Swans to extreme values and this ratio falls to 9% in residuals indicating that a proportion of that volatility was resulting from volatility clustering in the data.

The final hypothesis this chapter tested was the behaviour of Black Swans in foreign exchange markets in relation to the equity markets. The results in section 6 reveal that the probability of Black Swans in foreign exchange markets is distinctly contrasting to the probability of Black Swans in equity markets in every aspect tested i.e. trend when structural breaks are accounted for, evolution during highly volatile periods as well as skewness.

With regards to structural breaks, accounting for them in the daily returns of equity markets, with the exception of Iceland and Slovakia, all countries bared a 20% decline in the occurrence of Black Swans whereas in foreign exchange markets there was a 20% increase in half the sample and 21% decrease in the other half. In residuals, this antipodean trend between forex and equity markets endures at a lower potency i.e. in equity markets Black Swans occurred 12% less frequently and in forex markets, there was a 35% increase in the appearance of Black Swans. The findings indicate that removing auto-correlation from the data, ignoring latent non-linearity in equity markets overestimates volatility whereas in foreign exchange markets it underestimates it.

In reference to the evolution of the Black Swan to extreme value ratio, after correcting auto-dependence in the returns, in equity markets, the segment preceding the Global Financial Crisis had 32% more Black Swans to extreme values and this ratio declined to 10% in the succeeding segment. For foreign exchange markets, the preceding segment had 2% more Black Swans to extreme values and this reduced by an additional 9% in the succeeding segment.

Reducing Black Swans in the succeeding segment to the Global Financial Crisis fits in with the current literature (see (Uppal and Ullah Mangla, 2013), that studied five developed countries along with five leading emerging countries; (Iorgulescu, 2015) for 36 equity markets) that markets had begun to stabilize therefore reducing the probability of future Black Swans taking place. However, the declining proportion of extreme events that can be classified as Black Swans from the preceding segment culminating into the segment which includes the fall of the Lehman Brothers is contrary to the literature. This finding suggests that in general financial markets globally had begun to experience increased financial volatility long before the fall of the Lehman Brothers in 2008. While returns continued to exhibit a higher number of volatility clusters or Black Swan swarms due to amplified integration, spill-over and leverage effects, having accounted for them using the best fit ARMA-APARCH model effectively reduced the probability of heterogeneous Black Swans occurring within the crisis period (see (Iorgulescu, 2015) for empirical study on 36 markets from six continents showing higher integration between global financial markets even before the occurrence of the crisis as well as lower correlation during the crisis for markets in the Asia/Pacific region).

Finally with regards to skewness of the Black Swan distribution, while equity markets remained skewed to the left (56% of the total back swans occurred in the left tail of returns and 61% in the left tail of the residuals), foreign exchange markets depicted a persistent skewness to the right (54% on the right tail of the returns distribution and 57% on the right tail of the residuals distribution). This contrary trend between the two financial markets with regards to the probability of Black Swans indicates a possible opportunity for international portfolio diversification and mitigation of losses during crisis periods.

4.8 Conclusion

Financial models that have traditionally assumed normality in distribution of financial market returns have grossly underestimated the probability of extreme events while invalidating the inferences drawn from asset pricing models that follow that assumption. However, having being crippled by a barrage of financial crisis that were neither predicted nor accommodated in existing models, has led to the increasing awareness about the significance of information in the tails of the distribution i.e. the Gaussian distribution grossly underestimate the probability of extreme events.

In this chapter, the main goal of understanding the extreme tails of the returns distribution of the foreign exchange markets when there is a provision for structural breaks was achieved by looking at Black Swans and extreme values in 35 currency markets when there was a provision for breaks versus without with an additional scaffolding of the Global Financial Crisis. The chapter allows for an in-depth study of the homogeneity of Black Swans in the presence of structural breaks in the mean and/or variance dynamic of the underlying distribution along with corrections for auto-correlation among the data as well as any leverage effects by using the best-fit ARMA-APARCH models.

The focal contribution of the empirical results found that by accounting for latent non-linearity in the mean and/or variance dynamic of the forex returns, the skewness of the distribution was significantly reduced leading to a more symmetrical tail, similar to the equity market returns.

The empirical results provide a clarity regarding the nature of Black Swans and their evolution during the Global Financial Crisis. While the homogeneity of Black Swans in strongly rejected with the provision of structural breaks, the evolution of the Black Swan to extreme value ratio elucidates that much of the volatility in foreign exchange markets resulted from volatility clustering and leverage effects in the market. When each of these are accounted for, volatility essentially declines through the segments from ex-ante to ex-post the Global Financial Crisis.

These results have a strong implication with regards the risk hedging strategies that could be used by market participants as well as the potential for portfolio diversification between foreign exchange and equity markets. There is also an implication for asset pricing models that overestimate risk during crisis periods and underestimate risk during ordinary periods.

Chapter 5: Understanding the occurrence of highly improbable events in volatility dynamics of Forex market

5.1 Introduction

A popular method of measuring volatility without imposing parametric assumptions on the error term are stochastic volatility models. These assume that since volatility is latent i.e. unobservable, proxies such as absolute returns, squared returns and log squared returns can be used instead. What has not been explored yet, is the viability of these volatility proxies to study the occurrence of highly improbable events stemming from the volatility dynamics of the distribution.

An additional justification for studying highly improbable occurrences in these volatility proxies is that although the occurrence of an extreme event in returns will affect both the mean and variance of the distribution, a surge in the probability of the occurrence of a highly improbable event holds the potential to increase the variance without affecting the mean. Therefore, understanding the degree of overlap/difference in the occurrence of highly improbable events from mean and volatility dynamics individually could contribute to a clearer understanding of extreme risk; this could then be translated into asset pricing and forecasting models.

The following chapter will further this conclusion by examining the riskiness of risk i.e. highly improbable events in the volatility dynamics of forex and equity markets and compare it to highly improbable events in the return dynamics. It will carry out this investigation in two ways: first it will identify Black Swans and extreme values in each tail using volatility proxies and compare them to Black Swans and extreme values; second it will undertake the same investigation vis-à-vis an ARMA-APARCH model to include heteroscedasticity in the variance with an additional scaffolding of the Global Financial Crisis of 2008.

The rest of this article is organized as follows. Section II sets the context for the research with a literature review, followed by section III which provides the specific research questions this chapter seeks to answer. Section IV and section V make available the description of the data and the research methodology used to obtain results. The final segments of the chapter are section VI which presents the empirical results on the frequency of Black Swans and extreme returns that exist in the stock market, and section VII which discusses the implications for key market participants. Section VIII concludes.

5.2 Literature Review

The primary objective of this chapter is to scrutinize extreme volatility that results from the conditional variance of the returns using non-parametric measures and comparing it to the extreme volatility from the conditional means of the returns obtained using parametric measures.

Studying volatility is imperative from a risk management and asset pricing perspective however, in the past decade, volatility has also been explicitly used as the basis of financial assets since it is actively traded in the form of futures, options, and variance and volatility swaps with the key players being institutional investors, hedge funds and banks.

The study of volatility of volatility has substantial facets in literature but the depth of such studies remains emergent. The following sections will review the various facets of volatility of volatility starting with expected volatility of volatility followed by realized volatility of volatility and finally implied volatility. Each section will cover assumptions, methodologies, model alterations over time and performance. The review concludes with a section on extreme volatility of volatility which is of particular interest to the research questions of this chapter.

Expected volatility of volatility

One of the most recent strands is volatility of expected volatility; see a recent paper by Baltussen et al. (2018) studying the risk of volatility or in other words the volatility of expected volatility found that stocks with a lower volatility of volatility characteristic outperformed their riskier counterparts when studied from

1996 to 2016[8], i.e. the uncertainty of future volatility is an important predictor of stock prices.

In terms of modelling, a study that looked at the comparative effectiveness of GARCH and stochastic volatility models in forecasting the expected volatility of volatility found that SV models were only slightly more accurate than GARCH models specifically in case where financial assets display high volatility of volatility[9] on the Euro, Pound and Japanese Yen against the Dollar (Ding and Meade, 2010).

Realized volatility of volatility

Another popular strand is realized volatility which has led to modelling of "observable" volatility and predicting future volatility; see Corsi et al. (2008) that modelled volatility of realised volatility along with log realized variance of the residuals of a conventional time series from 1985 to 2004 while allowing for heteroscedasticity and clustering effects. Incorporating these properties improved the overall model-fit as well as the predictive performance of the model.

In terms of modelling, a seminal and fairly recent paper by Barndorff-Nielsen et al. (2010) developed a model-free way that decomposed realized variance into positive and negative semi-variances by adding high frequency intraday squared returns. Using this model led to improved realized variance forecasts (Patton and Sheppard, 2015), allow an understanding of cross-section equity returns (Bollerslev et al., 2017), and permitted a clearer untangling of positive and negative returns in the options market (Feunou and Okou, 2019)

Stochastic volatility of volatility

Stochastic volatility models remain one of the most popular methods in literature to model time-varying volatility of financial assets that also allowing for the inclusion of other stylized properties of volatility such as the clustering and leverage effect. One of the primary assumptions of SV models is that volatility is

[8] The study was across American and European stock markets and the results
[9] The GARCH model set the conditional variance as a function of lagged squared residuals and

unobservable and hence proxies are used to measure latent volatility such as squared returns, absolute returns, log squared returns, etc.

Over time, extensions of SV models allowed for the incorporation of long memory (Comte and Renault, 1998), non-linear mean reversion, stochastic leverage (Veraart and Veraart, 2012) as well as the implied volatility smile and asymmetries (Heston, 1993) also known as multifactor SV models.

Extreme volatility of volatility

The Global Financial Crisis of 2008 highlighted the importance of including tail risk hedging tools within portfolios such as index put options or volatility index futures and options.

A recent study by Park (2015) used the implied volatility index (VIX) options to explore volatility of volatility finding that it was a good proxy for forecasting for tail risk hedging returns in the presence of jump risk, asymmetries, skewness, liquidity, as well as variance risk premiums.

While the literature on tail risk is ever-expanding, yet there are no clear indicators of tail risk that emerge. This chapter contributes to the literature by examining the possibility of volatility of volatility as a tail risk indicator.

5.3 Research question

Conventional literature of financial risk measures the frequency and probability of highly improbable events in the mean dynamics of the financial markets. As a result, traditional financial models that focus on volatility resulting from a change in the mean dynamics do not allow for the inclusion of volatility that can result from a change in the volatility dynamics. Additionally, changes in the volatility dynamics might not be directly observable by a change in the mean and therefore traditional financial models can underestimate overall tail risk in the market.

Therefore, in order to measure the frequency of highly improbable events contributing to tail risk, this chapter will study the number of Black Swans and extreme events in volatility dynamics and then compare it to those in the mean dynamics. If the two are indeed different, the chapter will then analyse the degree of this difference to determine the importance of the inclusion of extreme volatility resulting from a change in the volatility dynamics.

This chapter, moreover, endeavours to provide a study of the behaviour of these Black Swans pre-crisis, during crisis and post-crisis using non-parametric measures of volatility. Specifically, I measure the evolution of tail risk in forex markets during the Global Financial Crisis resulting from a change in the volatility dynamics of the asset returns such as absolute returns, squared returns, and, log squared returns. To ensure the robustness of the results, each volatility measure is corrected for long memory, fat tails and unit roots by using first-order differentiation combined with the best-fit ARMA-APARCH model. An additional robustness check, that integrates the findings of the first two empirical chapters, is the incorporation of probable non-linearity's in the mean and/or variance dynamic of the underlying returns generating process that are known to create and upward bias in the kurtosis measures during a crisis.

The two key aspects of the research question embedded within this chapter are:

- Are highly improbable events from the volatility dynamics the same as the ones that arise from the mean dynamics in foreign exchange markets? If not, then what is the degree of similarity or lack thereof between these extreme events resulting from a change in volatility dynamics versus mean dynamics?
- What is the evolution of the occurrence of these extreme events resulting from the volatility dynamics of the forex market during the Global Financial Crisis of 2008?

These questions will be answered using two models: The first model will compare the frequency of Black Swans resulting from the volatility of volatility and then testing their similarity when using conditional mean while recognizing multiple structural breaks in the underlying process of both data sets. The second model will further improve estimates by using the residuals of both return-based measures and proxy-based measures obtained from the best fit ARMA-APARCH model to correct conditional heteroscedasticity and/or correlations between the values. The second model will additionally study the transformation of the Black Swans to extreme value during the Global Financial Crisis to understand the evolution of Black Swans in forex markets resulting from volatility dynamics and comparing it to the results to the equity market which are studied in the chapter 6.

5.4 Data

In order to understand the evolution of Black Swans in forex markets, daily returns of the currency market were obtained from the DataStream software for 31 countries denoted in the US Dollar – the same ones as in Chapter 4. These returns were converted into three volatility proxies: Absolute returns, squared returns and log returns. This section presents the descriptive statistics of each of the proxies finding evidence of severe kurtosis, skewness, and non-normality followed by a section that illustrates the stationarity of the chosen data sets.

Descriptive Statistics

The following table 18 lists the mean, standard deviation, skewness and kurtosis of each volatility proxy.

Table 18 – Summary Statistics of differenced Absolute returns (Forex Market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**
No. of obs.	5839	5839	5710	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	1.10%	0.71%	0.85%	0.48%
Skewness	0.47	-0.08	0.06	-0.14
Kurtosis	615.20	15.54	20.57	9.44
	Chilean Peso	**Chinese Yuan**	**Danish Krone**	**Euro**
No. of obs.	5839	2828	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.52%	0.12%	0.57%	0.57%
Skewness	-0.02	1.41	0.02	0.02
Kurtosis	8.89	92.62	2.60	2.59
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	4557	5839	4953	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.64%	0.03%	0.71%	0.34%
Skewness	-1.64	1.24	-0.58	0.13

Kurtosis	630.01	61.15	33.86	10.70
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	5839	4953	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	1.27%	0.47%	0.89%	0.85%
Skewness	-0.23	0.48	0.02	0.63
Kurtosis	74.86	20.86	903.26	120.99
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
No. of obs.	5611	5839	5839	5454
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.75%	1.07%	0.73%	0.66%
Skewness	0.18	6.14	0.03	0.04
Kurtosis	115.07	295.55	6.90	68.86
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
No. of obs.	5839	4742	5577	5271
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.66%	0.29%	0.71%	1.46%
Skewness	-0.02	-0.21	-0.06	7.11
Kurtosis	5.66	38.56	4.65	371.31
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**
No. of obs.	5839	2586	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.33%	0.76%	0.85%	0.78%
Skewness	0.16	-0.10	-0.13	1.64
Kurtosis	12.20	14.77	5.07	130.45
	Swedish Krona	**Taiwan New Dollar**	**UK Sterling**	
No. of obs.	5839	5839	5839	
Mean	0.00%	0.00%	0.00%	
Standard Dev.	0.64%	0.26%	0.49%	
Skewness	0.02	-0.54	0.02	
Kurtosis	3.44	29.33	4.95	

Table 19 – Summary Statistics of differenced Squared Returns (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**
No. of obs.	5839	5839	5710	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.32%	0.03%	0.05%	0.01%
Skewness	-0.01	0.07	-1.14	-0.36
Kurtosis	1197.70	299.53	252.71	196.74
	Chilean Peso	**Chinese Yuan**	**Danish Krone**	**Euro**
No. of obs.	5839	2828	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.01%	0.00%	0.01%	0.01%
Skewness	-0.42	2.80	0.03	0.05

Kurtosis	160.18	613.44	75.37	77.54
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	4557	5839	4953	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.11%	0.00%	0.04%	0.01%
Skewness	-0.27	3.82	-1.18	0.16
Kurtosis	2257.31	1069.91	880.84	115.43
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	5839	4953	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.20%	0.02%	0.24%	0.10%
Skewness	0.17	1.71	0.00	1.06
Kurtosis	308.10	179.78	2663.65	686.47
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
No. of obs.	5611	5839	5839	5454
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.06%	0.25%	0.02%	0.04%
Skewness	0.06	16.59	0.35	-0.09
Kurtosis	1055.58	2280.85	96.52	732.93
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
No. of obs.	5839	4742	5577	5271
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.02%	0.01%	0.02%	0.56%
Skewness	-0.22	-0.53	-0.90	8.75
Kurtosis	148.14	162.38	64.97	926.72
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**
No. of obs.	5839	2586	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.01%	0.03%	0.03%	0.10%
Skewness	1.21	-1.01	-2.63	1.83
Kurtosis	241.04	113.98	191.06	950.62
	Swedish Krona	**Taiwan New Dollar**	**UK Sterling**	
No. of obs.	5839	5839	5839	
Mean	0.00%	0.00%	0.00%	
Standard Dev.	0.02%	0.00%	0.01%	
Skewness	0.69	-8.00	-0.02	
Kurtosis	85.57	749.39	129.25	

Table 20 – Summary statistics of differenced Log squared returns (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**
No. of obs.	3653	5620	5257	5635
Mean	-1.61%	-0.14%	-0.70%	-0.82%
Standard Dev.	264.99%	293.06%	291.02%	297.05%
Skewness	0.04	0.05	0.01	0.00
Kurtosis	1.59	0.59	0.76	0.69

	Chilean Peso	Chinese Yuan	Danish Krone	Euro
No. of obs.	5506	2441	5697	5663
Mean	-1.30%	-2.75%	-0.26%	-0.26%
Standard Dev.	288.92%	280.79%	330.99%	317.76%
Skewness	0.02	0.12	0.01	-0.01
Kurtosis	0.70	0.85	0.99	0.55
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	3902	4753	4785	4867
Mean	-2.82%	-0.11%	0.23%	-0.16%
Standard Dev.	276.87%	258.72%	296.64%	267.92%
Skewness	-0.04	0.08	0.00	0.07
Kurtosis	0.60	0.66	0.48	0.61
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	4897	3741	3579	5611
Mean	-2.88%	0.47%	-4.24%	-0.69%
Standard Dev.	272.25%	237.40%	280.27%	294.71%
Skewness	0.06	0.02	0.08	0.02
Kurtosis	0.79	0.17	0.32	0.90
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
No. of obs.	3073	5557	5607	3649
Mean	-3.10%	-1.08%	-0.13%	-2.04%
Standard Dev.	273.82%	296.04%	302.37%	262.96%
Skewness	0.00	0.07	-0.02	0.00
Kurtosis	1.46	0.78	0.62	0.82
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
No. of obs.	5706	3791	5386	4832
Mean	-0.82%	-1.03%	0.78%	-2.17%
Standard Dev.	319.22%	267.52%	304.32%	310.08%
Skewness	-0.03	0.06	0.03	0.05
Kurtosis	1.07	0.54	1.02	1.41
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**
No. of obs.	5547	1508	5634	5143
Mean	-0.75%	-3.28%	-0.38%	-2.83%
Standard Dev.	291.83%	288.92%	315.86%	295.56%
Skewness	0.04	-0.03	0.00	0.08
Kurtosis	0.52	0.64	1.05	0.74

	Swedish Krona	Taiwan New Dollar	UK Sterling
No. of obs.	5705	5073	5661
Mean	-1.04%	-4.38%	-0.22%
Standard Dev.	312.45%	294.75%	307.99%
Skewness	0.00	0.07	-0.01
Kurtosis	1.02	0.65	0.50

For comparative purposes I provide the descriptive statistics of these proxies without differencing in Appendix 35, 36, and 37.

Volatility proxies – in first differences

One of the principal innovations of this study is that its volatility proxies (absolute, squared and log-squared returns) are examined in first differences. This transformation is very convenient in terms of modelling because it overcomes the problem of estimates based on bounded series (the absolute and squared returns are by construction positive and the log-squared returns are practically always negative). However, it is more than simply modelling convenience since differencing even series that are widely accepted as not having a unit root (i.e. over-differencing) is a suggested method to deal not only with high persistence in the series (see (Barros et al., 2016, Meese and Singleton, 1982, Wright, 1999, Baillie and Bollerslev, 2002, Taylor, 2008)) but also with a certain class of breaks namely location shifts (Hendry, 2003). These two are key features of the financial series – in fact a very large strand of literature attributes the very high or infinite persistence of GARCH-type models to the presence of unaccounted for breaks (Bissoondeeal et al., 2019, Karanasos et al., 2014a, Karanasos et al., 2016, Karanasos et al., 2014b, Karoglou, 2006a, Karoglou, 2010b) .

For completeness, I also provide the graphical evidence of stationarity, of the final data sets, in Appendices 13, 14 and 15.

5.5 Methodology

As before, there are two general models that I use, namely a completely non-parametric one and one based on the residuals of the best-fit ARMA-APARCH model – which is possible due to the aforementioned use of differencing the volatility proxies. It is worth noting at this stage that in the chosen setting, the ARMA part effectively models the evolution of the volatility of the returns; and the APARCH part, the evolution of the volatility of volatility, implying effectively a form of autoregressive kurtosis in the same spirit as Brooks et al (2005).

The return function can be specified as:

$$r_t = \varepsilon_t \cdot \mu, \qquad \varepsilon_t = \sigma_t \cdot z_t \qquad z_t \sim N(0,1) \tag{22}$$

Where the data in the underlying distribution is assumed to independent and identically distributed. If the model is correctly specified then

$$E_{t-1}(r_t^2) = E_{t-1}(\sigma^2 \cdot z^2) = \sigma_t^2 \tag{23}$$

This equation justifies the use of squared returns as an adequate proxy for understanding ex-post volatility.

However, the model that specifically addresses the research questions about the volatilities that arise from the mean and the variance can be specified as:

$$Var(r_t) = \sigma_t^2 = E(r_t^2) - \mu_t^2 \tag{24}$$

Wherein, the volatilities in the mean will inferred in the variance of the returns, and therefore can be appreciated however the volatilities arising in the variance are not axiomatic in the mean of the returns and therefore need to be investigated distinctively using volatility proxies such as squared returns, log squared returns, and absolute returns.

Within the ARMA-APARCH model, the conditional mean of the returns process can be defined by the ARMA (p, q) process of autoregressive order p and moving average order of n as:

$$r_t = \alpha_o + \sum_{i=1}^{p} \alpha_i \, r_{t-i} + \sum_{j=1}^{q} \beta_j \, \epsilon_{t-j} + \epsilon_t \tag{25}$$

In order to capture the heteroskedastic properties of the returns process as well as the leverage effects, the conditional variance of the returns process can be defined by the APARCH (m, n):

$$\varepsilon_t = z_t \sigma_t; \qquad z_t \sim N(0,1)$$

$$\sigma_t^\delta = \varphi_0 + \sum_{i=1}^{m} \psi_i \, (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{n} \omega_j \, (\sigma_{t-j})^\delta \tag{26}$$

In both cases, I am seeking to obtain the timing of the extreme events in the underlying volatility proxies; and then compare them to those obtained in the previous chapter. Each observation of the two that is not the same would effectively signify an extreme event that has its source on the volatility and not the mean returns stochastic process.

The remainder of this section is dedicated to providing some evidence of the literature that justifies the use of differencing the volatility proxies. It begins with the presence of unit roots and structural changes in the forex series, followed by the evidence of the presence of unit root and near unit root in volatility proxies for the forex market returns and concludes with the presence of high volatility persistence in forex returns and proxies.

Unit roots and Structural changes in Foreign exchange data

The earliest evidence on the existence of non-stationarity in exchange rates corrected with differencing[10] dates to the early 19th century (Bilson, 1980, Cornell, 1977, Cumby and Obstfeld, 1980, Fuller et al., 1981, Hakkio, 1981, Tryon, 1979, Hansen and Hodrick, 1980) until more formal testing for unit roots gained popularity due to the introduction of large sample distribution theory for unit roots by Fuller et al. (1981), Dickey and Fuller (1979) and Hasza and Fuller (1979). Where unit roots are found to be present, the standard asymptotic theory cannot be applied and will result in spurious statistical inferences if ignored. At this time, it became standard practice in financial modeling to test for stationarity using Unit root tests.

One of the elementary studies formally testing the presence of unit roots within foreign exchange data, using weekly spot and forward exchange rates of the US Dollar against the Swiss Franc, the Canada Dollar and the German Mark for the time period of 1976 to 1981 (Meese and Singleton, 1982) found conclusive evidence of the presence of unit roots; these were corrected using first-order differencing, following which the coefficients of the first-order lag autoregressive model were found to be characterised by random walk (Additionally, ample evidence on the non-stationarity of univariate time-series spot and forward exchange rate frameworks were pivoted by Cornell (1977), Mussa (1979), Corbae and Ouliaris (1986)[11]).

[10] Differencing in the form of time

[11] The study conducted univariate tests for the presence of unit roots in daily spot exchange rates tested for unit roots in Univariate tests reveal strong evidence for the presence of a unit root in the univariate time-series representation for seven daily spot and forward exchange rate series.

These investigations were furthered by Baillie and Bollerslev (2002) revealing that the time-dependent heteroscedasticity and severe kurtosis was more prevalent in daily exchange rate data tested using the Phillips-Perron tests (Phillips and Perron, 1988) on the spot and forward exchange rates of the US dollar against the currencies of UK, West Germany, France, Italy, Switzerland, Japan, and Canada[12].

By the late nineteenth century, one essential drawback of conventional unit root tests came to light - the integral disregard of structural breaks in the underlying volatility structure of the financial returns (see, for examples (Dickey and Fuller, 1981, Said and Dickey, 1985, Phillips and Perron, 1988, Kwiatkowski et al., 1992).

This was remedied by the development of advanced unit root tests that explicitly considered structural shifts in the constant terms and slope parameters by deriving a limiting distribution for the volatility test statistic (see, for examples, (Perron, 1989, Perron, 1990, Perron and Vogelsang, 1992). However, these advanced unit root test assumed homoscedasticity in the underlying variance structure of the data.

One of the seminal papers that highlighted the effect of shifts in innovation variance was by Hamori and Tokihisa (1997) which found evidence that heteroscedasticity in the variance structure leads to the invalidation of the conventional unit root asymptotes unless the heteroscedasticity follows a GARCH type specification that is stationary (i.e. the unconditional variance is well-defined and constant) (Kim and Schmidt, 1993, Seo, 1999, Ling et al., 2003).

However, empirically the assumption of mean reversion remains highly questionable and volatility in financial data displays persistent variation from its mean. This obstinate volatility variation in time series impairs the invariance principle due to size distortions (see (Boswijk, 2001, Kim et al., 2002, Cavaliere, 2005)). This has led to non-parametrically corrected versions of the unit root tests

[12] The presence of unit roots signifying time-varying conditional heteroscedasticity were confirmed using the Lagrange Multiplier (LM) tests which was highly significant for all the currencies tested.

that do not display size distortions due to permanent changes in the innovation variance (see (Cavaliere and Taylor, 2007, Beare, 2004, Boswijk, 2005, Beare, 2018); others recommend identifying the structural breakpoints within the variance structure of the time series before the testing of unit roots (for more evidence see (Kim et al., 2002).

Unit roots in proxies

Volatility itself is inherently unobservable, therefore the use of volatility proxies to gain understanding of the nature of volatility has become common practice in literature. In this case, volatility proxies such as squared returns, absolute returns and/or log squared returns can be considered as a conditionally unbiased estimator of latent conditional variance and can potentially increase the efficiency of the financial model and lead to superior parameter estimates. However, most studies have found evidence that squared returns and absolute returns are too noisy and highly persistent and therefore they have been arbitrated as inadequate estimators of latent volatility (Andersen and Bollerslev, 1998, Andersen et al., 2005, Hansen and Lunde, 2006, Patton, 2011).

This motivated the investigation of this noise, its nature and causation: typically found to be from structural breaks in the volatility process and the inability of conventional ARCH models to capture cross-sectional heteroscedasticity. For the former, many studies show that there are structural breaks in the volatility process and these breaks could be one of the causes of high persistence in the volatility process (Lobato and Savin, 1998, Granger and Hyung, 1999). For the latter causation of why volatility proxies could be misjudged as too noisy can be attributed to the restrictive nature of GARCH models that assume that conditional volatility depends only on lagged information. These models do not take cross-sectional information into account and therefore what is being classified as 'noise' could be arising from multiple cross-sectional factors if asset returns follow linear factor models as suggested by Fama and French. Hwang et al. (2007) found that by using the SVMSR (stochastic volatility Markov switching regime) model that allowed for structural breaks, the level of persistence in the data is reduced and the resulting estimators are more efficient. They found their results to be applicable to GARCH models that allow for structural breaks. While it is beyond the scope of this chapter to identify and test for cross-sectional heteroscedasticity in factors of the financial returns, the results from literature are being used as evidence to specify a model that identifies structural breaks as well as uses GARCH parameters to estimate latent volatility.

Having established the efficiency of volatility proxies once structural breaks are accounted for and cross-sectional heteroscedasticity has been corrected, many studies adopted absolute and squared returns to model asset returns; the earliest one can be traced back to Taylor (2008) where first lag coefficients of absolute and squared returns were used to test the hypothesis that return series are strictly stationary (the study used UK share index, US stock Index, UK agricultural futures, US corn futures among others). Finding conclusive evidence to reject this null hypothesis of a linear structure in returns led to revelation of the ability of using volatility proxies[13] to understand temporal properties of a return series (Cao and Tsay, 1992, Granger and Ding, 1995). For example: the study by Taylor (2008) found that correlation coefficients of lagged volatility proxies are almost always positive and larger than the correlation coefficients of the lagged values of the returns signifying higher volatility clustering in volatility proxies in comparison to returns caused by a variation in the conditional variance; Granger and Ding (1995) found that autocorrelation between lagged absolute returns of the S&P 500 index decline slowly when time decay (θ) is positive and that long-memory is strongest when $\theta = 1$, consequently suggesting that an exponential distribution is most appropriate to model risk.

These findings are particularly interesting to the current study as it allows for an enhanced understanding of the behaviour of extreme volatility during periods of high volatility. Additional studies have found evidence of the superior forecasting ability of volatility proxies, long-memory of absolute returns (Ding et al., 1993a)[14] which can be efficiently captured using APARCH models.

While the literature agrees unanimously on the heteroscedasticity present in the volatility of financial time series it remains divided on how the fluctuations in volatility should be modelled. There is the ARCH literature which assumes volatility

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[13] Absolute returns denoted as $|r_t|$ and squared returns denoted as $|r_t^2|$

[14] Data set included daily closing prices from the S&P 500 stock index from the period of 1928 to 1991.

is observable and stationary (see (Bollerslev et al., 1992, Bollerslev et al., 1994) for a detailed review of different ARCH models) and the alternative is stochastic volatility models that assume that volatility is unobservable hence must be modelled using volatility proxies (see (Hansen, 1995, Ruiz, 1994, Taylor, 1994) for a detailed review).

The stochastic volatility models assume that the log of squared time series returns follow an ARMA process and therefore the largest autoregressive root of the process is the same as that of the volatility process which indicates that it is possible to test for stationarity of innovations in volatility by conducting a unit root test on the log of squared returns. However, this process was not valid with large sample sizes since the data exhibited a large negative moving average which is indicative that conventional unit roots test suffer from size distortions (Schwert, 2002, Pantula, 1991, Harvey et al., 1994). This issue was resolved by proposed unit roots tests by Pantula (1991), Stock (1994), Wright (1999).

Using the seminal work of Wright (1999) who tested the hypothesis of nonstationary stochastic volatility in financial data with the help of proxies such as absolute returns, squared returns and log of squared returns[15] using the fractionally integrated stochastic volatility model that did not require restrictive assumptions regarding the specification for the error term distribution, proposed by Breidt et al. (1998), found strong evidence rejecting the presence of unit roots at all conventional significance levels, however could not reject the presence of near unit roots in the series conceding to the presence of long memory/considerable persistence in the asset returns.

High persistence in Volatility proxies

A final justification for using differencing in volatility proxies is that it allows for the correction of long memory in the financial data. Studies have shown that long-range persistence cannot be adequately modelled by autoregressive conditional

[15] The author used stock market data from 1982-1994 (S&P 500 index) as well as foreign exchange data (dollar to pound, dollar to mark and dollar to yen exchange rates) from 1986-1996.

heteroskedastic (ARCH), Generalized autoregressive conditional heteroskedastic (GARCH) and standard stochastic models (Breidt et al., 1998).

Numerous studies have found evidence that real exchange rates (DM and Yen against the US Dollar) contain innovations that can be classified as fractionally integrated with slow reversion to mean and therefore contain long memory properties (Caporale and Gil-Alana, 2004) (see (Dufrénot et al., 2008) for similar results for five European, namely France, UK, Netherlands, Germany and Portugal currencies using non-linear integration models; the study found evidence of unit roots in the series as well).

Long memory in volatility has also been estimated in various papers using semi-parametric estimates, the results of which further contribute to the existing pool of evidence in literature regarding its presence in financial asset returns (Lobato, 1999, Andersen and Bollerslev, 1997). While there are varied methods that exist, the most prevalent ones are the Long memory stochastic volatility model by Breidt et al. (1998) and the FIEGARCH model (Fractionally Integrated Exponential GARCH) by Bollerslev and Mikkelsen (1996). The LMSV model has a memory parameter that can be the GPH estimator by Geweke and Porter-Hudak (1983)[16], or the GSE parameter (Gaussian Semi-Parametric estimator) by Kunsch (1987). However, there also exists trifling evidence that in some cases these estimators make assumptions that are violated when tested on data. For example, the Long memory stochastic volatility model that uses the GPH estimator assumes that the data used is Gaussian (see (Andersen and Bollerslev, 1997, Deo and Hurvich, 2001)); the limiting distribution of the GSE estimator does not assume normality but that the data generation process is characterised by increments that are linear suggesting asymptotic normality.

Barros et al., 2016 is one of the very few studies that have found evidence of long memory as well as the presence of unit roots in the Chinese Yuan against the US Dollar.

[16] The GPH estimator models the log periodogram of a proxy series such as absolute returns, squared returns, and log squared returns using ordinary linear regression. The estimate is then incorporated into a stochastic volatility model to derive theoretical inferences.

5.6 Empirical Results

The following results will test the hypothesis of the homogeneity of the different clusters of Black Swans in return series when compared to differenced volatility proxies: namely absolute returns, squared returns, and log squared returns. It will then scrutinize the tails of these distributions with regards to the skewness to gain an understanding of the probability of occurrence of negative versus positive Black Swans.

The following section will begin by presenting model 1 outcomes and illustrate the degree of overlap of Black Swans between returns and the three proxies individually (with the inclusion of structural breaks in all data sets) along with other properties such as skewness and the final section of model 1 will conclude by summarizing the degree of overlap overall.

The second section of the empirical results will render the results of model 2 and illustrate the degree of overlap of Black Swans between residuals of returns and residuals of the volatility proxies along with the evolution of volatility of volatility during the Global Financial Crisis of 2008 as the focal point.

5.6.1 Model 1 outcomes

Summary of the innovation in extreme conditional variance between returns and differenced absolute returns of the daily foreign exchange market

The following table 21 compares the frequency of Black Swans in the daily returns of the foreign exchange market and compares it to the frequency of Black Swans in chosen volatility proxy – differenced absolute returns. Both data series are inclusive of the provision of structural breaks in the mean and/or variance dynamic which account for latent non-linearity in the underlying data structure. The table also tests for the homogeneity of Black Swans in the left and right tail thereby scrutinizing the skewness of the volatility distribution (see Appendix 19 for detailed results).

Table 21 – Difference in the frequency of Black Swans between returns and absolute returns (Forex Market)

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso
Right tail	28.77%	17.19%	21.13%	-30.75%	15.42%
Left tail	30.01%	-47.69%	-34.17%	0.00%	-23.64%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	-22.31%	-101.16%	-91.63%	-28.77%	-37.81%
Left tail	53.90%	13.35%	6.25%	0.00%	17.44%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	-17.59%	-22.31%	-4.26%	21.03%	-12.03%
Left tail	-17.59%	18.23%	-35.42%	13.06%	-3.06%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	-4.26%	23.92%	19.42%	14.66%	8.70%
Left tail	-32.38%	17.77%	-45.20%	-25.13%	0.00%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	0.00%	1.98%	-12.14%	26.42%	-18.23%
Left tail	-24.12%	-1.87%	-16.99%	-32.38%	5.41%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-17.59%	19.11%	25.38%	-45.20%	-20.07%
Left tail	-3.28%	-37.04%	-20.97%	3.39%	10.82%
	UK Sterling				
Right tail	-23.84%				
Left tail	-16.99%				

The returns of all 31 countries depict a difference in the frequency of Black Swans in returns as compared to absolute returns with the latter having a higher frequency in 76% of the cases excluding Pakistani Rupee and Russian Rouble which showed no difference. Also, on average, there were 6% additional Black Swans in the differenced absolute returns in comparison to the return series (representing an increase from 1.36% of trading days in the return series to 1.42% of the trading days in the absolute return series).

However, a nuanced analysis at the results divulges an antagonistic trend – while largely, there are a higher number of Black Swans in the absolute returns, yet 7 of the 29 countries maintain a higher frequency of Black Swans in the returns (1.68% of the trading days are Black Swans) as opposed to the absolute returns

(1.49% of the trading days are Black Swans) with the highest frequency of Black Swans within returns occurring in the Argentine Peso.

Summary of the innovation in extreme conditional variance between returns and squared returns of the daily foreign exchange market

The following table 22 compares the frequency of Black Swans in returns and contrasts it to the frequency of Black Swans in differenced squared returns of the foreign exchange market of 31 countries while allowing for non-linearity in the mean/variance structure. It also breaks down the frequency of Black Swans on the left and right tails of each distribution to observe skewness in the volatility structure (see Appendix 20 for detailed results).

Table 22 – Difference in the frequency of Black Swans between returns and squared returns (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	93.43%	-23.36%	7.41%	-92.43%	-21.36%
Left tail	65.68%	-93.83%	-39.30%	-66.33%	-64.19%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	109.86%	-160.94%	-145.53%	-78.85%	0.00%
Left tail	156.86%	-61.18%	-56.39%	-66.14%	32.85%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	-50.31%	-6.67%	39.47%	27.63%	-6.19%
Left tail	-26.83%	9.53%	13.35%	20.29%	13.88%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	9.10%	121.11%	32.09%	-42.05%	26.57%
Left tail	-28.03%	127.63%	-25.13%	-82.67%	27.09%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	-60.98%	71.29%	-53.63%	65.81%	-28.14%
Left tail	-75.91%	53.63%	-46.54%	15.91%	-23.36%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-10.92%	-4.26%	-8.27%	-111.44%	4.55%
Left tail	-6.45%	-41.69%	-77.32%	-77.32%	5.26%
	UK Sterling				
Right tail	-67.37%				
Left tail	-63.60%				

The frequency of Black Swans captured by returns and squared returns are not homogenous. In every country, there is a significant difference between the numbers of Black Swans in the returns series versus the differenced squared returns series which reiterates the conclusion that volatility arising from variance of returns is varied to the volatility arising from mean. Also, there are on average 11% more Black Swans in the differenced squared return series (where Black Swans accounted for 1.52% of the trading days) in comparison to returns (where Black Swans accounted for 1.31% of the trading days). It is a well-known fact that there are substantially more correlations between absolute returns and squared returns as compared to returns themselves (Taylor, 2008, Kariya et al., 1990, Ding et al., 1993b) which explains the higher frequency of Black Swans in the squared returns. This trend can be seen in 18 of the 31 countries within the data set. The remaining 13 countries depict higher percentage of Black Swans in returns (1.68% of the trading days) as opposed to squared returns (1.20% of the trading days).

The above results are consistent with the results from table 21 (absolute returns).

Summary of the innovation in extreme conditional variance between returns and log squared returns of the daily foreign exchange market

The following table 23 reports the results on the frequency of Black Swans in returns in comparison to differenced log squared returns of the foreign exchange market of 31 countries while accounting for structural changes in the mean and/or variance dynamic of the underlying series (see Appendix 21 for detailed results).

Table 23 – Difference in the frequency of Black Swans between returns and log squared returns (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	22.41%	60.36%	125.71%	29.29%	78.86%
Left tail	11.88%	14.41%	44.06%	54.64%	56.99%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	14.05%	-90.70%	12.35%	35.57%	100.72%
Left tail	142.15%	18.30%	63.27%	13.25%	114.13%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	73.87%	139.35%	94.47%	198.80%	156.89%
Left tail	73.87%	160.97%	143.35%	181.94%	240.09%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	84.44%	35.77%	79.13%	134.58%	138.99%
Left tail	59.62%	38.65%	51.01%	58.36%	127.77%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	25.14%	209.86%	22.11%	104.80%	99.85%
Left tail	-26.42%	305.33%	8.29%	49.50%	94.72%

	Solomon Isl. Dollar	South Africa Rand	South-Korean Won	Swedish Krona	Taiwan new Dollar
Right tail	271.88%	46.07%	84.25%	-11.42%	83.28%
Left tail	176.33%	11.27%	38.39%	24.25%	103.80%
	UK Sterling				
Right tail	51.91%				
Left tail	43.17%				

Among the three volatility proxies in the model, the most significant and counter-intuitive results emerge in the results comparing the frequency of Black Swans in returns and differenced log squared returns i.e. there are a higher frequency of Black Swans in returns as opposed to log squared returns.

Overall, there are 82% more Black Swans in returns (1.36% of the trading days were classified as Black Swans) as opposed to the log squared returns (0.61% of the trading days were identified as Black Swans). This trend is seen in currency returns of 29 out of the 31 countries in the data set; the only exceptions are Danish Krone and Norwegian Krone where there are 24% and 2% more Black Swans in the returns as opposed to the log squared returns respectively.

Degree of overlap between returns (mean dynamics) and the proxies (volatility dynamics) of the daily foreign exchange market

The following table presents the degree of similarity between the extreme events from mean dynamics versus those from the volatility dynamics of the returns' distribution.

	Returns
Returns	100%
Differenced Absolute returns	95.0%*
Differenced Squared returns	81.9%*
Differenced Log Squared returns	40.8%**

*Returns have higher frequency of Black Swans ** Proxies have higher frequency of Black Swans

It is clear from the summary that there is a slight discrepancy between extreme events in returns versus those in the proxies of the returns.

Accounting for volatility from mean dynamics plays an essential role in asset pricing and hedging strategies and results show that a change in the volatility (resulting from mean) is accounted for in traditional models. However, these parametric models do not account for the extreme events resulting from the volatility of volatility (resulting from volatility dynamics of the asset distribution). This disregard of changes in volatility of volatility could let to potential misspecification of risk faced by investors and market participants and hence hold the potential to be an integral indicator of overall financial risk.

With respect to log-squared returns, majority of scholarly literature tends to exclude it as a volatility proxy because it if the asset returns are close to zero, then transforming it to log-squared returns yields a highly negative number; if it is zero then the transformation is undefined. Such an extreme negative numbers tends to distort overall model estimates (Cavalcante and Assaf, 2004, Pereira, 2004, Christodoulakis and Satchell, 2005). The focus of this chapter is sizable distortion in financial markets due to a crisis, i.e. heightened tail risk from the volatility dynamics of asset returns during a crisis and not small innovations, therefore the focus will remain primarily on the results from absolute returns and squared returns.

5.6.2 Model 2 outcomes

Summary of the innovation in extreme variance between residuals of returns and differenced absolute returns of the daily foreign exchange market from best fit ARMA-APARCH model

The following table 24 determines the frequency of Black Swans in the residuals of the conditional volatility mean and variance of daily returns of the forex market of 31 countries along with the differenced absolute returns. Both data sets are inclusive of multiple structural breaks in the underlying returns process (see Appendix 22 for detailed results).

Table 24 – Difference in the frequency of Black Swans between residuals of returns and residuals of absolute returns (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	-143.01%	-71.93%	-66.43%	-120.48%	-100.30%
Left tail	295.20%	277.17%	125.17%	304.37%	309.02%

	Chinese Yuan	Danish Krone	Euro	Fijian Dollar	Hong Kong Dollar
Right tail	-128.82%	-126.94%	-156.95%	-57.34%	-75.96%
Left tail	270.77%	340.03%	333.13%	270.72%	263.80%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	-116.12%	-94.62%	-57.62%	-51.94%	-173.55%
Left tail	194.49%	244.18%	349.57%	170.41%	78.76%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	-65.69%	-43.44%	-51.74%	-75.72%	-66.33%
Left tail	50.98%	321.85%	187.11%	299.49%	189.58%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	-144.17%	-88.32%	-105.07%	-71.59%	-106.56%
Left tail	306.31%	363.67%	313.46%	198.02%	361.01%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-65.55%	-75.75%	-61.68%	-137.42%	-188.84%
Left tail	288.88%	256.43%	283.22%	321.84%	232.64%
	UK Sterling				
Right tail	-153.23%				
Left tail	321.80%				

The empirical results of table 24 can be summarized into two key trends: first, that overall there are fewer Black Swans in both residuals and absolute residuals in comparison to returns and absolute returns respectively which is consistent with the results in equity markets, second the skewness of the two distributions: residuals of returns are heavily skewed to the left whereas residuals of absolute returns are heavily skewed to the right.

Regarding the first result, there are 24% fewer Black Swans in residuals as compared to returns and 0.73% fewer Black Swans in residuals of absolute returns in comparison to absolute returns themselves. The decrease in the frequency of Black Swans in both data sets is because of the correction of conditional kurtosis in returns by using the best-fit ARMA-APARCH model i.e. volatility is higher in returns and absolute returns due to the common phenomena of volatility clustering in financial returns.

The second result entertains the possibility of equivalence of the distributions. Indeed, a comparison of the tails of the two distributions shows that the majority of the positive Black Swans exist in the absolute residuals (72%) whereas an overwhelming number of negative Black Swans exist in the residuals (91%).

In anticipation of the results of the next chapter, these results are consistent with equity market trends in residuals and absolute residuals.

Summary of the ratio of Black Swans to extreme values in the residuals of differenced Absolute returns obtained from the best fit ARMA-APARCH model

The following section, table 25, delves deeper into the nature of the volatility of volatility by considering the transformation in the ratio of Black Swans to extreme value during the pre- and post- Global Financial crisis periods. This will be accomplished by surveying the transformation in the ratio of Black Swans to extreme values in the residuals of absolute differenced returns of the foreign exchange markets of 34 countries ex ante and ex post the Global Financial Crisis of 2008. Specifically, it will study the ratio of Black Swans to extreme values in the segment before the crisis labelled as pre-crisis and after the crisis labelled as post crisis in the following table 25 while keeping the central time segment consistent to the fall of the Lehman Brothers on 15th of September, 2008. The dates of the chosen segments for each of the 34 countries are:

Table 25 – Structural breaks in the Forex market ex-ante and ex-post the GFC (2008)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-Crisis	18/07/2001	24/10/1997	09/06/2003	21/08/1998	06/07/2001
Crisis	08/07/2007	31/07/2007	08/09/2008	13/09/2007	10/01/2008
Post Crisis	21/12/2015	20/07/2009	29/06/2009	13/09/2010	13/05/2009
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-Crisis	NA	18/08/2004	18/08/2004	02/12/1998	24/09/2003
Crisis	20/07/2005	12/08/2008	12/08/2008	27/08/2006	21/11/2007
Post Crisis	20/02/2014	18/11/2011	18/11/2011	29/06/2009	12/01/2016
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-Crisis	27/05/1997	27/08/1998	26/10/1999	27/05/1997	22/07/2005
Crisis	26/03/2006	01/05/2008	21/10/2006	20/11/2007	10/03/2008

Post Crisis	11/06/2009	08/10/2013	23/06/2009	27/01/2009	02/12/2014
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-Crisis	22/12/1994	29/07/1998	26/02/2001	17/12/1997	12/07/2000
Crisis	06/12/2005	24/12/2002	26/11/2005	27/07/2007	05/12/2003
Post Crisis	26/05/2009		04/05/2009	20/07/2009	01/12/2008
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Pre-Crisis	26/08/1998	15/10/2001	26/05/1998	27/08/1998	01/12/1998
Crisis	18/01/2008	21/02/2008	07/08/2008	11/01/2006	14/04/2008
Post Crisis	24/08/2009	28/01/2009	09/08/2012	03/11/2014	21/03/2012
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-Crisis	23/06/2006	27/05/1998	11/08/1998	01/09/1998	20/10/1998
Crisis	06/11/2007	03/12/2001	17/03/2008	12/08/2008	02/03/2004
Post Crisis	08/07/2011	04/05/2009	19/05/2009	16/12/2011	15/11/2011
	UK Sterling				
Pre-Crisis	04/01/1994				
Crisis	28/06/2002				
Post Crisis	17/06/2010				

The following table 26 summarizes the change in the volatility of volatility before, during and after the Global Financial Crisis of 2008.

Table 26 – Difference in the frequency of Black Swans in residuals of absolute returns (forex market) ex-post and ex-ante the GFC (2008)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-crisis	0.00%	73.08%	71.43%	56.25%	73.53%
Crisis	1.39%	83.33%	16.67%	58.33%	75.00%
Difference	N/A	13.13%	-145.51%	3.63%	1.98%
Post-crisis		42.86%	67.86%	16.67%	37.50%
Difference	N/A	-66.49%	140.38%	-125.25%	-69.31%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-crisis	N/A	45.45%	54.55%	56.67%	50.00%
Crisis	58.70%	50.00%	55.56%	15.38%	84.38%
Difference	N/A	9.54%	1.83%	-130.42%	52.33%
Post-crisis	16.67%	66.67%	66.67%	35.71%	50.00%

Difference	-125.88%	28.77%	18.23%	84.24%	-52.33%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-crisis	80.00%	121.15%	81.25%	85.71%	121.74%
Crisis	68.18%	76.67%	77.78%	50.00%	66.67%
Difference	-15.99%	-45.75%	-4.36%	-53.89%	-60.21%
Post-crisis	50.00%	92.86%	54.55%	112.50%	50.00%
Difference	-31.01%	19.16%	-35.48%	81.09%	-28.77%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-crisis	100.00%	79.17%	75.00%	86.54%	127.78%
Crisis	77.78%	51.43%	67.50%	66.67%	96.15%
Difference	-25.13%	-43.14%	-10.54%	-26.09%	-28.44%
Post-crisis	55.56%	N/A	66.67%	64.29%	0.00%
Difference	-33.64%	N/A	-1.24%	-3.64%	N/A
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Pre-crisis	64.00%	79.41%	74.07%	100.00%	58.00%
Crisis	50.00%	83.33%	59.09%	88.10%	77.27%
Difference	-24.69%	4.82%	-22.59%	-12.67%	28.69%
Post-crisis	35.71%	90.00%	85.71%	100.00%	93.75%
Difference	-33.66%	7.70%	37.19%	12.67%	19.33%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-crisis	100.00%	95.00%	88.46%	69.23%	60.71%
Crisis	85.00%	70.00%	50.00%	50.00%	85.71%
Difference	-16.25%	-30.54%	-57.05%	-32.54%	34.49%
Post-crisis	100.00%	58.33%	42.86%	58.33%	77.78%
Difference	16.25%	-18.24%	-15.41%	15.41%	-9.71%
	UK Sterling				
Pre-crisis	67.39%				
Crisis	66.67%				
Difference	-1.07%				
Post-crisis	30.00%				
Difference	-79.86%				

The table 26 summarizes the change in the extreme tails of the residuals of the differenced absolute returns; it is important to note that while there is a change in extreme value constantly, in some cases there are no Black Swans identified within those extreme values, therefore in such cases the cell has been labelled as NA and ability to measure change in the ratio becomes limited.

The evidence presented in the table 26 can be categorized into two key results: first that the ratio of Black Swans to extreme values in the residuals of the differenced absolute returns decrease substantially from pre-crisis to the crisis period in majority of the countries in the dataset, second that the decline in the ratio becomes more significant from the crisis to the post-crisis time period with more than half the countries showing a consistent decline across all three time periods.

Regarding the first result, there were on average 40% fewer Black Swans to extreme values in the residuals of the differenced absolute returns for 20 of the 31 countries in the data set. This result is consistent with the results derived from the residuals of returns found in chapter 2 but the decline is more pronounced in the residuals of the volatility proxy.

The second result shows that out of these 20 countries; 10 continued to stabilize in the consecutive segment (with 28% fewer Black Swans to extreme values in the post crisis period), 8 displayed a higher ratio of Black Swans to extreme values (on average there were 51% more Black Swans to extreme values in the post-crisis period suggesting continued extreme volatility) and 2 were NA values (New Guinea Kina and Nigerian Naira). Of the remaining 11 countries, which demonstrated a higher ratio of Black Swans to extreme values in the crisis period as compared to the pre-crisis period 6 stabilized in the post crisis period (with the ratio of Black Swans to extreme value declining by 63%) while 4 continued to experience greater extreme volatility (with a rise in the ratio of Black Swans to extreme values of 19%) and the Argentine Peso was incomparable. Overall, the trend of declining tails of the extreme volatility distribution remains dominant with the ratio of Black Swans to extreme values decreasing to half of their number in the post-crisis period when compared to the crisis period in 16 of the countries in the data set.

Summary of the innovation in extreme variance between residuals of returns and differenced squared returns of the daily foreign exchange market from best fit ARMA-APARCH model

The following table 27 summarizes the frequency of Black Swans in residuals of daily forex returns of 31 countries and compares it to the frequency of Black Swans in residuals of the selected volatility proxy which is squared returns. It also takes an in-depth look at the tails of both the distributions to compare and contrast the number of positive and negative Black Swans (see Appendix 23 for detailed results).

Table 27 – Difference in the frequency of Black Swans in residuals of returns versus residuals of squared returns (Forex market)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	-109.93%	-92.23%	-77.88%	-170.99%	-96.43%
Left tail	120.33%	277.17%	153.94%	304.37%	239.70%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	-86.50%	-175.49%	-193.72%	-86.11%	-28.19%
Left tail	270.81%	340.03%	333.13%	270.72%	373.66%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	-153.02%	-67.22%	-9.61%	-40.61%	-100.64%
Left tail	333.22%	382.81%	340.04%	378.36%	309.02%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	-49.21%	33.18%	-40.62%	-92.58%	-22.03%
Left tail	201.39%	321.85%	61.84%	299.49%	258.90%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	-162.15%	-24.60%	-133.95%	-74.80%	-106.56%
Left tail	199.14%	202.73%	296.36%	175.71%	361.01%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	-48.11%	-108.16%	-85.14%	-180.06%	-59.74%
Left tail	288.88%	256.43%	283.22%	321.84%	232.64%
	UK Sterling				
Right tail	-179.26%				

Left tail	252.49%				

The results of the empirical table 27 above can be summarized into three key findings: first that that there are fewer Black Swans in the residuals of both returns and squared returns in comparison to the returns themselves; second residuals of squared returns have more Black Swans than the residuals of the returns and finally there are more negative Black Swans in residuals of squared returns in comparison to residuals of returns which have more positive Black Swans.

With respect to the first result, there are 24% fewer Black Swans in the residuals of returns and 9% fewer Black Swans in the residuals of squared returns. The significant decrease in the frequency of Black Swans in residuals of returns align with stylized properties of financial returns known as conditional heteroscedasticity which has been corrected here using the best fit ARMA-APARCH model.

Regarding the second result, even though the decrease in extreme volatility is higher in residuals of returns yet the residuals of squared returns continue to have more Black Swans in comparison to residuals of the returns i.e. 1.44% of trading days in the residuals of squared returns were Black Swans in comparison to 1.06% of the residuals of returns. This trend is seen in the forex market of 23 of the 31 countries in the data set which have 53% more Black Swans in the residuals of squared returns as compared to residuals of returns.

Finally, the third result which investigates the tails of extreme volatility of both distributions, shows that while there are more Black Swans in the right tail of the residuals of squared returns, the reverse is true for residuals of returns which have a higher incurrence of Black Swans in the left tail i.e. 0.48% of the residuals of returns were classified as negative Black Swans in comparison to 0.04% of the residuals of squared returns.

Summary of the ratio of Black Swans to extreme values in the residuals of differenced squared returns obtained from the best fit ARMA-APARCH model

The following table 28 captures the modification to the extreme tails of the unconditional measure of variance during the Global Financial Crisis of 2008, specifically, it reports the results of the hypothesis testing the homogeneity of Black Swans when markets are under stress.

Table 28 - Difference in the frequency of Black Swans in residuals of squared residuals (forex market) ex-post and ex-ante the GFC (2008)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-crisis	33.33%	88.46%	71.43%	114.58%	108.82%
Crisis	2.78%	41.67%	66.67%	106.25%	62.50%
Difference	-248.49%	-75.29%	-6.90%	-7.55%	-55.46%
Post-crisis	100.00%	42.86%	57.14%	87.50%	18.75%
Difference	358.35%	2.82%	-15.42%	-19.42%	-120.40%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-crisis		95.45%	100.00%	73.33%	50.00%
Crisis	39.13%	88.89%	83.33%	3.85%	59.68%
Difference	N/A	-7.13%	-18.23%	-294.79%	17.69%
Post-crisis	8.33%	88.89%	83.33%	71.43%	50.00%
Difference	-154.66%	0.00%	0.00%	292.16%	-17.69%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-crisis	115.00%	76.92%	37.50%	57.14%	21.74%
Crisis	77.27%	83.33%	44.44%	100.00%	63.89%
Difference	-39.76%	8.00%	16.99%	55.96%	107.80%
Post-crisis	100.00%	100.00%	40.91%	137.50%	50.00%
Difference	25.78%	18.23%	-8.29%	31.85%	-24.51%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-crisis	116.67%	100.00%	75.00%	103.85%	61.11%
Crisis	43.06%	5.88%	62.50%	83.33%	23.08%
Difference	-99.68%	-283.32%	-18.23%	-22.01%	-97.39%
Post-crisis	94.44%		80.00%	78.57%	100.00%
Difference	78.55%	N/A	24.69%	-5.88%	146.63%

	Norwegian Krone	Pakistan Rupee	Polish Zloty	Russian Rouble	Singapore Dollar
Pre-crisis	76.00%	20.59%	79.63%	100.00%	54.00%
Crisis	60.00%	100.00%	122.73%	102.38%	86.36%
Difference	-23.64%	158.05%	43.26%	2.35%	46.96%
Post-crisis	114.29%	50.00%	92.86%	150.00%	118.75%
Difference	64.44%	-69.31%	-27.89%	38.19%	31.85%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-crisis	87.50%	90.00%	107.69%	98.08%	14.29%
Crisis	45.00%	105.00%	112.50%	100.00%	100.00%
Difference	-66.50%	15.42%	4.37%	1.94%	194.59%
Post-crisis	120.00%	127.78%	64.29%	112.50%	33.33%
Difference	98.08%	19.63%	-55.96%	11.78%	-109.86%
	UK Sterling				
Pre-crisis	84.78%				
Crisis	57.14%				
Difference	-39.45%				
Post-crisis	70.00%				
Difference	20.29%				

The results in table 28 can summarized into three key trends: first that the ratio of Black Swans to extreme vales decreases in the crisis period as compared to the pre-crisis period, second that this trend is reversed in the post crisis period wherein there are more Black Swans within the extreme values and finally the trend seen in the residuals of the differenced squared returns of the forex market are antipodal to the equity markets across all segments.

With respect to the first result, overall there are 24% fewer Black Swans to extreme values in the crisis period as compared to the pre-crisis period but upon further examination there are two interesting findings: while the residuals of majority of the countries in the data set show significantly fewer Black Swans to extreme values in the crisis period (i.e. there are 83% fewer Black Swans to extreme values in the residuals of 17 of the 31 countries in the data set) yet there are 13 countries that display a reverse trend of 52% more Black Swans to extreme values in the crisis period as compared to the preceding segment.

Regarding the second result, of the 17 countries which displayed a dominant trend of a lower Black Swan to extreme value ratio in the crisis period as compared the preceding segment, only 4 continued to stabilize with 40% fewer Black Swans to extreme values post crisis; 12 displayed higher probability of Black Swans occurring after a crisis with 93% more Black Swans to extreme values in the post crisis segment, and the New Guinea Kina was incomparable.

The submissive trend of the 13 countries which displayed a higher ratio of Black Swans to extreme values during the crisis period, 6 continued to experience higher volatility with 25% more Black Swans to extreme value post crisis while 7 of the countries stabilized with 45% fewer Black Swans to extreme values in the segment succeeding the crisis. Overall, on average, there were 21% more Black Swans to extreme values post crisis which indicated a slow response to the financial crisis with extreme volatility persistently mounting in consecutive segments.

In anticipation of the results of the next chapter, when these results are compared to the equity markets, the asymmetric response to the Global Financial Crisis remains consistent with both markets reacting in an antipodal manner.

Summary of the innovation in extreme variance between residuals of returns and differenced log squared returns of the daily foreign exchange market obtained from best fit ARMA-APARCH model

The following table 29 summarizes the frequency of Black Swan in residuals[17] of returns and compares it to the residuals of differenced log squared returns of the forex market of 31 countries. It also looks at the tails of the volatility distribution

[17] Residuals are standardized and have been obtained by fitting the daily returns and differenced log squared daily returns with the best fit ARMA-APARCH model. Both returns series are inclusive of potential structural breaks in the underlying mean and/or variance dynamic that could be caused by economic, political and/or financial crisis in the respective country.

for both data sets, comparing and contrasting the number of positive and negative Black Swans in each series (see Appendix 24 for detailed results).

Table 29 – Difference in the frequency of Black Swans in the residuals of returns versus residuals of log squared returns

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Right tail	-25.19%	227.64%	242.70%	194.50%	226.80%
Left tail	-22.37%	-98.95%	-82.74%	-82.76%	-43.60%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Right tail	69.27%	288.95%	201.40%	199.17%	166.37%
Left tail	6.86%	-81.92%	-53.99%	-47.08%	84.60%
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Right tail	165.72%	145.16%	190.85%	252.55%	109.75%
Left tail	-30.64%	83.23%	66.23%	129.90%	78.73%
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Right tail	232.62%	203.66%	213.93%	272.91%	149.26%
Left tail	-135.35%	8.31%	-132.76%	-81.18%	79.71%
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Right tail	288.93%	197.33%	172.18%	241.97%	201.40%
Left tail	-112.95%	169.09%	-69.41%	-9.91%	-15.12%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Right tail	215.82%	253.83%	190.33%	230.21%	225.03%
Left tail	109.73%	-162.54%	-66.43%	-92.48%	21.61%
	UK Sterling				
Right tail	207.89%				
Left tail	-73.29%				

The key findings of the table 29 can be broken into two key themes: first comparing the average frequency of Black Swans in residuals to returns it is noticed that while residuals of returns have lower occurrences of Black Swans yet the frequency is higher in residuals of log squared returns, as compared to the log squared returns themselves; second while residuals of returns have higher frequency of Black Swans overall, residuals of returns have higher number of Black Swans in the right tail and residuals of log squared returns have a higher number of Black Swans in the left tail.

Regarding the first result, while the residuals from returns consistently show lower volatility in comparison to the returns themselves in stock and forex markets, residuals from volatility proxies do not show such consistently i.e. while residuals from absolute returns and squared returns depicted fewer Black Swans in comparison to the returns however residuals of log squared returns depict the reverse trend (0.61% of trading days in log squared returns were Black Swans whereas 0.83% of trading days were identified as Black Swans in log squared residuals). This result is keeping in mind the tenacious result that volatility proxies (both returns and residuals) have fewer Black Swans, on average, than the returns themselves. For example: 20 of the countries in the data set, have on average 79% more Black Swans in residuals in comparison to the log squared residuals.

The second result categorically focuses on the tails of volatility of residuals and log squared residuals. It is found that there is a significantly higher frequency of positive Black Swans in residuals (on average, 0.71% of trading days with positive returns were Black Swans once the effect of volatility clustering was corrected in comparison to 0.11% of trading days with positive log squared returns) and there are a slightly greater number of negative Black Swans in log squared residuals.

Summary of the ratio of Black Swans to extreme values in the residuals of differenced Log Squared returns obtained from the best fit ARMA-APARCH model

The following table 30 summarizes the transformation in the extreme tails of the unconditional volatility measure that has been corrected for serial correlation i.e. it analyses the change in ratio of Black Swans to extreme values in the residuals of differenced log squared returns of the forex market of 34 countries.

Table 30 - Difference in the frequency of Black Swans in residuals of log squared returns (forex market) ex-post and ex-ante the GFC (2008)

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**
Pre-crisis	0.00%	36.00%	42.86%	56.52%	29.41%
Crisis	57.14%	100.00%	33.33%	50.00%	25.00%

Difference	N/A	102.17%	-25.13%	-12.26%	-16.25%
Post-crisis	50.00%	50.00%	35.71%	40.91%	28.57%
Difference	-13.35%	-69.31%	6.90%	-20.07%	13.35%
	Chinese Yuan	**Danish Krone**	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**
Pre-crisis		63.64%	27.27%	29.17%	0.00%
Crisis	35.00%	72.22%	66.67%	50.00%	28.57%
Difference	N/A	12.66%	89.38%	53.90%	N/A
Post-crisis	33.33%	38.89%	37.50%	16.67%	0.00%
Difference	-4.88%	-61.90%	-57.54%	-109.86%	N/A
	Icelandic Krona	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**
Pre-crisis	20.00%	22.73%	42.86%	26.19%	8.33%
Crisis	50.00%	29.17%	23.33%	16.67%	31.25%
Difference	91.63%	24.95%	-60.80%	-45.20%	132.18%
Post-crisis	50.00%	41.67%	33.33%	7.14%	0.00%
Difference	0.00%	35.67%	35.67%	-84.73%	N/A
	Mexican Peso	**New Guinea Kina**	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
Pre-crisis	0.00%	56.25%	25.00%	36.00%	33.33%
Crisis	52.86%	44.44%	31.58%	50.00%	38.89%
Difference	N/A	-23.56%	23.36%	32.85%	15.42%
Post-crisis	50.00%		66.67%	50.00%	0.00%
Difference	-5.56%	N/A	74.72%	0.00%	N/A
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**	**Singapore Dollar**
Pre-crisis	72.92%	3.85%	53.85%	50.00%	37.50%
Crisis	40.00%	0.00%	40.91%	36.84%	68.18%
Difference	-60.04%	N/A	-27.48%	-30.54%	59.78%
Post-crisis	57.14%	25.00%	41.67%	50.00%	25.00%
Difference	35.67%	N/A	1.83%	30.54%	-100.33%
	Solomon Isl. Dollar	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**
Pre-crisis	16.67%	55.56%	32.61%	57.69%	33.33%
Crisis	35.71%	65.79%	33.33%	66.67%	47.37%
Difference	76.21%	16.91%	2.20%	14.46%	35.14%
Post-crisis	0.00%	82.35%	50.00%	29.17%	25.00%

Difference	N/A	22.46%	40.55%	-82.67%	-63.91%
	UK Sterling				
Pre-crisis	36.36%				
Crisis	42.86%				
Difference	16.43%				
Post-crisis	40.00%				
Difference	-6.90%				

The results from table 30 above can be summarized into three key trends: first that there are more Black Swans to extreme values in the crisis period as compared to the preceding segments, second that this ratio decline post crisis and finally that forex and equity markets have asymmetric responses to crisis.

With respect to the first result, majority of the countries in the data set have more Black Swans to extreme values in the crisis period as compared to the pre-crisis period, i.e. there are 47% more Black Swans within the extreme values during the segment that contains the fall of the Lehman Brothers as compared to the previous segment. However, there are a few countries in the data set whose forex market displayed a reverse trend, i.e. 9 of the 31 countries had 31% fewer Black Swans to extreme values.

Regarding the second result, of the 17 countries that had experienced higher probability of the occurrence of Black Swans in the crisis period, 5 were incomparable, 4 continued to encounter a great number of Black Swans within the extreme values (i.e. they had 35% more Black Swans post crisis) however the remaining 8 started to stabilize with 70% fewer Black Swans to extreme values. Of the 9 countries that had experienced fewer Black Swans during the crisis period only 2 continued to stabilize with 35% lesser Black Swans post crisis, 2 displayed on change in the ratio post crisis and the remaining 6 exhibited 18% more Black Swans to extreme values. Overall there were 15% fewer Black Swans to extreme values across all countries indicating that markets were at their worst during the crisis and had started to improve ex post the fall of the Lehman Brothers.

Finally, the response of the forex market across the 3 segments continues to remain contrary to the equity markets within which the residuals of the differenced log squared returns had experienced a polar trend ex post and ex ante the Global Financial Crisis.

Degree of overlap between residuals of returns (mean dynamics) and the residuals of proxies (volatility dynamics) of the daily foreign exchange market

The following table presents the degree of similarity or overlap between the extreme events from mean dynamics (residuals of returns) versus those from the volatility dynamics (residuals of volatility proxies) and while there is a significant level of overlap across the data sets, there is clear evidence that there is a degree of dissimilarity between the two.

	Residuals
Residuals	100%
Differenced Absolute residuals	66.8%*
Differenced Squared residuals	63.4%*
Differenced Log Squared residuals	69.5%**

*Returns have higher frequency of Black Swans ** Proxies have higher frequency of Black Swans

This table presents evidence that there is a difference in the extreme events from mean dynamics and volatility dynamics when heteroscedasticity is accounted for.

Accounting for volatility from mean dynamics plays an essential role in asset pricing and hedging strategies and it is well known in the literature that a change in the volatility (resulting from mean) is accounted for in traditional models however volatility reflected in returns by itself does not account for the extreme events resulting from the volatility of volatility (resulting from volatility dynamics). This disregard of changes in volatility of volatility could let to potential misspecification of risk faced by investors and market participants and hence possesses the potential to be an integral indicator of overall financial risk.

With respect to log-squared returns, majority of scholarly literature tends to exclude it as a volatility proxy because it if the asset returns are close to zero, then transforming it to log-squared returns yields a highly negative number; if it is zero then the transformation is undefined. Such an extreme negative numbers tends to distort overall model estimates (Cavalcante and Assaf, 2004, Pereira, 2004, Christodoulakis and Satchell, 2005). The focus of this chapter is sizable distortion in financial markets due to a crisis, i.e. heightened tail risk from the volatility dynamics of asset returns during a crisis and not small innovations, therefore the focus will remain primarily on the results from absolute returns and squared returns.

5.7 Discussion

With respect to the main research question that tests the degree of overlap/similarity between Black Swans in forex markets when using unconditional measures of volatility such as volatility proxies against the mean dynamics, the null is strongly rejected in all three of the proxies (when testing within returns of the volatility proxies as well as within the residuals of those proxies).

In the forex market, for two of the three volatility proxies (namely the residuals of absolute returns and residuals of squared returns), contain fewer Black Swans than the residuals that are derived from their respective counterparts. This result is clearly linked to the clustering effect evident in financial markets that has been filtered using the best-fit ARMA-APARCH model and therefore the probability of highly extreme returns is reduced substantially.

Another supplementary result is that the degree of decrease is higher from returns to residuals and much lower from volatility proxies to their respective residuals i.e. there is a 24% decrease in Black Swans from returns to residuals where as there is only a 0.73% and a 9% decrease in the number of Black Swans from the absolute returns and squared returns to their respective residuals. This signifies that there is a greater clustering effect in returns which is captured by the conditional heteroscedasticity models and that there is sparse negative correlation in volatility proxies to begin with. This is contrary to the evidence[18] found in

[18] The authors found significant positive autocorrelation within the absolute returns for long lags in the S&P stock market and proposed the use of the ARMA-APARCH model to model the conditional variance of absolute returns.

literature indicating that there is higher correlation in volatility proxies as compared to returns (Ding et al., 1993a, Taylor, 2008). However, it should be underlined that this is clearly because of differencing the series of the volatility proxies – in effect, already assuming there is either a unit root or at least a very high volatility persistence.

Log squared returns and residuals however present an antipodal trend with 31% more Black Swans in the residuals as opposed to the log squared returns. In anticipation of the results of the next chapter, these results are reciprocated in the equity markets as well further validating the rejection of the homogeneity of Black Swans in the presence of structural breaks and with the correction of conditional heteroscedasticity.

Scrutinizing the residuals of returns against the residuals of the volatility proxy, the emergent trend is that there are a greater frequency of Black Swans within the residuals of the volatility proxies in comparison to the returns i.e. there are 25% and 29% more Black Swans in the residuals of absolute returns and squared returns respectively in comparison to the residuals of the returns. While a possible explanation of this behavior could be that proxies tend of be noisy estimators, it could also signify latent volatility trends that are better captured by non-parametric volatility measures like proxies rather than the return-based measures. Once again, the log squared returns display a polar trend with 31% more Black Swans in the residuals of returns as compared to the residuals of the log squared returns.

Finally, examining the tails of the highly extreme returns once the clustering effect has been removed, reveal that the distribution of absolute residuals and squared residuals are skewed heavily to the right whereas the distribution of log squared residuals is skewed to the left.

With respect to the final question regarding the transformation of the Black Swans during the Global Financial Crisis when compared to the conceding and preceding segments, the trend is more gradual in comparison to the results obtained in the previous sections and remain mixed.

It is observed in the residuals of absolute and squared returns is that on average there are fewer Black Swans in crisis period as compared to the pre-crisis period which increases slightly post crisis within the residuals of the squared returns. The residuals of the log squared returns display a reverse trend with a higher ratio during the crisis period which declines in the consecutive segment.

A possible justification of these mixed results could be that the application of the ARMA-APARCH model averaged out most of the variability in the series, however, the model still rejects the hypothesis of homogeneity of Black Swans because the clusters differ in size irrespective of the presence of a dominant trend.

5.8 Conclusion

The empirical results of the chapter clearly indicate that extreme tail risk is not homogenous when it arises from mean and variance. In the returns, it was found that there was a significant difference in the frequency of Black Swans in the volatility proxies when compared to the frequency of Black Swans in the returns i.e. on average, in comparison to the returns, both absolute returns and squared returns depicted more Black Swans whereas log squared returns displayed less Black Swans.

A similar result is mirrored in the residuals of the returns when compared to the residuals of the proxies which were obtained using the best fit ARMA-APARCH model to control for the effect of stylized properties of financial returns such as volatility clustering. There is a significant difference in the frequency of Black Swans, i.e. on average, in comparison to the residuals of returns, residuals of absolute and squared returns have a higher number of Black Swans whereas residuals of log squared returns display an antagonistic trend.

From an investors point of view, these results signify the importance of including volatility from mean dynamics along with volatility dynamics to obtain a comprehensive picture of the overall tail risk present in the market during a crisis. It is also important for a market participant to be aware that regime shifts or structural breaks in the mean and/or variance structure of the returns could also lead to overly-inflated kurtosis and tail asymmetry measures if they are not accounted for.

By incorporating measures of structural breaks and obtaining inferences about tail risk from both mean and the variance of the returns, market participants can assess the true level of tail risk present in the market. This can be incorporated into asset pricing models and expected returns thereafter.

Chapter 6: Understanding the evolution of tail risk in the equity markets using unconditional volatility measures combined with the ARMA-APARCH model

6.1 Introduction

Understanding the underlying properties of a financial time series is pivotal to using the appropriate empirical models that incorporate those inherent characteristics in order to draw efficient inferences and make sound financial investment decisions. As a result, there are a multitude of financial models in the empirical literature (such as numerous variations of the auto-regressive conditional heteroscedasticity models, asset pricing models and extreme value theory models) that measure volatility in the stock markets by using innovations in returns as well as return proxies (such as absolute returns, squared returns, log squared returns and range). This is done to capture the cardinal conditions of financial markets returns such as heteroscedasticity, fat tails, negative skewness, etc.

While models that use innovations in returns to measure volatility remain popular in the financial literature, the debate continues to rage regarding the efficiency of those financial models that use volatility proxies to measure latent volatility of financial asset returns in Equity markets.

There is conflicting evidence on their suitability to measure extreme volatility in stock market when they are under stress. These contradictions are disconcerting, because models that use proxies could be used for practical applications by investors in decisions regarding timings of entering/leaving the markets, portfolio selection that matches investors risk preference with the appropriate return level and pricing of financial assets by deriving the premium based on variance estimates provided by the model (Forsberg and Bollerslev, 2002, Liu et al., 2012, Bee et al., 2016).

Therefore, while there are a myriad of papers that have focused on financial market volatility using traditional financial models in literature, the contribution of this chapter is noteworthy: it aims to model extreme tail risk arising from the variance of return distributions using the frequency of Black Swans measured via differenced volatility proxy measures. Specifically, this chapter will investigate extreme tail risk in equity markets by identifying black swans in the dynamics of volatility proxies and comparing them to those identified in the mean dynamics using a model specification that takes into account the possible presence of structural breaks in the mean and/or volatility dynamic of each of these series.

The rest of this chapter is organized as follows: Section II sets the context for the research with a literature review, section III provides the specific research questions this chapter seeks to answer, Section IV makes available the description of the data, and, section V presents the research methodology used to obtain results. The final segments of the papers are: section VI, which presents the empirical results on Black swans from innovations in the mean dynamics versus those from volatility dynamics that exist in the stock market, and, section VII discusses the implications of the empirical results for key market participants. Section VIII concludes.

6.2 Literature Review

Volatility itself is inherently unobservable, therefore the use of volatility proxies to gain understanding of the nature of volatility has become common practice in literature.

In this case, volatility proxies such as squared returns, absolute returns and/or log squared returns can be considered as a conditionally unbiased estimator of latent conditional variance and can potentially increase the efficiency of the financial models leading to superior parameter estimates.

However, some studies have found empirical evidence that certain volatility proxies such as squared returns and absolute returns are too noisy and highly persistent. As a result of this, these proxies have been arbitrated as inadequate estimators of latent volatility (Andersen and Bollerslev, 1998, Andersen et al., 2005, Hansen and Lunde, 2006, Patton, 2011).

Moreover, while volatility proxies such as absolute and squared returns became known to be adversely affected due to microstructure noise, it led to the popularization of range-based volatility estimators of which the Parkinson is first and most widely used (Parkinson, 1980) – captured by the simple expression $\left(\frac{1}{4\ \ln 2}\right)(\ln H_t - \ln L_t)^2$ wherein we take the difference between the Intraday High returns (H_t) and Intraday Low returns (L_t).

Consequently, a myriad of literature suggests that intra-day range is a more efficient method of estimating realized volatility because it captures intraday price fluctuations more accurately in comparison to squared or daily returns which depict low volatility if the closing price is similar to the opening price despite the presence of multiple extreme fluctuations during the trading day. For example, a study that allowed for structural breaks in the lagged log range of the S&P 500 data while using autoregressive conditional heteroscedasticity models found that the model is superior based on long range in-sample and out-sample forecasts (Brandt and Jones, 2006).

An additional reason, is that the distribution of log range, conditional on volatility, is known to be approximately Gaussian (Andersen and Bollerslev, 1998, Alizadeh et al., 2002).

Finally, the integrated variance of estimates from range based proxies are more precise when compared to the estimates based on absolute and squared returns (Christensen and Podolskij, 2007, Martens and Van Dijk, 2007).

The model specification of this chapter will combine every volatility proxy, including range, with conditional volatility models, specifically the best fit ARMA-APARCH model as these most aptly capture the well-known properties that time series depict such as volatility clustering, highly negative correlation between returns as well as long memory.

Another important consideration in extreme volatility modelling is to test for market efficiency in volatility proxies of stock returns, which is the rate of decay of arbitrage with the arrival of new information. If this decay is rapid, it indicates that the market has a short memory and therefore follows the martingale process.

However, if that is not the case then there is evidence of long-term correlation in volatility of volatility, i.e. there is a persistent statistical dependence between distant price observations of the time series indicating that the volatility at long lags have long memory properties, therefore, providing evidence against markets being efficient. In these cases, there is evidence of the presence of unit roots, near unit roots or long-term memory in the data set.

Consequently, an audit of the empirical literature revealed various studies that have found evidence against market efficiency by proving the manifestation of unit roots, near unit roots and/or long memory in the data. A study which established the presence of unit roots in absolute and squared returns in every equity market included in the data set (Assaf, 2016), (Cavalcante and Assaf, 2004) for the Brazilian stock market, (Kiliĉ, 2004) for the Istanbul equity market, (Kang and Yoon, 2007) for the Korean stock market). A more comprehensive study that tested the hypothesis of long range memory in a wider set of volatility proxies (namely absolute returns, squared returns, log squared returns, and range), in the equity markets of Portugal, Ireland, Italy, Greece and Spain, used the wavelet approach combined with the GARCH family of models finding significant asymmetric long range properties in the volatility of each of the equity markets (Kumar and Maheswaran, 2013).

The strengthening evidence on the presence of unit roots in volatilities of asset returns fuelled an interest in the origins and potential causes of these long memory properties, the one of interest is the possibility that these processes were non-linear and that was being wrongly perceived as unit root non-stationarity.

Here the literature divides into two strands: One that assume that non-linearity is being confused for non-stationarity (see (Diebold and Inoue, 2001, Davidson and

Sibbertsen, 2005) for further evidence and see (Granger and Hyung, 2004) for evidence that absolute returns of the S&P equity market display long memory persistence which could be explained by undetected structural breaks) and the other examines the possibility that process itself could be non-linear while displaying long memory (see a study that furthered that conclusion by examining the possibility that absolute returns in foreign exchange markets[19] can continue to display long memory properties while containing non linearity in the subsamples discovered that only three of the seven currencies displayed pure memory, the remaining appeared to display marginal nonlinearity along with long memory (Baillie and Kapetanios, 2007).

In conclusion, taking into consideration the multitude of evidence against market efficiency in volatility proxies, this chapter will be using first-order differences to account for the potential presence of unit roots, near unit roots and/or long memory in the data.

6.3 Research Question

Previous studies have focused on singular and/or at most binary measures of extreme volatility within the GARCH framework which leads to the need for a comprehensive and thorough analysis of the nature of Black Swans arising from mean and volatility dynamics demonstrated through the behaviour of global financial markets when undergoing financial collapse.

While chapters 3 and 4 have studied the probability of highly extreme returns occurring in the equity and forex markets respectively using returns and residuals from the best fit ARMA-APARCH model and tested the homogeneity of Black Swans when structural breaks are included in the model, this chapter studies the nature of Black Swans using unconditional volatility measures such as differenced volatility proxies when markets are undergoing extreme financial distress in trying to understand the difference in extreme events affecting mean dynamics versus those from the volatility dynamics.

[19] Daily absolute returns on seven major currencies vis-a-vis the U.S. dollar were studied, namely Canada, France, Germany, Italy, Japan, U.K., Argentina and Brazil.

It further tests the degree of similarity or overlap between Black Swans in mean dynamics versus volatility dynamics.

Therefore, it proposes to use absolute returns, squared returns, log squared returns and intraday range as volatility proxies to measure the behaviour of extreme values known as Black Swans overall and during periods of economic/financial crisis. It proposes to riposte to the following research questions:

The two key research questions embedded within this study are:

- Are highly improbable events from the volatility dynamics in equity markets the same as the ones that arise from the mean dynamics? If not, then what is the degree of similarity or lack thereof between these extreme events resulting from a change in volatility dynamics versus mean dynamics?

- What is the evolution of the occurrence of these extreme events resulting from the volatility dynamics of the forex market during the Global Financial Crisis of 2008?

These questions will be answered using two models: The first model will compare the evolution of Black Swans from mean dynamics (using conditional measures like returns) and compare the degree of similarity against Black Swans identified in the volatility dynamics (using proxy-based measures) while recognizing multiple structural breaks in the underlying process of both data sets.

The second model will further improve estimates by using the residuals of both return-based measures and proxy-based measures obtained from the best fit ARMA-APARCH model to correct conditional heteroscedasticity and/or correlations between the values that might lead improper inferences if there is a clustering effect.

The second model will additionally study the degree of overlay of the Black Swans to extreme values from volatility dynamics during the Global Financial Crisis to understand the evolution of Black Swans in forex markets while comparing the results to the equity markets which are studied in the chapter 5.

6.4 Methodology

The methodological approach adopted here is very similar to the one described in Chapter 5 and therefore I refer the reader to there for more details. Once again, I empirically test the behaviour of Black Swans in financial – here equity - markets and compare those captured in the returns process with those captured in the processes of volatility proxies namely the absolute returns, squared returns, range, and log squared returns.

However, it is worth noting that an interesting expansion that equity markets make available is that it is possible to also construct range-based estimators of volatility. In other words, unlike the forex markets, where our volatility proxies were based on the information set encompassed in the daily returns series, since the volatility proxies were effectively (non-linear) transformations of the return series, for equity markets we can also search for Black Swans in information that lies in the intraday returns.

Indeed, range-based volatility estimators are found to be highly efficient, in stochastic volatility models that estimate realized variance, when compared to return-based measures as well as volatility proxies such as squared returns, as they are known to being approximately Gaussian as well as being impervious to micro-structure noise (Alizadeh et al., 2002, Andersen and Bollerslev, 1998, Brandt and Diebold, 2003, Martens and Van Dijk, 2007, Fuertes et al., 2009). In this chapter I use the Parkinson range, the use of which is extant in literature as it is known to be an unbiased estimator of daily volatility which is supposed to be at least five time more efficient than squared returns (Parkinson, 1980) calculated as:

$$\text{Parkinson's range} = \left(\frac{1}{4\ln 2}\right)(\ln H_t - \ln L_t)^2 \qquad (27)$$

6.5 Data

Descriptive Statistics

The following table 31 presents the descriptive statistics of the differenced absolute returns for the equity markets.

Table 31 – Summary statistics of differenced Absolute returns (Equity market)

	Australia	Austria	Belgium	Canada	Chile	Czech
No. of obs.	6254	7922	9492	8949	6882	5771
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.84%	1.18%	0.87%	0.93%	0.95%	1.19%
Skewness	0.11	-0.08	-0.07	0.26	-0.07	0.60
Kurtosis	5.22	6.81	12.63	12.83	7.15	15.37
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
No. of obs.	6903	5208	7664	7530	13405	7209
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	1.02%	1.39%	1.44%	1.27%	1.09%	1.71%
Skewness	0.04	0.05	0.05	-0.11	0.07	0.15
Kurtosis	6.15	36.61	11.95	5.91	9.05	8.30
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
No. of obs.	6621	6100	8706	7585	4796	13405
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	1.42%	2.23%	1.18%	1.34%	1.40%	1.15%
Skewness	0.15	0.92	0.10	0.19	-0.09	0.15
Kurtosis	9.00	1862.95	9.66	6.97	3.98	11.09
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
No. of obs.	10797	4533	7403	8708	4014	7664
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	1.41%	1.50%	1.38%	1.20%	0.60%	1.32%
Skewness	0.14	-1.20	0.10	0.11	0.20	0.20
Kurtosis	8.98	68.29	7.23	8.40	7.16	12.00
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
No. of obs.	5763	6100	5917	2382	7401	7663
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	1.64%	1.02%	1.53%	1.51%	1.25%	1.27%
Skewness	-0.05	-0.17	-0.60	0.23	0.05	-0.08
Kurtosis	6.17	8.26	33.37	411.27	6.32	5.03
	Switzerland	**Turkey**	**UK**	**USA**		
No. of obs.	7013	7141	9730	13405		
Mean	0.00%	0.00%	0.00%	0.00%		
Standard Dev.	1.00%	2.27%	1.07%	0.94%		
Skewness	0.22	0.06	0.10	0.18		
Kurtosis	7.70	5.69	33.11	27.35		

Clearly, there is ample evidence of leptokurtosis and skewness in the data. The same holds true also for the squared returns – as depicted in the following table 32.

Table 32 – Summary statistics of differenced squared returns (Equity market)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech**
No. of obs.	6254	7922	9492	8949	6882	5771
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.03%	0.07%	0.04%	0.05%	0.04%	0.08%
Skewness	2.20	-0.59	-1.05	2.04	-0.98	6.45
Kurtosis	117.32	84.92	264.02	229.57	227.34	347.16
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
No. of obs.	6903	5208	7664	7530	13405	7209
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.05%	0.15%	0.11%	0.07%	0.06%	0.13%
Skewness	-0.61	0.15	0.15	-0.68	0.73	0.48
Kurtosis	214.59	397.75	301.26	72.02	200.47	150.63
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
No. of obs.	6621	6100	8706	7585	4796	13405
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.11%	2.19%	0.07%	0.08%	0.07%	0.07%
Skewness	2.14	0.02	1.68	0.37	-0.71	2.42
Kurtosis	189.25	2967.73	172.70	87.16	50.88	350.32
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
No. of obs.	10797	4533	7403	8708	4014	7664
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.09%	0.23%	0.09%	0.07%	0.02%	0.11%
Skewness	0.26	-12.47	1.31	0.91	0.56	4.20
Kurtosis	252.29	1709.68	107.70	106.52	139.70	937.53
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
No. of obs.	5763	6100	5917	2382	7401	7663
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.11%	0.05%	0.18%	0.48%	0.07%	0.07%
Skewness	-0.25	-1.10	-3.60	-0.29	-0.13	-0.62
Kurtosis	99.37	153.56	893.12	1164.17	145.33	61.27
	Switzerland	**Turkey**	**UK**	**USA**		
No. of obs.	7013	7141	9730	13405		
Mean	0.00%	0.00%	0.00%	0.00%		
Standard Dev.	0.05%	0.21%	0.08%	0.07%		
Skewness	0.99	0.40	0.29	0.03		
Kurtosis	138.34	62.67	830.71	3132.34		

Below is the descriptive statistics of the differenced log squared returns.

Table 33 – Summary statistics of differenced Log squared returns (Equity market)

	Australia	Austria	Belgium	Canada	Chile	Czech
No. of obs.	5909	7187	8729	8312	6298	5232
Mean	-0.47%	-2.93%	-2.00%	-2.05%	-1.31%	-1.61%
Standard Dev.	314.40%	318.99%	310.89%	310.70%	314.24%	293.32%
Skewness	-0.02	-0.02	0.06	0.02	-0.01	0.03
Kurtosis	0.97	1.78	1.73	1.30	1.77	0.89
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
No. of obs.	6407	4877	7165	7131	12534	6503
Mean	-0.99%	-1.48%	-1.44%	-1.47%	-2.27%	-3.75%
Standard Dev.	301.02%	306.84%	318.55%	329.92%	310.95%	320.72%
Skewness	0.03	0.02	0.01	0.04	-0.01	-0.02
Kurtosis	1.07	1.50	2.23	1.43	1.25	1.49
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
No. of obs.	6127	5382	7190	6768	4575	12129
Mean	-2.95%	-0.91%	-1.81%	-4.34%	-1.45%	-2.14%
Standard Dev.	315.34%	325.69%	320.04%	310.35%	315.50%	319.87%
Skewness	-0.05	0.06	-0.04	-0.01	0.01	-0.03
Kurtosis	1.70	1.25	2.02	1.21	1.17	1.40
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
No. of obs.	9724	4243	6829	8169	3764	7158
Mean	-3.54%	-2.19%	-2.60%	-1.69%	-0.80%	-1.28%
Standard Dev.	320.38%	311.38%	316.71%	310.35%	309.15%	307.61%
Skewness	-0.03	-0.01	-0.01	0.01	-0.04	-0.02
Kurtosis	1.03	0.96	1.16	1.05	1.75	0.90
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
No. of obs.	5287	5742	4450	2192	6933	7133
Mean	-2.73%	-3.13%	0.19%	-1.37%	-1.95%	-2.71%
Standard Dev.	325.14%	318.10%	348.72%	313.82%	318.08%	314.89%
Skewness	0.01	-0.05	0.02	0.07	-0.01	-0.01
Kurtosis	1.44	1.93	0.59	1.75	1.41	1.55
	Switzerland	**Turkey**	**UK**	**USA**		
No. of obs.	6576	6627	7728	12389		
Mean	-3.40%	-3.46%	-1.90%	-1.42%		
Standard Dev.	313.24%	305.42%	310.53%	320.46%		
Skewness	-0.05	0.04	-0.01	0.01		
Kurtosis	1.50	1.25	1.27	1.25		

Below is the descriptive statistics of the Parkinson's range for the returns.

Table 34 – Summary statistics of differenced Log squared returns (Equity market)

	Australia	Austria	Belgium	Canada	Chile	Czech

Mean	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
Standard Dev.	0.01%	0.03%	0.02%	0.02%	0.08%	0.03%
Skewness	8.88	8.80	9.55	22.14	46.25	12.53
Kurtosis	122.26	108.45	171.39	828.22	2229.92	259.31
	Denmark	**Finland**	**France**	**Germany**	**Greece**	**Iceland**
Mean	0.01%	0.03%	0.01%	0.01%	0.02%	0.01%
Standard Dev.	0.06%	0.11%	0.02%	0.03%	0.05%	0.06%
Skewness	18.26	14.46	6.19	8.51	20.91	18.26
Kurtosis	396.88	249.73	58.16	137.58	826.17	396.88
	Hungary	**Ireland**	**Israel**	**Italy**	**Japan**	**Luxembourg**
Mean	0.00%	0.02%	0.02%	0.07%	0.02%	0.01%
Standard Dev.	0.01%	0.03%	0.04%	0.48%	0.03%	0.03%
Skewness	3.88	13.99	26.45	10.51	4.89	14.76
Kurtosis	18.99	352.58	1115.33	130.66	36.12	373.90
	Mexico	**Netherlands**	**Norway**	**Portugal**	**Korea**	**Spain**
Mean	0.03%	0.01%	0.01%	0.02%	0.01%	0.02%
Standard Dev.	0.17%	0.04%	0.02%	0.04%	0.02%	0.03%
Skewness	15.05	17.63	7.37	6.98	6.95	7.91
Kurtosis	254.54	599.12	84.97	64.86	75.10	132.23
	Sweden	**Switzerland**	**Turkey**	**UK**	**USA**	
Mean	0.01%	0.02%	0.01%	0.02%	0.01%	
Standard Dev.	0.02%	0.03%	0.02%	0.03%	0.02%	
Skewness	8.03	8.11	15.69	5.24	11.66	
Kurtosis	129.60	108.71	442.73	39.95	214.15	

For comparative purposes, I provide the descriptive statistics of the un-differenced volatility proxies in Appendix 32, 33 and 34.

6.6 Empirical Results

The following section will begin by presenting model 1 outcomes and illustrate the degree of overlap of Black Swans between returns of 35 stock markets and the three proxies individually (with the inclusion of structural breaks in all data sets) along with other properties such as skewness and the final section of model 1 will conclude by summarizing the degree of overlap overall.

The second section of the empirical results will render the results of model 2 and illustrate the degree of overlap of Black Swans between residuals of returns and residuals of the volatility proxies along with the evolution of volatility of volatility during the Global Financial Crisis of 2008 as the focal point.

6.6.1 Model 1 Outcomes

Summary of the innovation in extreme variance between returns and differenced absolute returns of the daily stock exchange market

The following table 35 presents the frequency of Black Swans in daily stock returns of 34 countries and compares it to the frequency of Black Swans in differenced absolute returns. Both data sets, returns and absolute returns incorporate multiple structural breaks in the underlying mean and/or variance dynamics. The table also displays the skewness of both distributions by comparing and contrasting the frequency of Black Swans in the left and right tails (see Appendix 25 for detailed results).

Table 35 – Difference in the frequency of Black Swans between returns and absolute returns (Equity market)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	-69.31%	-29.27%	-102%	-21%	-14%
Left tail	34.83%	20.66%	-140%	17.59%	-9.53%
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
Right tail	-41%	-40.55%	0.00%	-25.7%	5.26%
Left tail	17.80%	21.83%	-13.06%	-8.70%	4.17%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Right tail	4.51%	-5.41%	-19.89%	-6.60%	-27.90%
Left tail	-1.38%	7.55%	-17.52%	14.31%	22.01%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	-39.73%	-38.80%	-21.50%	0.05%	-27.19%
Left tail	26.57%	14.84%	5.41%	17.44%	32.28%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Right tail	-1.80%	-47.96%	-42.70%	-28%	-44.47%
Left tail	14.17%	34.23%	-9.53%	35.28%	31.24%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Right tail	-42.90%	-7.28%	0.00%	-37.9%	-18.60%
Left tail	2.11%	29.78%	31.85%	6.78%	2.15%
	Switzerland	**Turkey**	**UK**	**USA**	
Right tail	-31.37%	-32.54%	1.38%	-7.23%	
Left tail	20.76%	-2.06%	-23.20%	2.56%	

The empirical findings of the table 35 can be summarized into three key results: first that there are a higher frequency of Black Swans in the absolute returns in comparison to the returns, second, there are comparatively more negative Black Swans in the absolute returns as compared to returns and more positive Black Swans in the returns as compared to absolute returns and finally that the returns distribution seems skewed to the right with more positive Black Swans where the absolute returns seems more normally distributed with approximately the same number of Black Swans in the left and right tails.

With respect to the first result, on average there are 5% more Black Swans in absolute returns (1.35% of the trading days were identified as Black Swans) as compared to the returns (1.29% of the trading days were classified as Black Swans). While this trend is consistent in 24 of the 34 countries, the remaining, namely, France, Germany, Greece, Iceland, Korea, Luxembourg, Mexico, Norway, Slovakia, and Slovenia have a higher frequency of Black Swans in returns as opposed to absolute returns.

Regarding the second result, examining the frequency of Black Swans in the tails individually for both data sets shows that while absolute returns have 25% more Black Swans in the left tail (0.73% of the trading days in absolute returns were Black Swans as compared to returns wherein 0.57% of the trading days were Black Swans); returns have 7% more Black Swans in the right tail.

Finally, the distribution of Black Swans for returns appears to be skewed to the right (56% of Black Swans appear on the right) whereas the distribution of Black Swans for absolute returns appears normal with an equivalent number of Black Swans on both tails.

Summary of the innovation in extreme variance between returns and squared returns of the daily stock exchange market

The following table 36 presents the frequency of Black Swans in the daily equity returns of 34 benchmark stock indices and parallels it to the frequency of Black Swans in the differenced squared returns while taking into account latent Non-linearities in the underlying mean and/or variance structure in both data sets. It also delves into the frequency of Black Swans in the left and right tails of both data sets comparing their ability to capture extreme values (see Appendix 26 for detailed results).

Table 36 – Difference in the frequency of Black Swans in returns versus squared returns (Equity market)

	Australia	Austria	Belgium	Canada	Chile
Right tail	-53.52%	-7.70%	0.00%	-3.25%	-26.20%
Left tail	-124.43%	-43.18%	-6.45%	-22.8%	-35.40%
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
Right tail	4.17%	-37.22%	26.47%	-31%	-61.90%
Left tail	-43%	-77.77%	27.33%	-31.2%	-79.08%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Right tail	-34.80%	-30%	-21.03%	-2.20%	-5.99%
Left tail	-49.90%	-27%	-14.84%	4.65%	-34.10%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	-12.52%	-50.40%	21%	-24.7%	10.92%
Left tail	-49.11%	-96.80%	-3.90%	-16.3%	-59.47%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Right tail	-36%	-23.18%	-22.30%	-7.80%	-41.36%
Left tail	-25.50%	-82.51%	-51.10%	-77%	-77.01%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Right tail	-4.08%	25.80%	20.07%	-24.5%	-56%
Left tail	-55.96%	3.85%	-13.35%	-68%	-64%
	Switzerland	**Turkey**	**UK**	**USA**	
Right tail	-21.06%	-33.35%	-15.60%	21.06%	
Left tail	-73.19%	-49.53%	-11.60%	7.80%	

The empirical results have identified three key manifestations of Black Swans: first, on average there are a higher number of Black Swans in the squared daily returns of equity markets; second that squared returns have a higher frequency of negative Black Swans that are not being captured by the returns series themselves and finally the returns series is more skewed in comparison to squared returns that have approximately the same number of Black Swans in the left and right tails.

Regarding the first result, there are on average 27% more Black Swans in the squared returns (1.71% of the trading days have Black Swans) in comparison to the returns (1.29% of the trading days have Black Swans) themselves. These results are consistent with the findings of Ding et al. (1993a) and Taylor (2008) that there are higher autocorrelation between squared returns as opposed to the returns themselves and the higher frequency of Black Swans in the squared returns can be rendered to that. However, looking at each country individually, 28 of the 34 countries show this trend and the remaining 6 countries, namely Estonia, Japan, Slovakia, Spain, USA and Iceland exhibit an antagonistic orientation i.e. there are a higher frequency of Black Swans in the returns as opposed to the squared returns.

With respect to the second result, the squared returns possess 42% more Black Swans in the left tails (0.85% of the trading days in squared returns and 0.57% of the trading days in returns) and 16% more Black Swans in the right tails as compared to the returns (0.86% of trading days in squared returns and 0.72% of trading days in returns). While serial negative volatility clustering is a stylized fact of financial returns, it is magnified in squared returns and hence the higher frequency of Black Swans in squared returns as compared to the returns alone.

Finally, concerning the final result, while the returns series seems skewed to the right with a slightly higher percentage of Black Swans in the right tails, the squared returns seem to be more normally distributed with an equal number of Black Swans in both tails. The highest percentage of difference in the frequency of Black Swans in the left tails capture by both data sets was in the Australian Stock market where returns had 0.24% of negative Black Swans and squared returns had 0.83% of negative Black Swans which is difference of 124%.

Summary of the innovation in extreme variance between returns and log squared returns of the daily stock exchange market

The following table 37 presents the frequency of Black Swans in the daily equity returns of 34 benchmark stock indices and counterparts it to the frequency of Black Swans in the differenced log squared returns while taking into account latent Non-linearities in the underlying mean and/or variance structure in both data sets. It also delves into the frequency of Black Swans in the left and right tails of both data sets comparing their ability to capture extreme values (see Appendix 27 for detailed results).

Table 37 – Difference in the frequency of Black Swans in returns versus log squared returns (Equity market)

	Australia	Austria	Belgium	Canada	Chile
Right tail	29.16%	60.92%	12.03%	41.57%	10.37%
Left tail	-48.42%	20.37%	5.93%	3.86%	16.62%
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
Right tail	79.80%	39.15%	43.74%	-23.4%	5.32%
Left tail	35.87%	-1.39%	58.49%	-18.2%	-5.44%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Right tail	-1.00%	29.30%	40.53%	18.49%	46.60%
Left tail	0.92%	30.20%	36.87%	29.17%	22.30%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	57.92%	27.60%	52.20%	54.93%	66.21%
Left tail	-8.93%	-9.85%	-1.66%	45.81%	-23.80%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Right tail	39.30%	48.13%	-28.70%	80.20%	49.19%
Left tail	18.90%	-1.13%	-49.20%	12.40%	8.81%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Right tail	34.50%	160%	-62.97%	71.30%	-0.57%
Left tail	-25.46%	120%	-77.63%	4.29%	10.70%
	Switzerland	**Turkey**	**UK**	**USA**	
Right tail	66.06%	30.00%	44.80%	28.33%	
Left tail	-25.54%	25.67%	39.70%	27.79%	

The empirical results comparing the frequency of Black Swans in returns and log squared returns exhibit exceedingly antipodean trends in comparison to the previous two volatility measures of absolute returns and squared returns i.e. there are a higher frequency of Black Swans in returns as opposed to the different log squared returns. The second inference that can be drawn about these results is that while it is more pronounced in forex market, nevertheless it holds in majority of the stock markets as well.

With respect to the first result, there are 24% more Black Swans in returns (1.29% of the trading days were identified as Black Swans) in comparison to the log squared returns (where only 1% of the trading days were Black Swans). This trend is seen in 29 out of the 34 countries in the data set. The exceptions are Australia, Germany, Finland, New Zealand, and Slovenia, where returns, on average, have 26% less Black Swans in comparison to log squared returns.

Summary of the innovation in extreme variance between returns and range of the daily stock exchange market

The following table 38 presents the summary of the frequency of Black Swans in daily returns and compares them to the frequency of Black Swans discovered in the differenced daily range of the equity market of 29 countries. It also analyses the frequency of these clusters in the tails of the distributions of both data sets, effectively providing results on the skewness of the data given that there exists a provision for latent Non-linearities in the mean and/or variance dynamic of the series resulting from potential and multiple structural breaks (see Appendix 28 for detailed results).

Table 38 – Difference in the frequency of Black Swans in returns versus range (Equity market)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	-127%	-19.78%	-58%	-16.90%	29.90%
Left tail	-32.67%	14.31%	-24%	8.60%	40.50%
	Czech	**Denmark**	**Finland**	**France**	**Germany**
Right tail	-47.49%	-30.20%	-41.87%	-82%	-52%
Left tail	11.11%	-2.33%	-32.66%	-52%	-42.90%
	Greece	**Iceland**	**Hungary**	**Ireland**	**Israel**
Right tail	-27%	-71.23%	8.84%	-15.75%	-109%
Left tail	-29%	-46.67%	12.49%	12.87%	-21%
	Italy	**Japan**	**Luxembourg**	**Mexico**	**Netherlands**
Right tail	-115%	-16%	-112.98%	-11.26%	-77.49%
Left tail	-47.20%	-0.40%	-16.74%	-18.42%	-6.86%
	Norway	**Portugal**	**Korea**	**Spain**	**Sweden**
Right tail	-92%	-89.10%	-22%	-55.95%	-41.94%
Left tail	-17%	-4.98%	-23%	3.62%	-49.40%
	Switzerland	**Turkey**	**U.K.**	**U.S.A.**	
Right tail	-26.05%	-59%	1.74%	-14%	
Left tail	23.83%	-42%	3.24%	-8.90%	

The results can be summarized into two key trends: first, there are a greater number of Black Swans' clusters in the range data set as opposed to the returns, second the data becomes more normalized in the range while it is skewed to the left for the returns.

Regarding the first result, on average, there are 29% fewer clusters of Black Swans in returns as compared to the range. This trend is seen in 25 of the 29 countries in the data set with the exception of Hungary, Switzerland, U.K., and Chile which had 13% more cluster of Black Swans in the returns as opposed to the range indicating that extreme tail risk arising from variance in returns is lower as compared to that arising from the mean.

With respect to the second result, analysing the tails independently, the distribution of the extreme volatility is skewed to the left in the returns i.e. with 56% of the total clusters existing on the left tail whereas the distribution is more normalized for the range i.e. 48% on the left tail and 53% of the right tail.

These results are similar to those of absolute and squared returns.

Degree of overlap between residuals of returns (mean dynamics) and the residuals of proxies (volatility dynamics) of the equity market

The following table presents the degree of similarity or overlap between the extreme events from mean dynamics (residuals of returns) versus those from the volatility dynamics (residuals of volatility proxies) and while there is a significant level of overlap across the data sets, there is clear evidence that there is a degree of dissimilarity between the two.

	Returns
Returns	100%
Differenced Absolute returns	85.7%*
Differenced Squared returns	54.3%*
Differenced Log Squared returns	78.4%**
Range	80.7%**

*Returns have higher frequency of Black Swans ** Proxies have higher frequency of Black Swans

This table presents evidence that there is a difference in the extreme events from mean dynamics and volatility dynamics when heteroscedasticity is accounted for.

Accounting for volatility from mean dynamics plays an essential role in asset pricing and hedging strategies and it is well known in the literature that a change in the volatility (resulting from mean) is accounted for in traditional models however volatility reflected in returns by itself does not account for the extreme events resulting from the volatility of volatility (resulting from volatility dynamics). This disregard of changes in volatility of volatility could let to potential misspecification of risk faced by investors and market participants and hence possesses the potential to be an integral indicator of overall financial risk.

With respect to log-squared returns, majority of scholarly literature tends to exclude it as a volatility proxy because it if the asset returns are close to zero, then transforming it to log-squared returns yields a highly negative number; if it is zero then the transformation is undefined. Such an extreme negative numbers tends to distort overall model estimates (Cavalcante and Assaf, 2004, Pereira, 2004, Christodoulakis and Satchell, 2005). The focus of this chapter is sizable distortion in financial markets due to a crisis, i.e. heightened tail risk from the volatility dynamics of asset returns during a crisis and not small innovations, therefore the focus will remain primarily on the results from absolute returns and squared returns.

6.6.2 Model 2 Outcomes

The following section of empirical results have been obtained from the application of model 2 wherein returns along with the volatility proxies have been filtered through the best fit ARMA-APARCH model in order to obtain residuals that have been corrected for conditional heteroscedasticity to understand the volatility of volatility in financial markets. Both returns and volatility proxies are regardful of multiple structural breaks in the underlying mean and/or variance dynamics of the returns process.

Summary of the innovation in extreme variance between residuals of returns and differenced absolute returns of the daily stock exchange market from best fit ARMA-APARCH model

The following table 39 summarizes the frequency of Black Swans in residual and differenced absolute residuals while accounting of latent Non-linearities in the underlying structure of the asset returns (see Appendix 29 for detailed results).

Table 39 – Difference in the frequency of Black Swans in residuals of returns versus residuals of absolute returns (Equity market)

Difference in the frequency of Black Swans in residuals of returns vs residuals of absolute returns	**Australia**	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	-236.80%	-154.11%	-144%	-167.5%	-113%
Left tail	321.78%	393.12%	415.80%	405.97%	346.50%
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
Right tail	-169.40%	-136.40%	-58.84%	-92.90%	-173.16%
Left tail	245.59%	343.34%	253.84%	313.50%	352.52%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Right tail	-120%	-89.22%	-120.10%	-83.71%	-98.10%
Left tail	404.30%	380.61%	376.07%	363.68%	418.90%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	-131.03%	-197%	-128.60%	-120.9%	-66.03%
Left tail	378.33%	54.53%	352.60%	313.49%	207.83%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Right tail	-147%	-188.82%	-176%	-148.3%	-174%
Left tail	299.40%	376.01%	257.90%	355.44%	313.46%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Right tail	-129.15%	-92.20%	-20.19%	-188%	-126.90%
Left tail	325.76%	418.90%	256.37%	371.30%	368.80%
	Switzerland	**Turkey**	**UK**	**USA**	
Right tail	-186.20%	-112.05%	-98.40%	-147%	
Left tail	358.24%	373.70%	385%	343.32%	

The empirical results of table 39 can be summarized into the following key trends: first, there are significantly fewer Black Swans in the residuals and absolute residuals compared to returns and absolute returns respectively; second that there is a higher frequency of Black Swans in differenced absolute residuals in comparison to the residuals and finally although there are higher Black Swans in absolute residuals overall, most of them are on the right tail, and negative Black Swans are occurring in the residuals.

With respect to the first result, residuals have 36% fewer Black Swans in comparison to returns and absolute residuals have approximately 8% fewer Black Swans as compared to the absolute returns. This result is expected since conditional heteroscedasticity models were used and therefore the removal of volatility clustering or dependence between returns has reduced the number of Black Swans overall. A corresponding conclusion that can be drawn from these results is that while there is serial correlation between returns and absolute returns, a counter conclusion is that it is lower within absolute returns in comparison to returns which could be a result of ARMA-APARCH models not being able to adequately capture the conditional heteroscedasticity between absolute returns due to divergent factors affecting extreme volatility.

With regards to the second result, comparing the frequency of Black Swans between residuals and absolute residuals there are 34% more Black Swans in the latter (1.25% of the trading days were Black Swans) than the former (0.90% of the trading days were identified as Black Swans). This result was seen in 32 out of the 34 countries in the data set with the exception of Slovenia and Luxembourg.

However, while the majority of Black Swans (99%) in absolute residuals occur on the right tail with very few on the left tail; an overwhelming number of negative Black Swans exist in residuals of returns (0.56% of the trading days) in comparison to residuals of absolute returns (0.03% of the trading days).

Summary of the ratio of Black Swans to extreme values in the residuals of differenced *Absolute returns obtained from the best fit ARMA-APARCH model*

The results so far have displayed the consistent trend of fewer Black Swans to extreme values in residuals of absolute returns in comparison to the residuals of returns of the stock market for 34 countries within the data set. This section will examine the behaviour of the volatility of volatility in the residuals of differenced absolute returns obtained using the best-fit ARMA-APARCH model by scrutinizing the change in the ratio of Black Swans to extreme values in segments before, during and after the Global Financial Crisis of 2008 with the central time component containing the fall of the Lehman Brothers. The dates of the segments examined are:

Table 40 – Structural breaks identified in residuals of absolute forex returns during, ex-ante and ex-post the GFC (2008)

Country	**Australia**	**Austria**	**Belgium**	**Canada**
Pre-Crisis	29/10/2001	08/10/1992	26/07/2007	29/10/1997
GFC	27/07/2007	30/07/2007	17/01/2008	21/08/2009
Post-Crisis	21/07/2009	09/11/2009	26/05/2009	06/01/2012
Country	**Chile**	**Czech**	**Denmark**	**Estonia**
Pre-Crisis	12/06/1998	07/04/1994	14/07/1997	04/06/1996
GFC	18/05/2007	24/06/2006	10/08/2007	22/10/2008
Post-Crisis	05/12/2011	15/06/2010	02/07/2009	05/12/2011
Country	**France**	**Germany**	**Greece**	**Hungary**
Pre-Crisis	15/04/2003	19/06/2003	27/09/2001	09/07/1993
GFC	17/01/2008	17/01/2008	25/06/2008	06/04/2005
Post-Crisis	18/05/2009	20/07/2009	16/10/2014	25/01/2012
Country	**Iceland**	**Ireland**	**Israel**	**Italy**
Pre-Crisis	26/08/2004	11/02/1988	05/04/1995	10/04/2003
GFC	12/12/2008	26/07/2007	23/04/2005	08/09/2008
Post-Crisis	08/03/2011	13/07/2010	21/09/2009	26/05/2009
Country	**Korea**	**Luxembourg**	**Mexico**	**Netherlands**
Pre-Crisis	14/03/1986	28/03/2003	09/01/2001	10/07/2003
GFC	02/05/2006	11/05/2007	19/10/2007	17/01/2008
Post-Crisis	23/07/2009	05/08/2010	28/07/2009	20/07/2009
Country	**New Zealand**	**Norway**	**Poland**	**Portugal**
Pre-Crisis	01/01/2001	17/12/1992	08/06/1995	28/02/2003
GFC	09/01/2008	16/05/2006	08/02/2005	11/01/2008
Post-Crisis	27/08/2009	06/08/2009	31/05/2010	11/12/2008
Country	**Slovenia**	**Spain**	**Sweden**	**Switzerland**
Pre-Crisis	04/04/2007	08/01/1988	13/04/2004	28/06/2004
GFC	20/11/2012	12/04/2006	28/07/2008	29/07/2008
Post-Crisis	20/06/2014	19/01/2009	08/07/2010	09/04/2010
Country	**Turkey**	**UK**	**USA**	**Finland**
Pre-Crisis	30/10/1998	13/02/1989	31/03/1998	22/07/2003
GFC	19/04/2006	19/07/2005	21/10/2006	27/07/2007
Post-Crisis	26/05/2010	21/07/2010	21/07/2010	21/07/2009
Country	**Japan**	**Slovakia**		
Pre-Crisis	23/02/1990	12/02/2003		
GFC	08/01/2008	11/09/2008		
Post-Crisis	22/05/2009	01/07/2010		

The results of this section will examine the homogeneity of Black Swans in the unconditional measure of extreme volatility during a crisis after negative serial correlation between the data points have been taken care of.

Table 41 - Difference in the ratio of Black Swans to extreme values in residuals of absolute returns ex-ante and ex-post the GFC (2008)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Pre-Crisis	46.67%	78.21%	59.26%	59.68%	50.00%
Crisis	33.33%	33.33%	50.00%	57.14%	53.23%
Difference	*-33.66%*	*-85.29%*	*-16.99%*	*-4.35%*	*6.26%*
Post Crisis	42.86%	37.50%	62.50%	62.50%	50.00%
Difference	*25.15%*	*11.79%*	*22.31%*	*8.97%*	*-6.26%*
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
Pre-Crisis	86.36%	57.41%	77.14%	77.27%	57.69%
Crisis	68.75%	40.00%	62.50%	58.33%	12.50%
Difference	*-22.80%*	*-36.13%*	*-21.05%*	*-28.12%*	*-152.94%*
Post Crisis	70.00%	66.67%	68.75%	37.50%	61.11%
Difference	*1.80%*	*51.09%*	*9.53%*	*-44.18%*	*158.69%*
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Pre-Crisis	50.00%	61.11%	86.67%	8.33%	73.53%
Crisis	37.50%	55.88%	64.71%	83.33%	31.25%
Difference	*-28.77%*	*-8.95%*	*-29.22%*	*230.29%*	*-85.57%*
Post Crisis	50.00%	83.33%	58.33%	70.00%	60.00%
Difference	*28.77%*	*39.96%*	*-10.38%*	*-17.43%*	*65.23%*
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Pre-Crisis	65.63%	75.00%	58.51%	76.47%	22.73%
Crisis	68.18%	44.44%	12.50%	38.89%	38.89%
Difference	*3.81%*	*-52.33%*	*-154.35%*	*-67.62%*	*53.71%*
Post Crisis	14.29%	83.33%	55.88%	56.25%	58.33%
Difference	*-156.26%*	*62.87%*	*149.75%*	*36.91%*	*40.54%*
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Pre-Crisis	100.00%	66.67%	57.89%	67.14%	80.56%
Crisis	71.43%	50.00%	40.00%	33.33%	61.36%
Difference	*-33.65%*	*-28.77%*	*-36.97%*	*-70.03%*	*-27.22%*
Post Crisis	50.00%	57.14%	59.09%	31.25%	66.67%
Difference	*-35.67%*	*13.35%*	*39.02%*	*-6.44%*	*8.30%*
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Pre-Crisis	57.69%	96.67%	10.00%	52.33%	53.33%

Crisis	50.00%	80.00%	40.00%	76.92%	83.33%
Difference	*-14.31%*	*-18.93%*	*138.63%*	*38.52%*	*44.63%*
Post Crisis	60.00%	83.33%	33.33%	42.31%	33.33%
Difference	*18.23%*	*4.08%*	*-18.24%*	*-59.77%*	*-91.64%*
	Switzerland	**Turkey**	**UK**	**USA**	
Pre-Crisis	50.00%	70.00%	66.07%	50.00%	
Crisis	50.00%	56.25%	58.62%	56.45%	
Difference	*0.00%*	*-21.87%*	*-11.96%*	*12.13%*	
Post Crisis	64.29%	56.25%	35.71%	35.71%	
Difference	*25.14%*	*0*	*-49.57%*	*-45.79%*	

The results of table 41 above can be summarized into two keys trends: first, that there is a considerable decline in the ratio of Black Swans to extreme values in the crisis period in comparison to the pre-crisis period and second that this trend was reversed significantly in the period following the crisis indicating a slow spread of extreme volatility in the stock markets.

Regarding the first result, 25 out of 34 countries in the data set report 44% fewer Black Swans to extreme values in the segment marked the beginning of the Global Financial Crisis in comparison to the segment preceding it. These results are consistent with the residuals of returns found in chapter 1, however, they are more pronounced in residuals of the absolute returns i.e. the reduction in Black Swans to extreme values was 32% in the residuals of the returns in comparison to 44% in residuals of differenced absolute returns. This trend of declining Black Swans to extreme values is persistent in the foreign exchange market as well which has implications for potential portfolio diversification benefits. For the remaining, 9 countries, Switzerland had no Black Swans within the extreme values, the other 8 had 66% more Black Swans per extreme values in the crisis period as compared to the pre-crisis period.

The second result indicates that out of the 25 countries that had fewer Black Swans in the crisis period as compared to the pre-crisis period, 6 continued to stabilize with the ratio of Black Swans to extreme values declining by 24% in the post-crisis period but the remaining 19 display a counterintuitive trend of higher ratio of Black

Swans to extreme values which has increased by 40%. Of the remaining 9 countries which has higher Black Swans in the crisis period as compared to the pre-crisis period, only Luxembourg displays a consistent increase across the three segments with 41% more Black Swans to extreme values post-crisis, 7 countries had higher Black Swans in the crisis period as compared to the pre-crisis period but stabilized in the post-crisis period with 65% fewer Black Swans to extreme values and the remaining one was incomparable. Overall, 12 of the countries in the data set had fewer Black Swans to extreme values in the post crisis period and the remaining 21 countries had higher Black Swans to extreme values.

A potential explanation of both the results is that while most countries had fewer Black Swans to extreme values in the pre-crisis period as compared to the crisis period, these increased significantly in the post-crisis period due to the slow spread of the effects of the financial crisis to these markets. These results are antipodal to the trend seen in foreign exchange markets resulting an asymmetric response to a severe financial crisis indicating a possible portfolio diversification benefit for investors.

Summary of the innovation in extreme variance between residuals of returns and differenced squared returns of the daily stock exchange market from best fit ARMA-APARCH model

The following table 42 summarizes the empirical results comparing the frequency of Black Swans in standardized residuals of returns and residuals of the volatility proxy namely the differenced squared returns. It further analyses the tails of both the distributions to find evidence of kurtosis. Both distributions are corrected for potential latent Non-linearities in the mean and/or variance dynamic (see Appendix 30 for detailed results).

Table 42 - Difference in the frequency of Black Swans in residuals of returns versus residuals of squared returns (Equity market)

Difference in the frequency of Black Swans in residuals of returns vs residuals of squared returns	**Australia**	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	-279.43%	-154.11%	-129.71%	-160.1%	-143.79%
Left tail	321.78%	213.94%	415.83%	405.97%	116.23%

	Czech	Denmark	Estonia	Finland	France
Right tail	-180.30%	-154.43%	-27.15%	-119.9%	-207.17%
Left tail	355.45%	343.34%	225.07%	382.77%	352.52%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Right tail	-156.12%	-122.86%	-122.21%	-39.16%	-84.79%
Left tail	404.25%	380.61%	376.07%	294.97%	349.59%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	-137.79%	-268.03%	-112.70%	-122.4%	-163.69%
Left tail	378.33%	294.32%	260.97%	382.81%	317.70%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Right tail	-168.32%	-218.34%	-185.34%	-181.7%	-228.79%
Left tail	79.70%	215.06%	270.71%	125.18%	244.15%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Right tail	-122.69%	-92.82%	-63.72%	-226.4%	-159.54%
Left tail	325.76%	407.97%	256.37%	210.37%	368.80%
	Switzerland	**Turkey**	**UK**	**USA**	
Right tail	-208.04%	-147.17%	-96.08%	-162.8%	
Left tail	358.25%	373.72%	384.95%	412.68%	

The results of table 42 can be précised into three key outcomes: first, there are significantly fewer Black Swans in residuals of returns and squared returns in comparison to returns and squared returns respectively; second, on average there are a greater number of Black Swans in squared residuals in comparison to residuals and finally residuals have a longer left tail when compared to the left tail of squared residuals whereas squared residuals have a considerably longer right tail compared to the right tail of residuals.

Regarding the first result, overall using an ARMA-APARCH model has significantly reduced the frequency of Black Swans in residuals and squared residuals i.e. there are 36% fewer Black Swans in residuals and almost 13% fewer Black Swans in squared residuals. This result confirms the observations of the significant reduction in residuals in comparison to the residuals from the volatility proxy which was witnessed in absolute residuals. It validates the result that there are alternate causes of volatility in the volatility proxy that cannot be captured adequately by GARCH class models.

With respect to the second result, there are 53% more Black Swans in squared residuals in comparison to residuals, i.e. in squared residuals, 1.51% of the trading days were identified as Black Swans whereas only 0.90% of the trading days in the residuals data set were identified as Black Swans. These results are consistent with the findings in the previous section wherein the residuals of the volatility proxy had higher Black Swans than the residuals of the returns themselves.

The final result delves into the individual tails of both the distributions wherein residuals of returns have an especially longer left tail with 94% of the total negative Black Swans and only 6% occurring the residuals of squared returns. With respect to the right tail, 80% of the total positive Black Swans occurred in the residuals of squared returns as opposed to the 20% that arose in the residuals of returns.

Summary of the ratio of Black Swans to extreme values in the residuals of differenced squared returns obtained from the best fit ARMA-APARCH model

The following table 43 demonstrates the transformation of volatility of volatility in the residuals of differenced squared equity returns of 34 countries ex post and ex ante the Global Financial Crisis of 2008.

Table 43 - Difference in the ratio of Black Swans to extreme values in residuals of squared returns ex-ante and ex-post the GFC (2008)

	Australia	Austria	Belgium	Canada	Chile
Pre-Crisis	96.67%	70.51%	77.78%	79.03%	100.00%
Crisis	50.00%	41.67%	50.00%	92.86%	54.84%
Difference	-65.92%	-52.61%	-44.18%	16.12%	-60.08%
Post-Crisis	85.71%	81.25%	87.50%	68.75%	75.00%
Difference	53.90%	66.78%	55.96%	-30.06%	31.31%
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
Pre-Crisis	81.82%	87.04%	61.43%	45.45%	92.31%
Crisis	78.13%	90.00%	50.00%	91.67%	87.50%
Difference	-4.62%	3.35%	-20.59%	70.14%	-5.35%
Post-Crisis	60.00%	73.33%	31.25%	106.25%	94.44%
Difference	-26.40%	-20.48%	-47.00%	14.76%	7.64%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Pre-Crisis	116.67%	113.89%	63.33%	8.33%	41.18%

Crisis	137.50%	94.12%	76.47%	16.67%	75.00%
Difference	16.43%	-19.07%	18.85%	69.31%	59.96%
Post-Crisis	125.00%	66.67%	70.83%	80.00%	80.00%
Difference	-9.53%	-34.48%	-7.66%	156.86%	6.45%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Pre-Crisis	78.13%	100.00%	71.28%	114.71%	31.82%
Crisis	84.09%	94.44%	87.50%	77.78%	94.44%
Difference	7.36%	-5.72%	20.51%	-38.85%	108.80%
Post-Crisis	21.43%	116.67%	97.06%	81.25%	100.00%
Difference	-136.72%	21.13%	10.37%	4.37%	5.72%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Pre-Crisis	150.00%	95.83%	57.89%	94.29%	116.67%
Crisis	92.86%	87.50%	90.00%	144.44%	115.91%
Difference	-47.96%	-9.10%	44.12%	42.66%	-0.65%
Post-Crisis	100.00%	121.43%	45.45%	93.75%	133.33%
Difference	7.41%	32.77%	-68.31%	-43.23%	-5.72%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Pre-Crisis	65.38%	120.00%	3.33%	77.91%	80.00%
Crisis	66.67%	60.00%	60.00%	115.38%	83.33%
Difference	1.94%	-69.31%	289.04%	39.28%	4.08%
Post-Crisis	57.50%	77.78%	50.00%	50.00%	50.00%
Difference	-14.79%	25.95%	-18.23%	-83.62%	-51.08%
	Switzerland	**Turkey**	**UK**	**USA**	
Pre-Crisis	50.00%	90.00%	66.07%	100.00%	
Crisis	80.00%	103.13%	81.03%	95.16%	
Difference	47.00%	13.61%	20.41%	-4.96%	
Post-Crisis	100.00%	56.25%	71.43%	107.14%	
Difference	22.31%	-60.61%	-12.62%	11.86%	

The key findings of table 43 can be summarized into three key trends: first, overall there the ratio of Black Swans to extreme values increases in the crisis period as compared to the pre-crisis period, second, this ratio declines slightly in the post-crisis period and finally, that while both of the above results are averages, delving deeper into them, shows counterintuitive results.

Regarding the first result, overall, there is an increase in the ratio of Black Swans to extreme values in the residuals of the differenced squared returns of 13%. However, scrutinizing the trend further reveals a divided trend, i.e. 19 countries had 47% more Black Swans contained within the extreme values in the segment recognized as the beginning of the Global Financial Crisis in comparison to the preceding segment yet for the remaining 15 countries in the data set, the trend is reversed, i.e. there are 30% fewer Black Swans to extreme values.

Regarding the second result, of the 19 countries which had a higher ratio of Black Swans to extreme values during the crisis period as compared to the pre-crisis period, 13 stabilized in the post crisis period with 43% fewer Black Swans to extreme values and the remaining 6 continued to experience extreme volatility in the succeeding segment with 36% more Black Swans to extreme values. Out of the remaining 15 countries in the data set which had a low ratio of Black Swans to extreme vales in the crisis period as compared to the pre-crisis period, only 3 continued to stabilize with 43% fewer Black Swans to extreme values in the succeeding period while the remaining 12 displayed antipodal trends of 28% more Black Swans to extreme values. Overall, there were 3% fewer Black Swans to extreme values when averaged across all the equity markets in the data set.

Summary of the innovation in extreme variance between residuals of returns and differenced log squared returns of the daily stock exchange market from best fit ARMA-APARCH model

The following table 44 presents the frequency of Black Swans in residuals[20] of returns and compares it to the frequency of Black Swans in residuals of differenced log squared returns. The table also presents the tails of the volatility distribution of both data sets to scrutinize the recurrence of positive and negative Black Swan cluster (see Appendix 31 for detailed results).

Table 34- Difference in the frequency of Black Swans in residuals of returns versus residuals of log squared returns (Equity market)

Difference in the frequency of Black Swans in residuals of returns vs residuals of log squared returns	**Australia**	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	34.75%	128.82%	143.53%	158.35%	129.68%
Left tail	-91.66%	-54.83%	-55.06%	-43.31%	-82.84%

[20] Residuals are standardized and have been obtained by fitting the daily returns and differenced log squared daily returns with the best fit ARMA-APARCH model. Both returns series are inclusive of potential structural breaks in the underlying mean and/or variance dynamic that could be caused by economic, political and/or financial crisis in the respective country.

	Czech	Denmark	Estonia	Finland	France
Right tail	167.30%	170.41%	311.29%	277.16%	102.84%
Left tail	-43.46%	-69.38%	-23.42%	-59.12%	-89.37%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
Right tail	212.77%	210.35%	172.24%	171.67%	219.65%
Left tail	-102.01%	-51.14%	-50.18%	-33.36%	-22.99%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
Right tail	219.65%	125.15%	177.03%	189.66%	270.69%
Left tail	-43.61%	-98.87%	-67.87%	-73.62%	-51.20%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
Right tail	156.72%	91.52%	138.52%	189.61%	250.70%
Left tail	-53.21%	-64.66%	-109.97%	-86.45%	-96.55%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
Right tail	299.30%	152.50%	102.39%	178.23%	150.31%
Left tail	-114.40%	77.30%	-76.78%	-80.67%	-73.09%
	Switzerland	**Turkey**	**UK**	**USA**	
Right tail	97.17%	194.96%	161.01%	99.32%	
Left tail	-77.85%	-57.99%	-80.90%	-83.79%	

The results of table 44 can be categorized into three key trends: first that while there are consistently fewer Black Swans in the residuals of returns yet residuals of the log squared returns depict a reverse trend; second, there are a greater frequency of Black Swans in residuals of the log squared returns in comparison to the residuals of the returns and finally that there are a higher number of negative Black Swans in the log squared residuals whereas the reverse is true for residuals which have a greater number of positive Black Swans.

Regarding the first result, while residuals of returns and other volatility proxies such as absolute returns and squared returns have fewer Black Swans in the residuals as opposed to the returns, the reverse is true for the residuals of log squared returns as there are 11% more Black Swans in log squared residuals as compared to log squared returns.

Additionally while the residuals of the daily differenced log squared returns of the forex market have a lower number of Black Swans than the residuals of the daily returns (i.e. there are 23% more Black Swans in the former as compared to the latter), equity markets are displaying a converse trends where residuals of the daily differenced log squared returns have a higher frequency of Black Swans in comparison to the residuals of the returns themselves (1.13% of the trading days in the residuals of the log squared returns were classified as Black Swans in comparison to 0.93% of the residuals of the daily returns). This result is seen in 29 of the 34 countries in the data set where residuals of log squared returns have 15% more Black Swans that the residuals of the returns.

The final result focuses on the tails of the volatility in returns and log squared returns once the effect of conditional heteroscedasticity has been corrected i.e. it measures the frequency of Black Swans on the left and right tail of the residuals of daily returns and compares it to the tails of the residuals of differenced log squared returns. The table shows that while there are more Black Swans in the left tail of the residuals of the log squared returns the contrary is true for residuals of returns which show a higher number of Black Swans in the right tail.

Summary of the ratio of Black Swans to extreme values in the residuals of differenced Log-Squared returns obtained from the best fit ARMA-APARCH model

The following table 45 illustrates the renovation of extreme volatility as evidenced by the residuals of the unconditional measure of choice, i.e. differenced log squared returns of the equity markets of 34 countries ex post and ex ante the Global Financial Crisis of 2008 defined here as the date of the fall of the Lehman Brothers. Keeping the central component of the crisis consistent across all markets, the table 46 lists the transition of the ratio of Black Swans to extreme values by comparing the crisis period to the preceding and succeeding segments respectively.

Table 45 - Difference in the ratio of Black Swans to extreme values in residuals of log squared returns ex-ante and ex-post the GFC (2008)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
1.Pre-Crisis	40.00%	58.57%	56.00%	51.72%	40.00%
2.Crisis	60.00%	66.67%	25.00%	33.33%	57.14%
3.Difference	40.55%	12.95%	-80.65%	-43.94%	35.67%

4.Post-Crisis	64.29%	50.00%	87.50%	78.57%	37.50%
5.Difference	6.90%	-28.77%	125.28%	85.75%	-42.12%
	Czech	**Denmark**	**Estonia**	**Finland**	**France**
1.Pre-Crisis	44.44%	60.00%	45.45%	60.00%	70.83%
2.Crisis	56.67%	80.00%	25.00%	50.00%	25.00%
3.Difference	24.29%	28.77%	-59.78%	-18.23%	-104.15%
4.Post-Crisis	40.00%	42.86%	75.00%	56.25%	55.56%
5.Difference	-34.83%	-62.42%	109.86%	11.78%	79.85%
	Germany	**Greece**	**Hungary**	**Iceland**	**Ireland**
1.Pre-Crisis	50.00%	55.88%	57.14%	45.45%	65.63%
2.Crisis	25.00%	53.13%	60.94%	41.67%	56.25%
3.Difference	-69.31%	-5.06%	6.43%	-8.70%	-15.42%
4.Post-Crisis	56.25%	50.00%	63.64%	50.00%	50.00%
5.Difference	81.09%	-6.06%	4.33%	18.23%	-11.78%
	Israel	**Italy**	**Japan**	**Korea**	**Luxembourg**
1.Pre-Crisis	75.00%	50.00%	71.43%	56.67%	54.55%
2.Crisis	47.50%	38.89%	37.50%	43.75%	62.50%
3.Difference	-45.68%	-25.13%	-64.44%	-25.87%	13.61%
4.Post-Crisis	0.00%	58.33%	53.33%	57.14%	50.00%
5.Difference	NA	40.55%	35.22%	26.71%	-22.31%
	Mexico	**Netherlands**	**New Zealand**	**Norway**	**Poland**
1.Pre-Crisis	25.00%	79.17%	58.33%	63.64%	67.65%
2.Crisis	60.53%	50.00%	50.00%	62.50%	54.76%
3.Difference	88.42%	-45.95%	-15.42%	-1.80%	-21.13%
4.Post-Crisis	50.00%	28.57%	70.00%	50.00%	33.33%
5.Difference	-19.11%	-55.96%	33.65%	-22.31%	-49.64%
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1.Pre-Crisis	84.62%	35.00%	60.71%	61.25%	60.71%
2.Crisis	16.67%	25.00%	112.50%	62.50%	54.55%
3.Difference	-162.47%	-33.65%	61.68%	2.02%	-10.71%
4.Post-Crisis	63.16%	12.50%	16.67%	62.50%	60.00%
5.Difference	133.22%	-69.31%	-190.9%	0.00%	9.53%
	Switzerland	**Turkey**	**UK**	**USA**	
1.Pre-Crisis	63.64%	57.14%	63.46%	0.00%	
2.Crisis	60.00%	46.67%	71.43%	53.45%	
3.Difference	-5.88%	-20.25%	11.83%	NA	
4.Post-Crisis	42.86%	46.67%	50.00%	41.67%	

5.Difference	-33.65%	0.00%	-35.67%	-24.90%	

[21]

The results provided in table 45 can be summarized into three key trends: first, that there is a smaller percentage of Black Swans in the pool of extreme values during the crisis period in comparison to the pre-crisis period, second that on average there are a slightly higher number of Black Swans to extreme values in the post crisis period and finally, the response of the equity markets is asymmetric to the response of the forex markets across the three segments.

Regarding the first result, there are on average 17% fewer Black Swans to extreme values in the crisis period as compared to the preceding segment however scrutinizing this trend further reveals that while majority of the countries presented fewer Black Swans (22 of the 34 countries in the data set had 40% fewer Black Swans to extreme values) there were a few countries that displayed the reverse trend (12 of the countries in the data set had 27% more Black Swans to extreme values in the crisis period).

With respect to the second result, of the 22 countries that experienced a lower probability of the occurrence of Black Swans during the crisis period, only 7 continued that trend with 31% fewer Black Swans to extreme value, the remaining 14 displayed 57% more volatility ex post the fall of the Lehman brothers. Of the 12 countries that showed a higher percentages of Black Swans to extreme values during the crisis segment, only 2 continued with the persistent trend with a slightly higher ratio of 6% more Black Swans to extreme values in the segment following the crisis, while 8 had started to stabilize with 55% fewer Black Swans to extreme value, 2 had no change the remaining were incomparable.

Finally, the equity market continues to display an antipodal trend across the three segments when compared to the forex market

[21] It is important to note that while there are a higher number of missing values in the results using residuals of the log squared returns, there are significant trends that support the results obtained from the other volatility proxies.

Summary of the innovation in extreme variance between residuals of returns and differenced range of the daily stock exchange market from best fit ARMA-APARCH model

The following table 46 presents the frequency of Black Swans in residuals[22] of daily returns and compares it to the frequency of Black Swans in the residuals of differenced range using the Parkinson's method. The table also presents the tails of the volatility distribution of both data sets to scrutinize the recurrence of positive and negative Black Swans (see Appendix 32 for detailed results).

Table 356 - Difference in the frequency of Black Swans in residuals of returns versus residuals of range (Equity market)

Difference in the frequency of Black Swans in residuals of returns vs residuals of range	**Australia**	**Austria**	**Belgium**	**Canada**	**Chile**
Right tail	-250.83%	-135.91%	-161.20%	-136.6%	-68.14%
Left tail	57.17%	79.56%	367.68%	167.23%	99.25%
	Czech	**Denmark**	**Finland**	**France**	**Germany**
Right tail	-163.21%	-116.94%	-109.31%	-218.60%	-133.71%
Left tail	138.42%	3.85%	175.48%	237.37%	197.64%
	Greece	**Iceland**	**Hungary**	**Ireland**	**Israel**
Right tail	-92.83%	-130.30%	-78.69%	-89.44%	-166.80%
Left tail	64.91%	75.05%	76.42%	342.04%	-28.15%
	Italy	**Japan**	**Luxembourg**	**Mexico**	**Netherlands**
Right tail	-230.98%	-96.07%	-157.38%	-138.8%	-213.06%
Left tail	92.94%	52.24%	58.79%	121.41%	277.27%
	Norway	**Portugal**	**Korea**	**Spain**	**Sweden**
Right tail	-186.30%	-149.32%	-139.10%	-196.7%	-134.51%
Left tail	61.82%	217.03%	89.50%	216.46%	137.52%
	Switzerland	**Turkey**	**U.K.**	**U.S.A.**	
Right tail	-156.92%	-113.50%	-95.33%	-153.30%	
Left tail	277.65%	46.82%	182.84%	61.52%	

[22] Residuals are standardized and have been obtained by fitting the daily returns and differenced log squared daily returns with the best fit ARMA-APARCH model. Both returns series are inclusive of potential structural breaks in the underlying mean and/or variance dynamic that could be caused by economic, political and/or financial crisis in the respective country.

The empirical results of table 46 can be summarized into two key findings: first, that on average, Black Swans are not homogenous between residuals of returns and range, second, that there are more negative clusters of Black Swans in residuals of returns and the trend is reversed in residuals of range.

Regarding the first result, there are 57% more clusters of Black Swans in residuals of range (wherein 1.53% of the trading days contained Black Swans) as compared to the residuals of the returns (wherein 0.86% of the trading days could be identified as Black Swans); a trend which is consistent across every country in the data set. This result is also similar to the results found in returns and range in section 6.6.1.

Regarding the second result, there are a higher frequency of clusters on the left tail of the residuals of returns i.e. there are more clusters of negative Black Swans in the residuals of returns (62% of the total clusters exist on the left tail); whereas in residuals of the range, the trend becomes antipodal i.e. there are a higher frequency of clusters on the right tail (88% of the total clusters are positive).

Summary of the ratio of Black Swans to extreme values in the residuals of differenced Range obtained from the best fit ARMA-APARCH model

The following table 47 illustrates the transformational of extreme volatility as evidenced by the residuals of the unconditional measure of choice, i.e. differenced Parkinson range of the equity markets of 27 countries ex post and ex ante the Global Financial Crisis of 2008 defined here as the date of the fall of the Lehman Brothers. Keeping the central component of the crisis consistent across all markets, the table lists the transition of the ratio of Black Swans to extreme values by comparing the crisis period to the preceding and succeeding segments respectively.

Table 47 - Difference in the ratio of Black Swans to extreme values in residuals of range ex-ante and ex-post the GFC (2008)

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**
Pre-Crisis	83.33%	52.78%	100.00%	70.69%	37.50%
Crisis	50.00%	100.00%	112.50%	41.67%	25.00%
Difference	-51.08%	63.91%	11.78%	-52.86%	-40.55%

Post-Crisis	21.43%	71.43%	100.00%	78.57%	83.33%
Difference	-84.73%	-33.65%	-11.78%	63.43%	-120.40%
	Czech	**Denmark**	**Finland**	**France**	**Germany**
Pre-Crisis	84.62%	70.00%	45.00%	62.50%	87.50%
Crisis	40.00%	60.00%	110.00%	94.44%	93.75%
Difference	-74.92%	-15.42%	89.38%	41.28%	6.90%
Post-Crisis	55.56%	63.33%	31.25%	125.00%	125.00%
Difference	32.85%	5.41%	-125.9%	28.03%	28.77%
	Greece	**Hungary**	**Ireland**	**Israel**	**Italy**
Pre-Crisis	105.00%	N/A	37.50%	N/A	50.00%
Crisis	60.00%	60.94%	87.50%	83.33%	105.56%
Difference	-55.96%	N/A	84.73%	N/A	74.72%
Post-Crisis	79.17%	45.45%	50.00%	100.00%	75.00%
Difference	27.72%	-29.31%	-55.96%	18.23%	-34.17%
	Japan	**Luxembourg**	**Mexico**	**Netherlands**	**Norway**
Pre-Crisis	37.50%	92.86%	75.00%	87.50%	93.75%
Crisis	23.33%	120.00%	89.47%	92.86%	87.50%
Difference	-47.45%	25.64%	17.65%	5.94%	-6.90%
Post-Crisis	125.00%	56.25%	0.00%	107.14%	83.33%
Difference	167.84%	-75.77%	N/A	14.31%	-4.88%
	Portugal	**Korea**	**Spain**	**Sweden**	**Switzerland**
Pre-Crisis	73.08%	76.67%	57.58%	77.27%	45.45%
Crisis	83.33%	43.75%	66.67%	100.00%	70.00%
Difference	13.13%	-56.10%	14.66%	25.78%	43.18%
Post-Crisis	97.37%	64.29%	75.00%	81.25%	78.57%
Difference	15.57%	38.48%	11.78%	-20.76%	11.55%
	UK	**USA**			
Pre-Crisis	57.69%	84.48%			
Crisis	71.43%	100.00%			
Difference	21.36%	16.86%			
Post-Crisis	91.67%	55.56%			
Difference	24.95%	-58.78%			

Overall, there were a higher number of Black Swans during the crisis as compared to the preceding segment and a lower number of Black Swans in the succeeding segment i.e. there are 6% more Black Swans during the crisis and 6% fewer in the post crisis segment.

With respect to the first phase, the dominant trend seen in 16 of the 27 countries in the data set is that there were a higher number of Black Swans during the crisis period in comparison to pre-crisis period i.e. there are 35% more clusters. For the remaining 9 countries, there were a higher frequency of clusters in the ex-ante the Global Financial Crisis; the remaining two countries did not have sufficient data for comparison. These results led to the overall effect of a greater number of clusters during the crisis indicating a higher probability to extremely extreme behaviour in the stock market.

With respect to the second result, the dominant trend was slightly different – on average the frequency of Black Swans had reduced by 6% but that result was seen in 12 of the 27 countries in the data set which experienced 55% fewer clusters, the remaining 14 countries continued to experience extreme volatility post the Global Financial Crisis with 35% more Black Swans.

Degree of overlap between residuals of returns (mean dynamics) and the residuals of proxies (volatility dynamics) of the equity market

The following table presents the degree of similarity or overlap between the extreme events from mean dynamics (residuals of returns) versus those from the volatility dynamics (residuals of volatility proxies) and while there is a significant level of overlap across the data sets, there is clear evidence that there is a degree of dissimilarity between the two.

	Returns
Returns	100%
Differenced Absolute returns	59.4%*
Differenced Squared returns	34.6%*
Differenced Log Squared returns	81.3%**
Range	91.2%**

*Returns have higher frequency of Black Swans ** Proxies have higher frequency of Black Swans

This table presents evidence that there is a difference in the extreme events from mean dynamics and volatility dynamics in equity markets when heteroscedasticity and structural breaks are accounted for.

While, accounting for volatility from mean dynamics plays an essential role in asset pricing and hedging strategies yet extreme volatility from innovations in the volatility dynamics is relatively unexplored territory. There is ample evidence in the literature that a change in the volatility (resulting from mean) is accounted for in traditional models however as is evident from these results, extreme volatility reflected in returns singularly does not account for the extreme events resulting from the volatility of volatility (i.e. those resulting from volatility dynamics). This disregard of changes in volatility of volatility could let to a potential misspecification of overall financial risk faced by investors and market participants and hence possesses the potential to become an integral indicator of overall market volatility.

With respect to log-squared returns, majority of scholarly literature tends to exclude it as a volatility proxy because it if the asset returns are close to zero, then transforming it to log-squared returns yields a highly negative number; if it is zero then the transformation is undefined. Such an extreme negative numbers tends to distort overall model estimates (Cavalcante and Assaf, 2004, Pereira, 2004, Christodoulakis and Satchell, 2005). The focus of this chapter is sizable distortion in financial markets due to a crisis, i.e. heightened tail risk from the volatility dynamics of asset returns during a crisis and not small innovations, therefore the focus will remain primarily on the results from absolute returns and squared returns.

6.7 Discussion

With respect to the main research question that tests the homogeneity of Black Swans in equity markets using unconditional measures of volatility such as volatility proxies, the null is strongly rejected in all four of the proxies when testing within returns of the volatility proxies as well as the residuals of those returns i.e. tail risk is not homogenous when using return-based measures as compared to proxy-based measures.

Range-based volatility estimators are found to be highly efficient, in stochastic volatility models that estimate realized variance, when compared to return-based measures as well as volatility proxies such as squared returns, as they are known to being approximately Gaussian as well as being impervious to micro-structure noise (Alizadeh et al., 2002, Andersen and Bollerslev, 1998, Brandt and Diebold, 2003, Martens and Van Dijk, 2007, Fuertes et al., 2009).

Specifically, the chapter used the Parkinson range, the use to which is extant in literature as it is known to be an unbiased estimator of daily volatility and five time more efficient than squared returns (Parkinson, 1980). Raju and Rangaswamy (2017) found that the conditional volatility of equity markets that are characterized by leptokurtosis and heteroscedasticity are modelled proficiently using intraday range-based estimators as compared to inter-day return-based estimators as they lead to superior one-day ahead forecasts (see (Maciel and Ballini, 2017) for similar results for the S&P 500 and Brazilian main stock market index; (Li and Hong, 2011) for proposing a range-based autoregressive volatility model that outperforms traditional return based GARCH-type models; (Gerlach et al., 2017) for generating superior tail risk forecasts in six financial market returns by estimating Expected Shortfall).

The results indicate that in the equity market, the results remain analogous to the forex market with three out of the four chosen volatility proxies (i.e. absolute returns, squared returns and range) displaying a lower number of Black Swans in their residuals that their returns' counterpart. This reduction in the probability of the manifestation of highly extreme values can be attributed to the implementation of model 2 which executes an asymmetric power ARCH model to capture the effect of time varying volatility of the chosen measures as well any potential leverage effects. It is important to note the degree of this decrease in order to comprehend the degree of volatility clustering/serial correlation captured by the best fit ARMA-APARCH model which tend to considerably increase when financial markets undergo extreme distress.

The empirical results of model two illustrate that the reduction in the frequency of Black Swans is greater in the residuals of returns in comparison to the residuals of absolute, squared returns, and range i.e. there is 34% reduction in the number of Black Swans in residuals derived from daily equity returns and a 8% and 13%

decrease in the incidence rate in the residuals of absolute returns and squared returns respectively and a 14% decrease in the frequency of clusters in the residuals of range as compared to the range itself. The log returns, however, demonstrate a reverse trend of greater Black Swans (11%) in the log squared residuals in comparison to the returns. A final auxiliary conclusion is that the decrease is more pronounced in the equity markets as compared to the forex market indicating greater clustering phenomena for which extant evidence exists in the existing literature.

Both equity and forex markets display symmetric trends with respect to the occurrence of Black Swans, however, the trend is more notable in equity markets. For example: similar to the forex markets, there are a higher number of Black Swans in the residuals of the volatility proxies in comparison to the residuals of the returns except the figures are higher i.e. there are 34% and 53% more Black Swans in residuals of absolute and squared equity returns respectively in comparison to their forex counterparts which had 24% and 29% fewer Black Swans. Residuals of log squared returns continue to depict antagonistic tendencies with 23% more Black Swans in the residuals of the log squared returns in comparison the residuals of the returns.

Examining the tails of the highly extreme returns once the clustering effect has been removed, reveal that the distribution of residuals from absolute returns, squared returns and range are skewed heavily to the right whereas the distribution of log squared residuals is skewed to the left.

With respect to the final question regarding the transformation of the Black Swans during the Global Financial Crisis when compared to the conceding and preceding segments, the dominant trend observed in the residuals of absolute and log squared returns is that on average there are fewer Black Swans in crisis period as compared to the pre-crisis period which increases slightly post-crisis. The residuals of the squared returns and range display a reverse trend with a higher ratio of clusters during the crisis period which declines in the consecutive segment. Therefore, Black Swans are not homogenous during the periods of extreme financial stress and are influenced by the market expectations and release of news.

6.8 Conclusion

By using non-parametric measures such as volatility proxies, namely, absolute returns, squared returns, log squared returns and Parkinson's range, the objective of this chapter was to elucidate the behaviour of the volatility of volatility in equity markets which are not sufficiently captured by parametric methods that primarily address volatilities that arise from mean. For robustness, first-order differences of volatility proxies have been taken to conduct the analysis in order to rectify and control the effect of unit roots, near unit roots and long-range persistence that are subsumed within them.

The empirical results of the chapter provide substantial evidence towards the research claim that volatilities arising in the variance are not axiomatic in the mean of the returns and therefore need to be investigated distinctively. For example: when comparing the frequency of Black Swans in returns with those in the volatilities proxies it is found that there are significant differences, rejecting the hypothesis of homogeneity of the volatility of volatility i.e. on average, in comparison to the mean, there were absolute returns, squared returns and range had a higher frequency of Black Swan clusters whereas log squared return had a higher frequency of them. This result is consubstantial to the foreign exchange market outcomes in chapter 5.

To further validate the outcomes and increase the efficiency of the model, residuals of returns and the respective volatility proxies the behaviour of Black Swans were investigated within the GARCH framework as well as various adaptations of volatility proxies within the GARCH framework. The outcomes of model 2 were identical to those found in the returns, i.e. on average, in comparison to the residuals of returns, residuals of all the volatility proxies consisted of a higher number of Black Swans.

Chapter 7: Conclusion

The behaviour of financial markets while undergoing extreme stress has gained immense popularity in scholarly literature over the past decade. This interest has stemmed partly from the inadequacy of current parametric financial models to understand the source of extreme volatility and the corresponding behaviour of financial markets and partly from the need to protect against such extreme movements in the market.

With respect to studying the extreme volatility in financial markets, two key strands of literature emerge. First, classical asset pricing models like CAPM and APT which have traditionally ignored extreme tail risk, i.e. the information that lies in the tails of an assets' distribution, by terming them 'outliers'. This was motivated by the seminal work of Edgeworth (1887) which showed that outliers can lead to biased coefficient estimates in the least squares regression. Consequently, most financial models identify these outliers (those that are 3 standard deviations from the mean) and then remedy them (also known as Winsorising).

The second most popular financial tool to understand and forecast volatility has been the ARMA-GARCH models introduced by Engle (1982) and Bollerslev (1986). These conditional heteroscedastic methods are frequently employed to correct inherent properties of financial returns such as long memory and volatility clustering. However, it is often observed that the standardized residuals obtained from these models continue to display excess kurtosis (Baillie and Bollerslev, 1989). This implies that the presence of outliers in returns series are not captured comprehensively by the GARCH models and hold further information with respect to tail risk (Balke and Fomby, 1994, Fiorentini and Maravall, 1996).

My thesis contributes primarily to this literature by focusing principally on the evolution of tail risk during the Global Financial Crisis of 2008 in the equity and forex markets. My research makes two contributions: first that unaccounted structural breaks in the mean/variance structure of asset returns can lead to excess kurtosis (tail risk) i.e. ignoring structural breaks overestimates the probability of the occurrence of an extreme event, and, two that there is a difference in tail risk that arises from the mean dynamics of asset returns and those that arise from the variance dynamics of asset returns.

Specifically, chapters three and four contribute to the literature by studying the evolution of tail risk when structural breaks in the mean and/or variance structure of the underlying asset distribution are taken into account. Additionally, chapters five and six delve deeper into the degree of asymmetry of tail risk that arises from the mean dynamics of asset returns in comparison to the volatility dynamics.

To test my hypotheses, I apply a superior mean-variance specification (ARMA-APARCH model that is inclusive of identifying structural breaks) to mean dynamics in chapters three (equity markets) and four (forex markets) then to volatility dynamics in chapters five (forex markets) and six (equity markets) while contextualizing it around the Global Financial Crisis of 2008.

The results of chapter three and four can be summarized as: first, the inclusion of structural breaks reduced kurtosis of the asset returns (i.e. the frequency of black swans reduced by 20% in returns and 12% in the residuals of Equity markets; in the foreign exchange markets there was a 5% decrease in the frequency of black swans in returns and 35% in the residuals); second, the clusters of black swans were not homogenous in nature (i.e. their frequency changed from one segment to the next justifying the use of structural breaks), finally, the incorporation of structural breaks significantly reduced tail asymmetry in the asset returns distribution (i.e. the distribution appeared to be normalised). Overall, accounting for structural breaks in equity and forex markets significantly reduced tail risk.

With regards to the global financial crisis of 2008, in both equity and forex markets, black swans were higher before the crisis and continued to decline over the two consecutive segments; one of which included the fall of the Lehman Brothers to mark the beginning of the crisis. This means that the markets were displaying signs of distress before the fall of the Lehman brothers and much of the heightened tail risk was attributable to the effect of conditional heteroscedasticity and unaccounted latent non-linearities in the mean and/or variance structure of the asset returns. Once these factors were accounted for, tail risk evidently declined in the following two segments. For example, in equity markets, the segment preceding the crisis had 32% more black swans in comparison to the segment during which the crisis took place. This continued to decline in the succeeding segment with 10% fewer black swans. Whereas in forex markets there were 2% more black swans in the preceding segment as compared to the crisis segment reducing to 9% less after the crisis segment.

Moving on to the results of chapters five and six: first, volatility proxies have a higher tail risk in comparison to returns in both equity and forex markets (this trend remained consistent even after the elimination of the volatility clustering effect using standardized residuals from the best fit ARMA-APARCH model); second, there was a difference in the tail risk from mean dynamics in comparison to the volatility dynamics in both forex and equity markets (the degree of overlap of tail risk between returns and proxies was 65% in equity and forex markets).

Specifically, these results highlight the inadequacy of parametric financial models, that estimate volatility from the mean dynamics of asset returns while remaining negligent to the volatility from the variance, in capturing the overall tail risk that exists in the market during a crisis.

The main implication of my research is that tail risk during the global financial crisis was influenced by various factors that are traditionally not incorporated into asset pricing models or conditional time series models, such as structural breaks in the mean and/or variance structure of the returns and overlooking tail risk arising from the variance of the returns.

With respect to these empirical findings, I believe my work will be of interest to investors and other financial market participants that deal with asset pricing, financial risk management, optimal portfolio selection, and/or option pricing and hedging. Specifically, the pricing of options incorporates a perceived crash risk which determines the subsequent returns at the date of maturity (Barro, 2006, Gabaix, 2008, Gabaix, 2012).

By studying the volatility of volatility, market participants could draw clearer inferences about tail risk arising from the volatility dynamics of the asset returns. For example, the price of an out of the money option is an indication of crash risk as perceived by the market. The estimate of this perception of tail risk can be improved with my model which incorporates a conditional time-varying crash risk factor.

My future work will be focused on developing a forecasting model of asset returns/prices that incorporates structural breaks in the underlying data while correcting it for conditional heteroscedasticity. Specifically, I am looking at developing a variable within an existing appropriate asset pricing model that will accommodate the effect of latent non-linearities in the mean and/or variance structure of the underlying returns distribution while incorporating tail risk from the mean and volatility dynamics of those returns. In short, a model that can contain an extreme tail risk measure that is reflective of the changes in the markets during a crisis with respect to returns.

References

ABID, F. & KAFFEL, B. 2018. Time–frequency wavelet analysis of the interrelationship between the global macro assets and the fear indexes. *Physica A: Statistical Mechanics and its Applications,* 490, 1028-1045.

AJAYI, R. A. & MOUGOUE, M. 1996. On the Dynamic Relation between Stock Prices and Exchange Rates. *Journal of Financial Research,* 19, 193-207.

ALIZADEH, S., BRANDT, M. W. & DIEBOLD, F. X. 2002. Range-based estimation of stochastic volatility models. *The Journal of Finance,* 57, 1047-1091.

ALLEN, D. E., SINGH, A. K. & POWELL, R. J. 2013a. EVT and tail-risk modelling: Evidence from market indices and volatility series. *North American Journal of Economics & Finance,* 26, 355-369.

ALLEN, D. E., SINGH, A. K. & POWELL, R. J. 2013b. EVT and tail-risk modelling: Evidence from market indices and volatility series. *North American Journal of Economics and Finance,* 26, 355-369.

ALOUI, C. 2007. Price and volatility spillovers between exchange rates and stock indexes for the pre-and post-euro period. *Quantitative Finance,* 7, 669-685.

ALOUI, C. & HAMIDA, H. B. 2014. Modelling and forecasting value at risk and expected shortfall for GCC stock markets: Do long memory, structural breaks, asymmetry, and fat-tails matter? *The North American Journal of Economics and Finance,* 29, 349-380.

ANDERSEN, T. G. & BOLLERSLEV, T. 1997. Heterogeneous information arrivals and return volatility dynamics: Uncovering the long-run in high frequency returns. *The journal of Finance,* 52, 975-1005.

ANDERSEN, T. G. & BOLLERSLEV, T. 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International economic review*, 885-905.

ANDERSEN, T. G., BOLLERSLEV, T. & MEDDAHI, N. 2005. Correcting the errors: Volatility forecast evaluation using high-frequency data and realized volatilities. *Econometrica,* 73, 279-296.

ANDREOU, E. & GHYSELS, E. 2002. Detecting multiple breaks in financial market volatility dynamics. *Journal of Applied Econometrics,* 17, 579-600.
APPADOO, S. S., THAVANESWARAN, A. & MANDAL, S. 2012. RCA model with quadratic GARCH innovation distribution. *Applied Mathematics Letters,* 25, 1452-1457.
AQUINO, R. Q. 2005. Exchange rate risk and Philippine stock returns: before and after the Asian financial crisis. *Applied Financial Economics,* 15, 765-771.
ARAGONÉS, J. R. & BLANCO, C. 2008. Incorporating correlation regimes in an integrated stressed risk modeling process. *Journal of Economics and Finance,* 32, 148-157.
ARAÚJO SANTOS, P. & FRAGA ALVES, M. I. 2013. Original article: Forecasting Value-at-Risk with a duration-based POT method. *Mathematics and Computers in Simulation,* 94, 295-309.
ASSAF, A. 2009. Extreme observations and risk assessment in the equity markets of MENA region: Tail measures and Value-at-Risk. *International Review of Financial Analysis,* 18, 109-116.
ASSAF, A. 2016. MENA stock market volatility persistence: Evidence before and after the financial crisis of 2008. *Research in International Business and Finance,* 36, 222-240.
AVEN, T. 2013. On the meaning of a black swan in a risk context. *Safety science,* 57, 44-51.
AVEN, T. 2015. Implications of black swans to the foundations and practice of risk assessment and management. *Reliability Engineering & System Safety,* 134, 83-91.
AVEN, T. & KROHN, B. S. 2014. A new perspective on how to understand, assess and manage risk and the unforeseen. *Reliability Engineering & System Safety,* 121, 1-10.
BAI, J. & PERRON, P. 1998. Estimating and testing linear models with multiple structural changes. *Econometrica*, 47-78.
BAILLIE, R. T. & BOLLERSLEV, T. 2002. The message in daily exchange rates: a conditional-variance tale. *Journal of Business & Economic Statistics,* 20, 60-68.
BAILLIE, R. T. & KAPETANIOS, G. 2007. Testing for neglected nonlinearity in long-memory models. *Journal of Business & Economic Statistics,* 25, 447-461.

BALKEMA, A. A. & DE HAAN, L. 1974. Residual Life Time at Great Age. Institute of Mathematical Statistics.

BALTUSSEN, G., VAN BEKKUM, S. & VAN DER GRIENT, B. 2018. Unknown Unknowns: Uncertainty About Risk and Stock Returns. *Journal of Financial and Quantitative Analysis,* 53, 1615-1651.

BAO, J., PAN, J. & WANG, J. 2011. The illiquidity of corporate bonds. *The Journal of Finance,* 66, 911-946.

BAO, Y., LEE, T.-H. & SALTOGLU, B. 2006. Evaluating predictive performance of value-at-risk models in emerging markets: a reality check. *Journal of Forecasting,* 25, 101-128.

BARNDORFF-NIELSEN, O., GRAWERT, S. & SHEPHARD, N. 2010. Measuring Downside Risk – Realized Semivariance*.

BEARE, B. 2004. Robustifying unit root tests to permanent changes in innovation variance. *Yale University, mimeographed*.

BEARE, B. K. 2018. Unit root testing with unstable volatility. *Journal of Time Series Analysis,* 39, 816-835.

BEE, M., DUPUIS, D. J. & TRAPIN, L. 2016. Realizing the extremes: Estimation of tail-risk measures from a high-frequency perspective. *Journal of Empirical Finance,* 36, 86-99.

BEINE, M., COSMA, A. & VERMEULEN, R. 2010. The dark side of global integration: Increasing tail dependence. *Journal of Banking & Finance,* 34, 184-192.

BEKIROS, S. & GEORGOUTSOS, D. 2005. Estimation of Value-at-Risk by extreme value and conventional methods: a comparative evaluation of their predictive performance. *Journal of International Financial Markets, Institutions and Money,* 15, 209-228.

BEKIROS, S. D. & GEORGOUTSOS, D. A. 2008a. The extreme-value dependence of Asia-Pacific equity markets. *Journal of Multinational Financial Management,* 18, 197-208.

BEKIROS, S. D. & GEORGOUTSOS, D. A. 2008b. Extreme Returns and the Contagion Effect between the Foreign Exchange and the Stock Market: Evidence from Cyprus. *Applied Financial Economics,* 18, 239-254.

BERGER, T. & MISSONG, M. 2014. Financial crisis, Value-at-Risk forecasts and the puzzle of dependency modeling. *International Review of Financial Analysis,* 33, 33-38.

BHATTACHARYYA, M. & RITOLIA, G. 2008. Conditional VaR using EVT – Towards a planned margin scheme. *International Review of Financial Analysis,* 17, 382-395.

BHATTI, M. I. & NGUYEN, C. C. 2012. Diversification evidence from international equity markets using extreme values and stochastic copulas. *Journal of International Financial Markets, Institutions & Money,* 22, 622-646.

BILSON, J. F. 1980. The" speculative efficiency" hypothesis. National Bureau of Economic Research Cambridge, Mass., USA.

BISSOONDEEAL, R. K., KAROGLOU, M. & BINNER, J. M. 2019. Structural changes and the role of monetary aggregates in the UK. *Journal of Financial Stability*.

BOGLE, J. C. 2008. Black Monday and black swans. *Financial Analysts Journal,* 64, 30-40.

BOLLERSLEV, T., CHOU, R. Y. & KRONER, K. F. 1992. ARCH modeling in finance: A review of the theory and empirical evidence. *Journal of econometrics,* 52, 5-59.

BOLLERSLEV, T., ENGLE, R. F. & NELSON, D. B. 1994. ARCH models. *Handbook of econometrics,* 4, 2959-3038.

BOLLERSLEV, T., LI, S. Z. & ZHAO, B. 2017. Good Volatility, Bad Volatility, and the Cross Section of Stock Returns. *Journal of Financial and Quantitative Analysis*, 1-57.

BOLLERSLEV, T. & MIKKELSEN, H. O. 1996. Modeling and pricing long memory in stock market volatility. *Journal of econometrics,* 73, 151-184.

BOSWIJK, H. P. 2001. Testing for a unit root with near-integrated volatility. Tinbergen Institute Discussion Paper.

BOSWIJK, H. P. 2005. Adaptive testing for a unit root with nonstationary volatility. *UvA-Econometrics Discussion Paper,* 7.

BRADLEY, B. O. & TAQQU, M. S. 2004. AN EXTREME VALUE THEORY APPROACH TO THE ALLOCATION OF MULTIPLE ASSETS. *International Journal of Theoretical & Applied Finance,* 7, 1031-1068.

BRANDT, M. W. & DIEBOLD, F. X. 2003. A no-arbitrage approach to range-based estimation of return covariances and correlations. National Bureau of Economic Research.

BRANDT, M. W. & JONES, C. S. 2006. Volatility forecasting with range-based EGARCH models. *Journal of Business & Economic Statistics,* 24, 470-486.

BREIDT, F. J., CRATO, N. & DE LIMA, P. 1998. The detection and estimation of long memory in stochastic volatility. *Journal of econometrics,* 83, 325-348.

BROWN, M. B. & FORSYTHE, A. B. 1974. Robust tests for the equality of variances. *Journal of the American Statistical Association,* 69, 364-367.

BRUNNERMEIER, M. K. 2009. Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic perspectives,* 23, 77-100.

BYSTRÖM, H. N. E. 2004. Managing extreme risks in tranquil and volatile markets using conditional extreme value theory. *International Review of Financial Analysis,* 13, 133-152.

CAO, C. Q. & TSAY, R. S. 1992. Nonlinear time-series analysis of stock volatilities. *Journal of Applied Econometrics,* 7, S165-S185.

CAPORALE, G. M. & GIL-ALANA, L. A. 2004. Fractional cointegration and real exchange rates. *Review of Financial Economics,* 13, 327-340.

CATANACH JR, A. H. & RAGATZ, J. A. 2010. 2008 Market Crisis: Black Swan, Perfect Storm or Tipping Point? *Bank Accounting & Finance,* 23, 20-26.

CAVALCANTE, J. & ASSAF, A. 2004. Long range dependence in the returns and volatility of the Brazilian stock market. *European Review of Economics and Finance,* 3, 22.

CAVALIERE, G. 2005. Unit root tests under time-varying variances. *Econometric Reviews,* 23, 259-292.

CAVALIERE, G. & TAYLOR, A. R. 2007. Testing for unit roots in time series models with non-stationary volatility. *Journal of Econometrics,* 140, 919-947.

CEROVIĆ, J. & KARADŽIĆ, V. 2015. Extreme value theory in emerging markets: Evidence from the Montenegrin stock exchange. *Economic Annals,* 60, 87-116.

CEROVIĆ, J., LIPOVINA-BOŽOVIĆ, M. & VUJOŠEVIĆ, S. 2015. A Comparative Analysis of Value at Risk Measurement on Emerging Stock Markets: Case of Montenegro. *Business Systems Research, Vol 6, Iss 1, Pp 36-55 (2015)*, 36.

CHARFEDDINE, L. & GUÉGAN, D. 2012. Breaks or long memory behavior: An empirical investigation. *Physica A: Statistical Mechanics and its Applications,* 391, 5712-5726.

CHAVEZ-DEMOULIN, V., DAVISON, A. C. & MCNEIL, A. J. 2005. Estimating Value-at-Risk: A Point Process Approach. *Quantitative Finance,* 5, 227-234.

CHEN, C. & TIAO, G. C. 1990. Random level-shift time series models, ARIMA approximations, and level-shift detection. *Journal of Business & Economic Statistics,* 8, 83-97.

CHEN, Q., GILES, D. E. & FENG, H. 2012. The Extreme-Value Dependence between the Chinese and Other International Stock Markets. *Applied Financial Economics,* 22, 1147-1160.

CHI, L. 2008. Credit Crisis, Asian Style. *International Economy,* 22, 74-77.

CHOI, D. F., FANG, V. & FU, T. Y. 2010. Volatility spillovers between New Zealand stock market returns and exchange rate changes before and after the 1997 Asian financial crisis. *Asian Journal of Finance & Accounting,* 1.

CHOLLETE, L., DE LA PEÑA, V. & LU, C.-C. 2012. International diversification: An extreme value approach. *Journal of Banking and Finance,* 36, 871-885.

CHRISTENSEN, K. & PODOLSKIJ, M. 2007. Realized range-based estimation of integrated variance. *Journal of Econometrics,* 141, 323-349.

CHRISTODOULAKIS, G. A. & SATCHELL, S. E. 2005. Forecast Evaluation in the Presence of Unobserved Volatility. *Econometric Reviews,* 23, 175-198.

CHRISTOFFERSEN, P. F. & DIEBOLD, F. X. 2000. How Relevant Is Volatility Forecasting for Financial Risk Management? *Review of Economics and Statistics,* 82, 12-22.

CHUNG, H. 2005. The contagious effects of the Asian financial crisis: some evidence from ADR and country funds. *Journal of Multinational financial management,* 15, 67-84.

CHUNHACHINDA, P., DANDAPANI, K., HAMID, S. & PRAKASH, A. J. 1997. Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking & Finance,* 21, 143-167.

CLAESSENS, S., DASGUPTA, S. & GLEN, J. 1995. Return behavior in emerging stock markets. *The World Bank Economic Review*, 131-151.
CLARK, E. & BACCAR, S. 2018. Modelling credit spreads with time volatility, skewness, and kurtosis. *Annals of Operations Research,* 262, 431-461.
COLES, S., HEFFERNAN, J. & TAWN, J. 1999. Dependence measures for extreme value analyses. *Extremes,* 2, 339-365.
COMMISSION, W. 2009. The Warwick Commission on international financial reform: in praise of unlevel playing fields. *Coventry: University of Warwick*.
COMTE, F. & RENAULT, E. 1998. Long memory in continuous-time stochastic volatility models. *Mathematical finance,* 8, 291-323.
CORBAE, D. & OULIARIS, S. 1986. Robust tests for unit roots in the foreign exchange market. *Economics Letters,* 22, 375-380.
CORNELL, B. 1977. Spot rates, forward rates and exchange market efficiency. *Journal of financial Economics,* 5, 55-65.
CORSI, F., MITTNIK, S., PIGORSCH, C. & PIGORSCH, U. 2008. The Volatility of Realized Volatility. *Econometric Reviews,* 27, 46-78.
COTTER, J. 2004. Downside risk for European equity markets. *Applied Financial Economics,* 14, 707-716.
COTTER, J. 2006. Extreme value estimation of boom and crash statistics. *The European Journal of Finance,* 12, 553-566.
COTTER, J. 2007. Varying the VaR for unconditional and conditional environments. *Journal of International Money and Finance,* 26, 1338-1354.
COTTER, J. & DOWD, K. 2011. Extreme global equity market risk. *Journal of Derivatives & Hedge Funds,* 17, 313-325.
COWEN, D. J. & ABUAF, D. 2010. Capital at Risk--A More Consistent and Intuitive Measure of Risk. *Journal of Financial Transformation*, 22-24.
CROTTY, J. 2009. Structural causes of the global financial crisis: a critical assessment of the 'new financial architecture'. *Cambridge Journal of Economics,* 33, 563-580.
CUMBY, R. E. & OBSTFELD, M. 1980. Exchange-rate expectations and nominal interest differentials: A test ofthe fisher hypothesis. National Bureau of Economic Research Cambridge, Mass., USA.

DA SILVA, A. & DE MELO MENDES, B. V. 2003. Value-at-risk and extreme returns in Asian stock markets. *International Journal of Business,* 8, 17-40.

DANIELSSON, J. & DE VRIES, C. G. 2000. Value-at-risk and extreme returns. *Annales d'Economie et de Statistique*, 239-270.

DANÍELSSON, J. & MORIMOTO, Y. 2000. *Forecasting extreme financial risk: a critical analysis of practical methods for the Japanese market*, Institute for Monetary and Economic Studies, Bank of Japan.

DAVIDSON, J. & SIBBERTSEN, P. 2005. Generating schemes for long memory processes: regimes, aggregation and linearity. *Journal of econometrics,* 128, 253-282.

DE JESÚS, R. & ORTIZ, E. 2011. Risk in emerging stock markets from Brazil and Mexico: Extreme value theory and alternative value at risk models.

DE MELO MENDES, B. V. & DE SOUZA, R. M. 2004. Measuring financial risks with copulas. *International Review of Financial Analysis,* 13, 27-45.

DEN HAAN, W. J. & LEVIN, A. 1998. Vector autoregressive covariance matrix estimation. *University of California, San Diego, manuscript*.

DEO, R. S. & HURVICH, C. M. 2001. On the log periodogram regression estimator of the memory parameter in long memory stochastic volatility models. *Econometric Theory,* 17, 686-710.

DIAMANDIS, P. F. & DRAKOS, A. A. 2011. Financial liberalization, exchange rates and stock prices: Exogenous shocks in four Latin America countries. *Journal of Policy Modeling,* 33, 381-394.

DICKEY, D. A. & FULLER, W. A. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association,* 74, 427-431.

DICKEY, D. A. & FULLER, W. A. 1981. Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica: Journal of the Econometric Society*, 1057-1072.

DIEBOLD, F. X. & INOUE, A. 2001. Long memory and regime switching. *Journal of econometrics,* 105, 131-159.

DIEBOLD, F. X., SCHUERMANN, T. & STROUGHAIR, J. D. 2000. Pitfalls and opportunities in the use of extreme value theory in risk management. *The Journal of Risk Finance,* 1**,** 30-35.
DIEOBOLD, F. X. 1986. Modeling the persistence of conditional variances: A comment. *Econometric Reviews,* 5**,** 51-56.
DIMITRAKOPOULOS, D. N., KAVUSSANOS, M. G. & SPYROU, S. I. 2010. Value at risk models for volatile emerging markets equity portfolios. *The Quarterly Review of Economics and Finance,* 50**,** 515-526.
DING, J. & MEADE, N. 2010. Forecasting accuracy of stochastic volatility, GARCH and EWMA models under different volatility scenarios. *Applied Financial Economics,* 20**,** 771-783.
DING, Z., GRANGER, C. W. & ENGLE, R. F. 1993a. A long memory property of stock market returns and a new model. *Journal of empirical finance,* 1**,** 83-106.
DING, Z., GRANGER, C. W. & ENGLE, R. F. J. J. O. E. F. 1993b. A long memory property of stock market returns and a new model. 1**,** 83-106.
DOMOWITZ, I., GLEN, J. & MADHAVAN, A. 1998. Country and currency risk premia in an emerging market. *Journal of Financial and Quantitative Analysis,* 33**,** 189-216.
DORNBUSCH, R. & FISCHER, S. 1980. Exchange rates and the current account. *The American Economic Review,* 70**,** 960-971.
DUFRÉNOT, G., LARDIC, S., MATHIEU, L., MIGNON, V. & PEGUIN-FEISSOLLE, A. 2008. Explaining the European exchange rates deviations: Long memory or non-linear adjustment? *Journal of International Financial Markets, Institutions and Money,* 18**,** 207-215.
ECONOMIST, T. 2013. The rich cousin. Available: https://www.economist.com/news/special-report/21570842-oil-makes-norway-different-rest-region-only-up-point-rich.
EMBRECHTS, P., KLÜPPELBERG, C. & MIKOSCH, T. 1997. *Modelling extremal events: for insurance and finance*, Springer Science & Business Media.
EMBRECHTS, P., RESNICK, S. I. & SAMORODNITSKY, G. 1999. Extreme value theory as a risk management tool. *North American Actuarial Journal,* 3**,** 30-41.

ENGLE, R. F. & NG, V. K. 1993. Measuring and testing the impact of news on volatility. *The journal of finance,* 48, 1749-1778.
ERGEN, I. 2014. Tail dependence and diversification benefits in emerging market stocks: an extreme value theory approach. *Applied Economics,* 46, 2215-2227.
ESTRADA, J. 2008. Investing in emerging markets: A black swan perspective.
ESTRADA, J. & VARGAS, M. 2015. Black swans, beta, risk, and return.
FAMA, E. F. 1965. The behavior of stock-market prices. *The journal of Business,* 38, 34-105.
FERNANDEZ, V. 2005. Risk management under extreme events. *International Review of Financial Analysis,* 14, 113-148.
FEUNOU, B. & OKOU, C. 2019. Good Volatility, Bad Volatility, and Option Pricing. *Journal of Financial & Quantitative Analysis,* 54, 695-727.
FISHER, R. A. & TIPPETT, L. H. C. Limiting forms of the frequency distribution of the largest or smallest member of a sample. Mathematical Proceedings of the Cambridge Philosophical Society, 1928. Cambridge Univ Press, 180-190.
FLYNN, D. B., GROSE, S. D., MARTIN, G. M. & MARTIN, V. L. 2005. PRICING AUSTRALIAN S&P200 OPTIONS: A BAYESIAN APPROACH BASED ON GENERALIZED DISTRIBUTIONAL FORMS.
FORSBERG, L. & BOLLERSLEV, T. 2002. Bridging the gap between the distribution of realized (ECU) volatility and ARCH modelling (of the Euro): the GARCH-NIG model. *Journal of Applied Econometrics,* 17, 535-548.
FRANKEL, J. 1983. Monetary and portfolio-balance models of exchange rate determination, in economic interdependence and flexible exchange rates. *MIT Press, Cambridge, MA*.
FRANKEL, J. A. & ROSE, A. K. 1996. Currency crashes in emerging markets: An empirical treatment. *Journal of international Economics,* 41, 351-366.
FUERTES, A.-M., IZZELDIN, M. & KALOTYCHOU, E. 2009. On forecasting daily stock volatility: The role of intraday information and market conditions. *International Journal of Forecasting,* 25, 259-281.

FULLER, W. A., HASZA, D. P. & GOEBEL, J. J. 1981. Estimation of the parameters of stochastic difference equations. *The Annals of Statistics*, 531-543.

FURIÓ, D. & CLIMENT, F. J. 2013. Extreme value theory versus traditional GARCH approaches applied to financial data: a comparative evaluation. *Quantitative Finance,* 13, 45-63.

GALBRAITH, J. W. & ZERNOV, S. 2009. Extreme dependence in the NASDAQ and S&P 500 composite indexes. *Applied Financial Economics,* 19, 1019-1028.

GENCAY, R. & SELCUK, F. 2004. Extreme value theory and Value-at-Risk: Relative performance in emerging markets. *International Journal of Forecasting,* 20, 287-303.

GERLACH, R., WALPOLE, D. & WANG, C. 2017. Semi-parametric Bayesian tail risk forecasting incorporating realized measures of volatility. *Quantitative Finance,* 17, 199-215.

GEWEKE, J. & PORTER-HUDAK, S. 1983. The estimation and application of long memory time series models. *Journal of time series analysis,* 4, 221-238.

GHORBEL, A. & TRABELSI, A. 2008. Predictive performance of conditional extreme value theory in value-at-risk estimation. *International Journal of Monetary Economics and Finance,* 1, 121-148.

GILLI, M. 2006. An application of extreme value theory for measuring financial risk. *Computational Economics,* 27, 207-228.

GONG, H., THAVANESWARAN, A. & SINGH, J. 2010. A BLACK-SCHOLES MODEL WITH GARCH VOLATILITY. *Mathematical Scientist,* 35.

GOURIEROUX, C. & JASIAK, J. 2001. Memory and infrequent breaks. *Economics Letters,* 70, 29-41.

GRANGER, C. W. & HYUNG, N. 1999. Occasional structural breaks and long memory.

GRANGER, C. W. & HYUNG, N. 2004. Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns. *Journal of empirical finance,* 11, 399-421.

GRANGER, C. W. J. & DING, Z. 1995. Some Properties of Absolute Return: An Alternative Measure of Risk. *Annales d'Économie et de Statistique*, 67-91.

GRIGOLETTO, M. & LISI, F. 2009. Looking for skewness in financial time series. *The Econometrics Journal,* 12, 310-323.
GUIDOLIN, M. & TIMMERMANN, A. 2008. International asset allocation under regime switching, skew, and kurtosis preferences. *The Review of Financial Studies,* 21, 889-935.
HAKKIO, C. S. 1981. The term structure of the forward premium. *Journal of Monetary Economics,* 8, 41-58.
HAMORI, S. & TOKIHISA, A. 1997. Testing for a unit root in the presence of a variance shift1. *Economics Letters,* 57, 245-253.
HAN, Y. W. 2016. Quantitative Comparisons on the Intrinsic Features of Foreign Exchange Rates between the 1920s and the 2010s: Case of the USD-GDP Exchange Rate.
HANSEN, B. E. 1995. Regression with nonstationary volatility. *Econometrica: Journal of the Econometric Society*, 1113-1132.
HANSEN, L. P. & HODRICK, R. J. 1980. Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *journal of Political Economy,* 88, 829-853.
HANSEN, P. R. & LUNDE, A. 2006. Consistent ranking of volatility models. *Journal of Econometrics,* 131, 97-121.
HARVEY, A., RUIZ, E. & SHEPHARD, N. 1994. Multivariate stochastic variance models. *The Review of Economic Studies,* 61, 247-264.
HARVEY, C. R. 1995. Predictable risk and returns in emerging markets. *Review of Financial studies,* 8, 773-816.
HARVEY, C. R. & SIDDIQUE, A. 1999. Autoregressive conditional skewness. *Journal of financial and quantitative analysis,* 34, 465-487.
HASZA, D. P. & FULLER, W. A. 1979. Estimation for autoregressive processes with unit roots. *The Annals of Statistics,* 7, 1106-1120.
HERRERA, R. & SCHIPP, B. 2013. Value at risk forecasts by extreme value models in a conditional duration framework. *Journal of Empirical Finance,* 23, 33-47.
HERRERA, R. & SCHIPP, B. 2014. Statistics of extreme events in risk management: The impact of the subprime and global financial crisis on the German stock market. *The North American Journal of Economics and Finance,* 29, 218-238.

HESTON, S. L. 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The review of financial studies,* 6, 327-343.

HILAL, S., POON, S.-H. & TAWN, J. 2011. Hedging the black swan: Conditional heteroskedasticity and tail dependence in S&P500 and VIX. *Journal of Banking & Finance,* 35, 2374-2387.

HILLEBRAND, E. 2005. Neglecting parameter changes in GARCH models. *Journal of Econometrics,* 129, 121-138.

HOOD, M. & MALIK, F. 2018. Estimating downside risk in stock returns under structural breaks. *International Review of Economics & Finance,* 58, 102-112.

HOQUE, M., UPPAL, J. Y. & ULLAH MANGLA, I. 2013. Extreme loss risk in financial turbulence–evidence from the global financial crisis. *Managerial Finance,* 39, 653-666.

HWANG, S., SATCHELL, S. E. & PEREIRA, P. L. V. 2007. How persistent is volatility? An answer with stochastic volatility models with markov regime switching state equations. *Journal of Business Finance & Accounting,* 34, 1002-24.

HYUNG, N. & DE VRIES, C. G. 2007. Portfolio selection with heavy tails. *Journal of Empirical Finance,* 14, 383-400.

IGLESIAS, E. M. 2012. An analysis of extreme movements of exchange rates of the main currencies traded in the Foreign Exchange market. *Applied Economics,* 44, 4631-4637.

INCI, A. C. & LEE, B. S. 2014. Dynamic Relations between Stock Returns and Exchange Rate Changes. *European Financial Management,* 20, 71-106.

INCLAN, C. & TIAO, G. C. 1994. Use of cumulative sums of squares for retrospective detection of changes of variance. *Journal of the American Statistical Association,* 89, 913-923.

IORGULESCU, F. 2015. Investigating contagion and market interdependence during the global financial crisis. *Central European Business Review,* 4, 31.

JAVED, F. 2011. Sensitivity of the causality in variance test to the GARCH (1, 1) parameters.

JI, P. I. & IN, F. 2010. The impact of the global financial crisis on the cross-currency linkage of LIBOR–OIS spreads. *Journal of*

International Financial Markets, Institutions and Money, 20, 575-589.

JONDEAU, E. & ROCKINGER, M. 2003a. Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements. *Journal of Economic dynamics and Control,* 27, 1699-1737.

JONDEAU, E. & ROCKINGER, M. 2003b. Testing for differences in the tails of stock-market returns. *Journal of Empirical Finance,* 10, 559-581.

JONDEAU, E. & ROCKINGER, M. 2004. Optimal Portfolio Allocation Under Higher Moments, forthcoming in Journal of the European Financial Management Association.

JORION, P. 1990. The exchange-rate exposure of US multinationals. *Journal of business*, 331-345.

JUDGE, G. G., HILL, R. C., GRIFFITHS, W., LUTKEPOHL, H. & LEE, T. C. 1982. Introduction to the Theory and Practice of Econometrics.

JUNG, R. C. & MADERITSCH, R. 2014. Structural breaks in volatility spillovers between international financial markets: Contagion or mere interdependence? *Journal of Banking & Finance,* 47, 331-342.

KABUNDI, A. & MUTEBA, J. M. 2011. Extreme Value at Risk: A Scenario for Risk Management. *South African Journal of Economics,* 79, 173-183.

KALLBERG, J. G., LIU, C. H. & PASQUARIELLO, P. 2005. An examination of the Asian crisis: regime shifts in currency and equity markets. *The Journal of Business,* 78, 169-211.

KAMINSKY, G. L. & SCHMUKLER, S. L. 1999. What triggers market jitters?: A chronicle of the Asian crisis. *Journal of international money and Finance,* 18, 537-560.

KANAS, A. 2000. Volatility spillovers between stock returns and exchange rate changes: International evidence. *Journal of Business Finance & Accounting,* 27, 447-467.

KANG, S. H. & YOON, S.-M. 2007. Long memory properties in return and volatility: Evidence from the Korean stock market. *Physica A: Statistical Mechanics and its Applications,* 385, 591-600.

KARANASOS, M., PARASKEVOPOULOS, A., ALI, F. M., KAROGLOU, M. & YFANTI, S. 2014a. Modelling Returns and Volatilities During Financial Crises: a Time Varying Coefficient Approach.
KARANASOS, M., PARASKEVOPOULOS, A. G., MENLA ALI, F., KAROGLOU, M. & YFANTI, S. 2014b. Modelling stock volatilities during financial crises: A time varying coefficient approach. *Journal of Empirical Finance,* 29, 113-128.
KARANASOS, M., YFANTI, S. & KAROGLOU, M. 2016. Multivariate FIAPARCH modelling of financial markets with dynamic correlations in times of crisis. *International Review of Financial Analysis,* 45, 332-349.
KARIMI, M. & VOIA, M. 2014. Identifying extreme values of exchange market pressure. *Empirical Economics,* 48, 1055-1078.
KARIYA, T., TSUKUDA, Y. & MARU, J. 1990. *Testing the random walk hypothesis for Japanese stock prices in S. Taylor's model*, HGB Alexander Research Foundation, University of Chicago.
KARMAKAR, M. & SHUKLA, G. K. 2015. Managing extreme risk in some major stock markets: An extreme value approach. *International Review of Economics & Finance,* 35, 1-25.
KAROGLOU, M. 2006a. On the detection of structural changes in volatility dynamics with applications. University of Leicester.
KAROGLOU, M. 2006b. The size and power of the CUSUM-type tests in detecting structural changes in financial markets volatility dynamics. *mimeograph, University of Leicester, Leicester*.
KAROGLOU, M. 2010a. Breaking down the non-normality of stock returns. *The European journal of finance,* 16, 79-95.
KAROGLOU, M. 2010b. Breaking down the non-normality of stock returns. *European Journal of Finance,* 16, 79.
KENNEDY, S. Australia’s response to the global financial crisis. Speech delivered at Australia Israel Leadership Forum, 2009.
KEYNES, J. M. 2016. *General theory of employment, interest and money*, Atlantic Publishers & Dist.
KILIĈ, R. 2004. On the long memory properties of emerging capital markets: evidence from Istanbul stock exchange. *Applied Financial Economics,* 14, 915-922.
KIM, K. & SCHMIDT, P. 1993. Unit root tests with conditional heteroskedasticity. *Journal of Econometrics,* 59, 287-300.

KIM, T.-H., LEYBOURNE, S. & NEWBOLD, P. 2002. Unit root tests with a break in innovation variance. *Journal of Econometrics,* 109**,** 365-387.

KITTIAKARASAKUN, J. & TSE, Y. 2011. Modeling the fat tails in Asian stock markets. *International Review of Economics & Finance,* 20**,** 430-440.

KOČENDA, E. 2005. Beware of breaks in exchange rates: evidence from European transition countries. *Economic Systems,* 29**,** 307-324.

KOEDIJK, K. G., SCHAFGANS, M. M. & DE VRIES, C. G. 1990. The tail index of exchange rate returns. *Journal of international economics,* 29**,** 93-108.

KOKOSZKA, P. & LEIPUS, R. 2000. Change-point estimation in ARCH models. *Bernoulli,* 6**,** 513-539.

KOSEOGLU, S. D. & CEVIK, E. I. 2013. Testing for causality in mean and variance between the stock market and the foreign exchange market: An application to the major central and eastern European countries. *Finance a Uver,* 63**,** 65.

KUMAR, D. & MAHESWARAN, S. 2013. Asymmetric long memory volatility in the PIIGS economies. *Review of Accounting and Finance,* 12**,** 23-43.

KUNSCH, H. R. Statistical aspects of self-similar processes. Proceedings of the First World Congress of the Bernoulli Society, 1987, 1987. VNU Science Press, 67-74.

KWIATKOWSKI, D., PHILLIPS, P. C., SCHMIDT, P. & SHIN, Y. 1992. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of econometrics,* 54**,** 159-178.

LAHMIRI, S., UDDIN, G. S. & BEKIROS, S. 2017. Clustering of short and long-term co-movements in international financial and commodity markets in wavelet domain. *Physica A: Statistical Mechanics and its Applications,* 486**,** 947-955.

LAMOUREUX, C. G. & LASTRAPES, W. D. 1990. Persistence in variance, structural change, and the GARCH model. *Journal of Business & Economic Statistics,* 8**,** 225-234.

LEADBETTER, M., LINDGREN, G. & ROOTZÉN, H. Extremes and Related Properties of Random Sequences and Processes. 1983. Springer, New York.

LEBARON, B. & SAMANTA, R. 2005. Extreme value theory and fat tails in equity markets.

LEDFORD, A. W. & TAWN, J. A. 1996. Statistics for near independence in multivariate extreme values. *Biometrika,* 83, 169-187.

LEDFORD, A. W. & TAWN, J. A. 1997. Modelling dependence within joint tail regions. *Journal of the Royal Statistical Society: Series B (Statistical Methodology),* 59, 475-499.

LEDFORD, A. W. & TAWN, J. A. 1998. Concomitant tail behaviour for extremes. *Advances in applied Probability,* 30, 197-215.

LEÓN, Á., RUBIO, G. & SERNA, G. 2005. Autoregresive conditional volatility, skewness and kurtosis. *The Quarterly Review of Economics and Finance,* 45, 599-618.

LEVENE, H. 1960. Contributions to Probability and Statistics 278–292. Stanford Univ. Press.

LI, H. & HONG, Y. 2011. Financial volatility forecasting with range-based autoregressive volatility model. *Finance Research Letters,* 8, 69-76.

LIN, C.-H. & KAO, T.-C. 2008. Multiple structural changes in the tail behavior: evidence from stock index futures returns. *Nonlinear Analysis: Real World Applications,* 9, 1702-1713.

LIN, H., WANG, J. & WU, C. 2011. Liquidity risk and expected corporate bond returns. *Journal of Financial Economics,* 99, 628-650.

LINDAAS, O. A. & PETTERSEN, K. A. 2016. Risk analysis and Black Swans: two strategies for de-blackening. *Journal of Risk Research,* 19, 1231-1245.

LING, S., LI, W. & MCALEER, M. 2003. Estimation and testing for unit root processes with GARCH (1, 1) errors: theory and Monte Carlo evidence. *Econometric Reviews,* 22, 179-202.

LIU, H.-C., CHIANG, S.-M. & CHENG, N. Y.-P. 2012. Forecasting the volatility of S&P depositary receipts using GARCH-type models under intraday range-based and return-based proxy measures. *International Review of Economics & Finance,* 22, 78-91.

LIU, M. 2000. Modeling long memory in stock market volatility. *Journal of Econometrics,* 99, 139-171.

LLEO, S. & ZIEMBA, W. T. 2015. The Swiss black swan bad scenario: Is Switzerland another casualty of the Eurozone crisis? *International Journal of Financial Studies,* 3, 351-380.

LOBATO, I. N. 1999. A semiparametric two-step estimator in a multivariate long memory model. *Journal of Econometrics,* 90, 129-153.

LOBATO, I. N. & SAVIN, N. E. 1998. Real and spurious long-memory properties of stock-market data. *Journal of Business & Economic Statistics,* 16, 261-268.

LONGIN, F. M. 2000. From value at risk to stress testing: The extreme value approach. *Journal of Banking & Finance,* 24, 1097-1130.

MACIEL, L. D. S. & BALLINI, R. 2017. Value-at-risk modeling and forecasting with range-based volatility models: empirical evidence / Modelagem e previsão do valor em risco com modelos de volatilidade baseada em variação: evidências empíricas. *Revista Contabilidade & Finanças*, 361.

MAGHYEREH, A. I. & AL-ZOUBI, H. A. 2006. Value-at-risk under extreme values: the relative performance in MENA emerging stock markets. *international journal of managerial finance,* 2, 154-172.

MANDELBROT, B. B. 1963. The variation of certain speculative prices. *Fractals and Scaling in Finance.* Springer.

MARSH, T. & PFLEIDERER, P. 2012. " Black Swans" and the Financial Crisis. *Review of Pacific Basin Financial Markets and Policies,* 15, 1250008.

MARTENS, M., DE POOTER, M. & VAN DIJK, D. J. 2004. Modeling and forecasting S&P 500 volatility: Long memory, structural breaks and nonlinearity.

MARTENS, M. & VAN DIJK, D. 2007. Measuring volatility with the realized range. *Journal of Econometrics,* 138, 181-207.

MCNEIL, A. J. & FREY, R. 2000. Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of empirical finance,* 7, 271-300.

MEESE, R. A. & SINGLETON, K. J. 1982. On unit roots and the empirical modeling of exchange rates. *the Journal of Finance,* 37, 1029-1035.

MIKOSCH, T. & STARICA, C. 1998. *Change of structure in financial time series, long range dependence and the GARCH model*, Citeseer.
MIZEN, P. 2008. The credit crunch of 2007-2008: a discussion of the background, market reactions, and policy responses. *Federal Reserve Bank of St. Louis Review,* 90.
MORANA, C. & BELTRATTI, A. 2004. Structural change and long-range dependence in volatility of exchange rates: either, neither or both? *Journal of Empirical Finance,* 11, 629-658.
MUN, K.-C. 2008. Effects of exchange rate fluctuations on equity market volatility and correlations: evidence from the Asian financial crisis. *Quarterly Journal of Finance and Accounting*, 77-102.
MUSSA, M. Empirical regularities in the behavior of exchange rates and theories of the foreign exchange market. Carnegie-Rochester Conference Series on Public Policy, 1979. Elsevier, 9-57.
NEWEY, W. K. & WEST, K. D. 1994. Automatic lag selection in covariance matrix estimation. *The Review of Economic Studies,* 61, 631-653.
OLSON, E., MILLER, S. & WOHAR, M. E. 2012. “Black Swans” before the “Black Swan” evidence from international LIBOR–OIS spreads. *Journal of International Money and Finance,* 31, 1339-1357.
OZUN, A., CIFTER, A. & YıLMAZER, S. 2010. Filtered extreme-value theory for value-at-risk estimation: evidence from Turkey. *The Journal of Risk Finance,* 11, 164-179.
PAGAN, A. R. & SCHWERT, G. W. 1990. Alternative models for conditional stock volatility. *Journal of econometrics,* 45, 267-290.
PANTULA, S. G. 1991. Asymptotic distributions of unit-root tests when the process is nearly stationary. *Journal of Business & Economic Statistics,* 9, 63-71.
PARK, Y.-H. 2015. Volatility-of-volatility and tail risk hedging returns. *Journal of Financial Markets,* 26, 38-63.
PARKINSON, M. 1980. The extreme value method for estimating the variance of the rate of return. *Journal of business*, 61-65.
PATTON, A. J. 2011. Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics,* 160, 246-256.

PATTON, A. J. & SHEPPARD, K. 2015. Good volatility, bad volatility: Signed jumps and the persistence of volatility. *Review of Economics and Statistics,* 97, 683-697.

PEREIRA, P. L. V. 2004. How persistent is volatility? An answer with stochastic volatility models with Markov regime switching state equations.

PERRON, P. 1989. The great crash, the oil price shock, and the unit root hypothesis. *Econometrica: Journal of the Econometric Society*, 1361-1401.

PERRON, P. 1990. Testing for a unit root in a time series with a changing mean. *Journal of Business & Economic Statistics,* 8, 153-162.

PERRON, P. & VOGELSANG, T. J. 1992. Testing for a unit root in a time series with a changing mean: corrections and extensions. *Journal of Business & Economic Statistics,* 10, 467-470.

PESARAN, M. H. & TIMMERMANN, A. 2004. How costly is it to ignore breaks when forecasting the direction of a time series? *International Journal of Forecasting,* 20, 411-425.

PHILLIPS, P. C. & PERRON, P. 1988. Testing for a unit root in time series regression. *Biometrika,* 75, 335-346.

PICKANDS, J. 1975. Statistical inference using extreme order statistics. *the Annals of Statistics*, 119-131.

POON, S.-H., ROCKINGER, M. & TAWN, J. 2003. Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *Review of financial studies,* 17, 581-610.

PREMARATNE, G. & BERA, A. K. 2000. Modeling asymmetry and excess kurtosis in stock return data.

QUINTOS, C., FAN, Z. & PHILLIPS, P. C. 2001. Structural change tests in tail behaviour and the Asian crisis. *The Review of Economic Studies,* 68, 633-663.

RAJU, K. & RANGASWAMY, S. 2017. Forecasting volatility in the Indian equity market using return and range-based models. *Applied Economics,* 49, 5027-5039.

RODRIGUES, P. M. & RUBIA, A. 2007. Testing for causality in variance under nonstationarity in variance. *Economics Letters,* 97, 133-137.

RUIZ, E. 1994. Quasi-maximum likelihood estimation of stochastic volatility models.

RYDÉN, T., TERÄSVIRTA, T. & ÅSBRINK, S. 1998. Stylized facts of daily return series and the hidden Markov model. *Journal of applied econometrics*, 217-244.

SAID, S. E. & DICKEY, D. A. 1985. Hypothesis testing in ARIMA (p, 1, q) models. *Journal of the American Statistical Association,* 80, 369-374.

SAMUEL, Y. M. Z. T. 2007. Value at risk and conditional extreme value theory via Markov regime switching models. *Journal of Futures Markets,* 28, 155-181.

SANSÓ, A., ARAGÓ, V. & CARRION, J. L. 2004. Testing for changes in the unconditional variance of financial time series. *Revista de Economía financiera,* 4, 32-53.

SCHICH, S. 2004. European stock market dependencies when price changes are unusually large. *Applied Financial Economics,* 14, 165-177.

SCHWERT, G. W. 2002. Tests for unit roots: A Monte Carlo investigation. *Journal of Business & Economic Statistics,* 20, 5-17.

SEABERG, R. B. 2009. A Flock of Black Swans. *Journal of Financial Planning,* 22, 64-70.

SEO, B. 1999. Distribution theory for unit root tests with conditional heteroskedasticity1. *Journal of Econometrics,* 91, 113-144.

SEYMOUR, A. J. & POLAKOW, D. A. 2003. A Coupling of Extreme-Value Theory and Volatility Updating with Value-at-Risk Estimation in Emerging Markets: A South African Test. *Multinational Finance Journal,* 7, 3-23.

SHESKIN, D. J. 2003. *Handbook of parametric and nonparametric statistical procedures*, crc Press.

SIEGEL, S. & TUKEY, J. W. 1960. A nonparametric sum of ranks procedure for relative spread in unpaired samples. *Journal of the American Statistical Association,* 55, 429-445.

SKINNER, R. K. 2010. Black Swans, Fat Tails, and Extreme Values Visit Energy Risk. *Natural Gas & Electricity,* 26, 1-7.

SO, M. K. & CHAN, R. K. 2014. Bayesian analysis of tail asymmetry based on a threshold extreme value model. *Computational Statistics & Data Analysis,* 71, 568-587.

SO, M. K. & WONG, C.-M. 2012. Estimation of multiple period expected shortfall and median shortfall for risk management. *Quantitative Finance,* 12, 739-754.
SOKAL, R. & ROHLF, F. 1995. Biometry: the principles and practice of statistics in biological sciences. *WH Free Company, New York, USA*.
STARICA, C. & GRANGER, C. 2005. Nonstationarities in stock returns. *Review of economics and statistics,* 87, 503-522.
STOCK, J. H. 1994. Unit roots, structural breaks and trends. *Handbook of econometrics,* 4, 2739-2841.
STRAETMANS, S. & CHAUDHRY, S. M. 2015. Tail risk and systemic risk of US and Eurozone financial institutions in the wake of the global financial crisis. *Journal of International Money and Finance,* 58, 191-223.
TALEB, N. N. 2007a. *The black swan: The impact of the highly improbable*, Random house.
TALEB, N. N. 2007b. Black swans and the domains of statistics. *The American Statistician,* 61, 198-200.
TAYLOR, J. B. & WILLIAMS, J. C. 2009. A black swan in the money market. *American Economic Journal: Macroeconomics,* 1, 58-83.
TAYLOR, S. J. 1994. Modeling stochastic volatility: A review and comparative study. *Mathematical finance,* 4, 183-204.
TAYLOR, S. J. 2008. *Modelling financial time series*, world scientific.
THOMAS, L. 2009. Thriving Norway provides an economics lesson. *The New York Times,* 13.
THOMAS, M. & REISS, R. 1997. *Statistical analysis of extreme values*, Springer.
TOTIĆ, S. & BOŽOVIĆ, M. 2015. Tail risk in emerging markets of Southeastern Europe. *Applied Economics,* 48, 1785-1798.
TRYON, R. W. 1979. Testing for rational expectations in foreign exchange markets.
TURGUTLU, E. & UCER, B. 2010. Is global diversification rational? Evidence from emerging equity markets through mixed copula approach. *Applied Economics,* 42, 647-658.
UPPAL, J. Y. & ULLAH MANGLA, I. 2013. Extreme loss risk in financial turbulence–evidence from the global financial crisis. *Managerial Finance,* 39, 653-666.

VAN DIJK, D., OSBORN, D. R. & SENSIER, M. 2005. Testing for causality in variance in the presence of breaks. *Economics Letters,* 89, 193-199.
VEE, D. A. N. N. C., GONPOT, P. N. & SOOKIA, N. U. H. 2014. Forecasting value-at-risk for frontier stock market indexes using GARCH-type models and extreme value theory: model validation for dynamic models. *The Journal of Risk Model Validation,* 8, 47.
VERAART, A. E. & VERAART, L. A. 2012. Stochastic volatility and stochastic leverage. *Annals of Finance,* 8, 205-233.
VILASUSO, J. & KATZ, D. 2000. Estimates of the likelihood of extreme returns in international stock markets. *Journal of Applied Statistics,* 27, 119-130.
VODRA, R. E. 2009. Fat Tails, Black Swans, and Serial Crises: Planning for a Volatile World. *Journal of Financial Planning,* 22, 44-46.
WAGNER, N. 2003. Estimating financial risk under time-varying extremal return behavior. *OR Spectrum,* 25, 317-328.
WALID, C., CHAKER, A., MASOOD, O. & FRY, J. 2011. Stock market volatility and exchange rates in emerging countries: A Markov-state switching approach. *Emerging Markets Review,* 12, 272-292.
WANG, Z., WU, W., CHEN, C. & ZHOU, Y. 2010. The exchange rate risk of Chinese yuan: Using VaR and ES based on extreme value theory. *Journal of Applied Statistics,* 37, 265-282.
WARDMAN, J. K. & MYTHEN, G. 2016. Risk communication: against the Gods or against all odds? Problems and prospects of accounting for Black Swans. *Journal of Risk Research,* 19, 1220-1230.
WERNER, T. & UPPER, C. 2004. Time variation in the tail behavior of Bund future returns. *Journal of Futures Markets,* 24, 387-398.
WRIGHT, J. H. 1999. Testing for a unit root in the volatility of asset returns. *Journal of Applied Econometrics,* 14, 309-318.
WYPLOSZ, C. 1996. Contagious Currency Crises: First Tests. *Scandinavian Journal of Economics,* 98, 463-484.
YANG, S.-Y. & DOONG, S.-C. 2004. Price and volatility spillovers between stock prices and exchange rates: empirical evidence from the G-7 countries. *International Journal of Business and Economics,* 3, 139.

YAU, H.-Y. & NIEH, C.-C. 2006. Interrelationships among stock prices of Taiwan and Japan and NTD/Yen exchange rate. *Journal of Asian Economics,* 17, 535-552.

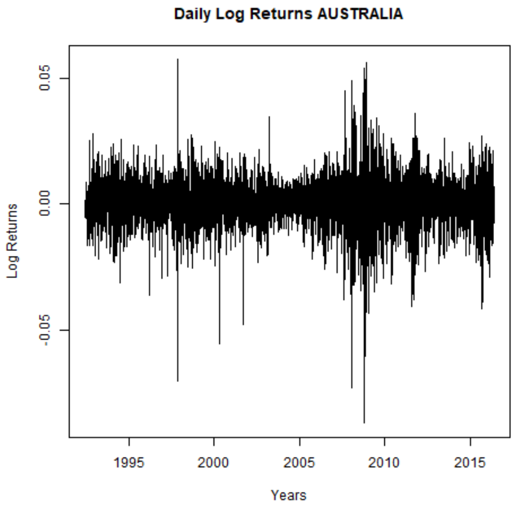
ZIKOVIC, S. & AKTAN, B. 2009. Global financial crisis and VaR performance in emerging markets: A case of EU candidate states-Turkey and Croatia.

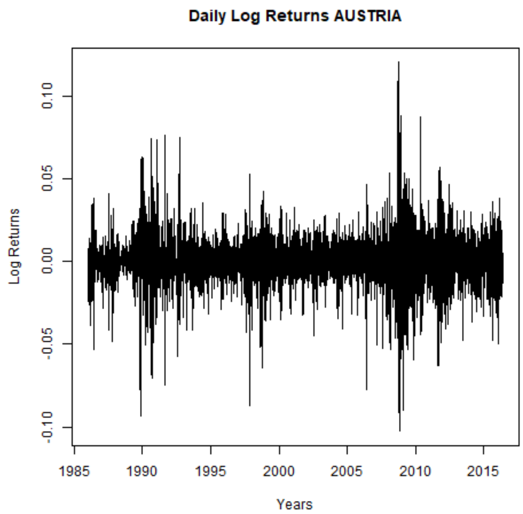
ZIVKOV, D., NJEGIC, J. & MILENKOVIC, I. 2015. Bidirectional Volatility Spillover Effect between the Exchange Rate and Stocks in the Presence of Structural Breaks in Selected Eastern European Economies. *Finance a Uver,* 65, 477.

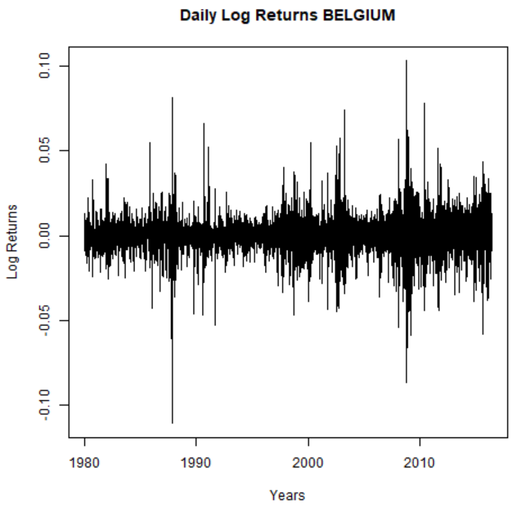
Appendices

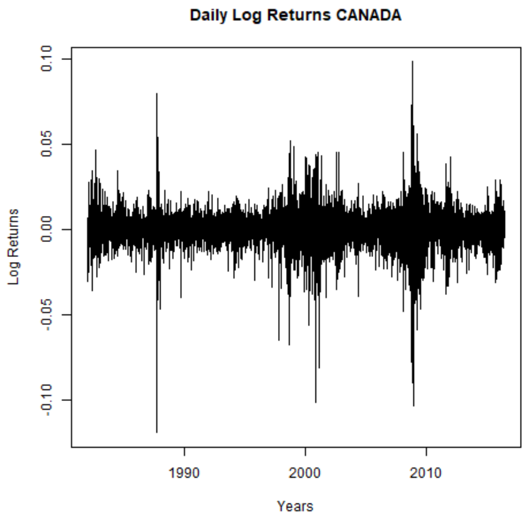
Appendix 1

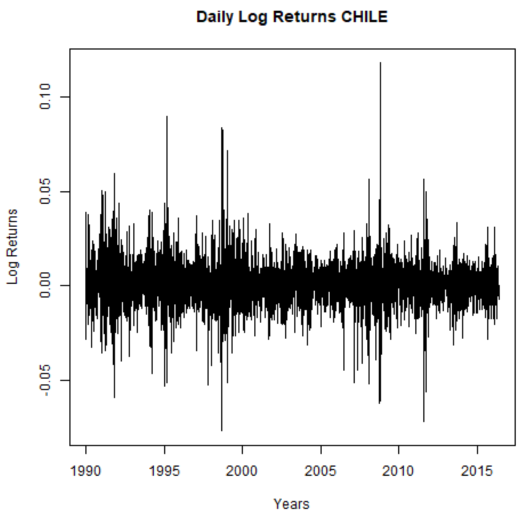
Time-series graphs of each series

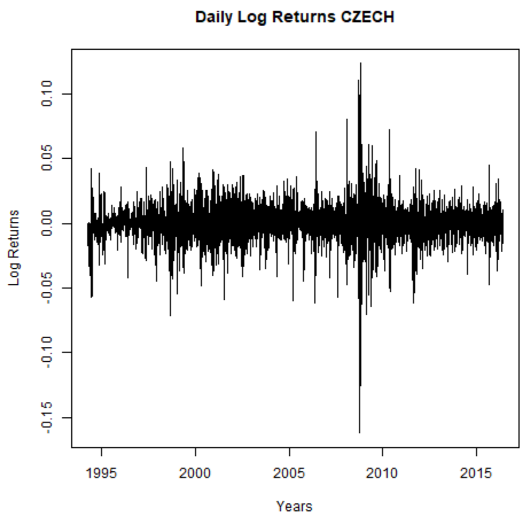


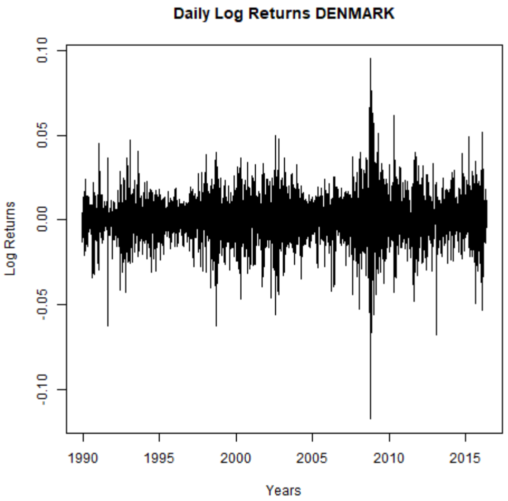


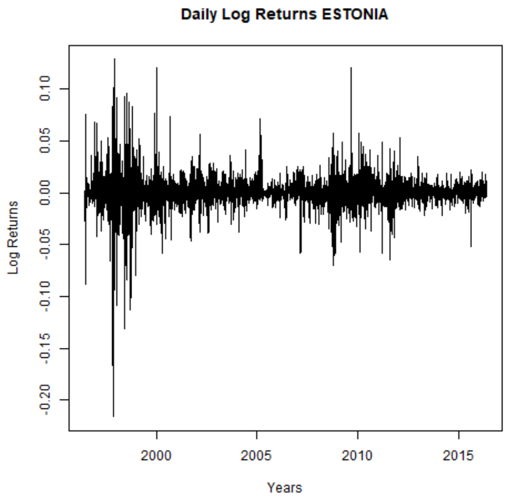


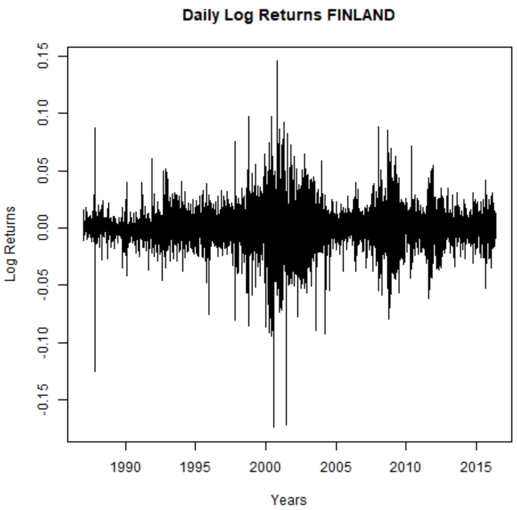


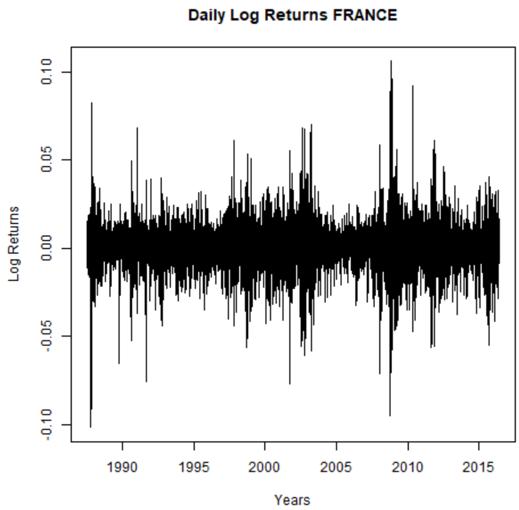


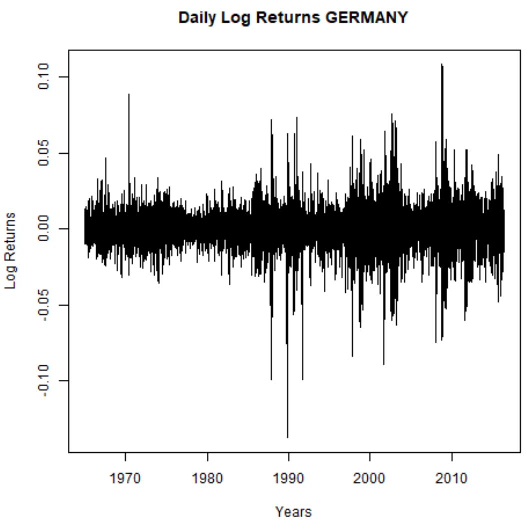


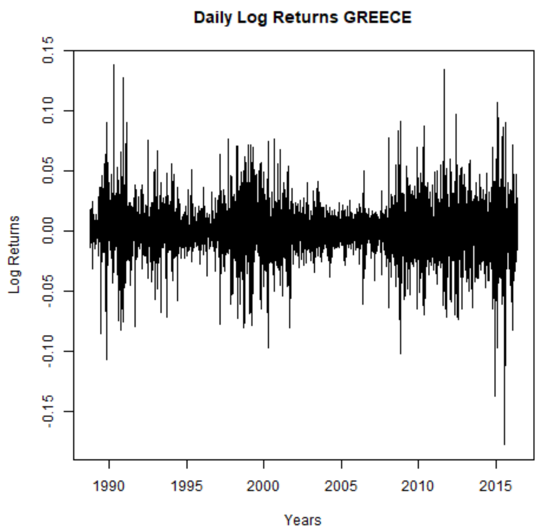

Daily Log Returns DENMARK
Log Returns
0.10
0.05
0.00
-0.05
-0.10
1990
1995
2000
2005
2010
2015
Years

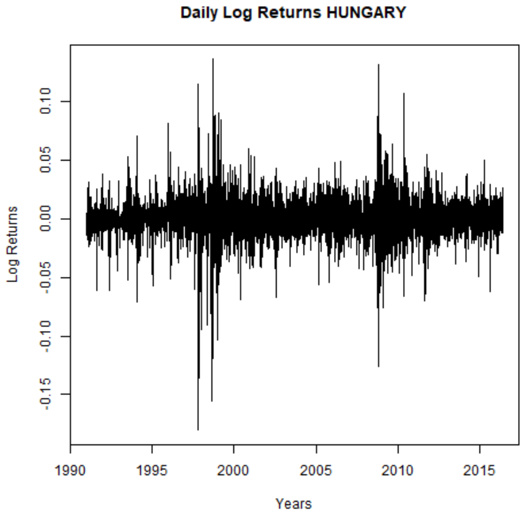

Daily Log Returns ESTONIA
Log Returns
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
2000
2005
2010
2015
Years

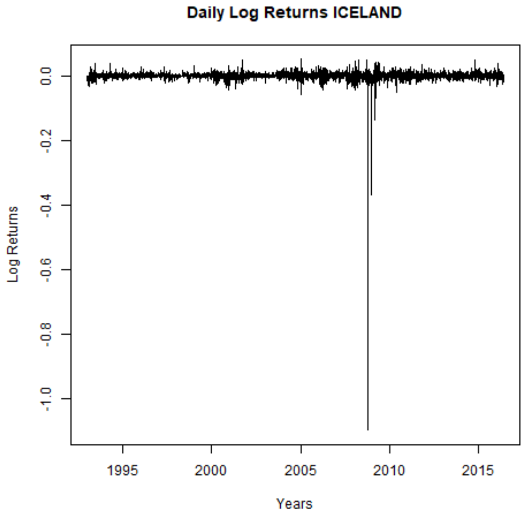

Daily Log Returns FINLAND
Log Returns
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
1990
1995
2000
2005
2010
2015
Years

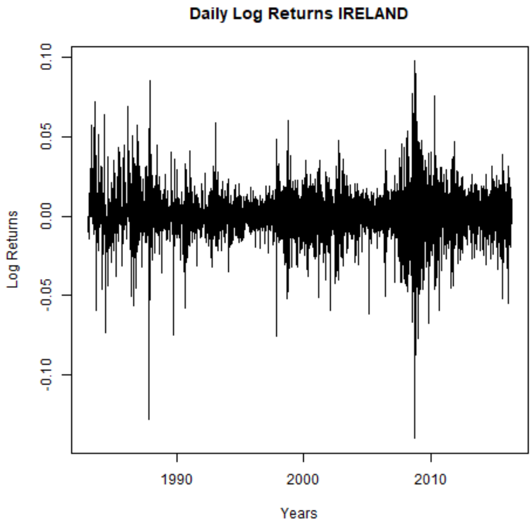

Daily Log Returns FRANCE
Log Returns
0.10
0.05
0.00
-0.05
-0.10
1990
1995
2000
2005
2010
2015
Years

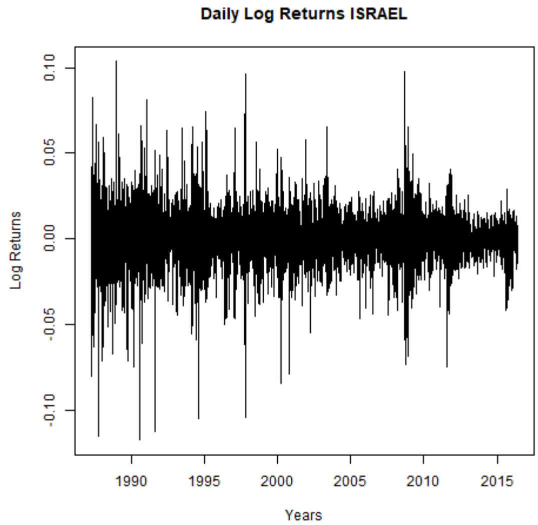

Daily Log Returns GERMANY
Log Returns
0.10
0.05
0.00
-0.05
-0.10
1970
1980
1990
2000
2010
Years

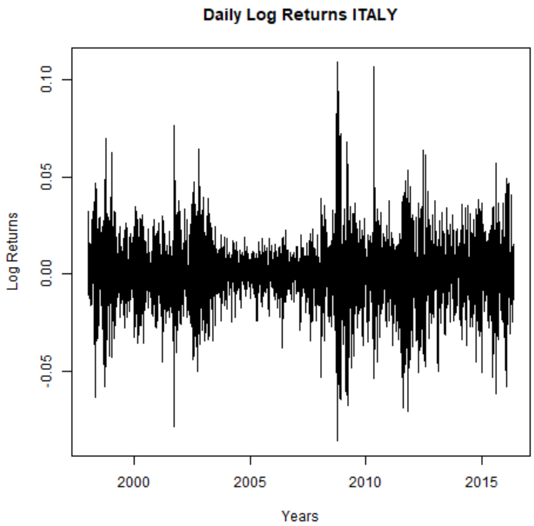

Daily Log Returns GREECE
Log Returns
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
1990
1995
2000
2005
2010
2015
Years

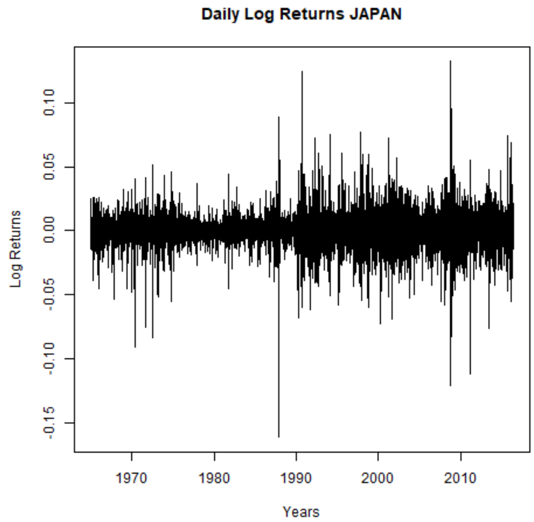

Daily Log Returns HUNGARY
Log Returns
Years

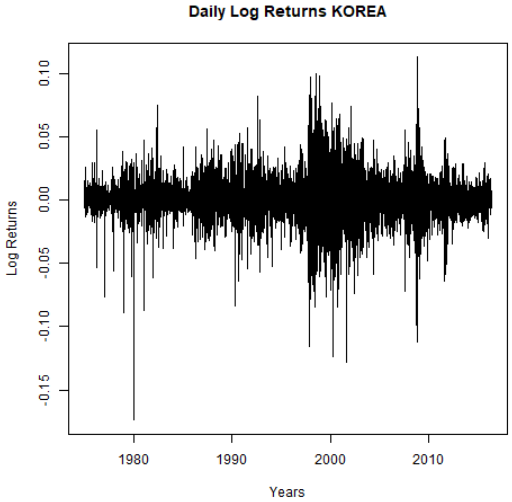

Daily Log Returns ICELAND
Log Returns
Years

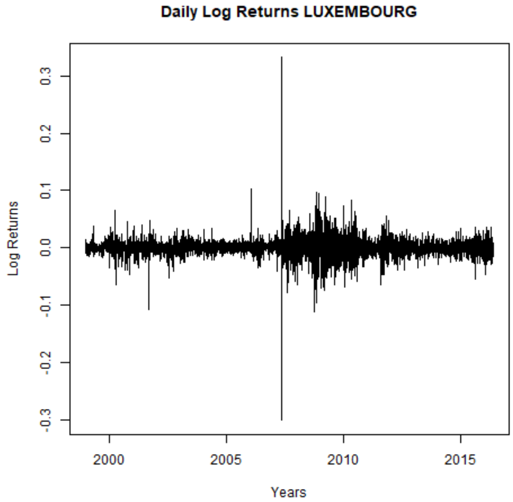

Daily Log Returns IRELAND
Log Returns
Years

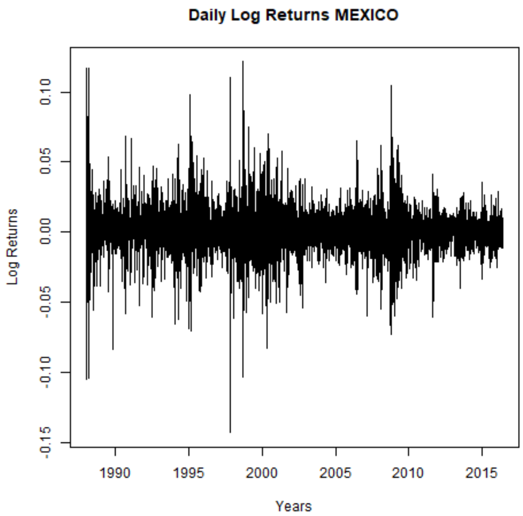

Daily Log Returns ISRAEL
Log Returns
Years

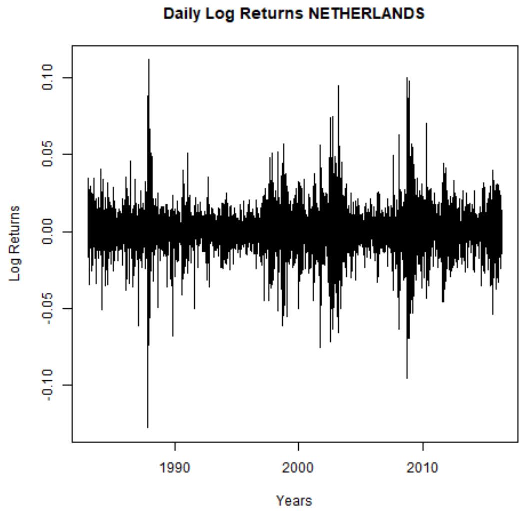

Daily Log Returns ITALY
Log Returns
Years

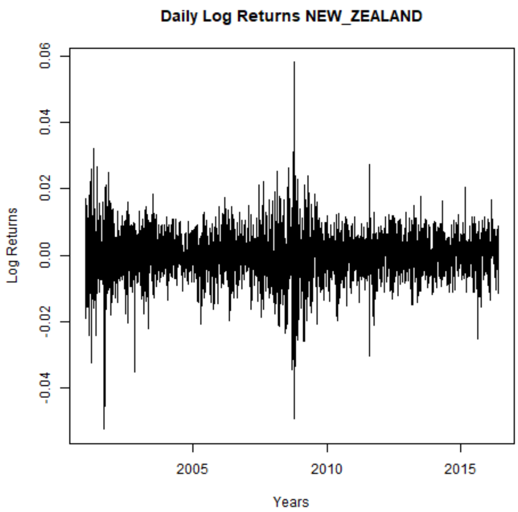

Daily Log Returns JAPAN
Log Returns
Years

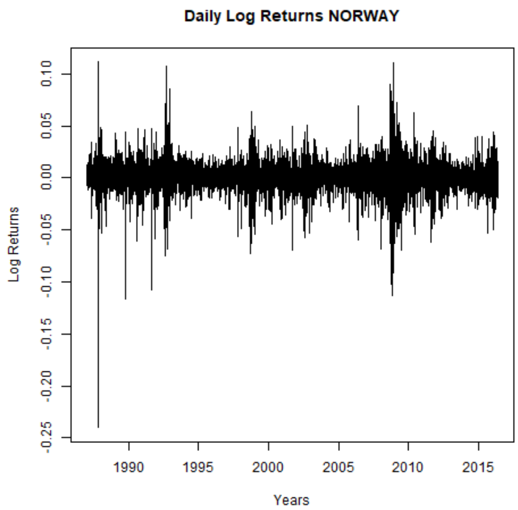

Daily Log Returns KOREA
Log Returns
Years
1980
1990
2000
2010
0.10
0.05
0.00
-0.05
-0.10
-0.15

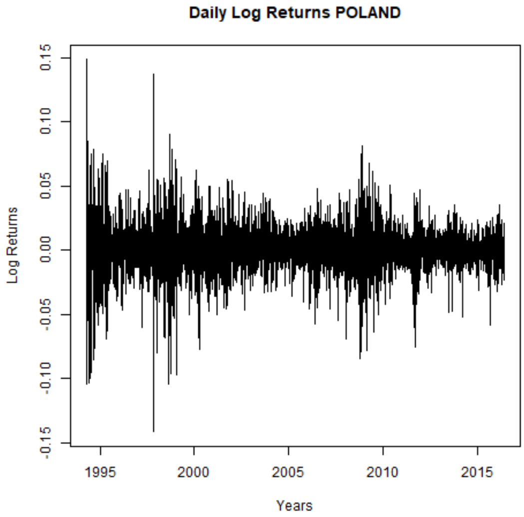

Daily Log Returns LUXEMBOURG
Log Returns
Years
2000
2005
2010
2015
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3

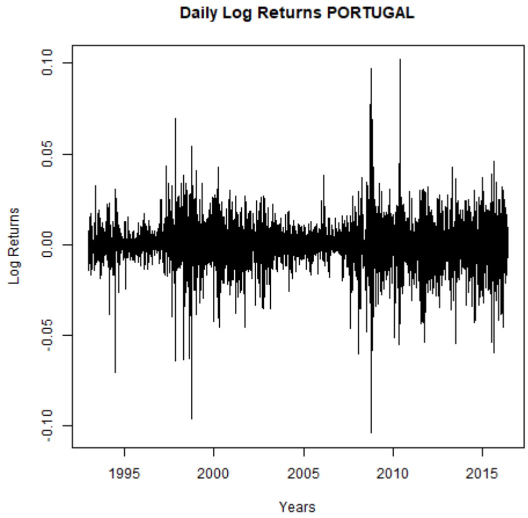

Daily Log Returns MEXICO
Log Returns
Years
1990
1995
2000
2005
2010
2015
0.10
0.05
0.00
-0.05
-0.10
-0.15

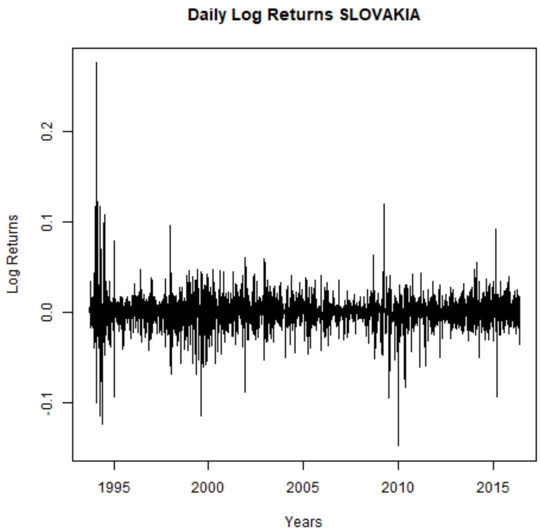

Daily Log Returns NETHERLANDS
Log Returns
Years
1990
2000
2010
0.10
0.05
0.00
-0.05
-0.10

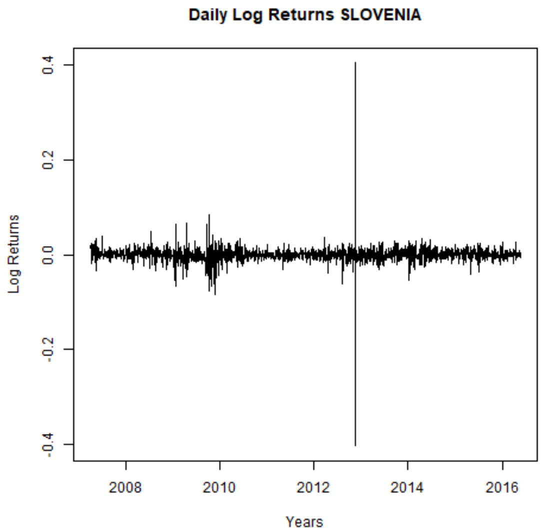

Daily Log Returns NEW_ZEALAND
Log Returns
Years
2005
2010
2015
0.06
0.04
0.02
0.00
-0.02
-0.04

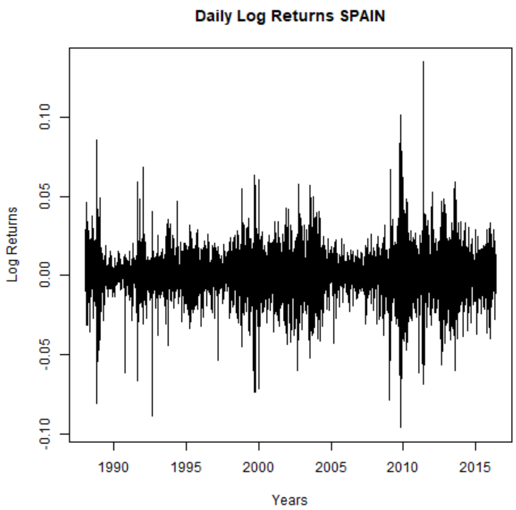

Daily Log Returns NORWAY
Log Returns
Years
1990
1995
2000
2005
2010
2015
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25

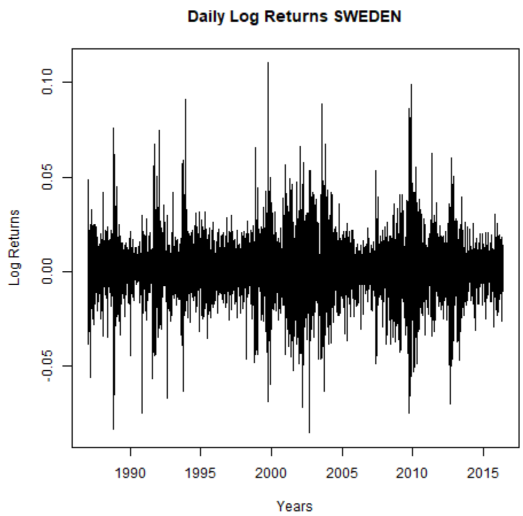

Daily Log Returns POLAND
Log Returns
Years
1995
2000
2005
2010
2015
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15

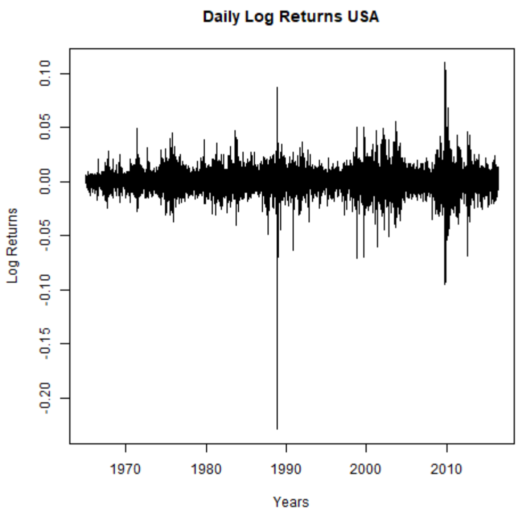

Daily Log Returns PORTUGAL
Log Returns
Years
1995
2000
2005
2010
2015
-0.10
-0.05
0.00
0.05
0.10

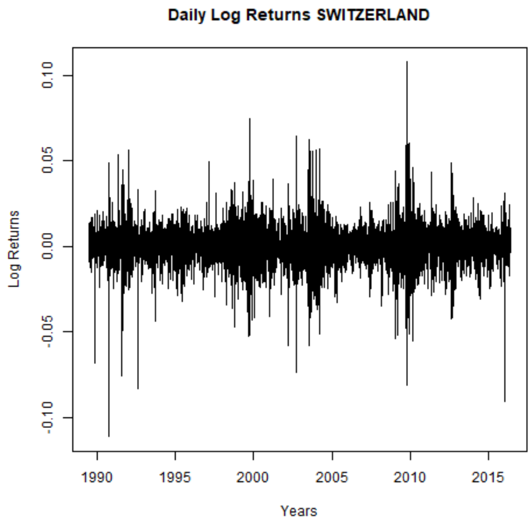

Daily Log Returns SLOVAKIA
Log Returns
Years
1995
2000
2005
2010
2015
-0.1
0.0
0.1
0.2

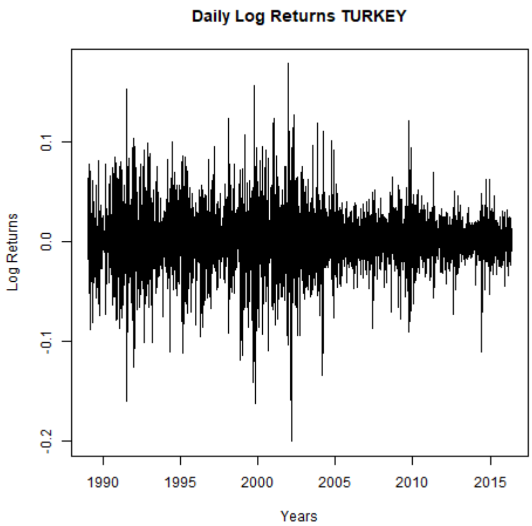

Daily Log Returns SLOVENIA
Log Returns
Years
2008
2010
2012
2014
2016
-0.4
-0.2
0.0
0.2
0.4

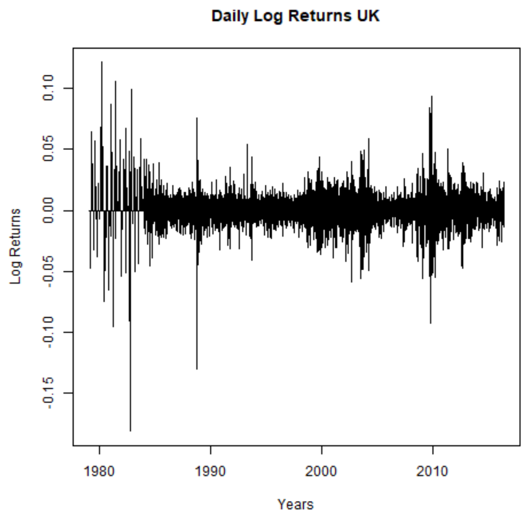

Daily Log Returns SPAIN
Log Returns
Years
1990
1995
2000
2005
2010
2015
-0.10
-0.05
0.00
0.05
0.10


Daily Log Returns SWEDEN
Log Returns
Years
1990
1995
2000
2005
2010
2015
-0.05
0.00
0.05
0.10





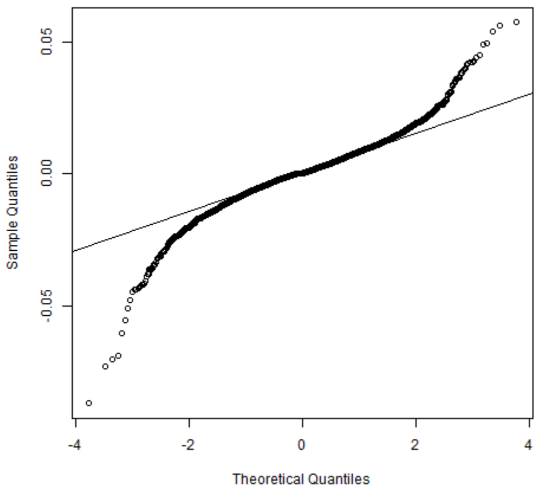




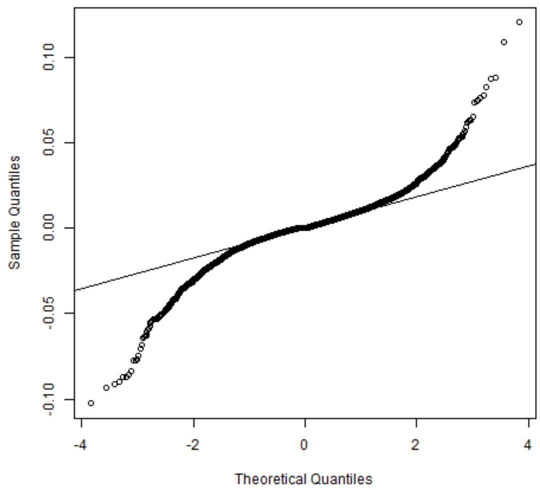
Appendix 2

QQ plots of daily log-returns

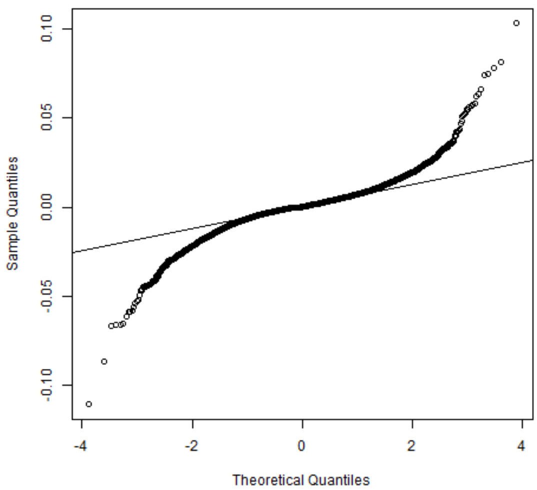
Q-Q plot AUSTRALIA



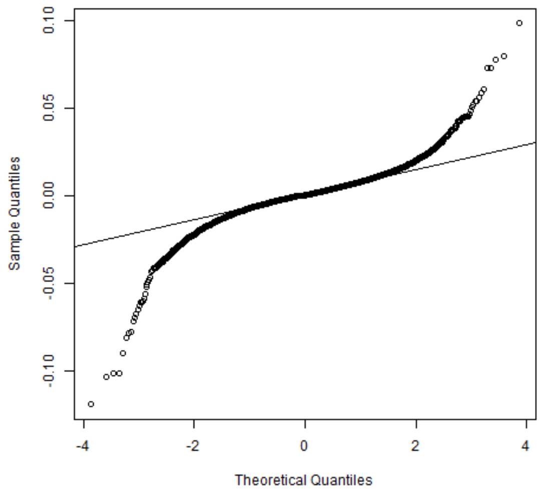
Q-Q plot AUSTRIA



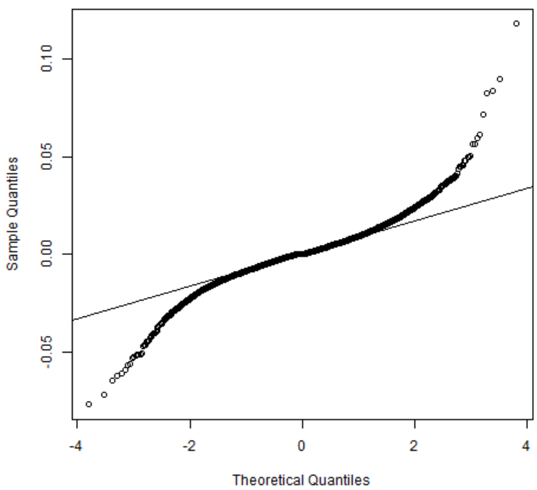
Q-Q plot BELGIUM



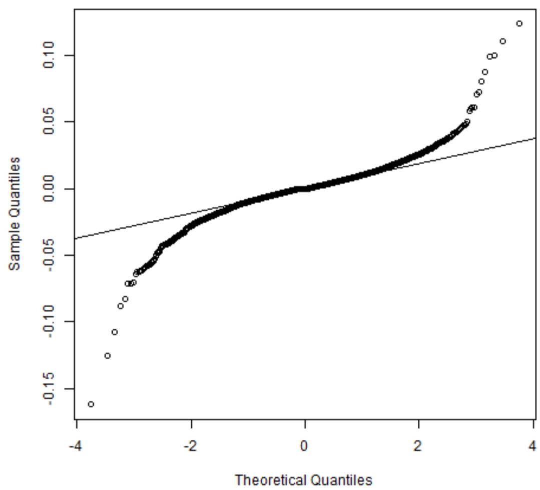
Q-Q plot CANADA

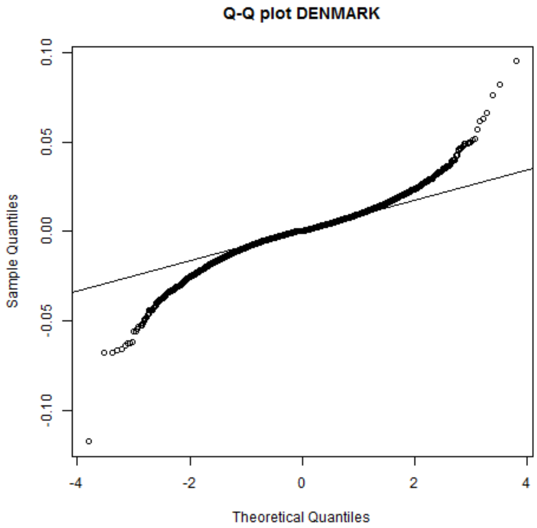


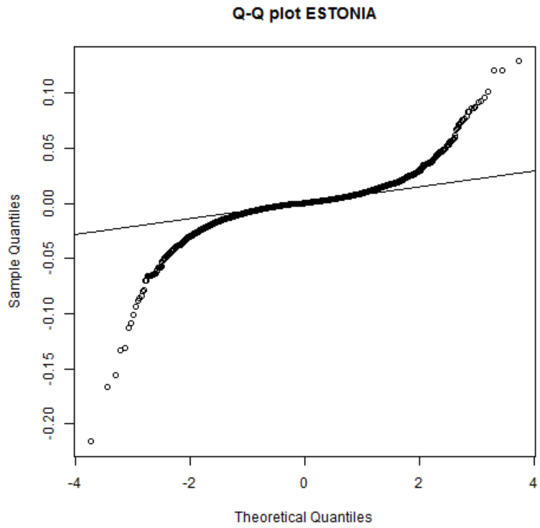
Q-Q plot CHILE

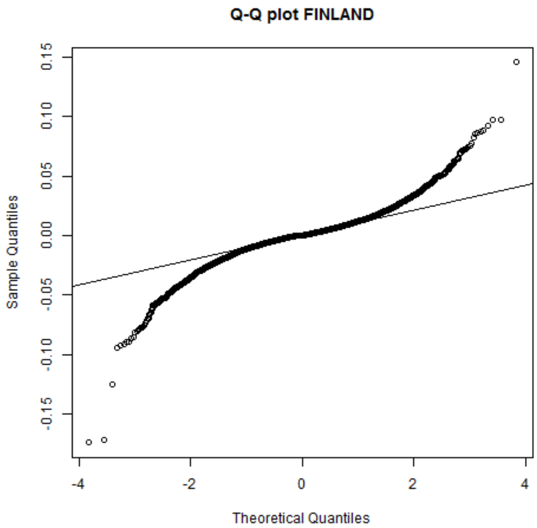


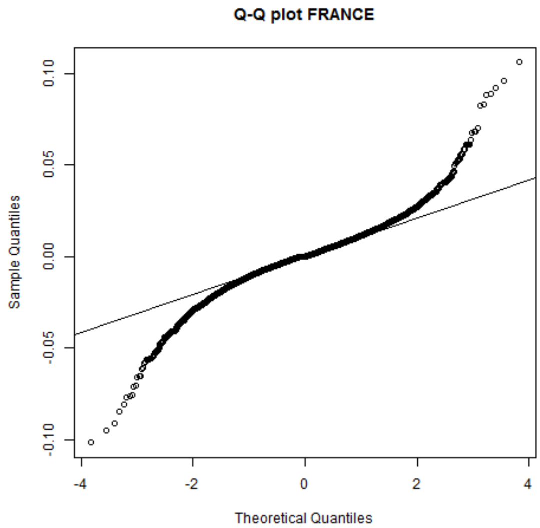
Q-Q plot CZECH

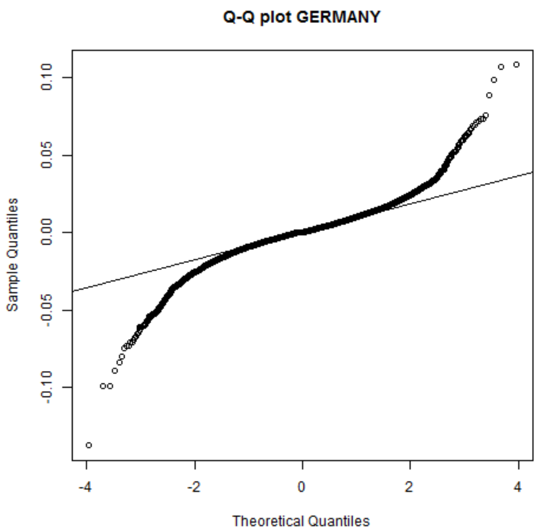


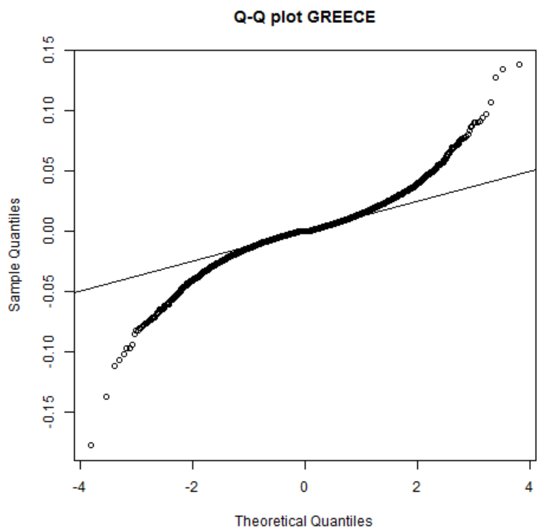

Q-Q plot DENMARK
Sample Quantiles
Theoretical Quantiles
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4

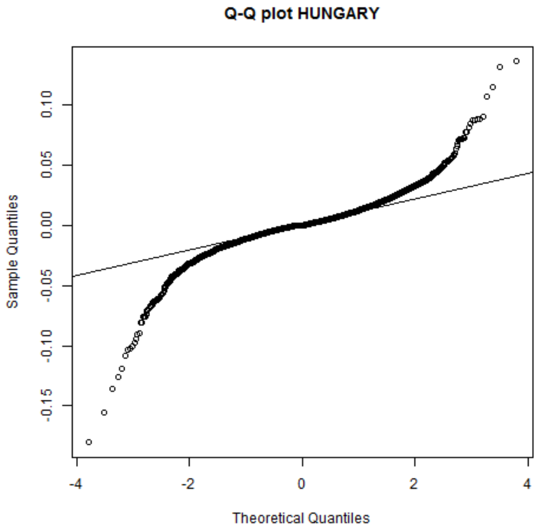

Q-Q plot ESTONIA
Sample Quantiles
Theoretical Quantiles
-0.20
-0.15
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4

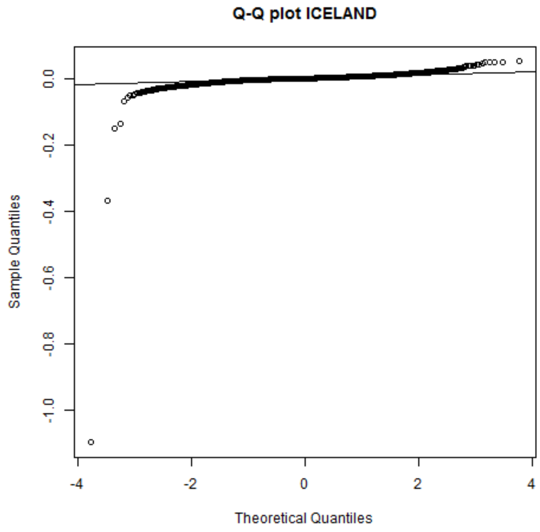

Q-Q plot FINLAND
Sample Quantiles
Theoretical Quantiles
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
-4
-2
0
2
4

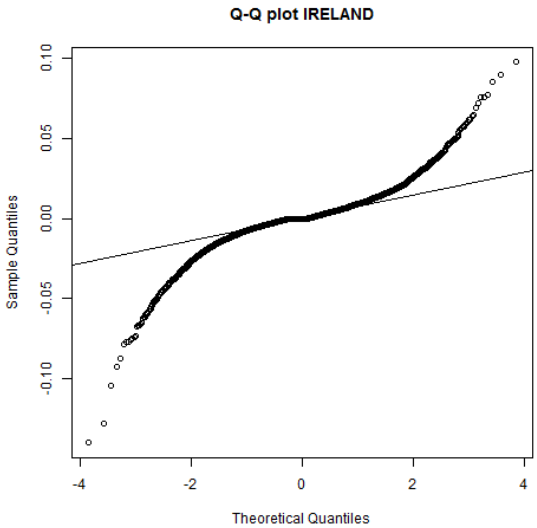

Q-Q plot FRANCE
Sample Quantiles
Theoretical Quantiles
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4

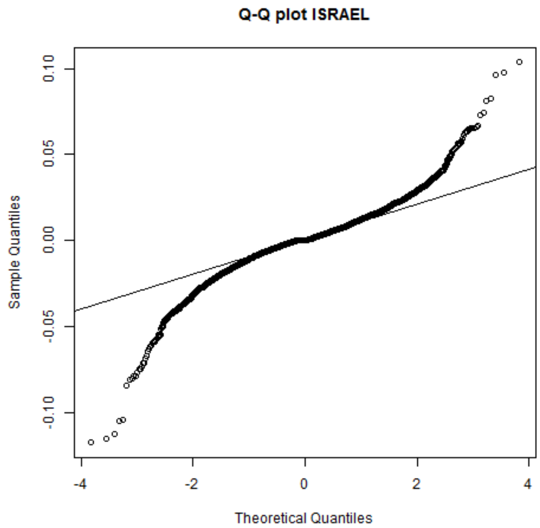

Q-Q plot GERMANY
Sample Quantiles
Theoretical Quantiles
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4

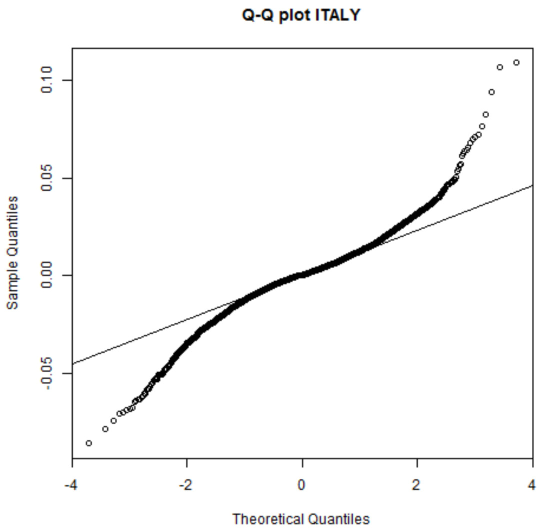

Q-Q plot GREECE
Sample Quantiles
Theoretical Quantiles
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
-4
-2
0
2
4

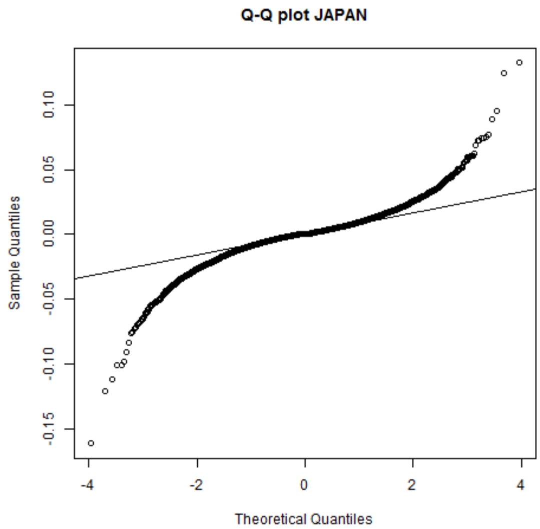

Q-Q plot HUNGARY
Sample Quantiles
-0.15 -0.10 -0.05 0.00 0.05 0.10
Theoretical Quantiles
-4 -2 0 2 4


Q-Q plot ICELAND
Sample Quantiles
-1.0 -0.8 -0.6 -0.4 -0.2 0.0
Theoretical Quantiles
-4 -2 0 2 4

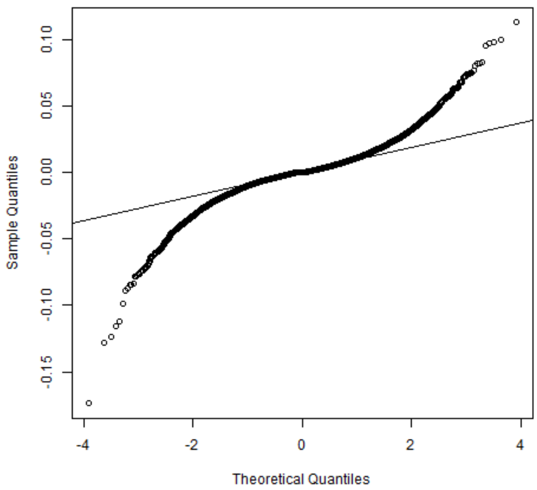

Q-Q plot IRELAND
Sample Quantiles
-0.10 -0.05 0.00 0.05 0.10
Theoretical Quantiles
-4 -2 0 2 4


Q-Q plot ISRAEL
Sample Quantiles
-0.10 -0.05 0.00 0.05 0.10
Theoretical Quantiles
-4 -2 0 2 4

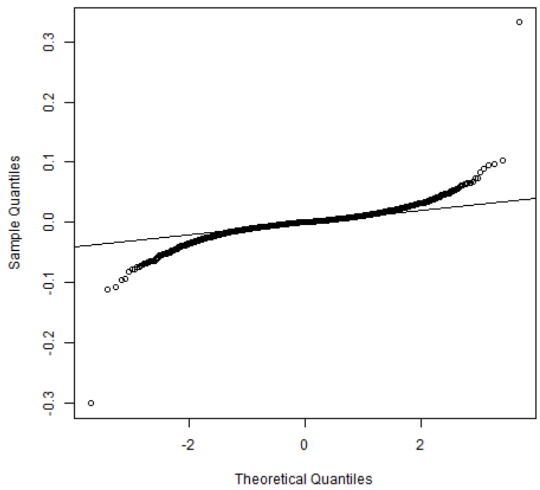

Q-Q plot ITALY
Sample Quantiles
-0.05 0.00 0.05 0.10
Theoretical Quantiles
-4 -2 0 2 4


Q-Q plot JAPAN
Sample Quantiles
-0.15 -0.10 -0.05 0.00 0.05 0.10
Theoretical Quantiles
-4 -2 0 2 4

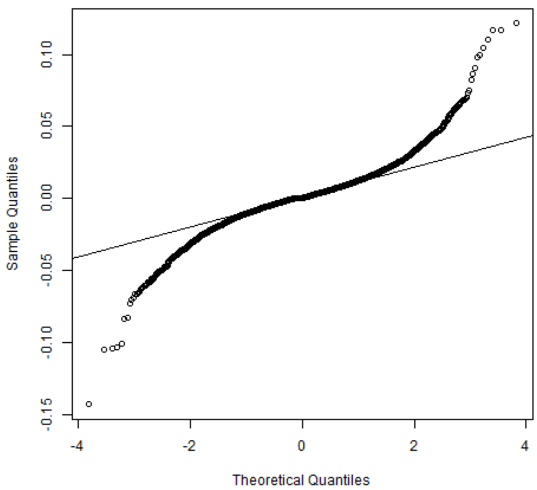
Q-Q plot KOREA



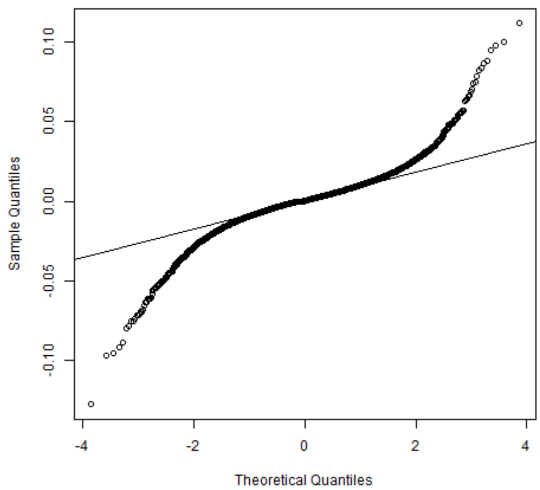
Q-Q plot LUXEMBOURG



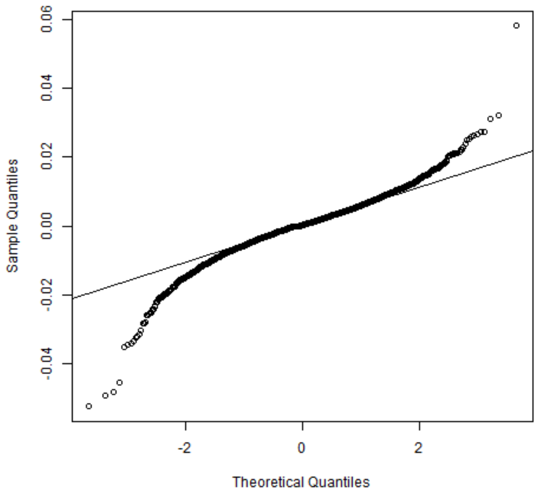
Q-Q plot MEXICO



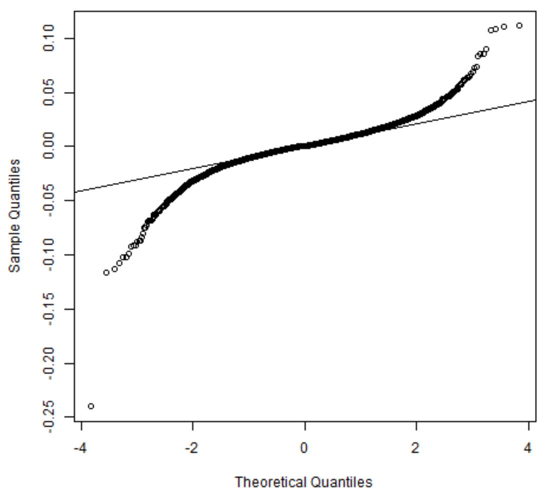
Q-Q plot NETHERLANDS

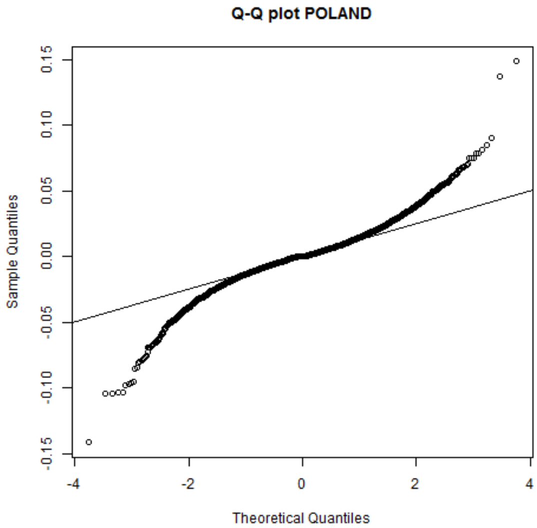
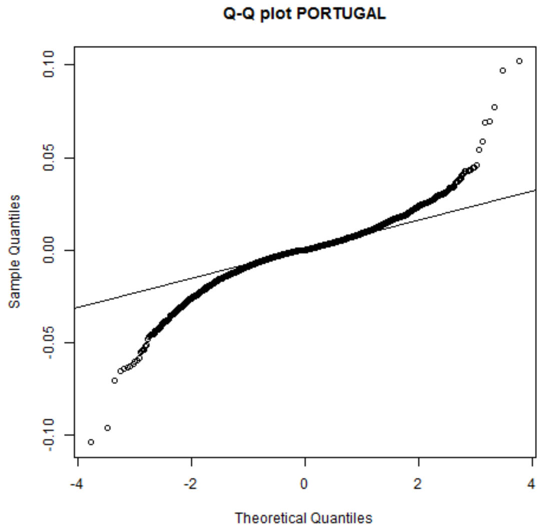
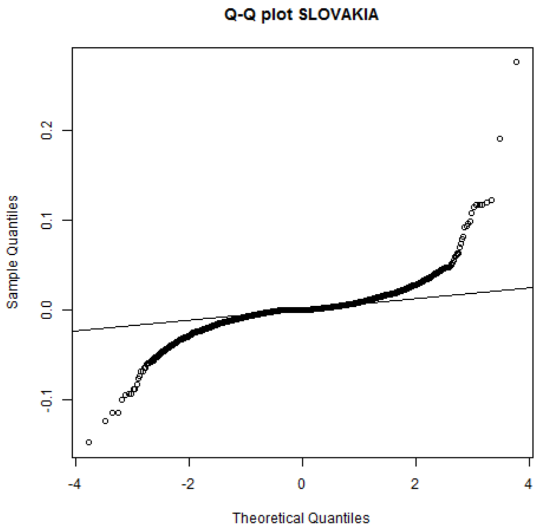
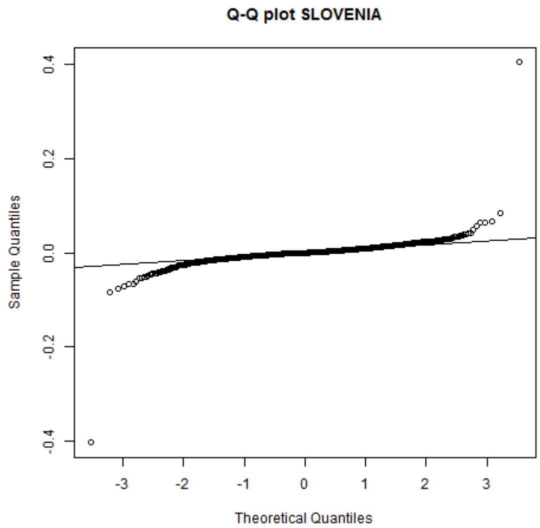
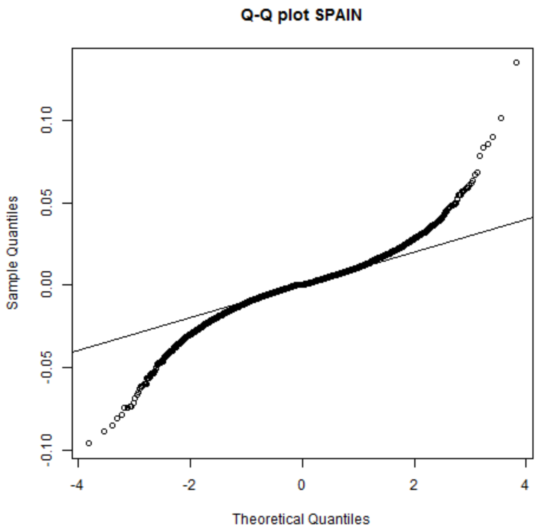
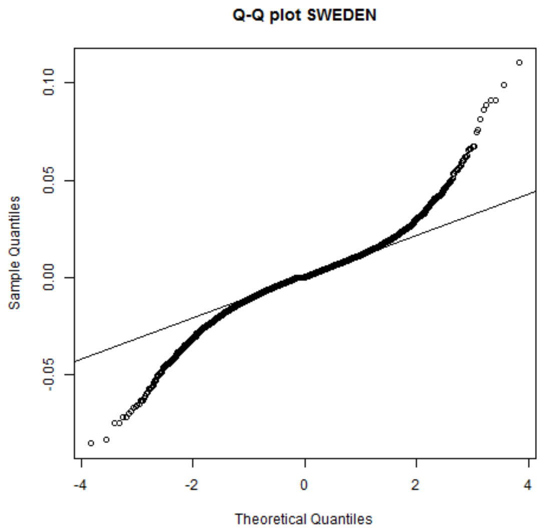


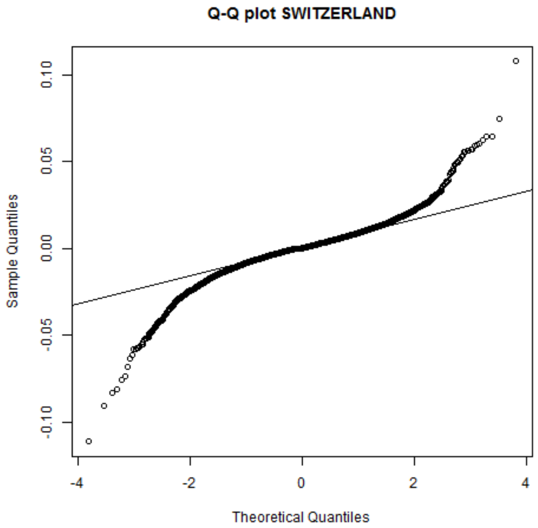
Q-Q plot NEW_ZEALAND

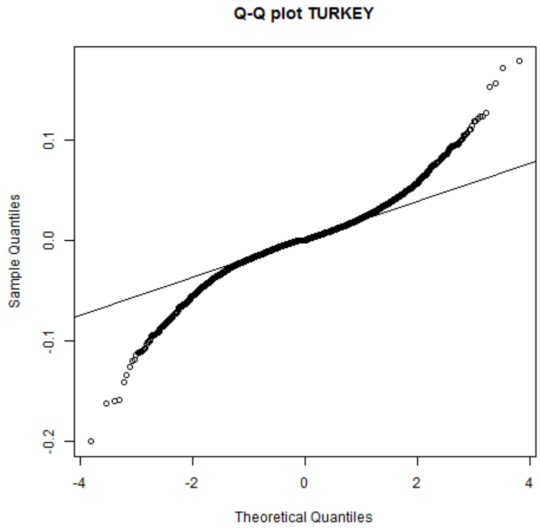


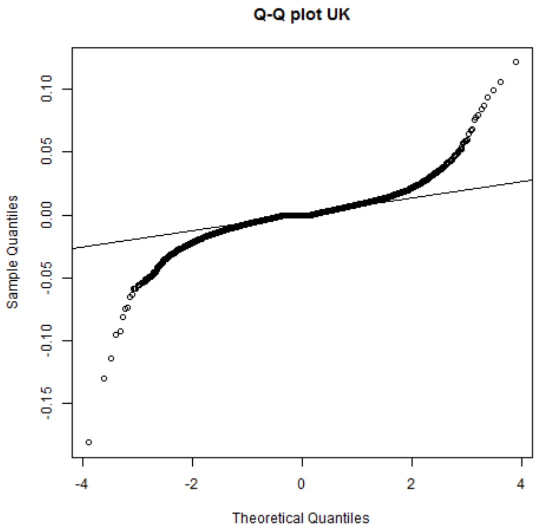
Q-Q plot NORWAY

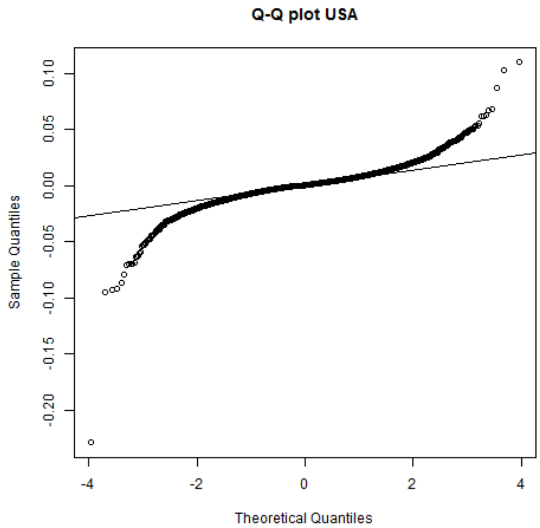



Q-Q plot POLAND
Sample Quantiles
Theoretical Quantiles
-0.15 -0.10 -0.05 0.00 0.05 0.10 0.15
-4 -2 0 2 4

Q-Q plot PORTUGAL
Sample Quantiles
Theoretical Quantiles
-0.10 -0.05 0.00 0.05 0.10
-4 -2 0 2 4

Q-Q plot SLOVAKIA
Sample Quantiles
Theoretical Quantiles
-0.1 0.0 0.1 0.2
-4 -2 0 2 4

Q-Q plot SLOVENIA
Sample Quantiles
Theoretical Quantiles
-0.4 -0.2 0.0 0.2 0.4
-3 -2 -1 0 1 2 3

Q-Q plot SPAIN
Sample Quantiles
Theoretical Quantiles
-0.10 -0.05 0.00 0.05 0.10
-4 -2 0 2 4

Q-Q plot SWEDEN
Sample Quantiles
Theoretical Quantiles
-0.05 0.00 0.05 0.10
-4 -2 0 2 4


Q-Q plot SWITZERLAND
Sample Quantiles
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4
Theoretical Quantiles

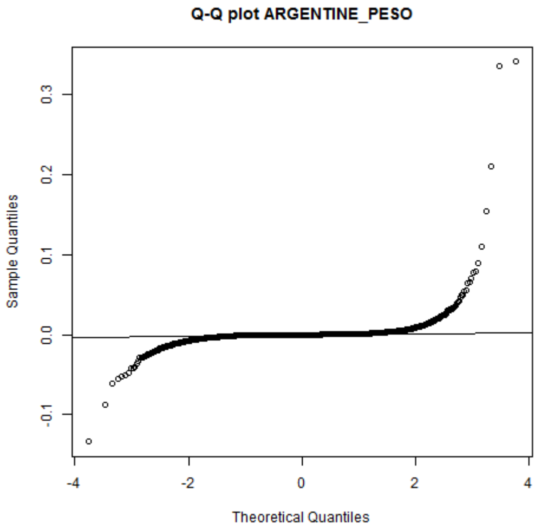

Q-Q plot TURKEY
Sample Quantiles
-0.2
-0.1
0.0
0.1
-4
-2
0
2
4
Theoretical Quantiles

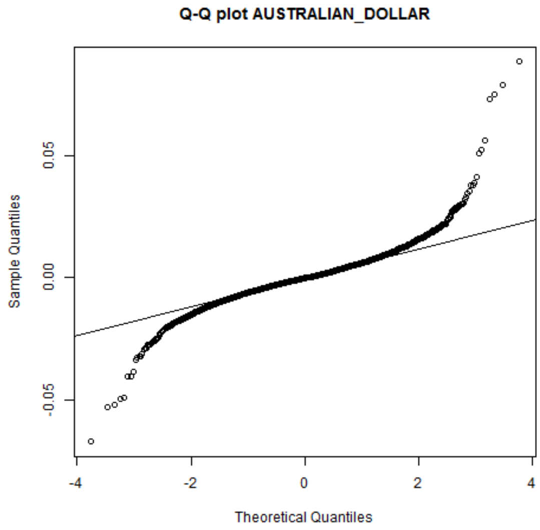

Q-Q plot UK
Sample Quantiles
-0.15
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4
Theoretical Quantiles

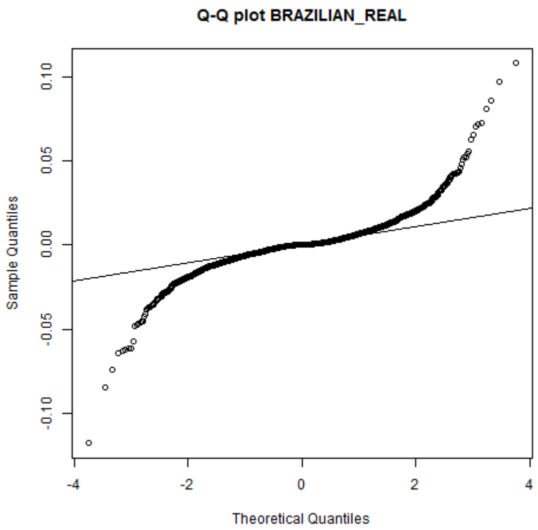

Q-Q plot USA
Sample Quantiles
-0.20
-0.15
-0.10
-0.05
0.00
0.05
0.10
-4
-2
0
2
4
Theoretical Quantiles

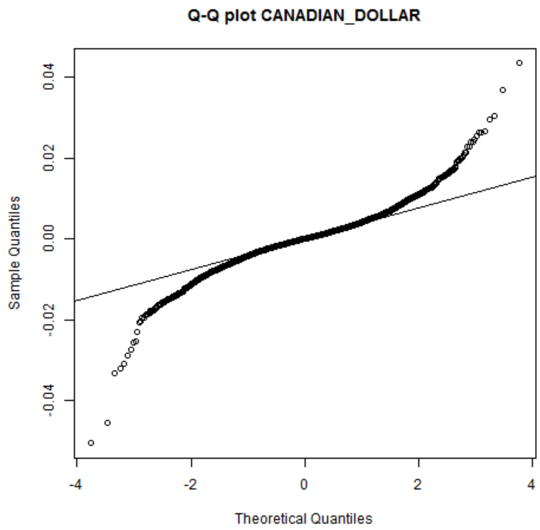
Appendix 3

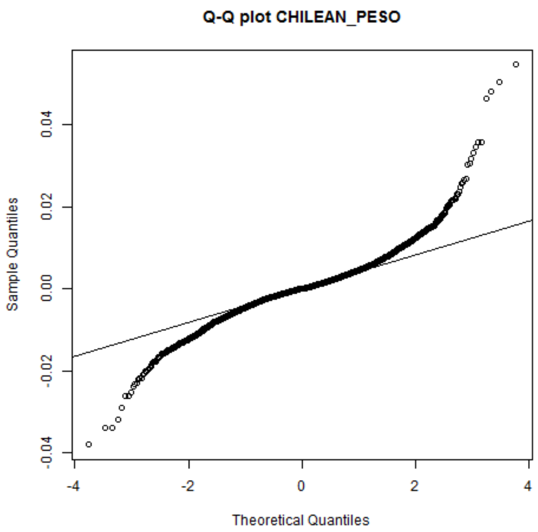
QQ plot of daily log returns of the forex indices

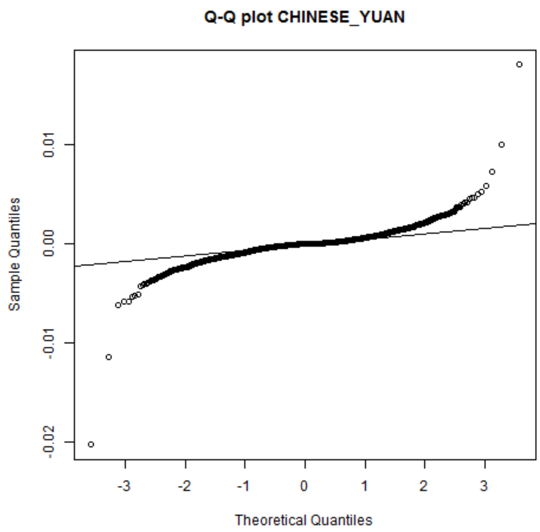


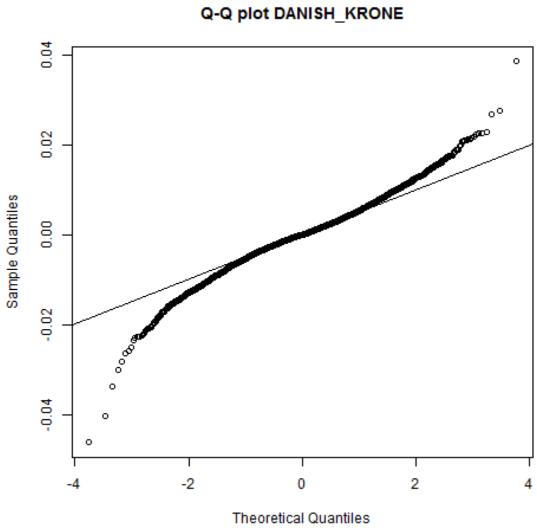


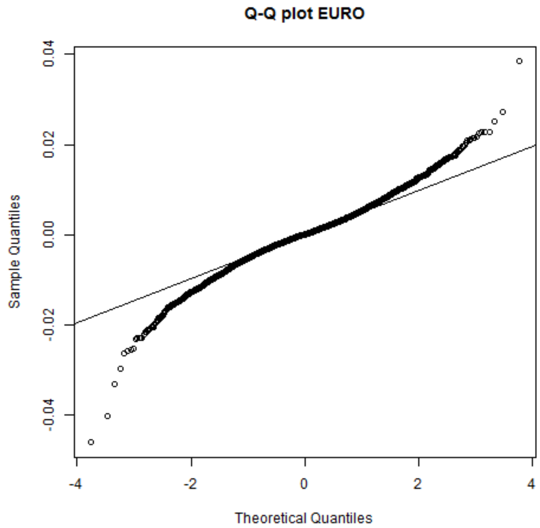


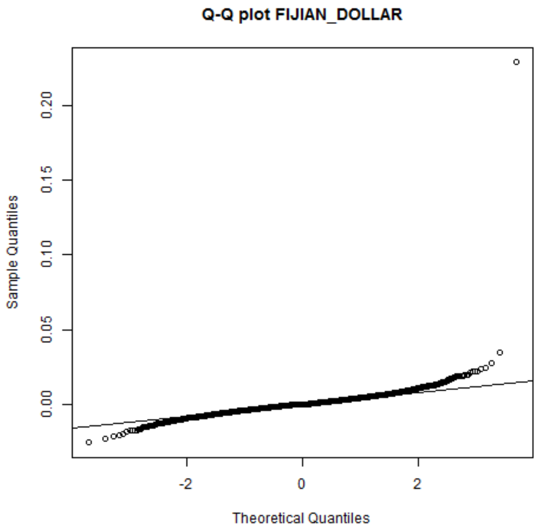


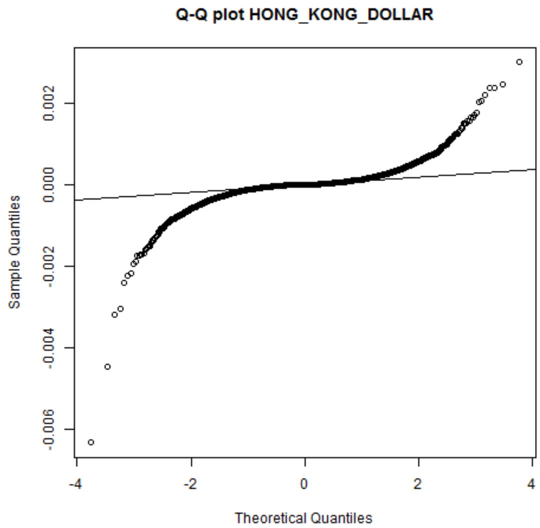


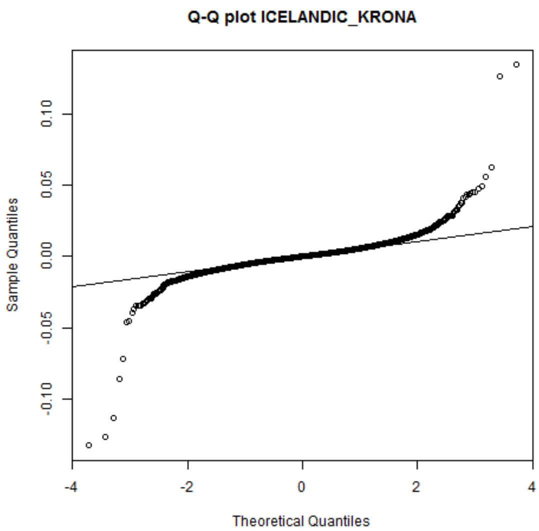


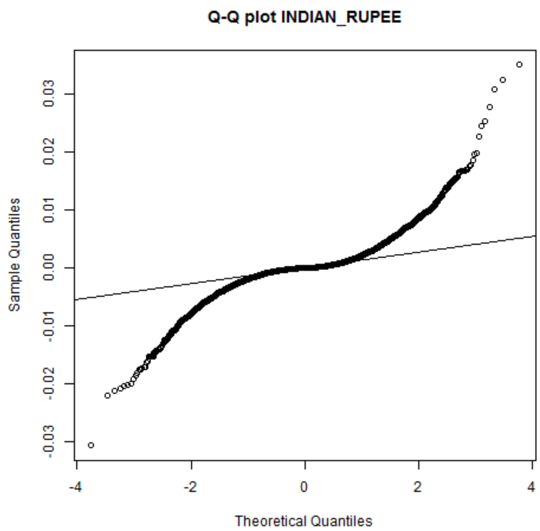

Q-Q plot DANISH_KRONE
Sample Quantiles
Theoretical Quantiles

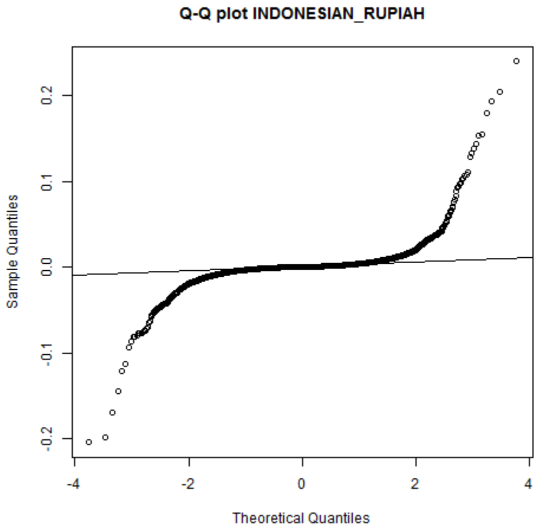

Q-Q plot EURO
Sample Quantiles
Theoretical Quantiles

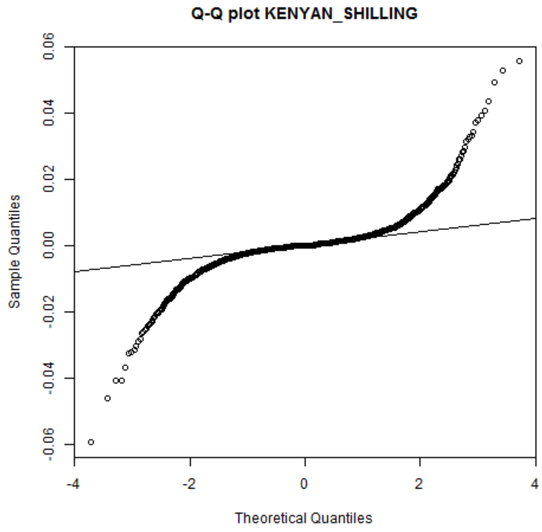

Q-Q plot FIJIAN_DOLLAR
Sample Quantiles
Theoretical Quantiles

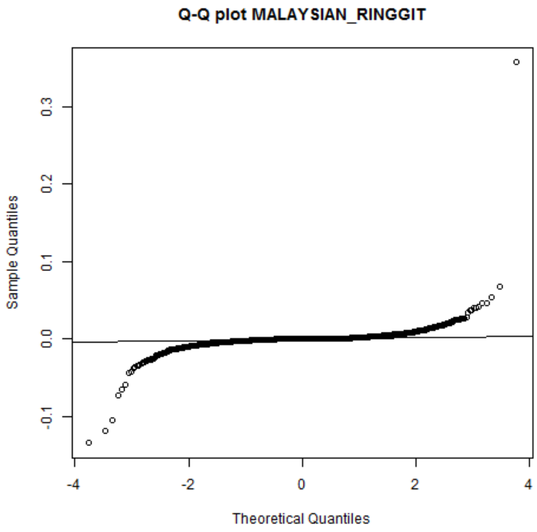

Q-Q plot HONG_KONG_DOLLAR
Sample Quantiles
Theoretical Quantiles

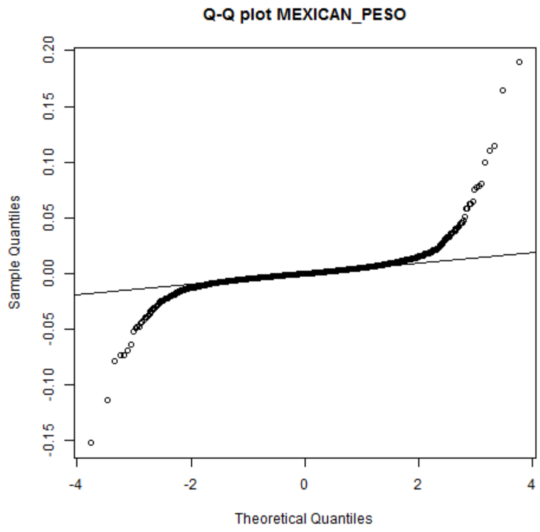

Q-Q plot ICELANDIC_KRONA
Sample Quantiles
Theoretical Quantiles

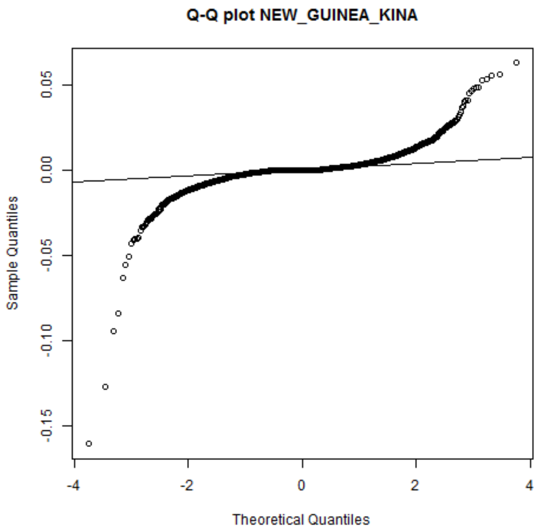

Q-Q plot INDIAN_RUPEE
Sample Quantiles
Theoretical Quantiles

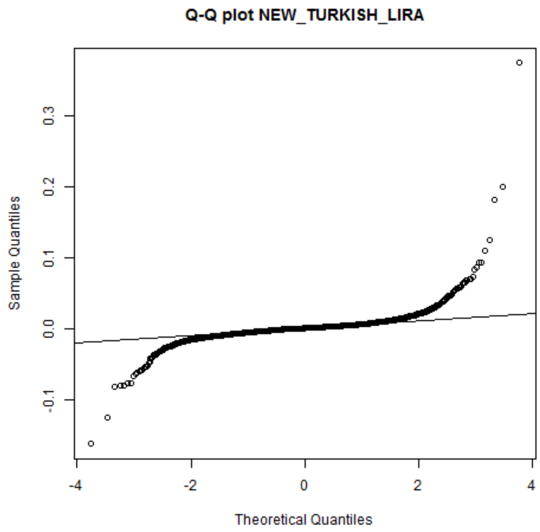

Q-Q plot INDONESIAN_RUPIAH
Sample Quantiles
Theoretical Quantiles
-0.2
-0.1
0.0
0.1
0.2
-4
-2
0
2
4

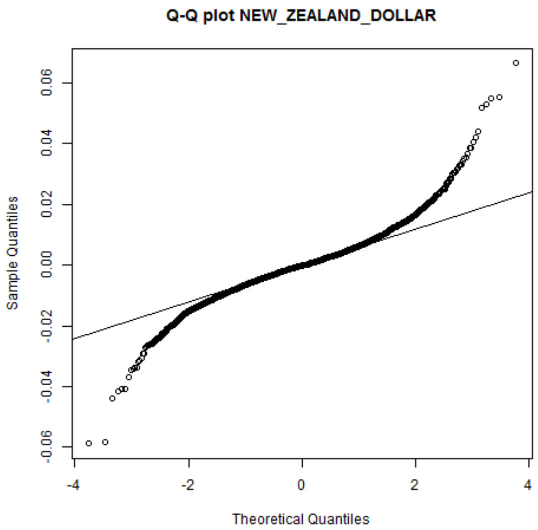

Q-Q plot KENYAN_SHILLING
Sample Quantiles
Theoretical Quantiles
-0.06
-0.04
-0.02
0.00
0.02
0.04
0.06
-4
-2
0
2
4

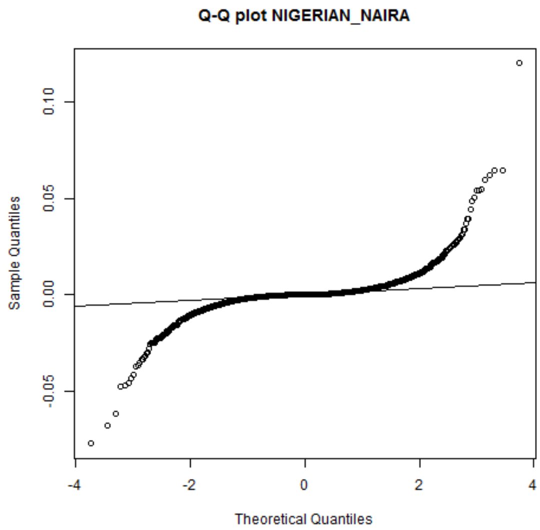

Q-Q plot MALAYSIAN_RINGGIT
Sample Quantiles
Theoretical Quantiles
-0.1
0.0
0.1
0.2
0.3
-4
-2
0
2
4

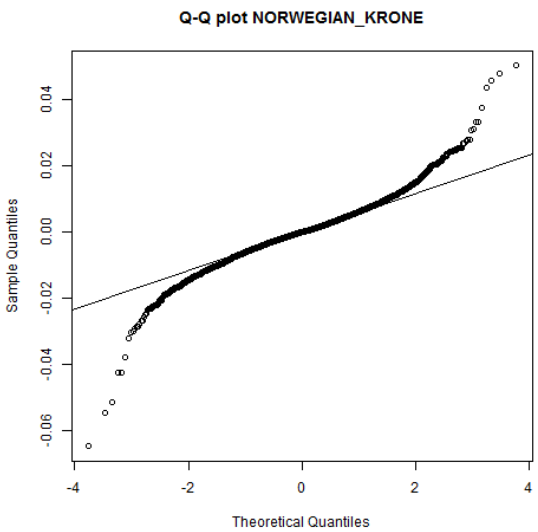

Q-Q plot MEXICAN_PESO
Sample Quantiles
Theoretical Quantiles
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
0.20
-4
-2
0
2
4

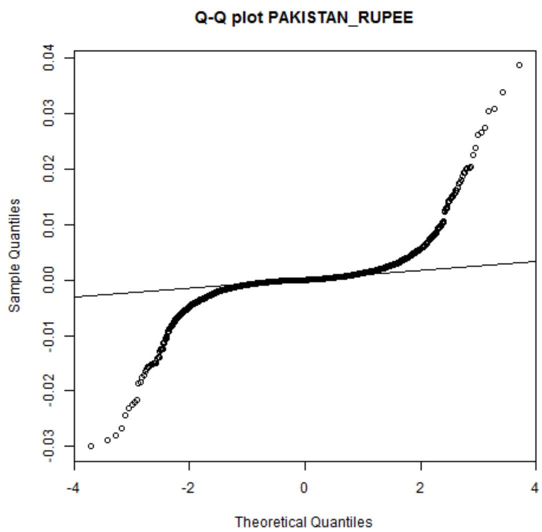

Q-Q plot NEW_GUINEA_KINA
Sample Quantiles
Theoretical Quantiles
-0.15
-0.10
-0.05
0.00
0.05
-4
-2
0
2
4

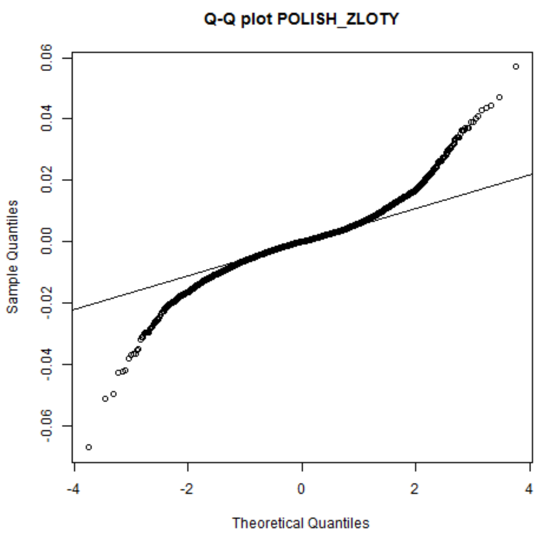

Q-Q plot NEW_TURKISH_LIRA
Sample Quantiles
Theoretical Quantiles
-0.1
0.0
0.1
0.2
0.3
-4
-2
0
2
4

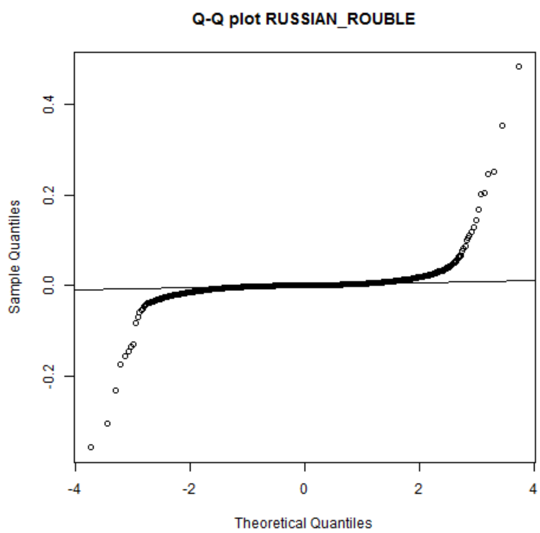

Q-Q plot NEW_ZEALAND_DOLLAR
Sample Quantiles
Theoretical Quantiles

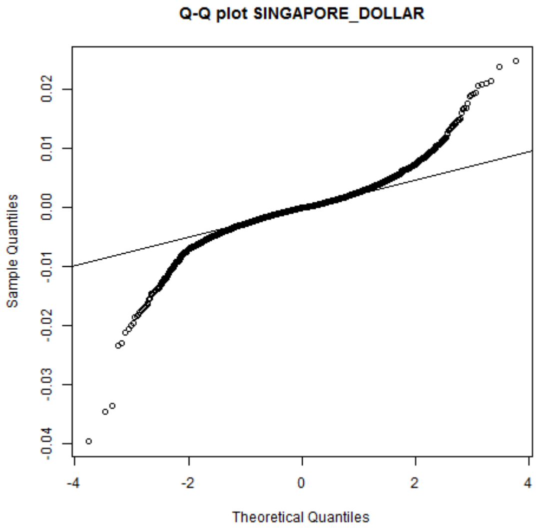

Q-Q plot NIGERIAN_NAIRA
Sample Quantiles
Theoretical Quantiles

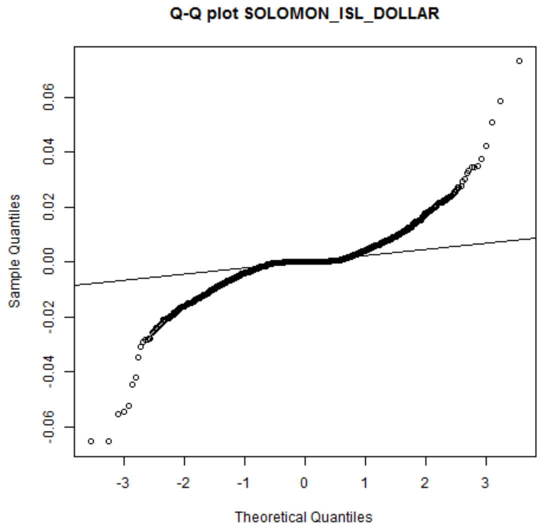

Q-Q plot NORWEGIAN_KRONE
Sample Quantiles
Theoretical Quantiles

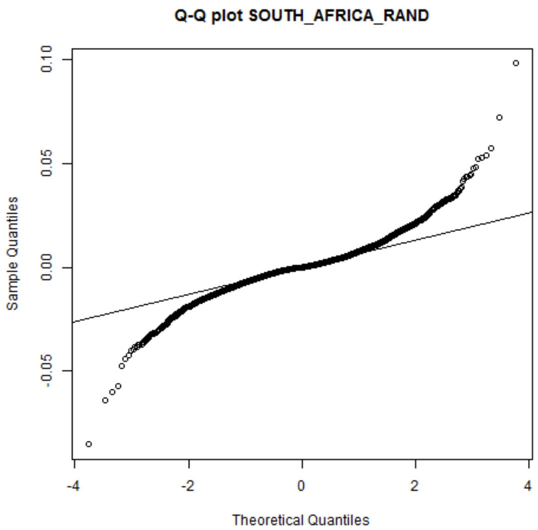

Q-Q plot PAKISTAN_RUPEE
Sample Quantiles
Theoretical Quantiles

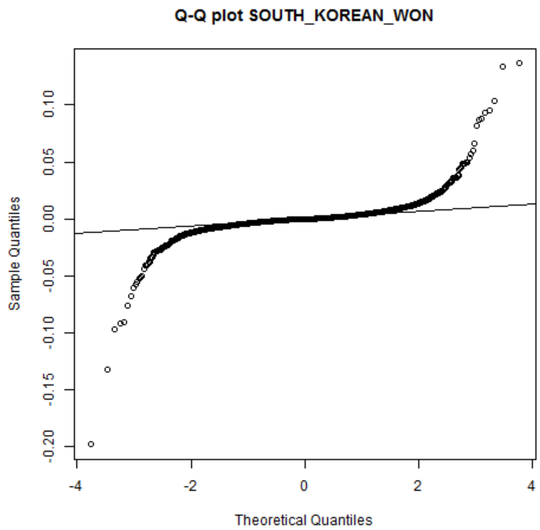

Q-Q plot POLISH_ZLOTY
Sample Quantiles
Theoretical Quantiles

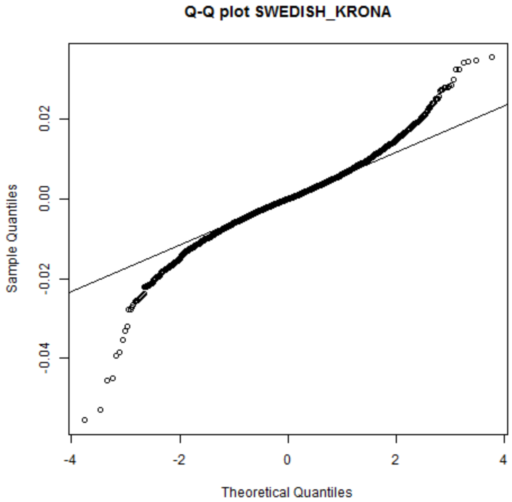

Q-Q plot RUSSIAN_ROUBLE
Sample Quantiles
Theoretical Quantiles

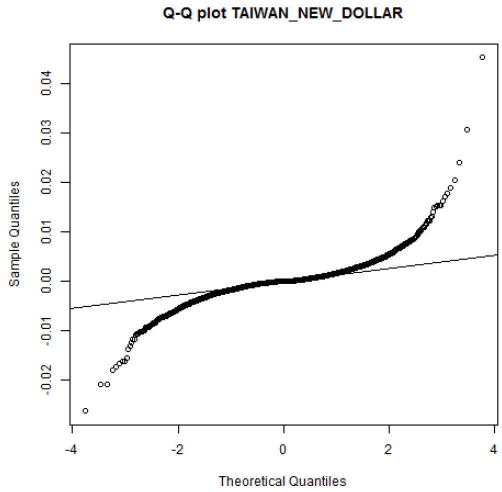

Q-Q plot SINGAPORE_DOLLAR
Sample Quantiles
Theoretical Quantiles

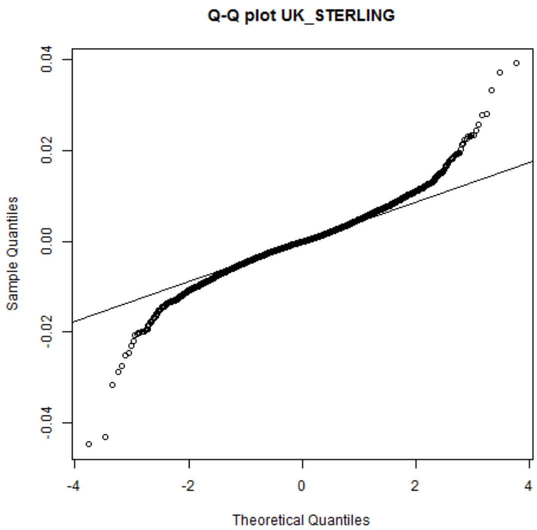

Q-Q plot SOLOMON_ISL_DOLLAR
Sample Quantiles
Theoretical Quantiles


Q-Q plot SOUTH_AFRICA_RAND
Sample Quantiles
Theoretical Quantiles


Q-Q plot SOUTH_KOREAN_WON
Sample Quantiles
Theoretical Quantiles

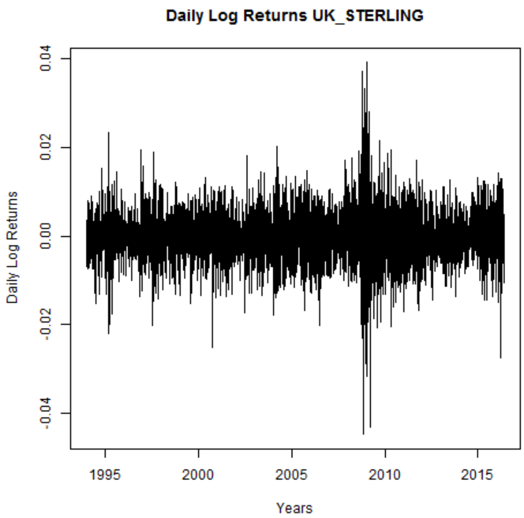

Q-Q plot SWEDISH_KRONA
Sample Quantiles
Theoretical Quantiles
-0.04
-0.02
0.00
0.02
-4
-2
0
2
4

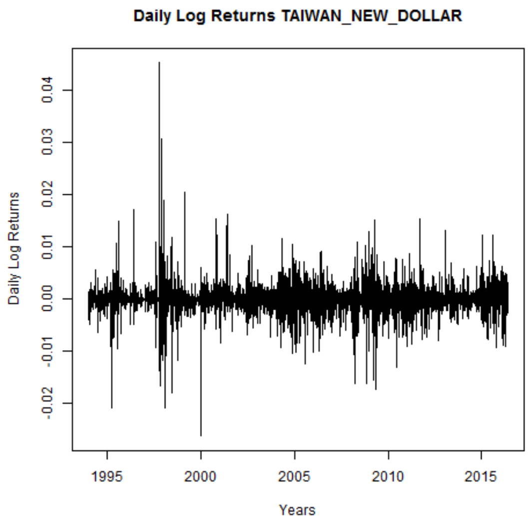

Q-Q plot TAIWAN_NEW_DOLLAR
Sample Quantiles
Theoretical Quantiles
-0.02
-0.01
0.00
0.01
0.02
0.03
0.04
-4
-2
0
2
4

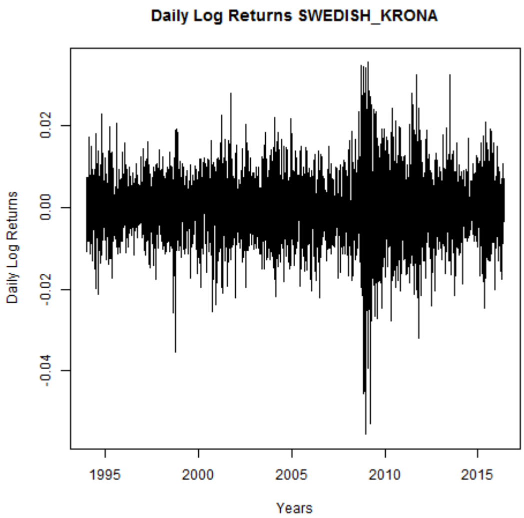

Q-Q plot UK_STERLING
Sample Quantiles
Theoretical Quantiles
-0.04
-0.02
0.00
0.02
0.04
-4
-2
0
2
4

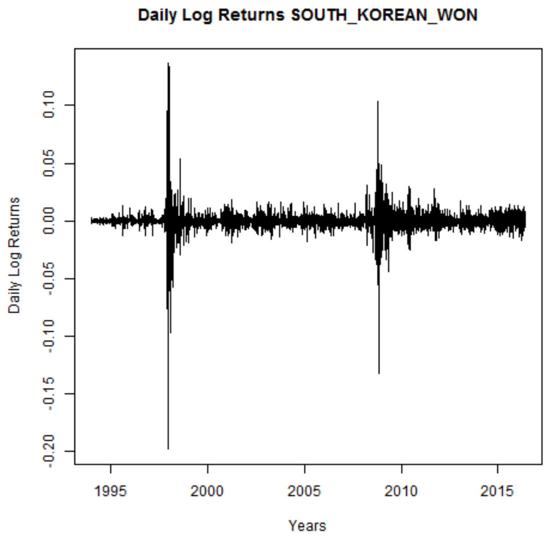
Appendix 4

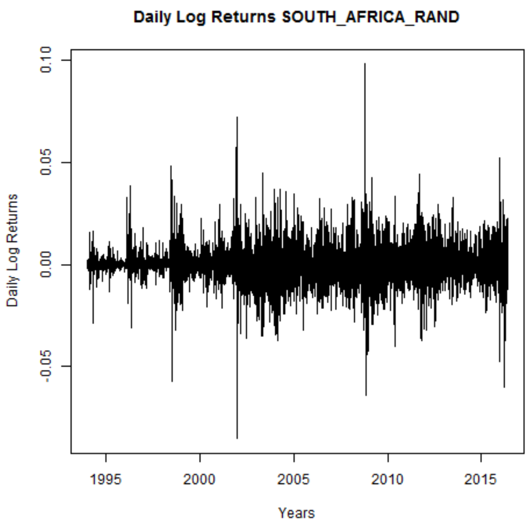
Evidence of stationarity (that returns oscillate around a common mean) for daily forex returns.

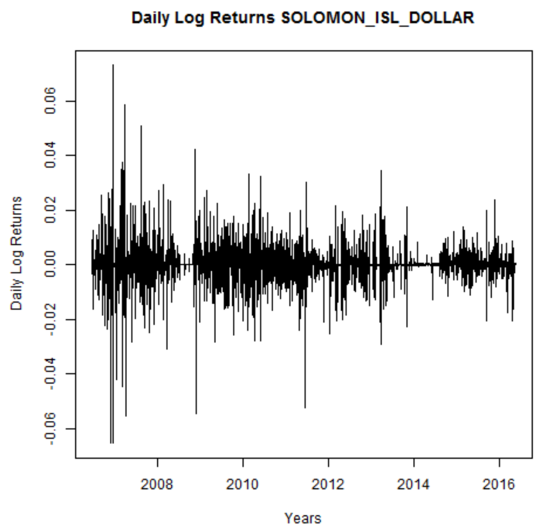


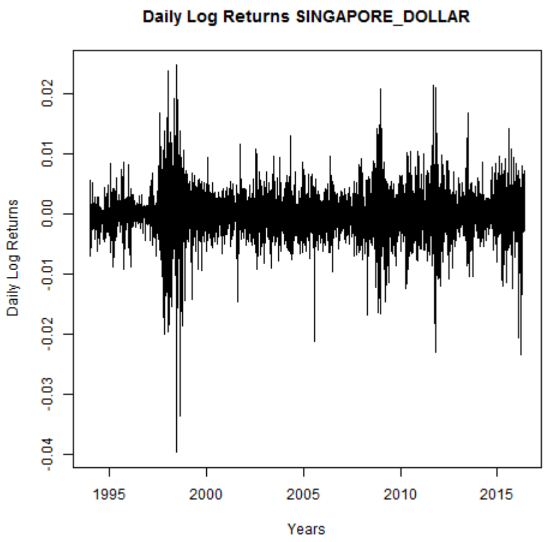


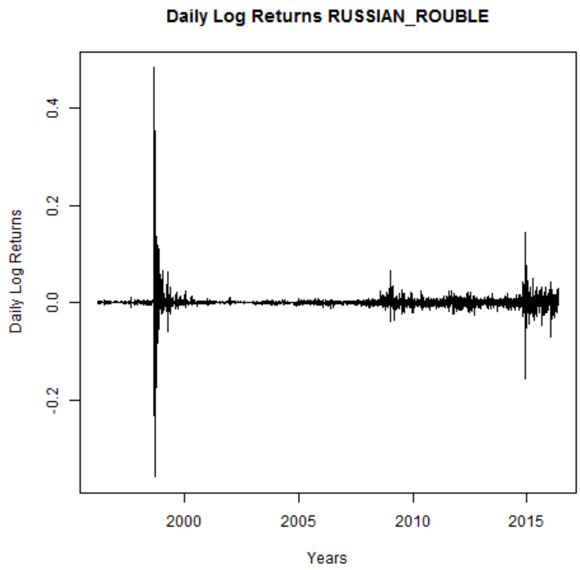


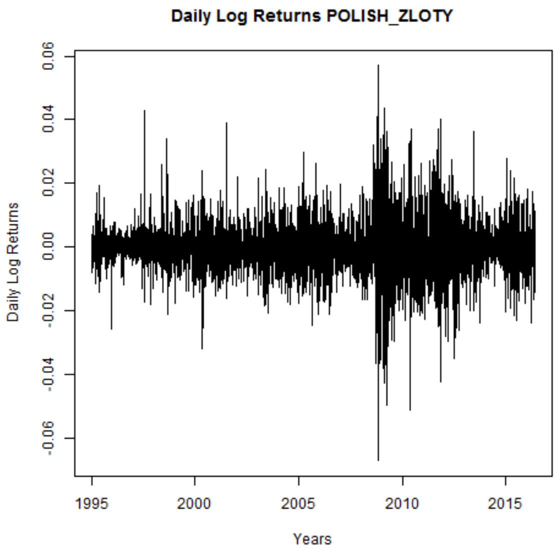


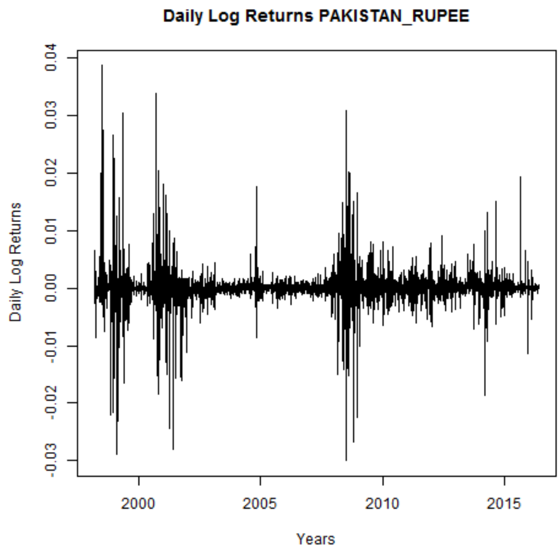


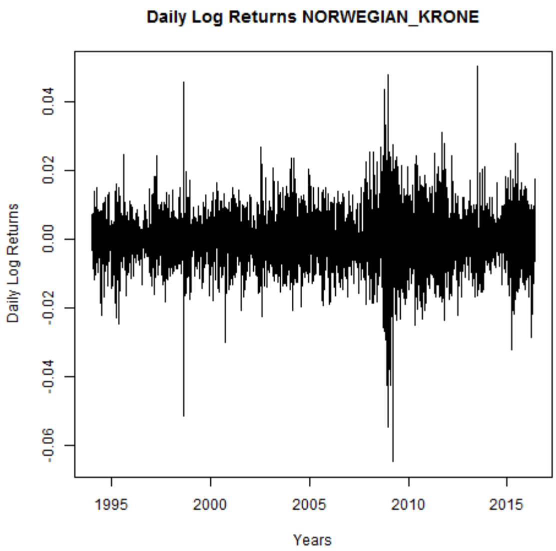


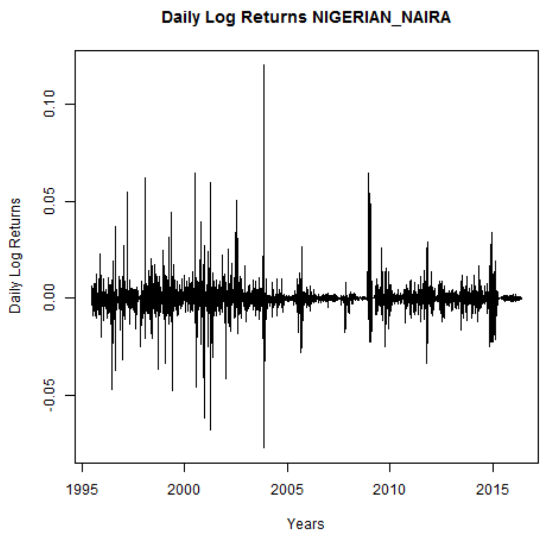

Daily Log Returns SINGAPORE_DOLLAR
Daily Log Returns
0.02
0.01
0.00
-0.01
-0.02
-0.03
-0.04
1995
2000
2005
2010
2015
Years

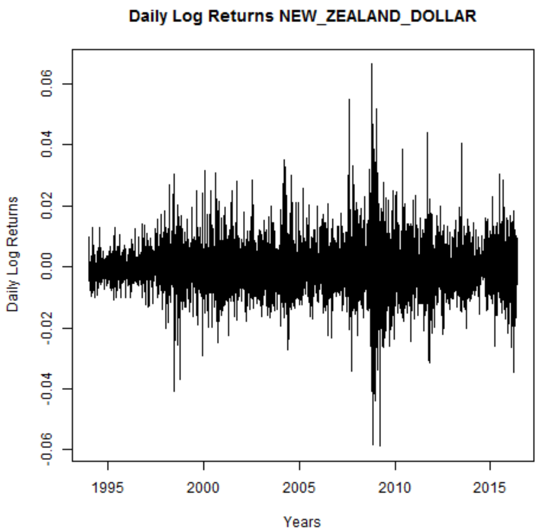

Daily Log Returns RUSSIAN_ROUBLE
Daily Log Returns
0.4
0.2
0.0
-0.2
2000
2005
2010
2015
Years

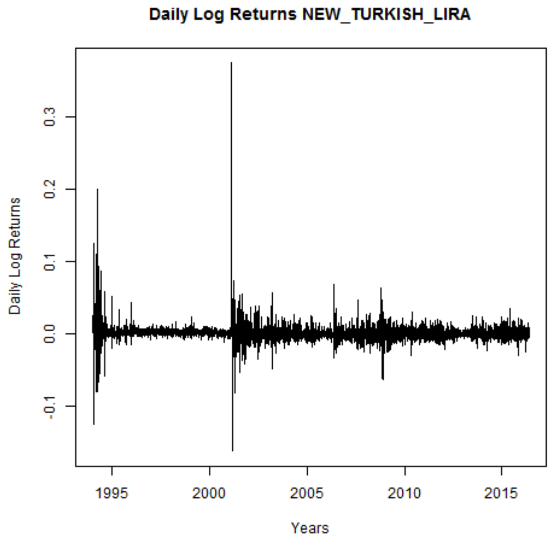

Daily Log Returns POLISH_ZLOTY
Daily Log Returns
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1995
2000
2005
2010
2015
Years

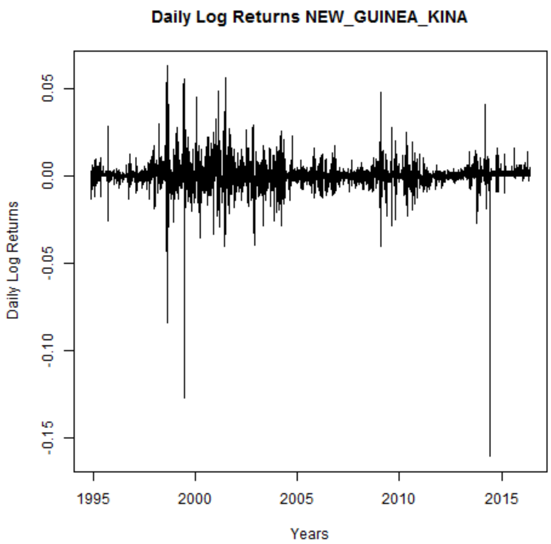

Daily Log Returns PAKISTAN_RUPEE
Daily Log Returns
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
2000
2005
2010
2015
Years

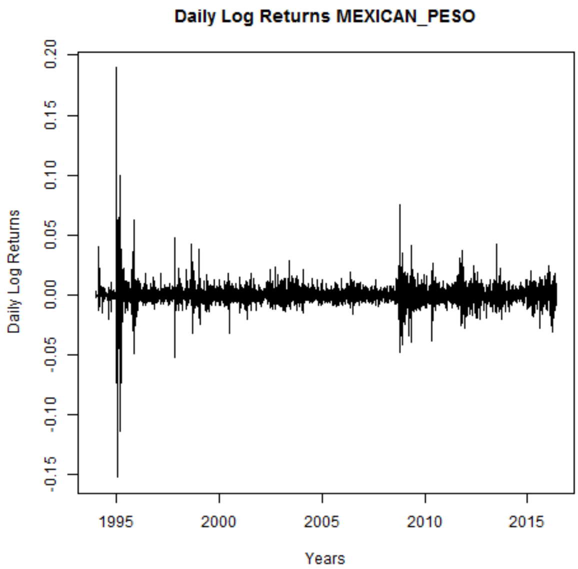

Daily Log Returns NORWEGIAN_KRONE
Daily Log Returns
0.04
0.02
0.00
-0.02
-0.04
-0.06
1995
2000
2005
2010
2015
Years

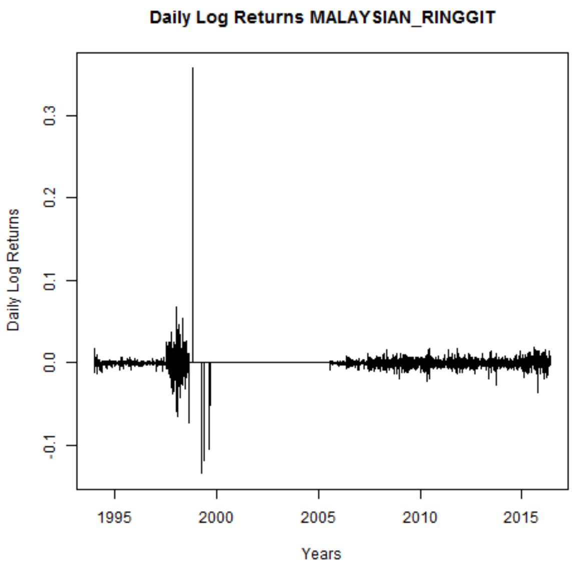

Daily Log Returns NIGERIAN_NAIRA
Daily Log Returns
0.10
0.05
0.00
-0.05
1995
2000
2005
2010
2015
Years

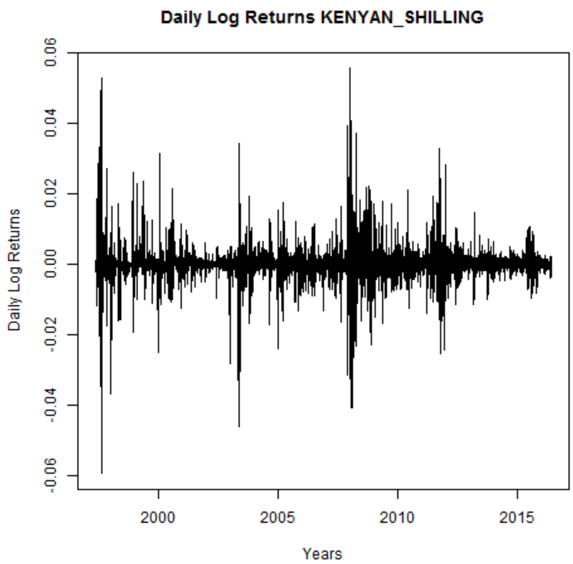

Daily Log Returns NEW_ZEALAND_DOLLAR
Daily Log Returns
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
1995
2000
2005
2010
2015
Years

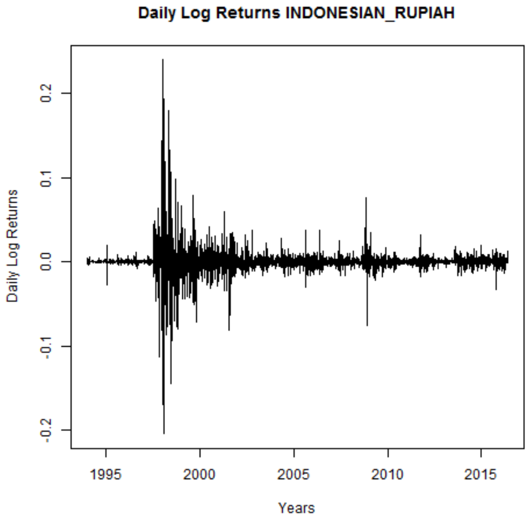

Daily Log Returns NEW_TURKISH_LIRA
Daily Log Returns
0.3
0.2
0.1
0.0
-0.1
1995
2000
2005
2010
2015
Years

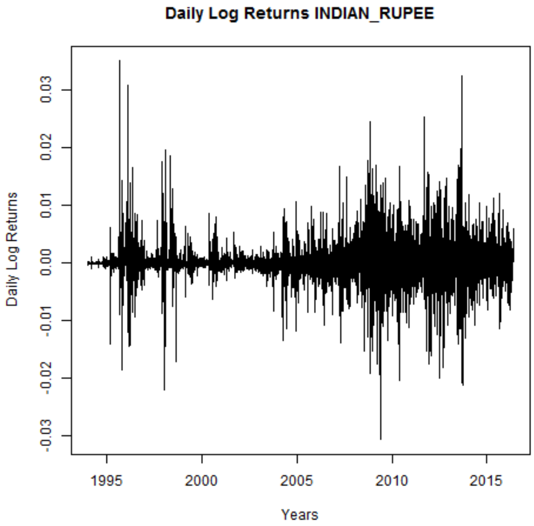

Daily Log Returns NEW_GUINEA_KINA
Daily Log Returns
0.05
0.00
-0.05
-0.10
-0.15
1995
2000
2005
2010
2015
Years

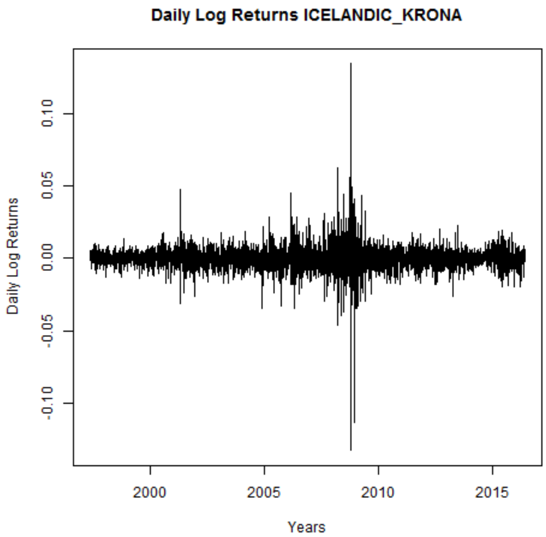

Daily Log Returns MEXICAN_PESO
Daily Log Returns
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
1995
2000
2005
2010
2015
Years

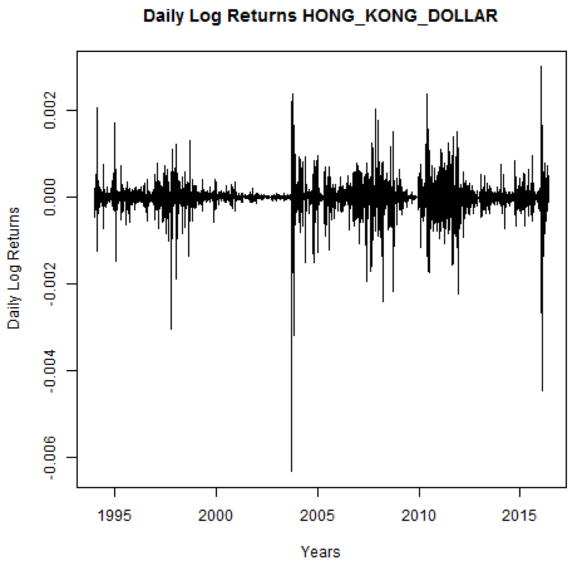

Daily Log Returns MALAYSIAN_RINGGIT
Daily Log Returns
0.3
0.2
0.1
0.0
-0.1
1995
2000
2005
2010
2015
Years

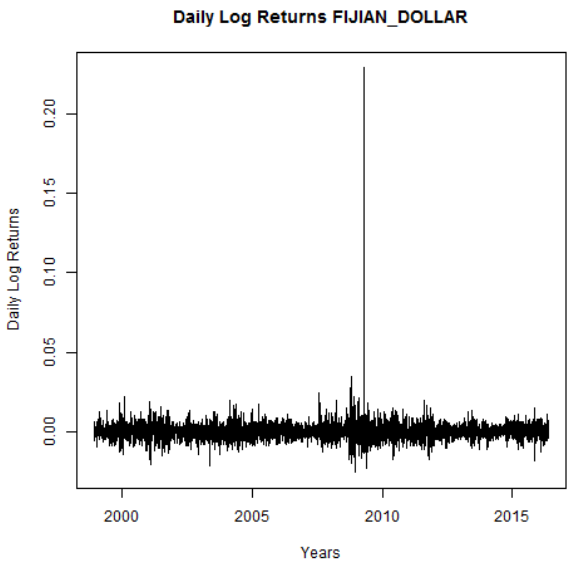

Daily Log Returns KENYAN_SHILLING
Daily Log Returns
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
2000
2005
2010
2015
Years

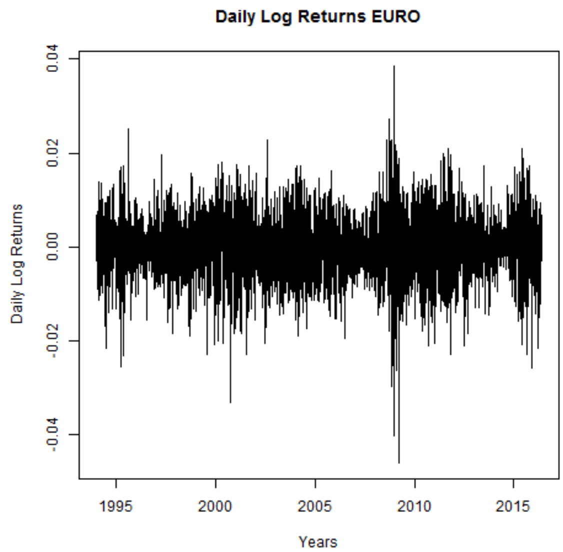

Daily Log Returns INDONESIAN_RUPIAH
Daily Log Returns
-0.2
-0.1
0.0
0.1
0.2
1995
2000
2005
2010
2015
Years

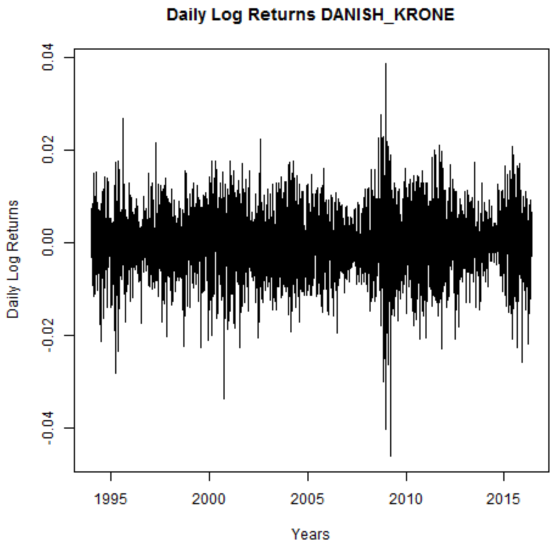

Daily Log Returns INDIAN_RUPEE
Daily Log Returns
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
1995
2000
2005
2010
2015
Years

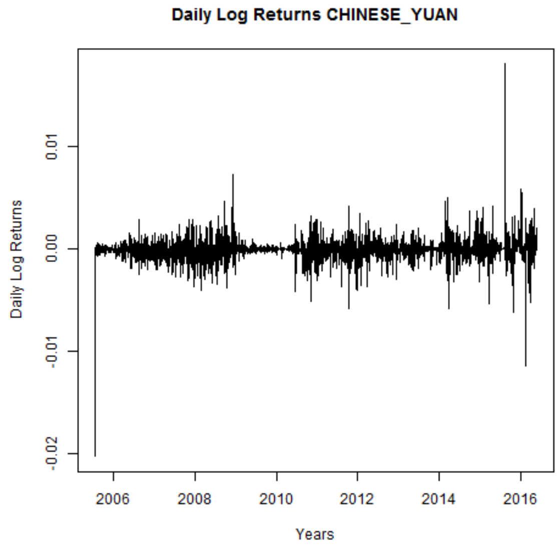

Daily Log Returns ICELANDIC_KRONA
Daily Log Returns
-0.10
-0.05
0.00
0.05
0.10
2000
2005
2010
2015
Years

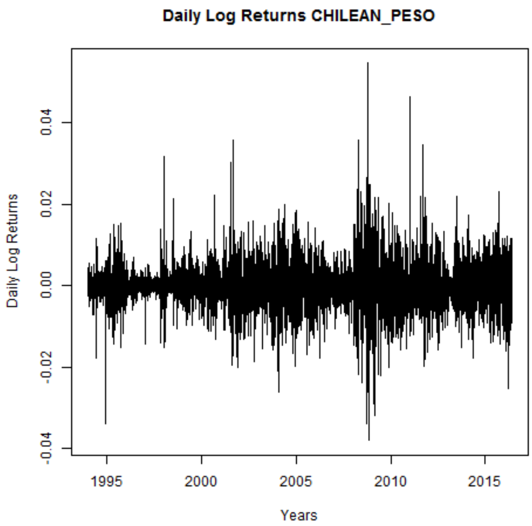

Daily Log Returns HONG_KONG_DOLLAR
Daily Log Returns
-0.006
-0.004
-0.002
0.000
0.002
1995
2000
2005
2010
2015
Years

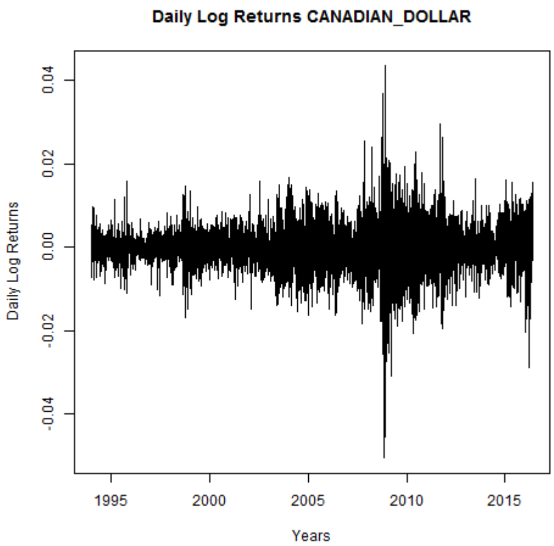

Daily Log Returns FIJIAN_DOLLAR
Daily Log Returns
0.00
0.05
0.10
0.15
0.20
2000
2005
2010
2015
Years

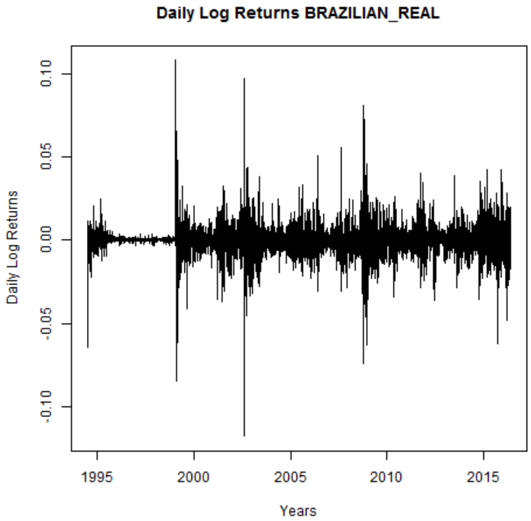

Daily Log Returns EURO
Daily Log Returns
-0.04
-0.02
0.00
0.02
0.04
1995
2000
2005
2010
2015
Years

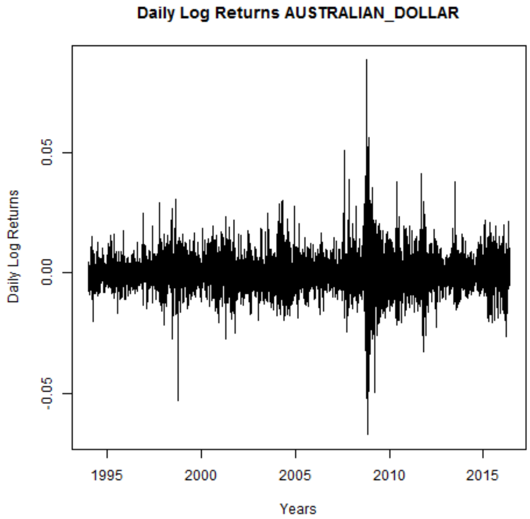

Daily Log Returns DANISH_KRONE
Daily Log Returns
0.04
0.02
0.00
-0.02
-0.04
1995
2000
2005
2010
2015
Years

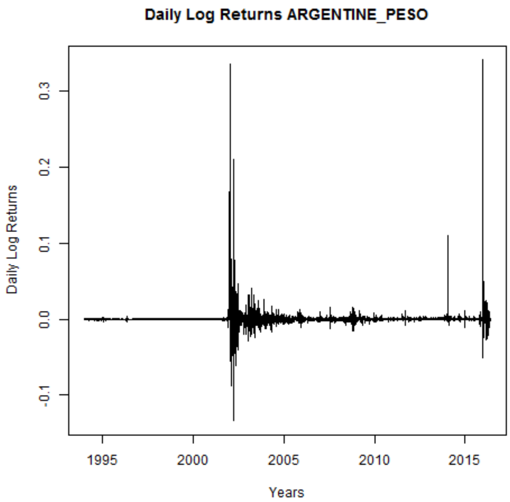

Daily Log Returns CHINESE_YUAN
Daily Log Returns
0.01
0.00
-0.01
-0.02
2006
2008
2010
2012
2014
2016
Years


Daily Log Returns CHILEAN_PESO
Daily Log Returns
0.04
0.02
0.00
-0.02
-0.04
1995
2000
2005
2010
2015
Years


Daily Log Returns CANADIAN_DOLLAR
Daily Log Returns
0.04
0.02
0.00
-0.02
-0.04
1995
2000
2005
2010
2015
Years


Daily Log Returns BRAZILIAN_REAL
Daily Log Returns
0.10
0.05
0.00
-0.05
-0.10
1995
2000
2005
2010
2015
Years


Daily Log Returns AUSTRALIAN_DOLLAR
Daily Log Returns
0.05
0.00
-0.05
1995
2000
2005
2010
2015
Years


Daily Log Returns ARGENTINE_PESO
Daily Log Returns
0.3
0.2
0.1
0.0
-0.1
1995
2000
2005
2010
2015
Years

Appendix 5

Summary of the frequency of the Black Swans in the full sample and segments (with breaks) of daily stock market returns

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech**
1. Ratio of Black Swans (Full Sample)	1.31% (82)	1.75% (139)	1.89% (150)	1.77% (140)	1.24% (98)	1.02% (81)
2. Ratio of Black Swans (Segmented Sample)	0.78% (49)	1.58% (125)	1.58% (125)	1.38% (109)	1.01% (80)	1.00% (79)
3. Difference between Black Swans clusters in full sample and segmented sample (%)	*51.49%*	*10.62%*	*18.23%*	*25.03%*	*20.29%*	*2.50%*
4. Ratio of Black – Right tail (Full Sample)	0.53% (33)	0.66% (52)	0.81% (64)	0.74% (59)	0.62% (49)	0.35% (28)
5. Ratio of Black Swans – Right tail (Segmented Sample)	0.24% (15)	0.63% (50)	0.76% (60)	0.59% (47)	0.50% (40)	0.38% (30)
6. Difference (%)	78.85%	3.92%	6.45%	22.74%	*20.29%*	-6.90%
7. Ratio of Black – Left tail (Full Sample)	0.78% (49)	1.10% (87)	1.09% (86)	1.02% (81)	0.62% (49)	0.67% (53)
8 Ratio of Black Swans – Left tail (Segmented Sample)	0.54% (34)	0.95% (75)	0.82% (65)	0.78% (62)	0.50% (40)	0.62% (49)
9. Difference (%)	36.55%	14.84%	28.00%	26.73%	*20.29%*	7.85%
	Italy	**Japan**	**Korea**	**Luxembourg**	**Mexico**	**Netherlands**
1. Ratio of Black Swans (Full Sample)	0.95% (75)	2.49% (197)	2.40% (190)	0.88% (70)	1.55% (123)	1.89% (150)
2. Ratio of Black Swans (Segmented Sample)	0.61% (48)	2.15% (170)	1.94% (154)	0.57% (45)	1.36% (108)	1.36% (108)
3. Difference (%)	*44.63%*	*14.74%*	*21.01%*	*44.18%*	*13.01%*	*32.85%*
4. Ratio of Black – Right tail (Full Sample)	0.35% (28)	1.03% (82)	1.16% (92)	0.35% (28)	0.80% (63)	0.80% (63)
5. Ratio of Black Swans – Right tail (Segmented Sample)	0.24% (19)	0.95% (75)	1.00% (79)	0.20% (16)	0.69% (55)	0.49% (39)
6. Difference (%)	38.78%	8.92%	15.23%	55.96%	13.58%	47.96%
7. Ratio of Black – Left tail (Full Sample)	0.59% (47)	1.45% (115)	1.24% (98)	0.53% (42)	0.76% (60)	1.10% (87)

8 Ratio of Black Swans – Left tail (Segmented Sample)	0.37% (29)	1.20% (95)	0.95% (75)	0.37% (29)	0.67% (53)	0.87% (69)
9. Difference (%)	48.29%	19.11%	26.75%	37.04%	12.41%	23.18%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Ratio of Black Swans (Full Sample)	1.19% (94)	1.29% (102)	1.69% (134)	1.29% (102)	2.32% (184)	1.57% (124)
2. Ratio of Black Swans (Segmented Sample)	1.07% (85)	1.12% (89)	1.07% (85)	1.11% (88)	1.77% (140)	1.38% (109)
3. Difference (%)	*10.06%*	*13.63%*	*45.52%*	*14.76%*	*27.33%*	*12.89%*
4. Ratio of Black – Right tail (Full Sample)	0.52% (41)	0.67% (53)	0.85% (67)	0.53% (42)	1.01% (80)	0.71% (56)
5. Ratio of Black Swans – Right tail (Segmented Sample)	0.43% (34)	0.58% (46)	0.52% (41)	0.49% (39)	0.86% (68)	0.68% (54)
6. Difference (%)	*18.72%*	*14.17%*	*49.11%*	*7.41%*	*16.25%*	*3.64%*
7. Ratio of Black – Left tail (Full Sample)	0.67% (53)	0.62% (49)	0.85% (67)	0.76% (60)	1.31% (104)	0.86% (68)
8 Ratio of Black Swans – Left tail (Segmented Sample)	0.64% (51)	0.54% (43)	0.56% (44)	0.62% (49)	0.91% (72)	0.69% (55)
9. Difference (%)	*3.85%*	*13.06%*	*42.05%*	*20.25%*	*36.77%*	*21.22%*
	New Zealand	**Norway**	**Poland**	**Portugal**	**Slovakia**	**Slovenia**
1. Ratio of Black Swans (Full Sample)	0.62% (49)	1.54% (122)	1.14% (90)	1.11% (88)	1.22% (97)	0.23% (18)
2. Ratio of Black Swans (Segmented Sample)	0.44% (35)	1.44% (114)	0.83% (66)	0.96% (76)	1.50% (119)	0.23% (18)
3. Difference (%)	33.65%	6.78%	31.02%	14.66%	-20.44%	0.00%
4. Ratio of Black – Right tail (Full Sample)	0.25% (20)	0.56% (44)	0.53% (42)	0.33% (26)	0.58% (46)	0.09% (7)
5. Ratio of Black Swans – Right tail (Segmented Sample)	0.19% (15)	0.50% (40)	0.32% (25)	0.35% (28)	0.67% (53)	0.09% (7)
6. Difference (%)	28.77%	9.53%	51.88%	-7.41%	-14.17%	0.00%
7. Ratio of Black – Left tail (Full Sample)	0.37% (29)	0.98% (78)	0.61% (48)	0.78% (62)	0.64% (51)	0.14% (11)

8 Ratio of Black Swans – Left tail (Segmented Sample)	0.25% (20)	0.93% (74)	0.52% (41)	0.61% (48)	0.83% (66)	0.14% (11)
9. Difference (%)	37.16%	5.26%	15.76%	25.59%	-25.78%	0.00%

	Ireland	**Israel**	**Switzerland**	**Hungary**	**Iceland**	**Spain**	**Sweden**
1. Ratio of Black Swans (Full Sample)	2.13% (169)	1.31% (104)	1.45% (115)	1.26% (100)	0.10% (8)	1.36% (108)	1.53% (121)
2. Ratio of Black Swans (Segmented Sample)	1.77% (140)	1.27% (101)	1.29% (102)	1.22% (97)	1.12% (89)	1.26% (100)	1.15% (91)
3. Difference (%)	*18.83%*	*2.93%*	*12.00%*	*3.05%*	*-241%*	*7.70%*	*28.49%*
4. Ratio of Black – Right tail (Full Sample)	0.91% (72)	0.58% (46)	0.58% (46)	0.58% (46)	0.01% (1)	0.59% (47)	0.69% (55)
5. Ratio of Black Swans – Right tail (Segmented Sample)	0.74% (59)	0.52% (41)	0.48% (38)	0.63% (50)	0.56% (44)	0.49% (39)	0.56% (44)
6. Difference (%)	19.91%	11.51%	19.11%	-8.34%	-378%	18.66%	22.31%
7. Ratio of Black – Left tail (Full Sample)	1.22% (97)	0.73% (58)	0.87% (69)	0.68% (54)	0.09% (7)	0.77% (61)	0.83% (66)
8 Ratio of Black Swans – Left tail (Segmented Sample)	1.02% (81)	0.76% (60)	0.81% (64)	0.59% (47)	0.57% (45)	0.77% (61)	0.59% (47)
9. Difference (%)	18.03%	-3.39%	7.52%	13.88%	-186%	0.00%	33.95%

	Turkey	**UK**	**USA**
1. Ratio of Black Swans (Full Sample)	1.53% (121)	1.91% (151)	2.32% (184)
2. Ratio of Black Swans (Segmented Sample)	1.10% (87)	1.74% (138)	2.01% (159)
3. Difference (%)	*32.99%*	*9.00%*	*14.60%*
4. Ratio of Black – Right tail (Full Sample)	0.78% (62)	1.00% (79)	1.19% (94)
5. Ratio of Black Swans – Right tail (Segmented Sample)	0.49% (39)	0.92% (73)	1.01% (80)
6. Difference (%)	46.36%	7.90%	16.13%
7. Ratio of Black – Left tail (Full Sample)	0.74% (59)	0.91% (72)	1.14% (90)
8 Ratio of Black Swans – Left	0.61% (48)	0.82% (65)	1.00% (79)

tail (Segmented Sample)			
9. Difference (%)	20.63%	10.23%	13.04%

Appendix 6

Summary of the ratio of Black Swans to extreme values in the full sample and segments (with breaks) of daily stock market returns

	Australia	Austria	Belgium	Canada	Chile	Czech
1. Ratio of Black Swans to Extreme Values (Full Sample)	(82/126) 65.08%	(139/160) 86.88%	(150/190) 78.95%	(140/180) 77.78%	(98/138) 71.01%	(81/116) 69.83%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(49/132) 37.12%	(125/164) 76.22%	(125/198) 63.13%	(109/188) 57.98%	(80/144) 55.56%	(79/118) 66.95%
3. Difference (%)	*56.14%*	*13.09%*	*22.36%*	*29.38%*	*24.55%*	*4.21%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(33/63) 52.38%	(52/80) 65%	(64/95) 67.37%	(59/90) 65.56%	(49/69) 71.01%	(28/58) 48.28%
5.8 Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(15/66) 22.73%	(50/82) 60.98%	(60/99) 60.61%	(47/94) 50%	(40/72) 55.56%	(30/59) 50.85%
6. Difference (%)	*83.50%*	*6.39%*	*10.58%*	*27.09%*	*24.55%*	*-5.19%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(49/63) 77.78%	(87/80) 108.5%	(86/95) 90.53%	(81/90) 90%	(49/69) 71.01%	(53/58) 31.38%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(34/66) 51.52%	(75/82) 91.46%	(65/99) 65.66%	(62/94) 65.96%	(40/72) 44.44%	(49/59) 83.05%
9. Difference (%)	*41.20%*	*17.31%*	*32.12%*	*31.08%*	*24.55%*	*9.56%*
	Italy	**Japan**	**Korea**	**Luxembourg**	**Mexico**	**Netherlands**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(75/96) 78.13%	(197/270) 72.96%	(190/216) 87.96%	(70/92) 76.09%	(123/150) 82%	(150/176) 85.23%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(48/102) 47.06%	(170/274) 62.04%	(154/224) 68.75%	(45/98) 45.92%	(108/160) 67.50%	(108/184) 58.70%
3. Difference (%)	*50.69%*	*16.21%*	*24.64%*	*50.50%*	*19.46%*	*37.30%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(28/48) 58.33%	(82/135) 60.74%	(92/108) 85.19%	(28/46) 60.87%	(63/75) 84%	(63/88) 71.59%
5 Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(19/51) 37.25%	(75/137) 54.74%	(79/112) 70.54%	(16/49) 32.65%	(55/80) 68.75%	(39/92) 42.36%
6. Difference (%)	*44.84%*	*10.39%*	*18.87%*	*62.28%*	*20.03%*	*52.40%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(47/48) 97.92%	(115/135) 85.19%	(98/108) 90.74%	(42/46) 91.30%	(60/75) 80%	(87/88) 98.86%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(29/51) 56.86%	(95/137) 69.34%	(75/112) 66.96%	(29/49) 59.18%	(53/80) 66.25%	(69/92) 75%
9. Difference (%)	*54.35%*	*20.58%*	*30.38%*	*43.36%*	*18.86%*	*27.63%*
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(94/140) 67.14%	(102/106) 96.23%	(134/154) 87.01%	(102/152) 67.11%	(184/270) 68.15%	(124/146) 84.93%

2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(85/140) 60.71%	(89/108) 82.41%	(85/160) 53.13%	(88/160) 55%	(140/274) 51.09%	(109/148) 73.65%
3. Difference (%)	*10.06%*	*15.50%*	*49.34%*	*19.89%*	*28.80*	*14.25%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(41/70) 58.57%	(53/53) 100%	(67/77) 87.01%	(42/76) 55.26%	(80/135) 59.26%	(56/73) 76.71%
5 Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(34/70) 48.57%	(46/54) 85.19%	(41/80) 51.25%	(39/80) 48.75%	(68/137) 49.64%	(54/74) 72.97%
6. Difference (%)	*18.72%*	*16.03%*	*52.93%*	*12.54%*	*17.72%*	*5.00%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(53/70) 75.71%	(49/53) 92.45%	(67/77) 87.01%	(60/76) 78.95%	(104/135) 77.04%	(68/73) 93.15%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(51/70) 72.86%	(43/54) 79.63%	(44/80) 55%	(49/80) 61.25%	(72/137) 52.55%	(55/74) 74.32%
9. Difference (%)	*3.85%*	*14.93%*	*45.87%*	*23.38%*	*38.24%*	*22.58%*

	New Zealand	**Norway**	**Poland**	**Portugal**	**Slovakia**	**Slovenia**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(49/82) 59.76%	(122/154) 79.22%	(90/116) 77.59%	(88/122) 72.13%	(97/120) 80.83%	(18/48) 37.50%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(35/88) 39.77%	(154/160) 71.25%	(66/76) 55%	(76/126) 60.32%	(119/124) 95.97%	(18/52) 34.62%
3. Difference (%)	*40.71%*	*10.60%*	*34.41%*	*17.89%*	*-17.16%*	*8.00%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(20/41) 48.78%	(44/77) 57.14%	(42/58) 72.41%	(26/61) 42.62%	(46/60) 76.67%	(7/24) 29.17%
5 Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(15/44) 34.09%	(40/80) 50%	(25/38) 41.67%	(28/63) 44.44%	(43/62) 85.48%	(7/26) 26.92%
6. Difference (%)	*35.83%*	*13.35%*	*55.27%*	*-4.18%*	*-10.89%*	*8.00%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(29/41) 70.73%	(78/77) 101.30%	(48/58) 82.76%	(62/61) 101.64%	(51/60) 85%	(11/24) 45.83%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(20/44) 45.45%	(74/80) 92.50%	(41/38) 68.33%	(48/63) 76.19%	(66/62) 106.45%	(11/26) 42.31%
9. Difference (%)	*44.22%*	*9.09%*	*19.15%*	*28.82%*	*-22.50%*	*8.00%*

	Ireland	**Israel**	**Switzerland**	**Hungary**	**Iceland**	**Spain**	**Sweden**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(169/176) 96.02%	(104/152) 68.87%	(115/142) 80.99%	(100/134) 74.63%	(8/122) 6.56%	(108/150) 72%	(121/154) 78.57%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(140/180) 77.78%	(101/158) 63.92%	(102/148) 68.92%	(97/136) 71.32%	(89/128) 69.53%	(100/154) 64.94%	(91/162) 56.17%
3. Difference (%)	*21.07%*	*7.46%*	*16.13%*	*4.53%*	*-236%*	*10.33%*	*33.56%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(72/88) 81.82%	(46/76) 60.53%	(46/71) 64.79%	(46/67) 68.66%	(1/61) 1.64%	(47/75) 62.67%	(55/77) 71.43%
5 Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(59/90) 65.56%	(41/79) 51.90%	(38/74) 51.35%	(50/68) 73.53%	(44/64) 68.75%	(39/77) 50.65%	(44/81) 54.32%

6. Difference (%)	*22.16%*	*15.38%*	*23.24%*	*-6.86%*	*-374%*	*21.29%*	*27.38%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(97/88) 110.23%	(58/76) 77.33%	(69/71) 97.18%	(54/67) 80.60%	(7/61) 11.48%	(61/75) 81.33%	(66/77) 85.71%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(81/90) 90%	(60/79) 75.95%	(64/74) 86.49%	(47/68) 69.12%	(45/64) 70.31%	(61/77) 79.22%	(47/81) 58.02%
9. Difference (%)	*20.27%*	*1.81%*	*11.66%*	*15.37%*	*-181 %*	*2.63%*	*39.02%*
	Turkey	**UK**	**USA**				
1. Ratio of Black Swans to Extreme Values (Full Sample)	(121/144) 84.03%	(151/196) 77.04%	(184/270) 68.15%				
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(87/146) 59.59%	(138/204) 67.65%	(159/274) 58.03%				
3. Difference (%)	*34.37%*	*13.00%*	*16.07%*				
7. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(62/72) 86.11%	(79/98) 80.61%	(94/135) 69.63%				
8 Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(39/73) 53.42%	(73/102) 71.57%	(80/137) 58.39%				
9. Difference (%)	*47.74%*	*11.90%*	*17.60%*				
4. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(59/72) 81.94%	(72/98) 73.47%	(90/135) 66.67%				
5. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(48/73) 65.75%	(65/102) 63.73%	(79/137) 57.66%				
6. Difference (%)	*22.01%*	*14.23%*	*14.51%*				

Appendix 7

Summary of the frequency of Black Swans in the residuals of equity returns using the best fit ARMA-APARCH model

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech**
1. Ratio of Black Swans (Full Sample)	0.67% (42)	0.93% (74)	1.11% (105)	1.61% (144)	0.78% (52)	0.90% (52)
2. Ratio of Black Swans (Segmented Sample)	0.50% (31)	0.95% (75)	(96) 1.01%	(79) 0.88%	(56) 0.81%	(51) 0.88%
3. Difference (%)	*30.37%*	*-1.34%*	*8.96%*	*60.04%*	*-3.64%*	*1.94%*
4. Ratio of Black – Right tail (Full Sample)	(10) 0.16%	(24) 0.30%	(36) 0.38%	(60) 0.67%	(26) 0.38%	(19) 0.33%
5. Ratio of Black Swans – Right tail (Segmented Sample)	(6) 0.10%	(24) 0.30%	(32) 0.34%	(21) 0.23%	(24) 0.35%	(16) 0.28%
6. Difference (%)	51.08%	0.00%	11.78%	104.98%	8.00%	17.19%
7. Ratio of Black – Left tail (Full Sample)	(32) 0.51%	(50) 0.63%	(69) 0.73%	(84) 0.94%	(28) 0.41%	(33) 0.57%
8 Ratio of Black Swans – Left tail (Segmented Sample)	(25) 0.40%	(51) 0.64%	(64) 0.67%	(58) 0.65%	(32) 0.46%	(35) 0.61%
9. Difference (%)	24.69%	-1.98%	7.52%	37.04%	-13.4%	-5.88%
	Italy	**Japan**	**Korea**	**Luxembourg**	**Mexico**	**Netherlands**
1. Ratio of Black Swans (Full Sample)	(33) 0.69%	(123) 0.92%	(88) 0.81%	(44) 0.97%	(66) 0.89%	(64) 0.73%
2. Ratio of Black Swans (Segmented Sample)	(26) 0.54%	(115) 0.86%	(86) 0.80%	(39) 0.86%	(64) 0.86%	(58) 0.67%
3. Difference (%)	*23.84%*	*6.73%*	*2.30%*	*12.06%*	*3.08%*	*9.84%*
4. Ratio of Black – Right tail (Full Sample)	(10) 0.21%	(43) 0.32%	(42) 0.39%	(20) 0.44%	(26) 0.35%	(18) 0.21%
5. Ratio of Black Swans – Right tail (Segmented Sample)	(7) 0.15%	(47) 0.35%	(40) 0.37%	(15) 0.33%	(24) 0.32%	(15) 0.17%
6. Difference (%)	35.67%	-8.89%	4.88%	28.77%	8.00%	18.23%
7. Ratio of Black – Left tail (Full Sample)	(23) 0.48%	(80) 0.60%	(46) 0.43%	(24) 0.53%	(40) 0.54%	(46) 0.53%
8 Ratio of Black Swans – Left tail (Segmented Sample)	(19) 0.40%	(68) 0.51%	(46) 0.43%	(24) 0.53%	(40) 0.54%	(43) 0.49%
9. Difference (%)	19.11%	16.3%	0.00%	0.00%	0.00%	6.74%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Ratio of Black Swans (Full Sample)	(52) 0.75%	(75) 1.44%	(65) 0.85%	(44) 0.58%	(91) 0.68%	(76) 1.05%
2. Ratio of Black Swans (Segmented Sample)	(53) 0.73%	(83) 1.59%	(78) 1.02%	(48) 0.64%	(99) 0.74%	(86) 1.19%
3. Difference (%)	*-1.90%*	*-10.1%*	*-18.23%*	*-8.70%*	*-8.43%*	*-12.36%*

4. Ratio of Black – Right tail (Full Sample)	(22) 0.32%	(37) 0.71%	(18) 0.23%	(13) 0.17%	(38) 0.28%	(39) 0.54%
5. Ratio of Black Swans – Right tail (Segmented Sample)	(22) 0.32%	(45) 0.86%	(32) 0.42%	(14) 0.19%	(42) 0.31%	(41) 0.57%
6. Difference (%)	*0.00%*	*-19.6%*	*-57.54%*	*-7.41%*	*-10.1%*	*-5.00%*
7. Ratio of Black – Left tail (Full Sample)	(30) 0.43%	(38) 0.73%	(47) 0.61%	(31) 0.41%	(53) 0.40%	(37) 0.51%
8 Ratio of Black Swans – Left tail (Segmented Sample)	(31) 0.45%	(38) 0.73%	(46) 0.60%	(34) 0.45%	(57) 0.43%	(45) 0.62%
9. Difference (%)	*-3.28%*	*0.00%*	*2.15%*	*-9.24%*	*-7.28%*	*-19.57%*

	New Zealand	**Norway**	**Poland**	**Portugal**	**Slovakia**	**Slovenia**
1. Ratio of Black Swans (Full Sample)	(27) 0.67%	(58) 0.76%	(46) 0.80%	(49) 0.80%	(99) 1.67%	(11) 0.46%
2. Ratio of Black Swans (Segmented Sample)	(23) 0.57%	(50) 0.72%	(36) 0.62%	(48) 0.79%	(111) 1.88%	(22) 0.92%
3. Difference (%)	16.03%	5.31%	24.51%	2.06%	-11.44%	-69.31%
4. Ratio of Black – Right tail (Full Sample)	(8) 0.20%	(21) 0.27%	(15) 0.26%	(25) 0.41%	(35) 0.59%	(2) 0.08%
5. Ratio of Black Swans – Right tail (Segmented Sample)	(8) 0.20%	(20) 0.2%	(13) 0.23%	(22) 0.36%	(45) 0.76%	(9) 0.38%
6. Difference (%)	0.00%	4.88%	14.31%	12.8%	-25.13%	-150.41%
7. Ratio of Black – Left tail (Full Sample)	(19) 0.47%	(37) 0.48%	(31) 0.54%	(24) 0.39%	(64) 1.08%	(9) 0.38%
8 Ratio of Black Swans – Left tail (Segmented Sample)	(15) 0.37%	(35) 0.46%	(23) 0.40%	(26) 0.43%	(66) 1.12%	(13) 0.55%
9. Difference (%)	23.64%	5.56%	29.85%	-8.00%	-3.08%	-36.77%

	Ireland	**Israel**	**Switzer land**	**Hungary**	**Iceland**	**Spain**	**Sweden**
1. Ratio of Black Swans (Full Sample)	(107) 1.23%	(80) 1.05%	(52) 0.74%	(72) 1.09%	(8) 0.13%	(52) 0.70%	(53) 0.69%
2. Ratio of Black Swans (Segmented Sample)	(111) 1.27%	(71) 0.94%	(50) 0.71%	(71) 1.07%	(77) 1.26%	(54) 0.73%	(67) 0.87%
3. Difference (%)	*-3.67%*	*12%*	*3.92%*	*1.40%*	*-226.4%*	*-3.8%*	*-23%*
4. Ratio of Black – Right tail (Full Sample)	(50) 0.57%	(29) 0.38%	(17) 0.24%	(30) 0.45%	(2) 0.03%	(17) 0.23%	(22) 0.29%
5. Ratio of Black Swans – Right tail (Segmented Sample)	(45) 0.52%	(27) 0.36%	(14) 0.20%	(28) 0.42%	(39) 0.64%	(13) 0.18%	(27) 0.35%
6. Difference (%)	10.54%	7.2%	19.42%	6.90%	-297%	27%	-21%
7. Ratio of Black – Left tail (Full Sample)	(57) 0.65%	(51) 0.67%	(35) 0.50%	(42) 0.63%	(6) 0.10%	(35) 0.47%	(31) 0.40%
8 Ratio of Black Swans – Left tail (Segmented Sample)	(66) 0.76%	(44) 0.58%	(36) 0.51%	(43) 0.65%	(39) 0.64%	(41) 0.55%	(40) 0.52%
9. Difference (%)	-14.66%	15%	-2.82%	-2.35%	-184.6%	-16%	-26%
	Turkey	**UK**	**USA**				

1. Ratio of Black Swans (Full Sample)	(75) 1.05%	(57) 0.59%	(94) 0.70%
2. Ratio of Black Swans (Segmented Sample)	(73) 1.03%	(96) 0.99%	(94) 0.70%
3. Difference (%)	*2.70%*	*-52%*	*0.00%*
4. Ratio of Black – Right tail (Full Sample)	(31) 0.43%	(32) 0.33%	(32) 0.24%
5. Ratio of Black Swans – Right tail (Segmented Sample)	(31) 0.43%	(49) 0.50%	(32) 0.24%
6. Difference (%)	0.00%	-43%	0.00%

Appendix 8

Summary of the ratio of Black Swans to extreme values in the residuals of daily equity returns obtained by using the best fit ARMA-APARCH model

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(42/126) 33.33%	(74/160) 46.25%	(105/190) 55.26%	(144/180) 80%	(54/138) 39.13%	(52/116) 44.83%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(31/132) 23.48%	(75/164) 45.73%	(96/198) 48.48%	(79/188) 42.02%	(56/144) 38.89%	(51/118) 43.22%
3. Difference (%)	*35.02%*	*1.13%*	*13.09%*	*64.39%*	*0.62%*	*3.65%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(10/63) 15.87%	(24/80) 30%	(36/95) 37.89%	(60/90) 66.67%	(26/69) 37.68%	(19/58) 32.76%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(6/66) 9.09%	(24/82) 29.27%	(32/99) 32.32%	(21/94) 22.34%	(24/72) 33.33%	(16/59) 27.12%
6. Difference (%)	*55.73%*	*2.47%*	*15.90%*	*109.33%*	*12.26%*	*18.89%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(32/63) 50.79%	(50/80) 62.50%	(69/95) 72.63%	(84/90) 93.33%	(28/69) 40.58%	(33/58) 56.90%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(25/66) 37.88%	(51/82) 62.20%	(64/99) 64.65%	(58/94) 61.70%	(32/72) 44.44%	(35/59) 59.32%
9. Difference (%)	*29.34%*	*0.49%*	*11.65%*	*41.39%*	*-9.10%*	*-4.17%*
	Italy	**Japan**	**Korea**	**Luxembourg**	**Mexico**	**Netherlands**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(33/96) 34.38%	(123/270) 45.56%	(88/216) 40.74%	(44/92) 47.83%	(66/150) 44%	(64/176) 36.36%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(26/102) 25.49%	(115/274) 41.97%	(86/224) 38.39%	(39/98) 39.80%	(64/160) 40%	(58/184) 31.52%
3. Difference (%)	*29.90%*	*8.20%*	*5.94%*	*18.38%*	*9.53%*	*14.29%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(10/48) 20.83%	(43/135) 31.85%	(42/108) 38.89%	(20/46) 43.48%	(26/75) 34.67%	(18/88) 20.45%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(7/51) 13.73%	(47/137) 34.31%	(40/112) 35.71%	(15/49) 30.61%	(24/80) 30%	(15/92) 16.30%

6. Difference (%)	*41.73%*	*-7.42%*	*8.52%*	*35.09%*	*14.46%*	*22.68%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(23/48) 47.92%	(80/135) 59.26%	(46/108) 42.59%	(24/46) 52.17%	(40/75) 53.33%	(46/88) 52.27%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(19/51) 37.25%	(68/137) 49.64%	(46/112) 41.07%	(24/49) 48.98%	(40/80) 50%	(43/92) 46.74%
9. Difference (%)	*25.17%*	*17.72%*	*3.64%*	*6.32%*	*6.45%*	*11.19%*
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(52/140) 37.14%	(75/106) 70.75%	(65/154) 42.21%	(44/152) 28.95%	(91/270) 33.70%	(76/146) 52.05%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(53/140) 37.86%	(83/108) 46.85%	(78/160) 48.75%	(48/160) 30%	(99/274) 36.13%	(86/148) 58.11%
3. Difference (%)	*-1.90%*	*-8.27%*	*-14.47%*	*-3.75%*	*-6.96%*	*-11%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(22/70) 31.43%	(37/53) 69.81%	(18/77) 23.38%	(13/76) 17.11%	(38/135) 28.15%	(39/73) 53.42%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(22/70) 31.43%	(45/54) 83.33%	(32/80) 40%	(14/80) 17.50%	(42/137) 30.66%	(41/74) 55.41%
6. Difference (%)	*0.00%*	*-17.71%*	*-53.71%*	*-2.28%*	*-8.54%*	*-3.64%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(30/70) 42.86%	(38/53) 71.70%	(47/77) 61.04%	(31/76) 40.79%	(53/135) 39.26%	(37/73) 50.68%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(31/70) 44.29%	(38/54) 70.37%	(46/80) 57.50%	(34/80) 42.50%	(57/137) 41.61%	(45/74) 60.81%
9. Difference (%)	*-3.28%*	*1.87%*	*5.97%*	*-4.11%*	*-5.81%*	*-18.21%*
	New Zealand	**Norway**	**Poland**	**Portugal**	**Slovakia**	**Slovenia**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(27/82) 32.93%	(58/154) 37.66%	(46/116) 39.66%	(49/122) 40.16%	(99/120) 82.50%	(11/48) 22.92%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(23/88) 26.14%	(50/160) 31.25%	(36/76) 47.37%	(48/126) 38.10%	(111/124) 89.52%	(22/52) 42.31%
3. Difference (%)	*23.10%*	*18.66%*	*-17.77%*	*5.29%*	*-8.16%*	*-61.31%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(8/41) 19.51%	(21/77) 27.27%	(15/58) 25.86%	(25/61) 40.98%	(35/60) 58.33%	(2/24) 8.33%

5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(8/44) 18.18%	(20/80) 25%	(13/38) 34.21%	(22/63) 34.92%	(45/62) 72.58%	(9/26) 34.62%
6. Difference (%)	*7.06%*	*8.70%*	*-27.98%*	*16.01%*	*-21.85%*	*-142.40%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(19/41) 46.34%	(37/77) 48.05%	(31/58) 53.45%	(24/61) 39.34%	(64/60) 106.67%	(9/24) 37.50%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(15/44) 34.09%	(35/80) 43.75%	(23/38) 60.53%	(26/63) 41.27%	(66/62) 106.45%	(13/26) 50%
9. Difference (%)	*30.70%*	*9.38%*	*-12.44%*	*-4.78%*	*0.20%*	*-28.77%*

	Ireland	**Israel**	**Switzerland**	**Hungary**	**Iceland**	**Spain**	**Sweden**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(107/176) 60.80%	(80/152) 52.63%	(52/142) 36.62%	(72/134) 53.73%	(8/122) 6.56%	(52/150) 34.65%	(53/154) 34.42%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(111/180) 61.67%	(71/158) 44.94%	(50/148) 33.78%	(71/136) 52.21%	(77/128) 60.16%	(54/154) 35.06%	(67/162) 41.36%
3. Difference (%)	*-1.42%*	*15.81%*	*8.06%*	*2.88%*	*-221.64%*	*-1.14%*	*-18.38%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(50/88) 56.82%	(29/76) 38.16%	(17/71) 23.94%	(30/67) 44.78%	(2/61) 3.28%	(17/75) 22.67%	(22/77) 28.57%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(45/90) 50%	(27/79) 34.18%	(14/74) 18.92%	(28/68) 41.18%	(39/64) 60.94%	(13/77) 16.88%	(27/81) 33.33%
6. Difference (%)	*12.78%*	*11.02%*	*23.55%*	*8.38%*	*-292.24%*	*29.46%*	*-15.42%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(57/88) 64.77%	51/76) 67.11%	(35/71) 49.30%	(42/67) 62.69%	(6/61) 9.84%	(35/75) 46.67%	(31/77) 40.26%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(66/90) 73.33%	(44/79) 55.70%	(36/74) 48.65%	(43/68) 63.24%	(38/64) 59.38%	(41/77) 53.25%	(40/81) 49.38%
9. Difference (%)	*-12.41%*	*18.64%*	*1.32%*	*-0.87%*	*-179.78%*	*-13.19%*	*-20.42%*

	Turkey	**UK**	**USA**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(75/144) 52.08%	(57/196) 29.08%	(94/270) 34.81%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(73/146) 50%	(96/204) 47.06%	(94/274) 34.31%

3. Difference (%)	*4.08%*	*-48.13%*	*1.47%*
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(31/72) 43.06%	(32/98) 32.65%	(32/135) 23.70%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(31/73) 42.47%	(49/102) 48.04%	(32/137) 23.36%
6. Difference (%)	*1.38%*	*-38.61%*	*1.47%*
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(44/72) 61.11%	(25/98) 25.51%	(62/135) 45.93%
8. Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(42/73) 57.53%	(47/102) 46.08%	(62/137) 45.26%
9. Difference (%)	*6.03%*	*-59.13%*	*1.47%*

Appendix 9

Summary of the frequency of the Black Swans in the full sample and segments (with breaks) of daily Foreign Exchange market returns

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso	Chinese Yuan	Danish Krone	Euro
1. Total Black Swans (Full Sample)	(48) 0.82%	(69) 1.18%	(92) 1.61%	(75) 1.28%	(72) 1.23%	(38) 1.34%	(55) 0.94%	(57) 0.98%
2. Total Black Swans (Segmented Samples)	(55) 0.94%	(56) 0.96%	(69) 1.21%	(59) 1.01%	(72) 1.23%	(36) 1.27%	(44) 0.75%	(47) 0.80%
3. Difference (%)	-13.6%	20.88 %	28.77 %	24%	0%	5.41%	22.31%	19.29 %
4. Positive Black Swans (Full Sample)	(32) 0.55%	(38) 0.65%	(53) 0.93%	(39) 0.67%	(43) 0.74%	(18) 0.64%	(22) 0.38%	(23) 0.39%
5. Positive Black Swans (Segmented Sample)	(28) 0.48%	(38) 0.65%	(42) 0.74%	(25) 0.43%	(42) 0.72%	(12) 0.42%	(12) 0.21%	(14) 0.24%
6. Difference	13.35%	0%	23.26 %	44.47 %	2.35%	40.55 %	60.61%	49.64 %
7. Negative Black Swans (Full Sample)	(16) 0.27%	(31) 0.53%	(39) 0.68%	(36) 0.62%	(29) 0.50%	(20) 0.71%	(33) 0.57%	(34) 0.58%
8. Negative Black Swans (Segmented Sample)	(27) 0.46%	(18) 0.31%	(27) 0.47%	(34) 0.58%	(30) 0.51%	(24) 0.85%	(32) 0.55%	(33) 0.57%
9. Difference	-52.3%	54.36 %	36.77 %	5.72%	-3.39%	-18%	3.08%	2.99%

	Fijian Dollar	Hong Kong Dollar	Icelandic Krona	Indian Rupee	Indonesian Rupiah	Kenyan Shilling	Malaysian Ringgit	Mexican Peso
1. Total Black Swans (Full Sample)	(31) 0.68%	(104) 1.78%	(63) 1.27%	(123) 2.11%	(97) 1.66%	(111) 2.24%	(54) 0.92%	(76) 1.30%
2. Total Black Swans (Segmented Samples)	(31) 0.68%	(87) 1.49%	(52) 1.05%	(124) 2.12%	(96) 1.64%	(107) 2.16%	(101) 1.73%	(80) 1.37%
3. Difference (%)	0%	17.85 %	19.19 %	-0.8%	1.04%	3.67%	-62.6%	- 5.13%
4. Positive Black Swans (Full Sample)	(21) 0.46%	(49) 0.84%	(35) 0.71%	(66) 1.13%	(45) 0.77%	(59) 1.19%	(25) 0.43%	(46) 0.60%
5. Positive Black Swans (Segmented Sample)	(15) 0.33%	(37) 0.63%	(26) 0.52%	(58) 0.99%	(46) 0.96%	(58) 1.16%	(47) 0.80%	(46) 0.62%
6. Difference	33.65%	28.09 %	29.73 %	12.9%	-21.87%	1.71%	-63.1%	0%
7. Negative Black Swans (Full Sample)	(10) 0.22%	(55) 0.94%	(28) 0.57%	(57) 0.98%	(52) 0.89%	(52) 1.05%	(29) 0.50%	(30) 0.19%
8. Negative Black Swans (Segmented Sample)	(16) 0.35%	(50) 0.86%	(26) 0.52%	(66) 1.13%	(40) 0.68%	(49) 0.99%	(54) 0.92%	(34) 0.26%
9. Difference	-44%	9.53%	7.41%	-15%	26.24%	5.94%	-62.2%	- 12.5%

	New Guinea Kina	New Turkish Lira	New Zealand Dollar	Nigerian Naira	Norwegian Krone	Pakistan Rupee	Polish Zloty	Russian Rouble
1. Total Black Swans (Full Sample)	(88) 1.57%	(76) 1.30%	(83) 1.42%	(104) 1.91%	(73) 1.25%	(97) 2.05%	(90) 1.61%	(38) 0.72%
2. Total Black Swans (Segmented Samples)	(90) 1.60%	(79) 1.35%	(72) 1.23%	(119) 2.18%	(47) 0.80%	(104) 2.19%	(58) 1.04%	(90) 1.71%
3. Difference (%)	-2.25%	-3.87%	14.22%	-13.5%	44.03%	-6.97%	43.94 %	-86.2%
4. Positive Black Swans (Full Sample)	(49) 0.87%	(50) 0.86%	(46) 0.79%	(55) 1.01%	(41) 0.70%	(53) 1.12%	(55) 0.99%	(25) 0.47%

5. Positive Black Swans (Segmented Sample)	(47) 0.84%	(51) 0.87%	(44) 0.75%	(60) 1.10%	(25) 0.43%	(51) 1.08%	(31) 57.33 %	(56) 1.06%
6. Difference	4.17%	-1.98%	4.45%	-8.70%	49.47%	3.85%	18.23 %	-80.7%
7. Negative Black Swans (Full Sample)	(39) 0.69%	(26) 0.45%	(37) 0.63%	(49) 0.90%	(32) 0.55%	(44) 0.93%	(35) 0.63%	(13) 0.25%
8. Negative Black Swans (Segmented Sample)	(43) 0.77%	(28) 0.48%	(28) 0.48%	(59) 1.08%	(22) 0.38%	(53) 1.12%	(27) 0.48%	(34) 0.64%
9. Difference	-9.76%	-7.41%	27.87%	-18.6%	37.47%	-18.6%	25.95 %	-94.1%

	Singapore Dollar	**Solomon Isl. Dollar**	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**	**UK Sterling**
1. Total Black Swans (Full Sample)	(102) 1.75%	(45) 1.74%	(99) 1.70%	(79) 1.35%	(64) 1.10%	(98) 1.68%	(64) 1.10%
2. Total Black Swans (Segmented Sample)	(78) 1.34%	(56) 2.16%	(75) 1.28%	(88) 1.51%	(51) 0.87%	(84) 1.44%	(53) 0.91%
3. Difference (%)	26.83%	-21.9%	27.76%	-10.79%	22.71%	15.42%	18.86%
4. Positive Black Swans (Full Sample)	(45) 0.77%	(24) 0.93%	(59) 1.01%	(44) 0.75%	(32) 0.55%	(52) 0.89%	(35) 0.60%
5. Positive Black Swans (Segmented Sample)	(40) 0.68%	(26) 1.01%	(46) 0.79%	(58) 0.99%	(21) 0.36%	(45) 0.77%	(26) 0.45%
6. Difference	11.78%	-8.00%	24.89%	-27.63%	42.12%	14.46%	29.73%
7. Negative Black Swans (Full Sample)	(57) 0.98%	(21) 0.81%	(40) 0.68%	(35) 0.60%	(32) 0.55%	(46) 0.79%	(29) 0.50%
8. Negative Black Swans (Segmented Sample)	(38) 0.65%	(30) 1.16%	(29) 0.50%	(30) 0.51%	(30) 0.51%	(39) 0.67%	(27) 0.46%
9. Difference	40.55%	-35.7%	32.16%	15.42%	6.45%	16.51%	7.15%

Appendix 10

Summary of the ratio of Black Swans to extreme values in the full sample and segments (with breaks) of daily Foreign Exchange market returns pre and post the Global Financial Crisis (2007-2008)

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso	Chinese Yuan	Danish Krone	Euro
1. Ratio of Black Swans to Extreme Values (Full Sample)	(48/118) 40.68%	(69/118) 58.47%	(92/116) 79.31%	(75/118) 63.56%	(72/118) 61.02%	(38/58) 65.52%	(55/118) 46.61%	(57/118) 48.31%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(55/118) 46.61%	(56/124) 45.16%	(69/122) 56.56%	(59/122) 48.36%	(72/124) 58.06%	(36/58) 62.07%	(44/122) 36.07%	(44/122) 38.52%
3. Difference (%)	-13.61%	25.84%	33.81%	27.33%	4.96%	5.41%	25.65%	22.62%
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(32/59) 54.24%	(38/59) 64.41%	(53/58) 91.38%	(39/59) 66.10%	(43/59) 72.88%	(18/29) 62.07%	(22/59) 37.29%	(23/59) 38.98%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(28/58) 48.28%	(38/62) 61.29%	(42/61) 68.85%	(25/61) 40.98%	(42/62) 67.74%	(12/29) 41.38%	(12/61) 19.67%	(14/61) 22.95%
6. Difference (%)	11.64%	4.96%	28.31%	47.80%	7.31%	40.55%	63.95%	52.98%
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(16/59) 27.12%	(31/59) 52.54%	(39/58) 67.24%	(36/59) 61.02%	(29/59) 49.15%	(20/29) 68.97%	(33/59) 55.93%	(34/59) 57.63%
8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(27/60) 45%	(18/62) 29.03%	(27/61) 44.26%	(34/61) 55.74%	(30/62) 48.39%	(24/29) 82.76%	(32/61) 52.46%	(33/61) 54.01%
9. Difference (%)	-50.64%	59.32%	41.82%	9.05%	1.57%	-18.23%	6.41%	6.32%
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(31/92) 33.70%	(104/118) 88.14%	(63/100) 63%	(123/118) 104.24%	(97/118) 82.20%	(111/100) 111%	(54/118) 45.76%	(76/118) 64.41%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(31/96) 32.29%	(87/125) 69.60%	(52/104) 50%	(124/122) 101.64%	(96/122) 78.69%	(107/104) 102.88%	(101/122) 82.79%	(80/124) 64.52%
3. Difference (%)	4.26%	23.61%	23.11%	2.52%	4.37%	7.59%	-59.28%	-0.17%
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(21/46) 45.65%	(49/59) 83.05%	(35/50) 70%	(66/59) 111.86%	(45/59) 76.27%	(59/50) 118%	(25/59) 42.37%	(46/59) 77.97%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(15/48) 31.25%	(37/62) 59.68%	(26/52) 50%	(58/61) 95.08%	(46/61) 91.80%	(58/52) 111.54%	(47/61) 77.05%	(46/62) 74.19%
6. Difference (%)	37.90%	33.05%	33.65%	16.25%	-18.54%	5.63%	-59.79%	4.96%
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(10/46) 21.74%	(55/59) 93.22%	(28/50) 56%	(57/59) 96.61%	(52/59) 88.14%	(52/50) 104%	(29/59) 49.15%	(30/59) 50.85%

8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(16/48) 33.33%	(50/62) 79.37%	(26/52) 50%	(66/61) 108.2 %	(40/61) 65.57 %	(49/52) 94.23 %	(54/61) 88.52 %	(34/62) 54.84 %
9. Difference (%)	-42.74%	16.09%	11.33 %	-13.33 %	29.57 %	9.86%	-58.84 %	-7.56 %

	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(88/114) 77.19%	(76/118) 64.41 %	(83/118) 70.34%	(104/110) 94.55%	(73/118) 61.86 %	(97/96) 101.04 %	(90/112) 80.36 %	(38/106) 35.85 %
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(90/114) 78.95%	(79/120) 65.83 %	(72/126) 57.14%	(119/113) 105.31 %	(47/126) 37.30 %	(104/102) 101.96 %	(58/116) 50%	(90/110) 81.82 %
3. Difference (%)	-2.25%	-2.19%	20.78%	-10.78%	50.59 %	-0.91%	47.45 %	-85.52 %
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(49/57) 85.96%	(50/59) 84.75 %	(46/59) 77.97%	(55/55) 100%	(41/59) 69.49 %	(53/48) 110.42 %	(55/56) 98.21 %	(25/53) 47.17 %
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(47/57) 82.46%	(51/60) 85%	(44/63) 69.84%	(60/57) 105.26 %	(25/63) 39.68 %	(51/51) 100%	(31/58) 53.45 %	(56/55) 101.82 %
6. Difference (%)	4.17%	-0.03%	11%	-5.13%	56.03 %	9.91%	60.84 %	-76.94 %
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(39/57) 68.42%	(26/59) 44.07 %	(37/59) 62.71%	(49/55) 89.09%	(32/59) 54.24 %	(44/48) 91.67%	(35/56) 62.50 %	(13/53) 24.53 %
8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(43/57) 75.44%	(28/60) 46.67 %	(28/63) 44.44%	(59/57) 105.36 %	(22/63) 34.92 %	(53/51) 103.92 %	(27/58) 46.55 %	(34/55) 61.82 %
9. Difference (%)	-9.76%	-5.73%	34.43%	-16.77%	44.03 %	-12.55%	29.46 %	-92.44 %

	Singapore Dollar	**Solomon Isl. Dollar**	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**	**UK Sterling**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(102/118) 86.44%	(45/52) 86.54%	(99/118) 83.90%	(79/118) 66.95%	(64/118) 54.24%	(98/118) 83.05%	(64/118) 54.24 %
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(78/124) 62.90%	(56/56) 100%	(75/124) 60.48%	(88/126) 69.84%	(51/120) 42.50%	(84/122) 68.85%	(53/122) 43.44 %
3. Difference (%)	31.79%	-14.46%	32.72%	-4.23%	24.39%	18.75%	22.19 %
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(45/59) 76.27%	(24/26) 92.31%	(59/59) 100%	(44/59) 74.58%	(32/59) 54.24%	(52/59) 88.14%	(35/59) 59.32 %
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(40/62) 64.52%	(26/28) 92.86%	(46/62) 74.19%	(58/63) 92.06%	(21/60) 35%	(45/61) 73.77%	(26/61) 42.62 %
6. Difference (%)	16.74%	-0.59%	29.85%	-21.07%	43.80%	17.79%	33.06 %
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(57/59) 96.61%	(21/26) 80.77%	(40/59) 67.80%	(35/59) 59.32%	(32/59) 54.24%	(46/59) 77.97%	(29/59) 49.15 %

8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(38/62) 61.29%	(30/28) 107.14%	(29/62) 46.77%	(30/63) 47.62%	(30/60) 50%	(39/61) 63.93%	(27/61) 44.26%
9. Difference (%)	45.51%	-28.26	37.12%	21.97%	8.13%	19.84%	10.48%

Appendix 11

Summary of the frequency of Black Swans in the residuals of forex returns obtained using the best fit ARMA-APARCH model

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**	**Chinese Yuan**	**Danish Krone**	**Euro**
1. Total Black Swans (Full Sample)	(4) 0.07%	(50) 0.86%	(1) 0.02%	(42) 0.72%	(52) 0.89%	(18) 0.64%	(44) 0.75%	(43) 0.74%
2. Total Black Swans (Segmented Samples)	(27) 0.46%	(55) 0.94%	(48) 0.84%	(42) 0.72%	(51) 0.87%	(23) 0.81%	(48) 0.82%	(43) 0.74%
3. Difference (%)	-191%	-9.53%	-387%	0%	1.94%	-24.5%	-8.70%	0%
4. Positive Black Swans (Full Sample)	(2) 0.03%	(41) 0.70%	(1) 0.02%	(20) 0.34%	(29) 0.50%	(10) 0.35%	(15) 0.26%	(14) 0.24%
5. Positive Black Swans (Segmented Sample)	(7) 0.12%	(39) 0.67%	(34) 0.60%	(21) 0.36%	(29) 0.50%	(8) 0.28%	(18) 0.26%	(15) 0.26%
6. Difference	-125%	5.00%	-353%	-4.88%	0%	22.3%	-18.2%	-6.90%
7. Negative Black Swans (Full Sample)	(2) 0.03%	(9) 0.15%	(0) 0.00%	(22) 0.38%	(23) 0.39%	(8) 0.28%	(29) 0.50%	(29) 0.50%
8. Negative Black Swans (Segmented Sample)	(20) 0.34%	(16) 0.27%	(14) 0.25%	(21) 0.36%	(22) 0.38%	(15) 0.53%	(30) 0.51%	(28) 0.48%
9. Difference	-230%	-57.5%	N/A	4.65%	4.45%	-62.9%	-3.39%	3.51%

	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**	**Indonesian Rupiah**	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
1. Total Black Swans (Full Sample)	(29) 0.64%	(76) 1.30%	(55) 1.11%	(78) 1.34%	(96) 1.64%	(80) 1.61%	(1) 0.02%	(46) 0.79%
2. Total Black Swans (Segmented Samples)	(37) 0.81%	(79) 1.35%	(49) 0.99%	(93) 1.59%	(87) 1.49%	(94) 1.90%	(37) 0.63%	(56) 0.96%
3. Difference (%)	-24.4%	-3.87%	11.55%	-17.6%	9.84%	-16.1%	-361%	-19.7%
4. Positive Black Swans (Full Sample)	(17) 0.37%	(35) 0.630%	(28) 0.57%	(49) 0.84%	(63) 1.08%	(51) 1.03%	(0) 0%	(33) 0.57%
5. Positive Black Swans (Segmented Sample)	(22) 0.48%	(37) 0.63%	(21) 0.42%	(47) 0.80%	(54) 0.92%	1.01%	(15) 0.26%	(41) 0.70%
6. Difference	-25.8%	-5.56%	28.8%	4.17%	15.4%	1.98%	N/A	-21.7%
7. Negative Black Swans (Full Sample)	(12) 0.26%	(41) 0.70%	(27) 0.55%	(29) 0.50%	(33) 0.57%	(29) 0.59%	(1) 0.02%	(13) 0.22%
8. Negative Black Swans (Segmented Sample)	(15) 0.33%	(42) 0.72%	(28) 0.57%	(46) 0.79%	(33) 0.57%	(44) 0.89%	(22) 0.38%	(15) 0.26%
9. Difference	-22.3%	-2.41%	-3.64%	-46.1%	0%	-41.7%	-309%	-14.3%

	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
1. Total Black Swans (Full Sample)	(66) 1.18%	(15) 0.26%	(64) 1.10%	(81) 1.48%	(46) 0.79%	(73) 1.54%	(54) 0.97%	(82) 1.56%
2. Total Black Swans (Segmented Samples)	(71) 1.27%	(47) 0.80%	(66) 1.13%	(89) 1.63%	(40) 0.68%	(74) 1.56%	(51) 0.91%	(74) 1.40%
3. Difference (%)	-7.30%	-114.21%	-3.08%	-9.42%	13.98%	-1.36%	5.72%	10.27%

4. Positive Black Swans (Full Sample)	(39) 0.69%	(14) 0.24%	(49) 0.84%	(50) 0.92%	(22) 0.38%	(39) 0.82%	(30) 0.54%	(52) 0.99%
5. Positive Black Swans (Segmented Sample)	(46) 0.82%	(34) 0.58%	(46) 0.79%	(49) 0.90%	(18) 0.31%	(36) 0.76%	(28) 0.50%	(45) 0.85%
6. Difference	-16.51%	-88.73%	6.32%	2.02%	20.07%	8.00%	6.90%	14.46%
7. Negative Black Swans (Full Sample)	(27) 0.48%	(1) 0.02%	(15) 0.26%	(31) 0.57%	(24) 0.41%	(34) 0.72%	(24) 0.43%	(30) 0.57%
8. Negative Black Swans (Segmented Sample)	(25) 0.45%	(13) 0.22%	(20) 0.34%	(40) 0.73%	(22) 0.38%	(38) 0.80%	(23) 0.41%	(29) 0.55%
9. Difference	7.70%	-256.49%	-28.77%	-25.49%	8.70%	-11.12%	4.26%	3.39%

	Singapore Dollar	**Solomon Isl. Dollar**	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**	**UK Sterling**
1. Total Black Swans (Full Sample)	(64) 1.10%	(50) 1.93%	(54) 0.92%	(84) 1.44%	(48) 0.82%	(94) 1.61%	(40) 0.68%
2. Total Black Swans (Segmented Sample)	(67) 1.15%	(44) 1.70%	(51) 0.87%	(64) 1.10%	(45) 0.77%	(79) 1.35%	(41) 0.70%
3. Difference (%)	-4.58%	12.78%	5.72%	27.19%	6.45%	17.38%	-2.47%
4. Positive Black Swans (Full Sample)	(30) 0.51%	(25) 0.97%	(54) 0.70%	(46) 0.79%	(20) 0.34%	(49) 0.84%	(17) 0.29%
5. Positive Black Swans (Segmented Sample)	(30) 0.51%	(26) 1.01%	(39) 0.65%	(47) 0.80%	(20) 0.34%	(38) 0.65%	(16) 0.27%
6. Difference	0%	-3.92%	7.60%	-2.15%	0%	25.42%	6.06%
7. Negative Black Swans (Full Sample)	(34) 0.58%	(25) 0.97%	(13) 0.22%	(38) 0.65%	(28) 0.48%	(45) 0.77%	(23) 0.39%
8. Negative Black Swans (Segmented Sample)	(37) 0.63%	(18) 0.70%	(13) 0.22%	(17) 0.29%	(25) 0.43%	(41) 0.70%	(25) 0.43%
9. Difference	-8.46%	32.85%	0%	80.44%	11.33%	9.31%	-8.34%

Appendix 12

Summary of the ratio of Black Swans to extreme values in the residuals of the best fit ARMA-APARCH model

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso	Chinese Yuan	Danish Krone	Euro
1. Ratio of Black Swans to Extreme Values (Full Sample)	(4/118) 3.39%	(50/118) 42.37%	(1/116) 0.86%	(42/118) 35.59%	(52/118) 44.07%	(18/58) 31.03%	(44/118) 37.29%	(43/118) 36.44%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(27/122) 22.13%	(55/124) 44.35%	(48/122) 39.34%	(42/122) 34.43%	(51/124) 41.13%	(23/58) 39.66%	(48/122) 39.34%	(43/122) 35.25%
3. Difference (%)	-187.62%	-4.57%	-382.08%	3.33%	6.90%	-24.51%	-5.37%	3.33%
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(2/59) 3.39%	(41/59) 69.42%	(1/58) 1.72%	(20/59) 33.90%	(29/59) 49.15%	(10/29) 34.48%	(15/59) 25.42%	(14/59) 23.73%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(7/61) 11.48%	(39/62) 62.90%	(34/61) 55.74%	(21/61) 34.43%	(29/62) 46.77%	(8/29) 27.59%	(18/61) 29.51%	(15/61) 24.59%
6. Difference (%)	-121.94%	9.96%	-347.59%	-1.55%	4.96%	22.31%	-14.90%	-3.57%
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(2/59) 3.39%	(9/59) 15.25%	(0/58) 0%	(22/59) 37.29%	(23/59) 38.98%	(8/29) 27.59%	(29/59) 49.15%	(29/59) 49.15%
8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(20/61) 32.79%	(16/62) 25.81%	(14/61) 22.95%	(21/61) 34.43%	(22/62) 35.48%	(15/29) 51.72%	(30/61) 49.18%	(28/61) 45.90%
9. Difference (%)	-226.92%	-52.58%	N/A	7.99%	9.40%	-62.86%	-0.06%	6.84%

	Fijian Dollar	Hong Kong Dollar	Icelandic Krona	Indian Rupee	Indonesian Rupiah	Kenyan Shilling	Malaysian Ringgit	Mexican Peso
1. Ratio of Black Swans to Extreme Values (Full Sample)	(29/92) 31.52%	(76/118) 64.41%	(55/100) 55%	(78/118) 66.10%	(96/118) 81.36%	(80/100) 80%	(1/118) 0.85%	(46/118) 38.98%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(37/96) 38.54%	(79/126) 62.70%	(49/104) 47.12%	(93/122) 76.23%	(87/122) 71.31%	(94/104) 90.38%	(37/122) 30.33%	(56/124) 45.16%
3. Difference (%)	-20.11%	2.69%	15.47%	-14.26%	13.18%	-12.20%	-357.76%	-14.17%
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(17/46) 36.96%	(35/59) 59.32%	(28/50) 56%	(49/59) 83.05%	(63/59) 106.78%	(51/50) 102%	(0/59) 0%	(33/59) 55.93%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(22/48) 45.83%	(37/62) 58.73%	(21/52) 40.38%	(47/61) 77.05%	(54/61) 88.52%	(50/52) 96.15%	(15/61) 24.59%	(41/62) 66.13%
6. Difference (%)	-21.53%	1%	32.69%	7.50%	18.75%	5.90%	N/A	-16.75%
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(12/46) 26.09%	(41/59) 69.49%	(27/50) 54%	(29/59) 49.15%	(33/59) 55.93%	(29/50) 58%	(1/59) 1.69%	(13/59) 22.03%

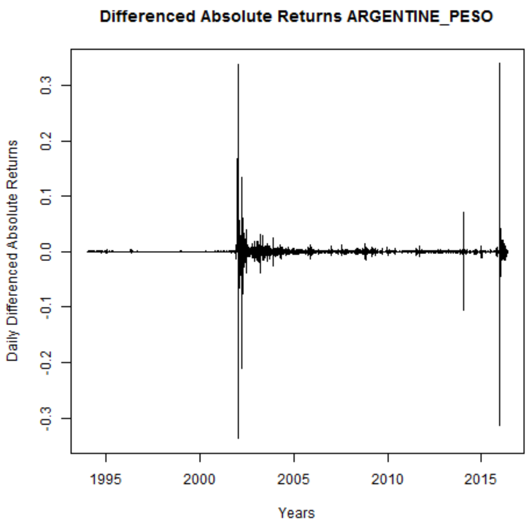
8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(15/48) 31.25%	(42/62) 66.67%	(28/52) 53.85%	(46/61) 75.41%	(33/61) 54.10%	(44/52) 84.62%	(22/61) 36.07%	(15/62) 24.19%
9. Difference (%)	-18.06%	4.15%	0.29%	-42.80%	3.33%	-37.77%	-305.77%	-9.35%

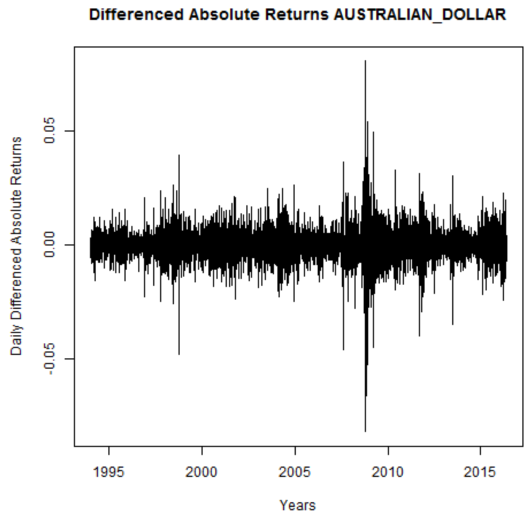
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(66/114) 57.89%	(15/118) 12.71%	(64/118) 54.24%	(181/110) 73.64%	(46/118) 38.98%	(73/96) 76.04%	(54/112) 48.21%	(82/106) 77.36%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(71/114) 62.28%	(47/120) 39.17%	(66/126) 52.38%	(89/113) 76.72%	(40/126) 31.75%	(74/102) 72.55%	(51/116) 43.97%	(74/110) 67.27%
3. Difference (%)	-7.30%	-112.53%	3.48%	-4.11%	20.54%	4.70%	9.22%	13.97%
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(39/57) 68.42%	(14/59) 23.73%	(49/59) 83.05%	(50/55) 90.91%	(22/59) 37.29%	(39/48) 81.25%	(30/56) 53.57%	(52/53) 98.11%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(46/57) 80.70%	(34/60) 56.67%	(46/63) 73.02%	(49/57) 84.48%	(18/63) 28.57%	(36/51) 70.59%	(28/58) 48.28%	(45/55) 81.82%
6. Difference (%)	-16.51%	-87.05%	12.88%	7.33%	26.63%	14.07%	10.41%	18.16%
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(27/57) 47.37%	(1/59) 1.69%	(15/59) 25.42%	(31/55) 56.36%	(24/59) 40.68%	(34/48) 70.83%	(24/56) 42.86%	(30/53) 56.60%
8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(25/57) 43.86%	(13/60) 21.67%	(20/63) 31.75%	(40/57) 68.97%	(22/63) 34.92%	(38/51) 74.51%	(23/58) 39.66%	(29/55) 52.73%
9. Difference (%)	7.70%	-254.81%	-22.21%	-20.18%	15.26%	-5.06%	7.77%	7.09%

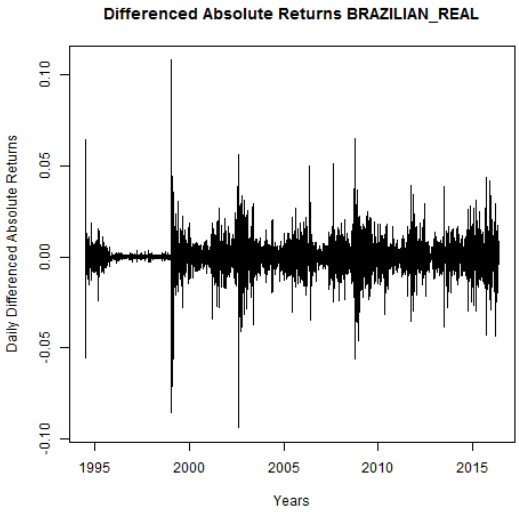
	Singapore Dollar	**Solomon Isl. Dollar**	**South Africa Rand**	**South-Korean Won**	**Swedish Krona**	**Taiwan new Dollar**	**UK Sterling**
1. Ratio of Black Swans to Extreme Values (Full Sample)	(64/118) 54.24%	(50/52) 96.15%	(54/118) 45.76%	(84/118) 71.19%	(48/118) 40.68%	(94/118) 79.66%	(40/118) 33.90%
2. Ratio of Black Swans to Extreme Values (Segmented Sample)	(67/124) 54.03%	(44/56) 78.57%	(51/124) 41.13%	(64/126) 50.79%	(45/120) 37.50%	(79/122) 64.75%	(41/122) 33.61%
3. Difference (%)	0.38%	20.19%	10.68%	33.75%	8.13%	20.72%	0.86%
4. Ratio of Black Swans to Extreme Values – Right tail (Full Sample)	(30/59) 50.85%	(25/26) 96.15%	(40/59) 69.49%	(46/59) 77.97%	(20/59) 33.90%	(49/59) 83.05%	(17/59) 28.81%
5. Ratio of Black Swans to Extreme Values – Right tail (Segmented Sample)	(30/62) 48.39%	(26/28) 92.86%	(29/62) 61.29%	(47/63) 74.60%	(20/60) 33.33%	(38/61) 62.30%	(16/61) 26.23%
6. Difference (%)	4.96%	3.49%	12.56%	4.41%	1.68%	28.76%	9.40%

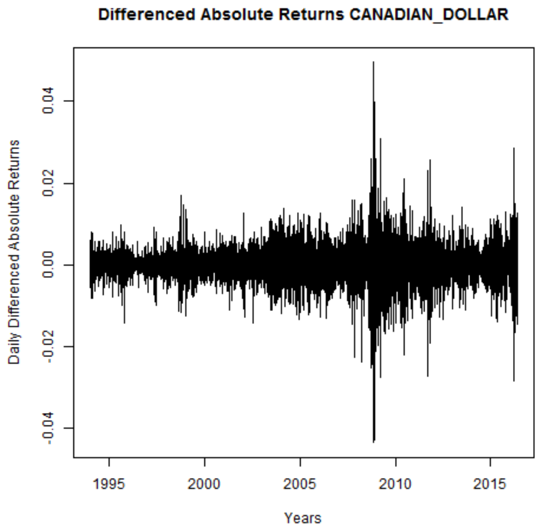
7. Ratio of Black Swans to Extreme Values – Left tail (Full Sample)	(34/59) 57.63%	(25/26) 96.15%	(59/59) 22.03%	(38/59) 64.41%	(28/59) 47.46%	(45/59) 76.27%	(23/59) 38.98 %
8 Ratio of Black Swans to Extreme Values – Left tail (Segmented Sample)	(37/62) 59.68%	(18/28) 64.29%	(46/62) 20.97%	(17/63) 26.98%	(25/60) 41.67%	(41/61) 67.21%	(25/61) 40.98 %
9. Difference (%)	-3.50%	40.26%	4.96%	87%	13.01%	12.64%	-5%

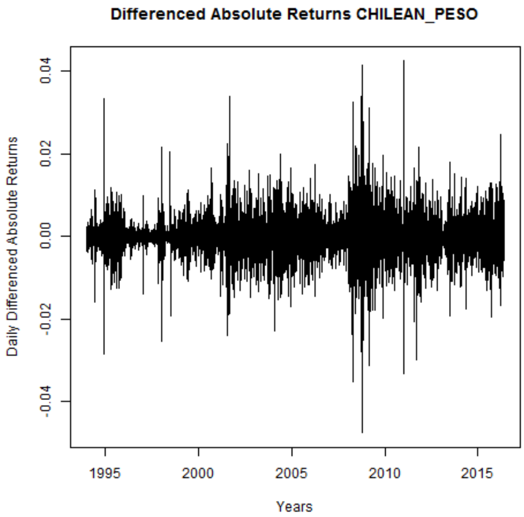
Appendix 13
Stationarity - Differenced Absolute Returns

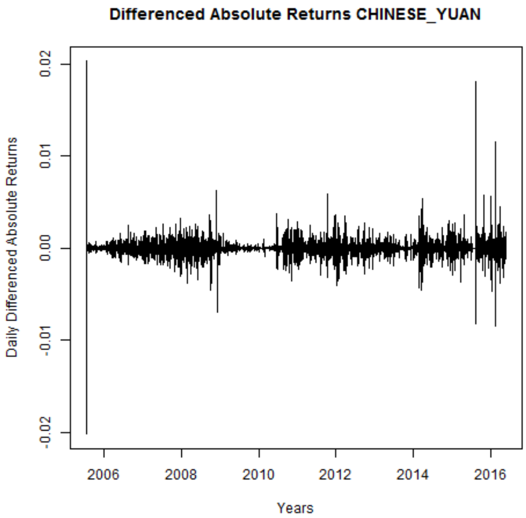


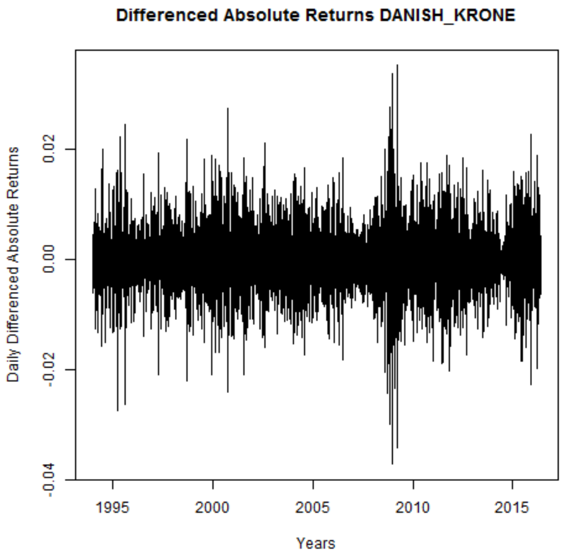








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Daily Differenced Absolute Returns
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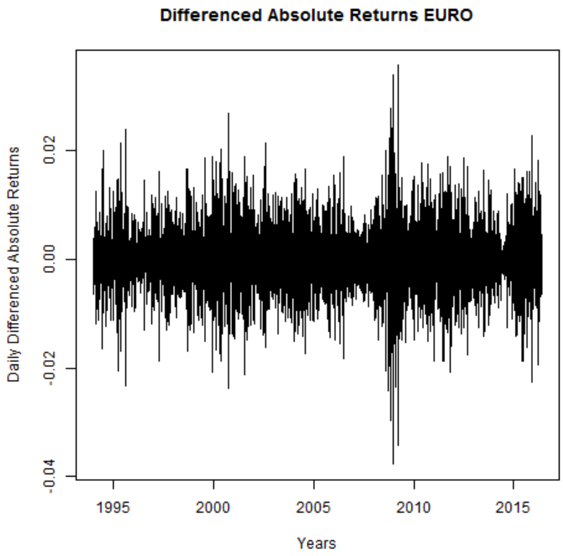
0.00
0.02
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns EURO
Daily Differenced Absolute Returns
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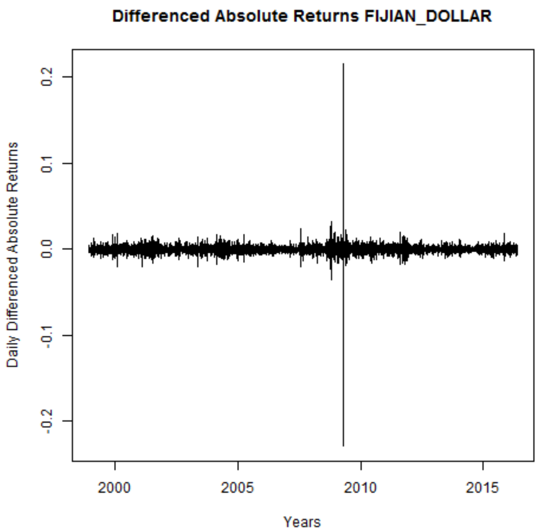
-0.02
0.00
0.02
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns FIJIAN_DOLLAR
Daily Differenced Absolute Returns
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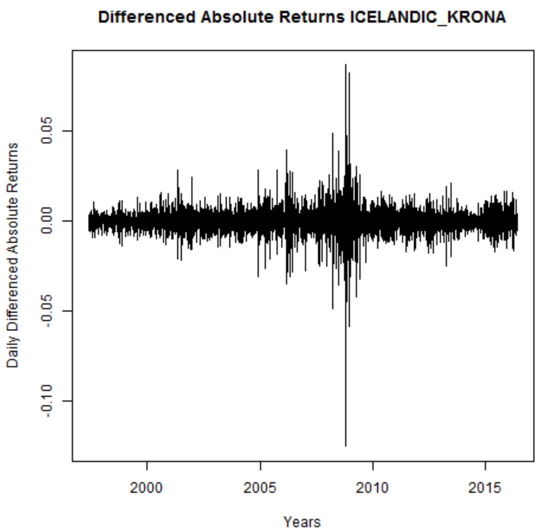
0.0
0.1
0.2
2000
2005
2010
2015
Years


Differenced Absolute Returns HONG_KONG_DOLLAR
Daily Differenced Absolute Returns
-0.004
-0.002
0.000
0.002
0.004
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns ICELANDIC_KRONA
Daily Differenced Absolute Returns
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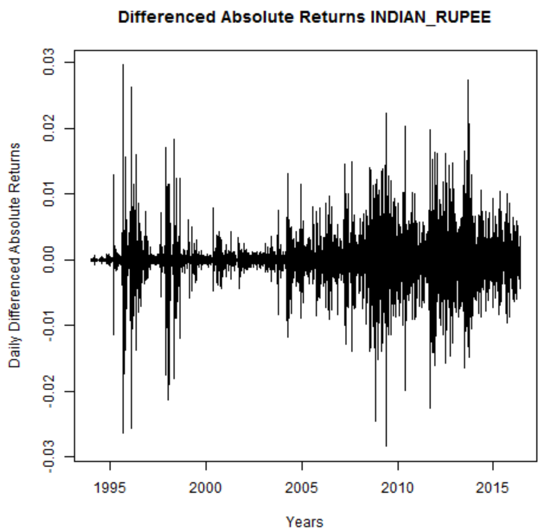
0.05
2000
2005
2010
2015
Years


Differenced Absolute Returns INDIAN_RUPEE
Daily Differenced Absolute Returns
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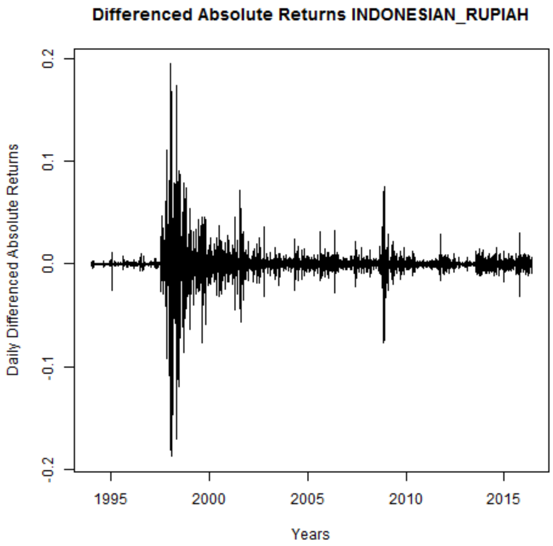
-0.01
0.00
0.01
0.02
0.03
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns INDONESIAN_RUPIAH
Daily Differenced Absolute Returns
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0.0
0.1
0.2
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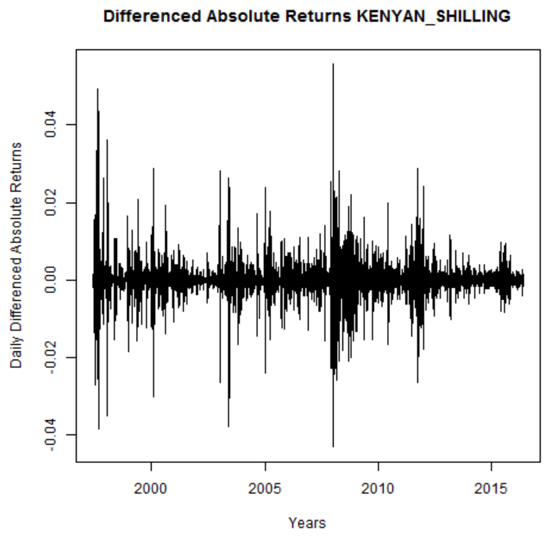
2000
2005
2010
2015
Years


Differenced Absolute Returns KENYAN_SHILLING
Daily Differenced Absolute Returns
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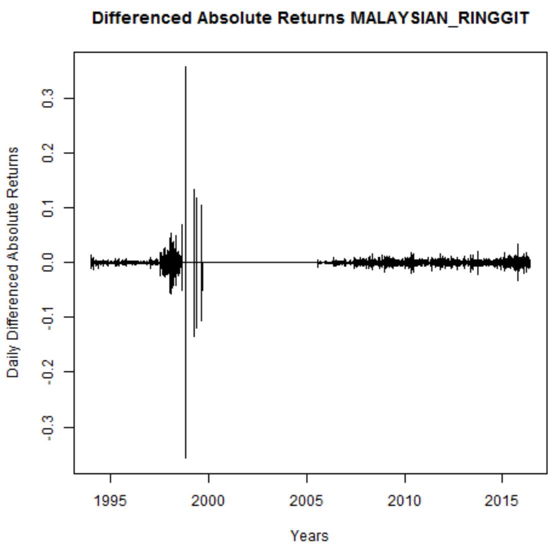
-0.02
0.00
0.02
0.04
2000
2005
2010
2015
Years


Differenced Absolute Returns MALAYSIAN_RINGGIT
Daily Differenced Absolute Returns
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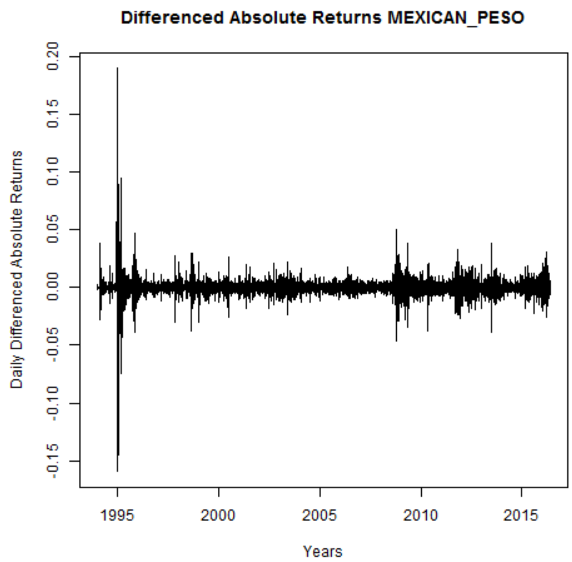
-0.1
0.0
0.1
0.2
0.3
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns MEXICAN_PESO
Daily Differenced Absolute Returns
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0.05
0.10
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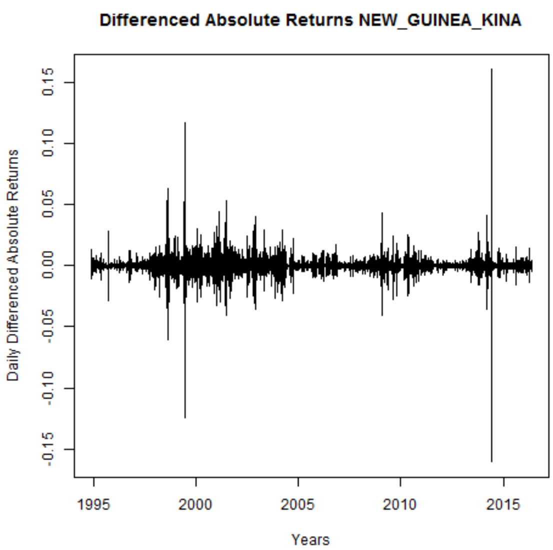
0.20
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns NEW_GUINEA_KINA
Daily Differenced Absolute Returns
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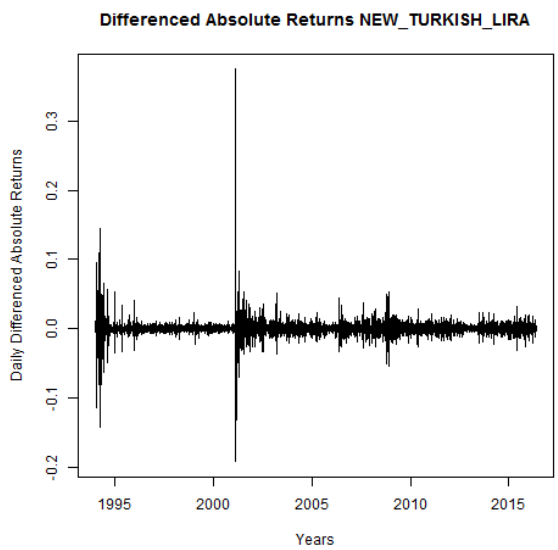
0.05
0.10
0.15
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns NEW_TURKISH_LIRA
Daily Differenced Absolute Returns
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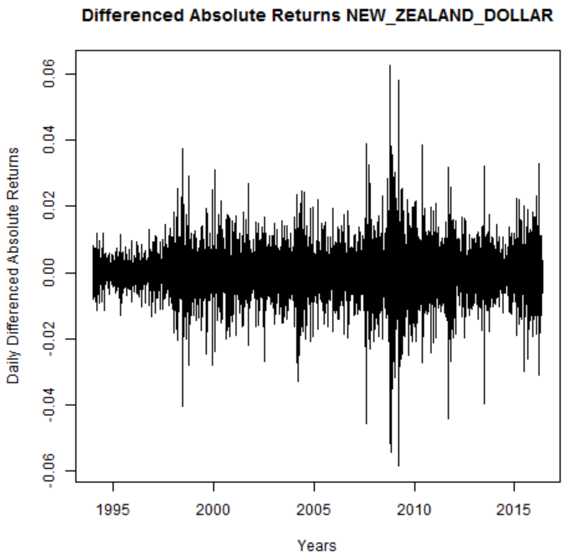
0.1
0.2
0.3
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns NEW_ZEALAND_DOLLAR
Daily Differenced Absolute Returns
Years
1995 2000 2005 2010 2015
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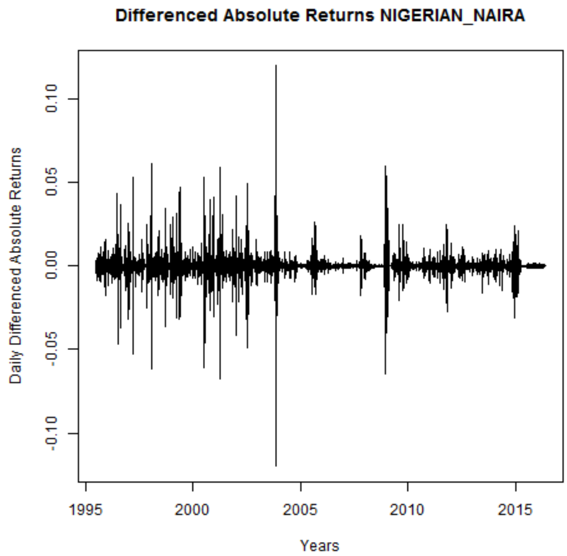
Differenced Absolute Returns NIGERIAN_NAIRA
Daily Differenced Absolute Returns
Years
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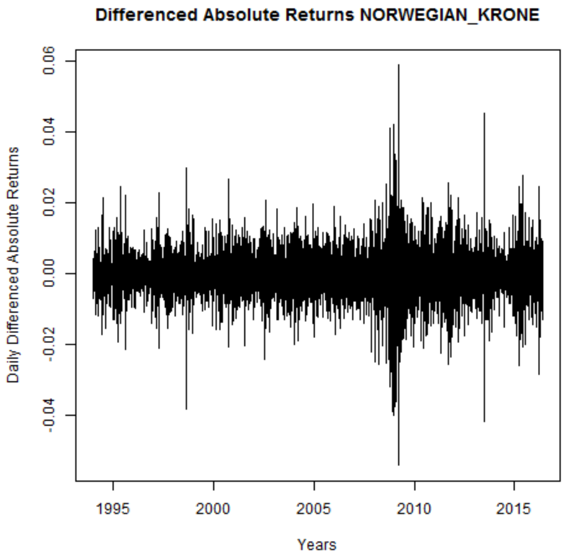
Differenced Absolute Returns NORWEGIAN_KRONE
Daily Differenced Absolute Returns
Years
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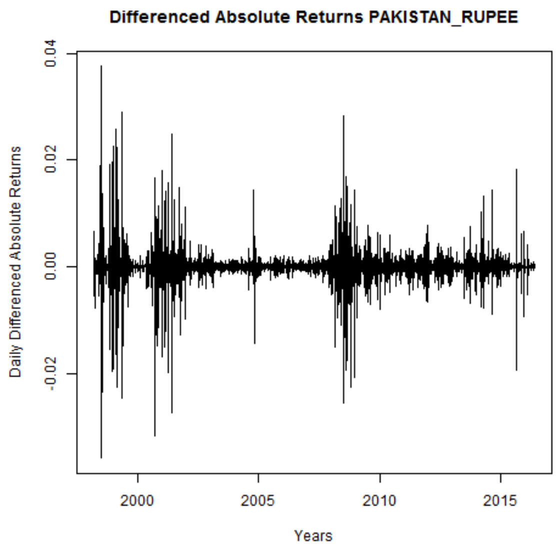
Differenced Absolute Returns PAKISTAN_RUPEE
Daily Differenced Absolute Returns
Years
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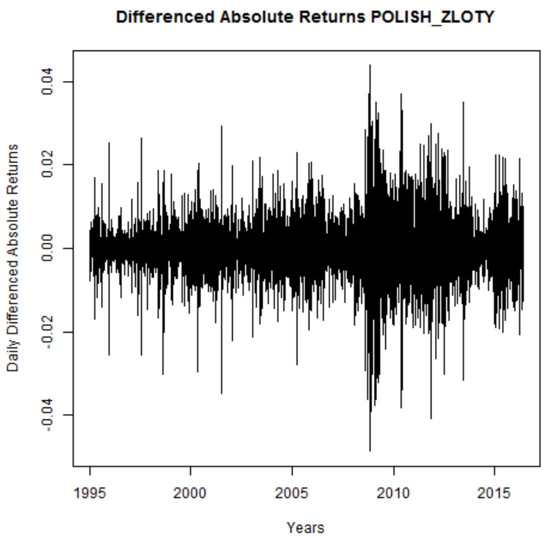
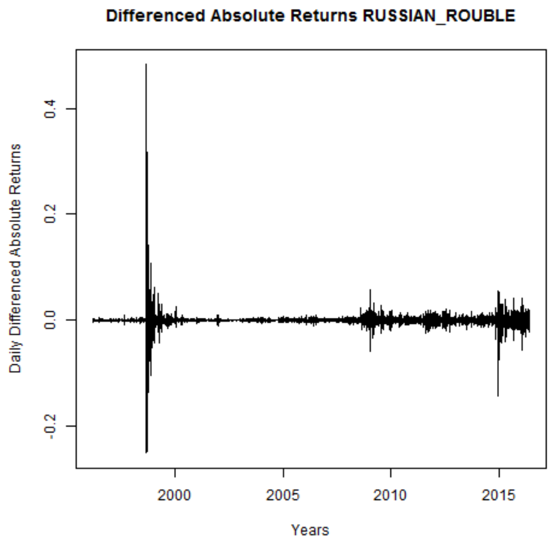
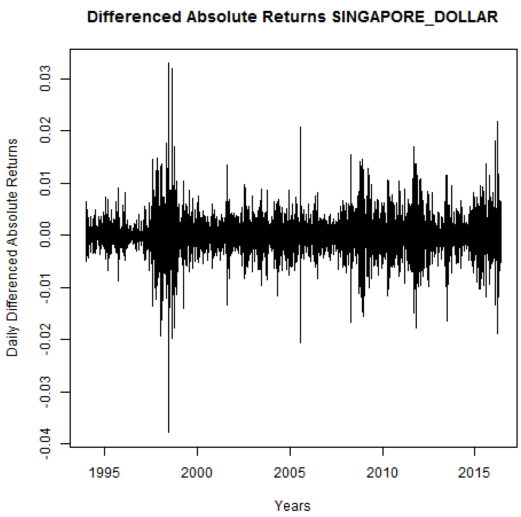
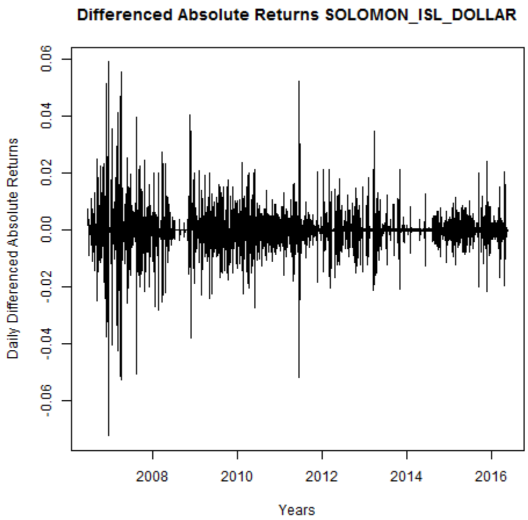
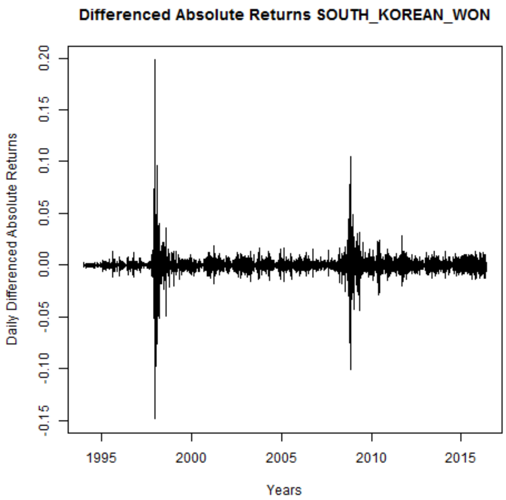
-0.02 0.00 0.02 0.04

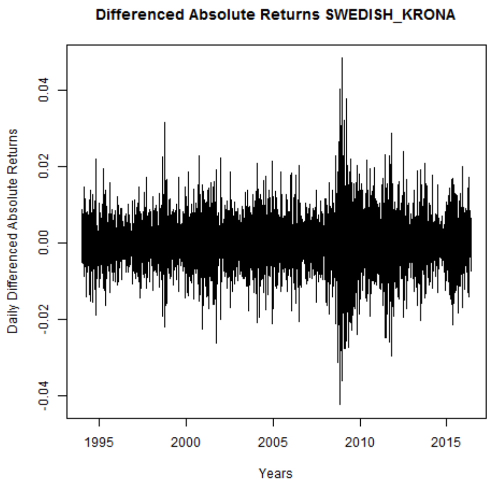

Differenced Absolute Returns POLISH_ZLOTY
Daily Differenced Absolute Returns
Years
1995 2000 2005 2010 2015
-0.04 -0.02 0.00 0.02 0.04

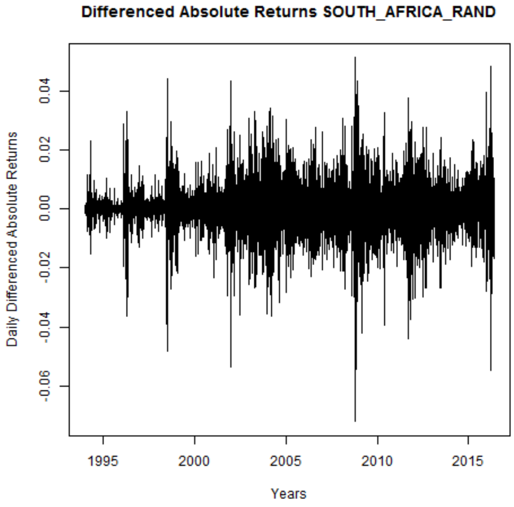

Differenced Absolute Returns RUSSIAN_ROUBLE
Daily Differenced Absolute Returns
Years
2000 2005 2010 2015
-0.2 0.0 0.2 0.4

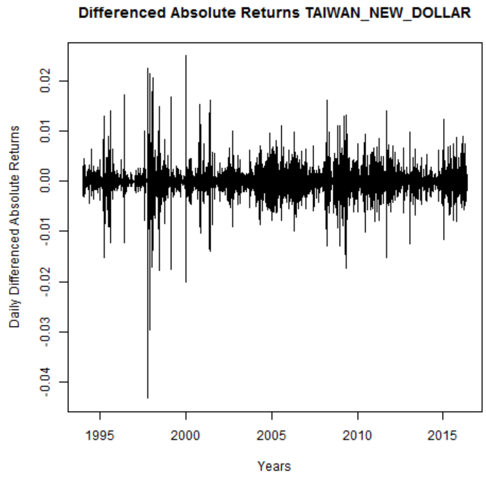

Differenced Absolute Returns SINGAPORE_DOLLAR
Daily Differenced Absolute Returns
Years

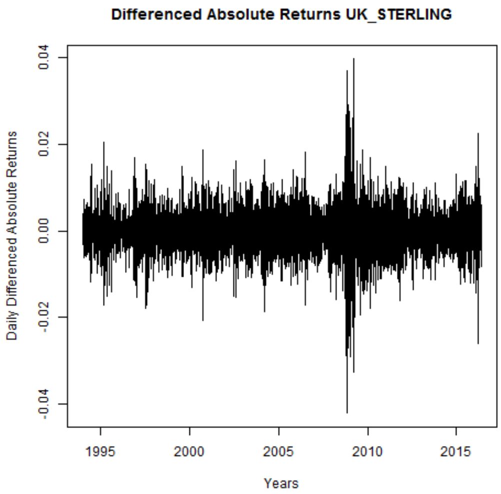

Differenced Absolute Returns SOLOMON_ISL_DOLLAR
Daily Differenced Absolute Returns
Years


Differenced Absolute Returns SOUTH_KOREAN_WON
Daily Differenced Absolute Returns
Years


Differenced Absolute Returns SWEDISH_KRONA
Daily Differenced Absolute Returns
Years

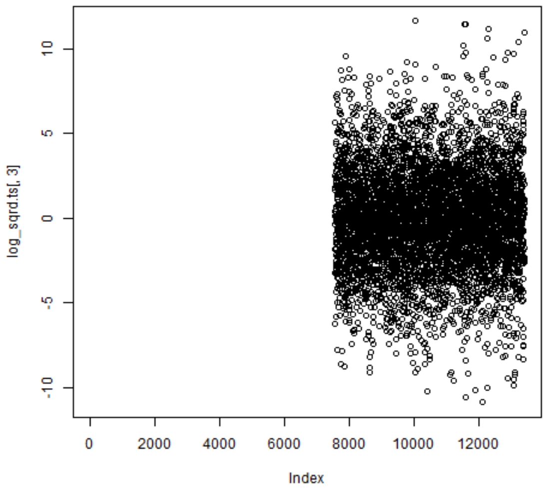

Differenced Absolute Returns SOUTH_AFRICA_RAND
Daily Differenced Absolute Returns
Years
-0.06
-0.04
-0.02
0.00
0.02
0.04
1995
2000
2005
2010
2015


Differenced Absolute Returns TAIWAN_NEW_DOLLAR
Daily Differenced Absolute Returns
Years
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
1995
2000
2005
2010
2015

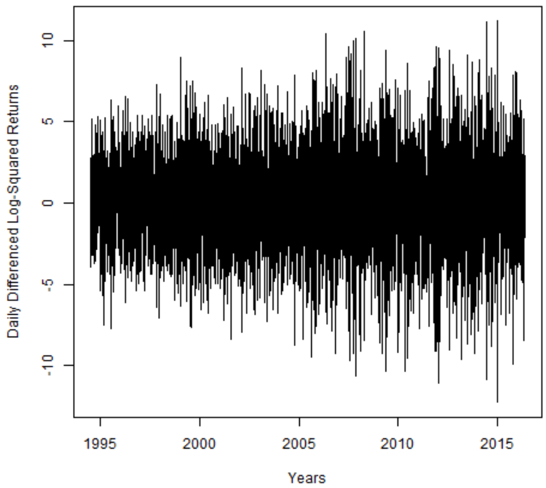

Differenced Absolute Returns UK_STERLING
Daily Differenced Absolute Returns
Years
-0.04
-0.02
0.00
0.02
0.04
1995
2000
2005
2010
2015

Appendix 14

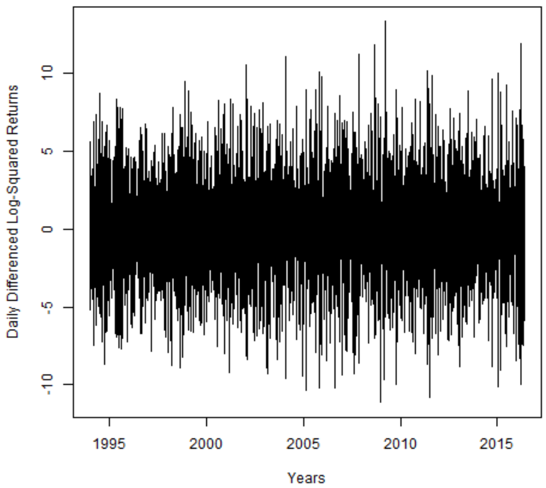
Differenced Log Squared Returns – Forex returns



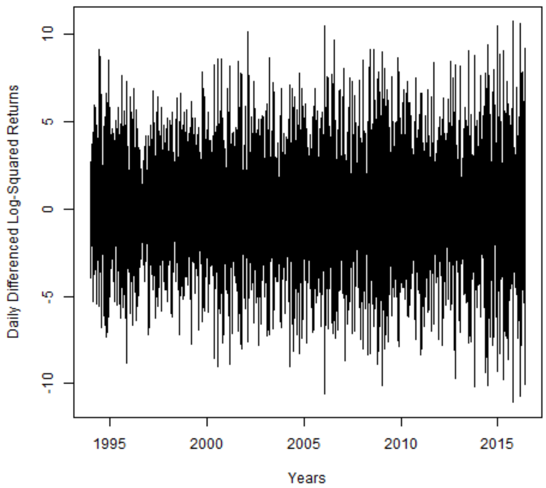
Differenced -Log-Squared Returns BRAZILIAN_REAL



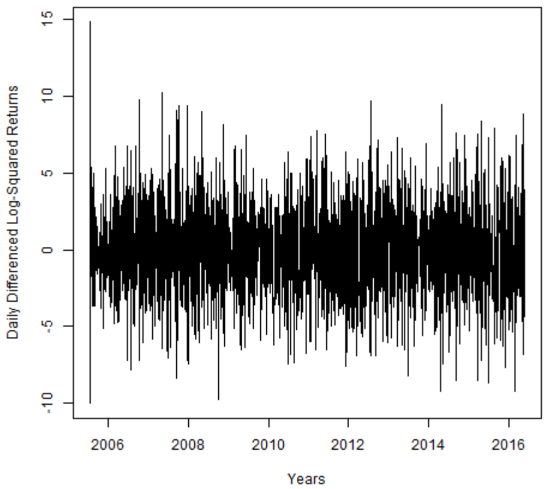
Differenced -Log-Squared Returns CANADIAN_DOLLAR



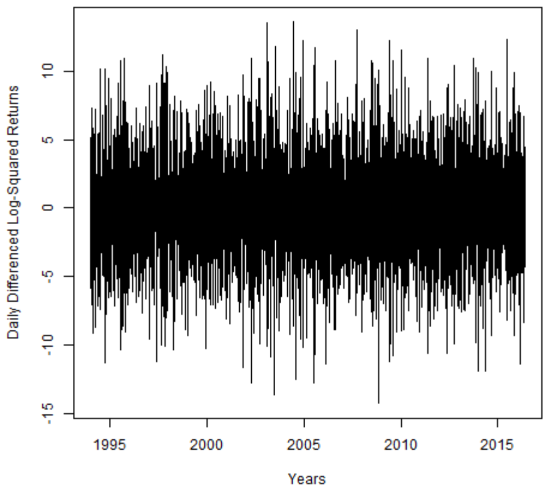
Differenced -Log-Squared Returns CHILEAN_PESO

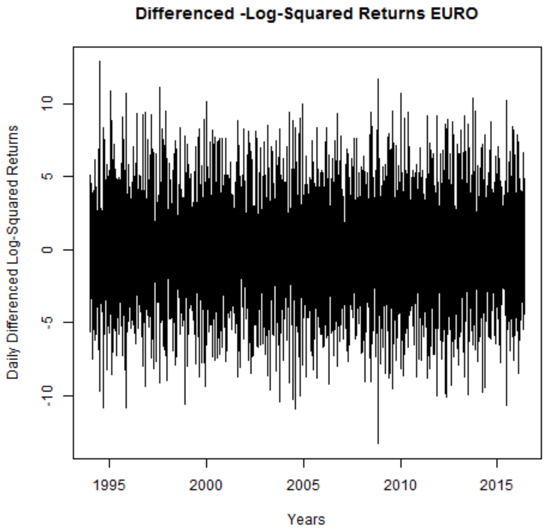


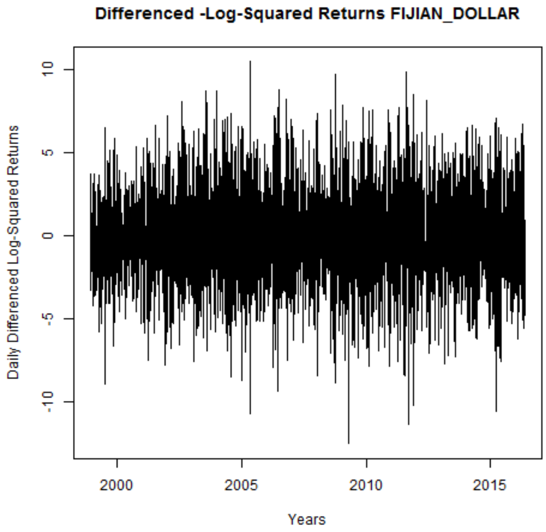
Differenced -Log-Squared Returns CHINESE_YUAN

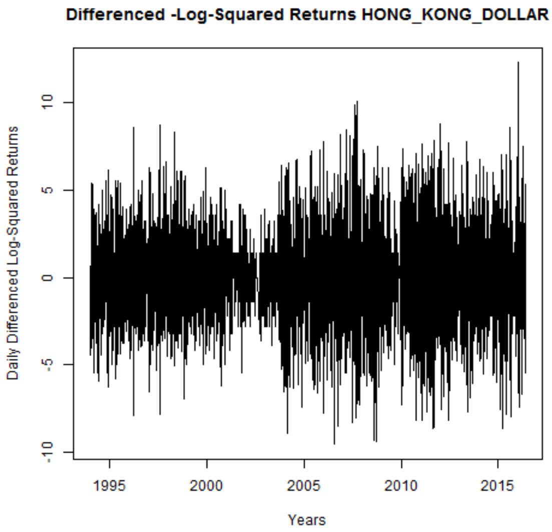


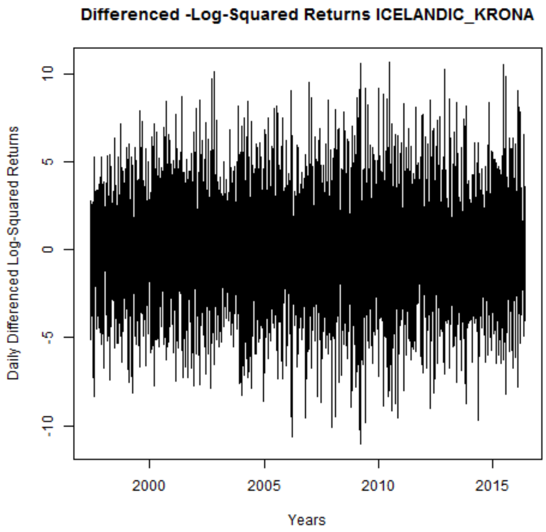
Differenced -Log-Squared Returns DANISH_KRONE

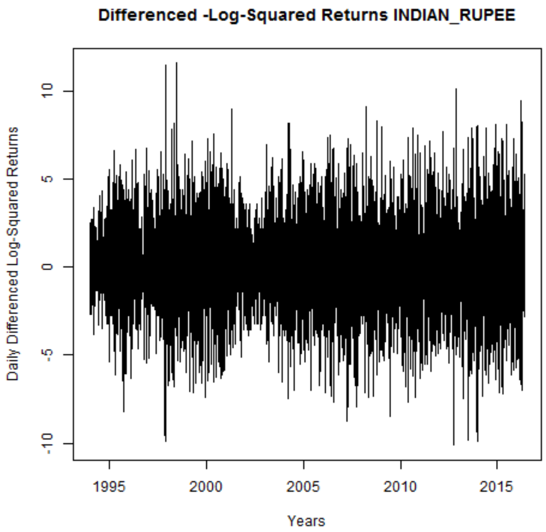


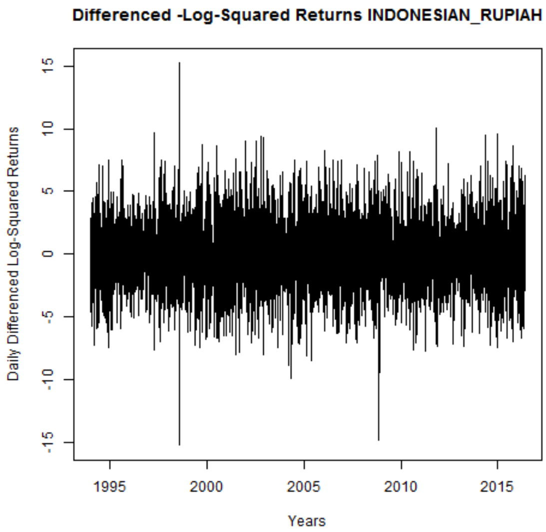

Differenced -Log-Squared Returns EURO
Daily Differenced Log-Squared Returns
Years

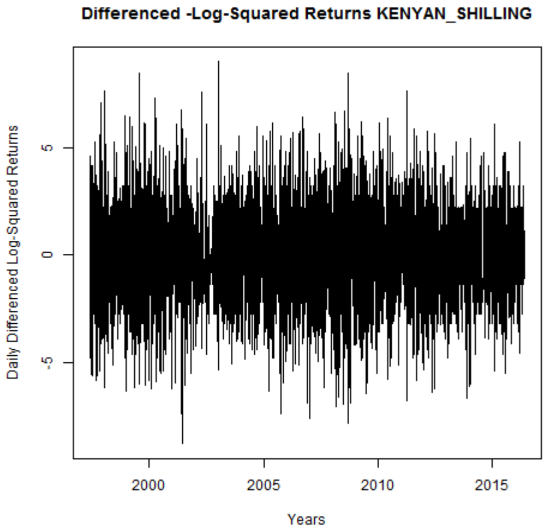

Differenced -Log-Squared Returns FIJIAN_DOLLAR
Daily Differenced Log-Squared Returns
Years

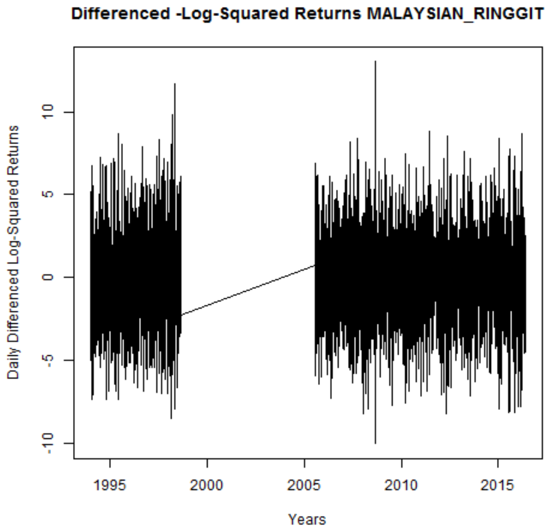

Differenced -Log-Squared Returns HONG_KONG_DOLLAR
Daily Differenced Log-Squared Returns
Years

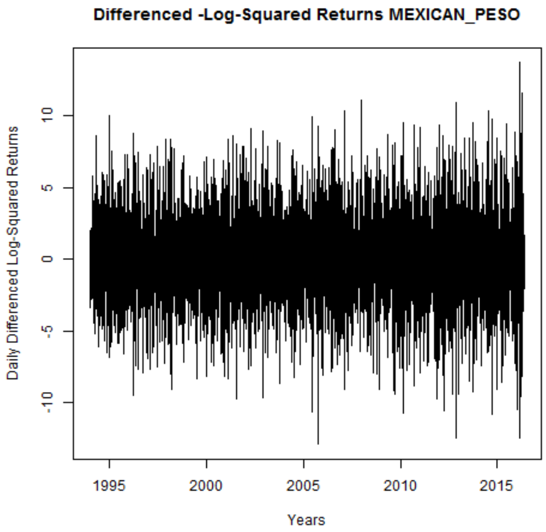

Differenced -Log-Squared Returns ICELANDIC_KRONA
Daily Differenced Log-Squared Returns
Years

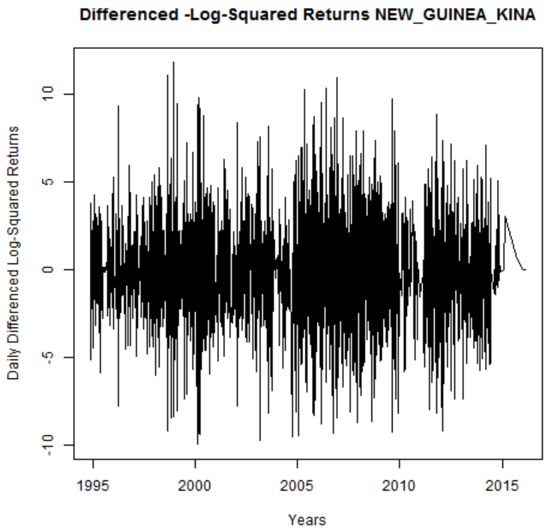

Differenced -Log-Squared Returns INDIAN_RUPEE
Daily Differenced Log-Squared Returns
Years

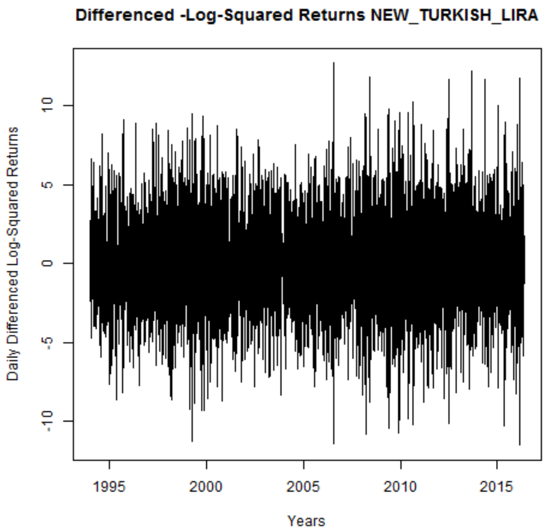

Differenced -Log-Squared Returns INDONESIAN_RUPIAH
Daily Differenced Log-Squared Returns
Years

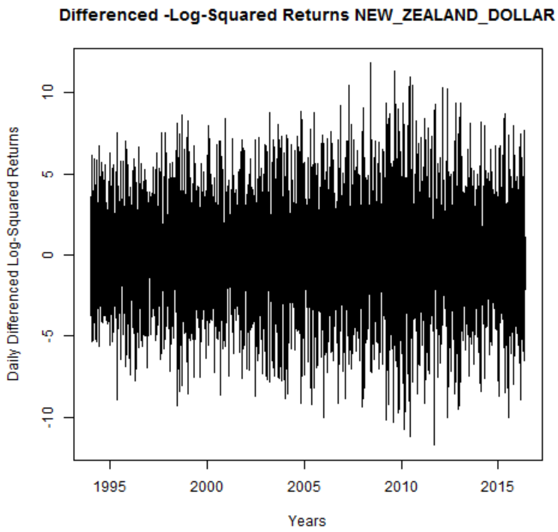

Differenced -Log-Squared Returns KENYAN_SHILLING
Daily Differenced Log-Squared Returns
Years
-5
0
5
2000
2005
2010
2015

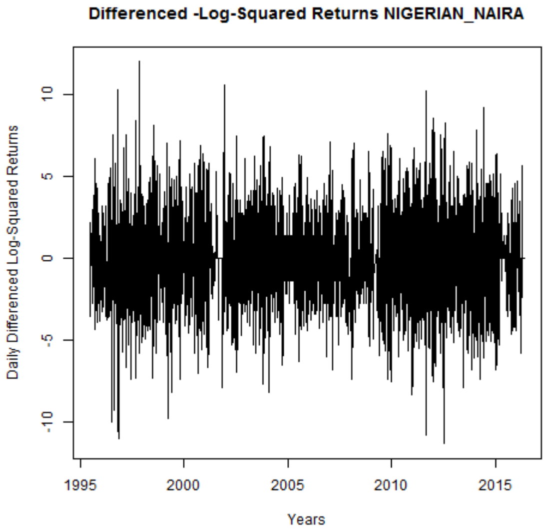

Differenced -Log-Squared Returns MALAYSIAN_RINGGIT
Daily Differenced Log-Squared Returns
Years
-10
-5
0
5
10
1995
2000
2005
2010
2015

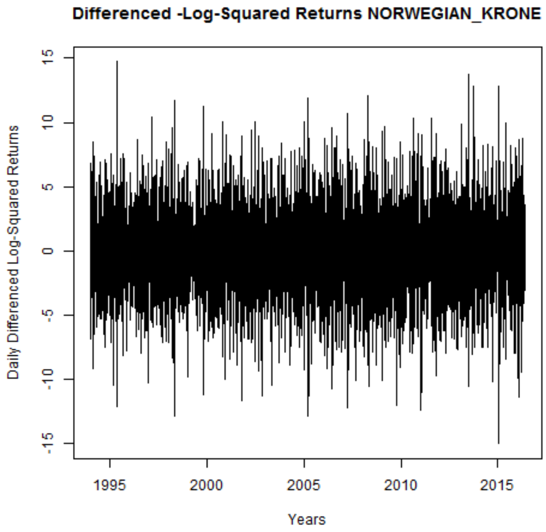

Differenced -Log-Squared Returns MEXICAN_PESO
Daily Differenced Log-Squared Returns
Years
-10
-5
0
5
10
1995
2000
2005
2010
2015

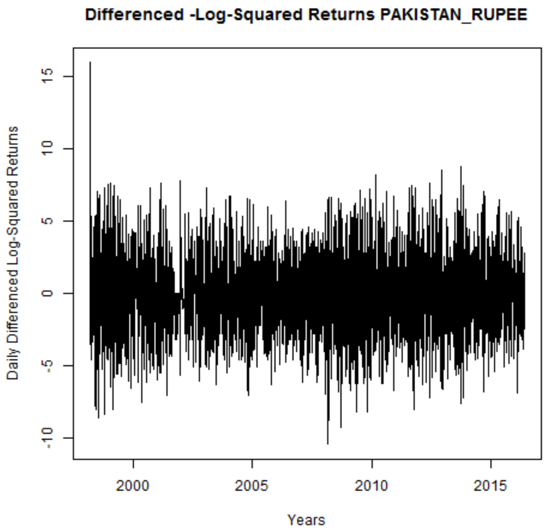

Differenced -Log-Squared Returns NEW_GUINEA_KINA
Daily Differenced Log-Squared Returns
Years
-10
-5
0
5
10
1995
2000
2005
2010
2015

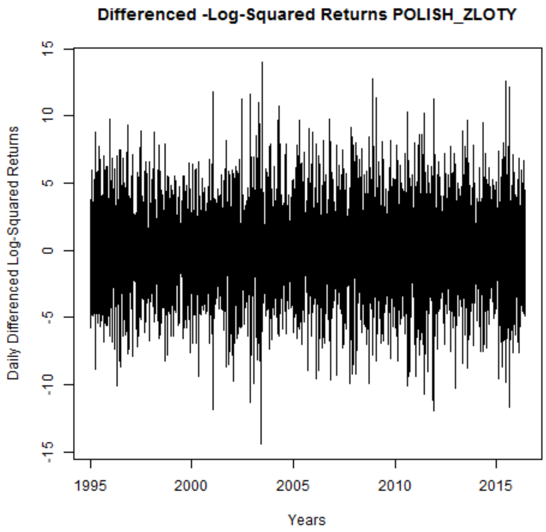

Differenced -Log-Squared Returns NEW_TURKISH_LIRA
Daily Differenced Log-Squared Returns
Years
-10
-5
0
5
10
1995
2000
2005
2010
2015

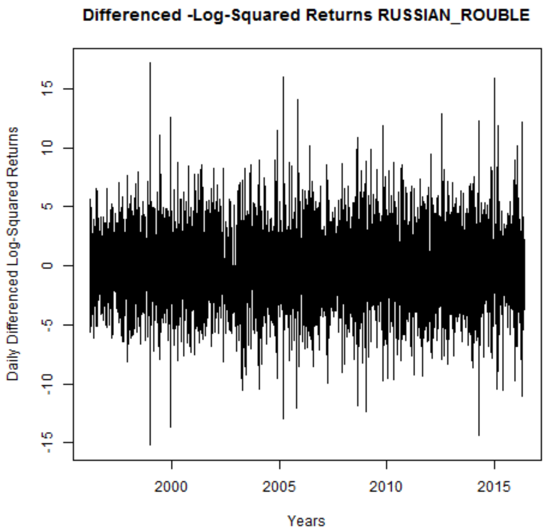

Differenced -Log-Squared Returns NEW_ZEALAND_DOLLAR
Daily Differenced Log-Squared Returns
Years
-10
-5
0
5
10
1995
2000
2005
2010
2015

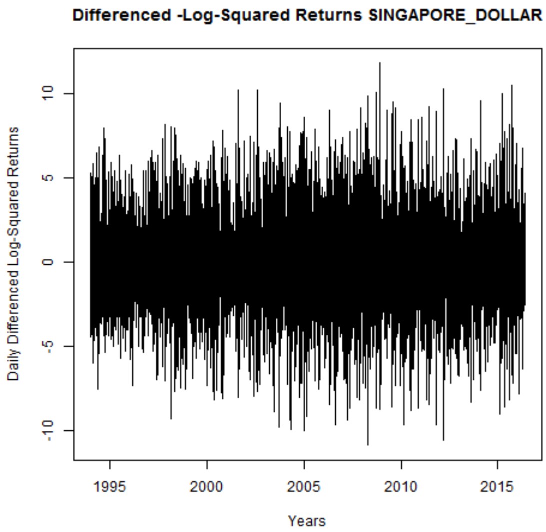

Differenced -Log-Squared Returns NIGERIAN_NAIRA
Daily Differenced Log-Squared Returns
Years

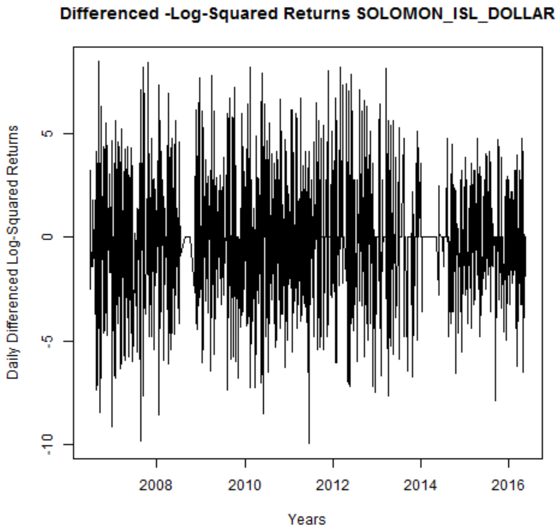

Differenced -Log-Squared Returns NORWEGIAN_KRONE
Daily Differenced Log-Squared Returns
Years

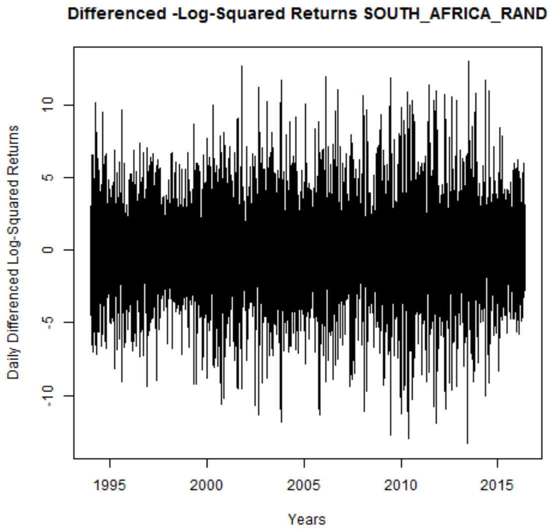

Differenced -Log-Squared Returns PAKISTAN_RUPEE
Daily Differenced Log-Squared Returns
Years

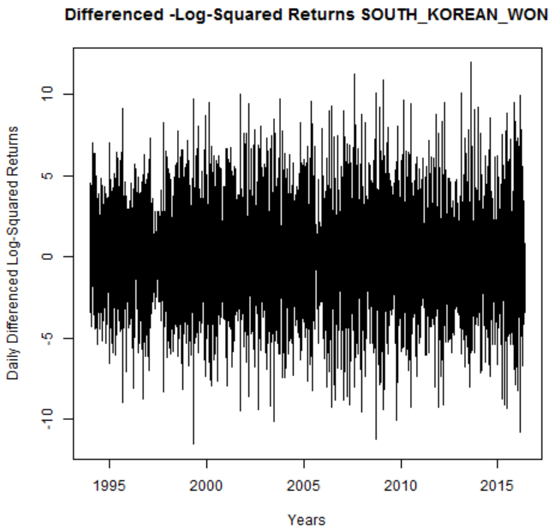

Differenced -Log-Squared Returns POLISH_ZLOTY
Daily Differenced Log-Squared Returns
Years

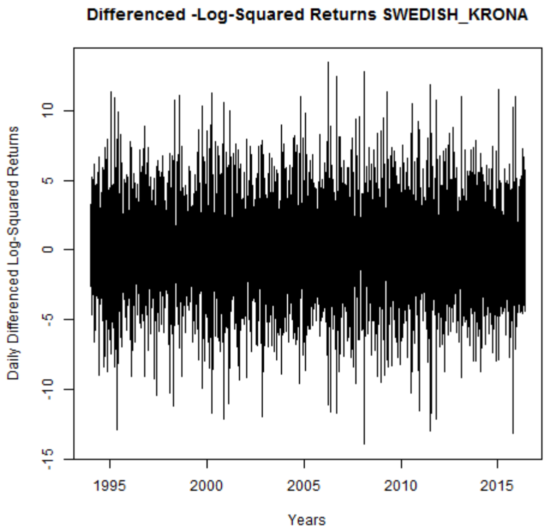

Differenced -Log-Squared Returns RUSSIAN_ROUBLE
Daily Differenced Log-Squared Returns
Years

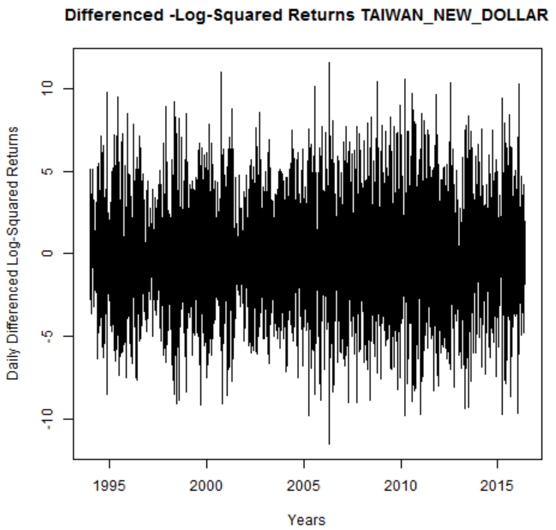

Differenced -Log-Squared Returns SINGAPORE_DOLLAR
Daily Differenced Log-Squared Returns
Years

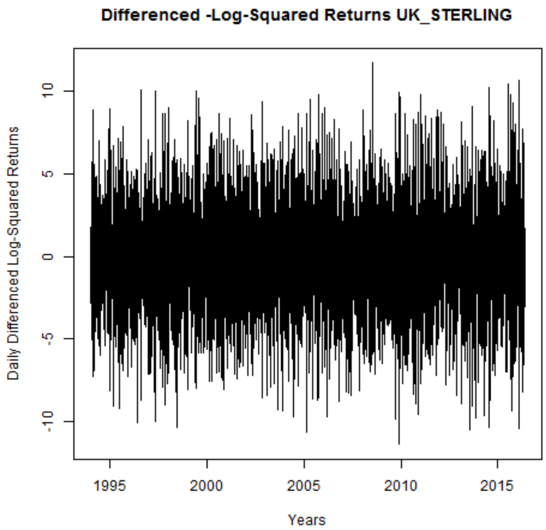

Differenced -Log-Squared Returns SOLOMON_ISL_DOLLAR
Daily Differenced Log-Squared Returns
5
0
-5
-10
2008
2010
2012
2014
2016
Years


Differenced -Log-Squared Returns SOUTH_AFRICA_RAND
Daily Differenced Log-Squared Returns
10
5
0
-5
-10
1995
2000
2005
2010
2015
Years


Differenced -Log-Squared Returns SOUTH_KOREAN_WON
Daily Differenced Log-Squared Returns
10
5
0
-5
-10
1995
2000
2005
2010
2015
Years

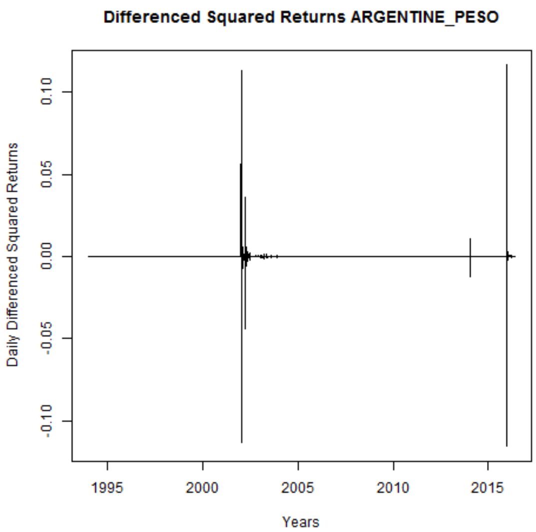

Differenced -Log-Squared Returns SWEDISH_KRONA
Daily Differenced Log-Squared Returns
10
5
0
-5
-10
-15
1995
2000
2005
2010
2015
Years

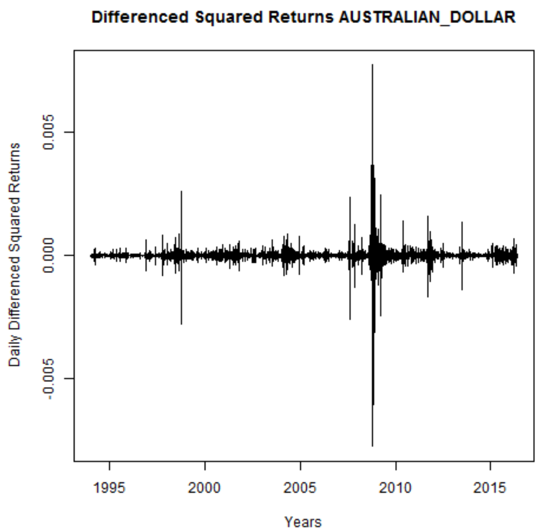

Differenced -Log-Squared Returns TAIWAN_NEW_DOLLAR
Daily Differenced Log-Squared Returns
10
5
0
-5
-10
1995
2000
2005
2010
2015
Years

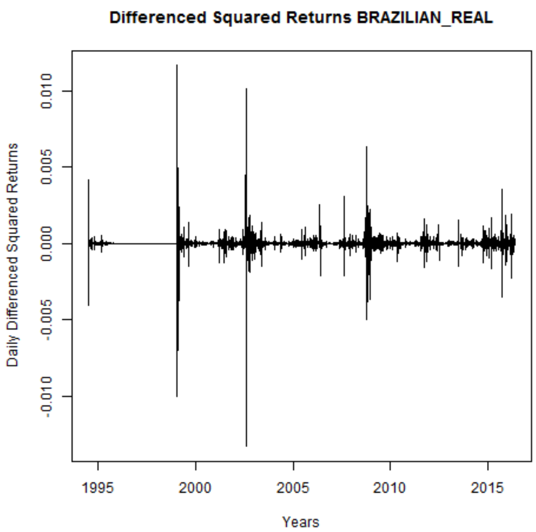

Differenced -Log-Squared Returns UK_STERLING
Daily Differenced Log-Squared Returns
10
5
0
-5
-10
1995
2000
2005
2010
2015
Years

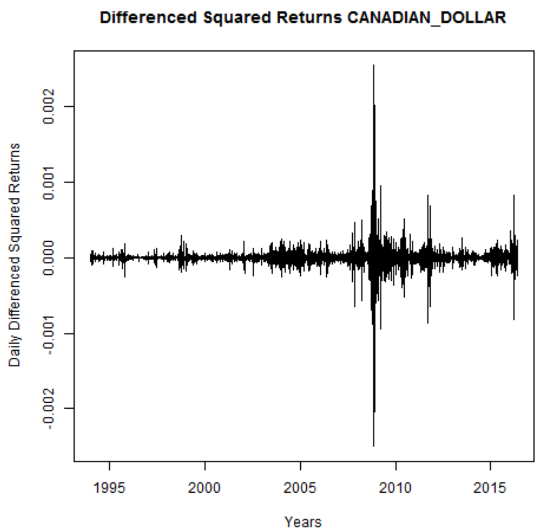
Appendix 15

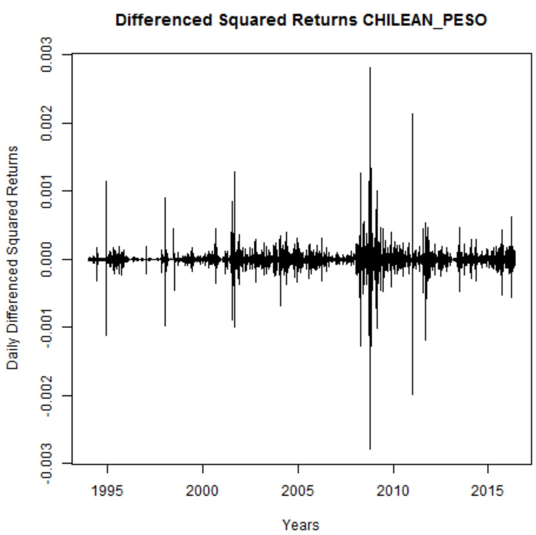
Differenced Squared Returns – Forex returns

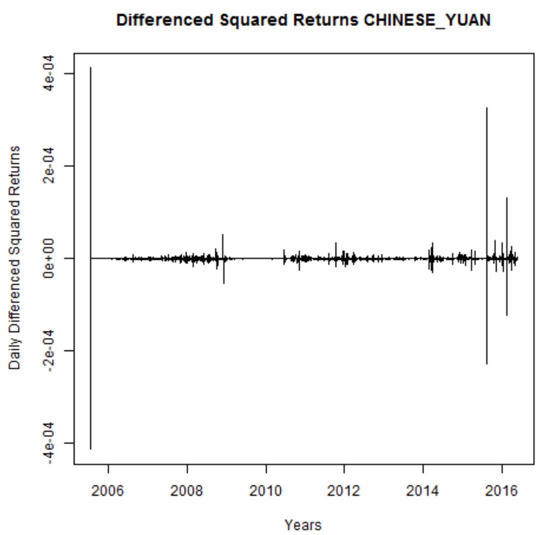


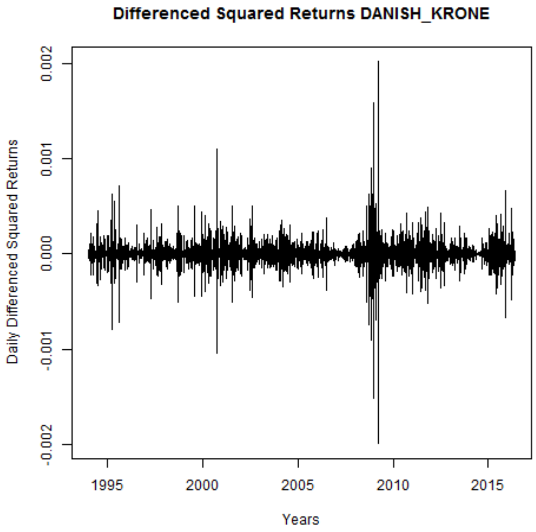


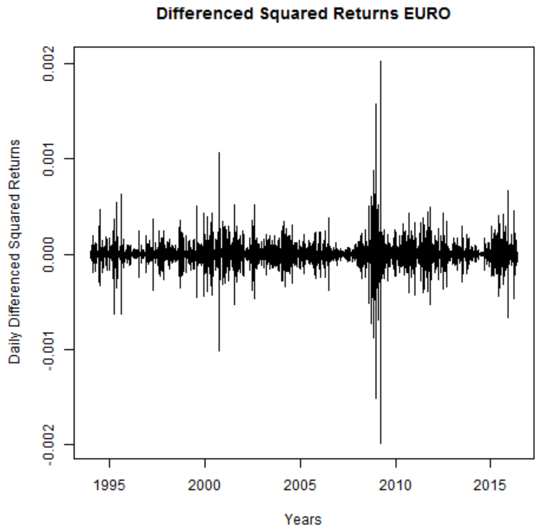


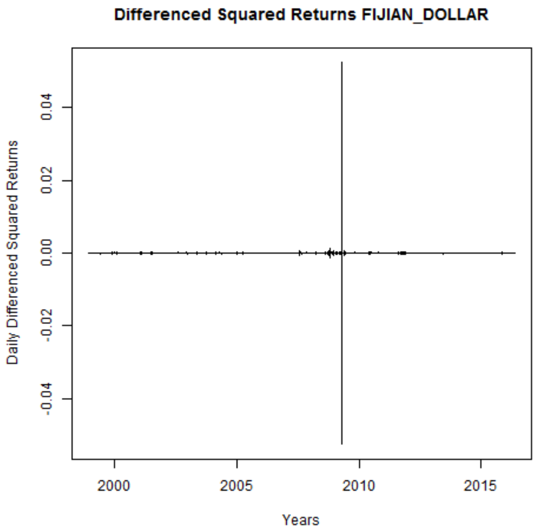


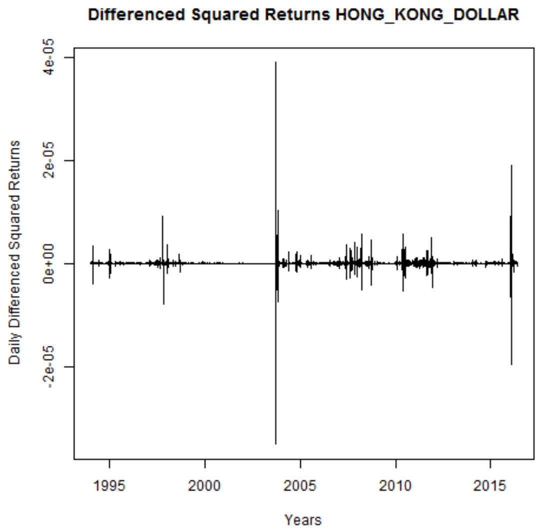


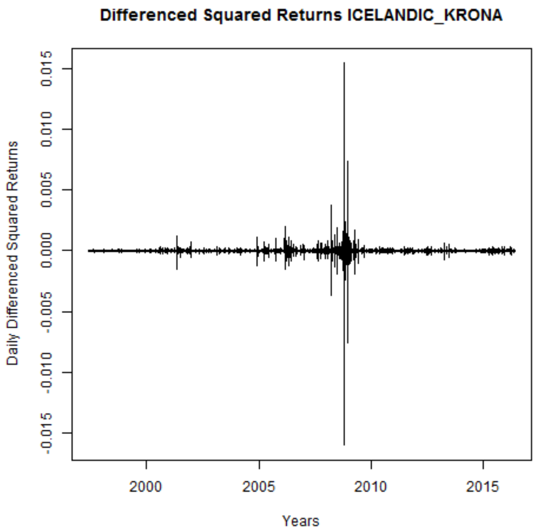


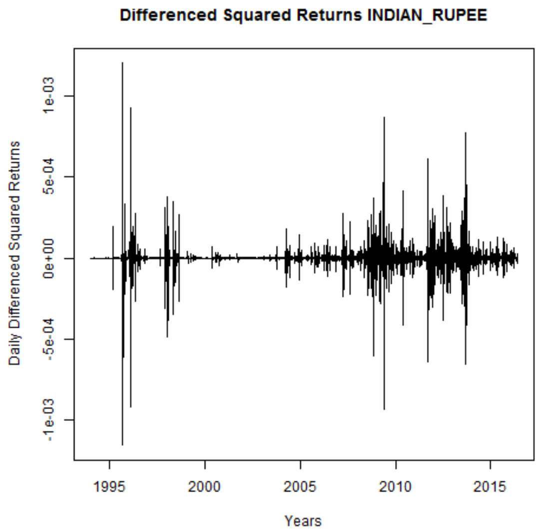

Differenced Squared Returns DANISH_KRONE
Daily Differenced Squared Returns
Years

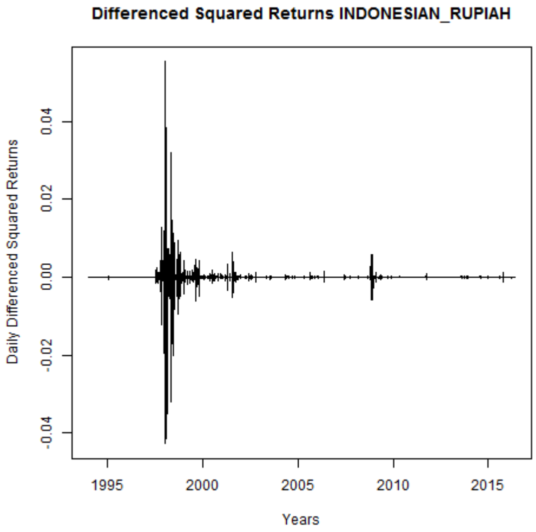

Differenced Squared Returns EURO
Daily Differenced Squared Returns
Years

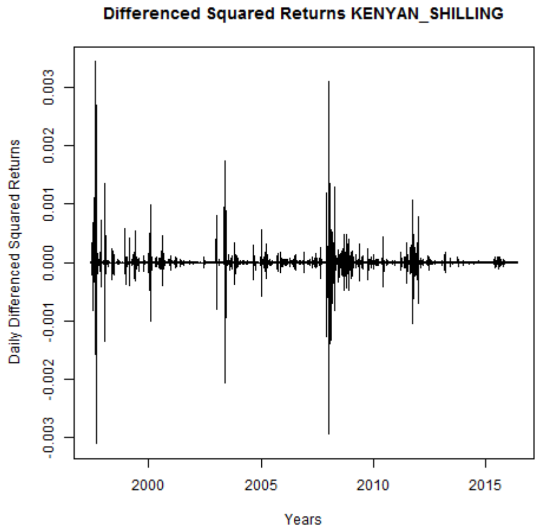

Differenced Squared Returns FIJIAN_DOLLAR
Daily Differenced Squared Returns
Years

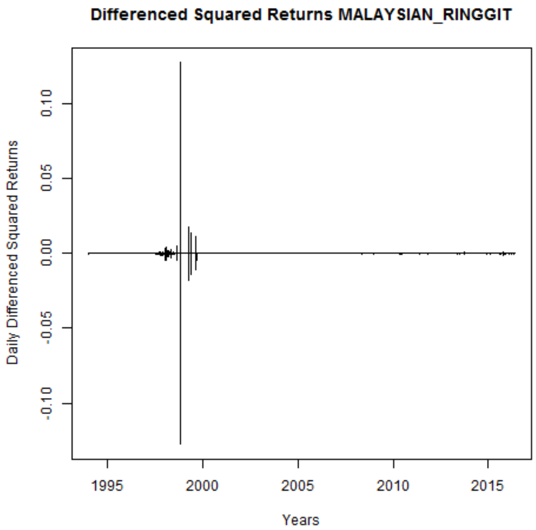

Differenced Squared Returns HONG_KONG_DOLLAR
Daily Differenced Squared Returns
Years

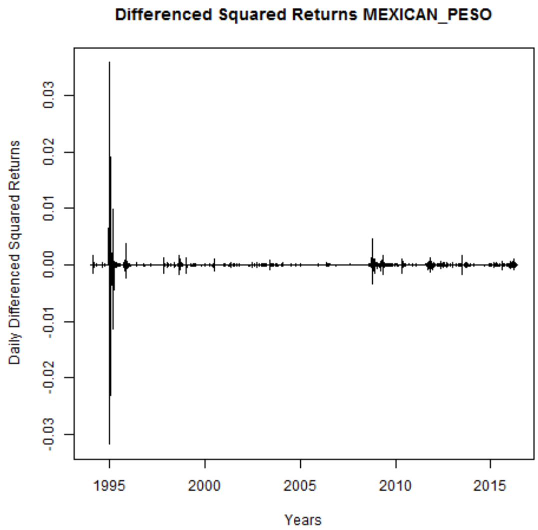

Differenced Squared Returns ICELANDIC_KRONA
Daily Differenced Squared Returns
Years

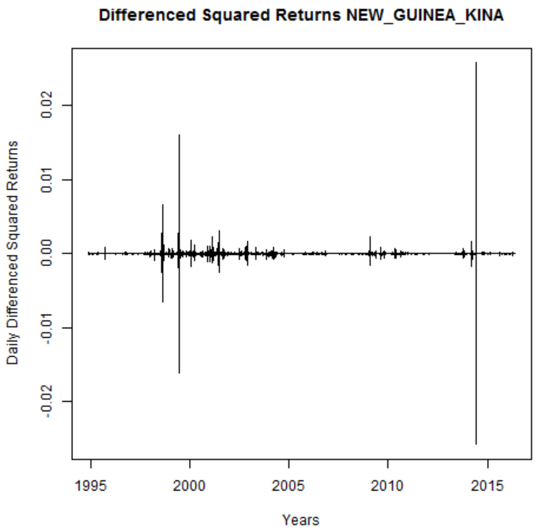

Differenced Squared Returns INDIAN_RUPEE
Daily Differenced Squared Returns
Years

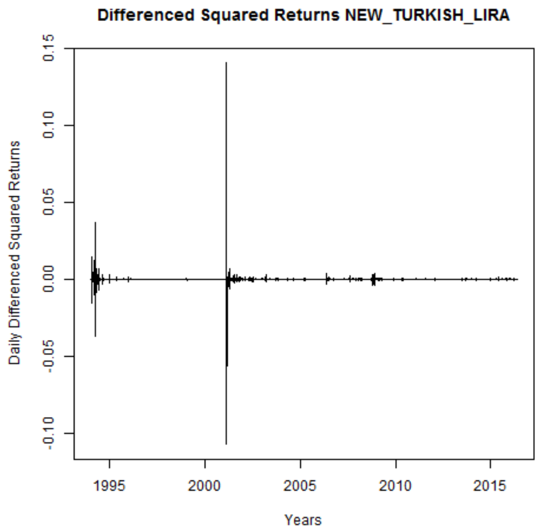

Differenced Squared Returns INDONESIAN_RUPIAH
Daily Differenced Squared Returns
0.04
0.02
0.00
-0.02
-0.04
1995
2000
2005
2010
2015
Years

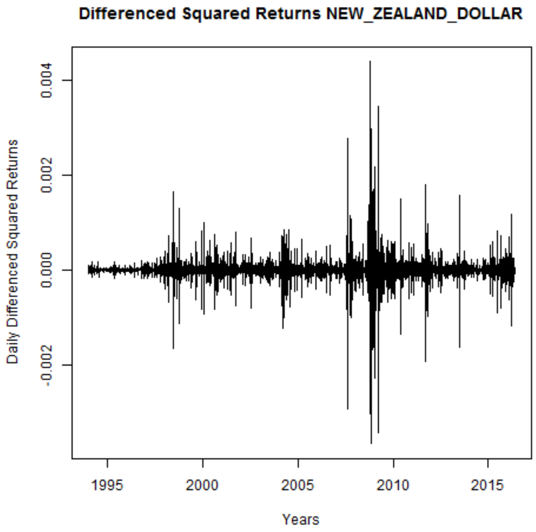

Differenced Squared Returns KENYAN_SHILLING
Daily Differenced Squared Returns
0.003
0.002
0.001
0.000
-0.001
-0.002
-0.003
2000
2005
2010
2015
Years

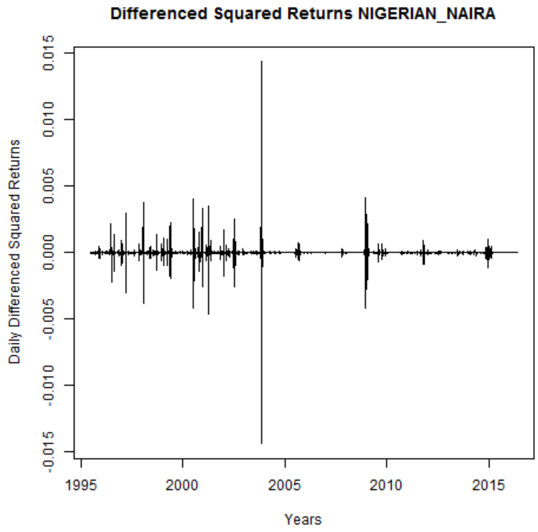

Differenced Squared Returns MALAYSIAN_RINGGIT
Daily Differenced Squared Returns
0.10
0.05
0.00
-0.05
-0.10
1995
2000
2005
2010
2015
Years

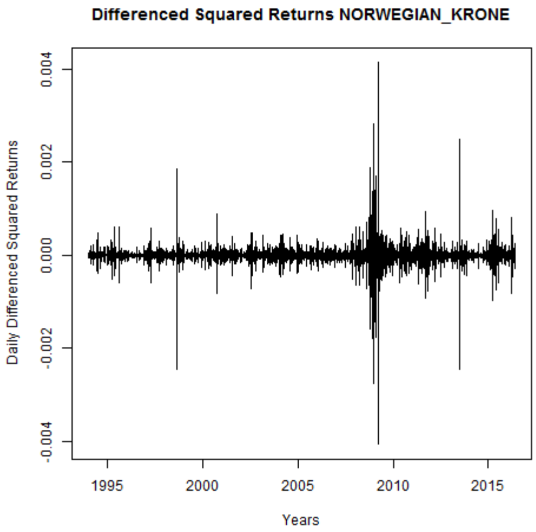

Differenced Squared Returns MEXICAN_PESO
Daily Differenced Squared Returns
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
1995
2000
2005
2010
2015
Years

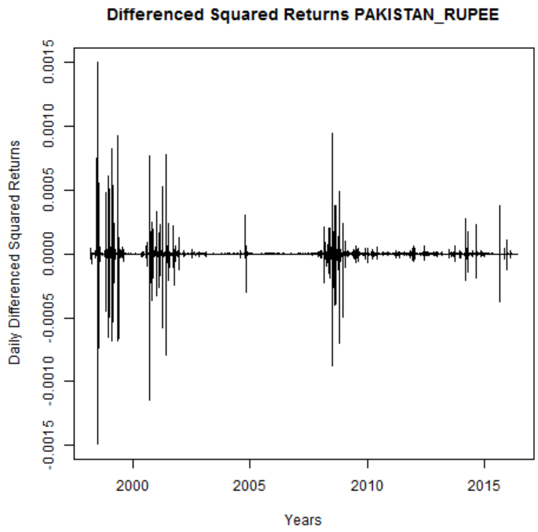

Differenced Squared Returns NEW_GUINEA_KINA
Daily Differenced Squared Returns
0.02
0.01
0.00
-0.01
-0.02
1995
2000
2005
2010
2015
Years

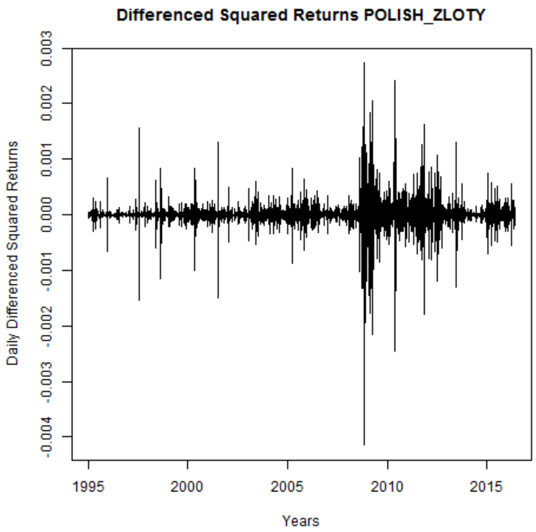

Differenced Squared Returns NEW_TURKISH_LIRA
Daily Differenced Squared Returns
0.15
0.10
0.05
0.00
-0.05
-0.10
1995
2000
2005
2010
2015
Years

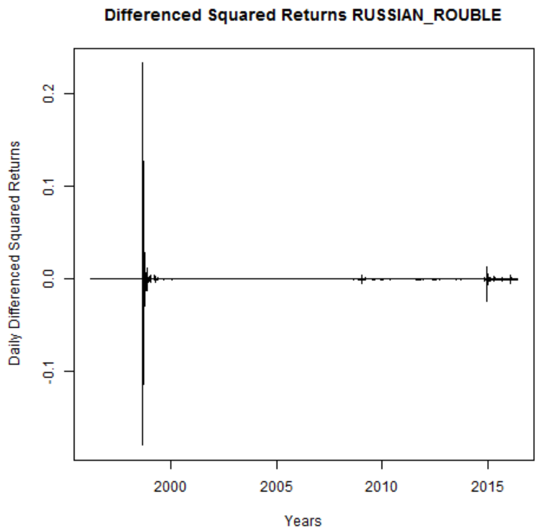

Differenced Squared Returns NEW_ZEALAND_DOLLAR
Daily Differenced Squared Returns
0.004
0.002
0.000
-0.002
1995
2000
2005
2010
2015
Years

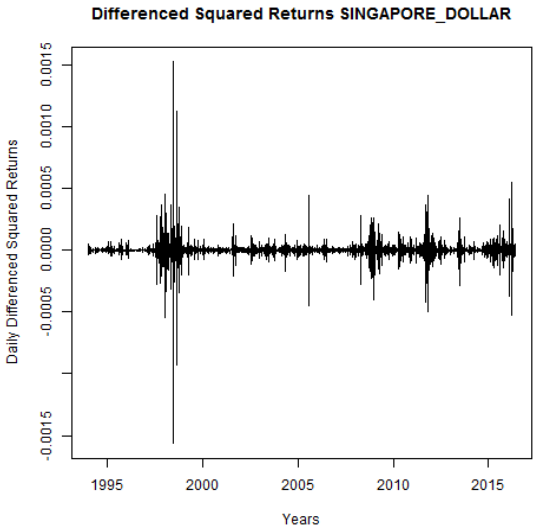

Differenced Squared Returns NIGERIAN_NAIRA
Daily Differenced Squared Returns
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
1995
2000
2005
2010
2015
Years

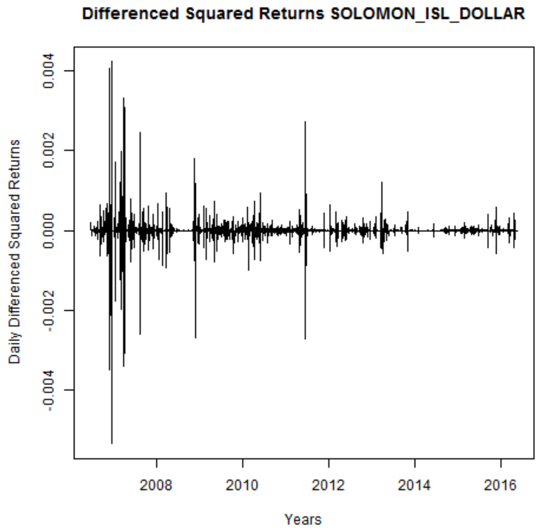

Differenced Squared Returns NORWEGIAN_KRONE
Daily Differenced Squared Returns
0.004
0.002
0.000
-0.002
-0.004
1995
2000
2005
2010
2015
Years

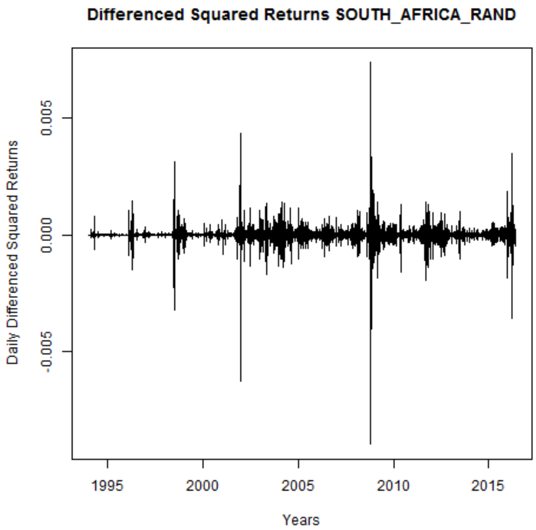

Differenced Squared Returns PAKISTAN_RUPEE
Daily Differenced Squared Returns
0.0015
0.0010
0.0005
0.0000
-0.0005
-0.0010
-0.0015
2000
2005
2010
2015
Years

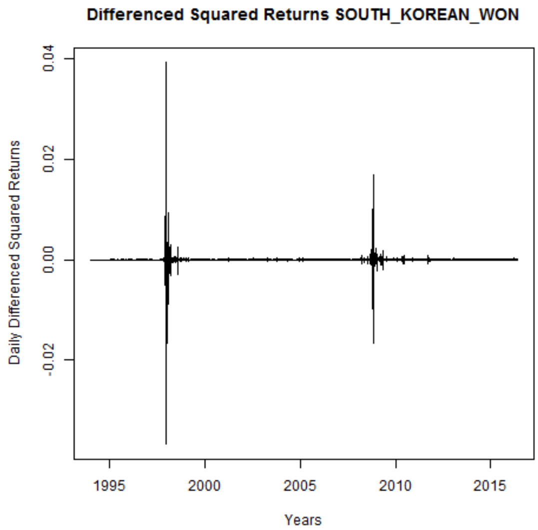

Differenced Squared Returns POLISH_ZLOTY
Daily Differenced Squared Returns
0.003
0.002
0.001
0.000
-0.001
-0.002
-0.003
-0.004
1995
2000
2005
2010
2015
Years

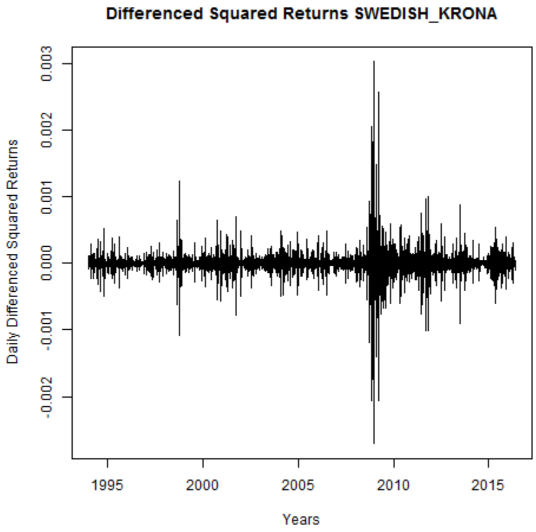

Differenced Squared Returns RUSSIAN_ROUBLE
Daily Differenced Squared Returns
0.2
0.1
0.0
-0.1
2000
2005
2010
2015
Years

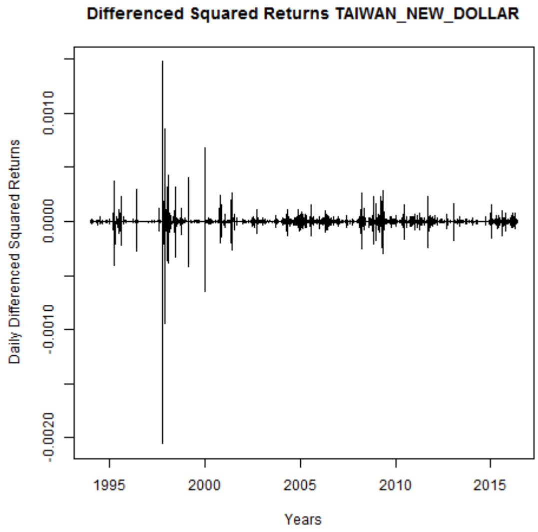

Differenced Squared Returns SINGAPORE_DOLLAR
Daily Differenced Squared Returns
Years

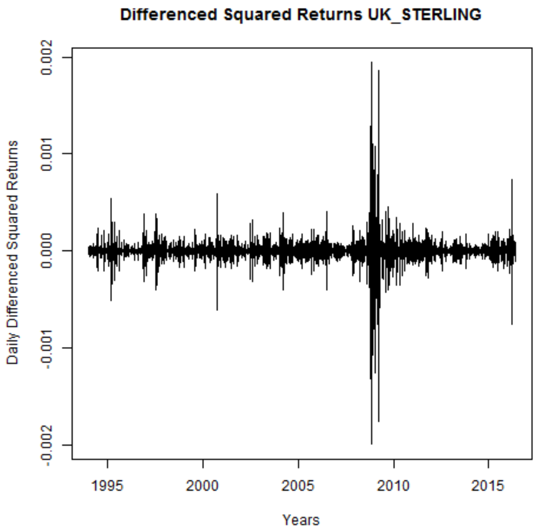

Differenced Squared Returns SOLOMON_ISL_DOLLAR
Daily Differenced Squared Returns
Years


Differenced Squared Returns SOUTH_AFRICA_RAND
Daily Differenced Squared Returns
Years


Differenced Squared Returns SOUTH_KOREAN_WON
Daily Differenced Squared Returns
Years

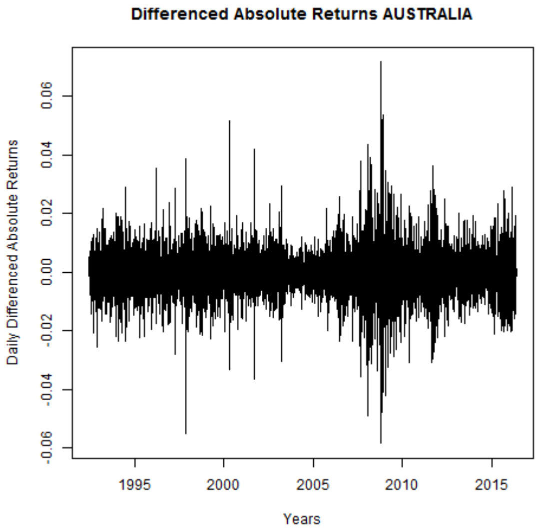

Differenced Squared Returns SWEDISH_KRONA
Daily Differenced Squared Returns
0.003
0.002
0.001
0.000
-0.001
-0.002
1995
2000
2005
2010
2015
Years

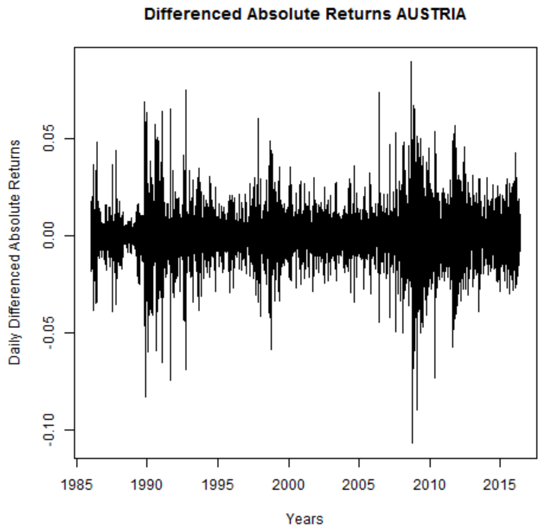

Differenced Squared Returns TAIWAN_NEW_DOLLAR
Daily Differenced Squared Returns
0.0010
0.0000
-0.0010
-0.0020
1995
2000
2005
2010
2015
Years

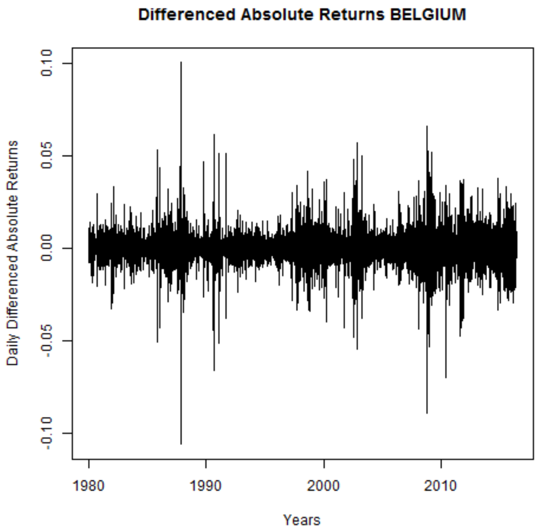

Differenced Squared Returns UK_STERLING
Daily Differenced Squared Returns
0.002
0.001
0.000
-0.001
-0.002
1995
2000
2005
2010
2015
Years

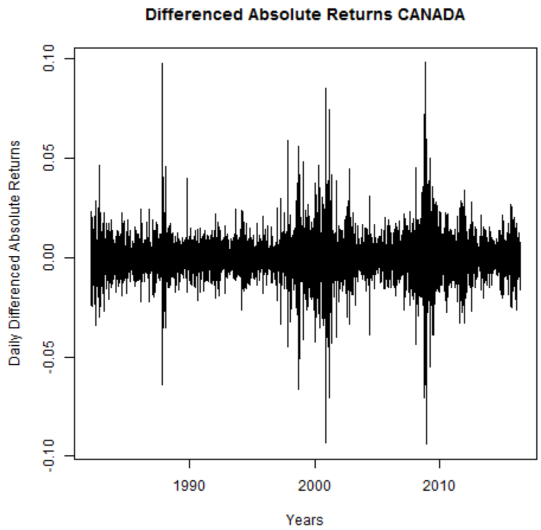
Appendix 16

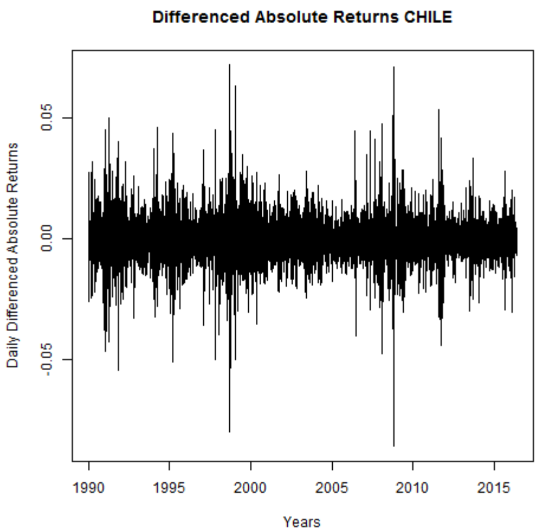
Evidence of Stationarity - Differenced Absolute Returns – Equity Market

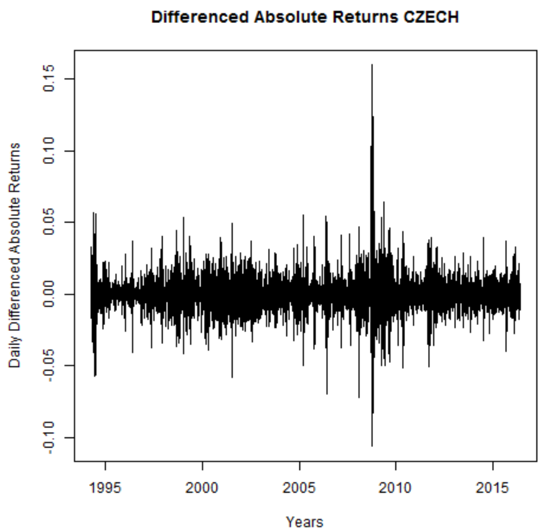


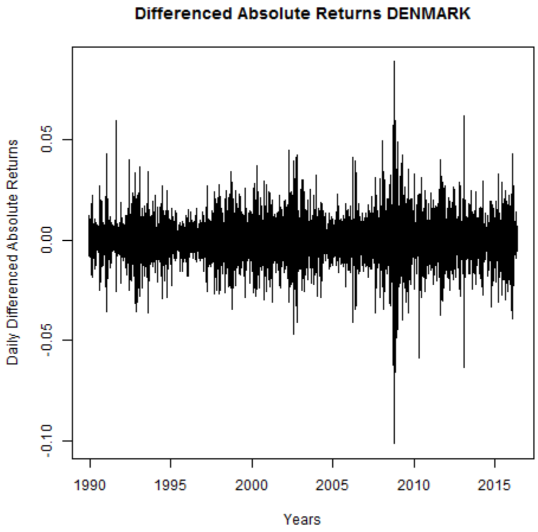


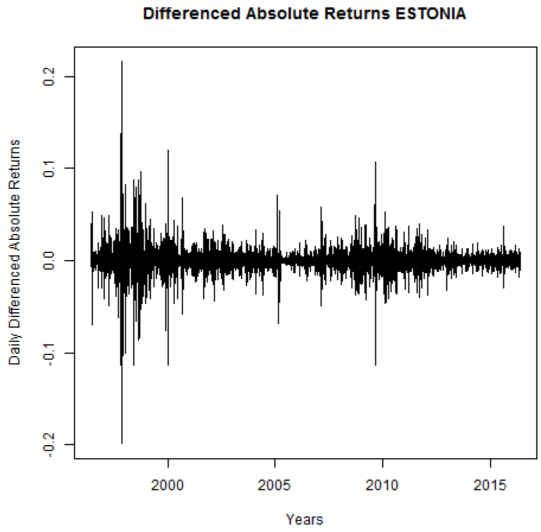


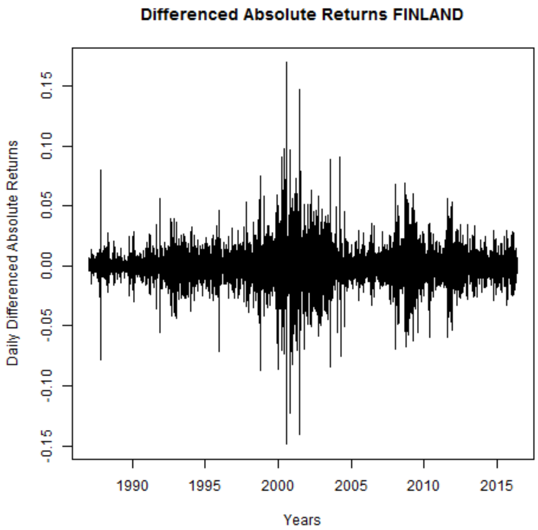


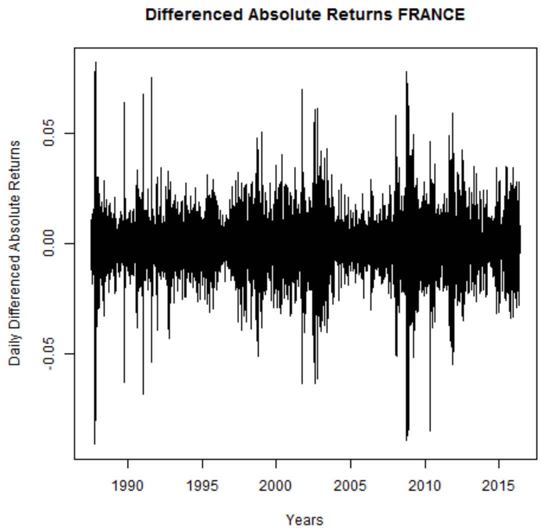


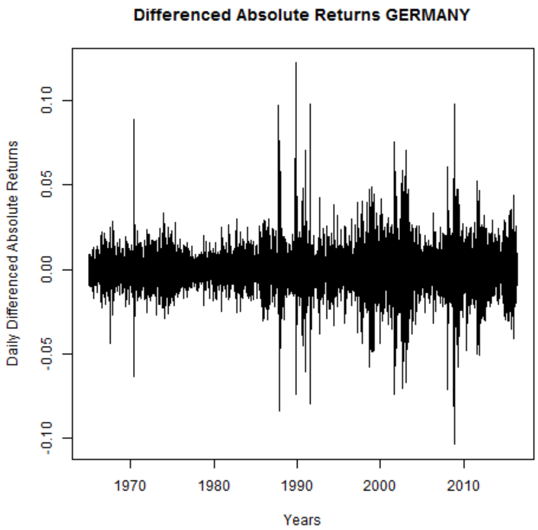


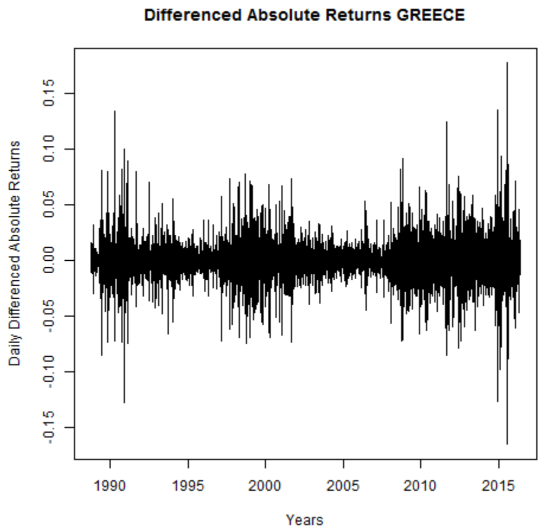

Differenced Absolute Returns DENMARK
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
1990
1995
2000
2005
2010
2015
Years

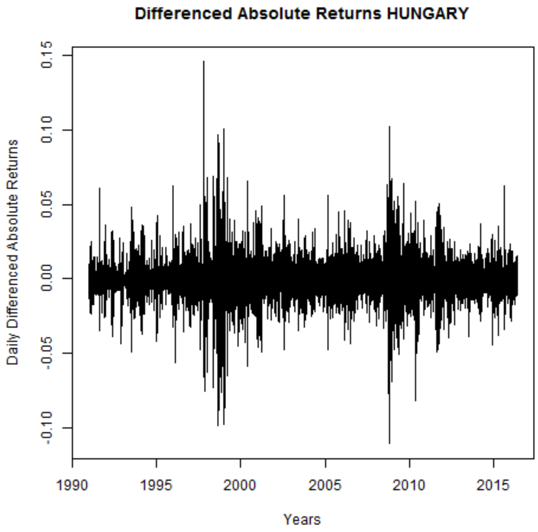

Differenced Absolute Returns ESTONIA
Daily Differenced Absolute Returns
-0.2
-0.1
0.0
0.1
0.2
2000
2005
2010
2015
Years

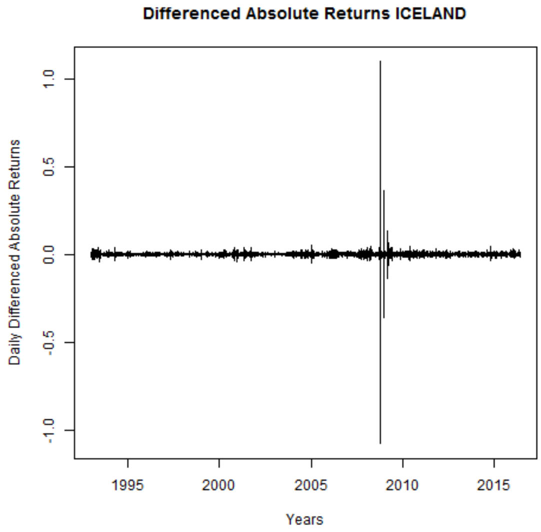

Differenced Absolute Returns FINLAND
Daily Differenced Absolute Returns
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
1990
1995
2000
2005
2010
2015
Years

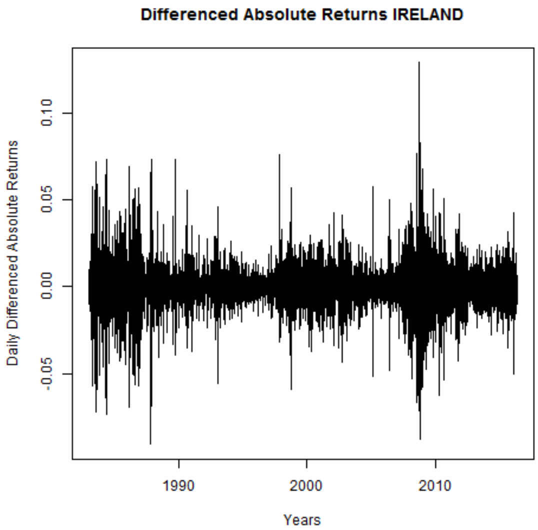

Differenced Absolute Returns FRANCE
Daily Differenced Absolute Returns
-0.05
0.00
0.05
1990
1995
2000
2005
2010
2015
Years

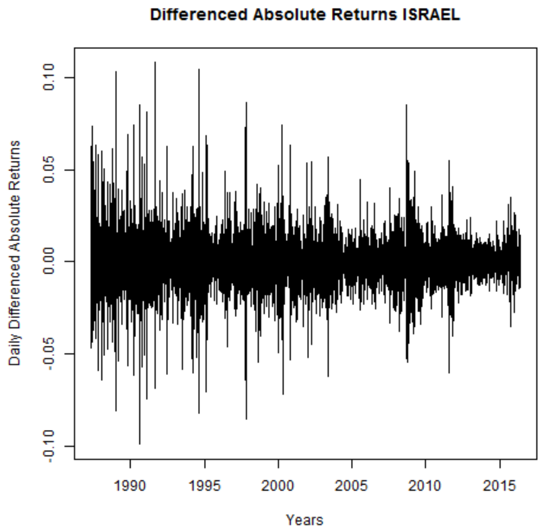

Differenced Absolute Returns GERMANY
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
1970
1980
1990
2000
2010
Years

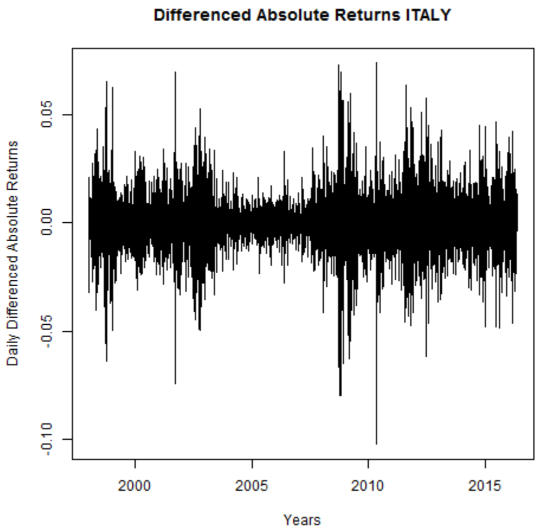

Differenced Absolute Returns GREECE
Daily Differenced Absolute Returns
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
1990
1995
2000
2005
2010
2015
Years

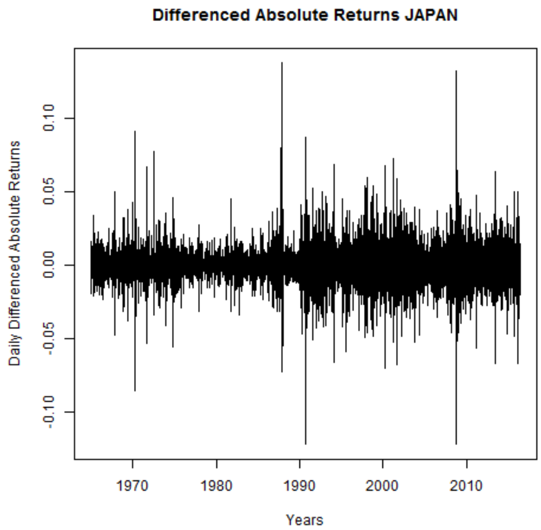

Differenced Absolute Returns HUNGARY
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
0.15
1990
1995
2000
2005
2010
2015
Years

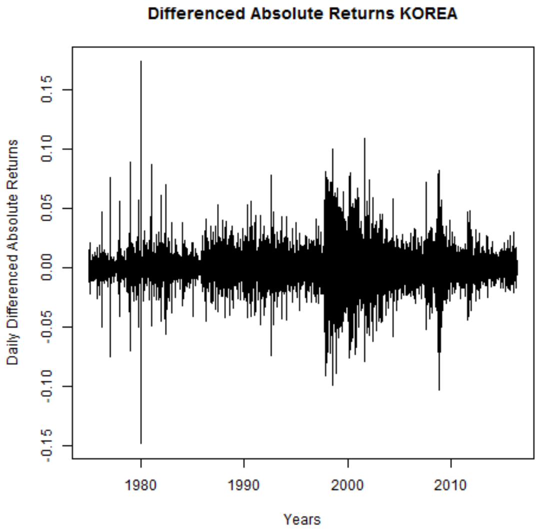

Differenced Absolute Returns ICELAND
Daily Differenced Absolute Returns
-1.0
-0.5
0.0
0.5
1.0
1995
2000
2005
2010
2015
Years

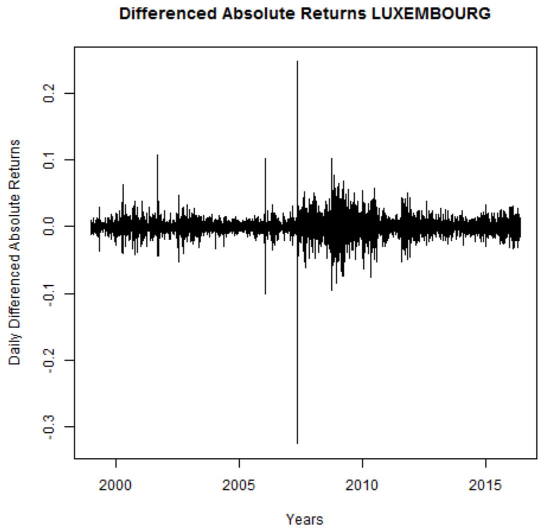

Differenced Absolute Returns IRELAND
Daily Differenced Absolute Returns
-0.05
0.00
0.05
0.10
1990
2000
2010
Years

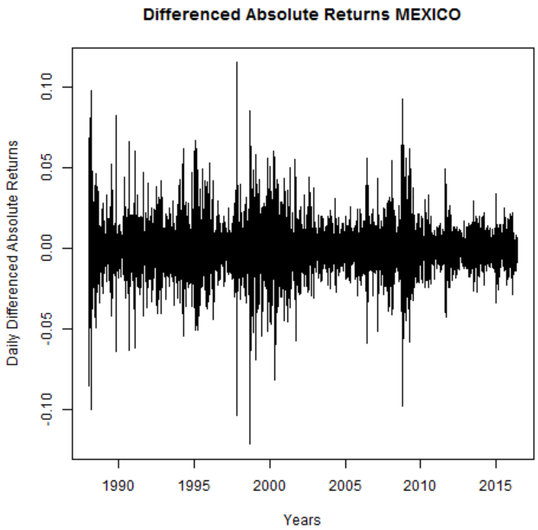

Differenced Absolute Returns ISRAEL
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
1990
1995
2000
2005
2010
2015
Years

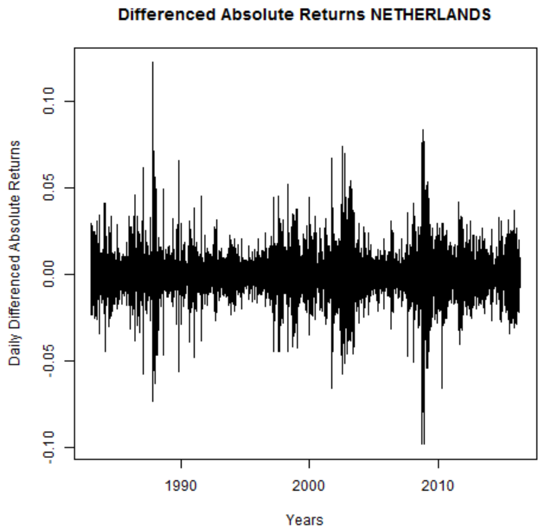

Differenced Absolute Returns ITALY
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
2000
2005
2010
2015
Years

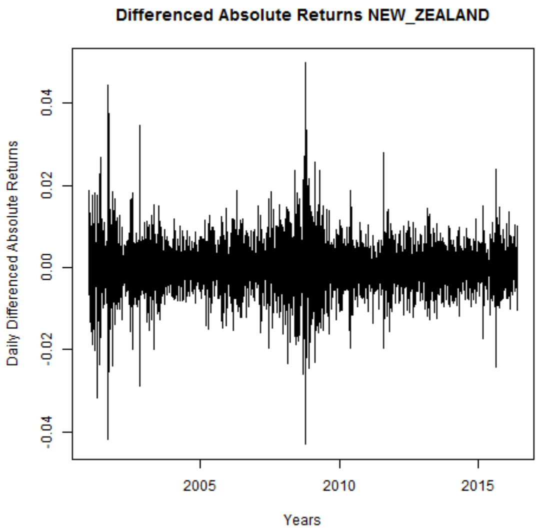

Differenced Absolute Returns JAPAN
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
1970
1980
1990
2000
2010
Years

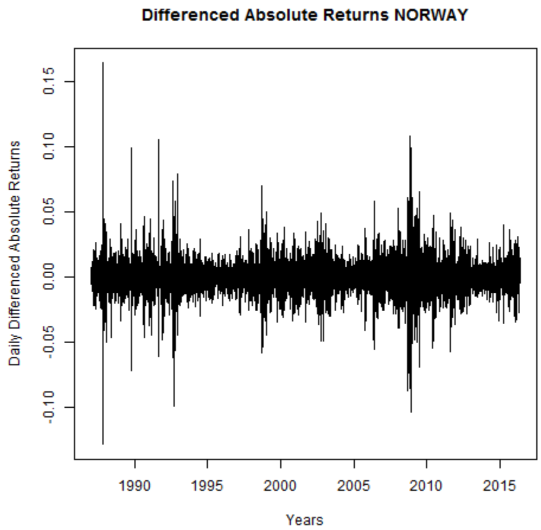

Differenced Absolute Returns KOREA
Daily Differenced Absolute Returns
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
1980
1990
2000
2010
Years

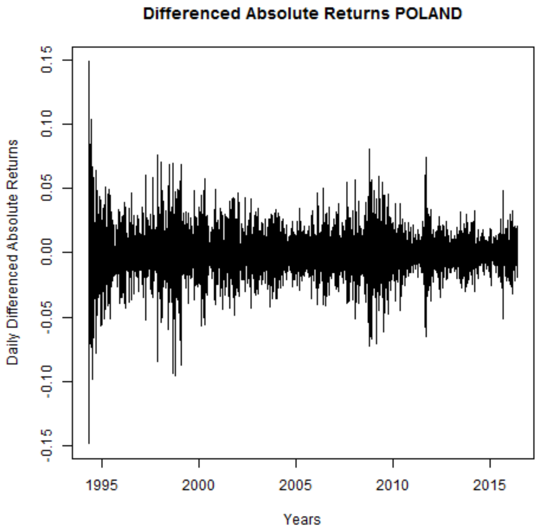

Differenced Absolute Returns LUXEMBOURG
Daily Differenced Absolute Returns
-0.3
-0.2
-0.1
0.0
0.1
0.2
2000
2005
2010
2015
Years

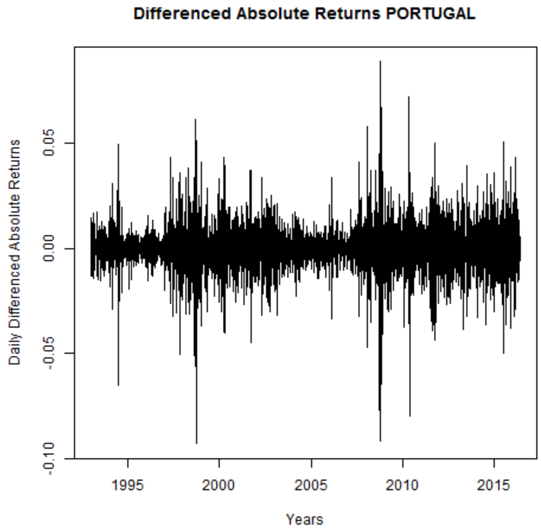

Differenced Absolute Returns MEXICO
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
1990
1995
2000
2005
2010
2015
Years

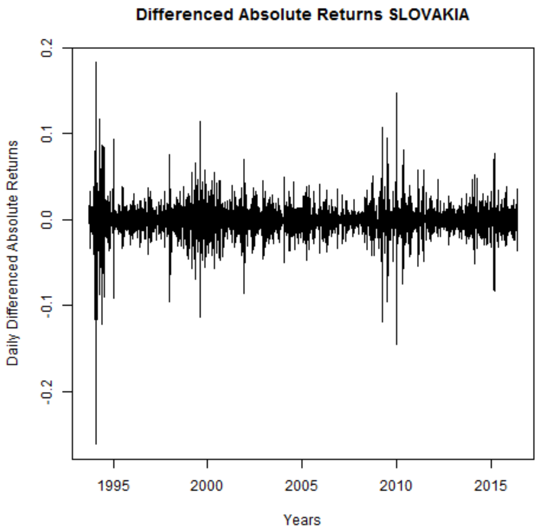

Differenced Absolute Returns NETHERLANDS
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
1990
2000
2010
Years

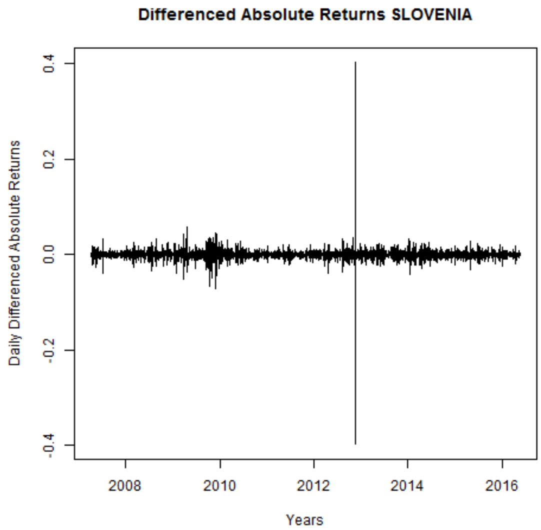

Differenced Absolute Returns NEW_ZEALAND
Daily Differenced Absolute Returns
-0.04
-0.02
0.00
0.02
0.04
2005
2010
2015
Years

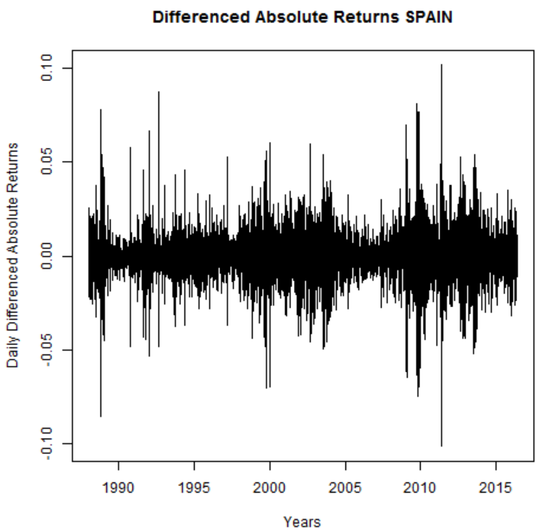

Differenced Absolute Returns NORWAY
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
0.15
1990
1995
2000
2005
2010
2015
Years

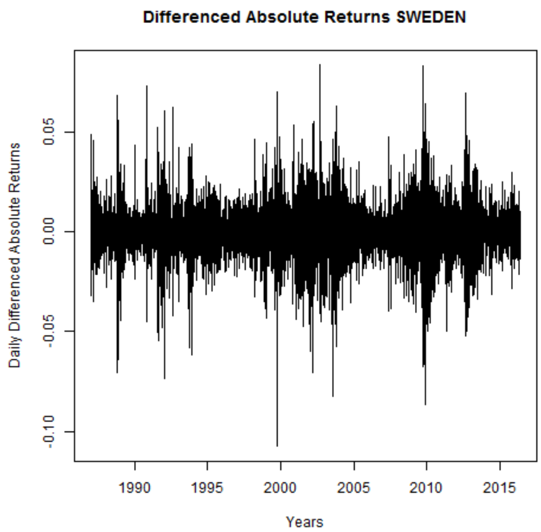

Differenced Absolute Returns POLAND
Daily Differenced Absolute Returns
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
1995
2000
2005
2010
2015
Years

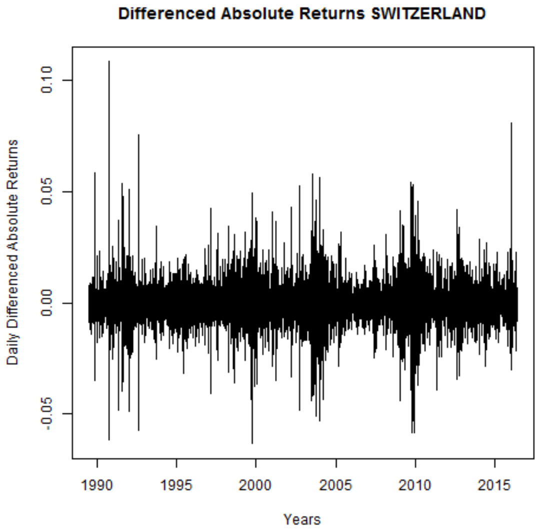

Differenced Absolute Returns PORTUGAL
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
1995
2000
2005
2010
2015
Years

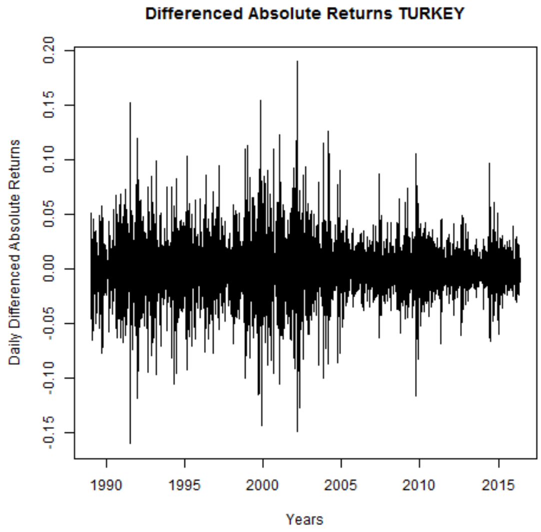

Differenced Absolute Returns SLOVAKIA
Daily Differenced Absolute Returns
-0.2
-0.1
0.0
0.1
0.2
1995
2000
2005
2010
2015
Years

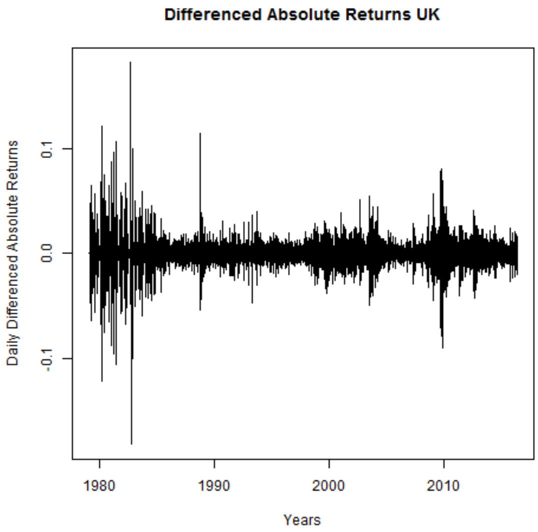

Differenced Absolute Returns SLOVENIA
Daily Differenced Absolute Returns
-0.4
-0.2
0.0
0.2
0.4
2008
2010
2012
2014
2016
Years

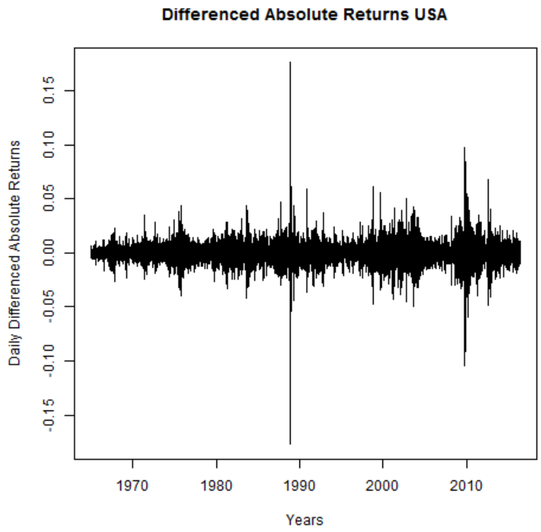

Differenced Absolute Returns SPAIN
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
0.10
1990
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns SWEDEN
Daily Differenced Absolute Returns
-0.10
-0.05
0.00
0.05
1990
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns SWITZERLAND
Daily Differenced Absolute Returns
-0.05
0.00
0.05
0.10
1990
1995
2000
2005
2010
2015
Years


Differenced Absolute Returns TURKEY
Daily Differenced Absolute Returns
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
0.20
1990
1995
2000
2005
2010
2015
Years

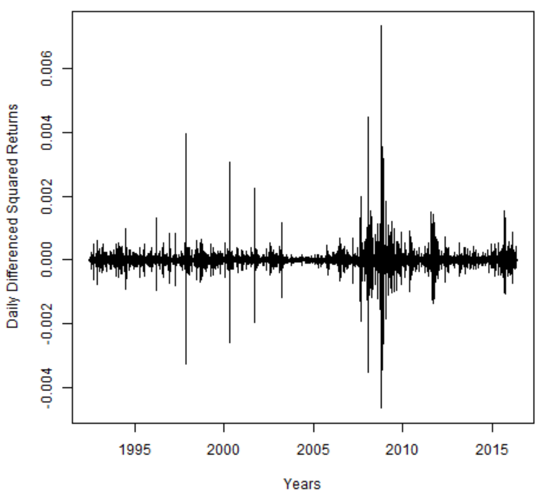

Differenced Absolute Returns UK
Daily Differenced Absolute Returns
-0.1
0.0
0.1
1980
1990
2000
2010
Years


Differenced Absolute Returns USA
Daily Differenced Absolute Returns
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
1970
1980
1990
2000
2010
Years

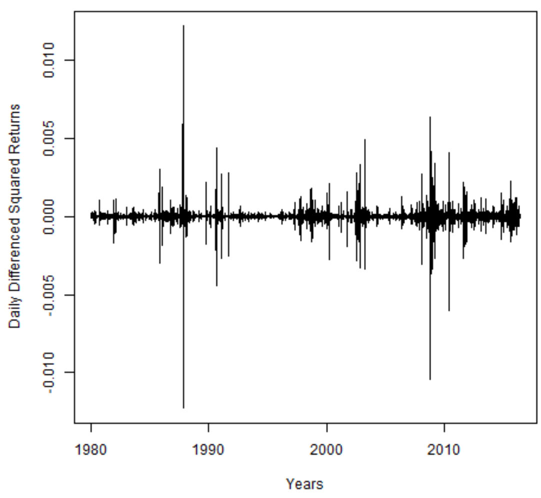
Appendix 17

Evidence of Stationarity - Differenced Squared Returns – Equity Market

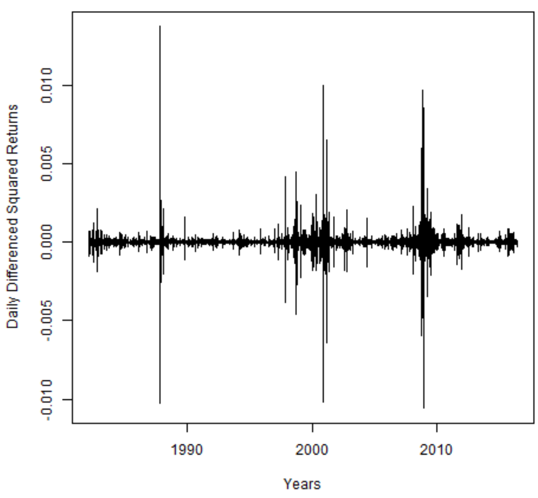
Differenced Squared Returns AUSTRALIA



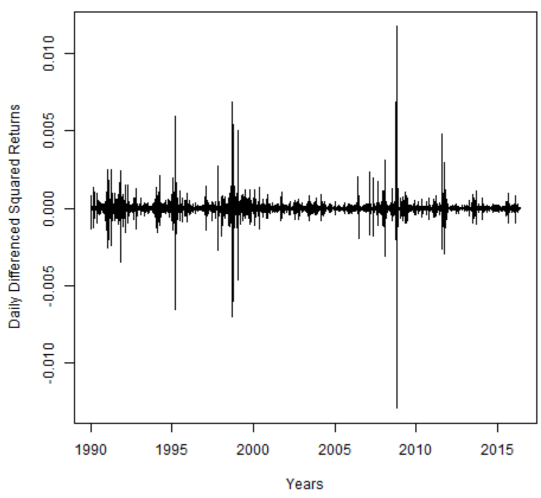
Differenced Squared Returns BELGIUM



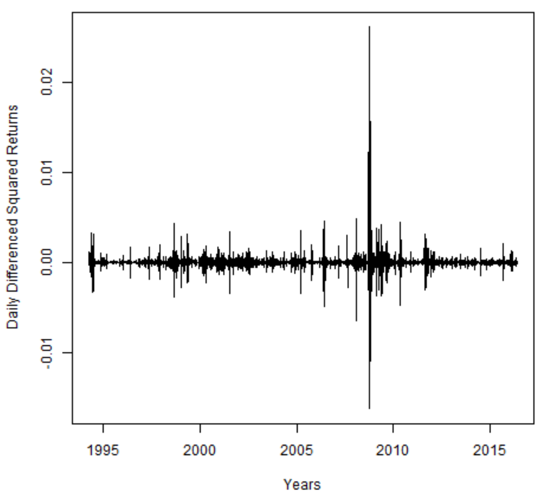
Differenced Squared Returns CANADA



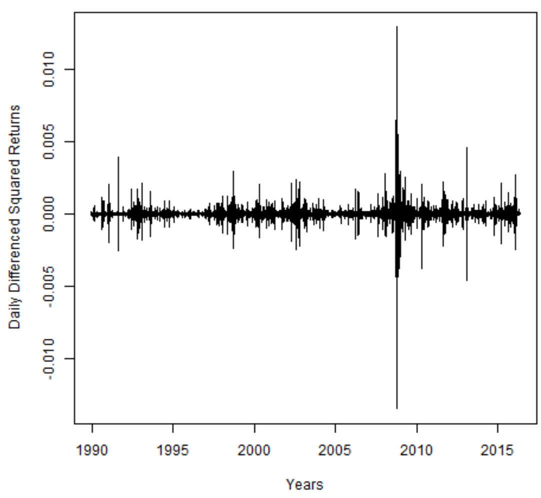
Differenced Squared Returns CHILE

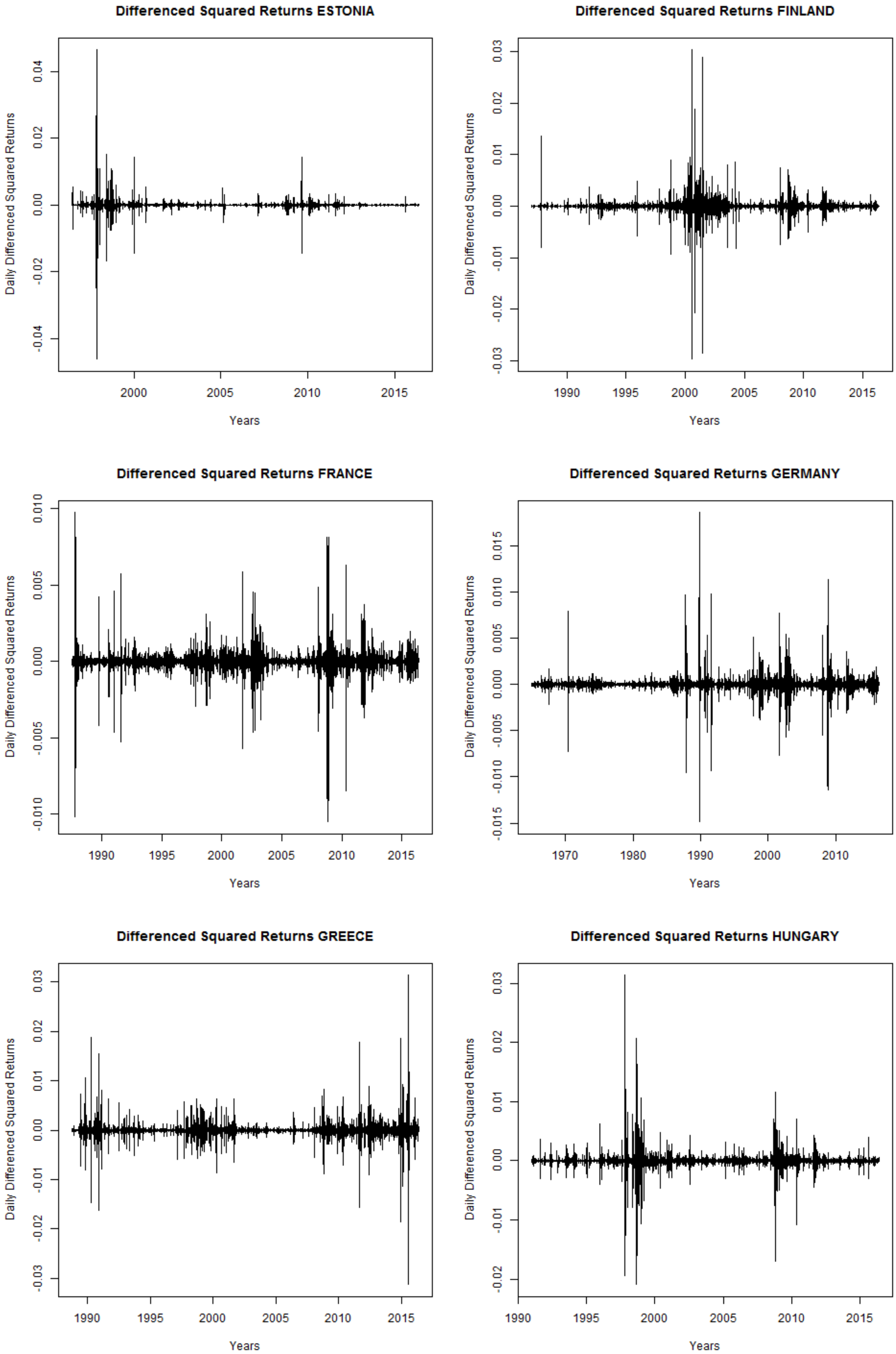


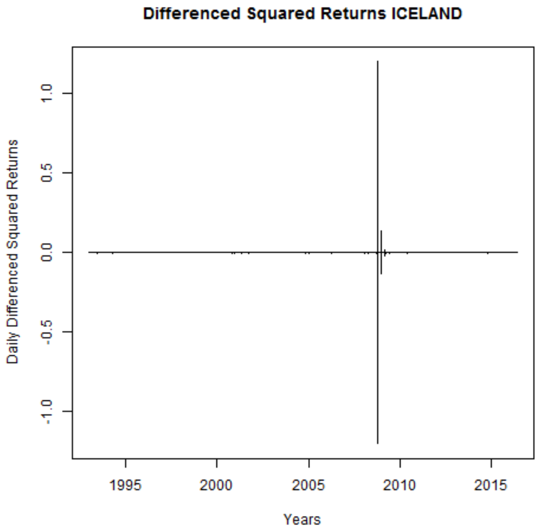
Differenced Squared Returns CZECH

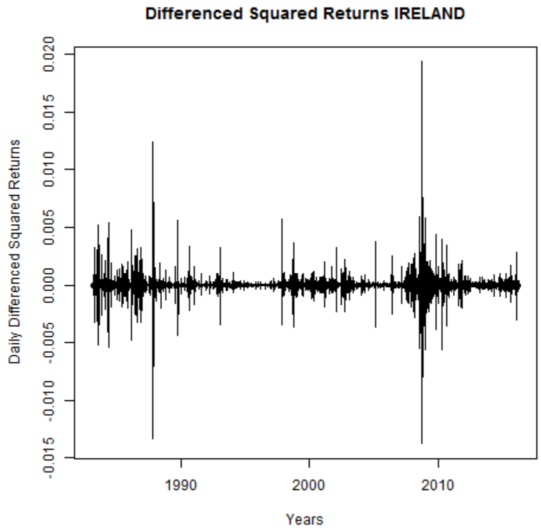


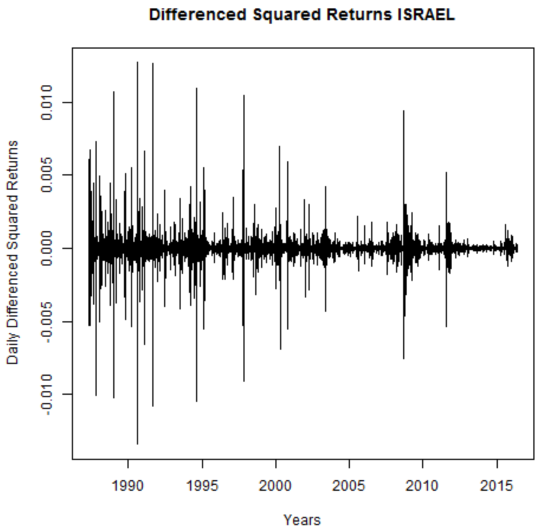
Differenced Squared Returns DENMARK

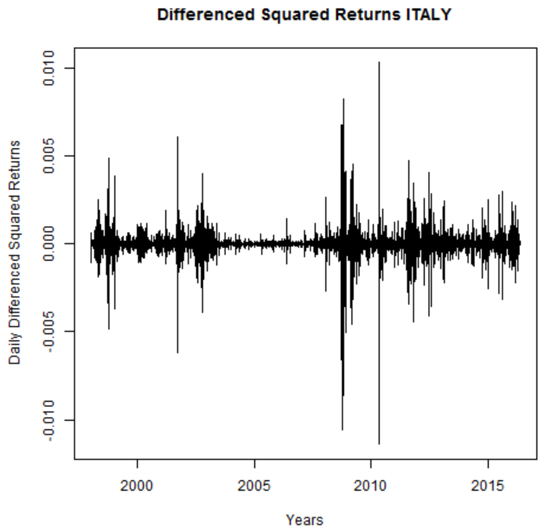


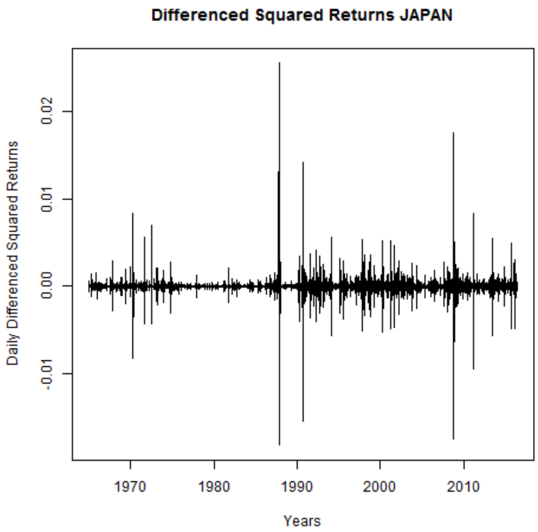

Differenced Squared Returns ESTONIA
Daily Differenced Squared Returns
Years
Differenced Squared Returns FINLAND
Daily Differenced Squared Returns
Years
Differenced Squared Returns FRANCE
Daily Differenced Squared Returns
Years
Differenced Squared Returns GERMANY
Daily Differenced Squared Returns
Years
Differenced Squared Returns GREECE
Daily Differenced Squared Returns
Years
Differenced Squared Returns HUNGARY
Daily Differenced Squared Returns
Years

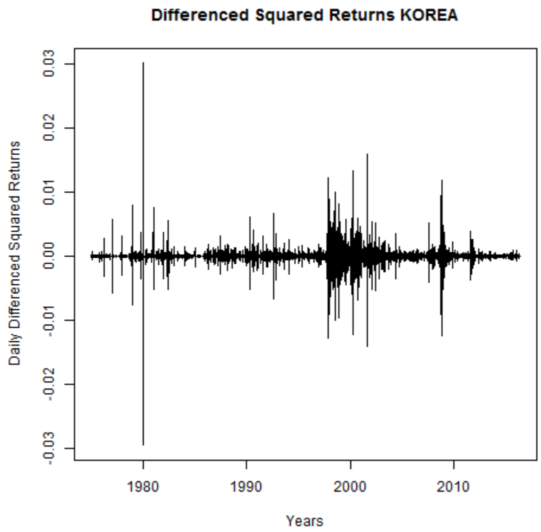

Differenced Squared Returns ICELAND
Daily Differenced Squared Returns
1.0
0.5
0.0
-0.5
-1.0
1995
2000
2005
2010
2015
Years

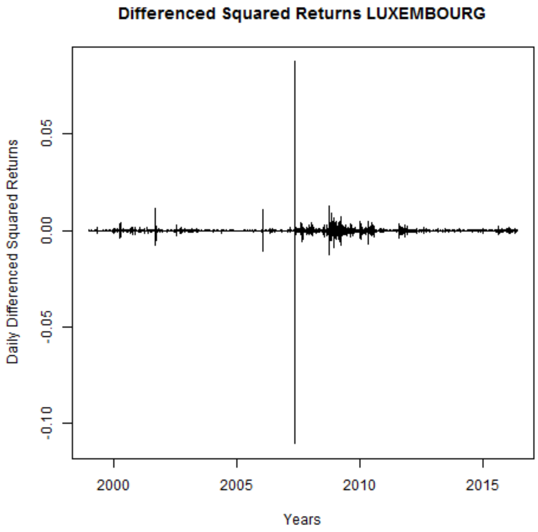

Differenced Squared Returns IRELAND
Daily Differenced Squared Returns
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
1990
2000
2010
Years

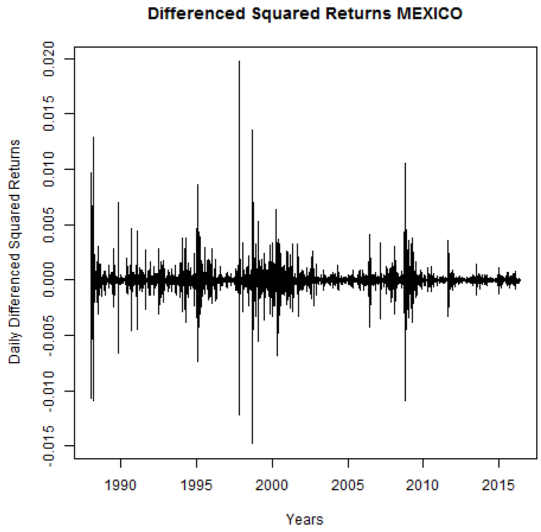

Differenced Squared Returns ISRAEL
Daily Differenced Squared Returns
0.010
0.005
0.000
-0.005
-0.010
1990
1995
2000
2005
2010
2015
Years

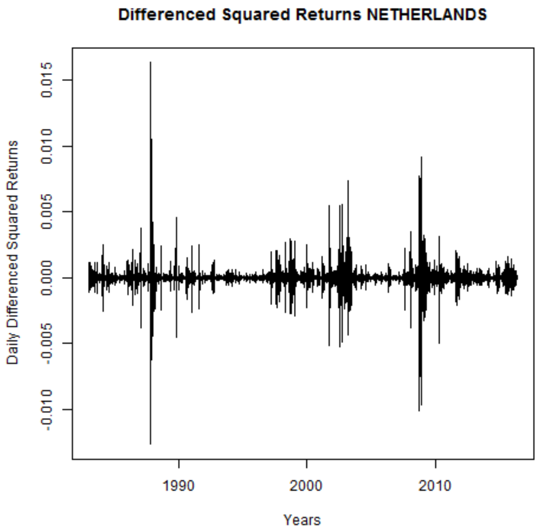

Differenced Squared Returns ITALY
Daily Differenced Squared Returns
0.010
0.005
0.000
-0.005
-0.010
2000
2005
2010
2015
Years

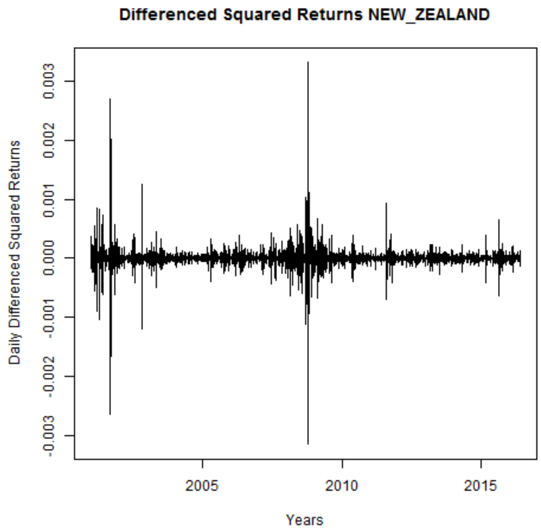

Differenced Squared Returns JAPAN
Daily Differenced Squared Returns
0.02
0.01
0.00
-0.01
1970
1980
1990
2000
2010
Years

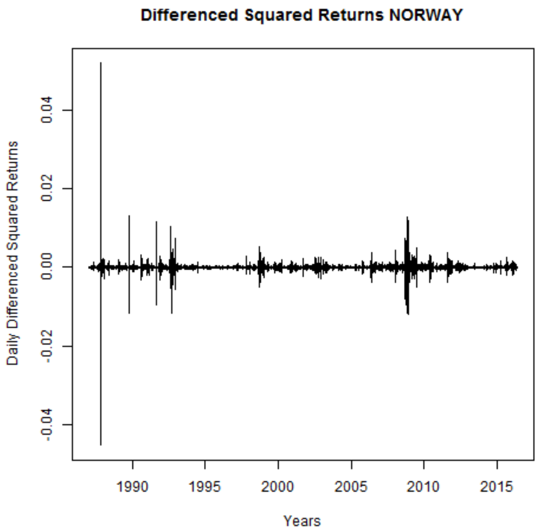

Differenced Squared Returns KOREA
Daily Differenced Squared Returns
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
1980
1990
2000
2010
Years

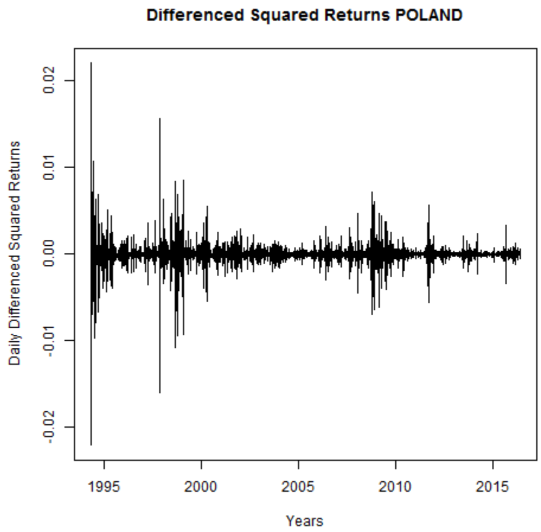

Differenced Squared Returns LUXEMBOURG
Daily Differenced Squared Returns
0.05
0.00
-0.05
-0.10
2000
2005
2010
2015
Years

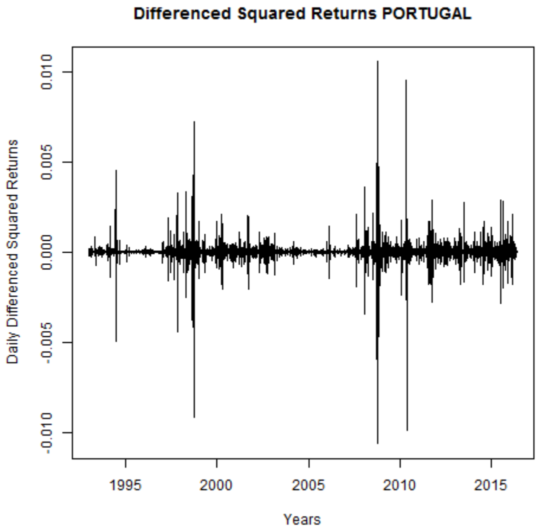

Differenced Squared Returns MEXICO
Daily Differenced Squared Returns
0.020
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
1990
1995
2000
2005
2010
2015
Years

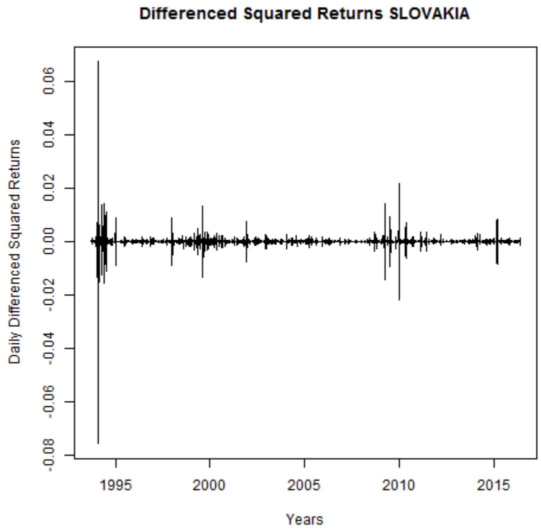

Differenced Squared Returns NETHERLANDS
Daily Differenced Squared Returns
0.015
0.010
0.005
0.000
-0.005
-0.010
1990
2000
2010
Years

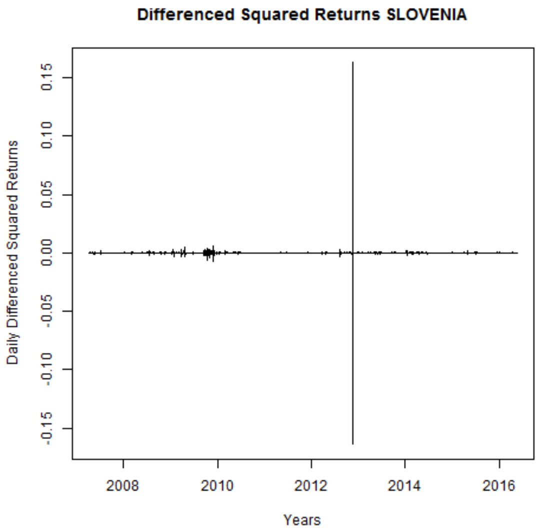

Differenced Squared Returns NEW_ZEALAND
Daily Differenced Squared Returns
0.003
0.002
0.001
0.000
-0.001
-0.002
-0.003
2005
2010
2015
Years

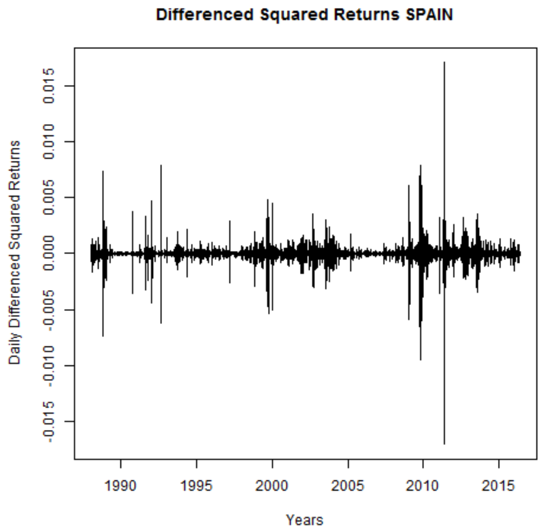

Differenced Squared Returns NORWAY
Daily Differenced Squared Returns
0.04
0.02
0.00
-0.02
-0.04
1990
1995
2000
2005
2010
2015
Years

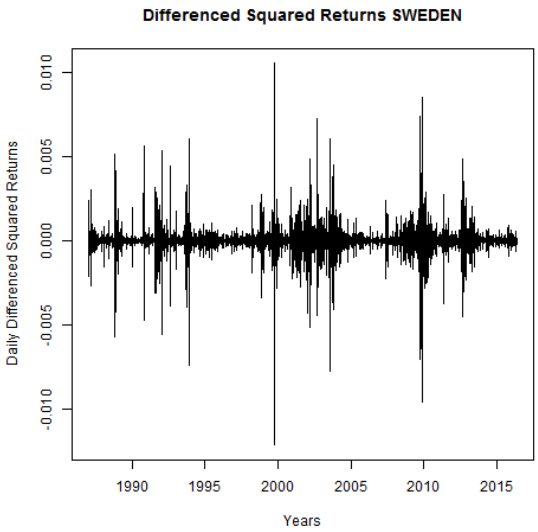

Differenced Squared Returns POLAND
Daily Differenced Squared Returns
0.02
0.01
0.00
-0.01
-0.02
1995
2000
2005
2010
2015
Years

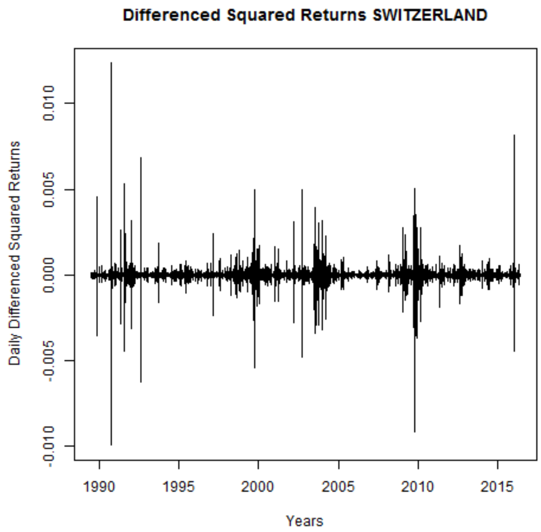

Differenced Squared Returns PORTUGAL
Daily Differenced Squared Returns
Years
0.010
0.005
0.000
-0.005
-0.010
1995
2000
2005
2010
2015

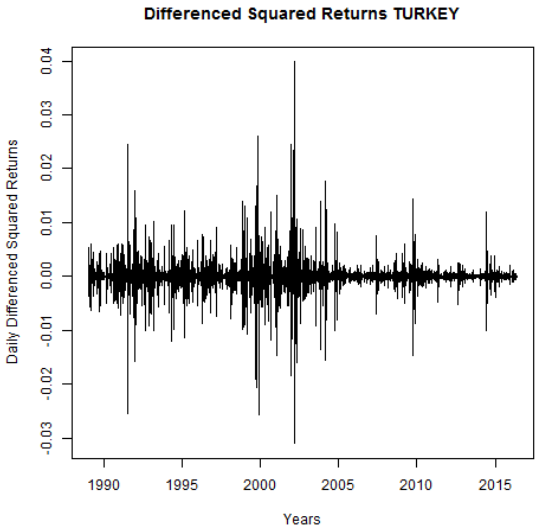

Differenced Squared Returns SLOVAKIA
Daily Differenced Squared Returns
Years
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
1995
2000
2005
2010
2015

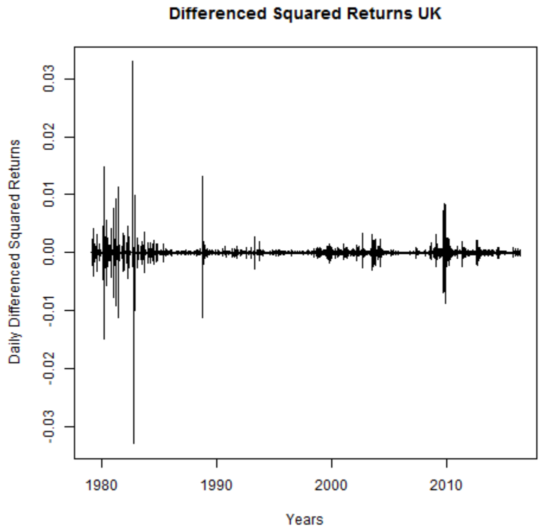

Differenced Squared Returns SLOVENIA
Daily Differenced Squared Returns
Years
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
2008
2010
2012
2014
2016

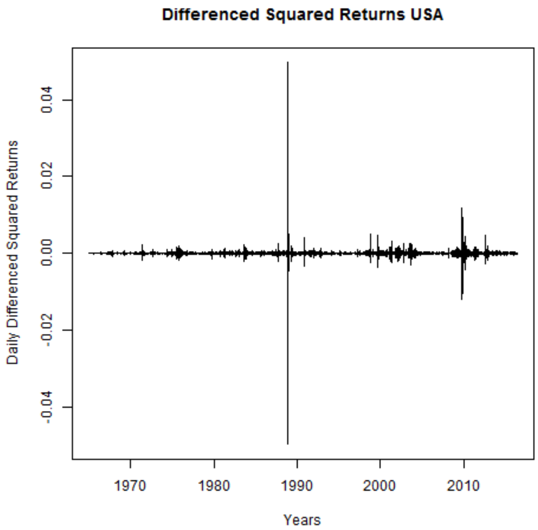

Differenced Squared Returns SPAIN
Daily Differenced Squared Returns
Years
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
1990
1995
2000
2005
2010
2015


Differenced Squared Returns SWEDEN
Daily Differenced Squared Returns
Years
0.010
0.005
0.000
-0.005
-0.010
1990
1995
2000
2005
2010
2015


Differenced Squared Returns SWITZERLAND
Daily Differenced Squared Returns
Years
0.010
0.005
0.000
-0.005
-0.010
1990
1995
2000
2005
2010
2015

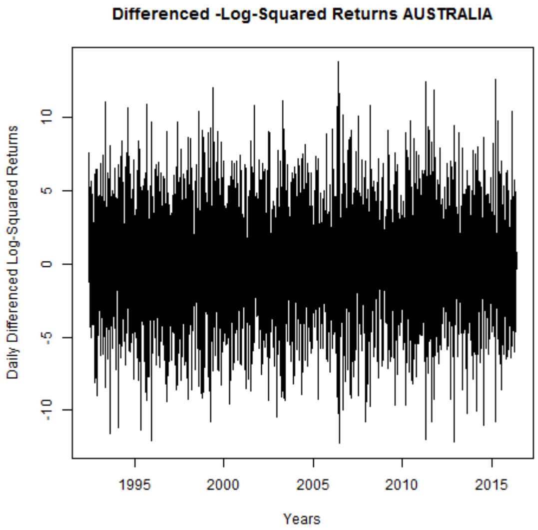

Differenced Squared Returns TURKEY
Daily Differenced Squared Returns
Years
0.04
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
1990
1995
2000
2005
2010
2015

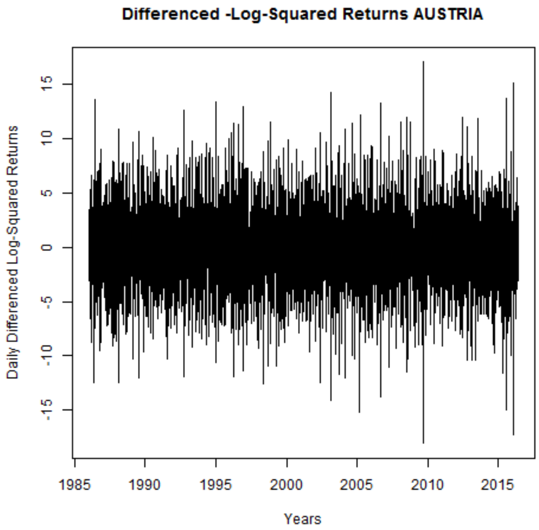

Differenced Squared Returns UK
Daily Differenced Squared Returns
Years
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
1980
1990
2000
2010

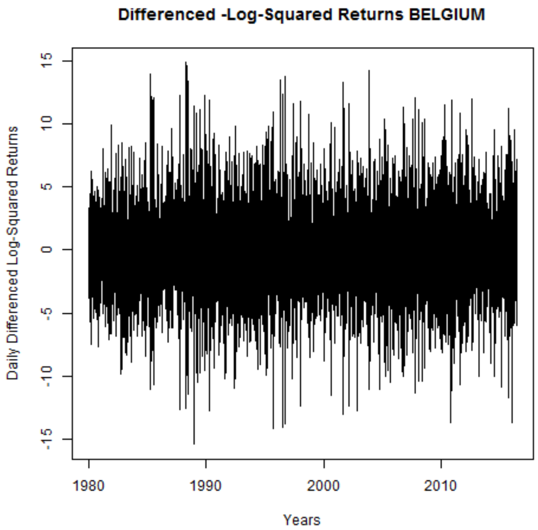

Differenced Squared Returns USA
Daily Differenced Squared Returns
Years
0.04
0.02
0.00
-0.02
-0.04
1970
1980
1990
2000
2010

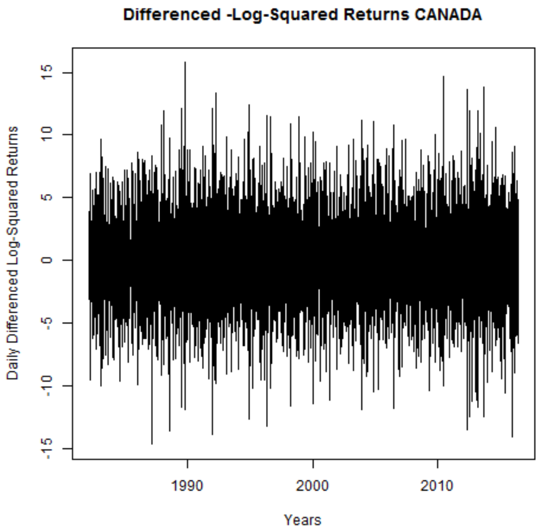
Appendix 18

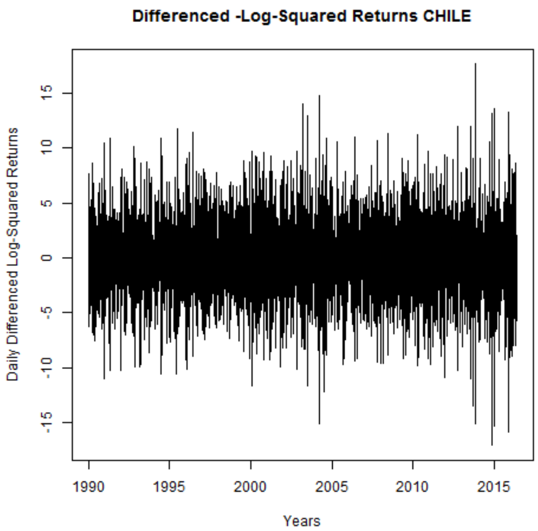
Evidence of Stationarity - Differenced Log Squared Returns – Equity returns

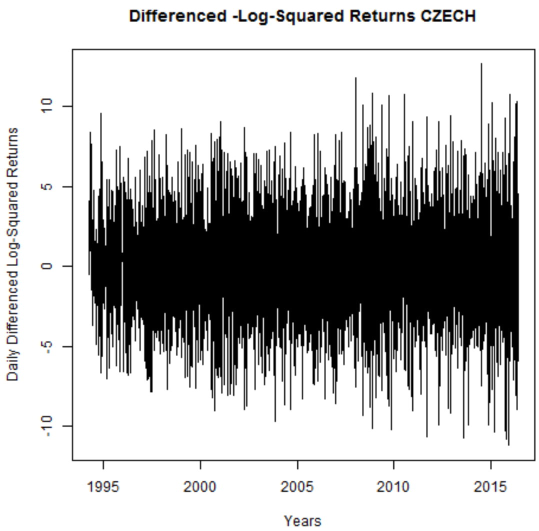


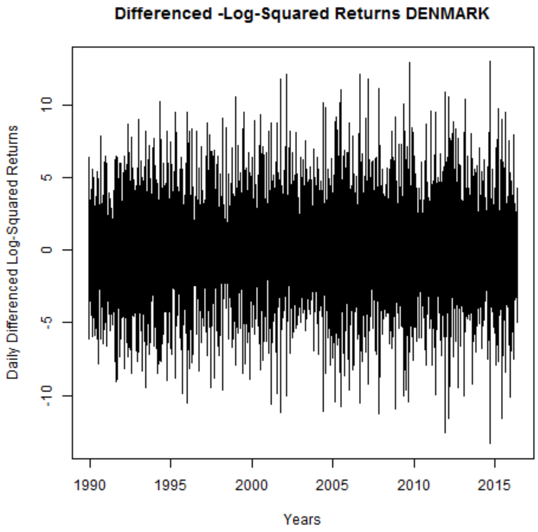


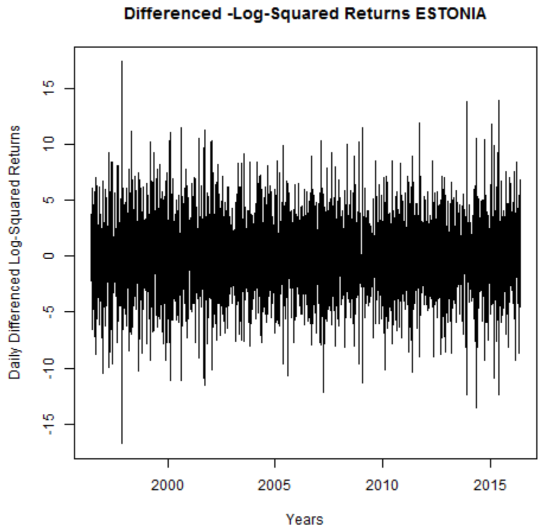


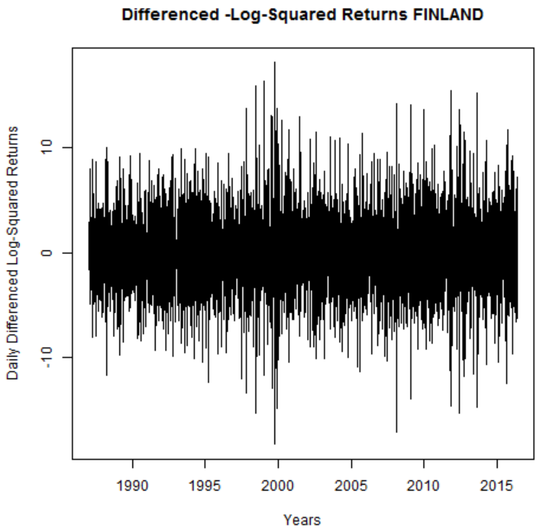


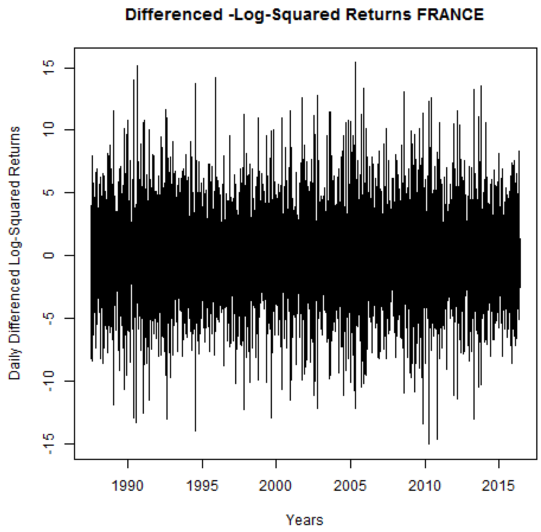


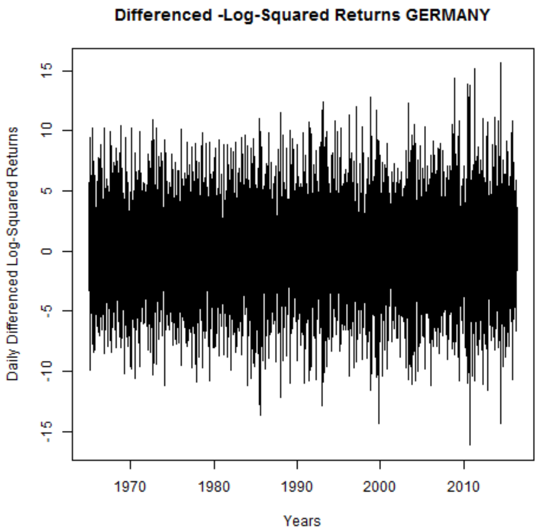


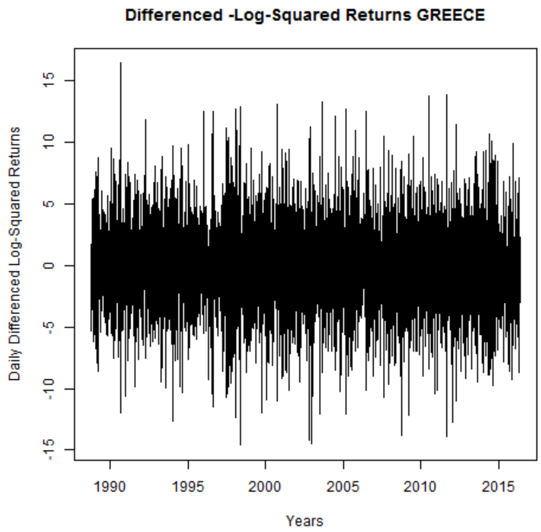

Differenced -Log-Squared Returns DENMARK
Daily Differenced Log-Squared Returns
-10 -5 0 5 10
1990 1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns ESTONIA
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
2000 2005 2010 2015
Years

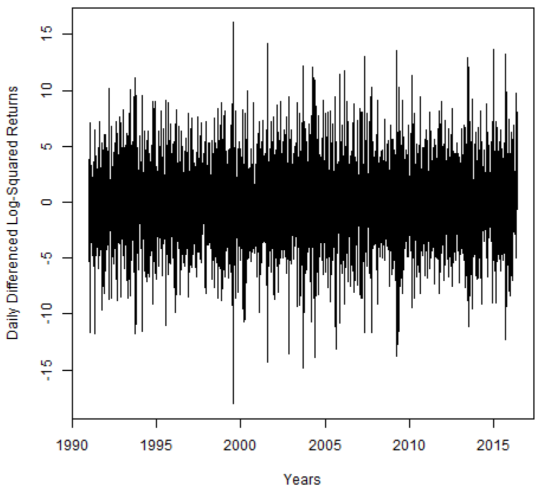

Differenced -Log-Squared Returns FINLAND
Daily Differenced Log-Squared Returns
-10 0 10
1990 1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns FRANCE
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1990 1995 2000 2005 2010 2015
Years

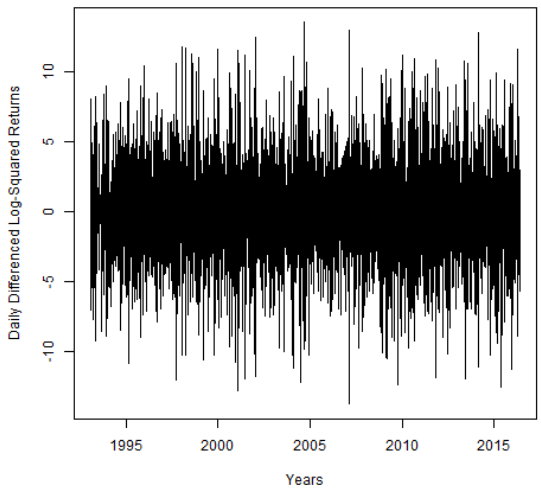

Differenced -Log-Squared Returns GERMANY
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1970 1980 1990 2000 2010
Years


Differenced -Log-Squared Returns GREECE
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1990 1995 2000 2005 2010 2015
Years

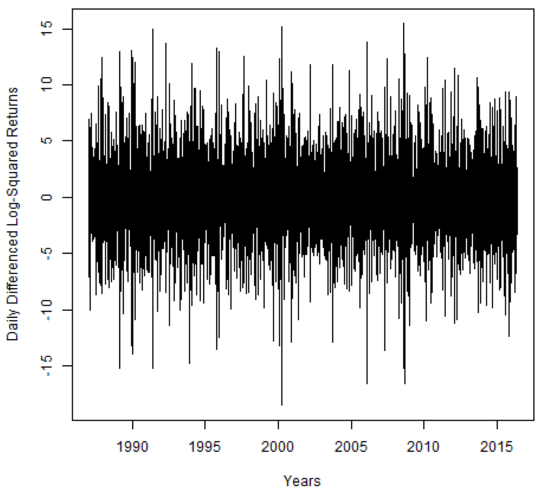
Differenced -Log-Squared Returns HUNGARY



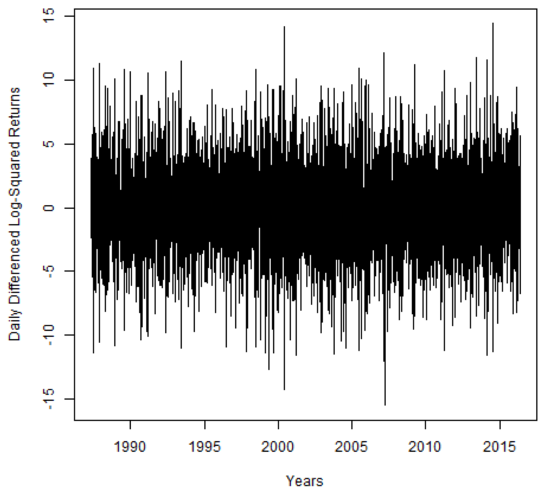
Differenced -Log-Squared Returns ICELAND



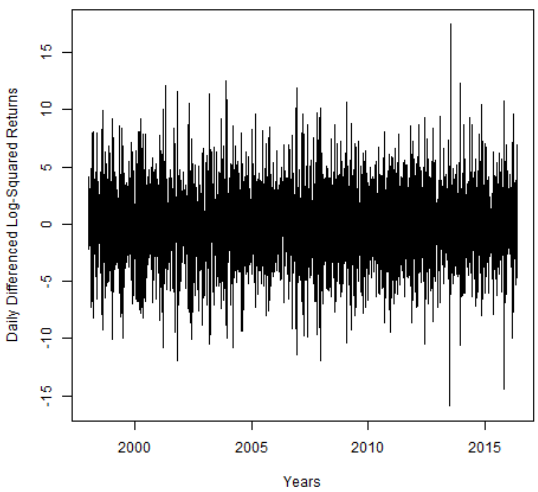
Differenced -Log-Squared Returns IRELAND



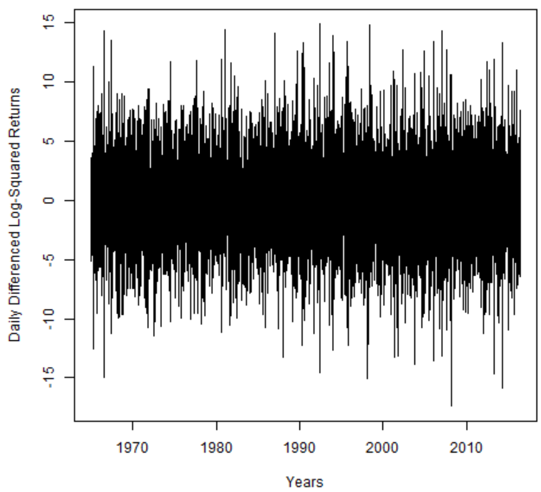
Differenced -Log-Squared Returns ISRAEL

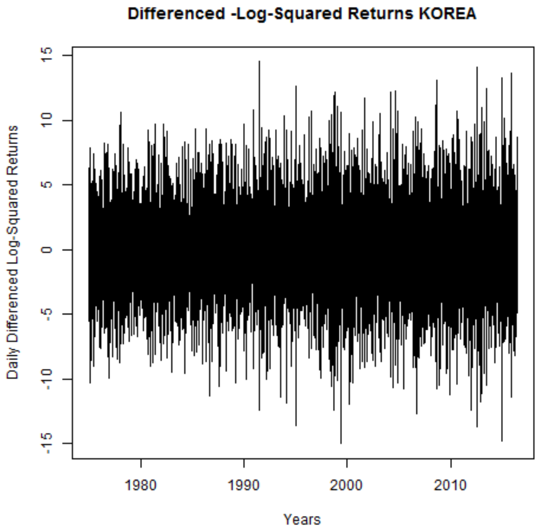


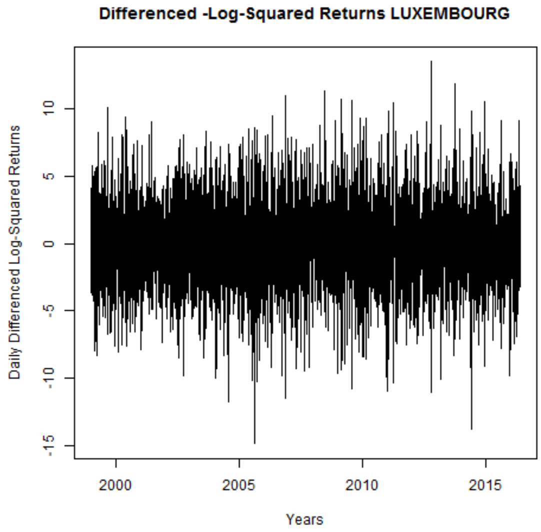
Differenced -Log-Squared Returns ITALY

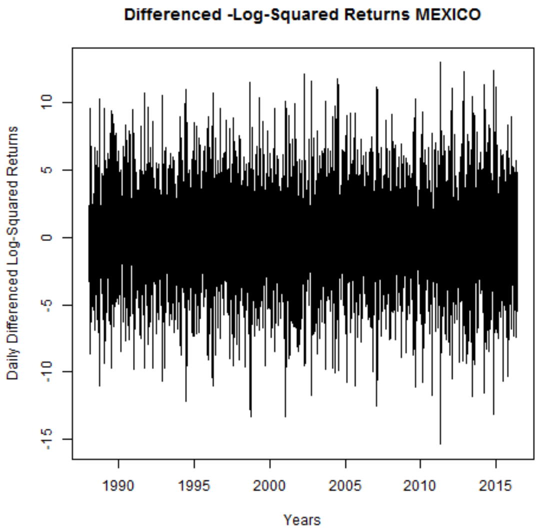


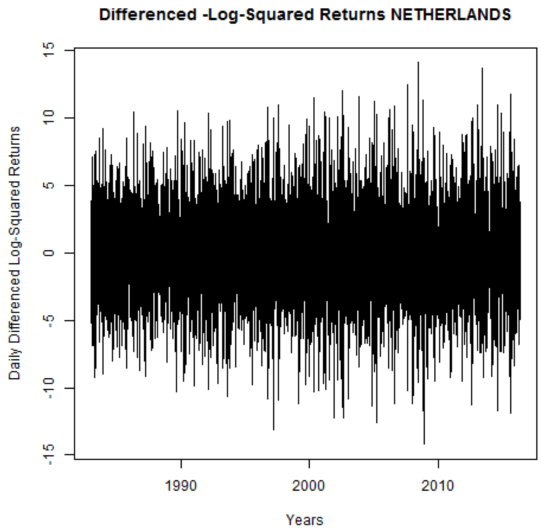
Differenced -Log-Squared Returns JAPAN

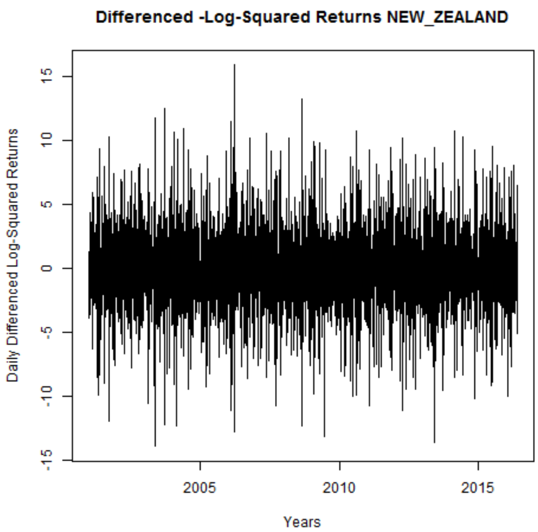


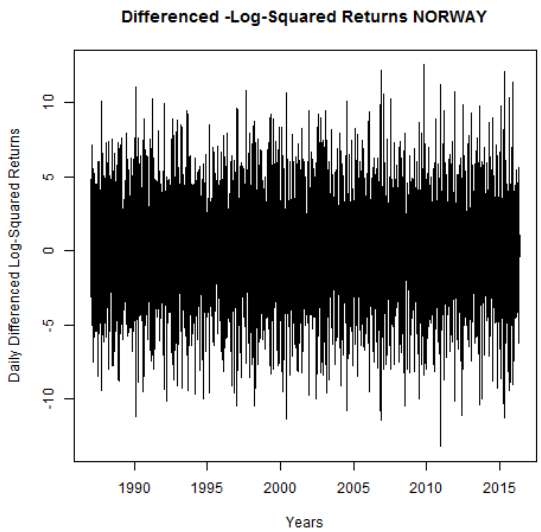

Differenced -Log-Squared Returns KOREA
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1980 1990 2000 2010
Years

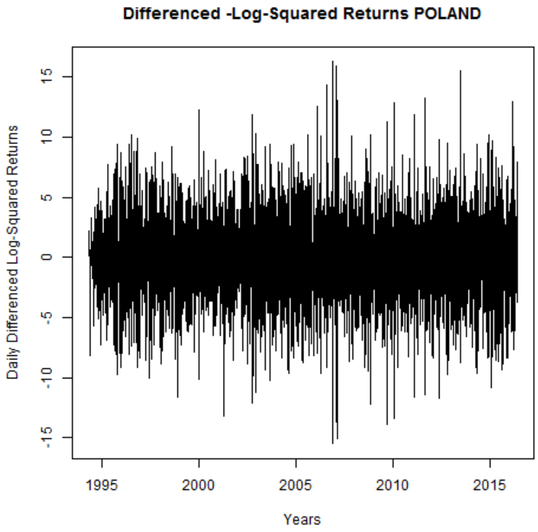

Differenced -Log-Squared Returns LUXEMBOURG
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10
2000 2005 2010 2015
Years

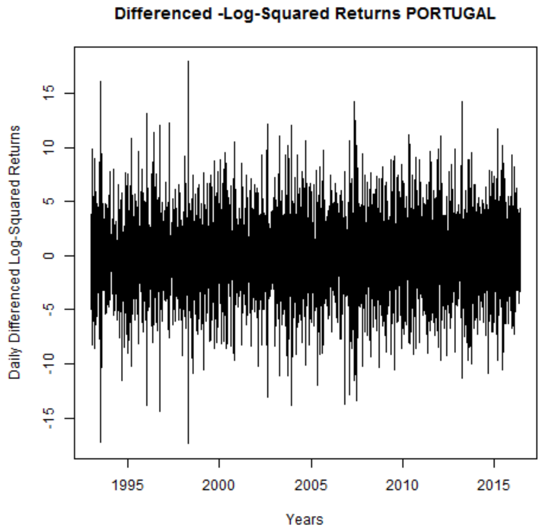

Differenced -Log-Squared Returns MEXICO
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10
1990 1995 2000 2005 2010 2015
Years

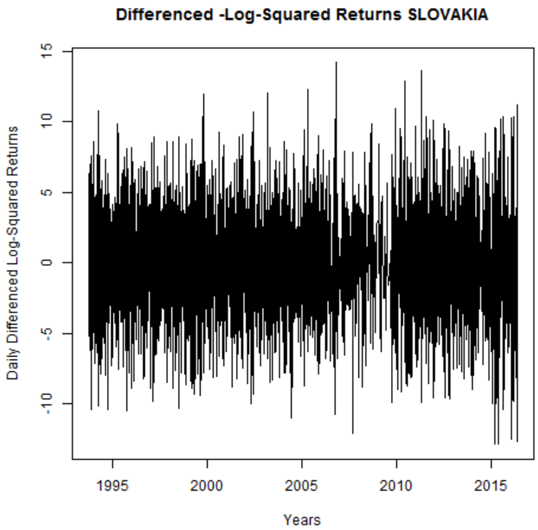

Differenced -Log-Squared Returns NETHERLANDS
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1990 2000 2010
Years

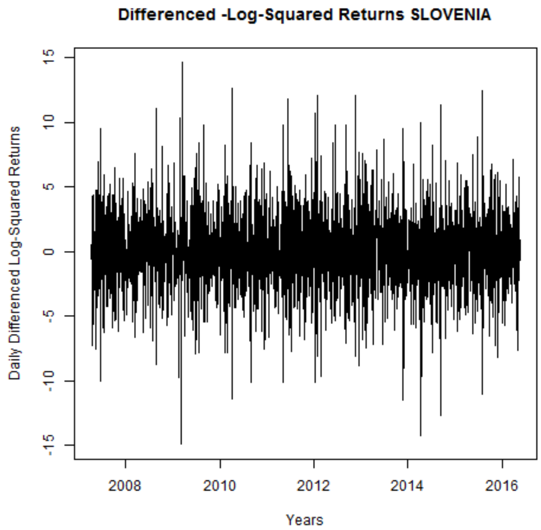

Differenced -Log-Squared Returns NEW_ZEALAND
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
2005 2010 2015
Years

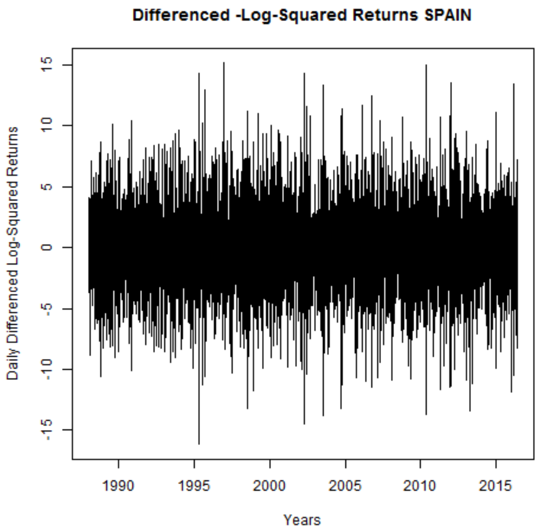

Differenced -Log-Squared Returns NORWAY
Daily Differenced Log-Squared Returns
-10 -5 0 5 10
1990 1995 2000 2005 2010 2015
Years

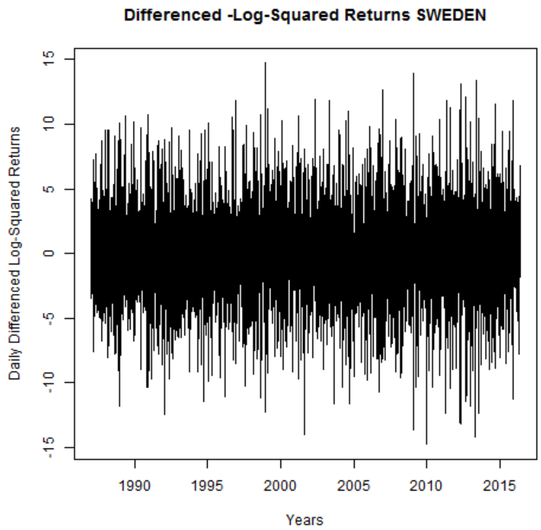

Differenced -Log-Squared Returns POLAND
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns PORTUGAL
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns SLOVAKIA
Daily Differenced Log-Squared Returns
-10 -5 0 5 10 15
1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns SLOVENIA
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
2008 2010 2012 2014 2016
Years


Differenced -Log-Squared Returns SPAIN
Daily Differenced Log-Squared Returns
-15 -10 -5 0 5 10 15
1990 1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns SWEDEN
Daily Differenced Log-Squared Returns
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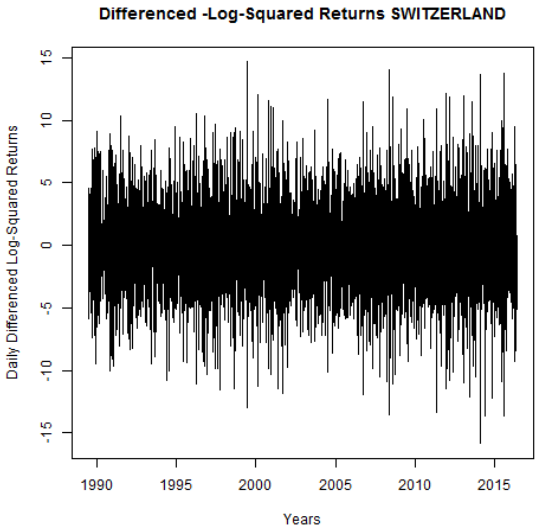
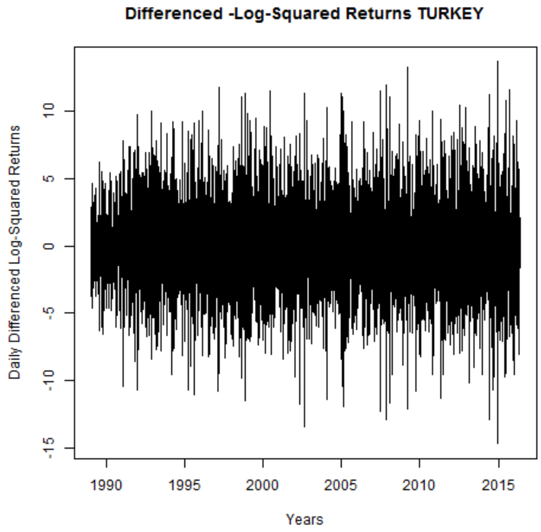
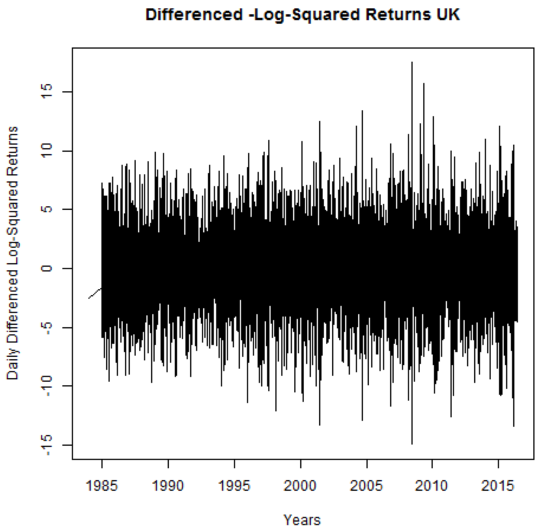
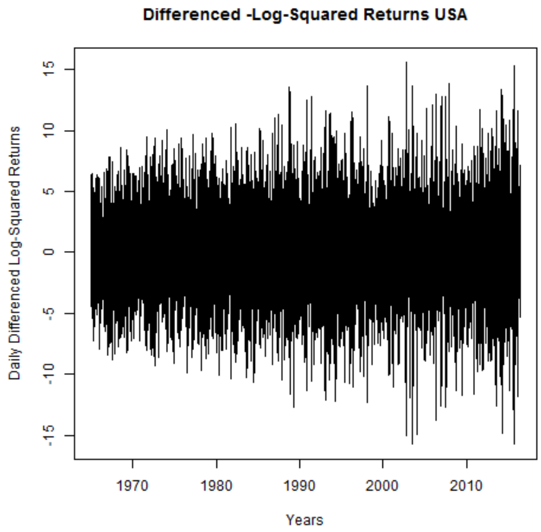
1990 1995 2000 2005 2010 2015
Years


Differenced -Log-Squared Returns SWITZERLAND
Daily Differenced Log-Squared Returns
Years
15
10
5
0
-5
-10
-15
1990
1995
2000
2005
2010
2015


Differenced -Log-Squared Returns TURKEY
Daily Differenced Log-Squared Returns
Years
10
5
0
-5
-10
-15
1990
1995
2000
2005
2010
2015


Differenced -Log-Squared Returns UK
Daily Differenced Log-Squared Returns
Years
15
10
5
0
-5
-10
-15
1985
1990
1995
2000
2005
2010
2015


Differenced -Log-Squared Returns USA
Daily Differenced Log-Squared Returns
Years
15
10
5
0
-5
-10
-15
1970
1980
1990
2000
2010

Appendix 19

Summary of the innovation in extreme variance between returns and differenced absolute returns of the daily foreign exchange market

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso	Chinese Yuan
1. Total Black Swans (Seg. Returns)	55 (0.94%)	56 (0.96%)	69 (1.21%)	59 (1.01%)	72 (1.23%)	36 (1.27%)
2. Total Black Swans (Seg. Diff. Abs. returns)	41 (0.70%)	61 (1.04%)	72 (1.26%)	68 (1.16%)	74 (1.27%)	29 (1.03%)
3. Difference (%)	29.38%	-8.55%	-4.26%	-14.20%	-2.74%	21.62%
4. Positive Black Swans (Seg. Returns)	28 (0.48%)	38 (0.65%)	42 (0.74%)	25 (0.43%)	42 (0.72%)	12 (0.42%)
5. Positive Black Swans (Seg. Diff. Abs. returns)	21 (0.36%)	32 (0.55%)	34 (0.60%)	34 (0.58%)	36 (0.62%)	15 (0.53%)
6. Difference (%)	28.77%	17.19%	21.13%	-30.75%	15.42%	-22.31%
7. Negative Black Swans (Seg. Returns)	27 (0.46%)	18 (0.31%)	27 (0.47%)	34 (0.58%)	30 (0.51%)	24 (0.85)
8. Negative Black Swans (Seg. Diff. Abs. returns)	20 (0.34%)	29 (0.50%)	38 (0.67%)	34 (0.58%)	38 (0.65%)	15 (0.50%)
9. Difference (%)	30.01%	-47.69%	-34.17%	0.00%	-23.64%	53.90%
	Danish Krone	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
1. Total Black Swans (Seg. Returns)	44 (0.75%)	47 (0.80%)	31 (0.68%)	87 (1.49%)	52 (1.05%)	124 (2.12%)
2. Total Black Swans (Seg. Diff. Abs. returns)	61 (1.04%)	66 (1.13%)	36 (0.79%)	96 (1.64%)	62 (1.25%)	121 (2.07%)
3. Difference (%)	-32.67%	-33.95%	-14.95%	-9.84%	-17.59%	2.45%
4. Positive Black Swans (Seg. Returns)	12 (0.21%)	14 (0.24%)	15 (0.33%)	37 (0.63%)	26 (0.52%)	58 (0.99%)
5. Positive Black Swans (Seg. Diff. Abs. returns)	33 (0.57%)	35 (0.60%)	20 (0.44%)	54 (0.92%)	31 (0.63%)	66 (1.13%)
6. Difference (%)	-101.16%	-91.63%	-28.77%	-37.81%	-17.59%	-22.31%
7. Negative Black Swans (Seg. Returns)	32 (0.55%)	33 (0.57%)	16 (0.35%)	50 (0.86%)	26 (0.52%)	66 (1.13%)
8. Negative Black Swans (Seg. Diff. Abs. returns)	28 (0.48%)	31 (0.53%)	16 (0.35%)	42 (0.72%)	31 (0.63%)	55 (0.94%)
9. Difference (%)	13.35%	6.25%	0.00%	17.44%	-17.59%	18.23%
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**	**New Guinea Kina**	**New Turkish Lira**
1. Total Black Swans (Seg. Returns)	96 (1.64%)	107 (2.16%)	101 (1.73%)	80 (1.37%)	90 (1.60%)	79 (1.35%)
2. Total Black Swans (Seg. Diff. Abs. returns)	105 (1.08%)	90 (1.82%)	109 (1.87%)	95 (1.63%)	73 (1.30%)	86 (1.47%)
3. Difference (%)	-8.96%	17.30%	-7.62%	-17.19%	20.94%	-8.49%
4. Positive Black Swans (Seg. Returns)	46 (0.79%)	58 (1.17%)	47 (0.80%)	46 (0.79%)	47(0.84%)	51 (0.87%)
5. Positive Black Swans (Seg. Diff. Abs. returns)	48 (0.82%)	47 (0.95%)	53 (0.91%)	48 (0.82%)	37 (0.66%)	42 (0.72%)
6. Difference (%)	-4.26%	21.03%	-12.03%	-4.26%	23.92%	19.42%
7. Negative Black Swans (Seg. Returns)	40 (0.69%)	49 (0.99%)	54 (0.92%)	35 (0.58%)	43 (0.77%)	28 (0.48%)
8. Negative Black Swans (Seg. Diff. Abs. returns)	57 (0.98%)	43 (0.87%)	56 (0.96%)	47 (0.80%)	36 (0.64%)	44 (0.75%)
9. Difference (%)	-35.42%	13.06%	-3.06%	-32.38%	17.77%	-45.20%
	New Zealand Dollar	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**

1. Total Black Swans (Seg. Returns)	72 (1.23%)	119 (2.18%)	47 (0.80%)	104 (2.19%)	58 (1.04%)	90 (1.71%)
2. Total Black Swans (Seg. Diff. Abs. returns)	74 (1.27%)	114 (2.09%)	53 (0.91%)	104 (2.19%)	67 (1.20%)	90 (1.71%)
3. Difference (%)	-2.74%	4.29%	-12.01%	0.00%	-14.42%	0.00%
4. Positive Black Swans (Seg. Returns)	44 (0.75%)	60 (1.10%)	25 (0.43%)	51 (1.08%)	31 (0.56%)	56 (1.06%)
5. Positive Black Swans (Seg. Diff. Abs. returns)	38 (0.65%)	55 (1.01%)	25 (0.43%)	50 (1.05%)	35 (0.63%)	43 (0.82%)
6. Difference (%)	14.66%	8.70%	0.00%	1.98%	-12.14%	26.42%
7. Negative Black Swans (Seg. Returns)	28 (0.48%)	59 (1.08%)	22 (0.38%)	53 (1.12%)	27 (0.48%)	34 (0.65%)
8. Negative Black Swans (Seg. Diff. Abs. returns)	36 (0.62%)	59 (1.08%)	28 (0.48%)	54 (1.14%)	32 (0.57%)	47 (0.89%)
9. Difference (%)	-25.13%	0.00%	-24.12%	-1.87%	-16.99%	-32.38%
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**	**Swedish Krona**	**Taiwan New Dollar**
1. Total Black Swans (Seg. Returns)	78 (1.34%)	56 (2.17%)	75 (1.28%)	88 (1.51%)	51 (0.87%)	84 (1.44%)
2. Total Black Swans (Seg. Diff. Abs. returns)	84 (1.44%)	62 (2.40%)	80 (1.37%)	82 (1.40%)	62 (1.06%)	90 (1.54%)
3. Difference (%)	-7.41%	-10.18%	-6.45%	7.06%	-19.53%	-6.90%
4. Positive Black Swans (Seg. Returns)	40 (0.69%)	26 (1.01%)	46 (0.79%)	58 (0.99%)	21 (0.36%)	45 (0.77%)
5. Positive Black Swans (Seg. Diff. Abs. returns)	48 (0.82%)	31 (1.20%)	38 (0.65%)	45 (0.77%)	33 (0.57%)	55 (0.94%)
6. Difference (%)	-18.23%	-17.59%	19.11%	25.38%	-45.20%	-20.07%
7. Negative Black Swans (Seg. Returns)	38 (0.65%)	30 (1.16%)	29 (0.50%)	30 (0.51%)	30 (0.51%)	39 (0.67%)
8. Negative Black Swans (Seg. Diff. Abs. returns)	36 (0.62%)	31 (1.20%)	42 (0.72%)	37 (0.63%)	29 (0.50%)	35 (0.60%)
9. Difference (%)	5.41%	-3.28%	-37.04%	-20.97%	3.39%	10.82%
	UK Sterling					
1. Total Black Swans (Seg. Returns)	53 (0.91%)					
2. Total Black Swans (Seg. Diff. Abs. returns)	65 (1.11%)					
3. Difference (%)	-20.41%					
4. Positive Black Swans (Seg. Returns)	26 (0.45%)					
5. Positive Black Swans (Seg. Diff. Abs. returns)	33 (0.57%)					
6. Difference (%)	-23.84%					
7. Negative Black Swans (Seg. Returns)	27 (0.46%)					
8. Negative Black Swans (Seg. Diff. Abs. returns)	32 (0.55%)					
9. Difference (%)	-16.99%					

Appendix 20

Summary of the innovation in extreme variance between returns and squared returns of the daily foreign exchange market

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso	Chinese Yuan
1. Total Black Swans (Seg. Returns)	55 (0.94%)	56 (0.96%)	69 (1.21%)	59 (1.01%)	72 (1.23%)	36 (1.27%)
2. Total Black Swans (Seg. Diff. Sqd returns)	25 (0.43%)	94 (1.61%)	79 (1.38%)	129 (2.21%)	109 (1.87%)	9 (0.32%)
3. Difference (%)	78.85%	-51.79%	-13.53%	-78.23%	-41.47%	138.63%
4. Positive Black Swans (Seg. Returns)	28 (0.48%)	38 (0.65%)	42 (0.74%)	25 (0.43%)	42 (0.72%)	12 (0.42%)
5. Positive Black Swans (Seg. Diff. Sqd returns)	11 (0.19%)	48 (0.82%)	39 (0.68%)	63 (1.08%)	52 (0.89%)	4 (0.14%)
6. Difference (%)	93.43%	-23.36%	7.41%	-92.43%	-21.36%	109.86%
7. Negative Black Swans (Seg. Returns)	27 (0.46%)	18 (0.31%)	27 (0.47%)	34 (0.58%)	30 (0.51%)	24 (0.85)
8. Negative Black Swans (Seg. Diff. Sqd returns)	14 (0.24%)	46 (0.79%)	40 (0.70%)	66 (1.13%)	57 (0.98%)	5 (0.18%)
9. Difference (%)	65.68%	-93.83%	-39.30%	-66.33%	-64.19%	156.86%
	Danish Krone	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
1. Total Black Swans (Seg. Returns)	44 (0.75%)	47 (0.80%)	31 (0.68%)	87 (1.49%)	52 (1.05%)	124 (2.12%)
2. Total Black Swans (Seg. Diff. Sqd returns)	119 (2.04%)	118 (2.02)	64 (1.40%)	73 (1.25%)	77 (1.55%)	122 (2.09%)
3. Difference (%)	-99.49%	-92.05%	-72.49%	17.54%	-39.26%	1.63%
4. Positive Black Swans (Seg. Returns)	12 (0.21%)	14 (0.24%)	15 (0.33%)	37 (0.63%)	26 (0.52%)	58 (0.99%)
5. Positive Black Swans (Seg. Diff. Sqd returns)	60 (1.03%)	60 (1.03%)	33 (0.72%)	37 (0.63%)	43 (0.87%)	62 (1.06%)
6. Difference (%)	-160.94%	-145.53%	-78.85%	0.00%	-50.31%	-6.67%
7. Negative Black Swans (Seg. Returns)	32 (0.55%)	33 (0.57%)	16 (0.35%)	50 (0.86%)	26 (0.52%)	66 (1.13%)
8. Negative Black Swans (Seg. Diff. Sqd returns)	59 (1.01%)	58 (0.99%)	31 (0.68%)	36 (0.62%)	34 (0.69%)	60 (1.03%)
9. Difference (%)	-61.18%	-56.39%	-66.14%	32.85%	-26.83%	9.53%
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**	**New Guinea Kina**	**New Turkish Lira**
1. Total Black Swans (Seg. Returns)	96 (1.64%)	107 (2.16%)	101 (1.73%)	80 (1.37%)	90 (1.60%)	79 (1.35%)
2. Total Black Swans (Seg. Diff. Sqd returns)	66 (1.13%)	84 (1.70%)	97 (1.66%)	87 (1.49%)	26 (0.46%)	73 (1.25%)
3. Difference (%)	37.47%	24.20%	4.04%	-8.39%	124.17%	7.90%
4. Positive Black Swans (Seg. Returns)	46 (0.79%)	58 (1.17%)	47 (0.80%)	46 (0.79%)	47(0.84%)	51 (0.87%)
5. Positive Black Swans (Seg. Diff. Sqd returns)	31 (0.53%)	44 (0.89%)	50 (0.86%)	42 (0.72%)	14 (0.25%)	37 (0.63%)
6. Difference (%)	39.47%	27.63%	-6.19%	9.10%	121.11%	32.09%
7. Negative Black Swans (Seg. Returns)	40 (0.69%)	49 (0.99%)	54 (0.92%)	35 (0.58%)	43 (0.77%)	28 (0.48%)
8. Negative Black Swans (Seg. Diff. Sqd returns)	35 (0.60%)	40 (0.81%)	47 (0.80%)	45 (0.77%)	12 (0.21%)	36 (0.62%)
9. Difference (%)	13.35%	20.29%	13.88%	-28.03%	127.63%	-25.13%
	New Zealand Dollar	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**

1. Total Black Swans (Seg. Returns)	72 (1.23%)	119 (2.18%)	47 (0.80%)	104 (2.19%)	58 (1.04%)	90 (1.71%)
2. Total Black Swans (Seg. Diff. Sqd returns)	131 (2.24%)	91 (1.67%)	93 (1.59%)	56 (1.18%)	96 (1.72%)	58 (1.10%)
3. Difference (%)	-59.85%	26.83%	-68.25%	61.90%	-50.39%	43.94%
4. Positive Black Swans (Seg. Returns)	44 (0.75%)	60 (1.10%)	25 (0.43%)	51 (1.08%)	31 (0.56%)	56 (1.06%)
5. Positive Black Swans (Seg. Diff. Sqd returns)	67 (1.15%)	46 (0.84%)	46 (0.79%)	25 (0.53%)	53 (0.95%)	29 (0.55%)
6. Difference (%)	-42.05%	26.57%	-60.98%	71.29%	-53.63%	65.81%
7. Negative Black Swans (Seg. Returns)	28 (0.48%)	59 (1.08%)	22 (0.38%)	53 (1.12%)	27 (0.48%)	34 (0.65%)
8. Negative Black Swans (Seg. Diff. Sqd returns)	64 (1.10%)	45 (0.83%)	47 (0.80%)	31 (0.65%)	43 (0.77%)	29 (0.55%)
9. Difference (%)	-82.67%	27.09%	-75.91%	53.63%	-46.54%	15.91%
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**	**Swedish Krona**	**Taiwan New Dollar**
1. Total Black Swans (Seg. Returns)	78 (1.34%)	56 (2.17%)	75 (1.28%)	88 (1.51%)	51 (0.87%)	84 (1.44%)
2. Total Black Swans (Seg. Diff. Sqd returns)	101 (1.73%)	61 (2.36%)	92 (1.58%)	128 (2.19%)	129 (2.21%)	80 (1.37%)
3. Difference (%)	-25.84%	-8.55%	-20.43%	-37.47%	-92.80%	4.88%
4. Positive Black Swans (Seg. Returns)	40 (0.69%)	26 (1.01%)	46 (0.79%)	58 (0.99%)	21 (0.36%)	45 (0.77%)
5. Positive Black Swans (Seg. Diff. Sqd returns)	53 (0.91%)	29 (1.12%)	48 (0.82%)	63 (1.08%)	64 (1.10%)	43 (0.74%)
6. Difference (%)	-28.14%	-10.92%	-4.26%	-8.27%	-111.44%	4.55%
7. Negative Black Swans (Seg. Returns)	38 (0.65%)	30 (1.16%)	29 (0.50%)	30 (0.51%)	30 (0.51%)	39 (0.67%)
8. Negative Black Swans (Seg. Diff. Sqd returns)	48 (0.82%)	34 (1.24%)	44 (0.75%)	65 (1.11%)	65 (1.11%)	37 (0.63%)
9. Difference (%)	-23.36%	-6.45%	-41.69%	-77.32%	-77.32%	5.26%
	UK Sterling					
1. Total Black Swans (Seg. Returns)	53 (0.91%)					
2. Total Black Swans (Seg. Diff. Sqd returns)	102 (1.75%)					
3. Difference (%)	-65.47%					
4. Positive Black Swans (Seg. Returns)	26 (0.45%)					
5. Positive Black Swans (Seg. Diff. Sqd returns)	51 (0.87%)					
6. Difference (%)	-67.37%					
7. Negative Black Swans (Seg. Returns)	27 (0.46%)					
8. Negative Black Swans (Seg. Diff. Sqd returns)	51 (0.87%)					
9. Difference (%)	-63.60%					

Appendix 21

Summary of the innovation in extreme variance between returns and log squared returns of the daily foreign exchange market

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar	Chilean Peso	Chinese Yuan
1. Total Black Swans (Seg. Returns)	55 (0.94%)	56 (0.96%)	69 (1.21%)	59 (1.01%)	72 (1.23%)	36 (1.27%)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.79% (29)	0.62% (35)	0.51% (27)	0.66% (37)	0.62% (34)	0.57% (14)
3. Difference (%)	17.10%	43.18%	85.56%	43.11%	69.16%	79.73%
4. Positive Black Swans (Seg. Returns)	28 (0.48%)	38 (0.65%)	42 (0.74%)	25 (0.43%)	42 (0.72%)	12 (0.42%)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.38% (14)	0.36% (20)	0.21% (11)	0.32% (18)	0.33% (18)	0.37% (9)
6. Difference (%)	22.41%	60.36%	125.71%	29.29%	78.86%	14.05%
7. Negative Black Swans (Seg. Returns)	27 (0.46%)	18 (0.31%)	27 (0.47%)	34 (0.58%)	30 (0.51%)	24 (0.85)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.41% (15)	0.27% (15)	0.30% (16)	0.34% (19)	0.29% (16)	0.20% (5)
9. Difference (%)	11.88%	14.41%	44.06%	54.64%	56.99%	142.15%
	Danish Krone	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
1. Total Black Swans (Seg. Returns)	44 (0.75%)	47 (0.80%)	31 (0.68%)	87 (1.49%)	52 (1.05%)	124 (2.12%)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.97% (55)	0.51% (29)	0.54% (21)	0.50% (24)	0.50% (24)	0.47% (23)
3. Difference (%)	-24.78%	45.22%	23.43%	108.21%	73.87%	150.27%
4. Positive Black Swans (Seg. Returns)	12 (0.21%)	14 (0.24%)	15 (0.33%)	37 (0.63%)	26 (0.52%)	58 (0.99%)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.51% (29)	0.21% (12)	0.23% (9)	0.23% (11)	0.25% (12)	0.25% (12)
6. Difference (%)	-90.70%	12.35%	35.57%	100.72%	73.87%	139.35%
7. Negative Black Swans (Seg. Returns)	32 (0.55%)	33 (0.57%)	16 (0.35%)	50 (0.86%)	26 (0.52%)	66 (1.13%)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.46% (26)	0.30% (17)	0.31% (12)	0.27% (13)	0.25% (12)	0.23% (11)
9. Difference (%)	18.30%	63.27%	13.25%	114.13%	73.87%	160.97%
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**	**New Guinea Kina**	**New Turkish Lira**
1. Total Black Swans (Seg. Returns)	96 (1.64%)	107 (2.16%)	101 (1.73%)	80 (1.37%)	90 (1.60%)	79 (1.35%)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.47% (23)	0.32% (12)	0.25% (9)	0.66% (37)	1.11% (34)	0.68% (38)
3. Difference (%)	125.29%	190.73%	192.84%	73.13%	37.14%	68.24%
4. Positive Black Swans (Seg. Returns)	46 (0.79%)	58 (1.17%)	47 (0.80%)	46 (0.79%)	47(0.84%)	51 (0.87%)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.31% (15)	0.16% (6)	0.17% (6)	0.34% (19)	0.59% (18)	0.40% (22)
6. Difference (%)	94.47%	198.80%	156.89%	84.44%	35.77%	79.13%
7. Negative Black Swans (Seg. Returns)	40 (0.69%)	49 (0.99%)	54 (0.92%)	35 (0.58%)	43 (0.77%)	28 (0.48%)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.16% (8)	0.16% (6)	0.08% (3)	0.32% (18)	0.52% (16)	0.29% (16)
9. Difference (%)	143.35%	181.94%	240.09%	59.62%	38.65%	51.01%
	New Zealand Dollar	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**

1. Total Black Swans (Seg. Returns)	72 (1.23%)	119 (2.18%)	47 (0.80%)	104 (2.19%)	58 (1.04%)	90 (1.71%)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.46% (26)	0.58% (21)	0.82% (47)	0.18% (7)	0.89% (48)	0.77% (37)
3. Difference (%)	97.80%	133.27%	-2.30%	247.47%	15.44%	80.19%
4. Positive Black Swans (Seg. Returns)	44 (0.75%)	60 (1.10%)	25 (0.43%)	51 (1.08%)	31 (0.56%)	56 (1.06%)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.20% (11)	0.27% (10)	0.33% (19)	0.13% (5)	0.45% (24)	0.37% (18)
6. Difference (%)	134.58%	138.99%	25.14%	209.86%	22.11%	104.80%
7. Negative Black Swans (Seg. Returns)	28 (0.48%)	59 (1.08%)	22 (0.38%)	53 (1.12%)	27 (0.48%)	34 (0.65%)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.27% (15)	0.30% (11)	0.49% (28)	0.05% (2)	0.45% (24)	0.39% (19)
9. Difference (%)	58.36%	127.77%	-26.42%	305.33%	8.29%	49.50%
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**	**Swedish Krona**	**Taiwan New Dollar**
1. Total Black Swans (Seg. Returns)	78 (1.34%)	56 (2.17%)	75 (1.28%)	88 (1.51%)	51 (0.87%)	84 (1.44%)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.50% (28)	0.20% (3)	0.94% (53)	0.78% (40)	0.81% (46)	0.57% (29)
3. Difference (%)	97.32%	238.74%	31.15%	66.15%	8.00%	92.29%
4. Positive Black Swans (Seg. Returns)	40 (0.69%)	26 (1.01%)	46 (0.79%)	58 (0.99%)	21 (0.36%)	45 (0.77%)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.25% (14)	0.00% (0)	0.50% (28)	0.43% (22)	0.40% (23)	0.34% (17)
6. Difference (%)	99.85%	271.88%	46.07%	84.25%	-11.42%	83.28%
7. Negative Black Swans (Seg. Returns)	38 (0.65%)	30 (1.16%)	29 (0.50%)	30 (0.51%)	30 (0.51%)	39 (0.67%)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.25% (14)	0.20% (3)	0.44% (25)	0.35% (18)	0.40% (23)	0.24% (12)
9. Difference (%)	94.72%	176.33%	11.27%	38.39%	24.25%	103.80%
	UK Sterling					
1. Total Black Swans (Seg. Returns)	53 (0.91%)					
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.57% (32)					
3. Difference (%)	47.36%					
4. Positive Black Swans (Seg. Returns)	26 (0.45%)					
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.26% (15)					
6. Difference (%)	51.91%					
7. Negative Black Swans (Seg. Returns)	27 (0.46%)					
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.30% (17)					
9. Difference (%)	43.17%					

Appendix 22

Summary of the innovation in extreme variance between residuals of returns and differenced absolute returns of the daily foreign exchange market from best fit ARMA-APARCH model

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**	**Chinese Yuan**
1. Total Black Swans (Seg. Residuals)	0.46% (27)	0.94% (55)	0.84% (48)	0.72% (42)	0.87% (51)	0.81% (23)
2. Total Black Swans (Seg. Diff. Abs. residuals)	0.50% (28)	1.39% (81)	1.23% (70)	1.20% (70)	1.35% (79)	1.03% (29)
3. Difference (%)	-11.52%	-38.80%	-37.83%	-22.84%	-19.59%	-11.55%
4. Positive Black Swans (Seg. Residuals)	0.12% (7)	0.67% (39)	0.60% (34)	0.36% (21)	0.50% (29)	0.28% (8)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	0.50% (28)	1.37% (80)	1.16% (66)	1.20% (70)	1.35% (79)	1.03% (29)
6. Difference (%)	-143.01%	-71.93%	-66.43%	-120.48%	-100.30%	-128.82%
7. Negative Black Swans (Seg. Residuals)	0.34% (20)	0.27% (16)	0.25% (14)	0.36% (21)	0.38% (22)	0.53% (15)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.02% (1)	0.07% (4)	0.02% (1)	0.02% (1)	0.04% (1)
9. Difference (%)	295.20%	277.17%	125.17%	304.37%	309.02%	270.77%
	Danish Krone	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
1. Total Black Swans (Seg. Residuals)	0.82% (48)	0.74% (43)	0.81% (37)	1.35% (79)	0.99% (49)	1.59% (93)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.10% (64)	1.23% (72)	0.86% (39)	1.41% (82)	1.43% (71)	2.14% (125)
3. Difference (%)	-13.20%	-23.02%	-3.42%	-1.66%	-16.15%	-12.87%
4. Positive Black Swans (Seg. Residuals)	0.31% (18)	0.26% (15)	0.48% (22)	0.63% (37)	0.42% (21)	0.80% (47)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.10% (64)	1.23% (72)	0.86% (39)	1.35% (79)	1.35% (67)	2.07% (121)
6. Difference (%)	-126.94%	-156.95%	-57.34%	-75.96%	-116.12%	-94.62%
7. Negative Black Swans (Seg. Residuals)	0.51% (30)	0.48% (28)	0.33% (15)	0.72% (42)	0.57% (28)	0.79% (46)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.02% (1)	0.02% (1)	0.05% (3)	0.08% (4)	0.07% (4)
9. Difference (%)	340.03%	333.13%	270.72%	263.80%	194.49%	244.18%
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**	**New Guinea Kina**	**New Turkish Lira**
1. Total Black Swans (Seg. Residuals)	1.49% (87)	1.90% (94)	0.63% (37)	0.96% (56)	1.27% (71)	0.80% (47)
2. Total Black Swans (Seg. Diff. Abs. residuals)	0.63% (37)	1.86% (92)	1.63% (95)	1.51% (88)	1.27% (71)	1.01% (59)
3. Difference (%)	85.41%	2.09%	-94.38%	-45.30%	-1.43%	-22.81%
4. Positive Black Swans (Seg. Residuals)	0.92% (54)	1.01% (50)	0.26% (15)	0.70% (41)	0.82% (46)	0.58% (34)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.65% (96)	1.70% (84)	1.46% (85)	1.35% (79)	1.27% (71)	0.98% (57)
6. Difference (%)	-57.62%	-51.94%	-173.55%	-65.69%	-43.44%	-51.74%
7. Negative Black Swans (Seg. Residuals)	0.57% (33)	0.89% (44)	0.38% (22)	0.26% (15)	0.45% (25)	0.22% (13)

8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.16% (8)	0.17% (10)	0.15% (9)	0.02% (1)	0.03% (2)
9. Difference (%)	349.57%	170.41%	78.76%	50.98%	321.85%	187.11%
	New Zealand Dollar	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
1. Total Black Swans (Seg. Residuals)	1.13% (66)	1.63% (89)	0.69% (40)	1.56% (74)	0.91% (51)	1.40% (74)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.68% (98)	1.85% (101)	1.30% (74)	1.86% (88)	1.45% (81)	1.82% (96)
3. Difference (%)	-40.63%	-12.78%	-65.66%	-17.41%	-46.35%	-26.10%
4. Positive Black Swans (Seg. Residuals)	0.79% (46)	0.90% (49)	0.31% (18)	0.76% (36)	0.50% (28)	0.85% (45)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.68% (98)	1.74% (95)	1.30% (74)	1.84% (87)	1.44% (80)	1.75% (92)
6. Difference (%)	-75.72%	-66.33%	-144.17%	-88.32%	-105.07%	-71.59%
7. Negative Black Swans (Seg. Residuals)	0.34% (20)	0.73% (40)	0.38% (22)	0.80% (38)	0.41% (23)	0.55% (29)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.11% (6)	0.02% (1)	0.02% (1)	0.02% (1)	0.08% (4)
9. Difference (%)	299.49%	189.58%	306.31%	363.67%	313.46%	198.02%
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**	**Swedish Krona**	**Taiwan New Dollar**
1. Total Black Swans (Seg. Residuals)	1.15% (67)	1.70% (44)	0.87% (51)	1.10% (64)	0.77% (45)	1.35% (79)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.49% (87)	1.94% (50)	1.41% (82)	1.51% (88)	1.35% (79)	1.66% (97)
3. Difference (%)	-27.35%	-14.92%	-47.56%	-31.95%	-57.59%	-20.61%
4. Positive Black Swans (Seg. Residuals)	0.51% (30)	1.01% (26)	0.65% (38)	0.80% (47)	0.34% (20)	0.65% (38)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.49% (87)	1.94% (50)	1.39% (81)	1.49% (87)	1.35% (79)	4.30% (93)
6. Difference (%)	-106.56%	-65.55%	-75.75%	-61.68%	-137.42%	-188.84%
7. Negative Black Swans (Seg. Residuals)	0.63% (37)	0.70% (18)	0.22% (13)	0.29% (17)	0.43% (25)	0.70% (41)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.04% (1)	0.02% (1)	0.02% (1)	0.02% (1)	104.30% (4)
9. Difference (%)	361.01%	288.88%	256.43%	283.22%	321.84%	232.64%
	UK Sterling					
1. Total Black Swans (Seg. Residuals)	0.70% (41)					
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.27% (74)					
3. Difference (%)	-60.48%					
4. Positive Black Swans (Seg. Residuals)	0.27% (16)					
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.27% (74)					
6. Difference (%)	-153.23%					
7. Negative Black Swans (Seg. Residuals)	0.43% (25)					
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)					

9. Difference (%)	321.80%					

Appendix 23

Summary of the innovation in extreme variance between residuals of returns and differenced squared returns of the daily foreign exchange market from best fit ARMA-APARCH model

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**	**Chinese Yuan**
1. Total Black Swans (Seg. Residuals)	0.46% (27)	0.94% (55)	0.84% (48)	0.72% (42)	0.87% (51)	0.81% (23)
2. Total Black Swans (Seg. Diff. Sqd residuals)	0.46% (27)	1.70% (99)	1.35% (77)	2.01% (116)	1.34% (78)	0.71% (19)
3. Difference (%)	-0.07%	-58.86%	-47.37%	-102.54%	-42.57%	13.98%
4. Positive Black Swans (Seg. Residuals)	0.12% (7)	0.67% (39)	0.60% (34)	0.36% (21)	0.50% (29)	0.28% (8)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	0.36% (21)	1.68% (98)	1.30% (74)	1.99% (116)	1.30% (76)	0.67% (19)
6. Difference (%)	-109.93%	-92.23%	-77.88%	-170.99%	-96.43%	-86.50%
7. Negative Black Swans (Seg. Residuals)	0.34% (20)	0.27% (16)	0.25% (14)	0.36% (21)	0.38% (22)	0.53% (15)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.10% (6)	0.02% (1)	0.05% (3)	0.02% (1)	0.03% (2)	0.04% (1)
9. Difference (%)	120.33%	277.17%	153.94%	304.37%	239.70%	270.81%
	Danish Krone	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
1. Total Black Swans (Seg. Residuals)	0.82% (48)	0.74% (43)	0.81% (37)	1.35% (79)	0.99% (49)	1.59% (93)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.80% (104)	1.80% (104)	1.16% (52)	0.86% (49)	1.98% (97)	1.59% (92)
3. Difference (%)	-78.36%	-89.36%	-36.03%	45.64%	-69.31%	-0.05%
4. Positive Black Swans (Seg. Residuals)	0.31% (18)	0.26% (15)	0.48% (22)	0.63% (37)	0.42% (21)	0.80% (47)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.78% (104)	1.78% (104)	1.14% (52)	0.84% (49)	1.96% (97)	1.58% (92)
6. Difference (%)	-175.49%	-193.72%	-86.11%	-28.19%	-153.02%	-67.22%
7. Negative Black Swans (Seg. Residuals)	0.51% (30)	0.48% (28)	0.33% (15)	0.72% (42)	0.57% (28)	0.79% (46)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.02% (1)	0.02% (1)	0.02% (1)	0.02% (1)	0.02% (1)	0.02% (1)
9. Difference (%)	340.03%	333.13%	270.72%	373.66%	333.22%	382.81%
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**	**New Guinea Kina**	**New Turkish Lira**
1. Total Black Swans (Seg. Residuals)	1.49% (87)	1.90% (94)	0.63% (37)	0.96% (56)	1.27% (71)	0.80% (47)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.02% (54)	1.52% (75)	0.70% (41)	1.18% (69)	0.59% (33)	0.99% (58)
3. Difference (%)	36.25%	21.20%	-12.76%	-20.98%	73.60%	-21.10%
4. Positive Black Swans (Seg. Residuals)	0.92% (54)	1.01% (50)	0.26% (15)	0.70% (41)	0.82% (46)	0.58% (34)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.02% (54)	1.52% (75)	0.70% (41)	1.15% (67)	0.59% (33)	0.87% (51)
6. Difference (%)	-9.61%	-40.61%	-100.64%	-49.21%	33.18%	-40.62%
7. Negative Black Swans (Seg. Residuals)	0.57% (33)	0.89% (44)	0.38% (22)	0.26% (15)	0.45% (25)	0.22% (13)

8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.02% (1)	0.02% (1)	0.02% (1)	0.03% (2)	0.02% (1)	0.12% (7)
9. Difference (%)	340.04%	378.36%	309.02%	201.39%	321.85%	61.84%
	New Zealand Dollar	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
1. Total Black Swans (Seg. Residuals)	1.13% (66)	1.63% (89)	0.69% (40)	1.56% (74)	0.91% (51)	1.40% (74)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.99% (116)	1.17% (64)	1.61% (94)	1.08% (51)	1.94% (91)	1.90% (100)
3. Difference (%)	-57.34%	32.85%	-85.54%	37.14%	-75.10%	-30.19%
4. Positive Black Swans (Seg. Residuals)	0.79% (46)	0.90% (49)	0.31% (18)	0.76% (36)	0.50% (28)	0.85% (45)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.99% (116)	1.12% (61)	1.56% (91)	0.97% (46)	1.92% (90)	1.80% (95)
6. Difference (%)	-92.58%	-22.03%	-162.15%	-24.60%	-133.95%	-74.80%
7. Negative Black Swans (Seg. Residuals)	0.34% (20)	0.73% (40)	0.38% (22)	0.80% (38)	0.41% (23)	0.55% (29)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.02% (1)	0.06% (3)	0.05% (3)	0.11% (5)	0.02% (1)	0.09% (5)
9. Difference (%)	299.49%	258.90%	199.14%	202.73%	296.36%	175.71%
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**	**Swedish Krona**	**Taiwan New Dollar**
1. Total Black Swans (Seg. Residuals)	1.15% (67)	1.70% (44)	0.87% (51)	1.10% (64)	0.77% (45)	1.35% (79)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.49% (87)	1.67% (43)	1.92% (112)	1.89% (110)	2.07% (121)	1.25% (73)
3. Difference (%)	-27.35%	2.14%	-79.62%	-55.17%	-99.79%	7.81%
4. Positive Black Swans (Seg. Residuals)	0.51% (30)	1.01% (26)	0.65% (38)	0.80% (47)	0.34% (20)	0.65% (38)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.49% (87)	1.63% (42)	1.92% (112)	1.89% (110)	2.07% (121)	1.18% (69)
6. Difference (%)	-106.56%	-48.11%	-108.16%	-85.14%	-180.06%	-59.74%
7. Negative Black Swans (Seg. Residuals)	0.63% (37)	0.70% (18)	0.22% (13)	0.29% (17)	0.43% (25)	0.70% (41)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.02% (1)	0.04% (1)	0.02% (1)	0.02% (1)	0.02% (1)	0.07% (4)
9. Difference (%)	361.01%	288.88%	256.43%	283.22%	321.84%	232.64%
	UK Sterling					
1. Total Black Swans (Seg. Residuals)	0.70% (41)					
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.68% (98)					
3. Difference (%)	-87.23%					
4. Positive Black Swans (Seg. Residuals)	0.27% (16)					
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.65% (96)					
6. Difference (%)	-179.26%					
7. Negative Black Swans (Seg. Residuals)	0.43% (25)					
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.03% (2)					

9. Difference (%)	252.49%					

Appendix 24

Summary of the innovation in extreme variance between residuals of returns and differenced log squared returns of the daily foreign exchange market from best fit ARMA-APARCH model

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**	**Chilean Peso**	**Chinese Yuan**
1. Total Black Swans (Seg. Residuals)	0.46% (27)	0.94% (55)	0.84% (48)	0.72% (42)	0.87% (51)	0.81% (23)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	0.93% (34)	0.84% (47)	0.67% (35)	0.91% (51)	0.67% (37)	0.74% (18)
3. Difference (%)	-23.11%	15.63%	31.51%	-19.50%	32.02%	24.47%
4. Positive Black Swans (Seg. Residuals)	0.12% (7)	0.67% (39)	0.60% (34)	0.36% (21)	0.50% (29)	0.28% (8)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.25% (9)	0.07% (4)	0.06% (3)	0.05% (3)	0.05% (3)	0.16% (4)
6. Difference (%)	-25.19%	227.64%	242.70%	194.50%	226.80%	69.27%
7. Negative Black Swans (Seg. Residuals)	0.34% (20)	0.27% (16)	0.25% (14)	0.36% (21)	0.38% (22)	0.53% (15)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.68% (25)	0.77% (43)	0.61% (32)	0.85% (48)	0.62% (34)	0.57% (14)
9. Difference (%)	-22.37%	-98.95%	-82.74%	-82.76%	-43.60%	6.86%

	Danish Krone	**Euro**	**Fijian Dollar**	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
1. Total Black Swans (Seg. Residuals)	0.82% (48)	0.74% (43)	0.81% (37)	1.35% (79)	0.99% (49)	1.59% (93)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.21% (69)	0.88% (50)	0.69% (27)	0.53% (25)	0.88% (42)	0.64% (31)
3. Difference (%)	-36.38%	-15.17%	31.43%	114.93%	15.31%	109.80%
4. Positive Black Swans (Seg. Residuals)	0.31% (18)	0.26% (15)	0.48% (22)	0.63% (37)	0.42% (21)	0.80% (47)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.02% (1)	0.04% (2)	0.08% (3)	0.15% (7)	0.08% (4)	0.23% (11)
6. Difference (%)	288.95%	201.40%	199.17%	166.37%	165.72%	145.16%
7. Negative Black Swans (Seg. Residuals)	0.51% (30)	0.48% (28)	0.33% (15)	0.72% (42)	0.57% (28)	0.79% (46)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	1.19% (68)	0.85% (48)	0.62% (24)	0.38% (18)	0.79% (38)	0.41% (20)
9. Difference (%)	-81.92%	-53.99%	-47.08%	84.60%	-30.64%	83.23%

	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**	**New Guinea Kina**	**New Turkish Lira**
1. Total Black Swans (Seg. Residuals)	1.49% (87)	1.90% (94)	0.63% (37)	0.96% (56)	1.27% (71)	0.80% (47)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	0.51% (25)	0.43% (16)	0.42% (15)	1.11% (62)	0.94% (29)	0.95% (53)
3. Difference (%)	124.60%	177.04%	90.17%	-10.29%	89.51%	-12.09%
4. Positive Black Swans (Seg. Residuals)	0.92% (54)	1.01% (50)	0.26% (15)	0.70% (41)	0.82% (46)	0.58% (34)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.16% (8)	0.11% (4)	0.14% (5)	0.07% (4)	0.20% (6)	0.07% (4)
6. Difference (%)	190.85%	252.55%	109.75%	232.62%	203.66%	213.93%

7. Negative Black Swans (Seg. Residuals)	0.57% (33)	0.89% (44)	0.38% (22)	0.26% (15)	0.45% (25)	0.22% (13)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.35% (17)	0.32% (12)	0.28% (10)	1.03% (58)	0.75% (23)	0.88% (49)
9. Difference (%)	66.23%	129.90%	78.73%	-135.35%	8.31%	-132.76%
	New Zealand Dollar	**Nigerian Naira**	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
1. Total Black Swans (Seg. Residuals)	1.13% (66)	1.63% (89)	0.69% (40)	1.56% (74)	0.91% (51)	1.40% (74)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	0.86% (48)	0.80% (29)	1.21% (69)	0.32% (12)	0.95% (51)	0.75% (36)
3. Difference (%)	31.76%	112.00%	-54.63%	181.84%	-0.09%	71.99%
4. Positive Black Swans (Seg. Residuals)	0.79% (46)	0.90% (49)	0.31% (18)	0.76% (36)	0.50% (28)	0.85% (45)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.05% (3)	0.30% (11)	0.02% (1)	0.13% (5)	0.09% (5)	0.08% (4)
6. Difference (%)	272.91%	149.26%	288.93%	197.33%	172.18%	241.97%
7. Negative Black Swans (Seg. Residuals)	0.34% (20)	0.73% (40)	0.38% (22)	0.80% (38)	0.41% (23)	0.55% (29)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.80% (45)	0.49% (18)	1.19% (68)	0.18% (7)	0.85% (46)	0.66% (32)
9. Difference (%)	-81.18%	79.71%	-112.95%	169.09%	-69.41%	-9.91%
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**	**Swedish Krona**	**Taiwan New Dollar**
1. Total Black Swans (Seg. Residuals)	1.15% (67)	1.70% (44)	0.87% (51)	1.10% (64)	0.77% (45)	1.35% (79)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	0.85% (47)	0.60% (9)	1.23% (69)	0.78% (40)	1.14% (65)	0.73% (37)
3. Difference (%)	35.36%	158.56%	-30.30%	46.90%	-36.83%	75.75%
4. Positive Black Swans (Seg. Residuals)	0.51% (30)	1.01% (26)	0.65% (38)	0.80% (47)	0.34% (20)	0.65% (38)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.07% (4)	0.20% (3)	0.05% (3)	0.14% (7)	0.04% (2)	0.08% (4)
6. Difference (%)	201.40%	215.82%	253.83%	190.33%	230.21%	225.03%
7. Negative Black Swans (Seg. Residuals)	0.63% (37)	0.70% (18)	0.22% (13)	0.29% (17)	0.43% (25)	0.70% (41)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.78% (43)	0.40% (6)	1.17% (66)	0.64% (33)	1.10% (63)	0.65% (33)
9. Difference (%)	-15.12%	109.73%	-162.54%	-66.43%	-92.48%	21.61%
	UK Sterling					
1. Total Black Swans (Seg. Residuals)	0.70% (41)					
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.48% (54)					
3. Difference (%)	-27.60%					
4. Positive Black Swans (Seg. Residuals)	0.27% (16)					
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.05% (2)					
6. Difference (%)	207.89%					
7. Negative Black Swans (Seg. Residuals)	0.43% (25)					

8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	1.42% (52)					
9. Difference (%)	-73.29%					

Appendix 25

Summary of the innovation in extreme variance between returns and differenced absolute returns of the daily stock exchange market

	Australia	Austria	Belgium	Canada	Chile	Czech
1. Total Black Swans (Seg. Returns)	0.78% (49)	1.58% (125)	1.32% (125)	1.22% (109)	1.16% (80)	1.37% (79)
2. Total Black Swans (Seg. Diff. Abs. returns)	0.86% (54)	1.62% (128)	1.41% (134)	1.23% (110)	1.31% (90)	1.49% (86)
3. Difference (%)	-9.72%	-2.37%	-6.95%	-0.91%	-11.8%	-8.49%
4. Positive Black Swans (Seg. Returns)	0.54% (34)	0.95% (75)	0.68% (65)	0.69% (62)	0.58% (40)	0.85% (49)
5. Positive Black Swans (Seg. Diff. Abs. returns)	0.48% (30)	0.85% (67)	1.76% (167)	0.65% (58)	0.67% (46)	0.78% (45)
6. Difference (%)	-69.31%	-29.27%	-102%	-21.03%	-14%	-41%
7. Negative Black Swans (Seg. Returns)	0.24% (15)	0.63% (50)	0.63% (60)	0.53% (47)	0.58% (40)	0.52% (30)
8. Negative Black Swans (Seg. Diff. Abs. returns)	0.38% (24)	0.77% (61)	2.78% (264)	0.58% (52)	0.64% (44)	0.71% (41)
9. Difference (%)	34.83%	20.66%	-140%	17.59%	-9.53%	17.8%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Total Black Swans (Seg. Returns)	1.23% (85)	1.71% (89)	1.11% (85)	1.17% (88)	1.04% (140)	1.51% (109)
2. Total Black Swans (Seg. Diff. Abs. returns)	1.33% (92)	1.82% (95)	1.32% (101)	1.12% (84)	1.03% (138)	1.50% (108)
3. Difference (%)	-7.91%	-6.52%	-17.25%	4.65%	1.44%	0.92%
4. Positive Black Swans (Seg. Returns)	0.74% (51)	0.83% (43)	0.57% (44)	0.65% (49)	0.54% (72)	0.76% (55)
5. Positive Black Swans (Seg. Diff. Abs. returns)	0.74% (51)	0.88% (46)	0.69% (53)	0.49% (37)	0.48% (65)	0.79% (57)
6. Difference (%)	-40.55%	0.00%	-25.7%	5.26%	4.51%	-5.41%
7. Negative Black Swans (Seg. Returns)	0.49% (34)	0.88% (46)	0.53% (41)	0.52% (39)	0.51% (68)	0.75% (54)
8. Negative Black Swans (Seg. Diff. Abs. returns)	0.59% (41)	0.94% (49)	0.63% (48)	0.62% (47)	0.54% (73)	0.71% (51)
9. Difference (%)	21.83%	-13.06%	-8.70%	4.17%	-1.38%	7.55%
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
1. Total Black Swans (Seg. Returns)	1.47% (97)	1.46% (89)	1.61% (140)	1.33% (101)	1.00% (48)	1.27% (170)

2. Total Black Swans (Seg. Diff. Abs. returns)	1.88% (117)	1.41% (86)	1.64% (143)	1.41% (107)	1.11% (53)	1.37% (183)
3. Difference (%)	-18.75%	3.43%	-2.12%	-5.77%	-9.91%	-7.37%
4. Positive Black Swans (Seg. Returns)	0.71% (47)	0.74% (45)	0.93% (81)	0.79% (60)	0.60% (29)	0.71% (95)
5. Positive Black Swans (Seg. Diff. Abs. returns)	0.98% (61)	0.77% (47)	0.90% (78)	0.80% (61)	0.58% (28)	0.69% (93)
6. Difference (%)	-19.89%	-6.60%	-27.9%	-39.73%	-38.8%	-21.5%
7. Negative Black Swans (Seg. Returns)	0.76% (50)	0.72% (44)	0.68% (59)	0.54% (41)	0.40% (19)	0.56% (75)
8. Negative Black Swans (Seg. Diff. Abs. returns)	0.90% (56)	0.64% (39)	0.75% (65)	0.61% (46)	0.52% (25)	0.67% (90)
9. Difference (%)	-17.52%	14.31%	22.01%	26.57%	14.84%	5.41%
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
1. Total Black Swans (Seg. Returns)	1.43% (154)	0.99% (45)	1.46% (108)	1.24% (108)	0.87% (35)	1.49% (114)
2. Total Black Swans (Seg. Diff. Abs. returns)	1.32% (142)	0.93% (42)	1.38% (102)	1.29% (112)	1.12% (45)	1.37% (105)
3. Difference (%)	8.11%	6.90%	5.72%	-3.64%	-25.1%	8.22%
4. Positive Black Swans (Seg. Returns)	0.69% (75)	0.64% (29)	0.72% (53)	0.79% (69)	0.50% (20)	0.97% (74)
5. Positive Black Swans (Seg. Diff. Abs. returns)	0.73% (79)	0.46% (21)	0.76% (56)	0.72% (63)	0.57% (23)	0.69% (53)
6. Difference (%)	0.05%	-27.19%	-1.80%	-47.96%	-42.7%	-28%
7. Negative Black Swans (Seg. Returns)	0.73% (79)	0.35% (16)	0.74% (55)	0.45% (39)	0.37% (15)	0.52% (40)
8. Negative Black Swans (Seg. Diff. Abs. returns)	0.58% (63)	0.46% (21)	0.62% (46)	0.56% (49)	0.55% (22)	0.68% (52)
9. Difference (%)	17.44%	32.28%	14.17%	34.23%	-9.53%	35.28%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1. Total Black Swans (Seg. Returns)	1.15% (66)	1.25% (76)	2.01% (119)	0.76% (18)	1.35% (100)	1.19% (91)
2. Total Black Swans (Seg. Diff. Abs. returns)	1.20% (69)	1.48% (90)	1.79% (106)	0.63% (15)	1.54% (114)	1.29% (99)
3. Difference (%)	-4.45%	-16.91%	11.57%	18.23%	-13.1%	-8.43%
4. Positive Black Swans (Seg. Returns)	0.71% (41)	0.79% (48)	1.12% (66)	0.46% (11)	0.82% (61)	0.61% (47)
5. Positive Black Swans (Seg. Diff. Abs. returns)	0.68% (39)	0.70% (43)	0.96% (57)	0.29% (7)	0.77% (57)	0.69% (53)

6. Difference (%)	-44.47%	-42.90%	-7.28%	0.00%	-37.9%	-18.6%
7. Negative Black Swans (Seg. Returns)	0.43% (25)	0.46% (28)	0.90% (53)	0.29% (7)	0.53% (39)	0.57% (44)
8. Negative Black Swans (Seg. Diff. Abs. returns)	0.52% (30)	0.77% (47)	0.83% (49)	0.34% (8)	0.77% (57)	0.60% (46)
9. Difference (%)	31.24%	2.11%	29.78%	31.85%	6.78%	2.15%
	Switzerland	**Turkey**	**UK**	**USA**		
1. Total Black Swans (Seg. Returns)	1.45% (102)	1.22% (87)	1.42% (138)	1.19% (159)		
2. Total Black Swans (Seg. Diff. Abs. returns)	1.48% (104)	1.44% (103)	1.58% (154)	1.22% (163)		
3. Difference (%)	-1.94%	-16.88%	-10.9%	-2.48%		
4. Positive Black Swans (Seg. Returns)	0.91% (64)	0.67% (48)	0.67% (65)	0.59% (79)		
5. Positive Black Swans (Seg. Diff. Abs. returns)	0.74% (52)	0.76% (54)	0.74% (72)	0.64% (86)		
6. Difference (%)	-31.37%	-32.54%	1.38%	-7.23%		
7. Negative Black Swans (Seg. Returns)	0.54% (38)	0.55% (39)	0.75% (73)	0.60% (80)		
8. Negative Black Swans (Seg. Diff. Abs. returns)	0.74% (52)	0.69% (49)	0.84% (82)	0.57% (77)		
9. Difference (%)	20.76%	-2.06%	-23.2%	2.56%		

Appendix 26

Summary of the innovation in extreme variance between returns and squared returns of the daily stock exchange market

	Australia	Austria	Belgium	Canada	Chile	Czech
1. Total Black Swans (Seg. Returns)	0.78% (49)	1.58% (125)	1.32% (125)	1.22% (109)	1.16% (80)	1.37% (79)
2. Total Black Swans (Seg. Diff. Sqd returns)	1.76% (110)	1.99% (158)	1.36% (129)	1.38% (123)	1.58% (109)	1.61% (93)
3. Difference (%)	-80.98%	-23.43%	-3.15%	-12.16%	-30.9%	-16%
4. Positive Black Swans (Seg. Returns)	0.54% (34)	0.95% (75)	0.68% (65)	0.69% (62)	0.58% (40)	0.85% (49)
5. Positive Black Swans (Seg. Diff. Sqd returns)	0.93% (58)	1.02% (81)	0.68% (65)	0.72% (64)	0.76% (52)	0.81% (47)
6. Difference (%)	-53.52%	-7.70%	0.00%	-3.25%	-26.2%	4.17%
7. Negative Black Swans (Seg. Returns)	0.24% (15)	0.63% (50)	0.63% (60)	0.53% (47)	0.58% (40)	0.52% (30)
8. Negative Black Swans (Seg. Diff. Sqd returns)	0.83% (52)	0.97% (77)	0.67% (64)	0.66% (59)	0.83% (57)	0.80% (46)
9. Difference (%)	-124.43%	-43.18%	-6.45%	-22.82%	-35.4%	-43%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Total Black Swans (Seg. Returns)	1.23% (85)	1.71% (89)	1.11% (85)	1.17% (88)	1.04% (140)	1.51% (109)
2. Total Black Swans (Seg. Diff. Sqd returns)	2.14% (148)	1.31% (68)	1.51% (116)	2.35% (177)	1.60% (214)	2.01% (145)
3. Difference (%)	-55.46%	26.91%	-31.1%	-69.88%	-42.4%	-29%
4. Positive Black Swans (Seg. Returns)	0.74% (51)	0.83% (43)	0.57% (44)	0.65% (49)	0.54% (72)	0.76% (55)
5. Positive Black Swans (Seg. Diff. Sqd returns)	1.07% (74)	0.63% (33)	0.78% (60)	1.21% (91)	0.76% (102)	1.03% (74)
6. Difference (%)	-37.22%	26.47%	-31%	-61.90%	-34.8%	-30%
7. Negative Black Swans (Seg. Returns)	0.49% (34)	0.88% (46)	0.53% (41)	0.52% (39)	0.51% (68)	0.75% (54)
8. Negative Black Swans (Seg. Diff. Sqd returns)	1.07% (74)	0.67% (35)	0.73% (56)	1.14% (86)	0.84% (112)	0.98% (71)
9. Difference (%)	-77.77%	27.33%	-31.2%	-79.08%	-49.9%	-27%
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
1. Total Black Swans (Seg. Returns)	1.47% (97)	1.46% (89)	1.61% (140)	1.33% (101)	1.00% (48)	1.27% (170)
2. Total Black Swans (Seg. Diff. Sqd returns)	1.75% (116)	1.44% (88)	1.94% (169)	1.78% (135)	2.04% (98)	1.16% (155)
3. Difference (%)	-17.89%	1.13%	-18.8%	-29.02%	-71.4%	9.24%
4. Positive Black Swans (Seg. Returns)	0.71% (47)	0.74% (45)	0.93% (81)	0.79% (60)	0.60% (29)	0.71% (95)
5. Positive Black Swans (Seg. Diff. Sqd returns)	0.88% (58)	0.75% (46)	0.99% (86)	0.90% (68)	1.00% (48)	0.57% (77)
6. Difference (%)	-21.03%	-2.20%	-5.99%	-12.52%	-50.4%	21%
7. Negative Black Swans (Seg. Returns)	0.76% (50)	0.72% (44)	0.68% (59)	0.54% (41)	0.40% (19)	0.56% (75)
8. Negative Black Swans (Seg. Diff. Sqd returns)	0.88% (58)	0.69% (42)	0.95% (83)	0.88% (67)	1.04% (50)	0.58% (78)
9. Difference (%)	-14.84%	4.65%	-34.1%	-49.11%	-96.8%	-3.9%
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
1. Total Black Swans (Seg. Returns)	1.43% (154)	0.99% (45)	1.46% (108)	1.24% (108)	0.87% (35)	1.49% (114)

2. Total Black Swans (Seg. Diff. Sqd returns)	1.75% (189)	1.21% (55)	1.99% (147)	2.02% (176)	1.25% (50)	2.17% (166)
3. Difference (%)	-20.48%	-20.07%	-30.8%	-48.84%	-35.7%	-38%
4. Positive Black Swans (Seg. Returns)	0.69% (75)	0.64% (29)	0.72% (53)	0.79% (69)	0.50% (20)	0.97% (74)
5. Positive Black Swans (Seg. Diff. Sqd returns)	0.89% (96)	0.57% (26)	1.03% (76)	1.00% (87)	0.62% (25)	1.04% (80)
6. Difference (%)	-24.69%	10.92%	-36%	-23.18%	-22.3%	-7.8%
7. Negative Black Swans (Seg. Returns)	0.73% (79)	0.35% (16)	0.74% (55)	0.45% (39)	0.37% (15)	0.52% (40)
8. Negative Black Swans (Seg. Diff. Sqd returns)	0.86% (93)	0.64% (29)	0.96% (71)	1.02% (89)	0.62% (25)	1.12% (86)
9. Difference (%)	-16.32%	-59.47%	-25.5%	-82.51%	-51.1%	-77%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1. Total Black Swans (Seg. Returns)	1.15% (66)	1.25% (76)	2.01% (119)	0.76% (18)	1.35% (100)	1.19% (91)
2. Total Black Swans (Seg. Diff. Sqd returns)	2.01% (116)	1.62% (99)	1.72% (102)	0.71% (17)	2.09% (155)	2.15% (165)
3. Difference (%)	-56.39%	-26.44%	15.4%	5.72%	-43.8%	-60%
4. Positive Black Swans (Seg. Returns)	0.71% (41)	0.79% (48)	1.12% (66)	0.46% (11)	0.82% (61)	0.61% (47)
5. Positive Black Swans (Seg. Diff. Sqd returns)	1.08% (62)	0.82% (50)	0.86% (51)	0.38 %(9)	1.05% (78)	1.07% (82)
6. Difference (%)	-41.36%	-4.08%	25.8%	20.07%	-24.5%	-56%
7. Negative Black Swans (Seg. Returns)	0.43% (25)	0.46% (28)	0.90% (53)	0.29% (7)	0.53% (39)	0.57% (44)
8. Negative Black Swans (Seg. Diff. Sqd returns)	0.94% (54)	0.80% (49)	0.86% (51)	0.34% (8)	1.04% (77)	1.08% (83)
9. Difference (%)	-77.01%	-55.96%	3.85%	-13.35%	-68%	-64%
	Switzerland	**Turkey**	**UK**	**USA**		
1. Total Black Swans (Seg. Returns)	1.45% (102)	1.22% (87)	1.42% (138)	1.19% (159)		
2. Total Black Swans (Seg. Diff. Sqd returns)	2.25% (158)	1.83% (131)	1.62% (158)	1.03% (138)		
3. Difference (%)	-43.76%	-40.93%	-13.5%	14.17%		
4. Positive Black Swans (Seg. Returns)	0.91% (64)	0.67% (48)	0.67% (65)	0.59% (79)		
5. Positive Black Swans (Seg. Diff. Sqd returns)	1.13% (79)	0.94% (67)	0.78% (76)	0.48% (64)		
6. Difference (%)	-21.06%	-33.35%	-15.6%	21.06%		
7. Negative Black Swans (Seg. Returns)	0.54% (38)	0.55% (39)	0.75% (73)	0.60% (80)		
8. Negative Black Swans (Seg. Diff. Sqd returns)	1.13% (79)	0.90% (64)	0.84% (82)	0.55% (74)		
9. Difference (%)	-73.19%	-49.53%	-11.6%	7.80%		

Appendix 27

Summary of the innovation in extreme variance between returns and log squared returns of the daily stock exchange market

	Australia	Austria	Belgium	Canada	Chile	Czech
1. Total Black Swans (Seg. Returns)	0.78% (49)	1.58% (125)	1.32% (125)	1.22% (109)	1.16% (80)	1.37% (79)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.80% (47)	1.03% (74)	1.20% (105)	0.96% (80)	1.02% (64)	0.75% (39)
3. Difference (%)	-1.51%	42.69%	9.06%	23.55%	13.5%	60.8%
4. Positive Black Swans (Seg. Returns)	0.54% (34)	0.95% (75)	0.68% (65)	0.69% (62)	0.58% (40)	0.85% (49)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.41% (24)	0.51% (37)	0.61% (53)	0.46% (38)	0.52% (33)	0.38% (20)
6. Difference (%)	29.16%	60.92%	12.03%	41.57%	10.37%	79.80%
7. Negative Black Swans (Seg. Returns)	0.24% (15)	0.63% (50)	0.63% (60)	0.53% (47)	0.58% (40)	0.52% (30)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.39% (23)	0.51% (37)	0.60% (52)	0.51% (42)	0.49% (31)	0.36% (19)
9. Difference (%)	-48.42%	20.37%	5.93%	3.86%	16.62%	35.87%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Total Black Swans (Seg. Returns)	1.23% (85)	1.71% (89)	1.11% (85)	1.17% (88)	1.04% (140)	1.51% (109)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	1.00% (64)	1.03% (50)	1.37% (98)	1.16% (83)	1.05% (131)	1.12% (73)
3. Difference (%)	20.92%	51.09%	-20.9%	0.41%	-0.07%	29.8%
4. Positive Black Swans (Seg. Returns)	0.74% (51)	0.83% (43)	0.57% (44)	0.65% (49)	0.54% (72)	0.76% (55)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.50% (32)	0.53% (26)	0.73% (52)	0.62% (44)	0.54% (68)	0.57% (37)
6. Difference (%)	39.15%	43.74%	-23.4%	5.32%	-1.00%	29.3%
7. Negative Black Swans (Seg. Returns)	0.49% (34)	0.88% (46)	0.53% (41)	0.52% (39)	0.51% (68)	0.75% (54)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.50% (32)	0.49% (24)	0.64% (46)	0.55% (39)	0.50% (63)	0.55% (36)
9. Difference (%)	-1.39%	58.49%	-18.2%	-5.44%	0.92%	30.2%
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
1. Total Black Swans (Seg. Returns)	1.47% (97)	1.46% (89)	1.61% (140)	1.33% (101)	1.00% (48)	1.27% (170)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	1.00% (61)	1.15% (62)	1.13% (81)	1.03% (70)	0.90% (41)	0.99% (120)
3. Difference (%)	38.63%	23.63%	35.6%	25.27%	11.1%	24.8%
4. Positive Black Swans (Seg. Returns)	0.71% (47)	0.74% (45)	0.93% (81)	0.79% (60)	0.60% (29)	0.71% (95)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.47% (29)	0.61% (33)	0.58% (42)	0.44% (30)	0.46% (21)	0.42% (51)
6. Difference (%)	40.53%	18.49%	46.6%	57.92%	27.6%	52.2%
7. Negative Black Swans (Seg. Returns)	0.76% (50)	0.72% (44)	0.68% (59)	0.54% (41)	0.40% (19)	0.56% (75)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.52% (32)	0.54% (29)	0.54% (39)	0.59% (40)	0.44% (20)	0.57% (69)
9. Difference (%)	36.87%	29.17%	22.3%	-8.93%	-9.85%	-1.66%
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
1. Total Black Swans (Seg. Returns)	1.43% (154)	0.99% (45)	1.46% (108)	1.24% (108)	0.87% (35)	1.49% (114)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.86% (84)	0.78% (33)	1.10% (75)	0.94% (77)	1.28% (48)	0.89% (64)

3. Difference (%)	50.15%	24.40%	28.4%	27.44%	-38.0%	50.9%
4. Positive Black Swans (Seg. Returns)	0.69% (75)	0.64% (29)	0.72% (53)	0.79% (69)	0.50% (20)	0.97% (74)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.40% (39)	0.33% (14)	0.48% (33)	0.49% (40)	0.66% (25)	0.43% (31)
6. Difference (%)	54.93%	66.21%	39.3%	48.13%	-28.7%	80.2%
7. Negative Black Swans (Seg. Returns)	0.73% (79)	0.35% (16)	0.74% (55)	0.45% (39)	0.37% (15)	0.52% (40)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.46% (45)	0.45% (19)	0.62% (42)	0.45% (37)	0.61% (23)	0.46% (33)
9. Difference (%)	45.81%	-23.80%	18.9%	-1.13%	-49.2%	12.4%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1. Total Black Swans (Seg. Returns)	1.15% (66)	1.25% (76)	2.01% (119)	0.76% (18)	1.35% (100)	1.19% (91)
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	0.83% (44)	1.15% (66)	0.49% (22)	1.51% (33)	0.91% (63)	1.14% (81)
3. Difference (%)	31.93%	8.06%	140%	-68.93%	39.6%	4.47%
4. Positive Black Swans (Seg. Returns)	0.71% (41)	0.79% (48)	1.12% (66)	0.46% (11)	0.82% (61)	0.61% (47)
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.44% (23)	0.56% (32)	0.22% (10)	0.87% (19)	0.40% (28)	0.62% (44)
6. Difference (%)	49.19%	34.50%	160%	-62.97%	71.3%	-0.57%
7. Negative Black Swans (Seg. Returns)	0.43% (25)	0.46% (28)	0.90% (53)	0.29% (7)	0.53% (39)	0.57% (44)
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.40% (21)	0.59% (34)	0.27% (12)	0.64% (14)	0.50% (35)	0.52% (37)
9. Difference (%)	8.81%	-25.46%	120%	-77.63%	4.29%	10.7%
	Switzerland	**Turkey**	**UK**	**USA**		
1. Total Black Swans (Seg. Returns)	1.45% (102)	1.22% (87)	1.42% (138)	1.19% (159)		
2. Total Black Swans (Seg. Diff. Log Sqd. returns)	1.17% (77)	0.92% (61)	0.93% (72)	0.90% (111)		
3. Difference (%)	21.68%	28.03%	42%	28.06%		
4. Positive Black Swans (Seg. Returns)	0.91% (64)	0.67% (48)	0.67% (65)	0.59% (79)		
5. Positive Black Swans (Seg. Diff. Log Sqd. returns)	0.47% (31)	0.50% (33)	0.43% (33)	0.44% (55)		
6. Difference (%)	66.06%	30.00%	44.8%	28.33%		
7. Negative Black Swans (Seg. Returns)	0.54% (38)	0.55% (39)	0.75% (73)	0.60% (80)		
8. Negative Black Swans (Seg. Diff. Log Sqd. returns)	0.70% (46)	0.42% (28)	0.50% (39)	0.45% (56)		
9. Difference (%)	-25.54%	25.67%	39.7%	27.79%		

Appendix 28

Summary of the innovation in extreme variance between returns and range of the daily stock exchange market

	Australia	Austria	Belgium	Canada	Chile	Czech Republic
1. Total Black Swans (Seg. Returns)	0.78% (49)	1.58% (125)	1.32% (125)	1.21% (189)	1.16% (82)	1.37% (79)
2. Total Black Swans (Seg. Diff. Range)	1.61% (64)	1.59% (93)	2.00% (117)	1.26% (93)	0.82% (19)	1.60% (63)
3. Difference (%)	-71.89%	-0.74%	-42%	-3.22%	35.1%	-15.32%
4. Positive Black Swans (Seg. Returns)	0.24% (15)	0.63% (51)	0.63% (62)	0.53% (47)	0.58% (41)	0.52% (30)
5. Positive Black Swans (Seg. Diff. Range)	0.85% (34)	0.77% (45)	1.13% (66)	0.62% (46)	0.43% (11)	0.84% (33)
6. Difference (%)	-127%	-19.78%	-58%	-16.9%	29.9%	-47.49%
7. Negative Black Swans (Seg. Returns)	0.54% (34)	0.95% (75)	0.68% (65)	0.69% (62)	0.58% (41)	0.85% (49)
8. Negative Black Swans (Seg. Diff. Range)	0.75% (34)	0.82% (48)	0.87% (51)	0.64% (47)	0.39% (9)	0.76% (30)
9. Difference (%)	-32.67%	14.31%	-24%	8.60%	40.5%	11.11%
	Denmark	**Finland**	**France**	**Germany**	**Greece**	**Iceland**
1. Total Black Swans (Seg. Returns)	1.23% (85)	1.11% (85)	1.17% (88)	1.04% (144)	1.51% (129)	1.46% (89)
2. Total Black Swans (Seg. Diff. Range)	1.42% (79)	1.61% (93)	2.27% (162)	1.68% (114)	1.99% (154)	2.65% (9)
3. Difference (%)	-14.42%	-37.21%	-66%	-47.5%	-28%	-59.57%
4. Positive Black Swans (Seg. Returns)	0.49% (34)	0.53% (41)	0.52% (39)	0.51% (68)	0.75% (54)	0.72% (44)
5. Positive Black Swans (Seg. Diff. Range)	0.67% (37)	0.81% (47)	1.18% (84)	0.85% (58)	0.98% (51)	1.47% (5)
6. Difference (%)	-30.20%	-41.87%	-82%	-52%	-27%	-71.23%
7. Negative Black Swans (Seg. Returns)	0.74% (51)	0.57% (44)	0.65% (49)	0.54% (72)	0.76% (55)	0.74% (45)
8. Negative Black Swans (Seg. Diff. Range)	0.76% (42)	0.80% (46)	1.09% (78)	0.82% (56)	1.02% (53)	1.18% (4)
9. Difference (%)	-2.33%	-32.66%	-52%	-42.9%	-29%	-46.67%
	Hungary	**Ireland**	**Israel**	**Italy**	**Japan**	**Luxembourg**
1. Total Black Swans (Seg. Returns)	1.47% (97)	1.60% (148)	1.33% (121)	1.00% (48)	1.26% (178)	0.99% (45)
2. Total Black Swans (Seg. Diff. Range)	1.32% (61)	1.61% (65)	2.59% (37)	2.22% (71)	1.36% (176)	1.85% (44)
3. Difference (%)	10.59%	-0.20%	-66%	-79.7%	-7.6%	-62.18%
4. Positive Black Swans (Seg. Returns)	0.75% (55)	0.68% (59)	0.54% (41)	0.40% (19)	0.56% (75)	0.35% (16)
5. Positive Black Swans (Seg. Diff. Range)	0.69% (32)	0.79% (32)	1.61% (23)	1.25% (41)	0.66% (48)	1.09% (26)
6. Difference (%)	8.84%	-15.75%	-109%	-115%	-16%	-112.98%
7. Negative Black Swans (Seg. Returns)	0.71% (47)	0.93% (81)	0.79% (61)	0.60% (29)	0.71% (95)	0.64% (29)
8. Negative Black Swans (Seg. Diff. Range)	0.63% (29)	0.82% (33)	0.98% (14)	0.97% (31)	0.71% (52)	0.76% (18)
9. Difference (%)	12.49%	12.87%	-21%	-47.2%	-0.4%	-16.74%
	Mexico	**Netherlands**	**Norway**	**Portugal**	**Korea**	**Spain**
1. Total Black Swans (Seg. Returns)	1.45% (198)	1.24% (108)	1.49% (114)	1.25% (76)	1.43% (154)	1.35% (112)
2. Total Black Swans (Seg. Diff. Range)	1.69% (116)	1.82% (118)	2.46% (61)	1.94% (86)	1.78% (92)	1.71% (178)
3. Difference (%)	-14.84%	-38.39%	-50%	-44.6%	-22%	-23.96%

4. Positive Black Swans (Seg. Returns)	0.74% (55)	0.45% (39)	0.52% (42)	0.46% (28)	0.73% (79)	0.53% (39)
5. Positive Black Swans (Seg. Diff. Range)	0.83% (57)	0.97% (63)	1.31% (32)	1.12% (46)	0.91% (47)	0.92% (58)
6. Difference (%)	-11.26%	-77.49%	-92%	-89.1%	-22%	-55.95%
7. Negative Black Swans (Seg. Returns)	0.72% (53)	0.79% (69)	0.97% (74)	0.79% (48)	0.69% (75)	0.82% (61)
8. Negative Black Swans (Seg. Diff. Range)	0.86% (59)	0.85% (55)	1.15% (28)	0.83% (34)	0.87% (45)	0.79% (55)
9. Difference (%)	-18.42%	-6.86%	-17%	-4.98%	-23%	3.62%
	Sweden	**Switzerland**	**Turkey**	**U.K.**	**U.S.A.**	
1. Total Black Swans (Seg. Returns)	1.19% (91)	1.45% (142)	1.22% (87)	1.42% (138)	1.19% (159)	
2. Total Black Swans (Seg. Diff. Range)	1.88% (114)	1.42% (89)	2.00% (49)	1.38% (147)	1.32% (191)	
3. Difference (%)	-45.86%	2.24%	-50%	2.44%	-11%	
4. Positive Black Swans (Seg. Returns)	0.57% (44)	0.54% (38)	0.55% (39)	0.75% (73)	0.59% (87)	
5. Positive Black Swans (Seg. Diff. Range)	0.87% (53)	0.70% (44)	0.98% (24)	0.74% (57)	0.68% (52)	
6. Difference (%)	-41.94%	-26.05%	-59%	1.74%	-14%	
7. Negative Black Swans (Seg. Returns)	0.61% (47)	0.91% (64)	0.67% (48)	0.67% (65)	0.59% (79)	
8. Negative Black Swans (Seg. Diff. Range)	1.01% (61)	0.72% (45)	1.02% (25)	0.64% (57)	0.64% (49)	
9. Difference (%)	-49.40%	23.83%	-42%	3.24%	-8.9%	

Appendix 29

Summary of the innovation in extreme variance between residuals of returns and differenced absolute returns of the daily stock exchange market from best fit ARMA-APARCH model

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech**
1. Total Black Swans (Seg. Residuals)	0.50% (31)	0.95% (75)	1.01% (96)	0.88% (79)	0.81% (56)	0.88% (51)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.02% (64)	1.43% (113)	1.42% (135)	1.25% (112)	1.08% (74)	1.56% (90)
3. Difference (%)	-72.60%	-41.05%	-34.7%	-34.98%	-27.9%	-56.89%
4. Positive Black Swans (Seg. Residuals)	0.10% (6)	0.30% (24)	0.34% (32)	0.23% (21)	0.35% (24)	0.28% (16)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.02% (64)	1.41% (112)	1.42% (135)	1.25% (112)	1.08% (74)	1.51% (87)
6. Difference (%)	-236.8%	-154.11%	-144%	-167.48%	-113%	-169.4%
7. Negative Black Swans (Seg. Residuals)	0.40% (25)	0.64% (51)	0.67% (64)	0.65% (58)	0.46% (32)	0.61% (35)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.01% (1)	0.01% (1)	0.01% (1)	0.01% (1)	0.05% (3)
9. Difference (%)	321.78%	393.12%	415.8%	405.97%	346.5%	245.59%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Total Black Swans (Seg. Residuals)	0.77% (53)	1.59% (83)	1.02% (78)	0.64% (48)	0.74% (99)	1.19% (86)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.25% (86)	1.61% (84)	1.08% (83)	1.05% (79)	1.04% (140)	1.39% (100)
3. Difference (%)	-48.46%	-1.26%	-6.30%	-49.94%	-34.7%	-15.14%
4. Positive Black Swans (Seg. Residuals)	0.32% (22)	0.86% (45)	0.42% (32)	0.19% (14)	0.31% (42)	0.57% (41)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.25% (86)	1.56% (81)	1.06% (81)	1.05% (79)	1.04% (139)	1.39% (100)
6. Difference (%)	-136.4%	-58.84%	-92.9%	-173.16%	-120%	-89.22%
7. Negative Black Swans (Seg. Residuals)	0.45% (31)	0.73% (38)	0.60% (46)	0.45% (34)	0.43% (57)	0.62% (45)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.01% (1)	0.06% (3)	0.03% (2)	0.01% (1)	0.01% (1)	0.01% (1)
9. Difference (%)	343.34%	253.84%	313.5%	352.52%	404.3%	380.61%
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
1. Total Black Swans (Seg. Residuals)	1.07% (71)	1.26% (77)	1.27% (111)	0.94% (71)	0.54% (26)	0.86% (115)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.42% (94)	1.48% (90)	1.38% (120)	1.32% (100)	1.04% (50)	1.28% (172)
3. Difference (%)	-28.11%	-15.68%	-7.85%	-34.34%	-65.5%	-40.29%
4. Positive Black Swans (Seg. Residuals)	0.42% (28)	0.64% (39)	0.52% (45)	0.36% (27)	0.15% (7)	0.35% (47)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.41% (93)	1.48% (90)	1.38% (120)	1.32% (100)	1.04% (50)	1.27% (170)
6. Difference (%)	-120.1%	-83.71%	-98.1%	-131.03%	-197%	-128.6%
7. Negative Black Swans (Seg. Residuals)	0.65% (43)	0.62% (38)	0.76% (66)	0.58% (44)	0.40% (19)	0.51% (68)

8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.02% (1)	0.01% (1)	0.01% (1)	0.23% (11)	0.01% (2)
9. Difference (%)	376.07%	363.68%	418.9%	378.33%	54.53%	352.60%
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
1. Total Black Swans (Seg. Residuals)	0.80% (86)	0.86% (39)	0.86% (64)	0.67% (58)	0.57% (23)	0.72% (55)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.26% (136)	0.71% (32)	1.43% (106)	1.15% (100)	1.16% (41)	1.15% (88)
3. Difference (%)	-45.89%	19.67%	-50.6%	-54.59%	-70.7%	-47.09%
4. Positive Black Swans (Seg. Residuals)	0.37% (40)	0.33% (15)	0.32% (24)	0.17% (15)	0.20% (8)	0.26% (20)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.24% (134)	0.64% (29)	1.41% (104)	1.14% (99)	1.16% (41)	1.15% (88)
6. Difference (%)	-120.9%	-66.03%	-147%	-188.82%	-176%	-148.3%
7. Negative Black Swans (Seg. Residuals)	0.43% (46)	0.53% (24)	0.54% (40)	0.49% (43)	0.37% (15)	0.46% (35)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (2)	0.07% (3)	0.03% (2)	0.01% (1)	0.03% (1)	0.01% (1)
9. Difference (%)	313.49%	207.83%	299.4%	376.01%	257.9%	355.44%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1. Total Black Swans (Seg. Residuals)	0.62% (36)	0.79% (48)	1.88% (111)	0.92% (22)	0.73% (54)	0.87% (67)
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.30% (75)	1.33% (81)	1.91% (113)	0.46% (11)	1.15% (85)	1.25% (96)
3. Difference (%)	-73.48%	-52.37%	-1.87%	69.19%	-45.4%	-36.06%
4. Positive Black Swans (Seg. Residuals)	0.23% (13)	0.36% (22)	0.76% (45)	0.38% (9)	0.18% (13)	0.35% (27)
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.29% (74)	1.31% (80)	1.91% (113)	0.46% (11)	1.15% (85)	1.25% (96)
6. Difference (%)	-174%	-129.15%	-92.2%	-20.19%	-188%	-126.9%
7. Negative Black Swans (Seg. Residuals)	0.40% (23)	0.43% (26)	1.12% (66)	0.55% (13)	0.55% (41)	0.52% (40)
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.02% (1)	0.02% (1)	0.02% (1)	0.04% (1)	0.01% (1)	0.01% (1)
9. Difference (%)	313.46%	325.76%	418.9%	256.37%	371.3%	368.80%
	Switzerland	**Turkey**	**UK**	**USA**		
1. Total Black Swans (Seg. Residuals)	0.71% (50)	1.02% (73)	0.99% (96)	0.70% (94)		
2. Total Black Swans (Seg. Diff. Abs. residuals)	1.30% (91)	1.33% (95)	1.35% (131)	1.05% (141)		
3. Difference (%)	-60.00%	-26.40%	-31.2%	-40.58%		
4. Positive Black Swans (Seg. Residuals)	0.20% (14)	0.43% (31)	0.50% (49)	0.24% (32)		
5. Positive Black Swans (Seg. Diff. Abs. residuals)	1.28% (90)	1.33% (95)	1.35% (131)	1.04% (139)		
6. Difference (%)	-186.2%	-112.05%	-98.4%	-146.95%		
7. Negative Black Swans (Seg. Residuals)	0.51% (36)	0.59% (42)	0.48% (47)	0.46% (62)		
8. Negative Black Swans (Seg. Diff. Abs. residuals)	0.01% (1)	0.01% (1)	0.01% (1)	0.01% (2)		
9. Difference (%)	358.24%	373.70%	385%	343.32%		

Appendix 30

Summary of the innovation in extreme variance between residuals of returns and differenced squared returns of the daily stock exchange market from best fit ARMA-APARCH model

	Australia	Austria	Belgium	Canada	Chile	Czech
1. Total Black Swans (Seg. Residuals)	0.50% (31)	0.95% (75)	1.01% (96)	0.88% (79)	0.81% (56)	0.88% (51)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.57% (98)	1.49% (118)	1.23% (117)	1.16% (104)	1.61% (111)	1.70% (98)
3. Difference (%)	-115.21%	-45.38%	-19.85%	-27.57%	-68.51%	-65.40%
4. Positive Black Swans (Seg. Residuals)	0.10% (6)	0.30% (24)	0.34% (32)	0.23% (21)	0.35% (24)	0.28% (16)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.57% (98)	1.41% (112)	1.23% (117)	1.16% (104)	1.47% (101)	1.68% (97)
6. Difference (%)	-279.43%	-154.11%	- 129.71%	-160.07%	-143.79%	- 180.30%
7. Negative Black Swans (Seg. Residuals)	0.40% (25)	0.64% (51)	0.67% (64)	0.65% (58)	0.46% (32)	0.61% (35)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.02% (1)	0.08% (6)	0.01% (1)	0.01% (1)	0.15% (10)	0.02% (1)
9. Difference (%)	321.78%	213.94%	415.83%	405.97%	116.23%	355.45%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Total Black Swans (Seg. Residuals)	0.77% (53)	1.59% (83)	1.02% (78)	0.64% (48)	0.74% (99)	1.19% (86)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.49% (103)	1.21% (63)	1.38% (106)	1.48% (111)	1.50% (201)	1.96% (141)
3. Difference (%)	-66.50%	27.51%	-30.76%	-83.95%	-70.87%	-49.50%
4. Positive Black Swans (Seg. Residuals)	0.32% (22)	0.86% (45)	0.42% (32)	0.19% (14)	0.31% (42)	0.57% (41)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.49% (103)	1.13% (59)	1.38% (106)	1.48% (111)	1.49% (200)	1.94% (140)
6. Difference (%)	-154.43%	-27.15%	- 119.86%	-207.17%	-156.12%	- 122.86%
7. Negative Black Swans (Seg. Residuals)	0.45% (31)	0.73% (38)	0.60% (46)	0.45% (34)	0.43% (57)	0.62% (45)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.01% (1)	0.08% (4)	0.01% (1)	0.01% (1)	0.01% (1)	0.01% (1)
9. Difference (%)	343.34%	225.07%	382.77%	352.52%	404.25%	380.61%
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
1. Total Black Swans (Seg. Residuals)	1.07% (71)	1.26% (77)	1.27% (111)	0.94% (71)	0.54% (26)	0.86% (115)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.45% (96)	0.98% (30)	1.23% (107)	1.41% (107)	2.13% (102)	1.12% (150)
3. Difference (%)	-30.21%	25.47%	3.61%	-41.11%	-136.81%	-26.61%
4. Positive Black Swans (Seg. Residuals)	0.42% (28)	0.64% (39)	0.52% (45)	0.36% (27)	0.15% (7)	0.35% (47)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.44% (95)	0.95% (29)	1.21% (105)	1.41% (107)	2.13% (102)	1.08% (145)
6. Difference (%)	-122.21%	-39.16%	-84.79%	-137.79%	-268.03%	- 112.70%

7. Negative Black Swans (Seg. Residuals)	0.65% (43)	0.62% (38)	0.76% (66)	0.58% (44)	0.40% (19)	0.51% (68)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.02% (1)	0.03% (1)	0.02% (2)	0.01% (1)	0.02% (1)	0.04% (5)
9. Difference (%)	376.07%	294.97%	349.59%	378.33%	294.32%	260.97%
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
1. Total Black Swans (Seg. Residuals)	0.80% (86)	0.86% (39)	0.86% (64)	0.67% (58)	0.57% (23)	0.72% (55)
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.26% (136)	1.72% (78)	1.99% (147)	1.59% (138)	1.27% (51)	1.74% (133)
3. Difference (%)	-45.89%	-69.43%	-83.30%	-86.80%	-79.73%	-88.39%
4. Positive Black Swans (Seg. Residuals)	0.37% (40)	0.33% (15)	0.32% (24)	0.17% (15)	0.20% (8)	0.26% (20)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.26% (136)	1.70% (77)	1.75% (129)	1.53% (133)	1.27% (51)	1.61% (123)
6. Difference (%)	-122.43%	-163.69%	-168.32%	-218.34%	-185.34%	-181.74%
7. Negative Black Swans (Seg. Residuals)	0.43% (46)	0.53% (24)	0.54% (40)	0.49% (43)	0.37% (15)	0.46% (35)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.01% (1)	0.02% (1)	0.24% (18)	0.06% (5)	0.02% (1)	0.13% (10)
9. Difference (%)	382.81%	317.70%	79.70%	215.06%	270.71%	125.18%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1. Total Black Swans (Seg. Residuals)	0.62% (36)	0.79% (48)	1.88% (111)	0.92% (22)	0.73% (54)	0.87% (67)
2. Total Black Swans (Seg. Diff. Sqd residuals)	2.26% (130)	1.23% (75)	1.92% (102)	0.71% (17)	1.76% (130)	1.75% (134)
3. Difference (%)	-128.49%	-44.68%	-2.54%	25.66%	-87.90%	-69.41%
4. Positive Black Swans (Seg. Residuals)	0.23% (13)	0.36% (22)	0.76% (45)	0.38% (9)	0.18% (13)	0.35% (27)
5. Positive Black Swans (Seg. Diff. Sqd residuals)	2.22% (128)	1.23% (75)	1.92% (102)	0.71% (17)	1.69% (125)	1.74% (133)
6. Difference (%)	-228.79%	-122.69%	-92.82%	-63.72%	-226.38%	-159.54%
7. Negative Black Swans (Seg. Residuals)	0.40% (23)	0.43% (26)	1.12% (66)	0.55% (13)	0.55% (41)	0.52% (40)
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.03% (2)	0.02% (1)	0.02% (1)	0.04% (1)	0.07% (5)	0.01% (1)
9. Difference (%)	244.15%	325.76%	407.97%	256.37%	210.37%	368.80%
	Switzerland	**Turkey**	**UK**	**USA**		
1. Total Black Swans (Seg. Residuals)	0.71% (50)	1.02% (73)	0.99% (96)	0.70% (94)		
2. Total Black Swans (Seg. Diff. Sqd residuals)	1.61% (113)	1.89% (135)	1.32% (128)	1.22% (164)		
3. Difference (%)	-81.64%	-61.52%	-28.83%	-55.69%		
4. Positive Black Swans (Seg. Residuals)	0.20% (14)	0.43% (31)	0.50% (49)	0.24% (32)		
5. Positive Black Swans (Seg. Diff. Sqd residuals)	1.60% (112)	1.89% (135)	1.32% (128)	1.22% (163)		
6. Difference (%)	-208.04%	-147.17%	-96.08%	-162.84%		

7. Negative Black Swans (Seg. Residuals)	0.51% (36)	0.59% (42)	0.48% (47)	0.46% (62)		
8. Negative Black Swans (Seg. Diff. Sqd residuals)	0.01% (1)	0.01% (1)	0.01% (1)	0.01% (1)		
9. Difference (%)	358.25%	373.72%	384.95%	412.68%		

Appendix 31

Summary of the innovation in extreme variance between residuals of returns and differenced log squared returns of the daily stock exchange market from best fit ARMA-APARCH model

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech**
1. Total Black Swans (Seg. Residuals)	0.50% (31)	0.95% (75)	1.01% (96)	0.88% (79)	0.81% (56)	0.97% (51)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.07% (63)	1.20% (86)	1.25% (109)	1.05% (87)	1.16% (73)	1.09% (57)
3. Difference (%)	-76.71%	-23.49%	-21.15%	-17.11%	-35.46%	-11.22%
4. Positive Black Swans (Seg. Residuals)	0.10% (6)	0.30% (24)	0.34% (32)	0.23% (21)	0.35% (24)	0.31% (16)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.07% (4)	0.08% (6)	0.08% (7)	0.05% (4)	0.10% (6)	0.06% (3)
6. Difference (%)	34.75%	128.82%	143.53%	158.35%	129.68%	167.30%
7. Negative Black Swans (Seg. Residuals)	0.40% (25)	0.64% (51)	0.67% (64)	0.65% (58)	0.46% (32)	0.67% (35)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	1.00% (59)	1.11% (80)	1.17% (102)	1.00% (83)	1.06% (67)	1.03% (54)
9. Difference (%)	-91.66%	-54.83%	-55.06%	-43.31%	-82.84%	-43.46%
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
1. Total Black Swans (Seg. Residuals)	0.83% (53)	1.70% (83)	1.09% (78)	0.67% (48)	0.79% (99)	1.32% (86)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.03% (66)	1.03% (50)	1.19% (85)	1.24% (88)	1.30% (163)	1.23% (80)
3. Difference (%)	-22.00%	50.62%	-8.69%	-60.74%	-49.92%	7.17%
4. Positive Black Swans (Seg. Residuals)	0.34% (22)	0.92% (45)	0.45% (32)	0.20% (14)	0.34% (42)	0.63% (41)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.06% (4)	0.04% (2)	0.03% (2)	0.07% (5)	0.04% (5)	0.08% (5)
6. Difference (%)	170.41%	311.29%	277.16%	102.84%	212.77%	210.35%
7. Negative Black Swans (Seg. Residuals)	0.48% (31)	0.78% (38)	0.64% (46)	0.48% (34)	0.45% (57)	0.69% (45)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.97% (62)	0.98% (48)	1.16% (83)	1.17% (83)	1.26% (158)	1.15% (75)
9. Difference (%)	-69.38%	-23.42%	-59.12%	-89.37%	-102.01%	-51.14%
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
1. Total Black Swans (Seg. Residuals)	1.16% (71)	1.43% (77)	1.54% (111)	1.05% (71)	0.57% (26)	0.95% (115)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.24% (76)	1.12% (60)	1.22% (88)	1.05% (71)	1.16% (53)	1.17% (142)
3. Difference (%)	-6.84%	24.85%	23.15%	-0.07%	-71.35%	-21.13%
4. Positive Black Swans (Seg. Residuals)	0.46% (28)	0.72% (39)	0.63% (45)	0.40% (27)	0.15% (7)	0.39% (47)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.08% (5)	0.13% (7)	0.07% (5)	0.04% (3)	0.04% (2)	0.07% (8)
6. Difference (%)	172.24%	171.67%	219.65%	219.65%	125.15%	177.03%
7. Negative Black Swans (Seg. Residuals)	0.70% (43)	0.71% (38)	0.92% (66)	0.65% (44)	0.42% (19)	0.56% (68)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	1.16% (71)	0.99% (53)	1.16% (83)	1.01% (68)	1.12% (51)	1.11% (134)
9. Difference (%)	-50.18%	-33.36%	-22.99%	-43.61%	-98.87%	-67.87%

	Korea	Luxembourg	Mexico	Netherlands	New Zealand	Norway
1. Total Black Swans (Seg. Residuals)	0.88% (86)	0.92% (39)	0.94% (64)	0.71% (58)	0.61% (23)	0.77% (55)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.05% (102)	0.94% (40)	1.07% (73)	1.08% (88)	1.25% (47)	1.20% (86)
3. Difference (%)	-17.11%	-2.65%	-13.30%	-41.80%	-71.57%	-44.80%
4. Positive Black Swans (Seg. Residuals)	0.41% (40)	0.35% (15)	0.35% (24)	0.18% (15)	0.21% (8)	0.28% (20)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.06% (6)	0.02% (1)	0.07% (5)	0.07% (6)	0.05% (2)	0.04% (3)
6. Difference (%)	189.66%	270.69%	156.72%	91.52%	138.52%	189.61%
7. Negative Black Swans (Seg. Residuals)	0.47% (46)	0.57% (24)	0.59% (40)	0.53% (43)	0.40% (15)	0.49% (35)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.99% (96)	0.94% (40)	1.00% (68)	1.00% (82)	1.20% (45)	1.16% (83)
9. Difference (%)	-73.62%	-51.20%	-53.21%	-64.66%	-109.97%	-86.45%
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
1. Total Black Swans (Seg. Residuals)	0.58% (36)	0.61% (48)	1.17% (111)	0.25% (22)	0.78% (54)	1.28% (67)
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	0.98% (58)	1.04% (75)	0.42% (37)	0.35% (29)	1.37% (86)	1.70% (89)
3. Difference (%)	-53.49%	-54.44%	101.41%	-35.09%	-55.48%	-28.49%
4. Positive Black Swans (Seg. Residuals)	0.21% (13)	0.28% (22)	0.47% (45)	0.10% (9)	0.19% (13)	0.52% (27)
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.02% (1)	0.01% (1)	0.10% (9)	0.04% (3)	0.03% (2)	0.11% (6)
6. Difference (%)	250.70%	299.30%	152.50%	102.39%	178.23%	150.31%
7. Negative Black Swans (Seg. Residuals)	0.37% (23)	0.33% (26)	0.70% (66)	0.15% (13)	0.60% (41)	0.76% (40)
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	0.97% (57)	1.03% (74)	0.32% (28)	0.31% (26)	1.33% (84)	1.59% (83)
9. Difference (%)	-96.55%	-114.40%	77.30%	-76.78%	-80.67%	-73.09%
	Switzerland	**Turkey**	**UK**	**USA**		
1. Total Black Swans (Seg. Residuals)	0.80% (50)	0.92% (73)	1.01% (96)	1.05% (94)		
2. Total Black Swans (Seg. Diff. Log Sqd. residuals)	1.34% (79)	1.00% (72)	1.22% (106)	1.73% (144)		
3. Difference (%)	-51.54%	-8.43%	-18.36%	-50.12%		
4. Positive Black Swans (Seg. Residuals)	0.22% (14)	0.39% (31)	0.52% (49)	0.36% (32)		
5. Positive Black Swans (Seg. Diff. Log Sqd. residuals)	0.08% (5)	0.06% (4)	0.10% (9)	0.13% (11)		
6. Difference (%)	97.17%	194.96%	161.01%	99.32%		
7. Negative Black Swans (Seg. Residuals)	0.58% (36)	0.53% (42)	0.50% (47)	0.69% (62)		
8. Negative Black Swans (Seg. Diff. Log Sqd. residuals)	1.25% (74)	0.95% (68)	1.11% (97)	1.60% (133)		
9. Difference (%)	-77.85%	-57.99%	-80.90%	-83.79%		

Appendix 32

Summary of the innovation in extreme variance between residuals of returns and differenced range of the daily stock exchange market from best fit ARMA-APARCH model

	Australia	**Austria**	**Belgium**	**Canada**	**Chile**	**Czech Republic**
1. Total Black Swans (Seg. Returns)	0.50% (31)	0.95% (75)	1.01% (96)	0.88% (79)	0.81% (56)	0.88% (51)
2. Total Black Swans (Seg. Diff. Range)	1.40% (56)	1.47% (86)	1.69% (99)	1.04% (77)	0.86% (20)	1.57% (62)
3. Difference (%)	-104.13%	-43.99%	-51.29%	-16.52%	-5.73%	-57.47%
4. Positive Black Swans (Seg. Returns)	0.10% (6)	0.30% (24)	0.34% (32)	0.23% (21)	0.35% (24)	0.28% (16)
5. Positive Black Swans (Seg. Diff. Range)	1.18% (47)	1.18% (69)	1.69% (99)	0.92% (68)	0.69% (16)	1.42% (56)
6. Difference (%)	-250.83%	-135.91%	-161.2%	-136.59%	-68.14%	-163.21%
7. Negative Black Swans (Seg. Returns)	0.40% (25)	0.64% (51)	0.67% (64)	0.65% (58)	0.46% (32)	0.61% (35)
8. Negative Black Swans (Seg. Diff. Range)	0.23% (9)	0.29% (17)	0.02% (1)	0.12% (9)	0.17% (4)	0.15% (6)
9. Difference (%)	57.17%	79.56%	367.68%	167.23%	99.25%	138.42%

	Denmark	**Finland**	**France**	**Germany**	**Greece**	**Iceland**
1. Total Black Swans (Seg. Returns)	0.77% (53)	1.02% (78)	0.64% (48)	0.74% (99)	1.19% (86)	1.26% (77)
2. Total Black Swans (Seg. Diff. Range)	1.46% (81)	1.35% (78)	1.70% (121)	1.25% (85)	1.77% (92)	2.35% (8)
3. Difference (%)	-64.16%	-28.21%	-97.86%	-52.79%	-39.18%	-62.27%
4. Positive Black Swans (Seg. Returns)	0.32% (22)	0.42% (32)	0.19% (14)	0.31% (42)	0.57% (41)	0.64% (39)
5. Positive Black Swans (Seg. Diff. Range)	1.03% (57)	1.25% (72)	1.65% (118)	1.19% (81)	1.44% (75)	2.35% (8)
6. Difference (%)	-116.94%	-109.31%	-218.6%	-133.71%	-92.83%	-130.30%
7. Negative Black Swans (Seg. Returns)	0.45% (31)	0.60% (46)	0.45% (34)	0.43% (57)	0.62% (45)	0.62% (38)
8. Negative Black Swans (Seg. Diff. Range)	0.43% (24)	0.10% (6)	0.04% (3)	0.06% (4)	0.33% (17)	0.29% (1)
9. Difference (%)	3.85%	175.48%	237.37%	197.64%	64.91%	75.05%

	Hungary	**Ireland**	**Israel**	**Italy**	**Japan**	**Luxembourg**
1. Total Black Swans (Seg. Returns)	1.07% (71)	1.27% (111)	0.94% (71)	0.54% (26)	0.86% (115)	0.86% (39)
2. Total Black Swans (Seg. Diff. Range)	1.23% (57)	1.29% (52)	2.66% (38)	1.63% (52)	1.22% (89)	1.89% (45)
3. Difference (%)	-13.83%	-1.10%	-104.3%	-109.87%	-34.98%	-78.74%
4. Positive Black Swans (Seg. Returns)	0.42% (28)	0.52% (45)	0.36% (27)	0.15% (7)	0.35% (47)	0.33% (15)
5. Positive Black Swans (Seg. Diff. Range)	0.93% (43)	1.26% (51)	1.89% (27)	1.47% (47)	0.92% (67)	1.60% (38)
6. Difference (%)	-78.69%	-89.44%	-166.8%	-230.98%	-96.07%	-157.38%
7. Negative Black Swans (Seg. Returns)	0.65% (43)	0.76% (66)	0.58% (44)	0.40% (19)	0.51% (68)	0.53% (24)
8. Negative Black Swans (Seg. Diff. Range)	0.30% (14)	0.02% (1)	0.77% (11)	0.16% (5)	0.30% (22)	0.29% (7)
9. Difference (%)	76.42%	342.04%	-28.15%	92.94%	52.24%	58.79%

	Mexico	Netherlands	Norway	Portugal	Korea	Spain
1. Total Black Swans (Seg. Returns)	0.86% (64)	0.67% (58)	0.72% (55)	0.79% (48)	0.80% (86)	0.73% (54)
2. Total Black Swans (Seg. Diff. Range)	1.46% (100)	1.48% (96)	1.93% (47)	1.65% (68)	1.66% (86)	1.32% (83)
3. Difference (%)	-52.32%	-79.93%	-98.82%	-74.29%	-73.64%	-59.25%
4. Positive Black Swans (Seg. Returns)	0.32% (24)	0.17% (15)	0.26% (20)	0.36% (22)	0.37% (40)	0.18% (13)
5. Positive Black Swans (Seg. Diff. Range)	1.30% (89)	1.45% (94)	1.68% (41)	1.61% (66)	1.49% (77)	1.26% (79)
6. Difference (%)	-138.75%	-213.06%	-186.3%	-149.32%	-139.1%	-196.72%
7. Negative Black Swans (Seg. Returns)	0.54% (40)	0.49% (43)	0.46% (35)	0.43% (26)	0.43% (46)	0.55% (41)
8. Negative Black Swans (Seg. Diff. Range)	0.16% (11)	0.03% (2)	0.25% (6)	0.05% (2)	0.17% (9)	0.06% (4)
9. Difference (%)	121.41%	277.27%	61.82%	217.03%	89.50%	216.46%
	Sweden	**Switzerland**	**Turkey**	**U.K.**	**U.S.A.**	
1. Total Black Swans (Seg. Returns)	0.87% (67)	0.71% (50)	1.02% (73)	0.99% (96)	0.70% (94)	
2. Total Black Swans (Seg. Diff. Range)	1.48% (90)	0.99% (62)	1.72% (42)	1.38% (107)	1.36% (103)	
3. Difference (%)	-52.93%	-32.90%	-51.94%	-33.85%	-65.89%	
4. Positive Black Swans (Seg. Returns)	0.87% (67)	0.71% (50)	1.02% (73)	0.99% (96)	0.70% (94)	
5. Positive Black Swans (Seg. Diff. Range)	1.35% (82)	0.96% (60)	1.35% (33)	1.31% (101)	1.11% (84)	
6. Difference (%)	-134.51%	-156.92%	-113.5%	-95.33%	-153.3%	
7. Negative Black Swans (Seg. Returns)	0.52% (40)	0.51% (36)	0.59% (42)	0.48% (47)	0.46% (62)	
8. Negative Black Swans (Seg. Diff. Range)	0.13% (8)	0.03% (2)	0.37% (9)	0.08% (6)	0.25% (19)	
9. Difference (%)	137.52%	277.65%	46.82%	182.84%	61.52%	

Appendix 33

Descriptive statistics of Absolute returns – Equity Markets

	Australia	Austria	Belgium	Canada	Chile	Czech
No. of obs.	6255	7923	9493	8950	6883	5772
Mean	0.67%	0.88%	0.64%	0.69%	0.77%	0.89%
Standard Dev.	0.67%	1.00%	0.74%	0.78%	0.81%	0.99%
Skewness	2.65	2.95	3.34	3.78	2.80	3.64
Kurtosis	13.85	14.70	20.90	27.88	15.85	28.18
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
No. of obs.	6904	5209	7665	7531	13406	7210
Mean	0.81%	0.87%	1.06%	0.97%	0.84%	1.25%
Standard Dev.	0.84%	1.23%	1.20%	0.99%	0.88%	1.37%
Skewness	2.60	4.85	3.08	2.60	3.00	2.58
Kurtosis	13.26	41.89	18.71	12.10	17.44	11.84

	Hungary	Iceland	Ireland	Israel	Italy	Japan
No. of obs.	6622	6101	8707	7586	4797	13406
Mean	1.06%	0.56%	0.76%	0.99%	1.10%	0.82%
Standard Dev.	1.22%	1.61%	0.95%	1.06%	1.11%	0.93%
Skewness	3.60	52.83	3.30	2.68	2.21	3.24
Kurtosis	24.61	3464.34	19.55	13.08	7.94	22.32
	Korea	**Luxembourg**	**Mexico**	**Netherlands**	**New Zealand**	**Norway**
No. of obs.	10798	4534	7404	8709	4015	7665
Mean	0.97%	1.04%	1.03%	0.88%	0.49%	1.00%
Standard Dev.	1.14%	1.32%	1.12%	0.99%	0.49%	1.12%
Skewness	2.95	7.28	2.95	3.13	2.64	3.94
Kurtosis	15.64	133.41	15.74	16.85	14.07	37.41
	Poland	**Portugal**	**Slovakia**	**Slovenia**	**Spain**	**Sweden**
No. of obs.	5764	6101	5918	2383	7402	7664
Mean	1.23%	0.79%	0.78%	0.83%	0.95%	1.00%
Standard Dev.	1.31%	0.85%	1.21%	1.44%	0.99%	1.03%
Skewness	2.52	2.84	5.73	17.92	2.64	2.43
Kurtosis	11.24	15.55	69.90	473.28	12.96	9.75
	Switzerland	**Turkey**	**UK**	**USA**		
No. of obs.	7014	7142	9731	13406		
Mean	0.78%	1.80%	0.67%	0.67%		
Standard Dev.	0.84%	1.88%	0.86%	0.76%		
Skewness	3.13	2.35	4.38	4.83		
Kurtosis	17.88	8.87	42.76	73.89		

Appendix 34

Descriptive statistics of Squared returns – Equity markets

	Australia	Austria	Belgium	Canada	Chile	Czech
No. of obs.	6057	7511	9049	8612	6572	5453
Mean	-1093%	-1047%	-1114.12%	-1094.00%	-1066%	-1032%
Standard Dev.	233%	247.73%	246.80%	235.65%	240.39%	227.49%
Skewness	-1.02	-1.09	-1.06	-1.00	-1.18	-0.83
Kurtosis	1.49	2.37	2.38	1.98	2.60	1.05
	Denmark	**Estonia**	**Finland**	**France**	**Germany**	**Greece**
No. of obs.	6599	5017	7368	7302	12899	6795
Mean	-1053%	-1075%	-1016.73%	-1023.85%	-1047%	-975.03%
Standard Dev.	228.4%	255.3%	250.53%	239.50%	232.2%	247.98%
Skewness	-0.92	-0.64	-1.05	-1.17	-1.06	-1.12
Kurtosis	1.45	0.97	2.32	2.41	2.03	2.42
	Hungary	**Iceland**	**Ireland**	**Israel**	**Italy**	**Japan**
No. of obs.	6323	5669	7595	7079	4661	12678
Mean	-1012%	-1148%	-1061.93%	-1012.90%	-1001%	-1061%
Standard Dev.	244.8%	251.4%	246.78%	237.72%	236.7%	246.02%
Skewness	-1.03	-0.91	-1.08	-1.05	-1.00	-1.01

Kurtosis	2.11	1.60	2.53	1.91	1.84	1.87
	Korea	**Mexico**	**Netherlands**	**New Zealand**	**Norway**	**Poland**
No. of obs.	10165	7096	8398	3866	7354	5490
Mean	-1030%	-1015%	-1044.66%	-1149.94%	-1017%	-977.11%
Standard Dev.	246.2%	241.7%	233.65%	231.22%	232%	243.34%
Skewness	-0.85	-1.02	-0.86	-1.26	-0.90	-1.10
Kurtosis	1.32	1.85	1.32	2.97	1.29	2.27
	Portugal	**Slovakia**	**Slovenia**	**Spain**	**Sweden**	**Switzerland**
No. of obs.	5897	4983	2267	7134	7358	6752
Mean	-1075%	-1075%	-1065.61%	-1027.67%	-1014%	-1067.76%
Standard Dev.	250.2%	269.7%	245.36%	238.62%	237.1%	235.92%
Skewness	-1.13	-0.75	-1.05	-1.09	-1.13	-1.08
Kurtosis	2.51	0.75	2.39	2.18	2.19	2.29
	Turkey	**UK**	**USA**	**Luxembourg**		
No. of obs.	6812	8035	12892	4371		
Mean	-897%	-1066%	-1101.53%	-1021.79%		
Standard Dev.	233.9%	234.8%	239.66%	245.81%		
Skewness	-0.97	-1.09	-0.98	-0.89		
Kurtosis	1.69	2.29	1.72	1.50		

Appendix 35

Descriptive stats - Log Squared returns – Forex markets

	Argentine Peso	**Australian Dollar**	**Brazilian Real**	**Canadian Dollar**
No. of obs.	4422	5730	5468	5737
Mean	-1416.86%	-1138.05%	-1162.86%	-1217.9%
Standard Dev.	281.30%	218.11%	276.60%	226.59%
Skewness	0.23	-0.76	-0.63	-0.77
Kurtosis	0.32	0.75	-0.04	0.71
	Chilean Peso	**Chinese Yuan**	**Danish Krone**	**Euro**
No. of obs.	5666	2568	5769	5751
Mean	-1198.61%	-1573.20%	-1177.98%	-1176.9%
Standard Dev.	225.36%	253.41%	237.64%	229.25%
Skewness	-0.69	-0.51	-1.16	-0.95
Kurtosis	0.51	-0.07	2.03	1.07
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	4185	5206	4870	5312
Mean	-1194.81%	-1891.61%	-1155.39%	-1376.3%
Standard Dev.	201.76%	246.10%	225.08%	279.09%
Skewness	-0.77	0.08	-0.69	-0.32
Kurtosis	0.78	-0.55	0.78	-0.41
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	5324	4264	3796	5721

Mean	-1241.64%	-1268.43%	-1273.31%	-1180.2%
Standard Dev.	285.27%	211.88%	260.52%	234.69%
Skewness	-0.01	0.14	-0.38	-0.73
Kurtosis	0.03	0.04	0.17	1.43
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
No. of obs.	3917	5688	5723	4336
Mean	-1219.75%	-1143.46%	-1130.98%	-1302.3%
Standard Dev.	248.65%	237.03%	223.45%	258.06%
Skewness	-0.47	-0.57	-0.79	-0.07
Kurtosis	0.56	1.00	0.67	-0.07
	South Korean Won	**Norwegian Krone**	**Pakistan Rupee**	**Polish Zloty**
No. of obs.	5456	5773	4191	5481
Mean	-1233.04%	-1147.73%	-1467.08%	-1145.2%
Standard Dev.	255.69%	233.40%	256.06%	234.72%
Skewness	-0.46	-1.08	0.09	-0.81
Kurtosis	0.38	1.85	-0.10	0.98
	Russian Rouble	**Singapore Dollar**	**Solomon Isl Dollar**	**South African Rand**
No. of obs.	5027	5692	1880	5736
Mean	-1287.59%	-1305.19%	-1180.61%	-1125.6%
Standard Dev.	305.47%	225.89%	261.18%	256.21%
Skewness	-0.29	-0.62	-0.19	-0.85
Kurtosis	0.36	0.42	-1.02	0.82
	Swedish Krona	**Taiwan New Dollar**	**UK Sterling**	
No. of obs.	5773	5387	5750	
Mean	-1142.94%	-1403.91%	-1199.22%	
Standard Dev.	226.36%	255.51%	226.73%	
Skewness	-1.09	-0.61	-0.92	
Kurtosis	1.96	0.40	0.93	

Appendix 36

Descriptive statistics for Absolute returns for forex markets

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar
No. of obs.	5840	5840	5711	5840
Mean	0.20%	0.53%	0.59%	0.37%
Standard Dev.	0.89%	0.56%	0.78%	0.38%

Skewness	23.55	3.88	4.15	2.79
Kurtosis	772.44	32.56	32.01	15.73
	Chilean Peso	**Chinese Yuan**	**Danish Krone**	**Euro**
No. of obs.	5840	2829	5840	5840
Mean	0.40%	0.067%	0.45%	0.44%
Standard Dev.	0.42%	0.096%	0.41%	0.41%
Skewness	2.87	7.25	1.85	1.85
Kurtosis	16.52	111.70	6.43	6.44
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	4558	5840	4954	5840
Mean	0.35%	0.01%	0.51%	0.21%
Standard Dev.	0.47%	0.03%	0.67%	0.30%
Skewness	24.77	6.75	8.02	3.15
Kurtosis	1153.72	95.08	120.26	15.63
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	5840	4954	5840	5840
Mean	0.51%	0.27%	0.24%	0.48%
Standard Dev.	1.26%	0.44%	0.69%	0.76%
Skewness	8.39	4.75	26.75	9.58
Kurtosis	100.33	34.46	1214.38	155.65
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
No. of obs.	5612	5840	5840	5455
Mean	0.31%	0.59%	0.56%	0.26%
Standard Dev.	0.61%	1.03%	0.56%	0.54%
Skewness	8.36	13.23	2.70	6.88
Kurtosis	140.70	348.39	13.82	82.43
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**
No. of obs.	5840	4743	5578	5272
Mean	0.52%	0.13%	0.55%	0.46%
Standard Dev.	0.50%	0.27%	0.58%	1.54%
Skewness	2.52	5.80	2.62	16.38
Kurtosis	13.23	45.97	11.47	363.11
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**
No. of obs.	5840	2587	5840	5840
Mean	0.24%	0.41%	0.65%	0.40%
Standard Dev.	0.27%	0.65%	0.70%	0.76%
Skewness	3.45	3.58	2.74	9.66
Kurtosis	22.18	21.57	15.13	154.00

	Swedish Krona	Taiwan New Dollar	UK Sterling	
No. of obs.	5840	5840	5840	
Mean	0.52%	0.16%	0.40%	
Standard Dev.	0.48%	0.22%	0.37%	
Skewness	2.21	4.76	2.34	
Kurtosis	9.47	50.14	11.67	

Appendix 37

Descriptive statistics for Squared Returns – Foreign exchange market

	Argentine Peso	Australian Dollar	Brazilian Real	Canadian Dollar
No. of obs.	5839	5839	5710	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.32%	0.03%	0.05%	0.01%
Skewness	-0.01	0.07	-1.14	-0.36
Kurtosis	1197.70	299.53	252.71	196.74
	Chilean Peso	**Chinese Yuan**	**Danish Krone**	**Euro**
No. of obs.	5839	2828	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.01%	0.00%	0.01%	0.01%
Skewness	-0.42	2.80	0.03	0.05
Kurtosis	160.18	613.44	75.37	77.54
	Fijian Dollar	**Hong Kong Dollar**	**Icelandic Krona**	**Indian Rupee**
No. of obs.	4557	5839	4953	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.11%	0.00%	0.04%	0.01%
Skewness	-0.27	3.82	-1.18	0.16
Kurtosis	2257.31	1069.91	880.84	115.43
	Indonesian Rupiah	**Kenyan Shilling**	**Malaysian Ringgit**	**Mexican Peso**
No. of obs.	5839	4953	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.20%	0.02%	0.24%	0.10%
Skewness	0.17	1.71	0.00	1.06
Kurtosis	308.10	179.78	2663.65	686.47
	New Guinea Kina	**New Turkish Lira**	**New Zealand Dollar**	**Nigerian Naira**
No. of obs.	5611	5839	5839	5454
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.06%	0.25%	0.02%	0.04%
Skewness	0.06	16.59	0.35	-0.09
Kurtosis	1055.58	2280.85	96.52	732.93
	Norwegian Krone	**Pakistan Rupee**	**Polish Zloty**	**Russian Rouble**

No. of obs.	5839	4742	5577	5271
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.02%	0.01%	0.02%	0.56%
Skewness	-0.22	-0.53	-0.90	8.75
Kurtosis	148.14	162.38	64.97	926.72
	Singapore Dollar	**Solomon Isl Dollar**	**South African Rand**	**South Korean Won**
No. of obs.	5839	2586	5839	5839
Mean	0.00%	0.00%	0.00%	0.00%
Standard Dev.	0.01%	0.03%	0.03%	0.10%
Skewness	1.21	-1.01	-2.63	1.83
Kurtosis	241.04	113.98	191.06	950.62
	Swedish Krona	**Taiwan New Dollar**	**UK Sterling**	
No. of obs.	5839	5839	5839	
Mean	0.00%	0.00%	0.00%	
Standard Dev.	0.02%	0.00%	0.01%	
Skewness	0.69	-8.00	-0.02	
Kurtosis	85.57	749.39	129.25	

Briefly overviewing the first two moments of the distributions across the data sets does not indicate non-normality; however scrutinizing the third and fourth moments reveal significant deviations from normality.