Zhi W., H. Liao and **A. Emrouznejad** (2021) Information representation of blockchain technology: Risk evaluation of investment by personalized quantifier with cubic spline interpolation, *Information Processing & Management, **58 (4):*** 102571. (doi)

Information representation of blockchain technology: Risk evaluation of investment by personalized quantifier with cubic spline interpolation

Zhi Wen [1], Huchang Liao [1,*], Ali Emrouznejad [2]

[1] *Business School, Sichuan University, Chengdu 610064, China*
[2] *Aston Business School, Aston University, Birmingham, UK*

Abstract

With the applications of blockchain technology in various fields, the research on blockchain has attracted much attention. Different from the researches focusing on specific applications of blockchain technology in a certain field, this study devotes to capturing the attitudes of investors regarding different risk criteria in blockchain technology investment decision making. We use personalized quantifiers to extract investors' preferences on each risk evaluation criterion. At present, the personalized quantifier that can reflect individual attitudes and behavior intentions have been fitted by various functions, but there are still limitations. In this regard, this paper introduces a cubic spline interpolation function to fit the personalized quantifier, and addresses the consistency of the personalized quantifier in the ordered weighted averaging aggregation. Moreover, we employ a qualitative information representation model called probabilistic linguistic term sets to express decision-makers' evaluations on each criterion. We give a case study to illustrate the usability of the proposed personalized quantifier in blockchain risk evaluation. The comparative analysis with other four types of personalized quantifiers shows that our proposed personalized quantifier with cubic spline interpolation has ideal geometric characteristics in terms of smooth curve and high fitting accuracy, thus having strong applicability. Further, we show that this method is relatively easy to operate.

Keywords: Information representation of blockchain; Blockchain technology investment; risk evaluation; personalized quantifier; cubic spline interpolation; probabilistic linguistic information

1 Introduction

Because of the multiple advantages in terms of dispersion, persistence, anonymity and auditability, the

* Corresponding author. E-mail addresses: wenzhi_456789@163.com (Z. Wen); liaohuchang@163.com (H.C. Liao), a.emrouznejad@aston.ac.uk (A. Emrouznejad).

blockchain technique has been widely applied in various fields such as finance, logistics, public service, insurance, medical treatment, and digital copyright (Zheng et al., 2018; Yang, 2019). However, most of the current researches focused on specific applications of blockchain technology in a certain field, while few researches paid attention to the decision-making problems related to the blockchain technology. In this respect, Bai and Sarkis (2020) proposed a hybrid group decision-making method based on hesitant fuzzy sets and the regret theory to evaluate the applicability of the blockchain technology in the field of supply chain. Different from this decision-making problem, to increase the possibility of social investment in blockchain technology in various fields so as to promote the development of the blockchain technology, this paper takes the position of investors to carry out blockchain risk evaluation and select the low-risk fields according to the attitudinal characteristics of investors.

For such a decision-making problem, how to aggregate the information regarding the performance of applying the blockchain technology in various fields under various risk criteria is the key to solve this problem. The ordered weighted averaging (OWA) operator proposed by Yager (1988) is a popular aggregation method in which a weight vector is associated with specific ordered positions instead of specific elements. Among various methods to determine the weight vector of the OWA operator, the quantifier-guided method has been widely studied (Yager, 1993; Yager, 1996; Yager, 2004; Liu, 2006; Zhou and Chen, 2011). However, as Guo (2014) pointed out, most quantifiers used in these researches were parametric quantifiers with pre-determined parameter values, which cannot reasonably reflect the attitude characteristics of all decision-makers. In a decision-making process, if the selected quantifier is not suitable for the decision-maker, it may lead to an unreliable aggregation value. To solve this problem, Guo (2014) proposed the personalized quantifier which is closely associated with a specific person and could be used as a tool to investigate and formalize the person’s decision attitude and behavior intention. In that study, the personalized quantifier was described as a piecewise linear function. However, the curve of the personalized quantifier fitted by the piecewise linear interpolation is rigid around the connection point, which leads to its smoothness and interpretability not high. Hence, Guo (2016) further adopted Bernstein polynomials to denote the personalized quantifier. Compared with the personalized quantifier with piecewise linear interpolation, the personalized quantifier with Bernstein polynomials improves the geometrical characteristics of the corresponding curve and the operability of the quantifier. Nevertheless, the higher-order polynomial interpolation is easy to produce Runge phenomenon, that is, a sharp oscillation occurs on the boundary of an interpolation interval. As a higher-order polynomial, the curve fitted by Bernstein polynomials is prone to errors to some extent. Furthermore, the interpolation condition within the unit interval cannot be satisfied by this personalized quantifier. To avoid the limitations of the above two types of personalized quantifiers simultaneously, Guo (2018) combined the Bernstein polynomials with an interpolation spline to generate a personalized quantifier which satisfies the interpolation condition and accelerates the global convergence. Deficiently, its operation procedure is complex and not suitable for the situation that the personalized quantifier needs to be generated based on a large number of alternative information.

In this sense, the current study aims to further investigate the personalized quantifier to bridge the above challenges. Motivated by the idea of using the cubic spline interpolation to depict functions in the research of Sobrie *et al*. (2019), in this study, we introduce the cubic spline interpolation function to fit the personalized quantifier. The cubic spline interpolation function is one of the commonly used curve characterization methods due to its advantages such as simple construction, convenient use and precise fitting. The proposed approach produces the same effect as the polynomial interpolation, but its advantage is that also overcomes the oscillation phenomenon that may occur in higher-order polynomial interpolation.

In addition, in generating the personalized quantifiers regarding personal preferences, the subjective expectations provided by decision-makers play an important role. Many relevant literature (Guo, 2014; 2016; 2018) formed personal expectations on criteria of decision-makers with crisp values. However, for the problems with complexity and uncertainty, such as the risk assessment, it is difficult for decision-makers to express their subjective expectations by providing specific values directly. Compared with crisp values, language is a better way to express human cognition. In this respect, the probabilistic linguistic terms set (PLTS, Pang et al., 2016) is a powerful tool to describe linguistic evaluations by assigning the corresponding probability of each possible linguistic term. Hence, in this study, the PLTS is used for decision-makers to express their subjective expectations on criteria easily and accurately.

In summary, the innovations of this study are highlighted as follows:

1. We introduce a cubic spline interpolation function to depict the personalized quantifiers of decision-makers. The personalized quantifier with cubic spline interpolation can not only satisfy the interpolation condition within a unit interval, but also maintain the smoothness and interpretability of the fitted curve, and thus can be easily understood and operated;
2. We use PLTSs to express the subjective expectations of decision-makers for the generation of personalized quantifiers, which is more accurate and reliable than using crisp values;
3. Based on the personalized quantifier with cubic spline interpolation, we carry out an investment risk evaluation of applying the blockchain technology in various fields, and then outstand the advantages of the proposed personalized quantifier by comparative analysis.

The framework of this study is arranged as follows: Section 2 briefly reviews the relevant knowledge. Section 3 introduces the cubic spline interpolation function to generate the personalized quantifier. Section 4 gives a case study to demonstrate the application of the proposed personalized quantifier in blockchain risk evaluation. Section 5 makes comparative analysis to highlight the advantages of the proposed personalized quantifier. Section 6 summarizes the research results of this study and future research directions.

2 Preliminaries

This section reviews the relevant knowledge of the PLTS, cubic spline interpolation function and quantifier-guided OWA operator.

2.1 Probabilistic linguistic term set

The PLTS was presented by Pang *et al.* (2016) to express human cognition by a set of ordered and continuous linguistic terms and the importance of each linguistic term is distinguished by assigning the corresponding probability to each linguistic term. Suppose that $S=\{s_\alpha \mid \alpha=0,1\cdots,\tau\}$ is a given linguistic term set. A PLTS can be defined as: $L_S(p)=\left\{s_\alpha^{(e)}(p^{(e)}) \mid s_\alpha^{(e)}\in S, p^{(e)}\geq 0, e=1,2,\cdots E, \sum_{e=1}^{E} p^{(e)}\leq 1\right\}$, in which $s_\alpha^{(e)}(p^{(e)})$ represents the e *th* linguistic term $s_\alpha^{(e)}$ connected with $p^{(e)}$, and E represents the number of linguistic terms in $L_S(p)$.

To achieve the handy calculation among PLTSs, Pang *et al.* (2016) defined a calculation formula based on the subscripts of linguistic terms to get the score of a PLTS. However, this kind of calculation method may cause the operation values to exceed the bounds of the given linguistic term set (Liao et al., 2019; Wu and Liao, 2019). In contrast, the calculation method based on linguistic scale functions may be more reasonable. Hence, this paper applies the calculation method based on linguistic scale functions defined by Wu and Liao (2019) to convert PLTSs into expectative values within the interval [0, 1]. If we suppose the semantics of linguistic terms corresponding to the linguistic scale function are uniformly distributed, then the expectative value function of a PLTS is (Wu and Liao, 2019):

$$EV(L_S(p))=\sum_{e=1}^{E}\left(\frac{\alpha^{(e)}}{\tau}p^{(e)}\right)/\sum_{e=1}^{E}p^{(e)} \tag{1}$$

2.2 Cubic spline interpolation function

A cubic spline interpolation function consists of a set of cubic polynomials with continuous second derivative over the entire interval (Sobrie et al., 2019).

Definition 1. Let the domain of $f(x)$ lie in the interval [a, b], and the values of the function at the given $m+1$ connection point x_i ($i=1,2,\cdots,m$) are $f(x_i)$. If there is a function $Y(x)$ satisfying:

1. It is a cubic polynomial on each interval $[x_i, x_{i+1}]$ ($i=1,2,\cdots,m$),
2. $Y(x_i)=f(x_i)$ ($i=1,2,\cdots,m$) on each connection point,
3. The second derivative continues on the interval [a, b], that is, $Y(x)$, $Y^{'}(x)$ and $Y^{''}(x)$ are

continuous on the interval [a, b].

Then, $Y(x)$ is called a cubic spline interpolation function. In addition, $Y''(x_0) = Y''(x_m) = 0$ is called the natural boundary condition, and the cubic spline interpolation function satisfying the natural boundary condition is called the natural spline interpolation function.

Let $Y(x_i) = f(x_i)$, $v_i = x_i - x_{i-1}$. Then,

$$Y(x) = P_{i-1}\frac{(x_i - x)^3}{6v_i} + P_i\frac{(x - x_{i-1})^3}{6v_i} + (f(x_{i-1}) - \frac{P_{i-1}}{6}v_i^2)\frac{(x_i - x)}{v_i} + (f(x_i) - \frac{P_i}{6}v_i^2)\frac{(x - x_{i-1})}{v_i},$$

$$x \in [x_{i-1}, x_i]\ i = 1, 2, \cdots, m \tag{2}$$

In this function, the system of equations determining the coefficients $P_0, P_1, \cdots, P_m$ is

$$\gamma_i P_{i-1} + 2P_i + \eta_i P_{i+1} = h_i,\ \ 0 < i < m \tag{3}$$

where $\gamma_i = \frac{v_i}{v_i + v_{i+1}}$, $\eta_i = \frac{v_{i+1}}{v_i + v_{i+1}} = 1 - \gamma_i$, and $h_i = \frac{6}{v_i + v_{i+1}}(\frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} - \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}})$.

This is a system of linear equations with $m+1$ unknowns and $m-1$ equations. To determine the values of $P_0, P_1, \cdots, P_m$ completely, we need to supplement it according to the boundary conditions at two end points of the interval [a, b]. For the sake of generality, we take the boundary condition as the natural boundary condition in this paper, that is, $P_0 = P_m = 0$.

2.3 Quantifier guided OWA aggregation

In many multi-criteria decision-making methods, the solution of a problem is required to perform as good as possible under most criteria. But, in fact, a decision-maker may have a specific expectation for the performance of an alternative under a criterion. Only if the solution of a problem reaches the expectation values under most criteria, the solution may satisfy the decision-maker. In this respect, Yager (1988) proposed the OWA operator guided by quantifiers. Since then, the close relationship between OWA operators and linguistic quantifiers have been studied (Yager, 1993). This was further extended by Yager (1996) to the applications of fuzzy sets to represent linguistic quantifiers in order to calculate the weight vector of an OWA operator. Yager (2004) used quantifiers to capture the attitude characteristics of decision-makers so as to determine the weight vector and aggregate information in a continuous interval form. Liu (2006) proposed two kinds of parameterized quantifiers with an exponential function and a piecewise linear function, respectively. Amin and Emrouznejad (2011) proposed a parametric OWA operator while Zhou and Chen (2011) employed quantifiers to derive the weight vector of a continuous generalized OWA operator. Jin *et al.* (2019) derived the weight vector for the data without clear linear order through the idea of quantifiers. In these studies, decision-makers provided a quantifier Q to express the proportion of each criterion required for a good solution, which showed the role of the quantifier Q in determining the weight vector of an OWA operator. For a survey of OWA applications, please see Emrouznejad and Marra (2014).

The mathematical explanation of an OWA aggregation guided by quantifiers is as follows:

Consider a set of criteria $c_1, c_2, \cdots, c_j, \cdots, c_n$ with a weight vector $(w_1, w_2, \cdots, w_j, \cdots, w_n)^T$ satisfying $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. An OWA operator can be expressed as:

$$F_{OWA}(c_1, c_2, \cdots, c_j, \cdots, c_n) = \sum_{j=1}^{n} w_j \hat{c}_j \tag{4}$$

where $\hat{c}_j$ represents the jth largest value of all the criteria. Since the weight vector can be generated by the quantifier Q provided by the decision-maker, the OWA operator can also be expressed as:

$$F_{OWA}(c_1, c_2, \cdots, c_j, \cdots, c_n) = \sum_{j=1}^{n} (Q(\frac{j}{n}) - Q(\frac{j-1}{n}))\hat{c}_j , \tag{5}$$

where

$$w_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n}) \quad \text{for} \quad j = 1, 2, \cdots, n . \tag{6}$$

Yager (1996) argued that the regular increasing monotone (RIM) quantifiers are the basis of other types of quantifiers. Accordingly, other types of quantifiers can be transformed by the RIM quantifiers, hence, in this paper, we focus on the RIM quantifiers which satisfy $Q(0) = 0$, $Q(1) = 1$, and if $a \geq b$ then $Q(a) \geq Q(b)$.

3 Personalized quantifier with cubic spline interpolation

In this section, we introduce a generation process of the personalized quantifier with cubic spline function and illustrate its applications.

3.1 The generation of the personalized quantifier with cubic spline interpolation

The creation of personalized quantifier is based on the information extracted from specific people about their decision-making attitudes or behavior intentions. For a multi-criteria decision-making problem with m alternatives and n criteria, it is necessary to extract the corresponding decision attitude from the probabilistic linguistic evaluation information provided by the decision-maker to construct the personalized quantifier associate with the decision-maker. In this regard, the following work can be carried out:

According to the given linguistic term set, the decision-maker gives the probabilistic linguistic evaluation information to *m* alternatives under each criterion, and then a probabilistic linguistic decision matrix corresponding to the decision-maker can be constructed as:

$$\begin{bmatrix} L_S^{11}(p) & L_S^{12}(p) & \cdots & L_S^{1j}(p) & \cdots & L_S^{1n}(p) \\ L_S^{21}(p) & L_S^{22}(p) & \cdots & L_S^{2j}(p) & \cdots & L_S^{2n}(p) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ L_S^{i1}(p) & L_S^{i2}(p) & \cdots & L_S^{ij}(p) & \cdots & L_S^{in}(p) \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ L_S^{m1}(p) & L_S^{m2}(p) & \cdots & L_S^{mj}(p) & \cdots & L_S^{mn}(p) \end{bmatrix}$$

where $L_S^{ij}(p)$ represents the PLTS of the i th alternative under the j th criterion. Afterwards, the decision-maker needs to provide subjective expectations in the form of PLTSs for each criterion, $\hat{L}_S^1(p), \hat{L}_S^2(p), \cdots, \hat{L}_S^j(p), \cdots, \hat{L}_S^n(p)$. By Eq. (1), we can calculate the expectation values of all elements in the probabilistic linguistic decision matrix $EV(L_S^{ij}(p))$ ($i=1,2,\cdots,m$, $j=1,2,\cdots,n$) and the subjective expectations provided by the decision-maker, $EV(\hat{L}_S^j(p))$ ($j=1,2,\cdots,n$).

According to the constrained non-linear optimization model established by Guo (2014) to minimize the total distance between the evaluation values of alternatives under each criterion and the subjection expectations, the attitudinal weight vector $(AW_1, AW_2, \cdots, AW_m)^T$ can be calculated as follows:

$$AW_i = \frac{1}{\sum_{j=1}^{n}(EV(L_S^{ij}(p)) - EV(\hat{L}_S^j(p)))^2 \sum_{k=1}^{m} \frac{1}{\sum_{j=1}^{n}(EV(L_S^{kj}(p)) - EV(\hat{L}_S^j(p)))^2}}, \text{ for } i=1,2,\cdots,m \tag{7}$$

where $\sum_{i=1}^{m} AW_i = 1$ and $AW_i \geq 0$ ($i=1,2,\cdots,m$).

The attitudinal weight vector can be regarded as the weight vector generated by the quantifiers. According to Eq. (7), the personalized quantifier about the decision-maker can be obtained by:

$$Q(\frac{i}{m}) = \sum_{k=1}^{i} AW_k, \quad i=1,2,\cdots,m \tag{8}$$

Based on these values, we can fit a smooth and continuous function by the cubic spline interpolation to express the personalized quantifier:

$$Q(x) = mP_{i-1}^{'} \frac{(\frac{i}{m} - x)^3}{6} + mP_i^{'} \frac{(x - \frac{i-1}{m})^3}{6} + m(Q(\frac{i-1}{m}) - \frac{P_{i-1}^{'}}{6m^2})(\frac{i}{m} - x) + m(Q(\frac{i}{m}) - \frac{P_i^{'}}{6m^2})(x - \frac{i-1}{m}),$$

$$x \in [\frac{i-1}{m}, \frac{i}{m}] \tag{9}$$

where $i=1,2,\cdots,m$, and the system of equations determining the coefficients $P_0^{'}, P_1^{'}, \cdots, P_m^{'}$ is

$$\frac{1}{2}P_{i-1}^{'} + 2P_i^{'} + \frac{1}{2}P_{i+1}^{'} = h_i^{'}, \quad 0 < i < m \tag{10}$$

where $h_i^{'} = 3m^2(Q(\frac{i+1}{m}) - 2Q(\frac{i}{m}) + Q(\frac{i-1}{m}))$ and $P_0^{'} = P_m^{'} = 0$.

3.2 Applications of the personalized quantifier

Based on the personalized quantifier generated in the previous section, we can apply them in two aspects: (1) the weight vector in the OWA operator can be derived. According to the aggregation results, we can evaluate, classify or rank a number of alternatives for such a decision-making problem. Particularly, after the personalized quantifier generated, if the numbers of criteria and alternatives in the decision-making problem change, the weight of each criterion can be deduced according to the generated quantifiers, instead of regenerating the quantifiers. In this sense, compared with the method of determining the weights of criteria based on pairwise comparisons, personalized quantifiers not only need less information from decision makers, but also have higher flexibility and efficiency. (2) The attitude characteristics (*AC*) of decision makers can be captured. We can get the *AC* value of a decision-maker according to the total area of the personalized quantifier in each interval, where

$$\begin{aligned}
AC &= \int_0^1 Q_i(x)\,dx = \sum_{i=1}^{m}\int_{\frac{i-1}{m}}^{\frac{i}{m}} Q_i(x)\,dx \\
&= \sum_{i=1}^{m}\int_{\frac{i-1}{m}}^{\frac{i}{m}}[P_{i-1}^{'}\frac{(\frac{i}{m}-x)^3}{6v_i^{'}} + P_i^{'}\frac{(x-\frac{i-1}{m})^3}{6v_i^{'}} + (Q(\frac{i-1}{m}) - \frac{P_{i-1}^{'}}{6}v_i^{'2})\frac{(\frac{i}{m}-x)}{v_i^{'}} + (Q(\frac{i}{m}) - \frac{P_i^{'}}{6}v_i^{'2})\frac{(x-\frac{i-1}{m})}{v_i^{'}}]dx \\
&= \sum_{i=1}^{m}[\frac{mP_{i-1}^{'}}{6}(-\frac{1}{4}(\frac{i}{m}-x)^4)\Big|_{\frac{i-1}{m}}^{\frac{i}{m}} + \frac{mP_i^{'}}{6}(\frac{1}{4}(x-\frac{i-1}{m})^4)\Big|_{\frac{i-1}{m}}^{\frac{i}{m}} + m(Q(\frac{i-1}{m}) - \frac{P_{i-1}^{'}}{6m^2})(-\frac{1}{2}(\frac{i}{m}-x)^2)\Big|_{\frac{i-1}{m}}^{\frac{i}{m}} \\
&\quad + m(Q(\frac{i}{m}) - \frac{P_i^{'}}{6m^2})(\frac{1}{2}(x-\frac{i-1}{m})^2)\Big|_{\frac{i-1}{m}}^{\frac{i}{m}}] \\
&= \sum_{i=1}^{m}[\frac{1}{2m}(Q(\frac{i-1}{m}) + Q(\frac{i}{m})) - \frac{P_{i-1}^{'} + P_i^{'}}{24m^3}] \\
&= 1 - \frac{1}{m}\sum_{i=1}^{m} iAW_i + \frac{1}{2m} - \frac{1}{12m^3}\sum_{i=1}^{m-1} P_i^{'}
\end{aligned} \tag{11}$$

According to the value of *AC*, we can divide the decision-making attitudes of decision-makers into several pre-set categories. For example, if the decision-making attitude of a decision-maker is supposed to be divided into three categories, negative, neutral and positive, in advance, then referring to the idea that each category is evenly distributed in the interval [0,1], the decision-making attitude with the *AC* value in [0,0.33] can be classified as negative, the decision-making attitude with the *AC* value in [0.34,0.66] can be classified as neutral, and the decision-making attitude with the *AC* value in [0.67,1] can be classified as positive. Decision-makers with positive attitude think that the performance of alternatives is better at a higher level, decision-makers with neutral attitude think that the performance of alternatives is better at a medium level, and those with negative attitude think that the performance of alternatives is better at a lower level.

Furthermore, in the case of large-scale group decision making (Tang & Liao, 2021), we classify the decision-makers according to the attitude characteristic of each decision-maker, and assign corresponding weights to different types of decision-makers. For example, if the performance value of an alternative is considered to be the larger the better, then, the decision-makers with positive attitude will be assigned higher weights, and those with negative attitude will be assigned lower weights.

3.3 The consistency of the OWA aggregation derived by *Q*

To prove the applicability of the personalized quantifier with the cubic spline interpolation in OWA aggregation, we test the consistency of the OWA aggregation guided by the personalized quantifier. The content of the test is inspired by Guo (2016).

Let Q and Q' be two personalized quantifiers with the cubic spline interpolation deduced from the attitudinal weight vectors $AW=(AW_1, AW_2, \cdots, AW_m)^T$ and $AW'=(AW_1^{'}, AW_2^{'}, \cdots, AW_m^{'})^T$. In Eq. (11) on the calculation of AC, since the value of $\frac{1}{12m^3}\sum_{i=1}^{m-1} P_i^{'}$ is small, the default value of it is close to 0 here (the calculation of AC' value is the same). For AW and AW', let $Q(i/m)=\sum_{k=1}^{i} AW_k$ and $Q'(i/m)=\sum_{k=1}^{i} AW_k^{'}$ ($i=1, 2, \cdots, m$). F_{OWA} and $F_{OWA}^{'}$ are the OWA operators guided by Q and Q', respectively.

Theorem 1. If Q'' is deduced from $AW''=(AW_m, AW_{m-1}, \cdots, AW_1)^T$ (the reverse order of AW), then, the attitudinal characteristic related to Q'' is: $AC''=1-AC$.

Proof. Similar to that of the *Theorem 1* in Guo (2014).

Theorem 2. If $Q(i/m) \geq Q'(i/m)$, then: 1) $Q(x) \geq Q'(x)$ ($x \in [0,1]$); 2) $AC \geq AC'$; 3) $F_{OWA} \geq F_{OWA}^{'}$.

Proof. First, according to Eq. (9), $Q(x)$ increases with the increase of $Q((i-1)/m)$ and $Q(i/m)$. Similarly, $Q'(x)$ increases with the increase of $Q'((i-1)/m)$ and $Q'(i/m)$. Since $Q(i/m) \geq Q'(i/m)$ which represents $Q(i-1/m) \geq Q'(i-1/m)$, we can derive $Q(x) \geq Q'(x)$.

Secondly, according to Eq. (11), $AC=\sum_{i=1}^{m}[\frac{1}{2m}(Q(\frac{i-1}{m})+Q(\frac{i}{m}))-\frac{P_{i-1}^{'}+P_i^{'}}{24m^3}]$. Since $\frac{P_{i-1}^{'}+P_i^{'}}{24m^3}$ $=\frac{1}{12m^3}\sum_{i=1}^{m-1} P_i^{'} \approx 0$, $AC=\sum_{i=1}^{m}[\frac{1}{2m}(Q(\frac{i-1}{m})+Q(\frac{i}{m}))]$. Similarly, $AC'=\sum_{i=1}^{m}[\frac{1}{2m}(Q'(\frac{i-1}{m})+Q'(\frac{i}{m}))]$. $Q(i-1/m) \geq Q'(i-1/m)$ can be derived by

$Q(i/m) \geq Q'(i/m)$, which can further derive $\sum_{i=1}^{m}[\frac{1}{2m}(Q(\frac{i-1}{m})+Q(\frac{i}{m}))] \geq \sum_{i=1}^{m}[\frac{1}{2m}(Q'(\frac{i-1}{m})+Q'(\frac{i}{m}))]$. Thus, we can deduce $Q(x) \geq Q'(x)$.

Finally, Similar to that of the first point of *Theorem 2* in Guo (2016).

Theorem 3. If $Q(x) \geq Q'(x)$ ($x \in [0,1]$), then: 1) $F_{OWA} \geq F_{OWA}^{'}$; 2) $AC \geq AC'$.

Proof. First, Since $Q(x) \geq Q'(x)$, $\forall x \in [0,1]$, we have $Q(\frac{j}{n}) - Q(\frac{j-1}{n}) \geq Q'(\frac{j}{n}) - Q'(\frac{j-1}{n})$ ($j = 1, 2, \cdots, n$), which represents $\sum_{j=1}^{n}(Q(\frac{j}{n}) - Q(\frac{j-1}{n}))\hat{c}_j = \sum_{j=1}^{n}(Q'(\frac{j}{n}) - Q'(\frac{j-1}{n}))\hat{c}_j$, where $\hat{c}_j$ is a non-negative value. According to Eq. (5), we can deduce $F_{OWA} \geq F_{OWA}^{'}$.

Secondly, It is obvious that $Q(i/m) \geq Q'(i/m)$ derived by $Q(x) \geq Q'(x)$ ($x \in [0,1]$). Then, $AC \geq AC'$ can be obtained based on *Theorem 2*.

Theorem 4. If $F_{OWA} \geq F_{OWA}^{'}$, then $Q(x) \geq Q'(x)$ ($x \in [0,1]$).

Proof. Similar to that of *Theorem 5* in Guo (2016).

Theorem 5. If $AW_u AW_v^{'} \geq AW_u^{'} AW_v$ for any $v \geq u$ ($v, u = 1, 2, \cdots, m$), then $Q(i/m) \geq Q'(i/m)$.

Proof. Similar to that of *Theorem 2* in Liu and Han (2008).

Theorem 6. If $AW_j - AW_{j+1} \geq AW_j^{'} - AW_{j+1}^{'}$ for any $j = 1, 2, \cdots, m-1$, then $Q(i/m) \geq Q'(i/m)$.

Proof. Similar to that of *Theorem 3* in Liu and Han (2008).

4 Case study: Blockchain risk evaluation

In this section, a case study on the blockchain risk evaluation is given to illustrate the applicability of the personalized quantifier with the cubic spline interpolation. The influence of initial information change on the decision results is also considered.

4.1 Case description

Blockchain represents a different paradigm of transaction and record handling, which is mainly used to promote authentication and trust between various entities (Berdik, 2021). From the perspective of application, blockchain is a distributed shared ledger and database, which has the characteristics of decentralization, tamper-resistant, immutability, whole process trace, traceability, collective maintenance, openness and transparency. These characteristics enable the blockchain to solve the problem of information asymmetry and realize the cooperative trust and concerted action among multiple agents. In recent years, more and more attention has been paid to the blockchain technology, and the blockchain technique has been widely used in many fields. For example, Li et al. (2020) utilized the blockchain technique to develop a novel public auditing scheme for verifying data integrity in cloud storage; Zhao et al. (2020) used the blockchain technique to construct a novel privacy-preserving remote data integrity checking scheme for an Internet of Things information management system; Chen et al. (2020) applied the blockchain technology to determine the integrity of fake news; Baniata et al. (2021) combined the blockchain technology with fog computing to establish a task scheduling model.

At present, the investment capital of blockchain is gradually rising, and the investment density is increasing. The capital supply of investment side is expected to promote the further development of the blockchain technology. Nevertheless, the risks confronted with the applications of the blockchain technology in various fields are different. Investors may face such a decision-making problem: how to evaluate the risks confronted with the applications of the blockchain technology in various fields and choose the field with the lowest investment risk to invest. In the process of solving such a decision-making problem, it may be necessary to capture the attitude characteristics of decision-makers to ensure the reliability of decision-making results.

Suppose an investor intends to invest in the application of blockchain technology in a certain field. The candidate fields are finance, logistics, public service, insurance, medical treatment and digital copyright, represented as A_1, A_2, A_3, A_4, A_5, A_6, respectively. The risk evaluation criteria for the applications of blockchain technology are lack of return on investment (c_1), lack of business value (c_2), inadequate understanding of potential technology (c_3) and unsustainable usage scenarios (c_4) (Drljevic *et al*., 2019). A linguistic term set for blockchain risk evaluation is given as $\{s_0: \textit{extremely high}, s_1: \textit{very high}, s_2: \textit{high}, s_3: \textit{slightly high}, s_4: \textit{medium}, s_5: \textit{slightly low}, s_6: \textit{low}, s_7: \textit{very low}, s_8: \textit{extremely low}\}$.

4.2 Solve the problem by personalized quantifier with cubic spline interpolation

First, the investor gives the subjective expectation information in form of PLTSs for the four criteria, $\hat{L}_S^1(p)=\{s_7(0.8), s_8(0.2)\}$, $\hat{L}_S^2(p)=\{s_6(0.4), s_7(0.6)\}$, $\hat{L}_S^3(p)=\{s_3(0.5), s_4(0.3)\}$ and $\hat{L}_S^4(p)=\{s_4(0.2), s_5(0.3), s_6(0.4)\}$, and provides the probabilistic linguistic evaluation information for each field under each criterion according to the given linguistic term set. The probabilistic linguistic evaluation matrix for the problem is formed as:

$$\begin{array}{c|llll} & c_1 & c_2 & c_3 & c_4 \\ A_1 & \{s_4(0.2), s_5(0.3), s_6(0.4)\} & \{s_7(0.7)\} & \{s_3(0.6), s_4(0.3)\} & \{s_6(0.4), s_7(0.3), s_8(0.2)\} \\ A_2 & \{s_5(0.6)\} & \{s_6(0.3), s_7(0.5)\} & \{s_2(0.5), s_3(0.5)\} & \{s_4(0.3), s_5(0.6)\} \\ A_3 & \{s_5(0.5), s_6(0.5)\} & \{s_7(0.4), s_8(0.2)\} & \{s_4(0.7), s_5(0.2)\} & \{s_3(0.8)\} \\ A_4 & \{s_6(0.2), s_7(0.5), s_8(0.1)\} & \{s_4(0.3), s_5(0.4)\} & \{s_2(0.8)\} & \{s_4(0.5), s_5(0.3)\} \\ A_5 & \{s_6(0.4), s_7(0.3)\} & \{s_5(0.2), s_6(0.6)\} & \{s_1(0.6)\} & \{s_2(0.7), s_3(0.3)\} \\ A_6 & \{s_6(0.8)\} & \{s_5(0.1), s_6(0.3), s_7(0.4)\} & \{s_1(0.5), s_2(0.5)\} & \{s_5(0.6), s_6(0.4)\} \end{array}$$

By Eq. (1), the expected values of these probabilistic linguistic information are calculated as follows:

$$EV(\hat{L}_S^1(p)) = 0.900,\quad EV(\hat{L}_S^2(p)) = 0.825,\quad EV(\hat{L}_S^3(p)) = 0.422,\quad EV(\hat{L}_S^4(p)) = 0.653.$$

$$\begin{bmatrix} 0.653 & 0.875 & 0.417 & 0.847 \\ 0.625 & 0.828 & 0.313 & 0.583 \\ 0.688 & 0.917 & 0.528 & 0.375 \\ 0.859 & 0.571 & 0.250 & 0.547 \\ 0.804 & 0.719 & 0.125 & 0.288 \\ 0.750 & 0.797 & 0.188 & 0.675 \end{bmatrix}$$

Suppose that we take the evaluation information of the first five fields as the data information for generating the personalized quantifier of the investor. The decision matrix can be obtained by rearranging the values under each criterion in descending order:

$$\begin{bmatrix} 0.859 & 0.917 & 0.528 & 0.847 \\ 0.804 & 0.875 & 0.417 & 0.583 \\ 0.688 & 0.828 & 0.313 & 0.547 \\ 0.653 & 0.719 & 0.250 & 0.375 \\ 0.625 & 0.571 & 0.125 & 0.288 \end{bmatrix}$$

By Eq. (7), we can derive the attitudinal weight vector $(0.169, 0.600, 0.147, 0.056, 0.028)^T$. Then, based on Eq. (8), the values with respect to the personalized quantifier can be obtained as $Q_1(0) = 0$, $Q_1(\frac{1}{5}) = 0.169$, $Q_1(\frac{2}{5}) = 0.769$, $Q_1(\frac{3}{5}) = 0.916$, $Q_1(\frac{4}{5}) = 0.972$ and $Q_1(1) = 1$. The function values of the personalized quantifier with cubic spline interpolation are displayed in Table 1.

Table 1. The function values of the personalized quantifier

	x_0	x_1	x_2	x_3	x_4	x_5
x_i	0	0.2	0.4	0.6	0.8	1
$Q_1(x_i)$	0	0.169	0.769	0.916	0.972	1

To fit the corresponding cubic spline interpolation function, we need to establish a system of equations to obtain the coefficient values in the interpolation function. By Eq. (3), we can calculate that $\gamma_i^{'} = \eta_i^{'} = 0.5$ ($i = 1, 2, 3, 4, 5$), $h_1^{'} = 32.325$, $h_2^{'} = -33.975$, $h_3^{'} = -6.825$ and $h_4^{'} = -2.1$. So, the established equations are as follows:

$$\begin{cases} 2P_1^{'} + 0.5P_2^{'} = 32.325 \\ 0.5P_1^{'} + 2P_2^{'} + 0.5P_3^{'} = -33.975 \\ 0.5P_2^{'} + 2P_3^{'} + 0.5P_4^{'} = -6.825 \\ 0.5P_3^{'} + 2P_4^{'} = -2.1 \end{cases}$$

Solving this system of equations, we can get that $P_1^{'}$ = 21.959, $P_2^{'}$ = -23.183, $P_3^{'}$ = 2.824 and $P_4^{'}$ = -1.231. Taking these values into Eq. (2), the personalized quantifier with cubic spline interpolation function about the investor can be generated as:

$$Q_1(x) = \begin{cases} 18.299x^3 + 0.113x\text{ , } 0 \le x \le 0.2 \\ -37.618x^3 + 33.550x^2 - 6.597x + 0.447\text{ , } 0.2 \le x \le 0.4 \\ 21.672x^3 - 37.598x^2 + 21.862x - 3.347\text{ , } 0.4 \le x \le 0.6 \\ -3.379x^3 + 7.494x^2 - 5.211x + 2.075\text{ , } 0.6 \le x \le 0.8 \\ 1.026x^3 - 3.078x^2 + 3.177x - 0.125\text{ , } 0.8 \le x \le 1 \end{cases}$$

This personalized quantifier can be plotted as Fig. 1. According to this personalized quantifier and Eq. (6), the weights of five criteria used for the OWA aggregation can be derived as follows:

$$w_1 = Q_1(\frac{1}{n}) - Q_1(0) = Q_1(0.25) - 0 = 0.307\text{ ,}$$

$$w_2 = Q_1(\frac{2}{n}) - Q_1(\frac{1}{n}) = Q_1(0.5) - Q_1(0.25) = 0.893 - 0.307 = 0.586\text{ ,}$$

$$w_3 = Q_1(\frac{3}{n}) - Q_1(\frac{2}{n}) = Q_1(0.75) - Q_1(0.5) = 0.957 - 0.893 = 0.064\text{ ,}$$

$$w_4 = Q_1(\frac{4}{n}) - Q_1(\frac{3}{n}) = Q_1(1) - Q_1(0.75) = 1 - 0.957 = 0.043\text{ .}$$

Then, the OWA aggregation values of all alternatives can be derived by Eq. (5): $F_{OWA}^{(1)}(A_1)$ = 0.825, $F_{OWA}^{(1)}(A_2)$ = 0.671, $F_{OWA}^{(1)}(A_3)$ = 0.735, $F_{OWA}^{(1)}(A_4)$ = 0.644, $F_{OWA}^{(1)}(A_5)$ = 0.692, $F_{OWA}^{(1)}(A_6)$ = 0.735. Thus, the ranking of these fields under the guidance of the subjective expectation of the investor is $A_1 > A_6 > A_3 > A_5 > A_2 > A_4$, which implies that the field with the lowest risk is finance.

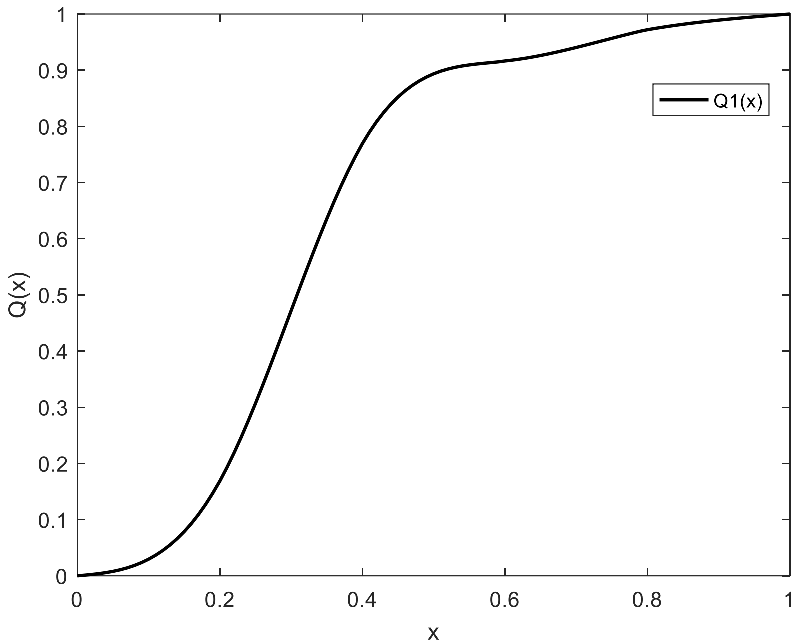

Fig. 1. The personalized quantifier based on the information of the first five fields

In addition, the AC value with regard to the personalized quantifier can be calculated by Eq. (11). The result is $AC_1 = 1 - \frac{1}{m}\sum_{i=1}^{m} iAW_i + \frac{1}{2m} - \frac{1}{12m^3}\sum_{i=1}^{m-1} P_i^{'} = 0.665$. If we divide the decision-making attitudes into three categories: negative, neutral and positive, then, the investment attitude of the investor is classified as neutral. The above results show that the investors should invest more in the applications of the blockchain technology in the financial field under the guidance of the decision-making attitude of preferring the alternative with medium risk.

4.3 Sensitive analysis

Below we make sensitive analysis about the above decision-making process.

Adding an evaluation criterion

Suppose that we add another risk evaluation criterion, namely, the inadequate personal privacy protection (c_5). The probabilistic linguistic evaluation information of each alternative under this criterion is also provided by the investor and expressed as $L_S^{15}(p) = \{s_5(0.4), s_6(0.5)\}$, $L_S^{25}(p) = \{s_7(0.8)\}$, $L_S^{35}(p) = \{s_6(0.6), s_7(0.3)\}$, $L_S^{45}(p) = \{s_4(0.2), s_5(0.3), s_6(0.4)\}$, $L_S^{55}(p) = \{s_5(0.6), s_6(0.4)\}$ and $L_S^{65}(p) = \{s_6(0.7)\}$, respectively. By Eq. (1), we can calculate the expected value of these information as $EV(L_S^{15}(p)) = 0.694$, $EV(L_S^{25}(p)) = 0.875$, $EV(L_S^{35}(p)) = 0.792$, $EV(L_S^{45}(p)) = 0.653$, $EV(L_S^{55}(p)) = 0.675$ and $EV(L_S^{65}(p)) = 0.75$. According to the personalized quantifier $Q_1(x)$ and Eq. (6), we can derive the weights of five criteria as $W_1^{'} = 0.169$, $W_2^{'} = 0.600$, $W_3^{'} = 0.147$ and $W_4^{'} = 0.056$, $W_5^{'} = 0.028$. Then, the

OWA aggregation values of all alternatives can be calculated as: $F_{OWA}^{(2)}(A_1) = 0.806$, $F_{OWA}^{(2)}(A_2) = 0.778$, $F_{OWA}^{(2)}(A_3) = 0.771$, $F_{OWA}^{(2)}(A_4) = 0.659$, $F_{OWA}^{(2)}(A_5) = 0.686$, $F_{OWA}^{(2)}(A_6) =0.738$. The ranking of various fields under the guidance of the subjective expectations of the investor is $A_1 > A_2 > A_3 > A_6 > A_5 > A_4$.

Compared with the ranking result obtained in Section 4.2, although the lowest risk field is still finance, the ranking of alternatives A_2 and A_6 has changed greatly. This means that changes in the number of criteria will affect the final ranking result. Fortunately, the change in the numbers of criteria will not lead to entire solution process being used from scratch. In the case of adding or removing criteria, we can still easily get a solution with stability through the personalized quantifier generated from a part of the information selected at the beginning.

<u>*Changing the number of alternatives for personalized quantifier generation*</u>

Suppose that we take the evaluation information of the first four fields as the data information for generating personalized quantifier about the investor. The decision matrix can be obtained by rearranging the values under each criterion in descending order:

$$\begin{bmatrix} 0.859 & 0.917 & 0.528 & 0.847 \\ 0.688 & 0.875 & 0.417 & 0.583 \\ 0.653 & 0.828 & 0.313 & 0.547 \\ 0.625 & 0.571 & 0.250 & 0.375 \end{bmatrix}$$

From Eqs. (7) and (8), we can derive the attitudinal weight vector $(0.326, 0.367, 0.229, 0.078)^T$. The values with respect to the personalized quantifier can be obtained as $Q_2(0) = 0$, $Q_2(\frac{1}{4}) = 0.326$, $Q_2(\frac{2}{4}) = 0.693$, $Q_2(\frac{3}{4}) = 0.922$ and $Q_2(1) = 1$. The personalized quantifier associated with the investor can be generated by the cubic spline interpolation as:

$$Q_2(x) = \begin{cases} 1.161x^3 + 1.231x\,, 0 \le x \le 0.25 \\ -3.182x^3 + 3.257x^2 + 0.417x + 0.068\,, 0.25 \le x \le 0.5 \\ 0.110x^3 - 1.680x^2 + 2.886x - 0.344\,, 0.5 \le x \le 0.75 \\ 1.911x^3 - 5.733x^2 + 5.926x - 1.104\,, 0.75 \le x \le 1 \end{cases}$$

The geometric demonstration of this personalized quantifier is shown in Fig. 2, from which we can find that the personalized quantifier $Q_2(x)$ is smoother than $Q_1(x)$ presented in Section 4.2. The reason for this is that the personalized quantifier $Q_2(x)$ is generated based on the smaller number of alternatives compared with $Q_1(x)$. According to this personalized quantifier and Eq. (5), the OWA aggregation values of all alternatives can be derived as: $F_{OWA}^{(3)}(A_1) = 0.778$, $F_{OWA}^{(3)}(A_2) = 0.657$, $F_{OWA}^{(3)}(A_3) = 0.702$, $F_{OWA}^{(3)}(A_4) = 0.634$, $F_{OWA}^{(3)}(A_5) = 0.602$, $F_{OWA}^{(3)}(A_6) = 0.704$. The ranking of the various fields is $A_1 > A_6 > A_3 > A_2 > A_4 > A_5$, which is similar to the ranking derived from the personalized quantifier

$Q_1(x)$, and the least risky field remains finance field. This shows that in the process of personalized quantifier generation, the number of alternatives may not make much difference to the final ranking result. In the case of a large number of alternatives, we can consider to intercept a small number of alternatives to generate the personalized quantifier to get the ranking of all alternatives.

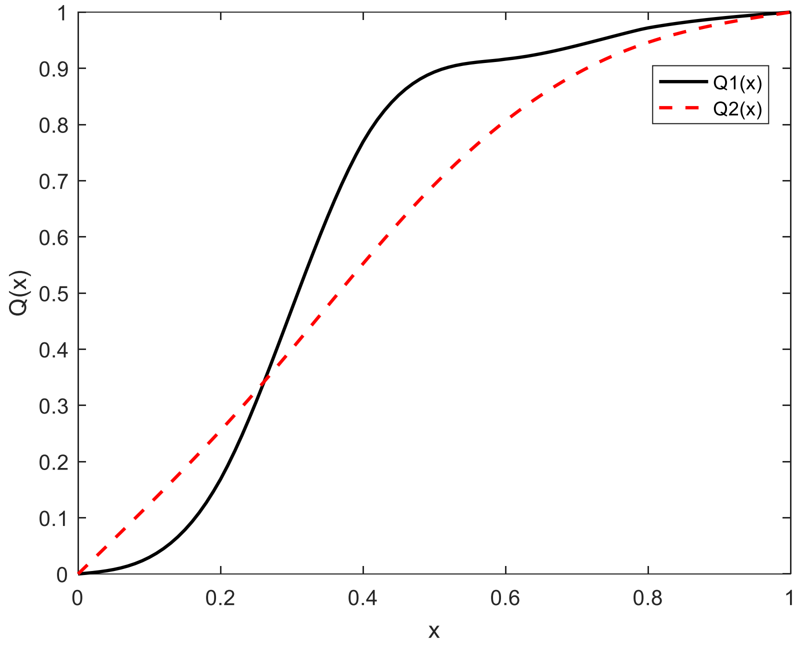

Fig. 2. The comparison of personalized quantifiers $Q_1(x)$ and $Q_2(x)$

Moreover, the AC value associated with this personalized quantifier generated from four alternatives can be measured as 0.616. The investor's investment attitude is classified as neutral, which is the same as the AC value with respect to the personalized quantifier generated by five alternatives. This means that the change of the number of alternatives has little effect on capturing the attitude characteristics of decision-makers.

5 Comparative analysis

Based on the data in Section 4, this section compares the personalized quantifier with cubic spline interpolation with the personalized quantifier with piecewise linear interpolation, personalized quantifier with Bernstein polynomials, and personalized quantifier by Bernstein polynomials combined with interpolation spline, respectively.

5.1 The personalized quantifier with piecewise linear interpolation

The personalized quantifier with piecewise linear interpolation was proposed by Guo (2014), who used the following piecewise linear function to denote the personalized quantifier:

$$Q_3(x) = (mx - i)AW_i + Q(\frac{i}{m}),\ \ x \in [\frac{i-1}{m}, \frac{i}{m}],\ \ 0 < i < m$$

Based on the function values of the personalized quantifier in Table 1, the personalized quantifier with piecewise linear interpolation can be generated as:

$$Q_3(x) = \begin{cases} 0.845x,\ 0 \le x \le 0.2 \\ 3x - 0.431,\ 0.2 \le x \le 0.4 \\ 0.735x + 0.475,\ 0.4 \le x \le 0.6 \\ 0.28x + 0.748,\ 0.6 \le x \le 0.8 \\ 0.14x + 0.86,\ 0.8 \le x \le 1 \end{cases}$$

The geometric demonstration of this personalized quantifier is shown in Fig. 3. The OWA weight vector is derived by this personalized quantifier and Eq. (6) as $(0.319, 0.524, 0.115, 0.042)^T$. Then, the OWA aggregation values of all alternatives can be calculated as: $F_{OWA}^{(4)}(A_1) = 0.816$, $F_{OWA}^{(4)}(A_2) = 0.672$, $F_{OWA}^{(4)}(A_3) = 0.730$, $F_{OWA}^{(4)}(A_4) = 0.647$, $F_{OWA}^{(4)}(A_5) = 0.672$, $F_{OWA}^{(4)}(A_6) = 0.733$. The ranking result can be obtained as $A_1 > A_6 > A_3 > A_5 > A_2 > A_4$, which is the same as the result derived from the personalized quantifier with cubic spline interpolation $Q_1(x)$. Furthermore, we compute the *AC* value associated with this personalized quantifier as:

$$AC_3 = \int_0^1 Q_i(x)\,dx = 1 - \frac{1}{m}\sum_{i=1}^{m} iAW_i + \frac{1}{2m} = 0.665\,.$$

This value is the same as the *AC* value corresponding to the personalized quantifier with cubic spline interpolation, and it is also classified as neutral. However, from Fig. 3, we can find that the curve of the personalized quantifier $Q_3(x)$ is rigid at the connection point and thus its interpretability is not high. By contrast, the curve of the proposed personalized quantifier $Q_1(x)$ is smoother.

5.2 The personalized quantifier with Bernstein polynomials

The personalized quantifier with Bernstein polynomials was introduced by Guo (2016), who applied the following Bernstein polynomials to represent the personalized quantifier to improve the geometrical characteristics of the personalized quantifier denoted by the piecewise linear function:

$$Q_4(x) = \sum_{i=0}^{m} Q(\frac{i}{m}) C_m^i x^i (1-x)^{m-i},\ 0 \le x \le 1,\ \ 0 < i < m$$

Based on the function values of the personalized quantifier in Table 1, the personalized quantifier with piecewise linear interpolation can be generated as:

$$Q_4(x) = -1.545x^5 + 6.23x^4 - 8.84x^3 + 4.31x^2 + 0.845x,\ 0 \le x \le 1$$

The geometric demonstration of this personalized quantifier is shown in Fig. 3. It is clear to see from this figure that the smoothness of the function image is improved. By this personalized quantifier and Eq. (6), the OWA weight vector can be derived as $(0.365, 0.371, 0.197, 0.067)^T$, and the OWA aggregation value of each alternative can be calculated as: $F_{OWA}^{(5)}(A_1) = 0.790$, $F_{OWA}^{(5)}(A_2) = 0.670$, $F_{OWA}^{(5)}(A_3)$ = 0.719, $F_{OWA}^{(5)}(A_4)$ = 0.650, $F_{OWA}^{(5)}(A_5)$ = 0.625, $F_{OWA}^{(5)}(A_6)$ = 0.715. The ranking result $A_1 > A_3 > A_6 > A_2 > A_4 > A_5$ can be obtained, which is different from the result derived by the personalized quantifier with cubic spline interpolation $Q_1(x)$. Moreover, the AC value of this personalized quantifier can be computed as

$$AC_4 = \int_0^1 Q_i(x)\,dx = 1 - \frac{1}{m+1}\sum_{i=1}^{m} iAW_i = 0.638\,.$$

It is also classified as neutral. However, from Fig. 3, we can see that there is a large gap in the fitted function at the given connection points. For example, the value of the fitted personalized quantifier should be 0.169 at x = 0.2, but the value of the personalized quantifier $Q_4(x)$ at x = 0.2 is 0.28. In this sense, this personalized quantifier cannot satisfies the interpolation condition within a unit interval. Hence, the accuracy of the personalized quantifier fitted by Bernstein polynomials is not high. By contrast, the accuracy of the proposed personalized quantifier $Q_1(x)$ is higher.

5.3 The personalized quantifier by Bernstein polynomials combined with interpolation spline

The personalized quantifier by Bernstein polynomials combined with the interpolation spline was proposed by Guo (2018), who introduced a sequence of piecewise nonlinear polynomials with an adjustable degree to denote the personalized quantifier:

$$Q_5(x) = \begin{cases} (2m)^{2t}\sum_{k=0}^{2t} G_i(\frac{i-1}{m}+\frac{k}{4mt})C_{2t}^k(x-\frac{i-1}{m})^k(\frac{i-1/2}{m}-x)^{2t-k},\ x\in[\frac{i-1}{m},\frac{i-1/2}{m}] \\ (2m)^{2t}\sum_{k=0}^{2t} G_i(\frac{i-1/2}{m}+\frac{k}{4mt})C_{2t}^k(x-\frac{i-1/2}{m})^k(\frac{i}{m}-x)^{2t-k},\ x\in[\frac{i-1/2}{m},\frac{i}{m}] \end{cases},\quad 0<i<m$$

where t is any positive integer (t is usually equal to 1 for ease of calculation), and the value of $G_i(x)$ depends on the piecewise linear function plot composed of points ($\frac{i-1}{m}$, $Q(\frac{i-1}{m})$), ($\frac{i-3/4}{m}$, $Q(\frac{i-1}{m})$), ($\frac{i-1/2}{m}$, $\frac{1}{2}(Q(\frac{i-1}{m})+Q(\frac{i}{m}))$), ($\frac{i-1/4}{m}$, $Q(\frac{i}{m})$) and ($\frac{i}{m}$, $Q(\frac{i}{m})$), in which the $Q(x)$ value corresponding to x is the value of $G_i(x)$.

Based on the function values of the personalized quantifier in Table 1, the personalized quantifier Bernstein polynomials combined with the interpolation spline can be generated as:

$$Q_5(x)=\begin{cases} 8.45x^2\ ,\ 0\le x\le 0.1 \\ -8.45x^2+3.38x-0.169\ ,\ 0.1\le x\le 0.2 \\ 30x^2-12x+1.369\ ,\ 0.2\le x\le 0.3 \\ \text{-}30x^2+24x-4.031\ ,\ 0.3\le x\le 0.4 \\ 7.35x^2-5.88x+1.945\ ,\ 0.4\le x\le 0.5 \\ \text{-}7.35x^2+8.82x-1.73\ ,\ 0.5\le x\le 0.6 \\ 2.8x^2-3.36x+1.924\ ,\ 0.6\le x\le 0.7 \\ \text{-}2.8x^2+4.48x-0.802\ ,\ 0.7\le x\le 0.8 \\ 1.4x^2-2.24x+1.868\ ,\ 0.8\le x\le 0.9 \\ \text{-}1.4x^2+2.8x-0.4\ ,\ 0.9\le x\le 1 \end{cases}$$

The geometric demonstration of this personalized quantifier is shown in Fig. 3, and we can find that the fitting function image of this personalized quantifier has great fluctuation than other personalized quantifiers. The OWA weight vector is $(0.244,\ 0.598,\ 0.141,\ 0.017)^T$. The OWA aggregation values of all alternatives are: $F_{OWA}^{(6)}(A_1)=0.819$, $F_{OWA}^{(6)}(A_2)=0.663$, $F_{OWA}^{(6)}(A_3)=0.716$, $F_{OWA}^{(6)}(A_4)$ = 0.632, $F_{OWA}^{(6)}(A_5)$ = 0.669, $F_{OWA}^{(6)}(A_6)$ = 0.741. The ranking result is $A_1 > A_6 > A_3 > A_5 > A_2 > A_4$, which is the same as the result derived by the personalized quantifier with the cubic spline interpolation $Q_1(x)$. The AC value associated with this personalized quantifier is the same as AC_3.

In summary, the personalized quantifier by Bernstein polynomials combined with the interpolation spline has a large amount of calculation and the calculation process is relatively complex, so it is not suitable for the situation that the personalized quantifier needs to be generated based on a large number of alternative information. By contrast, the proposed personalized quantifier $Q_1(x)$ is more applicable.

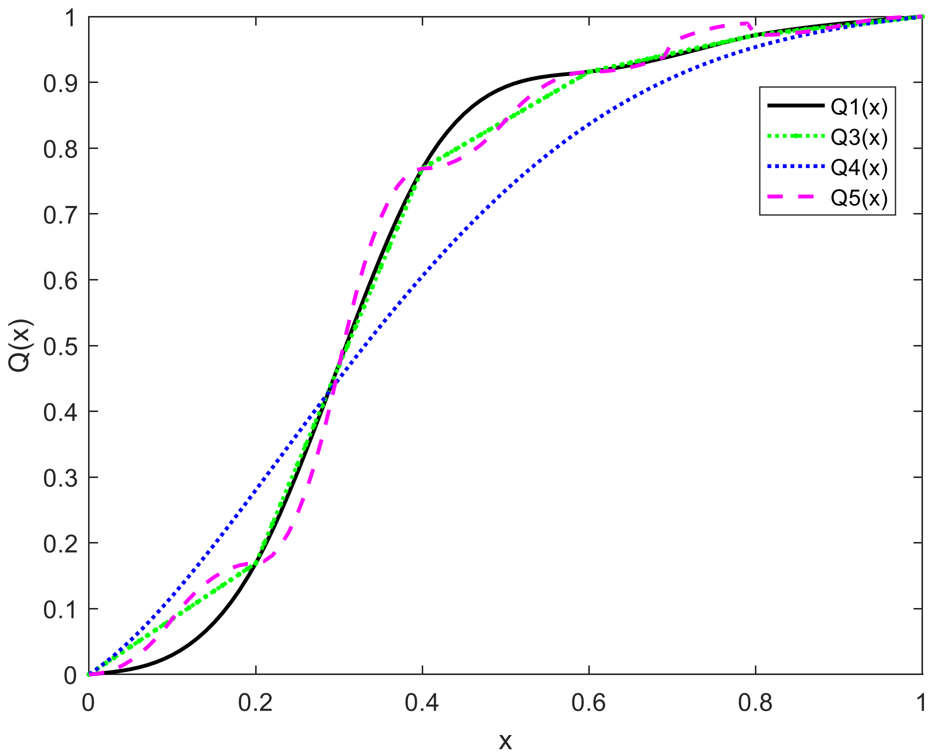

Fig. 3. The comparison of personalized quantifiers generated by different fitting functions

According to the above comparison with the four types of the personalized quantifiers, it can be demonstrated that the proposed personalized quantifier with the cubic spline interpolation not only improves the geometric characteristics in terms of smooth curve and high fitting accuracy, but also is relatively easy to operate.

6 Conclusions and directions for future research

Different from the literature on specific applications of the blockchain technique in various fields, this paper is about information representation of blockchain technology from position of investors to carry out blockchain risk evaluation. This study introduced the personalized quantifier with cubic spline interpolation to extract the investor's preferences for each blockchain risk evaluation criterion and determined the field with the lowest risk for the investor. Compared with other decision-making methods, this method guided by the personalized quantifiers of decision-makers highlights the attitude characteristics of them, while the change in the numbers of criteria and alternatives will not lead to the whole solution process starting from scratch. A case study was given to demonstrate the flexible applicability of the proposed personalized quantifier in blockchain risk evaluation. From the comparative analysis, we can find that the personalized quantifier with cubic spline interpolation not only has good smoothness, but also has good approximation to the inserted function as a whole when the connection points are gradually encrypted, and the corresponding derivative values converge to the derivative of the inserted function.

One of the limitations in this study is consideration of a personalized quantifier related to one kind of decision-making attitude of a decision-maker. Researchers interested could extend the approach presented in this study to describe the corresponding personalized quantifier for other decision-making attitude assumptions of the decision-maker or the decision-making attitudes of multiple decision-makers. It is also suggested to extend this idea to consider the introduction of personalized quantifiers in large-scale group decision making (Tang & Liao, 2021) to classify decision-makers.

Acknowledgements

The work was supported by the National Natural Science Foundation of China under Grant 71771156, 71971145 and the Chengdu Planning Project of Social Science under Grant YY2320191038.

References

Amin, G. R. and A. Emrouznejad (2011). Parametric aggregation in ordered weighted averaging. *International Journal of Approximate Reasoning*, *52*(6), 819-827. https://doi.org/10.1016/j.ijar.2011.02.004.

Bai, C. G., & Sarkis, J. (2020). A supply chain transparency and sustainability technology appraisal model for blockchain technology. *International Journal of Production Research*, *58*(7), 2142-2162. https://doi.org/10.1080/00207543.2019.1708989.

Baniata, H., Anaqreh, A., & Kertesz, A. (2021). PF-BTS: A privacy-aware fog-enhanced blockchain-assisted task scheduling. *Information Processing & Management*, *58*(1), 102393. https://doi.org/10.1016/j.ipm.2020.102393.

Berdik, D., Otoum, S., Schmidt, N., Porter, D., & Jararweh, Y. (2021). A survey on blockchain for information systems management and security. *Information Processing & Management*, *58*(1), 102397. https://doi.org/10.1016/j.ipm.2020.102397.

Chen, Q., Srivastava, G., M.Parizi, R., Aloqaily, M., & AlRidhawi, I. (2020). An incentive-aware blockchain-based solution for internet of fake media things. *Information Processing & Management*. *57*(6), 102370. https://doi.org/10.1016/j.ipm.2020.102370.

Drljevic, N., Aranda, D. A., & Stantchev, V. (2019). Perspectives on risks and standards that affect the requirements engineering of blockchain technology. *Computer Standards & Interfaces*, 103409. https://doi.org/10.1016/j.csi.2019.103409.

Emrouznejad, A. & Marianna M. (2014). Ordered weighted averaging operators 1988-2014: A citation-based literature survey. *International Journal of Intelligent Systems*, *29*(11), 994-1014. https://doi.org/10.1002/int.21673.

Guo, K. H., & Xu, H. (2018). Personalized quantifier by Bernstein polynomials combined with interpolation spline. *International Journal of Intelligent Systems*, 33, 1507-1533. https://doi.org/10.1002/int.21991.

Guo K. H. (2014). Quantifier induced by subjective expected value of sample information. *IEEE Transactions on Cybernetics*, *44*(10), 1784-1794. https://doi.org/10.1109/TCYB.2013.2295316.

Guo K. H. (2016). Quantifier induced by subjective expected value of sample information with Bernstein polynomials. *European Journal of Operational Research*, 254, 226-235. https://doi.org/10.1016/j.ejor.2016.03.015.

Jin, L., Mesiar, R., & Yager, R. R. (2019). Ordered weighted averaging aggregation on convex poset. *IEEE Transactions on Fuzzy Systems*, *27*(3), 612-617. https://doi.org/10.1109/tfuzz.2019.2893371.

Liao, H. C., Qin, R., Gao, C. Y., Wu, X. L., Hafezalkotob, A., & Herrera, F. (2019). Score-HeDLiSF: A score function of hesitant fuzzy linguistic term set based on hesitant degrees and linguistic scale functions: An application to unbalanced hesitant fuzzy linguistic MULTIMOORA. *Information Fusion*, 48, 39-54. https://doi.org/10.1016/j.inffus.2018.08.006.

Li, J., Wu, J., Jiang, G., & Srikanthan, T. (2020). Blockchain-based public auditing for big data in cloud storage. *Information Processing & Management*, *57*(6), 102382. https://doi.org/10.1016/j.ipm.2020.102382 .

Liu, X. W. (2006). Some properties of the weighted OWA operator. *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, *36*(1), 118-127. https://doi.org/10.1109/tsmca.2005.854496.

Liu, X. W., & Han, S. L. (2008). Orness and parameterized RIM quantifier aggregation with OWA operators: A summary. *International Journal of Approximate Reasoning*, *48*(1), 77-97. https://doi.org/10.1016/j.ijar.2007.05.006.

Pang, Q., Wang, H., & Xu, Z. S. (2016). Probabilistic linguistic term sets in multi-attribute group decision making. *Information Science*, 369, 128-143. https://doi.org/10.1016/j.ins.2016.06.021.

Sobrie, O., Gillis, N., Mousseau, V., & Pirlot, M. (2018). UTA-poly and UTA-splines: Additive value functions with polynomial marginals. *European Journal of Operational Research*, *264*(2), 405-418. https://doi.org/10.1016/j.ejor.2017.03.021.

Tang, M., & Liao, H.C. (2021). From conventional group decision making to large-scale group decision making: what are the challenges and how to meet them in big data era? A state-of-the-art survey. *Omega*, 102141. https://doi.org/10.1016/j.omega.2019.102141.

Wu, X. L., Liao, H. C., Xu, Z. S., Hafezalkotob, A., & Herrera, F. (2018). Probabilistic linguistic

MULTIMOORA: A multicriteria decision making method based on the probabilistic linguistic expectation function and the improved Borda rule. *IEEE Transportation Fuzzy System*, *26*(6), 3688-3702. https://doi.org/10.1109/tfuzz.2018.2843330.

Wu, X. L., & Liao, H. C. (2019). A consensus-based probabilistic linguistic gained and lost dominance score method. *European Journal of Operational Research*, 272, 1017-1027. https://doi.org/10.1016/j.ejor.2018.07.044.

Yager, R. R. (1993). Families of OWA operators. *Fuzzy Sets and Systems*, *59*(2), 125-148. https://doi.org/10.1016/0165-0114(93)90194-m.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on systems, Man, and cybernetics*, *18*(1), 183-190. https://doi.org/001S-9472/88/0100-0183$01.00.

Yager, R. R. (2004). OWA aggregation over a continuous interval argument with applications to decision making.” *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics) 34*(5), 1952-1963. https://doi.org/10.1109/tsmcb.2004.831154.

Yager, R. R. (1996). Quantifier guided aggregation using OWA operators. *International Journal of Intelligent Systems*, *11*(1), 49-73. https://doi.org/0884-8173/%/010049-25.

Yang, L. (2019). The blockchain: State-of-the-art and research challenges. *Journal of Industrial Information Integration*, 15, 80-90. https://doi.org/10.1016/j.jii.2019.04.002.

Zhao, Q., Chen, S., Liu, Z., Baker, T., & Zhang, Y. (2020). Blockchain-based privacy-preserving remote data integrity checking scheme for IOT information systems. *Information Processing & Management*, *57*(6), 102355. https://doi.org/10.1016/j.ipm.2020.102355.

Zheng, Z. B., Xie, S. A., Dai, H. N., Chen, X. P., & Wang, H. M. (2018). Blockchain challenges and opportunities: A survey. *International Journal of Web and Grid Services*, *14*(4), 352-375. https://doi.org/10.1504/IJWGS.2018.095647.

Zhou, L. G., & Chen, H. Y. (2011). Continuous generalized OWA operator and its application to decision making. *Fuzzy Sets and Systems*, *168*(1), 18-34. https://doi.org/10.1016/j.fss.2010. 05.009.