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The trade-off in facility location and facility efficiency in supply chain network: A DEA approach

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ABSTRACT

Purpose

This paper aims to obtain an optimal DEA efficiency score simultaneously with facility location pattern for two-stage supply chain. As we explained in the literature there is a lack in the multi-objective facility location and facility efficiency in supply chain network design.

Design/ methodology

First, network DEA has been introduced as a possible model to solve this problem. Secondly, we have presented a solution approach based on Benders decomposition algorithm (BDA) to deal with the large-scale size and last compare the result of the solution found via BDA with the result found from original problem via CPLEX.

Findings

Result of mathematical modeling shows that while DEA calculate the efficiency score but other objective in respect to total cost have been ignored. In supply change it is important to locate minimum facilities (plant and warehouse) to transship and at the same time assign products to retailers in an efficient way. Moreover, proposed solution approach (Benders decomposition algorithm) depicts that this algorithm has better performance than CPLEX in large scale.

Originality

Applicability and originality of the model has been explored with a real case study to validate the solution approach. In this case study we use a simulation-based data for manufacturing located in Klang area in Selangor province in Malaysia which is skilled in fabrication of various alloys and many other exotic materials. The company is the first and only Malaysian manufactur covering all range of primary instrumentation for Flow, Temperature and Level Measurement and aims to be a leader in South East Asia, Middle East, Europe markets.

Keywords: Supply chain networks; Data Envelopment Analysis; optimization, Benders decomposition algorithm, CPLEX.

1. INTRODUCTION

Multi-objective problem is a challenging key in design of supply chain network which consider some aspects such as cost, time, coverage, environment, social, etc: (Wang et al., 2011, Tang et al., 2016, Rahimi et al., 2016, Pishvaee et al., 2014, Rahimi et al., 2017, Ahmadi-Javid and Ghandali, 2014, Golany et al., 2002, Rocha et al., 2017, Yadav and Sharma, 2016, Latha Shankar et al., 2012, Desport et al., 2017). Wang et al. (2011) Studied a bi-objective mixed integer programming model for green supply chain network. It is known as the first model in this issue in which is included environmental investment decision considering the objective cost in the supply chain design, and normal constraint (NC) method has been used to find optimal solutions. Cruz and Matsypura (2009) have proposed a model for the supply chain network considering social objective through the environmental decision. Their work covered multi-criteria decisions in the all supply chain elements including manufacturers, retailers, and distributors as a decision-maker. Their framework covered three objectives including net returns, risk, waste emission; they developed a model which has been shown by Nagurney and Toyasaki (2003). Cruz and Matsypura (2009) have reviewed the importance of environmental issues and its role completely is supply chain. They derived the optimality conditions, the equilibrium state of the supply chain network, and derive the equivalent finite-dimensional variation inequality. Furthermore, managers must have exact and enough information of organization's performance during designing a cost-efficient supply chain. Regarding this, Data Envelopment Analysis has been studied by many scholars (Omrani and Soltanzadeh, 2016, Tajbakhsh and Hassini, 2015, Thomas et al., 2002, Xu et al., 2009, Emrouznejad and Yang, 2018, Charnes et al., 1978).

The first model in a combination of facility efficiency and facility location-allocation introduced in Klimberg and Ratick (2008). According to Klimberg and Ratick (2008), there are some conflicts between the results from DEA facility efficiency, facility location, and transportation cost. When the model optimizes DEA efficiency the most challenging task of the proposed model is how to locate a facility and serves customer's demand and at the same time maximize DEA efficiency. In other words, when minimizing cost is considered, DEA efficiency is ignored, with this, the most important task is to locate a facility and to assign demand such that proposed model optimize total cost. Integrated DEA efficiency and uncertainty location-allocation model has studied in Moheb-Alizadeh et al. (2011), while Moheb-Alizadeh et al. (2011) discussed multi-criteria DEA in facility location-allocation in fuzzy environment, Mitropoulos et al. (2013) used DEA method for pre-assessing efficiency of DMUs then applied efficiency score found in a multi-objective pure integer programming method.

(Klimberg and Ratick, 2008) introduced the first model that integrate both facility efficiency and facility location-allocation, while (Moheb-Alizadeh et al., 2011) discussed multi-criteria decision making and DEA in facility location-allocation using fuzzy environment. (Mitropoulos et al., 2013) applied DEA for pre-assessing efficiency of DMUs then used the DEA efficiency scores found in a multi-objective pure integer programming method for location analysis.

However, there is a lack in the literature of multi-objective facility location and facility efficiency in supply chain network design. Because of the complex nature of supply chain, the more challenging it becomes to be measured effectively. The high number of facilities, locations, and capacity of the facilities affects the performance of supply chain.

Therefore, there are disadvantages in dealing with conventional DEA model, since DEA maximize the operational efficiency and ignore some other aspects as explained above, further standard DEA model ignores the inter-link between different components of supply change. For this modern production system with a large collection of interlink processes which are treats like a network that should be used. In supply chain, the location of warehouses are very important, since the effective and functional warehouse is the basic objective of warehouse design in the supply chain. In fact, speed, quality and price in serving the demand of customer are a crucial task for effective warehouses. About this, considering the design of plant and warehouse simultaneously with their interactions is a crucial task for decision makers. Thus, because of existing activity links between facilities in the supply chain, conventional DEA model is not able to measure its performance and find a source of inefficiency.

The rest of this paper is organized as follow. Section 2 and section 3 briefly describe facility location in supply chain and data envelopment analysis, respectively. Combined facility location in supply chain and network DEA are explained in Section 4. Section 5 presents numerical example considering a real case study. Finally, the conclusion and direct for future research are provided in the section 6.

2. MODELS

Next section briefly discusses some concept regarding facility location and Data Envelopment Analysis.

1.1. Facility location

Facility location is a mixed integer programming problem concerning to place a facility in a location to minimize cost. In supply chain it is assumed, there are several facilities including warehouses which tend to serve some retailers in such way cost of delivering products to retailers should be minimized, while all demands of retailers are satisfied. In other words, supply chain includes stages such as supplier, plant/manufacturer, warehouses/distributor, retailer, and customer. Facility location decisions have a significant role in network design for the supply chain. Mathematical formulation of this problem includes two type of decision variable, binary variable referring to decisions of building a facility in potential locations and continuous decision variables referring to number of commodities which deliver between facilities. One of the basic forms of capacitated facility location is (Daskin et al., 2005):

$$Min \sum_i \sum_j c_{ij} z_{ij} \qquad (1)$$

subject to

$$\sum_j z_{ij} = h_i \quad i \in I, \qquad (2)$$

$$\sum_{i \in I} z_{ij} \geq b_j X_{ij} \quad j \in J, \qquad (3)$$

$$z_{ij} \geq 0 \quad i \in I; j \in J, \qquad (4)$$

Where:

I: Set of plants
J: Set of customers
Parameters include:
c_{ij} : Fixed cost of transporting between plant i and customer j
h_i : Capacity of plant i
b_j : Demand of customer j

Decision variable includes:
z_{ij} : Amount of product which transports between plant I and customer j
X_{ij} : a binary variable (1 if customer j served by plant i; 0 otherwise)

The objective function (1) minimizes the total cost (fixed facility cost & shipment). Equation (2) shows limits total products should be transported between facilities and customers for every customer. Constraint (3) limits total products should be transported to each facility. Equation (4) is a simple non-negative equation.

1.2. Data Envelopment Analysis

Data Envelopment Analysis (DEA) is one of the mathematical programmings that has been widely used for measuring the performance of decision-making units (DMUs) that have same inputs and outputs. One of the advantages of DEA is that it does not need to convert input or output measures to some common metric, i.e., they remain in their natural form. Based on model, following DEA model is considered:

$$Max \frac{\sum_{r=1}^{R} u_r y_{rj}}{\sum_{i=1}^{I} v_i x_{ij}} \tag{5}$$

subject to

$$\sum_{r=1}^{R} u_r y_{rj} - \sum_{i=1}^{I} v_i x_{ij} \leq 0 \;, \forall j \tag{6}$$

$$u_r \geq 0, \forall r \tag{7}$$

$$v_i \geq 0, \forall i \tag{8}$$

Where:

i= 1. . . I: Inputs used at DMU,
r= 1. . . R: Outputs produced at DMU,
j= 1. . . J; DMUs.
Parameters:
y_{rj} : Quantity of the rth output for DMU_j ,
x_{ij} : Quantity of the ith input for DMU_j .
Decision variables:
u_r : The weight allocated to the rth output,
v_i : The weight allocated to the ith input.

3. AN INTEGRATED FACILITY LOCATION AND NETWORK DEA IN SUPPLY CHAIN

In this paper we use plant warehouses and retailers to design the proposed supply chain network that encompass manufacturing production systems in which some decision-making units (DMUs) produce intermediate products which are considered as inputs in the subsequent stage. In the DEA literature where some outputs of stage 1 become inputs for DMUs in the second stage we refer to network DEA models. To clarify this, consider a generic two-stage network DEA as shown in Fig.1, where it is assumed that each DMU (j=1,…,J) has I inputs ($X_{ij;}$ i=1,..., I) and D outputs ($Z_{dj;}$ d=1,…, D) in the first stage. These outputs (Z_{dj}) then become inputs in the second stage. Simultaneously there are another set of inputs (Z_{hj} ; h=1,…, H) to the second stage which they produce the final outputs (Y_{rj} ; r=1,…, R).

Our proposed model tries to optimize facility location cost in supply chain network including fixed and transportation costs and, simultaneously, optimize facility efficiency measured by DEA. Although there are many publications on evaluation of supply chain including location and efficiency in the literature (Azadeh and Alem, 2010, Hamdouch, 2011, Zeballos et al., 2014, Tajbakhsh and Hassini, 2015) but there is no clear research that considered these two objectives simultaneously when measuring the performance the supply chain.

---------------------------------[Figure 1 – about here] ---------------------------------

To construct the model assume there are two stages, including plant, warehouse which serves customers; in our supply chain network model.

– Facilities (plant and warehouse) are capacitated, and this amount is a parameter.

– The amount of products ship between plant, warehouse, and retailers are variable.

– In the network DEA model, output from the first stage is inputs to the second stage. Also, there is an additional input for the second stage.

It should be noted that in this paper a DMU assumed to be locating facility (plant and warehouse). For example, if there are K facilities and J candidate sites, we have $K \times J$ DMU. According to (Klimberg and Ratick, 2008) the second objective function calculates the maximum efficiency for all DMUs, simultaneously.

Based on aforementioned assumptions, the combined facility location and DEA model is formulated as follows:

I: Set of retailers (i=1, …I)
J: Set of candidate plant locations (j=1, …J)
E: Set of opened warehouses (e=1,…,E)
W: set of candidate warehouse locations (w=1,…W)
P: Set of opened plants (p=1,…,P)
T: Set of inputs of plants (t=1,…,T)
F: Set of outputs of warehouses (f=1,…F)
L: Set of products (l=1,…,L)

Objective functions:

$$Min \sum_p \sum_j f_{pj}.a_{pj} + \sum_w \sum_e f_{ew}.s_{ew} + \sum_j \sum_w \sum_l c_{jw}.b_{jwl} + \sum_l \sum_w \sum_i c_{wi}.b_{wil} \qquad (9)$$

The first objective function, (9), minimizes the total fixed cost of facilities and transportation of shipment from plant to warehouse and retailers.

$$Max \sum_p \sum_j 1 - d_{pj} + \sum_w \sum_e 1 - d_{ew} \qquad (10)$$

The second objective function, (10), maximizes the total sum of the efficiency of plants and warehouses which are considered as DMUs. d_{pj} and d_{ew} are defined as the inefficiency of DMUs in facility efficiency (Klimberg and Ratick, 2008).

<u>Constraints</u>:

Constraint (11) implies all requirements of demands for retailers must be fulfilled.

$$\sum_{e} y_{ei} \geq 1 \ \ \forall i \tag{11}$$

Equation (12) shows that maximum one plant should be located in every candidate location.

$$\sum_{p} a_{pj} \leq 1 \ \ \forall j \tag{12}$$

Constraint (13) implies that every plant should be located exactly in one candidate location.

$$\sum_{j} a_{pj} = 1 \ \ \forall p \tag{13}$$

Equation (14) and (15) are same as (12) and (13) for warehouses and candidate location.

$$\sum_{w} s_{ew} = 1 \ \ \forall e \tag{14}$$

$$\sum_{e} s_{ew} \leq 1 \ \ \forall w \tag{15}$$

It is clear that before locating warehouses at candidate site which serves demands for retailers, that warehouse should be located at that candidate location; this is satisfied by constraint (16).

$$y_{wi} \leq \sum_{e} s_{ew} \ \ \forall w, i \tag{16}$$

Constraint (17) implies that sum of finished products transported from all warehouses to every customer should be equal to demand of that customer.

$$\sum_{e} b_{eil} = De_{li} \ \ \forall i, l \tag{17}$$

The maximum of these assigned units must be equal to either the demand for that customer or capacity of that warehouse site in candidate location w. This necessity satisfies by constraint (18).

$$b_{wil} \leq \min[De_{li}, cap_{ewl}].\sum_e s_{ew} \quad \forall i, w \tag{18}$$

Constraint (19) implies that sum of product ship from all plant for every product must be equal to sum-product transport to all retailers by every warehouse.

$$\sum_i b_{eil} = \sum_p b_{pel} \quad \forall e, l \tag{19}$$

If plant p be located at location j then its inputs for resource t is x_{pjt}, and its output is y_{pjt}. Regarding this issue, the sum of weighted inputs for open facilities (plant and warehouse) is equal to 1 (Klimberg and Ratick, 2008). The proposed model is a network DEA with additional input in the second stage (Cook and Zhu, 2014), then these requirements are satisfied by constraints (20) and (21).

$$\sum_q u_{pjq}.y_{pjq} + d_{pj} = a_{pj} \quad \forall p, j \tag{20}$$

$$\sum_q u_{ewq}.y_{ewq} + d_{ew} = s_{ew} \quad \forall e, w \tag{21}$$

It is obvious the weighted sum of the each output of open plant and warehouse cannot be more than value 1 (note that a_{pj} and s_{ew} are binary variables). This is satisfied by constraints (22) and (23).

$$u_{pjq}.y_{pjt} \leq a_{pj} \quad \forall p, j, t \tag{22}$$

$$u_{ewq}.y_{ewt} \leq s_{ew} \quad \forall e, w, t \tag{23}$$

Constraint (24) implies that sum of weighted outputs must be less than the sum of weighted inputs (weights for each DMU j should be examined with the inputs/output vectors for all other homogeneous DMU).

$$\sum_f u_{pjq}.y_{pkf} - \sum_t v_{pjt}.x_{pht} \leq 0 \quad \forall p, j, k \neq j, h \neq j \tag{24}$$

Constraints (25) and (26) show that the sum of each corresponding DMU‘s weighted inputs to be equal 1.

$$\sum_t v_{pjt}.x_{pjt} = a_{pj} \quad \forall p, j \tag{25}$$

$$\sum_t v_{ewt}.x_{ewt} = s_{ew} \quad \forall e, w \tag{26}$$

The maximum amount of product transport from every warehouse to every customer must be equal to demand of that customer. This requirement is satisfied by constraint (27).

$$b_{wil} \le \sum_{e} s_{ew}.De_{li} \quad \forall w,i,l \tag{27}$$

Constraint (27) forces only open plants ships commodities to warehouses.

$$\sum_{w}\sum_{l} b_{jwl} \le \sum_{p} a_{pj} * M \quad \forall j \tag{28}$$

Constraint (29) depicts if plant p open at location j at least 1 product should be transported from that plant. (Moheb-Alizadeh, Rasouli et al. 2011).

$$\sum_{w} b_{jwl} \ge a_{pj} \quad \forall p, j, l \tag{29}$$

Constraint (30) is same as (29) for warehouses.

$$\sum_{i} b_{wil} \ge s_{ew} \quad \forall e, w, l \tag{30}$$

Equations (31) - (34) show no negatively and binary variables.

$$d, u, v, b \ge 0 \tag{31}$$

$$a, s, y \in \{0,1\} \tag{32}$$

$$u_{pj}, v_{pj} \ge \varepsilon x_{pj} \tag{33}$$

$$u_{ew}, v_{ew} \ge \varepsilon s_{ew} \tag{34}$$

Where:

<u>Parameters</u>

f_{pj} : Fixed cost of locating plant p at site j

f_{ew} : Fixed cost of locating warehouse e at site w

C_{jw} : Transportation cost between candidate site j and w

C_{wi} : Transportation cost between candidate site w and retailer i

x_{pjt} : Input t for plant P at site j

x_{ewt} : Input t for warehouse e at site w

y_{few}: Output f for locating warehouse e at site w

$y_{f\,pi}$: Output f for locating plant p at site i

De_{li} : The demand of retailer i for product l

cap_{mnl} : The capacity of facility m locate at site n for product l

Positive decision variables:

b_{jwl} : The number of products that transport between sites j and w

b_{wil} : The quantity of products that transport between site w and retailer i

d_{pj} : Value of inefficiency for plant p located at site j

d_{ew} : Value of inefficiency for warehouse e located at site w

v_{mnt} : Relating weight of each t th input for each facility m locate at site n

u_{mnq} : Relating weight for each q th output for each facility m locate at site n

Binary decision variables

a_{pj} : 1 if plant p locate at site j; 0 otherwise

s_{ew} : 1 if warehouse e locate at site w; 0 otherwise

y_{wi} : 1 if candidate warehouse w serve retailer I; 0 otherwise.

2. AN APPLICATION IN MANUFACTURING

In this section we explain the applicability of the model by applying it in a manufacturing industry. For this purpose we gathered data from "ERATECH Manufacturing" located in Klang area in Selangor province in Malaysia. Then we used simulation-based data according to initial information we gathered. . ERATECH is skilled in fabrication of various alloys and many other exotic materials, using the right process and procedures paired with the latest equipment, ensuring that the products are of the highest quality. Their experienced and technically equipped sales team, engineers and production staff are dedicated, motivated and result-oriented. ERATECH prides itself on the ability to meet express deliveries, supply quality products and their commitment to excellence service, working around the clock to meet requirements of customers. ERATECH is the first and the only Malaysian manufacturer covering all range of primary instrumentation for Flow, Temperature and Level Measurement and aims to be a leader in South East Asia, Middle East, Europe markets. Major products of Eratech include Orifice Plates, Restriction Orifice Plates, Venture Tubes, Wedge Meter, Flow Nozzle, Averaging Pitot Tubes, Orifice Meter Runs, and Venture Nozzle. ERATECH is a member of MOGSC (Malaysian Oil and Gas Service Council) and has a close relation with MIDA (Malaysian Industrial Development Authority), SSIC (Selangor State Investment Centre), and MPK (Majlis Perbandaran Klang).

Our proposed model is suitable for two stage supply chain networks. In this application, how to site the plants and warehouses in the capacitated candidate location has been selected with the view of how to assign demands of retailers, so also, we present how efficiency affect location pattern for supply chain network. Locating facility in any potential location is considered as stage 1 of the process where inputs to this stage include: capital cost, labor cost, production cost, while finished products are considered as output in the first stage, which then will be turned to input in the second stage. Hence, in the second stage for plants and warehouses, finished products from the first stage and transportation cost are considered to be inputs while customers served by distribution are the final outputs for warehouses (Figure 2).

--------------------------------[Figure 2 – about here] --------------------------------

--------------------------------[Table 1 – about here] ---------------------------------

Table 1 shows product list (products 1 to 8) and demand for each customer ($i=i_1,\ldots, i_{18}$).

Tables 2 and 3 show the cost of locating candidate plants and warehouses. The idea is to establish four plants ($p=p_1,\ldots,p_4$) and warehouses ($e=e_1,\ldots,e_4$) considering eight potential locations respectively ($j=j_1,\ldots,j_8$; $w=w_1,\ldots,w_8$).

--------------------------------[Table 2 – about here] ---------------------------------

--------------------------------[Table 3 – about here] ---------------------------------

Table 4 shows the transportation cost between candidate plants and warehouses while Table 5 presents transportation cost between candidate warehouses and customers.

--------------------------------[Table 4 – about here] ---------------------------------

--------------------------------[Table 5 – about here] ---------------------------------

Tables 6, 7 and 8 show results that are obtained from the sum weighted method. In Table 6 we can see which plant is connected to which warehouse using different set of weights.

--------------------------------[Table 6 – about here] ---------------------------------

In Table 6 weights have been set from zero to one. However, in some cases, different weights results in the same location for each candidate location meaning that different weights result in same Pareto points in some cases.

--------------------------------[Table 7 – about here] ---------------------------------

Where: $j=j_1,...,j_8$ (plants), $w=w_1,...,w_8$ (warehouses), $i=i_1,...,i_8$ (customers) (in the case $w_1=w_2=0.5$, in the weighted sum)

Table 8 depicts divisional efficiency for each DMU, overall efficiency by an average of division scores and products of division scores for different weights. As it can be seen, the w1 coefficient depicts weight for cost objective such that sum of weights (w for cost objective and w2=1-w1 for efficiency objective) for both objectives is assumed 1.

--------------------------------[Table 8 – about here] ---------------------------------

As seen in this table, weights have changed from 0 to 1. Average DEA score and Geometric DEA scores vs. total cost are measured. Figures 3 and 4 depict Pareto optimality points of Table 9.

--------------------------------[Figure 3 – about here] --------------------------------

--------------------------------[Figure 4 – about here] --------------------------------

As it is clear from Fig. 3 and 4 there are four Pareto optimal points by using weighted sum method. Total cost includes facility location and transportation cost. In the DEA model we used output-oriention approach. The overall efficiency of the supply chain as a two-stage network can be measured by the two average scores and geometric mean of divisions for each DMU. When DEA objective is optimized, another one (cost objective) increase significantly since the proposed model attempt to locate a minimum number of facilities and allocate customer's demands to minimum available warehouse such that it maximize the DEA efficiency score at the same time.

We have tested the proposed approach with different size in the range of case study which using design of experiments, which is known as a powerful tool for improving quality. This has been done with changing values for some input variables and observing of the related responses. Most important objectives of the design of experiments include identifying the best level of variables, obtaining variables and their impacts on output responses and control of these variables in responses. In this paper full factorial design is applied which is useful when facing a large number of experiments. In full factorial design, we considered four significant factors including: the number of products, number of customers, number of working plants, and number of working warehouses. For each factor, three different levels are designed resulting in 81 experiments. As it is shown the proposed model is a kind of NP-hard problem, after running all experiments for all cases, it is observed that increasing number of binary variables cause to increase the time to get an optimal solution.

In this application, we proposed an exact solution based on Benders decomposition algorithm which could find exact Pareto optimal solution in reasonable time. Benders algorithm first introduced by (Benders, 1962). There are several advantages for using Benders decomposition in comparison to other optimization methods especially metaheuristic: the BDA algorithm relies on strong algebra concept such that its convergence analytically is proven and can achieve to the optimal solution (Pishvaee et al., 2014). In this algorithm, it is possible to adjust optimality gap precisely when it is needed. Another advantage of BDA is that other methods could be employed with BDA (Poojari and Beasley, 2009). In this case, this algorithm works as follow:

In BDA instead of solving the original complex problem, the problem is decomposed into a master problem and sub-problem. These two problems are solved iteratively by using the solution of one in the other while the optimal solution is achieved. The procedure of Benders decomposition algorithm has depicted in Figure 5.

--------------------------------[Figure 5 – about here] --------------------------------

Therefore, the master problem is presented in equation (35) to (41):

$$Min \ A \times (\sum_p \sum_j f_{pj} a_{pj} + \sum_w \sum_e f_{ew} s_{ew}) \tag{35}$$

subject to

$$\sum_e y_{ei} \geq 1 \ \ \forall i \tag{36}$$

$$\sum_p a_{pj} \leq 1 \ \ \forall j \tag{37}$$

$$\sum_j a_{pj} = 1 \ \ \forall p \tag{38}$$

$$\sum_w s_{ew} = 1 \ \ \forall e \tag{39}$$

$$\sum_e s_{ew} \leq 1 \ \ \forall w \tag{40}$$

$$y_{wi} \leq \sum_e s_{ew} \ \ \forall w, i \tag{41}$$

Where A and $1-A$ are weights for aggregating objective functions in equation (9) and (10) where two objectives (9) and (10) are aggregated then the binary section is selected and are considered for the master problem in BDA. Equation (35) to (41) are the same as equations (11) to (16).

Primal sub-problem is as below:

$$Min\ A\times(\sum_j\sum_w\sum_l c_{jw}b_{jwl}+\sum_l\sum_w\sum_i c_{wi}b_{wil})-(1-A)\times(\sum_p\sum_j d_{pj}-1+\sum_w\sum_e d_{ew}-1) \quad (42)$$

The objective function (42) is linear part of aggregated functions (9) and (10).

$$\sum_w b_{wil}=De_{li}\ \ \forall i,l \quad (43)$$

Equation (43) to (62) are same as (17) to (34) respectively such that integer variables have been fixed for primal sub-problem.

$$b_{wil}\le \min[De_{li},cap_{ewl}].\sum_e \overline{s_{ew}}\ \ \forall i,w,l \quad (44)$$

$$\sum_i b_{wil}=\sum_j b_{jwl}\quad \forall w,l \quad (45)$$

$$\sum_q u_{pjq}y_{pjq}+d_{pj}=\overline{a_{pj}}\ \ \forall p,j \quad (46)$$

$$\sum_q u_{ewq}y_{ewq}+d_{ew}=\overline{s_{ew}}\ \ \forall e,w \quad (47)$$

$$u_{pjq}y_{pjt}\le\overline{a_{pj}}\ \ \forall p,j,t \quad (48)$$

$$u_{ewq}y_{ewt}\le\overline{s_{ew}}\ \ \forall e,w,t \quad (49)$$

$$\sum_f u_{pjq}y_{pkf}-\sum_t v_{pjt}.x_{pht}\le 0\quad \forall p,j,k\ne j,h\ne j \quad (50)$$

$$\sum_t v_{pjt}x_{pjt}=\overline{a_{pj}}\quad \forall p,j \quad (51)$$

$$\sum_t v_{ewt}x_{ewt}=\overline{s_{ew}}\quad \forall e,w \quad (52)$$

$$b_{wil}\le\sum_e\overline{s_{ew}}De_{li}\ \ \forall w,i,l \quad (53)$$

$$\sum_w\sum_l b_{jwl}\le\sum_p\overline{a_{pj}}y_{pj}\ M\ \ \forall j \quad (54)$$

$$u_{pj}\ge\varepsilon\overline{a_{pj}} \quad (55)$$

$$u_{ew}\ge\varepsilon\overline{s_{ew}} \quad (56)$$

$$v_{pjt}\ge\varepsilon\overline{a_{pj}}\quad \forall p,j,t \quad (57)$$

$$v_{ewf}\ge\varepsilon\overline{s_{ew}}\quad \forall e,w,f \quad (58)$$

$$\sum_{w} b_{jwl} \geq \bar{a}_{pj} \quad \forall p, j, l \tag{59}$$

$$\sum_{i} b_{wil} \geq \bar{s}_{ew} \quad \forall e, w, l \tag{60}$$

$$d, u, v, b \geq 0 \tag{61}$$

$$a, s, y \in \{0,1\} \tag{62}$$

Where: $\bar{s}_{ew}, \bar{a}_{pj}, \bar{y}_{ei}$ are binary fixed variables that found from the master problem.

To find dual of primal sub-problem the following dual variables have been introduced:

Dual variables $q_{1(i,\ l)}$, $q_{2(i,\ w,\ l)}$, $q_{3(w,l)}$, $q_{4(p,j)}$, $q_{5(e,w)}$, $q_{6(p,j,t)}$, $q_{7(e,w,t)}$, $q_{9(p,j)}$, $q_{10(e,w)}$, $q_{11(w,i,l)}$, $q_{12(j)}$, $uu_{1(p,j)}$,$uu_{2(e,w)}$,$vv_{1(p,j,t)}$,$vv_{2\ (e,w,f)}$, $vv_{3(p,j,l)}$, $vv_{4\ (e,w,l)}$ are introduced for equations (43) to (62) respectively . Then dual of sub-problem is considered as follow:

$$\begin{aligned} Max\ z = & \sum_{i}\sum_{l} q_{1(il)} \times De_{(li)} - \sum_{i}\sum_{w}\sum_{l} q_{2(iwl)} \times De_{(li)} \times \sum_{e}\sum_{w} \overline{s_{ew}} \\ & + \sum_{w}\sum_{w} q_{4(pj)} \times \overline{a_{pj}} + \sum_{w}\sum_{w} q_{5(ew)} \times \overline{s_{ew}} - \sum_{p}\sum_{j}\sum_{t} q_{6(pjt)} \times \overline{a_{pj}} - \sum_{e}\sum_{w}\sum_{t} q_{7(ewt)} \times \overline{s_{ew}} \\ & + \sum_{p}\sum_{j} q_{9(pj)} \times \overline{a_{pj}} + \sum_{e}\sum_{w} q_{10(ew)} \times \overline{s_{ew}} - \sum_{j} q_{12(j)} \times M \times \sum_{p}\sum_{j} \overline{a_{pj}} \\ & + \sum_{p}\sum_{j} uu_{1(pj)} \times \varepsilon \times \overline{a_{pj}} + \sum_{e}\sum_{w} uu_{2(ew)} \times \varepsilon \times \overline{s_{ew}} + \sum_{p}\sum_{j}\sum_{t} vv_{1(pjt)} \times \varepsilon \times \overline{a_{pj}} \\ & + \sum_{e}\sum_{w}\sum_{f} vv_{2(ewf)} \times \varepsilon \times \overline{s_{ew}} + \sum_{p}\sum_{j}\sum_{l} vv_{3(pjl)} \times \overline{a_{pj}} + \sum_{e}\sum_{w}\sum_{l} vv_{4(ewl)} \times \overline{s_{ew}} \\ & + A \times (\sum_{p}\sum_{j} f_{pj} \times \overline{a_{pj}} + \sum_{e}\sum_{w} f_{ew} \times \overline{a_{ew}}) \end{aligned} \tag{63}$$

subject to

$$q_{1(il)} - q_{2(iwl)} - q_{3(wl)} - q_{11(wil)} + vv_{4(ewl)} \leq AC_{2(wi)} \tag{64}$$

$$q_{3(wl)} + vv_{3(pjl)} - q_{12(j)} \leq AC_{1(jw)} \tag{65}$$

$$-\ q_{4(pj)} \leq -\ (1-A) \tag{66}$$

$$-\ q_{5(ew)} \leq -\ (1-A) \tag{67}$$

Equations (63) to (67) are a dual form of equations (42) to (62) with respect to dual variables defined earlier. Then optimality cut is added to the master problem as below:

$$
\begin{aligned}
& Min \;\; z \\
& z \geq A \times (\sum_{p}\sum_{j} f_{pj} \times x_{pj} + \sum_{e}\sum_{w} f_{ew} \times s_{ew} + \sum_{i}\sum_{l} \overline{qq}_{1(il)} \times De_{(il)} \\
& + \sum_{i}\sum_{w}\sum_{l} \overline{qq}_{2(iwl)} \times De_{(li)} \times De_{(il)} \times \sum_{e}\sum_{w} s_{ew} + \sum_{p}\sum_{j} \overline{qq}_{4\,(pj)} \times a_{pj} + \\
& \sum_{e}\sum_{w} \overline{qq}_{5(ew)} \times s_{ew} + \sum_{p}\sum_{j} \overline{qq}_{6\,(pj)} \times a_{pj} - \sum_{e}\sum_{w}\sum_{t} \overline{q}_{7\,(ewt)} \times s_{ew} + \sum_{p}\sum_{j} \overline{qq}_{9\,(pj)} \times a_{pj} \\
& + \sum_{e}\sum_{w} \overline{qq}_{10\,(ew)} \times s_{ew} - \sum_{j} \overline{qq}_{12(j)} \times M \times \sum_{p}\sum_{j} a_{pj} + \sum_{p}\sum_{j} \overline{uu}_{1\,(pj)} \times \varepsilon \times \overline{a_{pj}} \\
& + \sum_{e}\sum_{w} \overline{uu}_{2\,(ew)} \times \varepsilon \times s_{ew} + \sum_{p}\sum_{j}\sum_{t} \overline{vv}_{1\,(pjt)} \times \varepsilon \times \overline{a_{pj}} \\
& + \sum_{e}\sum_{w}\sum_{f} \overline{vv}_{2\,(ewf)} \times \varepsilon \times \overline{s_{ew}} + \sum_{p}\sum_{j}\sum_{l} \overline{vv}_{3\,(pjl)} \times \overline{a_{pj}} + \sum_{e}\sum_{w}\sum_{l} \overline{vv}_{4\,(ewl)} \times \overline{s_{ew}}
\end{aligned}
\tag{68}
$$

Constraints for the feasibility cut is given below:

$$
\begin{aligned}
& \sum_{i}\sum_{l} \overline{qq}_{1(il)} \times De_{(il)} \\
& + \sum_{i}\sum_{w}\sum_{l} \overline{qq}_{2(iwl)} \times De_{(li)} \times De_{(il)} \times \sum_{e}\sum_{w} s_{ew} + \sum_{p}\sum_{j} \overline{qq}_{4\,(pj)} \times a_{pj} + \\
& \sum_{e}\sum_{w} \overline{qq}_{5\,ew} \times s_{ew} + \sum_{p}\sum_{j} \overline{qq}_{6\,(pj)} \times a_{pj} - \sum_{e}\sum_{w}\sum_{t} \overline{q}_{7\,(ewt)} \times s_{ew} + \sum_{p}\sum_{j} \overline{qq}_{9\,(pj)} \times a_{pj} \\
& + \sum_{e}\sum_{w} \overline{qq}_{10\,(ew)} \times s_{ew} - \sum_{j} \overline{qq}_{12\,(j)} \times M \times \sum_{p}\sum_{j} x_{pj} + \sum_{p}\sum_{j} \overline{uu}_{1\,(pj)} \times \varepsilon \times \overline{a_{pj}} \\
& + \sum_{e}\sum_{w} \overline{uu}_{2\,(ew)} \times \varepsilon \times s_{ew} + \sum_{p}\sum_{j}\sum_{t} \overline{vv}_{1\,(pjt)} \times \varepsilon \times \overline{a_{pj}} \\
& + \sum_{e}\sum_{w}\sum_{f} \overline{vv}_{2\,(ewf)} \times \varepsilon \times \overline{s_{ew}} \;\; \sum_{p}\sum_{j}\sum_{l} \overline{vv}_{3\,(pjl)} \times \overline{a_{pj}} + \sum_{e}\sum_{w}\sum_{l} \overline{vv}_{4\,(ewl)} \times \overline{s_{ew}} \leq 0
\end{aligned}
\tag{69}
$$

Where $\overline{qq}_1, \overline{qq}_4, \overline{qq}_9, \overline{q}_7, \overline{vv}_1, \overline{vv}_2, \overline{vv}_3, \overline{vv}_4, \overline{qq}_5, \overline{uu}_1, \overline{uu}_2, \overline{qq}_{12}, \overline{qq}_7, \overline{qq}_{10}, \overline{qq}_2, \overline{qq}_6$ are solutions found from dual sub-problem given in equation (63).

Equation (68) and (69) are optimality and feasibility cuts, respectively, which are added to the initial master problem. To validate the proposed approach, one test in the range of case study has been examined which include 5 capacitated plants (p_1,…p_5) with 8 potential locations (j_1,…,j_8), 7 warehouse facilities (e_1,…,e_7) with 10 potential locations (w_1,…,w_{10}) and 120 external retailers are considered as customers.

Table 9 presents results of the trade-off between total cost and DEA efficiency score for open facilities. The same as Table 8, there are Pareto points for these two objectives (cost and DEA).

--------------------------------[Table 9 – about here] ---------------------------------

Presentation in the Table 9 validates that proposed model for large-scale size, however, to validate the efficiency of the proposed approach for the mentioned model in large-scale size, more experiments are carried out, and results are presented in Table 10.

4. RESULTS AND DISCUSSION

In this section we present the performance of BDA against an original problem with CPLEX. CPLEX is a solver that has been developed by ILOG and could solve different difficult problems such as linear, mixed-integer and quadratic programming. CPLEX contains a primal simplex algorithm, a dual simplex algorithm a network optimizer and for problems which possess integer variables uses a branch and bound algorithm. To illustrate more about the efficiency of BDA algorithm, we have done several extra experiments, and the results are presented in Table 10.

---------------------------------[Table 10– about here] ----------------------------------

For evaluating and analyzing the performance of the BDA, GAMS 24.7.3 optimization software is used to solve the decomposed model which include positive and binary MP model. Then the original model is also solved by CPLEX. Moreover, two other cases are applied in our experiments. The required data and parameters for extra tests are selected randomly but in the range of case study's parameters. The size of test problems is large than the studied case. Furthermore, we set the optimality gap of algorithm equal to zero. The related found results are presented in Table 10.

Generally, after three experiments for three different weights including (1,0), (0,1), and (0.5,0.5), it is observed that BDA for solving the mentioned problem is 9.78 times quicker than CPLEX solver in GAMS. Hence, this results shows the BDA algorithm for solving the proposed model is significantly more time efficient than CPLEX from solving the original problem.

5. CONCLUSION AND DIRECTION FOR FUTURE RESEARCH

While there is a lack in the literature of multi-objective facility location and facility efficiency in supply chain network design, this paper considered to optimize DEA efficiency and facility and transportation cost for supply chain network simultaneously. In this paper, a two-stage supply chain with additional input in the second stage has been considered, and a mathematical model was developed to show a trade-off between facility cost and facility efficiency. As it is shown, while one objective (DEA efficiency) optimize, another objective (total cost) may fails because the model tries to locate minimum facilities (plant and warehouse) to transship and assign products to retailers resulting in increased transportation cost and the total cost for a long period. Hence the proposed approach could help to assess performance of supply chain form different angle simultaneously. It is also note that in large-scale size that classic multi-objective approach fails to solve this model in a reasonable time. To deal with this problem, here we have presented a suitable algorithm based on Benders decomposition which could find the solution much faster than the CPLEX. Managers must have exact and enough information of organization's performance during designing a cost-efficient supply chain. The proposed model provides an important framework for managers in monitoring and planning their supply chain operations and can significantly help them in making supply chains more efficient. As a recommendation for future study, it is worth to explore the closed supply chain considering Data Envelopment Analysis as well as uncertainty condition. Regarding solution method, it is suggested to work with more decomposition approach or even combination of Benders decomposition with other solution especially Meta heuristic algorithm.

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Table 1: Product list and demand points

	Demand (Kg)																	
Products	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}	i_{11}	i_{12}	i_{13}	i_{14}	i_{15}	i_{16}	i_{17}	i_{18}
$Product_1$	37	61	96	45	43	45	43	55	65	34	54	33	54	23	44	65	66	76
$Product_2$	64	53	54	78	29	27	35	52	27	44	28	31	59	60	67	30	60	76
$Product_3$	24	73	21	33	26	27	53	59	47	68	55	43	61	49	22	72	21	68
$Product_4$	26	65	40	26	56	79	71	79	74	37	68	50	26	68	38	45	74	56
$Product_5$	69	26	68	60	68	41	20	66	48	63	66	61	66	21	56	42	37	44
$Product_6$	52	33	76	77	68	76	74	66	72	39	63	24	51	35	55	29	72	50
$Product_7$	53	36	54	41	62	68	23	33	47	50	52	27	33	53	37	56	51	75
$Product_8$	46	45	53	38	66	22	55	59	56	58	27	44	52	71	76	63	57	59

Table 2: Fixed cost of opening plants in different candidate location (thousand RM)

	j_1	j_2	j_3	j_4	j_5	j_6	j_7	j_8
p_1	350	630	240	420	380	430	450	530
p_2	520	420	360	450	430	510	740	510
P_3	420	520	280	670	360	310	640	340
P_4	360	480	650	720	780	680	630	520

Table 3: Fixed cost of opening warehouses in different candidate location (thousand RM)

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
e_1	472	267	312	433	405	493	385	289
e_2	400	440	284	413	343	351	392	304
e_3	442	392	471	440	359	383	363	391
e_4	485	377	498	348	345	380	334	373

Table 4: Transportation cost between plants and warehouses (RM)

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
j_1	2025	5250	5060	2450	3265	2453	4150	3260
j_2	2016	3734	3683	2827	3241	4782	2276	3305
j_3	4345	3405	3522	4443	4522	2238	2397	4158
j_4	3980	3365	3298	4220	4836	3961	3067	2790
j_5	3560	2873	3596	4257	4586	3903	2608	4428
j_6	4364	3886	2276	2213	2145	4471	4596	3680
j_7	3772	4005	4145	4559	3954	3950	2155	3704
j_8	2817	3903	4424	3328	2779	4014	4893	4088

Table 5: Transportation cost between warehouses and customers (RM)

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8	i_9	i_{10}	i_{11}	i_{12}	i_{13}	i_{14}	i_{15}	i_{16}	i_{17}	i_{18}
w_1	805	850	598	850	990	931	869	528	823	962	500	564	697	761	681	969	874	900
w_2	792	924	612	623	958	986	554	595	549	861	539	927	662	936	802	934	955	896
w_3	966	898	960	614	676	924	963	677	778	673	899	639	548	636	588	699	922	975
w_4	765	709	793	694	795	570	611	506	558	869	923	714	500	745	655	959	717	814
w_5	829	972	881	721	846	676	518	752	585	761	957	833	508	986	940	633	709	548
w_6	850	896	809	605	833	654	519	834	769	632	867	565	763	782	728	628	512	565
w_7	942	550	958	804	610	818	834	819	546	858	817	812	771	768	994	988	939	659
w_8	734	746	611	615	604	971	910	526	535	575	575	554	628	696	932	937	881	1000

Table 6: Potential location vs. different weights

w_1	w_2	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
1	0	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_7
0.9	0.1	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_7
0.8	0.2	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_7
0.7	0.3	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_7
0.6	0.4	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_4
0.5	0.5	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_4
0.4	0.6	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_4
0.3	0.7	j_3	j_2	j_6	j_1	w_2	w_3	w_5	w_4
0.2	0.8	j_3	j_4	j_6	j_1	w_2	w_3	w_1	w_4
0.1	0.9	j_3	j_4	j_8	j_1	w_2	w_3	w_1	w_4
0	1	j_7	j_8	j_4	j_3	w_2	w_3	w_1	w_4

Table 7: Location links

j_1	w_2, w_3, w_4, w_5
j_2	w_2, w_3, w_4, w_5
j_3	w_2, w_3, w_4, w_5
j_6	w2, w_3, w_4, w_5
w_2	i_2, i_3, i4, i5, i_6, i_7, i_8, i_9, i_{10}, i_{11}, i_{13}, i_{14},i_{17},i_{18}
w_3	i_1, i_2, i_3, i_4, i_5, i_7, i_8, i_9, i_{10}, i_{11}, i_{12}, i_{13}, 1_{14}, i_{17}
w_4	i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_{10}, i_{11}, i_{12}, i_{13}, i_{14}, i_{15}, i_{16}, i_{17}, i_{18}
w_5	i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8, i_9, i_{11}, i_{12}, i_{13}, i_{14}, i_{15}, i_{16}, i_{17}, i_{18}

Table 8: Results of weighted sum method, total sum efficiency, divisional efficiency, product of divisions

					Efficiency score							
w_1	**w_2**	**Total Cost $ 10 million**	**Average DEA**	**Geomean divisions**	**DMU_1**	**DMU_2**	**DMU_3**	**DMU_4**	**DMU_5**	**DMU_6**	**DMU_7**	**DMU_8**
1	0	3.10	0.56	0.425197	1	0.41	0.6225	0.23	1	1	0.14	0.13
0.9	0.1	3.10	0.65	0.570469	1	0.5352	0.8151	0.3759	1	1	0.36	0.19
0.8	0.2	3.11	0.65	0.570469	1	0.5352	0.8151	0.3759	1	1	0.36	0.19
0.7	0.3	3.11	0.65	0.570469	1	0.5352	0.8151	0.3759	1	1	0.36	0.19
0.6	0.4	3.11	0.76	0.704085	1	0.5352	0.8151	0.3759	1	1	0.3683	1
0.5	0.5	3.11	0.76	0.704085	1	0.5352	0.8151	0.3759	1	1	0.3683	1
0.4	0.6	3.11	0.76	0.704085	1	0.5352	0.8151	0.3759	1	1	0.3683	1
0.3	0.7	3.11	0.76	0.704085	1	0.5352	0.8151	0.3759	1	1	0.3683	1
0.2	0.8	3.11	0.87	0.843125	1	0.8334	0.8151	0.3759	1	1	1	1
0.1	0.9	3.22	0.90	0.864949	1	0.8334	1	0.3759	1	1	1	1
0	1	4.82	0.96	0.966881	0.8108	0.975	0.9662	1	1	1	1	1

Table 9: The results under various weights of objective functions

w_1	w_2	Total Cost $100 Million	Arithmetic Average DEA	Geometric Average DEA	Efficiency score											
					DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU9	DMU10	DMU11	DMU12
1	0	3.855012	0.4789	0.39398	0.415	0.236	0.6225	0.2336	0.325	0.6914	0.408	0.14	0.13	0.8646	0.6807	1
0.9	0.1	3.891404	0.946633	0.934367	0.9811	0.5352	1	1	1	1	1	1	1	0.8433	1	1
0.8	0.2	3.855119	0.931217	0.918574	0.9811	0.5352	0.815	1	1	1	1	1	1	0.8433	1	1
0.7	0.3	4.081022	0.990792	0.990698	0.9811	0.9759	0.9662	1	1	1	1	1	1	1	0.9663	1
0.6	0.4	3.891402	0.959383	0.94744	0.9811	0.5352	1	1	1	1	1	1	1	1	0.9963	1
0.5	0.5	3.891402	0.959383	0.94744	0.9811	0.5352	1	1	1	1	1	1	1	1	0.9963	1
0.4	0.6	3.891402	0.959383	0.94744	0.9811	0.5352	1	1	1	1	1	1	1	1	0.9963	1
0.3	0.7	3.891402	0.959383	0.94744	0.9811	0.5352	1	1	1	1	1	1	1	1	0.9963	1
0.2	0.8	3.870577	0.946733	0.934648	0.8594	0.5352	0.9662	1	1	1	1	1	1	1	1	1
0.1	0.9	3.873206	0.946733	0.934648	0.8594	0.5352	0.9662	1	1	1	1	1	1	1	1	1
0	1	6.439649	0.9936	0.993533	0.9811	0.9759	0.9662	1	1	1	1	1	1	1	1	1

Table 10: A Comparison between BDA and CPLEX for different sizes

Problem size $\|p\|*\|j\|*\|e\|*\|w\|*\|i\|$	**Weights on OFs**	**BDA Time(s)**	**CPLEX Time (s)**
$\|5\|*\|8\|*\|7\|*\|10\|*\|120\|$	(1,0)	16.360	28.9903
	(0,1)	16.105	119.008
	(0.5,0.5)	18.078	360.37.789
$\|7\|*\|10\|*\|9\|*\|11\|*\|140\|$	(1,0)	26.681	141.967
	(0,1)	26.399	126.94
	(0.5,0.5)	28.184	202.078
$\|9\|*\|11\|*\|11\|*\|12\|*\|140\|$	(1,0)	21.469	118.477
	(0,1)	18.253	236
	(0.5,0.5)	17.273	874.661
Total time		188.802	1848.121

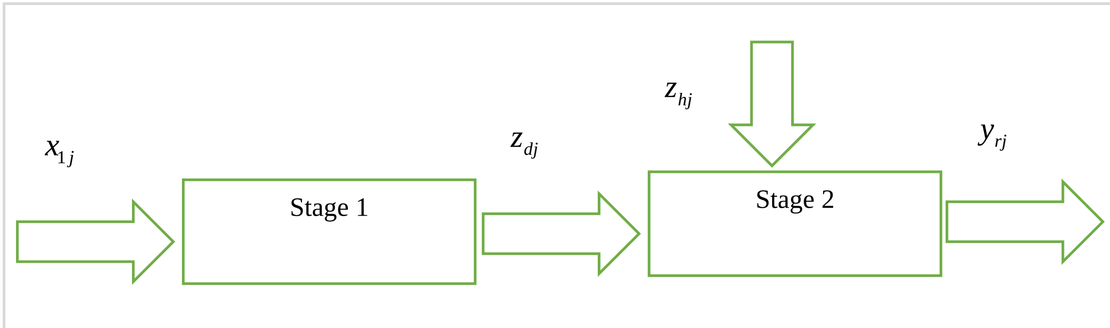

Figure 1: Sample of the two-stage process (Cook and Zhu, 2014Cook and Zhu, 2014)

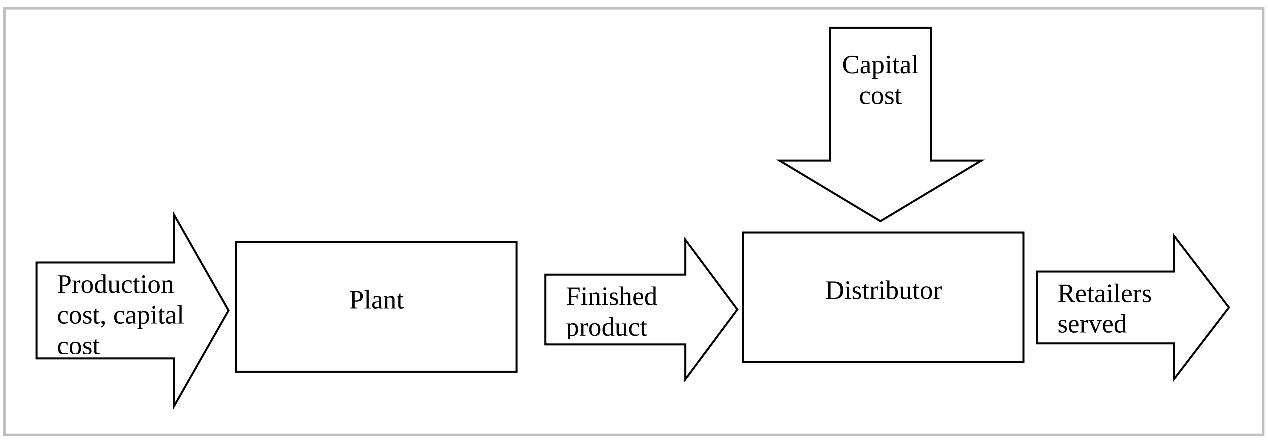

Figure 2: NDEA from case study

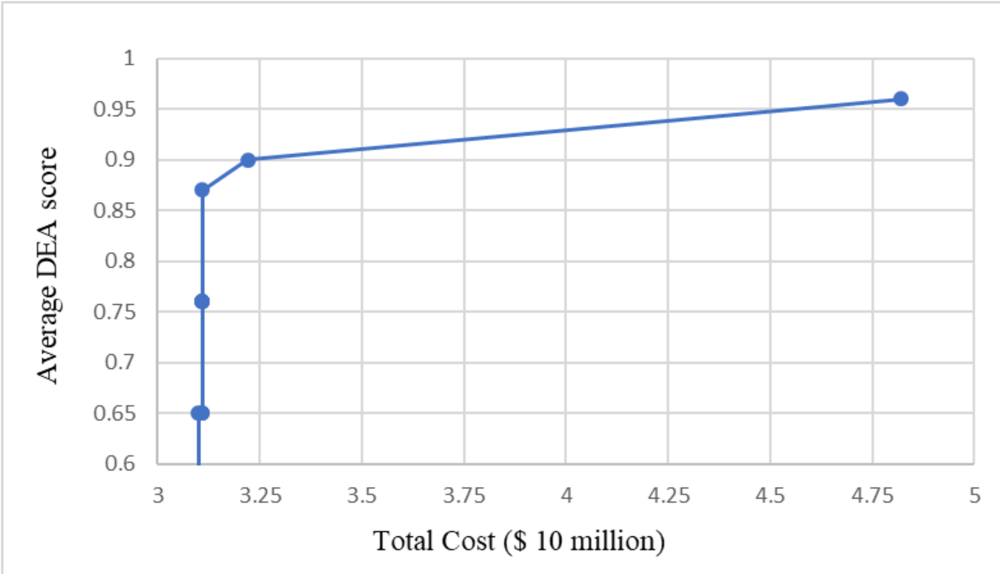

Figure 3: Tradeoff between facility location and transportation cost and sum DEA efficiency score for supply chain gained by an average of division scores

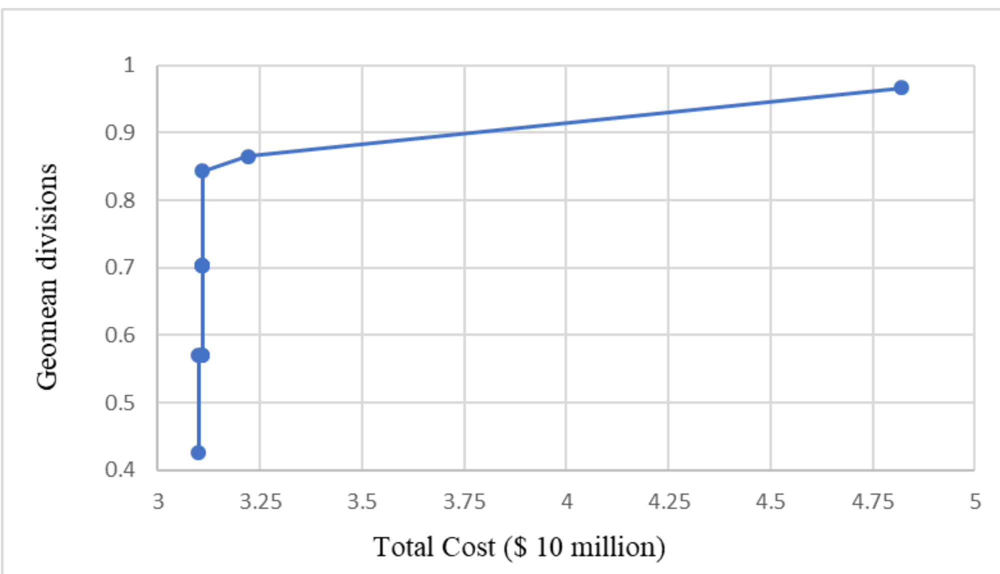

Figure 4: Tradeoff between facility location and transportation cost and DEA efficiency obtained by Geomean of division scores

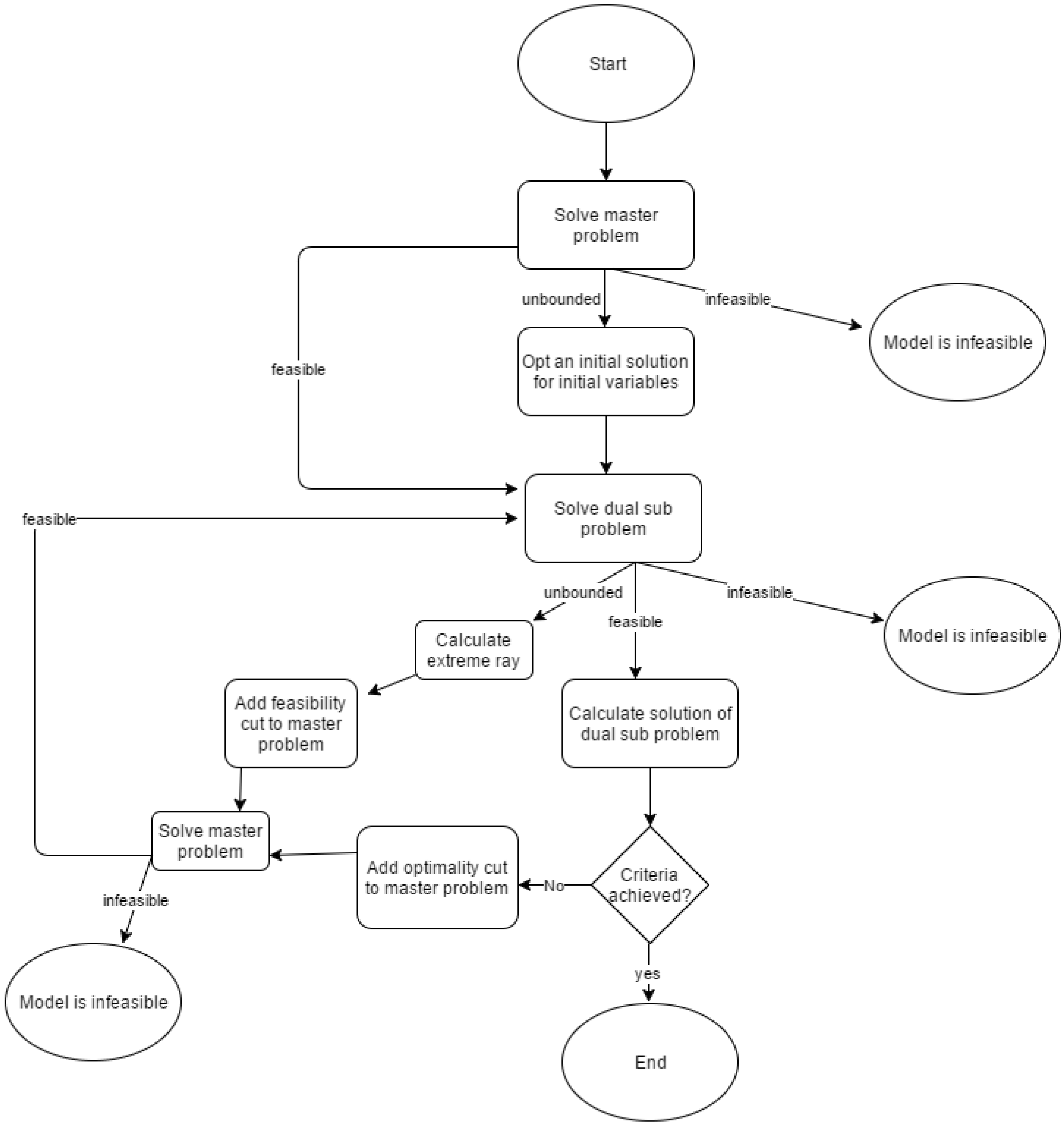

Figure 5: Solution Procedure Flowchart