Journal of the Operational Research Society

ISSN: 0160-5682 (Print) 1476-9360 (Online) Journal homepage: https://www.tandfonline.com/loi/tjor20

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To cite this article: Feng Li, Ali Emrouznejad, Guo-liang Yang & Yongjun Li (2020) Carbon emission abatement quota allocation in Chinese manufacturing industries: An integrated cooperative game data envelopment analysis approach, Journal of the Operational Research Society, 71:8, 1259-1288, DOI: 10.1080/01605682.2019.1609892

To link to this article: https://doi.org/10.1080/01605682.2019.1609892

Published online: 05 Jul 2019.

JOURNAL OF THE OPERATIONAL RESEARCH SOCIETY
2020, VOL. 71, NO. 8, 1259–1288
https://doi.org/10.1080/01605682.2019.1609892

ORIGINAL ARTICLE

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Carbon emission abatement quota allocation in Chinese manufacturing industries: An integrated cooperative game data envelopment analysis approach

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ABSTRACT

The Chinese government announced to cut its carbon emissions intensity by 60%–65% from its 2005 level. To realize the national abatement commitment, a rational allocation into its subunits (i.e. industries, provinces) is eagerly needed. Centralized allocation models can maximize the overall interests, but might cause implementation difficulty and fierce resistance from individual subunits. Based on this observation, this article will address the carbon emission abatement quota allocation problem from decentralized perspective, taking the competitive and cooperative relationships simultaneously into account. To this end, this article develops an integrated cooperative game data envelopment analysis (DEA) approach. We first investigate the relative efficiency evaluation by taking flexible carbon emission abatement allocation plans into account, and then define a super-additive characteristic function for developing a cooperative game among units. To calculate the nucleolus-based allocation plan, a practical computation procedure is developed based on the constraint generation mechanism. Further, we present a two-layer way to allocate the CO_2 abatement quota into different sub-industries and further different provinces in Chinese manufacturing industries. The empirical results show that five sub-industries (Processing of petroleum, coking and processing of nuclear fuel; Smelting and pressing of ferrous metals; Manufacture of non-metallic mineral products; Manufacture of raw chemical materials and chemical product; Smelting and pressing of non-ferrous metals) and two provinces (Guangdong and Shandong) will be allocated more than 10% of the total national carbon emission abatement quota.

ARTICLE HISTORY
Received 26 February 2019
Accepted 4 April 2019

KEYWORDS
Data envelopment analysis; carbon emission allocation; cooperative game; Chinese manufacturing industries; nucleolus

1. Introduction

Along with the economic development in the past few decades, the climate change has become one of the most important international issues. The global warming would cause some disastrous consequences and threaten the survival and development of all human beings (Feng, Chu, Ding, Bi, & Liang, 2015; Guo et al., 2010; Wu, Chu, & Liang, 2016). To protect and govern the environment, the greenhouse gas emission abatement has been put on the agenda. In fact, many countries and international organizations have exerted continuous efforts to reduce their greenhouse gas emissions in reacting to global warming (Soytas & Sari, 2009; Yu, Wei, & Wang, 2014). Furthermore, it is widely acknowledged that the primary component of greenhouse gases is CO_2, which contributes more than 50% to the atmospheric warming (IPCC, 2007), thus special attention should be paid to the CO_2 emissions and its control strategy. As the largest energy consumer and carbon emitter in the world, China faces an enormous pressure to cut its carbon emission level (Wang, Zhang, Wei, & Yu, 2013; Wei, Ni, & Du, 2012; Yu et al., 2014; Zhang, Wang, & Da, 2014). In June 2015, at the Climate Change Summit held in Paris, the Chinese government submitted its voluntary carbon emissions abatement plan to United Nations, which committed to reduce its CO_2 emissions by 60%–65% per unit gross domestic product (GDP) from the 2005 level.[1] It is a great responsibility and contribution that China has announced for the humankind and international society. However, to realize the reduction commitment, a reasonable allocation of the national goal to different industries and provinces is of vital importance and necessity. For this

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reason, we would study the carbon emission abatement (CEA) quota allocation problem in China.

The carbon emission abatement allocation is a special resource allocation problem, which uses certain mechanisms to allocate the total CEA quota across the decision making units (DMUs) (Sun, Wu, Liang, Zhong, & Huang, 2014). It is notable that lots of previous articles have addressed the CEA allocation problem, while almost all studies adopt a centralized perspective (Chiu, Lin, Su, & Liu, 2015; Gomes & Lins, 2008; Lozano, Villa, & Brännlund, 2009; Wu, Du, Liang, & Zhou, 2013). The optimal allocation mechanism from the perspective of the central decision maker will indeed maximize the overall interests, but also ignore the interests of individuals. That is to say, the centralized allocation plan would harm some DMUs' interests and cause suboptimal results for these DMUs (Feng et al., 2015). There is no doubt that in practice the centralized allocation plan might cause implementation difficulty and fierce resistance from DMUs and even a possibility of departures from the centralized allocation plan. For instance, the United States has announced to withdraw from the 2015 Paris Agreement saying that "the Paris accord will undermine (the U.S.) economy" and "puts (the U.S.) at a permanent disadvantage" (Chakraborty, 2017). Under this circumstance, the solution is to develop decentralized approaches. Based on this observation, we will try to address the carbon emission abatement quota allocation problem from a decentralized perspective.

Note in addition that there exist competitive and cooperative relationships simultaneously among related units in determining the carbon emission abatement quota allocation plan. On one hand, since the abatement quota allocated to each DMU is regarded as a constraint restricting its carbon emission permits (Wu et al., 2016), the individual DMU would compete with each other to minimize its allocated carbon emission abatement amount. On the other hand, some DMUs may also have enough incentives to establish alliances with others to achieve collaborative carbon emission abatement (Zhang et al., 2014), as the cooperation of carbon emission abatement is helpful to share relevant resources and curb the costs while cutting the carbon emission level. It is clear that considering the competitive and cooperative relationships among all DMUs simultaneously would provide valuable insights and generate a carbon emission abatement quota allocation plan that is more acceptable and stable. However, few works has been done on this topic. Motivated by this idea, a game theoretical approach would be of vital significance, and it is possible to cause satisfied results for each DMU through using game-based approaches.

In this article, we will use the famous nonparametric mathematical programming method, data envelopment analysis (DEA), to address the carbon emission abatement quota allocation problem in Chinese manufacturing industries. The most significant advantage of DEA methodology is that it requires no pre-specification of production functions and can also handle desirable and undesirable outputs simultaneously, which is highly fit with the circumstance of carbon emission abatement allocation problem. We consider that the national commitment to reduce its carbon emissions intensity by 60%–65% is essentially a total amount of carbon dioxide emission abatement quota. To address it, we develop an integrated cooperative game DEA approach to explore the carbon emission abatement allocation issue in Chinese manufacturing industries. To this end, we first investigate the relative efficiency evaluation by taking the flexible carbon emission abatement quota allocation plans into account. Analysis shows that any DMU can be efficient and all DMUs can also be simultaneously efficient with a common set of weights and allocation plans. Further, it involves in a phenomenon called the egoist's dilemma (Nakabayashi & Tone, 2006) in determining a unique carbon emission abatement quota allocation plan, which implies also the game space. Afterwards, we define a super-additive characteristic function and develop a practical computation procedure to calculate the nucleons solution based on the constraint generation mechanism of Hallefjord, Helming, and Jørnsten (1995). Finally, we apply the integrated cooperative game DEA approach to allocate the carbon emission abatement quota in Chinese manufacturing industries in 2012 through a two-layer way, and analysis shows that the cooperative game DEA approach can generate a well-defined and acceptable allocation plan upon negotiations. In particular, five sub-industries (Processing of petroleum, coking and processing of nuclear fuel; Smelting and pressing of ferrous metals; Manufacture of non-metallic mineral products; Manufacture of raw chemical materials and chemical product; Smelting and pressing of non-ferrous metals) and two provinces (Guangdong and Shandong) will undertake more than ten percent of the total national carbon emission abatement commitment quota.

The major contribution of this article can be summarized as bellows: first, we develop an integrated cooperative game DEA approach, which takes the competitive and cooperative relationships among all DMUs into account. As a result, the generated allocation mechanism is considered as fair enough and all DMUs have motivations to accept the allocation scheme in the sense of compromise.

Second, we present a two-layer allocation framework, namely, it allocates the national abatement goal to different manufacturing industries in the first stage, and in the second stage it further allocates the abatement share of each industry into different provinces. This article is different from existing literature that focuses mainly on provincial or regional allocation studies. Third, we applied the integrated cooperative game DEA approach to the empirical study of Chinese manufacturing industries. Hence, it presents a feasible way for Chinese government to realize its carbon emission abatement commitment that is submitted to United Nations ahead of the Paris Climate Change Summit, 2015. On the theoretical aspect, this article develops a new approach to address the carbon emission abatement quota allocation problem from a decentralized perspective, which is different from existing literature with mainly centralized models. On the application aspect, it solves a real-world problem and provides practical findings and implications.

The remainder of this article is organized as follows. In Section 2, we survey a relevant literature review. In Section 3, we introduce the real-world problem and summarize mathematical notations. Later, Section 4 proposes a cooperative game DEA approach and also a computation procedure of nucleolus solution based on the constraint generation mechanism. Afterwards, we present a two-layer empirical study of allocating the carbon emission abatement quota in Chinese manufacturing industries in Section 5. Eventually, Section 6 concludes this article and provides some perspectives.

2. Literature review

This article will address the carbon emission abatement allocation problem in Chinese manufacturing industries through integrating data envelopment analysis and game theory. Hence, there are mainly three relevant research streams, i.e. DEA and game DEA approach, DEA-based resource allocation study, and environmental performance and carbon emission abatement research.

2.1. DEA and game DEA approach

DEA, known as a famous nonparametric method to evaluate the relative efficiency of peer DMUs that convert multiple inputs into multiple outputs, was first introduced by Charnes, Cooper, and Rhodes (1978) with constant returns to scale (CRS) assumption and further extended by Banker, Charnes, and Cooper (1984) with variable returns to scale (VRS) assumption. The underlying logic of DEA methodology is that there exists an ideal performance (i.e. efficient production possibility surface) that can be used to assess the relative efficiency of individual DMUs. To this end, a convex combination of a set of comparable and homogeneous DMUs is calculated to construct an efficiency frontier. Then each DMU can be projected onto the frontier, and the certain DMU is evaluated by comparing itself to its projection on that frontier.

Since its seminal work in Charnes et al. (1978) and Banker et al. (1984), the DEA methodology has attracted more and more attention from scholars all over the world, and the DEA methodology and its applications have been extensively studied in the literature (Emrouznejad, 2014; Emrouznejad & Yang, 2018). On the application aspect, the literature has witnessed DEA-based approaches in different areas from public sector such as universities, hospitals, sports, and disaster relief operations to private sector such as banks, supply chains, manufacturing industries, and mergers and acquisitions (An, Meng, Ang, & Chen, 2018; Li, Liang, Li, & Emrouznejad, 2018a; Li, Zhu, & Zhuang, 2018e). On the methodology aspect, many innovative concepts and models have been proposed to extend and enrich the DEA theory. For example, Sexton, Silkman, and Hogan (1986) suggested using peer appraisal to replace traditional self-evaluation in cross-efficiency approach, and peer DMUs' optimal relative weights are used to evaluate other DMUs' efficiency. Andersen and Petersen (1993) proposed a novel super-efficiency approach, which excludes the evaluated DMU while constructing the efficiency frontier. Both the cross-efficiency approach and super-efficiency approach are supposed to improve DEA's discrimination power among efficient DMUs. In view of the fact that traditional DEA methods do not take DMUs' internal production system into account, Färe and Grosskopf (1996) proposed a network DEA model which opens the "black-box." Another two most important approaches are directional distance function (Chung, Färe, & Grosskopf, 1997) and slacks-based measure (Tone, 2001), both of which relax the proportional input extraction and output expansion requirement. The former requires the input extraction and output expansion in a given direction, while the later immediately estimates the input and output slacks without explicit direction. Additionally, some integrated approaches are also proposed. For example, Tone and Tsutsui (2009) firstly extended the lacks-based measure (SBM) model to network situations. Li, Shi, Emrouznejad, Xie, and Liang (2018g) proposed a network SBM approach for evaluating the environmental performance of Chinese industrial systems. Arabi, Munisamy, and Emrouznejad (2015) proposed a slacks-based distance function approach and applied it to calculate the Malmquist–Luenberger productivity index. Kao and Liu (2018) studied the cross-efficiency measurement and decomposition for both series and parallel production system.

The traditional DEA methodology maximizes the individual utility without considering the impacts of other DMUs' decisions, however, in circumstances associated with conflicts of interests each DMU must pay attention to other DMUs involved and corresponding impacts, that is, all DMUs should make a consensus and transigent decision. After all, there always exist direct or indirect competitions among all DMUs (Liang, Wu, Cook, & Zhu, 2008a). To take the competition and cooperation into account, some game DEA approaches and its applications have been studied. Banker (1980) provided a two-person zero-sum game to interpret the DEA efficiency, and that work was further extended to a constrained version by Banker, Charnes, Cooper, and Clarke (1989). Liang et al. (2008a) proposed a DEA game cross efficiency method to address the non-uniqueness of cross efficiency scores, and proved that the optimal cross-efficiency scores construct a Nash equilibrium. Further, Wu, Liang, Yang, and Yan (2009a) and Wu, Liang, Yang (2009b) adopted a Nash bargaining game and a cooperative game, respectively, to improve the traditional cross efficiency methods. Liang, Cook, and Zhu (2008b) proposed both a centralized cooperative game DEA model and a non-cooperative game DEA model for evaluating series-linked two-stage network processes. Omrani, Beiragh, and Kaleibari (2015) combined principal component analysis technique with conventional DEA models to reduce the number of inputs and outputs, and further combined the bargaining game with DEA model to obtain more realistic efficiency results. Li, Zhu, Chen, and Xue (2018b) unified the unbalanced evaluation standard in cross-efficiency method, and further proposed a game-like iterative procedure to obtain the optimal balanced cross-efficiency scores. Omrani, Shafaat, and Alizadeh (2019) integrated data envelopment analysis and cooperative game for evaluating energy efficiency of transportation sector in Iran. To see more studies on game DEA approaches the readers are encouraged to Chen, Liang, and Yang (2006), Du, Liang, Chen, Cook, and Zhu (2011), Wu and Liang (2012), etc.

This article will propose an integrated cooperative game DEA approach for the carbon emission abatement quota allocation problem. The proposed approach is also based on a directional distance function concept, and we consider both desirable outputs and undesirable outputs simultaneously in the game framework.

2.2. DEA-based resource allocation study

Resource allocation problem has traditionally become one of the most important application areas of DEA methodology (Fang & Zhang, 2008; Korhonen & Syrjänen, 2004; Li, Song, Dolgui, & Liang, 2017b). Among the literature, most studies are brought in a centralized environment (Asmild, Paradi, & Pastor, 2009; Lozano & Villa, 2004). For example, Fang and Zhang (2008) allocated variable resources to DMUs by maximizing both the total efficiency from centralized decision-making environment and the individual efficiency for each DMU. Lotfi, Noora, Jahanshahloo, Gerami, and Mozaffari (2010) and Lotfi, Nematollahi, Behzadi, Mirbolouki, and Moghaddas (2012) addressed the centralized resource allocation problem using enhanced Russell models. Given a capital budget constraint, Lozano, Villa, and Canca (2011) proposed a series of centralized DEA models for individual and collective output target setting, input reallocation and additional input acquisitions. Fang (2013) proposes a generalized DEA model that integrated the Lozano and Villa (2004) method and the Asmild et al. (2009) model as a special case to address the resource allocation problem. Pachkova (2009) introduced transfer costs of resources into the resource allocation problem, which is realized by a price matrix. Lozano (2014) proposed a SBM model for fixed cost and common revenue allocation in a centralized environment. Fang (2016) proposed a centralized resource allocation approach based on revenue efficiency, and the allocation plan was determined by maximizing the total output revenue. Ding, Chen, Wu, and Wei (2018) addressed the centralized fixed cost allocation problem by considering technology heterogeneity for different DMUs. An, Chen, Xiong, Wu, and Liang (2017) studied the intermediate output setting problem by considering fairness concern in a two-stage system.

There are also some studies that use parametric DEA approaches for resource allocation problems. For these studies, the efficiency frontier is supposed to have a specific hyperbolic shape. Avellar, Milioni, and Rabello (2007) made the first attempt to allocate a new fixed input across DMUs by considering a spherical frontier. Interestingly, Avellar et al. (2007) obtained a straightforward formula to calculate the resource amount for each DMU, and all DMUs will be finally efficient. Then Avellar, Milioni, Rabello, and Simão (2010) extended the same approach to another case where an already existing input resource will be reallocated across a set of DMUs. Milioni, de Avellar, Rabello, and De Freitas (2011b) further extended the Avellar et al. (2007) approach from input resource allocation to output target setting, and the authors studied both new fixed output setting and existing output resetting. Guedes, Milioni, de Avellar, and Silva (2012) proposed a new adjusted spherical frontier DEA model for input allocation, which has an important feature called coherence, implying that the generated allocation plan will be relatively stable and will not change with a small data modification. Milioni, de Avellar, and Gomest al. (2011a) proposed another parametric DEA model where the efficiency frontier has an ellipsoidal shape. Silva, Milioni, and Teixeira (2018) generalized the previous parametric DEA approaches for fairly allocating a new and fixed output under a centralized environment. Their new model can not only incorporate value judgments, but also be useful under increasing, constant, and decreasing returns to scale (RTS) properties. As Li, Zhu, and Liang (2019b) commented that the parametric DEA approaches can solve the resource allocation problem very easily, but the key focus is that whether it is acceptable to predefine a hyperbolic frontier or not.

Except for the above studies, some studies integrated the resource allocation and target setting into one problem. For example, Athanassopoulos (1995) applied goal programming and DEA to integrate target setting and resource allocation in multilevel planning problems. They extended the traditional efficiency-based orientation of DEA model to generate resource allocation and target setting scheme. Bi, Ding, Luo, and Liang (2011) studied the resource allocation and target setting for parallel production systems, and the authors tried to maximize the efficiency scores for all DMUs as well as the worst DMU under a set of common weights. Li et al. (2017b) addressed the resource allocation and target setting problem on the basis of two principles of efficiency invariance and common weights. In general the authors would give two possible allocation plans, with one emphasizing on efficiency invariance and the other on common weights.

Fixed cost is also a special resource that has attracted lots of research attention (Beasley, 2003; Cook & Kress, 1999). The first DEA-based fixed cost allocation research was proposed by Cook and Kress (1999), where two basic principles, efficiency-invariance and input Pareto optimality, were suggested. The efficiency-invariance principle is significant since the fixed allocation should not be utilized by any DMU to improve its performance. On the contrary, Beasley (2003) claimed that all DMUs will find the allocation plan be acceptable as it can realize an efficiency of one as compared with peer DMUs. Further, Beasley (2003) proposed a nonlinear problem to maximize the average efficiency score across all DMUs. Li et al. (2013) and Si et al. (2013) proved that all DMUs can be simultaneously efficient by considering the allocated cost as a new independent input and maximize the efficiency score as possible. Li et al. (2019b) proposed a novel non-egoistic principle for allocating a total fixed cost in a decentralized environment, which suggests that each DMU should propose its non-egoistic allocation proposal by allocating the maximal cost to itself. Li, Zhu, and Chen (2019) studied the fixed cost allocation problem by taking the internal two-stage network into account, and all DMUs' operation sizes are used to generated the final allocation plan such that it is consistent with the current input consumptions and output productions from a size point of view.

Note in particular that some game-DEA approaches were also proposed for the resource allocation problem. Nakabayashi and Tone (2006) studied a phenomenon called the "egoist's dilemma," and proposed a series of games to allocate cost and benefit. Du, Cook, Liang, and Zhu (2014) considered all DMUs as players, and suggested a game cross-efficiency iterative procedure to allocate fixed cost and input resource. The allocation plan will be obtained until all DMUs have the maximal cross-efficiency score (as the authors themselves indicated that the score will be one). Li, Zhu, and Liang (2018d) developed a game-DEA cross efficiency approach for allocating a total fixed cost across a set of competing DMUs, in which the final allocation plan is derived from the fair Shapley values and associated common weights. Li, Li, Emrouznejad, Liang, and Xie (2019c) also integrated DEA and cooperative game theory to allocate a total fixed cost. Both Li et al. (2018d, 2019c) provide us good lesson and reference, while this article will consider a reduction allocation problem and integrate undesirable outputs with desirable outputs in the cooperative game DEA allocation approach.

This article will also address a kind of resource allocation problem, but the target is on the carbon emission abatement quota. Such a problem not only involves undesirable outputs, but also is an output reduction allocation problem. In addition, this article will use an integrated cooperative game DEA approach to generate the carbon emission abatement quota allocation scheme. More importantly, this article adopts a decentralized perspective for the carbon emission abatement quota allocation problem, which is different from existing literature with mainly centralized perspectives.

2.3. Environmental performance and carbon emission abatement research

The last research stream focuses on the study of environmental performance and carbon emission abatement, which has received substantial research attention recently (Feng et al., 2015; Yu et al., 2014; Zhou & Ang, 2008; Zhou, Ang, & Poh, 2008). Herein we focus only on DEA-based studies for reference. The traditional DEA methodology assumes that outputs have to be maximized and inputs have to be minimized (Scheel, 2001), whereas when we address the environmental performance there always exist undesirable outputs (Chung et al., 1997), which is desired to be minimized. Through modelling undesirable outputs, lots of articles have addressed the environmental performance. Reinhard, Lovell, and Thijssen (2000) used both stochastic frontier approach and DEA approaches to estimate the environmental efficiency scores for Dutch dairy farms. Korhonen and Luptacik (2004) measured technical efficiency (the relation of desirable outputs to inputs) and ecological efficiency (the relation of desirable outputs to undesirable outputs), and then combined the two efficiencies to assess the eco-efficiency analysis of 24 power plants in a European country. Bian and Yang (2010) extended the Shannon-DEA procedure to obtain a comprehensive efficiency measure that simultaneously appraises DMUs' resource and environment performance. Further, the authors applied their approach to address the resource and environment efficiency evaluation problem of 30 Chinese provinces. Zhou, Ang, and Poh (2006), Zhou, Poh, and Ang (2007), and Zhou et al. (2008) proposed a series of environmental DEA technologies to measure environmental performance. For example, Zhou et al. (2006) proposed two slacks-based efficiency measures for environmental performance, with one being a composite index with a higher discriminating power, and the other being used to estimate the impacts of environmental regulations. Zhou et al. (2008) proposed a non-radial DEA approach for environmental performance evaluation, which involves a non-radial DEA-based model for multilateral environmental performance comparisons and a non-radial Malmquist environmental performance index for modeling the change of environmental performance over time. Song, Fisher, Wang, and Cui (2018) addressed the environmental performance evaluation in big data context. Sueyoshi and Yuan (2015) evaluated the regional environmental performance by incorporating PM2.5 and PM10 as undesirable outputs, and results show that the Chinese government should distribute more resources to northwestern cities. Wu, Zhu, Yin, and Song (2017) used an improved DEA approach for evaluating the Chinese regional total-factor energy and environmental efficiency, in which the authors considered the total-factor energy and environmental efficiency as a joint production framework involving both non-energy inputs and energy inputs, as well as desirable outputs and undesirable outputs.

To improve environmental performance and relieve the impacts of global warming, many countries and international organizations have conducted efforts to reduce its carbon emission levels, and the studies in the literature are also abundant. Sun et al. (2014) proposed two DEA models to allocate the carbon emissions among several paper mills, with one from the centralized perspective and another from the individual perspective. Feng et al. (2015) proposed a two-step procedure to obtain a comprehensive CEA allocation. For the first step, the author proposed improved DEA-based centralized allocation models under VRS and CRS assumptions, and then they used two compensation schemes to generate the centralized allocation schemes within the second step. Wu et al. (2016) proposed a DEA-based closest target technique to set target and allocate the CEA amount among 20 APEC economies. In 2009, the Chinese government announced to cut its carbon emission intensity by 40%–45% from the 2005 level, Yi, Zou, Guo, Wang, and Wei (2011), Wei et al. (2012), Wang et al. (2013), Yu et al. (2014), and Zhang and Hao (2015) proposed a series of provincial allocation mechanism to realize the reduction commitment. Yi et al. (2011) considered the per capita GDP, accumulated fossil fuel related CO_2 emissions and energy consumption per unit of industrial added value as indicators of emission reduction capacity, responsibility and potential, respectively, and further allocated the national CO_2 intensity reduction target to different provinces according to its different indicator values. Wei et al. (2012) applied an extended SBM model to estimate the CO_2 reduction potential and marginal abatement costs for Chinese 29 provinces, and further proposed an abatement capacity index using weighted equity and efficiency indexes to realize the regional allocation of carbon dioxide abatement in China. Wang et al. (2013) used an improved zero sum gains DEA optimization model to generate an efficient emission allowance allocation scheme on provincial level for China by 2020. Yu et al. (2014) addressed the provincial allocation of carbon emission reduction targets in China based on

Table 1. The CO_2 emission/GIOV in China from 2004 to 2012.

Year	CO_2 emission	Gross industrial output value (GIOV)	CO_2 emission/GIOV	CPI
2004	2,322,703.8950	19,396.1056	1.1975	81.8313
2005	2,535,271.3660	21,783.5740	1.1638	85.0227
2006	2,754,416.4470	27,457.1670	1.0032	86.5673
2007	2,932,353.4260	35,363.0840	0.8292	87.8369
2008	3,251,515.2580	44,135.8360	0.7367	92.0238
2009	3,411,188.4130	47,919.9720	0.7119	97.4532
2010	3,700,797.2980	60,955.8500	0.6071	96.7834
2011	3,950,889.9570	73,398.4010	0.5383	100.0000
2012	4,134,711.6380	80,925.5132	0.5109	105.4706

Source: China Statistical Yearbooks 2005–2013, China Energy Statistical Yearbook.
According to OECD statistics, we set CPI Index 2010 = 100.

particle swarm optimization algorithm, fuzzy c-means clustering algorithm, and Shapley decomposition approach. Then Yu et al. (2014) clustered all provinces into four classes based on relevant carbon emission factors and concluded that more carbon emission reduction amount would be allocated to provinces with large total emissions and high emission intensity. Emrouznejad, Yang, and Amin (2019) studied the same problem of this article, while they used an inverse DEA approach and neglected the competitive and cooperative relationships among different sub-level industries and provinces.

If the total reduction amount is set to zero, the CEA allocation problem would be reduced to a carbon emission reallocation problem. Lozano et al. (2009) proposed a three-phase approach to allocate the emission permit, namely, maximizing aggregated desirable outputs, minimizing undesirable total emissions and minimizing the consumption of input resources. Gomes and Lins (2008) reformulated a zero sum gains (ZSG) DEA model to reallocate CO_2 emissions. Further, Wang et al. (2013) and Chiu et al. (2015) used ZSG-DEA models to allocate CO_2 emissions permits in some regions. Miao, Geng, and Sheng (2016) used a non-radial ZSG-DEA model to allocate CO_2 emissions into different provinces in China. Readers can refer to Zhou and Wang (2016) for an overview of carbon dioxide emission allocation studies.

Additionally, some game DEA approaches are proposed to address the CEA allocation problem. Wu et al. (2013) proposed a bargaining DEA approach to generate the reduction scheme. In their article, all DMUs compete with each other to minimize its reduction in carbon emission permits, while the central authority maximizes the overall efficiency. In addition, the undesirable output (i.e. carbon emissions) in that article was considered as an input. Filar and Gaertner (1997) used the Shapley value solution of cooperative game theory to allocate carbon quotas among four regions. Yu et al. (2014) and Zhang et al. (2014) applied also the Shapley value concept when they addressed regional allocation of carbon emissions quota in China, but these studies took the Shapley value as a supplementary tool rather than the main method, and didn't base on the cooperative game theory.

This article bases itself upon a realistic and real problem of industrial and provincial allocation of national carbon emission abatement quota in China. We will study the industrial allocation scheme as well as regional allocation scheme, which is different from existing literature with only provincial allocation. We will propose an integrated cooperative game DEA approach by taking the game relationship among sub-level industries and provinces into account and is supposed to obtain fair and stable allocation results.

3. Problem description and mathematical notation

3.1. Problem background

In June 2015, in the climate conference in France, Chinese Premier Li Keqiang announced China's latest voluntary reduction commitment: the CO_2 emissions will reach the peak at about 2030 and seek to reach it as early as possible. China aims to cut its greenhouse gas emission intensity by 60%–65% (per unit of gross domestic product (GDP)) from the 2005 level. In manufacturing industries, the Gross Industrial Output Value (GIOV) plays the same role as GDP for the country. Thus we can propose CO_2 reduction goal as CO_2 emission/GIOV decreases 60% to 65% based on the level of 2005. The indicators of CO_2 emission/GIOV in China from 2004 to 2012 are listed in the following Table 1 (CO_2 emission is in the unit of thousand tons, Gross industrial output value is in the unit of billion Yuan).

It should be noted that, in order to ensure the comparability, we transform the value of GIOV to constant price in 2010 using the Consumer Price Index (CPI) of China, as shown in the last column of Table 1. The CPI data is derived from OECD statistics (2010). Therefore in this article

Table 2. Notation summary.

Notations	Description
$h = 1, \ldots, 31$	Index of different sub-level industries
$l = 1, \ldots, 31$	Index of different provinces
$i = 1, \ldots, m$	Index of inputs
$r = 1, \ldots, s$	Index of desirable outputs
$j, o = 1, \ldots, n$	Index of decision making units (DMU)
x_{ij}	Value of inputs of DMU_j
y_{rj}	Value of desirable outputs of DMU_j
b_j	Value of undesirable output of DMU_j
λ_j, ξ_j	Intensity variables
θ	Expansion proportion of outputs in BCC model
η	Measure of the directional distance function
v_i, υ_i	Relative weight of input i
u_r, μ_r	Relative weight of desirable output r
ω_1, ω_2, ω	Relative weight of the undesirable output
u_0, μ_0	Free variable reflecting the RTS property
$g = (g_y, g_b)$	Direction vector
E_o^*	Optimal original efficiency of DMU_o
e_o^*	Optimal post-allocation efficiency of DMU_o
η^*	Optimal directional distance function measure
ψ_o^*	Optimal inefficiency measure
R	Total fixed undesirable output abatement quota
R_j	Undesirable output abatement amount of DMU_j
$\bar{y}_{rj}$,$\tilde{y}_{rj}$	The change amount of desirable output r of DMU_j
R_j^{max}	Maximal undesirable output reduction level of DMU_j with desirable output $\bar{y}_{rj}$
T_{1j}, T_{2j}, T_j	The variation of undesirable output reduction amount of DMU_j
$\vec{R}_j/\overleftarrow{R}_j$	Maximal/minimal undesirable output reduction level of DMU_j with the efficient allocation set
$N = \{1, \ldots, n\}$	Large coalition of all DMU
K	Coalition of DMUs
$C_K = (c_1, \ldots, c_n)'$	Vector used to construct coalition K
$\vec{R}_K/\overleftarrow{R}_K$	Maximal/minimal quota allocated to coalition K
$V(K)$	Characteristic function
$Z = (z_1, \ldots, z_n)$	Pre-nucleolus imputation
$e(K, Z)$	Excess value of coalition K on imputation Z
$\delta(Z)$	Vector of excess values
ξ	Least maximal excess value
ρ_K	Dual variable
β_j	Dual variable
α	Dual variable
τ	Dual variable
π_j	Dual variable
χ_j	Dual variable
O^{max}	The maximal unsatisfied value
d_j	0–1 variable
f_j	Instrumental variable depending on d_j
M	A large enough positive value

we set the goal to decrease 60% to 65% of the level of CO_2 emission/GIOV in 2012 based on that in 2005. Thus CO_2 emission/GIOV in 2012 should be in the range of [0.4073, 0.4655]. However the real ratio of CO_2 emission/GIOV reaches 0.5109. If Chinese government achieves the goal of the CO_2 emission in 2012, the CO_2 emission in 2012 should be [3296467.6860, 3767391.6410] in the unit of thousand tons. However, the real amount of CO_2 emission in manufacturing industries in China is 4134711.6380 (thousand tons). Thus the CO_2 emission reduction gap should be [367319.9970–838243.9520] in the unit of thousand tons, and this is the national allocation target that will be addressed in this article.

Based on the above government goal, we use a two-layer dataset to allocate the CO_2 emission quota into different sub-level industries and then further allocate them into different provinces. The first layer is to allocate the national reduction quota CO_{2total} to different sub-level manufacturing industries, say CO_{2h}, where $h = 1, \ldots, 31$ denotes different sub-level manufacturing industries such that $\sum_{h=1}^{31} CO_{2h} = CO_{2total}$. The second layer is to further allocate the reduction quota for each industry CO_{2h} into different provinces, say CO_{2hl}, where $l = 1, \ldots, 31$ denotes different provinces. Finally, it holds that $\sum_{l=1}^{31}\sum_{h=1}^{31} CO_{2hl} = CO_{2total}$.

The problem studied in this article is how to allocate the total national government goal into different sub-level industries and then further different provinces in an equitable manner. The motivation of this article is the decentralized perspective and game relationship among different sub-level industries and different provinces. Therefore, we try to propose an integrated cooperative game DEA approach for this task.

3.2. Mathematical notations

For a better understanding of this article and subsequent mathematical problems, here we give a comprehensive summary of all mathematical notations in Table 2.

4. Mathematical model

4.1 Preliminary

For a mathematical modelling purpose, we follow a common framework in DEA literature to consider a set of n homogeneous and comparable peer DMUs, with each consuming m inputs to produce s outputs. Denote the input and output variables for $DMU_j\ (j = 1, \ldots, n)$ as $x_{ij}\ (i = 1, \ldots, m)$ and $y_{rj}\ (r = 1, \ldots, s)$, respectively. The production possible set (PPS) with VRS assumption and free disposability of inputs and outputs can be expressed as formula (1).

$$PPS = \left\{ (x_i, y_r) \left| \begin{array}{l} \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, i = 1, \ldots, m \\ \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_r, r = 1, \ldots, s \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, j = 1, \ldots, n \end{array} \right. \right\} \tag{1}$$

where λ_j is the intensity variable used for constructing the efficiency frontier, and the constraint $\sum_{j=1}^{n} \lambda_j = 1$ implies the VRS assumption. Based on the PPS in formula (1), the following model proposed by Banker et al. (1984) can be used to estimate the relative efficiency score of $DMU_o\ (o = 1, \ldots, n)$.

$$
\begin{aligned}
\theta_o^* &= \mathrm{Max}\,\theta \\
\text{s.t. } & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \quad i = 1, \ldots, m \\
& \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge \theta y_{ro}, \quad r = 1, \ldots, s \\
& \textstyle\sum_{j=1}^{n} \lambda_j = 1, \;\; \lambda_j \ge 0, j = 1, \ldots, n.
\end{aligned} \tag{2}
$$

Model (2) estimates the maximal proportional output expansion yet remaining the input consumption. If it is impossible to proportionally increase outputs, then the optimal objective function of model (2) would be one (namely, $\theta_o^* = 1$) and the evaluated DMU would be identified as DEA-efficient. Otherwise, $\mathrm{DMU}_o\,(o = 1, \ldots, n)$ is inefficient for cases where $\theta_o^* > 1$. Note in particular that model (2) will reduce to the first DEA model, known as Charnes-Banker-Rhodes model (Charnes et al., 1978), if we withdraw the constraint $\sum_{j=1}^{n} \lambda_j = 1$ and impose only non-negative requirementss on intensity variables.

It is notable that model (2) considers only traditional outputs, which means that outputs are maximized as possible (Scheel, 2001), while undesirable outputs like waste or pollution are also generated frequently in the production process (Färe, Grosskopf, Lovell, & Pasurka, 1989; Seiford & Zhu, 2002). With the consideration of undesirable output b_j for $\mathrm{DMU}_j\,(j = 1, \ldots, n)$, a core task is identifying methods to handle undesirable outputs (Song et al., 2018). The existing research can be mainly divided into two groups in terms of disposability assumptions (Chen & Delmas, 2012; Li, Li, Zhao, & Zhu, 2018h; Song, An, Zhang, Wang, & Wu, 2012). The first group considers that the undesirable outputs are joint weakly or strongly disposable with desirable outputs, hence it adopts weak or strong disposability assumption (Seiford & Zhu, 2002; Zhou, Ang, & Wang, 2012). The second category thinks that the undesirable outputs can be reduced by reducing output production or increasing input consumption, which implies natural and managerial disposability assumption (Li, Zhu, & Zhuang, 2018e; Sueyoshi & Goto, 2012a, 2012b). With deep consideration for our application in this article, we will use the most common used weak disposability assumption. To this end, the PPS assuming free disposability of inputs and desirable outputs and weak disposability of undesirable outputs can be written as below (Kuosmanen, 2005; Kuosmanen & Podinovski, 2009).

$$
\mathrm{PPS}^{\mathrm{un}} = \left\{ (x_i, y_r, b) \left| \begin{array}{l} \sum_{j=1}^{n} (\lambda_j + \xi_j) x_{ij} \le x_i, i = 1, \ldots, m \\ \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_r, r = 1, \ldots, s \\ \sum_{j=1}^{n} \lambda_j b_j = b \\ \sum_{j=1}^{n} (\lambda_j + \xi_j) = 1, \lambda_j, \xi_j \ge 0, j = 1, \ldots, n \end{array} \right. \right\} \tag{3}
$$

As compared with the traditional PPS in formula (1), in formula (3) both desirable outputs and undesirable outputs of $\mathrm{DMU}_j\,(j = 1, \ldots, n)$ are weighted by the non-disposed intensity variable λ_j, whereas the inputs of $\mathrm{DMU}_j\,(j = 1, \ldots, n)$ are weighted by the sum of the disposed intensity variable ξ_j and non-disposed intensity variable λ_j. In addition, the VRS assumption is ensured by summing the total disposed intensity variable ξ_j and non-disposed intensity variable λ_j to one, namely, $\sum_{j=1}^{n} (\lambda_j + \xi_j) = 1$.

Based on the PPS in formula (3), we can propose a directional distance function (DDF) model with weak disposability assumption to calculate maximal proportional desirable output expansion and undesirable output reduction simultaneously. When $\mathrm{DMU}_o\,(o = 1, \ldots, n)$ is under consideration, the above idea is formulated as model (4).

$$
\begin{aligned}
\eta_o^* &= \mathrm{Max}\,\eta \\
\text{s.t. } & \textstyle\sum_{j=1}^{n} (\lambda_j + \xi_j) x_{ij} \le x_{io}, \;\; i = 1, \ldots, m \\
& \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} + \eta \cdot y_{ro}, \;\; r = 1, \ldots, s \\
& \textstyle\sum_{j=1}^{n} \lambda_j b_j = b_o - \eta \cdot b_o \\
& \textstyle\sum_{j=1}^{n} (\lambda_j + \xi_j) = 1 \\
& \lambda_j, \xi_j \ge 0, \;\; j = 1, \ldots, n.
\end{aligned} \tag{4}
$$

In model (4), the direction vector is set as $g = (g_y, g_b) = (y_{ro}, -b_o)$. In addition, η is a measure of inefficiency on both desirable output expansion and undesirable output reduction, thus an optimal efficiency score can be calculated as $E_o^* = 1 - \eta_o^*$, where η_o^* is the optimal solution of η and derived from model (4). It is clear that η^* is no more than one and no less than zero, thus the optimal efficiency score $E_o^*\,(o = 1, \ldots, n)$ ranges from zero to unity.

The following model (5) is a dual of model (4).

$$
\begin{aligned}
\psi_o^* &= \mathrm{Min} \textstyle\sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} + \omega_1 b_o - \omega_2 b_o + u_0 \\
\text{s.t. } & \textstyle\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + \omega_1 b_j - \omega_2 b_j + u_0 \ge 0, \;\; j = 1, \ldots, n \\
& \textstyle\sum_{i=1}^{m} v_i x_{ij} + u_0 \ge 0, \;\; j = 1, \ldots, n \\
& \textstyle\sum_{r=1}^{s} u_r y_{ro} + \omega_1 b_o - \omega_2 b_o \ge 1 \\
& v_i, u_r, \omega_1, \omega_2 \ge 0, \;\; i = 1, \ldots, m; r = 1, \ldots, s; u_0 \text{ free.}
\end{aligned} \tag{5}
$$

Here $v_i\,(i = 1, \ldots, m)$, $u_r\,(r = 1, \ldots, s)$, ω_1, ω_2 and u_0 are dual variables derived from model (4), and are also unknown decision variables in model (5). Note in particular that this article focuses on the carbon emission which is a kind of undesirable output, thus it is required that at least $\omega_1 > 0$ or $\omega_2 > 0$, otherwise the undesirable output will make no sense if it holds $\omega_1 = 0$ and $\omega_2 = 0$ simultaneously. Without loss of generality, we assume that $\omega_1 > 0$ and $\omega_2 \ge 0$, and another case ($\omega_1 \ge 0$ and $\omega_2 > 0$) can be studied in a similar way. Each dual variable measures the level of efficiency increment due to a unit increase on according input-output bundle. According to the dual theory, it holds $\eta_o^* = \psi_o^*$ for all $\mathrm{DMU}_o\,(o = 1, \ldots, n)$, and the efficiency score can also be computed as $E_o^* = 1 - \psi_o^*$.

4.2. Performance evaluation with flexible CEA allocation plan

Now consider that the central government would cut its total carbon dioxide emissions by R. Then the problem comes out of how to reduce the undesirable output from peer DMUs in a fair way (Wu et al., 2013). Suppose that each $\mathrm{DMU}_j\,(j=1,\ldots,n)$ will be reduced an amount of R_j from its current carbon emissions level b_j such hat

$$\sum\nolimits_{j=1}^{n} R_j = R, R_j \geq 0. \quad (6)$$

The above Equation (6) guarantees that the individual reduced amounts $R_j\,(j=1,\ldots,n)$ precisely sum to the total carbon emissions abatement quota R.

To address the performance evaluation with flexible CEA allocation plan, the feasibility of CEA allocation plan should be investigated in advance. To guarantee that the reduced amounts R_j is feasible such that the reduced operating units fall within the PPS which is constructed by observed DMUs and defined in Formula (3), it must hold that $(x_{io}, y_{ro}+\bar{y}_{ro}, b_o-R_o)\in \mathrm{PPS}^{\mathrm{un}}$ for any $\mathrm{DMU}_o\,(o=1,\ldots,n)$, namely,

$$\begin{aligned}
&\sum\nolimits_{j=1}^{n} (\lambda_j+\xi_j)x_{ij} \leq x_{io}, i=1,\ldots,m\\
&\sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}+\bar{y}_{ro}, r=1,\ldots,s\\
&\sum\nolimits_{j=1}^{n} \lambda_j b_j = b_o - R_o\\
&\sum\nolimits_{j=1}^{n} (\lambda_j+\xi_j) = 1\\
&\lambda_j, \xi_j \geq 0, j=1,\ldots,n.
\end{aligned} \quad (7)$$

Here $\bar{y}_{ro}\,(r=1,\ldots,s)$ and R_o are unknown values and free. Note in particular that by inserting $\bar{y}_{ro}$ on desirable outputs, we are able to estimate the feasible undesirable output reduction level through changing desirable outputs within given PPS. This idea fits with the observation in practice that reducing undesirable outputs is usually associated with affecting the desirable outputs.

It is clear that the minimum reduction amount of $\mathrm{DMU}_o\,(o=1,\ldots,n)$ is zero, which means that all DMUs must reduce its carbon emission amount or it cannot increase its carbon emission amount anymore. Further, the maximum emission reduction level of $\mathrm{DMU}_o\,(o=1,\ldots,n)$, donated as $R_o^{\max}$, can be computed by model (8). Additionally, by setting $\bar{y}_{ro}\geq 0$ we can estimate the feasible undesirable output reduction level yet remaining unreduced desirable outputs. If $\bar{y}_{ro}\,(r=1,\ldots,s)$ is free we will estimate the feasible undesirable output reduction level by sacrificing desirable outputs.

$$\begin{aligned}
&R_o^{\max} = \mathrm{Max}\, R_o\\
&\text{s.t.} \sum\nolimits_{j=1}^{n} (\lambda_j+\xi_j)x_{ij} \leq x_{io}, i=1,\ldots,m\\
&\quad \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}+\bar{y}_{ro}, r=1,\ldots,s\\
&\quad \sum\nolimits_{j=1}^{n} \lambda_j b_j = b_o - R_o\\
&\quad \sum\nolimits_{j=1}^{n} (\lambda_j+\xi_j) = 1\\
&\quad \lambda_j, \xi_j \geq 0, j=1,\ldots,n.
\end{aligned} \quad (8)$$

Therefore, the feasible reduction amount R_j of any $\mathrm{DMU}_j\,(j=1,\ldots,n)$ would be placed within the interval $[0, R_j^{\max}]$ such that $0\leq R_j\leq R_j^{\max}$. Without loss of generality, we assume that the total reduction quota R can be fully covered by the sum of all maximum emission reductions, that is, $\sum\nolimits_{j=1}^{n} R_j^{\max}\geq R$, otherwise the carbon dioxide emissions abatement goal R is considered as infeasible within the current PPS and the central government should adjust its carbon dioxide emissions abatement goal. In addition, we have Theorem 1.

Theorem 1. $R_j^{\max} = b_j\,(j=1,\ldots,n)$.

Proof. See Appendix A.

The above Theorem 1 is an intuitive result. The underlying logic behind Theorem 1 is that any DMU can completely reduce its undesirable outputs at the cost of removing its desirable outputs. In other words, all DMUs have potentials to reduce its undesirable outputs level within the PPS. Therefore, in this article we consider all DMUs as carbon emission abatement targets.

With consideration of the carbon emission abatement allocation plan $(R_1,\ldots,R_n)$, model (5) is rewritten as model (9) to calculate the possible efficiency score of $\mathrm{DMU}_o\,(o=1,\ldots,n)$.

$$\begin{aligned}
&\hat{\psi}_o^* = \mathrm{Min} \sum\nolimits_{i=1}^{m} v_i x_{io} - \sum\nolimits_{r=1}^{s} u_r(y_{ro}+\bar{y}_{ro}) + \omega_1(b_o-R_o) - \omega_2(b_o-R_o) + u_0\\
&\text{s.t.} \sum\nolimits_{i=1}^{m} v_i x_{io} - \sum\nolimits_{r=1}^{s} u_r(y_{ro}+\bar{y}_{ro}) + \omega_1(b_o-R_o) - \omega_2(b_o-R_o) + u_0 \geq 0\\
&\quad \sum\nolimits_{i=1}^{m} v_i x_{ij} - \sum\nolimits_{r=1}^{s} u_r y_{rj} + \omega_1 b_j - \omega_2 b_j + u_0 \geq 0,\ j=1,\ldots,n\\
&\quad \sum\nolimits_{i=1}^{m} v_i x_{ij} + u_0 \geq 0,\ j=1,\ldots,n\\
&\quad \sum\nolimits_{r=1}^{s} u_r y_{ro} + \omega_1 b_o - \omega_2 b_o \geq 1\\
&\quad 0\leq R_j\leq b_j,\ j=1,\ldots,n\\
&\quad \sum\nolimits_{j=1}^{n} R_j = R\\
&\quad v_i, u_r, \omega_2 \geq 0,\ i=1,\ldots,m; r=1,\ldots,s; \omega_1>0, \bar{y}_{ro} \text{ and } u_0 \text{ free.}
\end{aligned} \quad (9)$$

However, model (9) is a nonlinear programming, therefore we can transform it into a linear programming problem as given in model (10).

$$\begin{aligned}
&\hat{\psi}_o^* = \text{Min} \sum_{i=1}^m v_i x_{io} - \sum_{r=1}^s u_r y_{ro} - \sum_{r=1}^s \bar{Y}_{ro} + \omega_1 b_o - T_{1o} - \omega_2 b_o + T_{2o} + u_0 \\
&\text{s.t. } \sum_{i=1}^m v_i x_{io} - \sum_{r=1}^s u_r y_{ro} - \sum_{r=1}^s \bar{Y}_{ro} + \omega_1 b_o - T_{1o} - \omega_2 b_o + T_{2o} + u_0 \geq 0 \\
&\quad \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} + \omega_1 b_j - \omega_2 b_j + u_0 \geq 0,\ j = 1, \ldots, n \\
&\quad \sum_{i=1}^m v_i x_{ij} + u_0 \geq 0,\ j = 1, \ldots, n \\
&\quad \sum_{r=1}^s u_r y_{ro} + \omega_1 b_o - \omega_2 b_o \geq 1 \\
&\quad 0 \leq T_{1j} \leq \omega_1 b_j,\ j = 1, \ldots, n \\
&\quad 0 \leq T_{2j} \leq \omega_2 b_j,\ j = 1, \ldots, n \\
&\quad \sum_{j=1}^n T_{1j} = \omega_1 R, \sum_{j=1}^n T_{2j} = \omega_2 R \\
&\quad v_i, u_r, \omega_2 \geq 0,\ i = 1, \ldots, m; r = 1, \ldots, s; \omega_1 > 0, \bar{Y}_{ro} \text{ and } u_0 \text{ free.}
\end{aligned} \tag{10}$$

Solving model (10) once for each $\text{DMU}_o\,(o = 1, \ldots, n)$ determines an optimal efficiency score with flexible carbon emission abatement allocation schemes. Suppose that the optimal solution of model (10) is $(v_i^{o*}, u_r^{o*}, T_{1j}^{o*}, T_{2j}^{o*}, \omega_1^{o*}, \omega_2^{o*}, u_0^{o*}, \bar{Y}_{ro}^{o*}, \forall i, r, j)$ when $\text{DMU}_o\,(o = 1, \ldots, n)$ is evaluated, then the possible efficiency is calculated as $e_o^* = 1 - (\sum_{i=1}^m v_i^{o*} x_{io} - \sum_{r=1}^s u_r^{o*} y_{ro} - \sum_{r=1}^s \bar{Y}_{ro}^{o*} + \omega_1^{o*} b_o - T_{1o}^{o*} - \omega_2^{o*} b_o + T_{2o}^{o*} + u_0^{o*})$, which is associated with a carbon emission abatement allocation scheme $R_j^{o*} = T_{1j}^{o*}/\omega_1^{o*}\,(j=1,\ldots,n)$.

Based on model (10), some useful and valuable conclusions are developed.

Theorem 2. *The optimal objective function of model (10) is always zero.*

Proof. See Appendix B.

Theorem 2 implies that $\text{DMU}_o\,(o = 1, \ldots, n)$ can be identified as DEA efficient through determining a flexible carbon emission abatement allocation scheme and selecting a set of optimal relative weights. Put it differently, by incorporating the carbon emission abatement allocation plan into efficiency evaluation it is possible for any DMU to separately maximize its efficiency score to one and realize the efficient status through laying itself on the efficiency frontier of the PPS defined by the observed DMUs. This provides also a possibility to propose an efficient allocation scheme of the total carbon emission abatement quota. To this end, we first provide Theorem 3 as below.

Theorem 3. *All DMUs can be simultaneously efficient with a certain carbon emission abatement allocation scheme under a set of common weights.*

Proof. See Appendix C.

Theorem 2 suggests that a certain DMU can be efficient through incorporating flexible carbon emission abatement allocation plans, while Theorem 3 steps further that there exist carbon emission abatement allocation schemes that can make all DMUs simultaneously efficient under a set of common weights. Based on this observation, we can have an efficient allocation set which contains all carbon emission abatement allocation schemes that can make all DMUs simultaneously efficient. The insight behind this finding is that any DMU has enough incentives to accept such efficient allocation schemes as it will benefit from the allocation process to improve its efficiency assessment result. It is also similar with the efficiency maximization pursuance of Beasley (2003) who argued that "each DMU can clearly see that… it is a fair and equitable one–it enables them to achieve maximum efficiency in comparison to their peers using exactly the same weights." The efficient allocation set is given by Corollary 1.

Corollary 1. *The efficient carbon emission abatement allocation scheme can be denoted as following System (11) under a set of common weights:*

$$\begin{aligned}
&\sum_{i=1}^m \upsilon_i x_{ij} - \sum_{r=1}^s \mu_r y_{rj} - \sum_{r=1}^s \tilde{y}_{rj} + b_j - R_j - \omega b_j + T_j \\
&\quad + \mu_0 = 0, j = 1, \ldots, n \\
&\sum_{j=1}^n R_j = R \\
&\sum_{j=1}^n T_j = \omega R \\
&0 \leq R_j \leq b_j, j = 1, \ldots, n \\
&\upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \ldots, m; r = 1, \ldots, s; j = 1, \ldots, n; \\
&\quad \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned} \tag{11}$$

Proof. See Appendix D.

It is clear that there are $(m + s + n \cdot s + n + n + 2)$ variables in System (11), but only $(n + 2)$ equations. Therefore, there exist flexibilities to determine a unique carbon emission abatement allocation scheme. Based on the efficient carbon emission abatement allocation scheme presented in System (11), the following model (12) is developed to calculate the maximal/minimal carbon emission abatement level of each $\text{DMU}_o\,(o = 1, \ldots, n)$.

$$
\begin{aligned}
&\vec{R}_o/\overleftarrow{R}_o = \text{Max/Min}\, R_o \\
&\text{s.t.}\;\; R_j = \textstyle\sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} \\
&\qquad\quad + b_j - \omega b_j + T_j + \mu_0, j = 1, \ldots, n \\
&\quad \textstyle\sum_{j=1}^{n} R_j = R \\
&\quad \textstyle\sum_{j=1}^{n} T_j = \omega R \\
&\quad \textstyle\sum_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \ldots, s \\
&\quad 0 \leq R_j \leq b_j, j = 1, \ldots, n \\
&\quad \upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad\quad j = 1, \ldots, n; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned}
\tag{12}
$$

Notably, we add an additional constraint $\sum_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \ldots, s$ in model (12), which is used to guarantee that the total desirable outputs are unreduced yet reducing the undesirable outputs. After all, a certain carbon emission abatement allocation scheme is unacceptable if it sacrifices desirable outputs, and a real instance is the withdrawal of United States from the 2015 Paris Agreement (Chakraborty, 2017). It is clear that the maximal carbon emission abatement level from $\text{DMU}_j\,(j = 1, \ldots, n)$ is $\vec{R}_j$, otherwise not all DMUs can be simultaneously efficient with the carbon emission abatement allocation scheme. Similarly, it must ensure that the carbon emission abatement level of $\text{DMU}_j\,(j = 1, \ldots, n)$ is at least $\overleftarrow{R}_j$. Model (12) not only shows the efficient carbon emission abatement interval of DMUs, but also shows a phenomenon like egoist's dilemma (Nakabayashi & Tone, 2006), implying that $\sum_{j=1}^{n} \overleftarrow{R}_j < R < \sum_{j=1}^{n} \vec{R}_j$. Then the problem comes out of how to generate a unique carbon emissions abatement scheme using game-theoretical models.

4.3. An integrated cooperative game DEA approach

Here in this subsection we develop an integrated cooperative game DEA approach for the carbon emission abatement quota allocation problem. To this end, we first construct a coalition $K \subset N = \{1, \ldots, n\}$ of n DMUs. Then the input-output vector of coalition K can be denoted as $X_K = (X_1, \ldots, X_n)C_K$, $Y_K = (Y_1, \ldots, Y_n)C_K$ and $B_K = (B_1, \ldots, B_n)C_K$, where for $DMU_j\,(j = 1, \ldots, n)$ it holds $X_j = (x_{1j}, \ldots, x_{mj})'$, $Y_j = (y_{1j}, \ldots, y_{sj})'$, $B_j = (b_j)'$, and $C_K = (c_1, \ldots, c_n)'$ is a non-negative vector with each component $C_{Kj} = c_j\,(j = 1, \ldots, n)$ being zero or one. The coalition vector $C_K = (c_1, \ldots, c_n)'$ indicates the establishment of coalitions of n DMUs. For example, $C_{\{d\}} = \left(\underbrace{0, \ldots, 0}_{d-1}, 1, 0, \ldots, 0\right)'$ is the original DMU_d, $C_{\{d,k\}} = \left(\underbrace{0, \ldots, 0}_{d-1}, 1, \underbrace{0, \ldots, 0}_{k-d-1}, 1, 0, \ldots, 0\right)'$ represents the coalition of DMU_d and DMU_k, and $C_N = (1, \ldots, 1)'$ represents the grand coalition of all DMUs.

For any coalition $K \subset N = \{1, \ldots, n\}$, we can formulate model (13) to calculate its collective max/min carbon emission abatement level.

$$
\begin{aligned}
&\vec{R}_K/\overleftarrow{R}_K = \text{Max/Min} \textstyle\sum_{j=1}^{n} C_{Kj} R_j \\
&\text{s.t.}\;\; R_j = \textstyle\sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} \\
&\qquad\quad + b_j - \omega b_j + T_j + \mu_0, j = 1, \ldots, n \\
&\quad \textstyle\sum_{j=1}^{n} R_j = R \\
&\quad \textstyle\sum_{j=1}^{n} T_j = \omega R \\
&\quad \textstyle\sum_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \ldots, s \\
&\quad 0 \leq R_j \leq b_j, j = 1, \ldots, n \\
&\quad \upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad\quad j = 1, \ldots, n; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned}
\tag{13}
$$

Based on the optimal solution and objective function of model (13), we give a characteristic function of coalition $K \subset N = \{1, \ldots, n\}$ as follows:

Definition 1. For any subset $\mathrm{K} \subset \mathrm{N} = \{1, \ldots, \mathrm{n}\}$, the characteristic function is calculated as $\mathrm{V(K)} = \sum_{\mathrm{j \epsilon K}} \vec{\mathrm{R}}_{\mathrm{j}} - \vec{\mathrm{R}}_{\mathrm{K}}$.

The first term $\sum_{j\epsilon K} \vec{R}_j$ represents the sum of individual maximum carbon emission abatement amount of DMUs in coalition K, while the second term $\vec{R}_K$ represents the maximum collective carbon emission abatement level of coalition K. It is clear that the formula $\sum_{j\epsilon K} \vec{R}_j - \vec{R}_K$ measures the improvement potentials by establishing the coalition K. The characteristic function promises an assurance level. Traditionally, we assigns zero to empty set, $V(\emptyset) = 0$. Further, we have $V(N) = \sum_{j\epsilon N} \vec{R}_j - \vec{R}_N = \sum_{j\epsilon N} \vec{R}_j - R > 0$, where the latter equation is the result of full cover of the total carbon emission abatement quota. Theorem 4 shows the super-additive property of the characteristic function, which promises the potential gains from cooperation and the feasibility to allocate the total carbon dioxide emission abatement quota from the perspective of cooperative game framework.

Theorem 4. *The characteristic function $V(K)$ satisfies super-additive, that is, for any two coalitions $K, L \subseteq N = \{1, \ldots, n\}$ and $K \cap L = \emptyset$, it holds $V(K) + V(L) \leq V(K \cup L)$.*

Proof. See Appendix E.

The super-additive property implies that all DMUs would like to participate into the grand coalition and it is possible to persuade DMUs into an agreement to generate a carbon emission abatement allocation scheme. The next problem is how to solve the game (N, V) and accordingly generate the carbon emission abatement allocation scheme.

4.4. Nucleolus-based solution and computation procedure

There are many solution concepts for the cooperative game, such as kernel, core, nucleolus, stable set, bargaining set, Shapley value, τ-value, etc. (Lozano, 2012; Nakabayashi & Tone, 2006). The decision maker should select the game solution depending on the context in which the problem is located. In this article, we take the nucleolus-based solution as an example to show how to obtain the game-based allocation plan. In addition, since the nucleolus does more favor to those vulnerable DMUs/coalitions, it implies a pessimistic attitude and corresponds to the defining way of the characteristic function. Besides, there is always one and only one unique nucleolus. Therefore, in this subsection we will discuss the nucleolus solution and its computation procedure.

For the sake of calculating the nucleolus solution and according allocation scheme, let us consider a pre-imputation or a carbon emission abatement quota allocation vector $Z = (z_1, \ldots, z_n)$, satisfies the following three rationalities (Nakabayashi & Tone, 2006):

1. Individual rationality: $\vec{R}_j - z_j \geq V(\{j\}), j = 1, \ldots, n$.
2. Coalition rationality: $\sum_{j \in K} \vec{R}_j - \sum_{j \in K} z_j \geq V(K), K \subset N = \{j = 1, \ldots, n\}$
3. Collective rationality: $\sum_{j=1}^{n} \vec{R}_j - \sum_{j=1}^{n} z_j = V(N) = \sum_{j=1}^{n} \vec{R}_j - R$.

In cooperative game theory, the pre-imputation vector $Z = (z_1, \ldots, z_n)$ represents an allocation scheme of the total carbon dioxide emission abatement quota R. The individual rationality ensures the non-negative improvements from the worst results for each DMU, and the coalition rationality guarantees the non-negative improvements from the worst results for all coalitions, and the last collective rationality promises a full cover of the total reduction quota R. As suggested by Lozano (2012) that, the coalition rationality can also guarantee the stability of the imputation, since the aggregated gains assigned to its members is not less than the coalition minimal gains, i.e. the characteristic function. Further, a stable imputation leaves no incentives for any DMUs to break the grand coalition and form a sub-coalition.

For calculating the nucleolus, an importation concept based on the pre-imputation vector $Z = (z_1, \ldots, z_n)$ is given as below:

Definition 2. For the cooperative game (N, V), the excess value of coalition $K \subset N = \{j = 1, \ldots, n\}$ on the pre-imputation vector $Z = (z_1, \ldots, z_n)$ is calculated as $e(K, Z) = \left(\sum_{j \in K} \vec{R}_K - \sum_{j \in K} z_j\right) - V(K)$.

The excess value measures the "unsatisfactory degree" or "unhappiness" relative to the guarantee level, i.e. the characteristic function. The nucleolus solution to the cooperative game is defined by Schmeidler (1969) as the optimal imputation that can minimize the excess value for all coalitions by lexicographical order. By sorting all excess values of coalitions $K \subseteq N = \{1, \ldots, n\}$ in non-descending order, a vector can be defined as

$$\delta(Z) = (\vartheta_1(Z), \ldots, \vartheta_{2^n-2}(Z)) = (e(K_1, Z), \ldots, e(K_{2^n-2}, Z))$$

where $e(K_1, Z) \leq \ldots \leq e(K_{2^n-2}, Z)$.

Then, the nucleolus solution can be interpreted as below:

$$\Theta(\varrho) = \{\varrho \in Z | \delta(\varrho) \leq \delta(\sigma), \forall \sigma \in Z\}. \tag{14}$$

where the pre-imputation vector $Z = (z_1, \ldots, z_n)$ is the set of all feasible distribution for the cooperative game (N, V). The nucleolus ϱ is the one that lexicographically minimizes the excess values for all coalitions.

To calculate the nucleolus of the cooperative game *(N, V)*, a min–max model based on the general concept framework of Maschler, Peleg, and Shapley (1979) is developed as below.

$$\begin{aligned}
&\underset{\upsilon_i, \mu_r, z_j, \omega, T_j, \tilde{y}_{rj}, \mu_0}{\text{Min}} \; \underset{K \subseteq N}{\text{Max}}\, e(K, Z) \\
&\text{s.t.} \; \sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} + b_j - \omega b_j \\
&\qquad + T_j + \mu_0 = z_j, j = 1, \ldots, n \\
&\quad \sum_{j=1}^{n} z_j = R \\
&\quad \sum_{j=1}^{n} T_j = \omega R \\
&\quad \sum_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \ldots, s \\
&\quad 0 \leq z_j \leq b_j, j = 1, \ldots, n \\
&\quad \upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad j = 1, \ldots, n; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned} \tag{15}$$

Let $\xi = \text{Max}_{K \subseteq N}\, e(K, Z)$, then model (15) is transformed into a linear program in model (16).

$$\begin{aligned}
&\text{Min}\, \xi \\
&\text{s.t.} \left(\sum_{j \in K} \vec{R}_j - \sum_{j \in K} z_j\right) - V(K) \leq \xi, K \subseteq N = \{1, \ldots, n\} \\
&\quad \sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} + b_j - \omega b_j \\
&\qquad + T_j + \mu_0 = z_j, j = 1, \ldots, n \\
&\quad \sum_{j=1}^{n} z_j = R \\
&\quad \sum_{j=1}^{n} T_j = \omega R \\
&\quad \sum_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \ldots, s \\
&\quad 0 \leq z_j \leq b_j, j = 1, \ldots, n \\
&\quad \upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \ldots, m; r = 1, \ldots, s; \\
&\qquad j = 1, \ldots, n; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned} \tag{16}$$

4.4.1. A practical computation procedure of nucleolus

Normally, the computation of the nucleolus is done by computing the characteristic function for each coalition and following the iterative procedure of Fromen (1997). However, when the number of coalitions is big, it is impractical to compute the characteristic function for each coalition and then a constraint generation mechanism suggested by Hallefjord et al. (1995) would be of vital significance. Therefore, this subsection would propose a practical computation procedure based on the work of Hallefjord et al. (1995).

First of all, we limit the first constraint of model (16) to a subset $K\epsilon\Omega$, where for simplification Ω can be the n singleton coalitions, namely, $\Omega=\{\{1\},\dots,\{n\}\}$. It is clear that we can easily calculate the characteristic function $V(K)$, $K\in\Omega$. Then we solve model (16) in a simplified version, which is formulated as model (17).

$$
\begin{aligned}
&\text{Min } \xi\\
&\text{s.t. } \left(\textstyle\sum_{j\in K}\vec{R}_j-\sum_{j\in K}z_j\right)-V(K)\le\xi, K\in\Omega\\
&\quad \textstyle\sum_{i=1}^{m}\upsilon_i x_{ij}-\sum_{r=1}^{s}\mu_r y_{rj}-\sum_{r=1}^{s}\tilde{y}_{rj}+b_j-\omega b_j\\
&\qquad +T_j+\mu_0=z_j, j=1,\dots,n\\
&\quad \textstyle\sum_{j=1}^{n}z_j=R\\
&\quad \textstyle\sum_{j=1}^{n}T_j=\omega R\\
&\quad \textstyle\sum_{j=1}^{n}\tilde{y}_{rj}\ge 0, r=1,\dots,s\\
&\quad 0\le z_j\le b_j, j=1,\dots,n\\
&\quad \upsilon_i,\mu_r,\omega,T_j\ge 0, i=1,\dots,m; r=1,\dots,s;\\
&\qquad j=1,\dots,n;\tilde{y}_{rj}\text{ and }\mu_0\text{ free.}
\end{aligned}
\tag{17}
$$

Solving model (17) determines a series of optimal solutions denoted as $\left(z_j^*,\upsilon_i^*,\mu_r^*,T_j^*,\tilde{y}_{rj}^*,\omega^*,\ \mu_0^*,\xi^*,\right.$ $\forall j,i,r)$. Note that the nucleolus is rather hard to compute in general cases and it is easy to compute it incorrectly due to multiple optimal solutions of model (16) and model (17) and the absence of relevance of duality. Following the approach of Guajardo and Jörnsten (2015), we formulate the dual form of model (17), which is given in model (18).

$$
\begin{aligned}
&\text{Max } \textstyle\sum_{\in\Omega}\rho_K\vec{R}_K-\sum_{j=1}^{n}\beta_j b_j+\tau R-\sum_{j=1}^{n}\pi_j b_j\\
&\text{s.t. } \textstyle\sum_{j=1}^{n}\beta_j x_{ij}\le 0, i=1,..,m\\
&\quad \textstyle\sum_{j=1}^{n}\beta_j y_{rj}\ge 0, r=1,..,s\\
&\quad \beta_j-\chi_r\le 0, r=1,\dots s; j=1,\dots,n\\
&\quad -\textstyle\sum_{K\in\Omega:j\in K}\rho_K+\beta_j-\tau+\pi_j\ge 0, j=1,\dots,n\\
&\quad \textstyle\sum_{j=1}^{n}\beta_j b_j+\alpha R\ge 0\\
&\quad \beta_j+\alpha\le 0, j=1,\dots,n\\
&\quad \textstyle\sum_{j=1}^{n}\beta_j=0\\
&\quad \textstyle\sum_{K\in\Omega}\rho_K=1\\
&\quad \rho_K,\pi_j,\chi_r\ge 0, K\in\Omega, j=1,\dots,n; r=1,\dots,s;\\
&\qquad \beta_j,\tau\text{ and }\alpha\text{ free.}
\end{aligned}
\tag{18}
$$

Solving model (18) determines a solution $\left(\rho_K^*,\beta_j^*,\tau^*,\alpha^*,\pi_j^*,\chi_j^*\right)$, which will be corresponding with the optimal objective function of model (17), namely, ξ^*. According to the duality theory, a positive value of the optimal dual variable implies the establishment of equality of inequality constraint in the primary problem. That is to say, if $\rho_K^*>0$ then it holds exactly $\vec{R}_K-\sum_{j\in K}z_j^*=\xi^*$ for $K\epsilon\Omega$. If there exists $\rho_K^*=0\,(K\epsilon\Omega)$, which implies that model (17) might have multiple optimal solutions, then we will continue to repeatedly solve model (17) and model (18) to obtain a unique solution, denoted as $\left(z_j^*,\upsilon_i^*,\mu_r^*,T_j^*,\tilde{y}_{rj}^*,\omega^*,\mu_0^*,\xi^*,\forall j,i,r\right)$. At this time, it holds $\vec{R}_K-\sum_{j\in K}z_j^*=\xi^*$ for all K belonging to Ω. Acknowledging that Ω contains the n singleton coalitions, hence we have $\vec{R}_j-z_j^*=\xi^*\,(j=1,\dots,n)$. Further, we are able to identify the most unsatisfied coalition among all nonempty subset of N, that is, $\text{Max}_{\substack{K\notin\Omega\\K\neq\emptyset,N}}\left[\left(\sum_{j\in K}\vec{R}_j-\sum_{j\in K}z_j^*\right)-V(K)-\xi^*\right]$. By simplification, it is $\text{Max}_{\substack{K\notin\Omega\\K\neq\emptyset,N}}\left(\vec{R}_K-\sum_{j\in K}z_j^*-\xi^*\right)$. Reconsidering formula (13) which contains the basic idea of computing $\vec{R}_K$, the above problem can be formulated as model (19) in a similar way of Hallefjord et al. (1995).

Table 3. A simple example.

DMU	x_1	x_2	y	b
1	5	9	7	8
2	8	6	7	7
3	9	8	6	8
4	6	10	8	10
5	11	8	7	9

Table 4. Preliminary results.

DMU	Original efficiency Model (4)	Post efficiency Model (10)	Minimal reduction Model (12)	Maximal reduction Model (12)
1	1.0000	1.0000	0	8.0000
2	1.0000	1.0000	0	7.0000
3	0.8571	1.0000	0	8.0000
4	1.0000	1.0000	0	10.0000
5	0.9333	1.0000	0	9.0000

$$
\begin{aligned}
O^{\max} = \ & \mathrm{Max} \sum\nolimits_{j=1}^{n} f_j - \sum\nolimits_{j=1}^{n} d_j z_j^* - \xi^* \\
\text{s.t.} \ & z_j = \sum\nolimits_{i=1}^{m} \upsilon_i x_{ij} - \sum\nolimits_{r=1}^{s} \mu_r y_{rj} - \sum\nolimits_{r=1}^{s} \tilde{y}_{rj} + b_j \\
& \quad -\omega b_j + T_j + \mu_0, j = 1, \dots, n \\
& \sum\nolimits_{j=1}^{n} z_j = R \\
& \sum\nolimits_{j=1}^{n} T_j = \omega R \\
& \sum\nolimits_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \dots, s \\
& 0 \leq z_j \leq b_j, j = 1, \dots, n \\
& 1 \leq \sum\nolimits_{j=1}^{n} d_j \leq n-1 \\
& f_j \leq M \cdot d_j, j = 1, \dots, n \\
& f_j \leq z_j, j = 1, \dots, n \\
& \upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \dots, m; r = 1, \dots, s; j = 1, \dots, n \\
& d_j \in \{0, 1\}; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned} \tag{19}
$$

In above model (19), $d_j\,(j = 1, \dots, n)$ is the jth component of the non-negative coalition vector $(d_1, \dots, d_n)'$ with d_j being one if DMU_j is a member of the coalition and being zero if not a member of that coalition. M is a large enough positive value ensuring that f_j will be zero if $d_j = 0$ and f_j will be a positive value if $d_j = 1$. The first five constraints are used to ensure the efficient allocation scheme and unreduced desirable outputs. The sixth constraint guarantees that the most unsatisfied coalition is a real nonempty subset of N. Finally, the seventh and eighth constraints can ensures that f_j will be either zero or z_j, which reflects the calculation of $\vec{R}_K$ in model (13). The optimal solution of model (19) can be used to generate a new constraint which will be added to model (17). Suppose that model (19) has an optimal solution $\left(\xi^*, z_j^{K^*}, c_j^{K^*}, d_j^{K^*}, \upsilon_i^{K^*}, \mu_r^{K^*}, \mu_0^{K^*}, T_j^{K^*}, \omega^{K^*}, \tilde{y}_{rj}^{K^*}, f_j^{K^*}, \forall j, i, r\right)$, then the constraint $\vec{R}_{K^*} - \sum_{j \in K^*} z_j \leq \xi^*$ is added to the simplified model (17), and model (19) is resolved. The above idea is formulated as model (20).

$$
\begin{aligned}
& \underset{\upsilon_i, \mu_r, z_j}{\mathrm{Min}} \ \xi \\
& \text{s.t.} \ \left(\sum\nolimits_{j \in K} \vec{R}_j - \sum\nolimits_{j \in K} z_j\right) - V(K) \leq \xi, K \in \Omega \\
& \qquad \vec{R}_{K^*} - \sum\nolimits_{j \in K^*} z_j \leq \xi \\
& \sum\nolimits_{i=1}^{m} \upsilon_i x_{ij} - \sum\nolimits_{r=1}^{s} \mu_r y_{rj} - \sum\nolimits_{r=1}^{s} \tilde{y}_{rj} + b_j - \omega b_j \\
& \qquad + T_j + \mu_0 = z_j, j = 1, \dots, n \\
& \sum\nolimits_{j=1}^{n} z_j = R \\
& \sum\nolimits_{j=1}^{n} T_j = \omega R \\
& \sum\nolimits_{j=1}^{n} \tilde{y}_{rj} \geq 0, r = 1, \dots, s \\
& 0 \leq z_j \leq b_j, j = 1, \dots, n \\
& \upsilon_i, \mu_r, \omega, T_j \geq 0, i = 1, \dots, m; r = 1, \dots, s; \\
& \qquad j = 1, \dots, n; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.}
\end{aligned} \tag{20}
$$

Model (20) and model (19) will be solved repeatedly until the termination condition, $O^{\max} = \sum_{j=1}^{n} f_j^{K^*} - \sum_{j=1}^{n} d_j^{K^*} z_j^* - \xi^* \leq 0$, is held. Under this circumstance, the constraint $V\left(\sum_{j \in K} \vec{R}_j - \sum_{j \in K} z_j\right) - (K) \leq \xi$ in original model (16) is satisfied for all nonempty coalitions of DMUs, $K \subseteq N = \{1, \dots, n\}$. Note in particular that an additional constraint $\sum_{\{j | d_j^{K^*} = 0\}} d_j + \sum_{\{j | d_j^{K^*} = 1\}} (1 - d_j) \geq 1$ would be also added in the computation procedure to prevent the coalition K^* from later iteration process. In addition, the constraints of model (16) are all satisfied, and the optimal objective function of model (16) can be calculated through a practical computation procedure. This procedure is helpful for a lexicographic optimization of model (16), and the unique nucleolus solution corresponds to the final carbon emission abatement quota allocation scheme.

4.5. An illustrative application

To illustrate the usefulness of the proposed cooperative game DEA approach, in this subsection we provide a small case. In Table 3, there are five DMUs with two inputs (x_1 and x_2), one undesirable output (y) and one undesirable output (b).

By solving model (4) once for each DMU, we can find three efficient DMUs (DMU_1, DMU_2, and DMU_4) and two inefficient DMUs (DMU_3 and DMU_5), and the efficiency scores are listed in the second column of Table 4.

The current total undesirable output is 42, and for simplification we consider an undesirable output reduction goal of 15 across five DMU, namely, $R = 15$. In addition, through solving model (10) for each individual DMU we can calculate the possible post-abatement-allocation efficiency from each DMU's decentralized perspective, as given in the third column of Table 4. It can be learned from Table 4 that it is possible for any DMU to reach the highest efficiency score of unity by taking the undesirable output reduction into account. This finding is also shown by Theorem 2. Further, all DMUs can be simultaneously efficient by considering undesirable output reduction goal, but there exist huge flexibilities in the undesirable output reduction plans. For a first validation of this flexibility, we can look at each DMU's minimal and maximal reduction amount $(\overleftarrow{R}_j, \vec{R}_j)$ derived from model (12) and given in the last two columns of Table 4. It is clear that each DMU can completely reduce its undesirable output level yet ensuring all DMUs are efficient and the total desirable outputs are unreduced. In addition, the total maximal reduction amount across five DMUs are larger than the total reduction goal as $42 > 15$. On the contrary, each DMU might be reduced a relatively small undesirable output amount and also making all DMUs

Table 5. The constraint generation process for the minimal excess value.

Iteration no.	ξ^*	Solution z_1^*	z_2^*	z_3^*	z_4^*	z_5^*	O^{max}	Constrained coalition
1	1.2000	2.8000	2.3000	2.8000	3.8000	3.3000	2.6000	{1,2,3,5}
2	2.5000	3.0000	1.0000	4.0000	2.5000	4.5000	3.5000	{1,2,4}
3	2.5000	4.0000	3.5000	1.5000	2.5000	4.5000	3.5000	{3,4,5}
4	3.6667	4.0000	1.1667	1.6667	3.6667	1.6667	2.3333	{2,3,4}
5	3.6667	4.0000	1.1667	4.0000	3.6667	3.3333	2.3333	{2,4,5}
6	3.6667	2.8333	2.3333	2.8333	3.6667	3.3333	0.0000	{1,3,4}
7	3.6667	2.8333	2.2333	2.8333	3.6667	3.3333	0.0000	{1,4,5}
8	3.6667	2.8333	2.3333	2.8333	3.6667	3.3333	−0.1667	{1,2,5}

Table 6. Dataset for 31 sub-level manufacturing industries.

DMU	x_1	x_2	x_3	y	u
DMU_1	2345.4120	4042.8384	27,505.4652	5268.1562	34,957.9171
DMU_2	1000.9680	2017.8770	16,213.1919	1590.2686	26,929.2787
DMU_3	1117.6840	1502.0504	11,800.8564	1361.0131	15,294.9203
DMU_4	708.4340	214.6285	2474.1824	791.3484	1622.8970
DMU_5	2047.9980	5502.0328	63,570.0966	3225.4091	41,235.7124
DMU_6	998.5240	4694.9556	8610.8837	1761.7822	4699.3819
DMU_7	559.8740	2828.8480	5742.2737	1135.2394	1608.2115
DMU_8	444.1300	1377.0669	11,526.3821	1051.5143	8392.0546
DMU_9	354.5860	1231.7698	1994.0713	576.6864	821.7364
DMU_{10}	1186.2730	1622.0111	38,461.4019	1284.6778	90,191.8334
DMU_{11}	378.0740	892.9594	4000.2502	461.5240	847.3745
DMU_{12}	508.6820	2247.8443	2804.5532	1025.8159	887.9110
DMU_{13}	2093.8780	813.9607	181,154.4352	3948.9324	2,040,675.7347
DMU_{14}	5338.2090	5070.4290	369,955.4389	6809.4352	499,314.0437
DMU_{15}	1576.8510	2147.7536	16,086.3055	1769.6635	17,149.1155
DMU_{16}	573.7440	498.1379	15,579.9525	677.0168	14,290.6147
DMU_{17}	1615.1240	3436.6337	38,971.3981	2473.5281	16,876.7012
DMU_{18}	3540.7840	5747.2893	294,009.2348	4523.7342	504,976.8358
DMU_{19}	5818.3480	3339.2290	596,681.0204	6951.5540	605,161.6462
DMU_{20}	2810.9660	2201.7215	148,290.1179	3847.8902	132,226.1401
DMU_{21}	1941.0900	3742.9294	38,543.4384	2968.9096	8704.7872
DMU_{22}	3149.3590	4736.7444	34,658.9286	3875.0892	6892.3558
DMU_{23}	2640.3520	3635.8722	17,818.3895	2930.3186	9931.4776
DMU_{24}	4039.9580	4304.9022	27,606.6976	5058.3485	14,469.2241
DMU_{25}	1869.4570	2104.7694	11,495.3636	1644.7536	7011.1171
DMU_{26}	4231.7440	6878.0743	23,290.7460	5549.9724	10,996.4708
DMU_{27}	4642.7820	10,255.5320	26,667.4979	7092.0568	6346.0115
DMU_{28}	584.4970	1155.8961	3112.6044	677.3105	723.7331
DMU_{29}	172.0230	418.9237	16,164.7095	209.0727	10,800.6134
DMU_{30}	141.2020	166.1615	1073.5639	294.5942	328.2493
DMU_{31}	117.0750	149.6450	813.4804	89.8976	347.5371

efficient, and the sum of minimal reduction amount across five DMUs are smaller than the total reduction goal as $0 < 15$. Such a phenomenon is called as egoist's dilemma (Nakabayashi & Tone, 2006), and game-based approaches are useful to generate a unique and fair undesirable output reduction plan.

To use the integrated cooperative game DEA approach, we first solve model (16) for n singleton coalitions, namely model (17). Before that we calculate the maximal reduction amount $\vec{R}_j$ (as shown in the last column of Table 4) and the characteristic function $V(K)$ for n singleton coalitions, $V(1) = V(2) = V(3) = V(4) = V(5) = 0$.

As shown in Table 5, solving model (17) for the first round determines an optimal objective function of $\xi^* = 1.2000$ and an imputation $(z_1^*, z_2^*, z_3^*, z_4^*, z_5^*) = (2.8000, 2.3000, 2.8000, 3.8000, 3.3000)$.

Note in particular that solving model (18) determines five positive dual variables (ρ_1, ρ_2, ρ_3, ρ_4, and ρ_5), implying that the previous optimal solution of model (17) is unique (we omit to report the results of dual formulas in later section for simplification). Further, based on the optimal solution of model (17) we solve model (19) to find the most unsatisfied coalition {1, 2, 3, 5} with an optimal objective function $O^{\max} = 2.6000$. Therefore, two constraints $\vec{R}_{\{1,2,3,5\}} - z_1 - z_2 - z_3 - z_5 \le \xi$ and $-d_1 - d_2 - d_3 + d_4 - d_5 \ge -3$ will be generated and added. In the second iteration, we solve model (20) to compute the solution corresponding to the current minimized excess value. As a result we obtain an optimal objective function of $\xi^* = 2.5000$ and a solution $(z_1^*, z_2^*, z_3^*, z_4^*, z_5^*) = (3.0000, 1.0000, 4.0000, 2.5000, 4.5000)$. Making use of this objective function and solution $(z_1^*, z_2^*, z_3^*, z_4^*, z_5^*)$, model (19) yields that another coalition {1, 2, 5} must be taken into account as it reached the largest unsatisfied value $O^{\max} = 3.5000$.

The above iterative process will obtain an optimal objective function of $\xi^* = 3.6667$ and a solution $(z_1^*, z_2^*, z_3^*, z_4^*, z_5^*) = (2.8333, 2.3333, 2.8333, 3.6667, 3.3333)$ in the sixth round. At this time the most

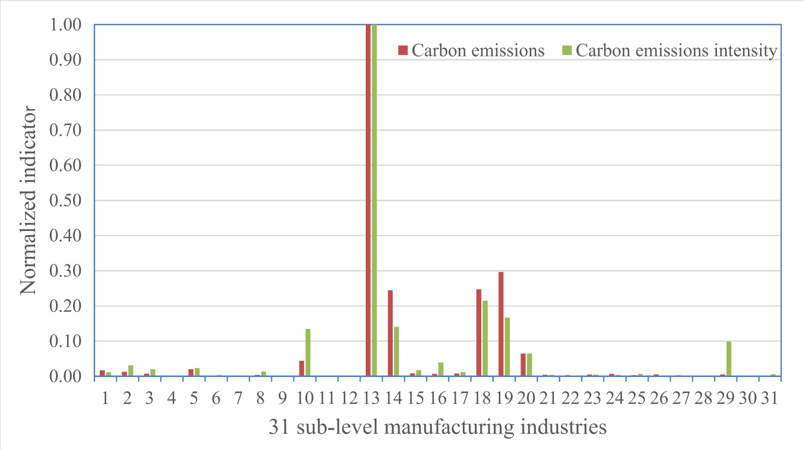

Figure 1. Normalized carbon emissions and carbon emission intensity.

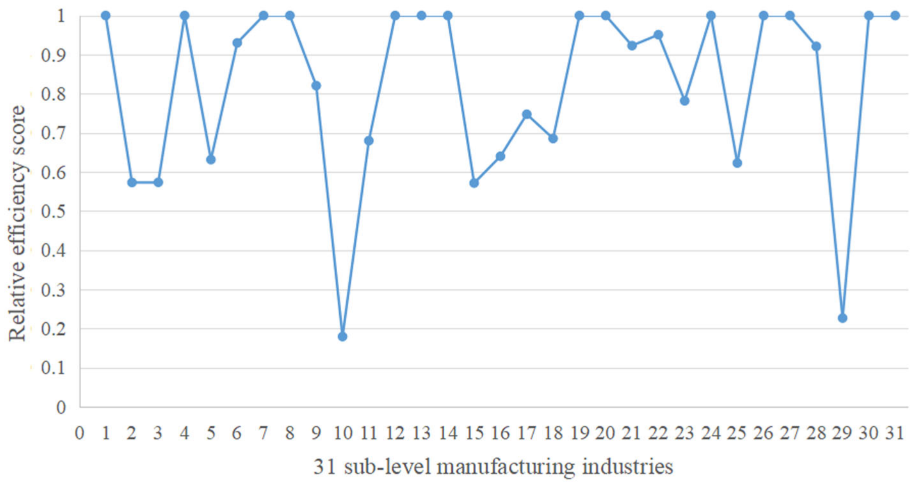

Figure 2. Relative efficiency of 31 sub-level manufacturing industries.

Table 7. CO_2 abatement quota allocation for 31 sub-level manufacturing industries.

DMUs	Allocation	DMUs	Allocation	DMUs	Allocation
DMU_1	2517.0333	DMU_{11}	0	DMU_{21}	8704.7872
DMU_2	2961.8097	DMU_{12}	0	DMU_{22}	0
DMU_3	2961.8148	DMU_{13}	11,3323.2930	DMU_{23}	0
DMU_4	0	DMU_{14}	10,7622.0579	DMU_{24}	0
DMU_5	0	DMU_{15}	2961.8123	DMU_{25}	2137.2264
DMU_6	0	DMU_{16}	7572.5680	DMU_{26}	0
DMU_7	0	DMU_{17}	16,876.7012	DMU_{27}	3184.2017
DMU_8	0	DMU_{18}	107,844.4474	DMU_{28}	723.7331
DMU_9	0	DMU_{19}	113,323.2930	DMU_{29}	2739.4202
DMU_{10}	2739.4254	DMU_{20}	104,588.3497	DMU_{30}	0
				DMU_{31}	0

unsatisfied coalition {1, 3, 4} is identified. In addition, the optimal objective function of model (19) reaches 0.0000, implying that we have found the minimal excess value and the constraint generation process for the minimal excess value terminates.

We then continue to identify the allocation scheme corresponding to the nucleolus. For these identified coalitions in Table 5 it holds equalities $\vec{R}_K - \sum_{j\epsilon K} z_j = \xi^*$. Take coalition {1, 3, 4} for example, it holds $13 - z_1 - z_3 - z_4 = 3.6667$. For coalition {2, 4, 5} it holds $13 - z_2 - z_4 - z_5 = 3.6667$. Solving those equalities simultaneously determines a unique solution (2.8333, 2.3333, 2.8333, 3.6667, 3.3333), and that solution is the nucleolus-based allocation scheme. Interestingly, DMUs with less undesirable outputs will be allocated with a smaller abatement amount.

5. Empirical study in Chinese manufacturing industries

In this section, we will apply the integrated cooperative game DEA approach to address a real-world problem, that is, allocating the carbon emission abatement quota in Chinese manufacturing industries. As discussed in previous Section 3, the CO_2

Table 8. Relationship associated with the allocation results.

DMU	Emission percentage (%)	Rank	Reduction percentage (%)	Rank	Allocation percentage (%)	Rank
DMU_1	0.8574	7	7.2002	15	0.4176	15
DMU_2	0.6807	8	10.9985	14	0.4914	12
DMU_3	0.3744	9	19.3647	11	0.4914	10
DMU_{10}	2.0804	6	3.0373	17	0.4545	13
DMU_{13}	48.5998	1	5.5532	16	18.8000	1
DMU_{14}	12.9683	3	21.5540	9	17.8542	4
DMU_{15}	0.3623	11	17.2709	13	0.4914	11
DMU_{16}	0.3653	10	52.9898	5	1.2563	8
DMU_{17}	0.3615	12	100.0000	1	2.7998	6
DMU_{18}	12.8063	4	21.3563	10	17.8911	3
DMU_{19}	14.1514	2	18.7261	12	18.8000	1
DMU_{20}	3.1934	5	79.0981	4	17.3509	5
DMU_{21}	0.221	14	100.0000	1	1.4441	7
DMU_{25}	0.1681	15	30.4834	7	0.3546	16
DMU_{27}	0.159	16	50.1764	6	0.5283	9
DMU_{28}	0.0124	17	100.0000	1	0.1201	17
DMU_{29}	0.2273	13	25.3636	8	0.4545	14

Table 9. Summary statistics of 31 provinces for 31 sub-industries.

Statistics	x_1	x_2	x_3	y_1	u
Min	0.0000	0.0000	0.0000	0.0000	0.0000
Max	1548.8482	3376.5480	104,242.2540	2286.5395	276,300.7450
Average	60.9982	88.6511	2312.7701	81.2663	4917.0858
Std.ev	116.1812	192.9952	7365.3472	168.0707	116.1812

emission reduction gap should be [367319.9970, 838243.9520] in the unit of thousand tons. Since it is difficult to allocate an interval quota, without loss of generality we consider the average value, $(367,319.9970 + 838,243.9520)/2 = 602,781.9745$, as the carbon emissions abatement quota in this research.

5.1. Data and preliminary analysis

In this subsection, a sub-level industrial dataset in 2012 is considered. There are 31 sub-industries with three inputs (total assets in billion Yuan, x_1; annual average employment personnel in thousand person, x_2; and total energy consumption in thousand tons of standard coal equivalent, x_3), one desirable output (gross output value of industry in billion Yuan, y) and one undesirable output (CO_2 emission in thousand tons, u), as given in Table 6 (The codes for two-digit Chinese manufacturing industries can be found in Appendix F).

First of all, we give some preliminary analysis for 31 sub-level manufacturing industries. For an overview of carbon emissions and intensity, we normalize the carbon emissions and carbon emissions intensity indicators. To this end, the normalized carbon emissions of a certain sub-level manufacturing industry is calculated as (real carbon emissions − B)/(A − B), where A is the maximal carbon emissions and B is the minimal emissions across 31 sub-level manufacturing industries. The normalized carbon emissions intensity indicators can be calculated in a similar way and Figure 1 shows the normalized carbon emissions and carbon emission intensity for 31 sub-level manufacturing industries. It can be learned from Figure 1 that most carbon emissions are mainly derived from six sub-level manufacturing industries (DMU_{10}, Manufacture of Paper and Paper Products; DMU_{13}, Processing of Petroleum, Coking and Processing of Nuclear Fuel; DMU_{14}, Manufacture of Raw Chemical Materials and Chemical Products; DMU_{18}, Manufacture of Non-metallic Mineral Products; DMU_{19}, Smelting and Pressing of Ferrous Metals; DMU_{20}, Smelting and Pressing of Non-ferrous Metals), of which the carbon emissions are relatively larger than other industries. Most seriously, DMU_{13} (Processing of Petroleum, Coking and Processing of Nuclear Fuel), which has the largest carbon emissions among 31 sub-level manufacturing industries, even accounts for 48.5998% of the total carbon emissions in Chinese manufacturing industries. Therefore, for the sake of reducing the carbon emissions in China and realizing the national carbon emissions abatement commitment special attention should be paid to these sub-level manufacturing industries. In addition, we can learn from Figure 1 that most sub-level manufacturing industries with large carbon emissions are also with high carbon emissions intensity. Three significant exceptions are DMU_{10} (Manufacture of Paper and Paper Products), DMU_{16} (Manufacture of Chemical Fibres) and DMU_{29} (Other Manufacture), which have a normalized carbon emissions intensity indicator much larger than its normalized carbon emissions indicator.

Then we can use model (4) to assess the relative efficiency of 31 sub-level manufacturing industries, as given in Figure 2. It is clear that 14 sub-level manufacturing industries are efficient in the sense that they cannot proportionally expand their gross output value of industry and shrink the carbon emissions. Furthermore, two sub-level manufacturing industries (DMU_{10}, Manufacture of Paper and Paper Products; DMU_{29}, Other Manufacture) have the least two relative efficiency scores, which is

Table 10. Results of regional CEA allocation scheme (unit: thousand tons).

Regions	I_1	I_2	I_3	I_{10}	I_{13}	I_{14}
Beijing	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Tianjin	223.7132	504.0956	0.0000	0.0000	0.0000	0.0000
Hebei	0.0000	0.0000	90.0918	0.0000	0.0000	0.0000
Shanghai	0.0000	0.0000	89.1932	0.0000	0.0000	0.0000
Jiangsu	0.0000	0.0000	230.0905	0.0000	0.0000	0.0000
Zhejiang	0.0000	0.0000	118.9122	0.0000	0.0000	11,687.1846
Fujian	0.0000	0.0000	144.6297	0.0000	0.0000	0.0000
Shandong	0.0000	0.0000	324.8134	0.0000	49,499.2709	0.0000
Guangdong	0.0000	0.0000	94.7229	0.0000	0.0000	16,604.6209
Hainan	0.0000	0.0000	61.5814	0.0000	0.0000	0.0000
Liaoning	628.6325	0.0000	107.5477	1066.2830	0.0000	0.0000
Jilin	0.0000	0.0000	112.9420	0.0000	0.0000	0.0000
Heilongjiang	0.0000	0.0000	0.0000	0.0000	10,990.8629	0.0000
Anhui	322.7990	0.0000	1.1022	0.0000	0.0000	0.0000
Jiangxi	0.0000	0.0000	196.4356	1198.3350	0.0000	0.0000
Henan	0.0000	0.0000	39.1846	0.0000	0.0000	0.0000
Hubei	0.0000	1719.4612	94.7229	474.8074	0.0000	29,510.7443
Hunan	1341.8887	438.4614	23.1890	0.0000	27,667.7339	0.0000
Shanxi	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Inner Mongolia	0.0000	0.0000	230.0905	0.0000	0.0000	0.0000
Guangxi	0.0000	0.0000	145.6301	0.0000	0.0000	0.0000
Chongqing	0.0000	0.0000	61.2714	0.0000	0.0000	0.0000
Sichuan	0.0000	0.0000	324.8134	0.0000	0.0000	35,739.9891
Guizhou	0.0000	0.0000	56.6404	0.0000	0.0000	0.0000
Yunnan	0.0000	0.0000	66.0042	0.0000	0.0000	0.0000
Tibet	0.0000	0.0000	23.0872	0.0000	0.0000	0.0000
Shaanxi	0.0000	0.0000	33.4514	0.0000	0.0000	0.0000
Gansu	0.0000	299.7915	126.0718	0.0000	0.0000	0.0000
Qinghai	0.0000	0.0000	41.7131	0.0000	0.0000	0.0000
Ningxia	0.0000	0.0000	85.7997	0.0000	25,165.4252	14,079.5184
Xinjiang	0.0000	0.0000	38.0824	0.0000	0.0000	0.0000

consistent with the previous result that the two manufactures have its carbon emissions intensity indicator much larger than corresponding carbon emissions indicator.

5.2. Sub-industrial allocation

Using the mathematical models in Section 4 and following a similar computation process in Section 4.5, we can calculate the nucleolus-based allocation of the national carbon emissions abatement quota across 31 sub-level manufacturing industries. In particular, we impose a reduction upper in this section by considering the reduction difficulty. The results of CO_2 abatement quota allocation for 31 sub-level manufacturing industries are given in Table 7 (the computation process is not presented here for its large-scale).

For the first sight of Table 7, it can be learned that only some sub-level manufacturing industries will be allocated with carbon abatement responsibility, while fourteen sub-level manufacturing industries (DMU_4, DMU_5, DMU_6, DMU_7, DMU_8, DMU_9, DMU_{11}, DMU_{12}, DMU_{22}, DMU_{23}, DMU_{24}, DMU_{26}, DMU_{30}, and DMU_{31}) don't have to reduce its carbon emission level. Further, DMU_{13} (Processing of petroleum, coking and processing of nuclear fuel) and DMU_{19} (Smelting and pressing of ferrous metals) were allocated with the most carbon emissions abatement responsibility, reaching 113,323.2930 (thousand tons) and 18.8000% of the total carbon emissions abatement quota. In addition, five DMUs attract our attention deeply for its large shares in the whole sample, namely, DMU_{13} (Processing of petroleum, coking and processing of nuclear fuel), DMU_{14} (Manufacture of raw chemical materials and chemical product), DMU_{18} (Manufacture of non-metallic mineral products), DMU_{19} (Smelting and pressing of ferrous metals) and DMU_{20} (Smelting and pressing of non-ferrous metals). These five sub-industries produced only 32.2291% of the total GIOV in 2012 but 91.4781% of the total CO_2 emissions, and account for 90.6964% of the total carbon emission abatement quota. The key to control the carbon emissions and to realize the national reduction commitment is to manage these five sub-industries, so to speak. Once the carbon emissions abatement is well done in these five sub-industries, the national carbon emissions abatement task will be easily implemented.

To investigate the relationship between current carbon emissions and allocated carbon emissions abatement quota, Table 8 lists the CO_2 emissions percentage (CO_2 emissions/total CO_2 emissions), reduction percentage (the allocated quota/CO_2 emissions), and abatement quota allocation percentage (allocated quota/total abatement quota) and corresponding descending rankings for 17 sub-level manufacturing industries that are allocated with a positive carbon emission abatement amount.

From Table 8, we can learn that the sub-industries with higher CO_2 emission percentage are most

Table 11. Results of regional CEA allocation scheme (unit: thousand tons).

Regions	I_{15}	I_{16}	I_{17}	I_{18}	I_{19}	I_{20}
Beijing	312.2816	0.0000	32.2274	0.0000	0.0000	0.0000
Tianjin	0.0000	3.6336	163.3004	0.0000	0.0000	270.4847
Hebei	0.0000	251.9519	514.2943	924.9577	0.0000	544.0242
Shanghai	0.0000	0.0000	672.0801	14,868.7929	0.0000	1253.9031
Jiangsu	0.0000	920.9691	1780.0800	0.0000	0.0000	8737.7880
Zhejiang	0.0000	966.9343	1982.1896	17,579.1939	0.0000	5521.7351
Fujian	0.0000	177.4540	393.1660	0.0000	0.0000	57.5132
Shandong	0.0000	865.8477	3866.5647	0.0000	0.0000	19,377.8286
Guangdong	0.0000	357.0427	3490.1712	74,471.5030	25,755.3523	7388.6808
Hainan	0.0000	35.8871	40.8632	0.0000	0.0000	0.0000
Liaoning	0.0000	302.5258	533.2964	0.0000	37,107.5793	865.1980
Jilin	1871.7447	574.2076	155.2591	0.0000	0.0000	344.2977
Heilongjiang	0.0000	78.7690	0.0000	0.0000	0.0000	693.9880
Anhui	0.0000	421.8767	187.5374	0.0000	0.0000	423.8788
Jiangxi	0.0000	374.3625	0.0000	0.0000	0.0000	440.9990
Henan	0.0000	354.3782	383.7191	0.0000	0.0000	14,240.5976
Hubei	777.7860	171.5063	38.1489	0.0000	0.0000	732.4983
Hunan	0.0000	67.3061	0.0000	0.0000	0.0000	2330.9355
Shanxi	0.0000	5.9966	0.0000	0.0000	24,785.0413	8069.3588
Inner Mongolia	0.0000	0.0000	148.2670	0.0000	0.0000	1262.4656
Guangxi	0.0000	0.0000	126.4290	0.0000	22,108.4996	9852.1529
Chongqing	0.0000	0.0000	266.5483	0.0000	0.0000	836.7945
Sichuan	0.0000	614.4358	717.0869	0.0000	0.0000	3037.5572
Guizhou	0.0000	0.0000	931.1748	0.0000	0.0000	3454.2565
Yunnan	0.0000	98.6850	80.8533	0.0000	0.0000	2543.6940
Tibet	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Shaanxi	0.0000	19.3078	283.3317	0.0000	3566.8206	7095.2553
Gansu	0.0000	46.3543	90.1125	0.0000	0.0000	4736.3018
Qinghai	0.0000	0.0000	0.0000	0.0000	0.0000	118.3759
Ningxia	0.0000	0.0000	0.0000	0.0000	0.0000	157.4397
Xinjiang	0.0000	863.1363	0.0000	0.0000	0.0000	200.3473

Table 12. Results of regional CEA allocation scheme (unit: thousand tons).

Regions	I_{21}	I_{25}	I_{27}	I_{28}	I_{29}
Beijing	37.0665	11.7220	0.0000	1.7815	0.0000
Tianjin	123.1876	33.2374	135.1479	0.0000	4.7496
Hebei	826.7085	55.6882	21.2433	45.9428	11.3411
Shanghai	327.0516	62.4681	601.8260	6.1090	27.3648
Jiangsu	1635.1095	281.8791	601.8260	168.4141	129.1209
Zhejiang	817.5394	109.9419	540.6590	146.6941	224.1699
Fujian	0.0000	0.0000	0.0000	0.0000	48.6643
Shandong	1635.1095	281.8791	601.8260	162.3051	54.0832
Guangdong	1635.1095	244.3248	601.8260	168.4141	662.1570
Hainan	12.0564	0.0000	0.0000	1.0957	0.0000
Liaoning	159.2857	80.0864	0.0000	4.1874	53.7470
Jilin	16.6236	38.8625	0.0000	3.0695	85.4812
Heilongjiang	7.4108	158.6444	0.0000	1.5355	0.5222
Anhui	39.0455	8.9641	0.0000	0.0000	0.0000
Jiangxi	3.8453	8.8393	0.0000	0.0000	0.0000
Henan	181.0752	37.5543	39.9237	3.6403	10.0296
Hubei	123.5324	0.0000	0.0000	0.0000	19.5879
Hunan	133.5340	41.8797	0.0000	3.0158	18.1144
Shanxi	159.0355	13.2928	0.0000	0.0000	1308.2723
Inner Mongolia	45.4263	0.2250	0.0000	0.0000	12.1344
Guangxi	112.2681	29.9922	0.0000	1.6428	3.7036
Chongqing	289.1818	117.9709	0.0000	1.5578	0.0000
Sichuan	224.1255	387.8118	0.0000	2.4687	13.5617
Guizhou	27.7776	2.9274	0.0000	0.4084	2.7338
Yunnan	29.6217	19.5659	0.0000	0.0000	6.9867
Tibet	0.0000	0.0000	0.0000	0.0000	0.0000
Shaanxi	58.3171	89.7181	39.9237	1.4504	15.3631
Gansu	24.0946	19.7510	0.0000	0.0000	27.5315
Qinghai	0.0000	0.0000	0.0000	0.0000	0.0000
Ningxia	0.0000	0.0000	0.0000	0.0000	0.0000
Xinjiang	21.6477	0.0000	0.0000	0.0000	0.0000

likely to be allocated larger proportions of the total CO_2 emissions abatement quota. Actually, most sub-industries are ranked at the same place or antero-posterior position as they process the CO_2 emissions percentage (DMU_3, DMU_{13}, DMU_{14}, DMU_{15},

Table 13. CO_2 abatement quota allocated to provinces (10 thousand tons).

Provinces	Allocation	Provinces	Allocation
Beijing	395.0790	Hubei	33,662.7956
Tianjin	1461.5499	Hunan	32,066.0585
Hebei	3286.2439	Shanxi	34,340.9973
Shanghai	17,908.7889	Inner Mongolia	1698.6088
Jiangsu	14,485.2774	Guangxi	32,380.3184
Zhejiang	39,695.1540	Chongqing	1573.3246
Fujian	821.4271	Sichuan	41,061.8507
Shandong	76,669.5284	Guizhou	4475.9190
Guangdong	131,473.9251	Yunnan	2845.4106
Hainan	151.4837	Tibet	23.0872
Liaoning	40,908.3691	Shaanxi	11,202.9392
Jilin	3202.4876	Gansu	5370.0089
Heilongjiang	11931.7327	Qinghai	160.0890
Anhui	1405.2036	Ningxia	39,488.1829
Jiangxi	2222.8166	Xinjiang	1123.2137
Henan	15,290.1027		

DMU_{18}, DMU_{19}, DMU_{20}, DMU_{25}, DMU_{28}, and DMU_{29}). Acknowledging that the units with larger carbon emissions naturally show stronger carbon reduction capacity or higher carbon reduction potentials, it is reasonable to allocate larger shares of the total carbon emissions abatement quota to these units. This result is also consistent with Yu et al. (2014) and Zhang et al. (2014).

However, the reduction percentage (the allocated quota/CO_2 emissions) varies from one industry to another industry, with the maximal and minimal reduction percentage being 100.0000% (Manufacture of Rubber and Plastics Products, DMU_{17}; Manufacture of Metal Products, DMU_{21}; Manufacture of Measuring Instruments and

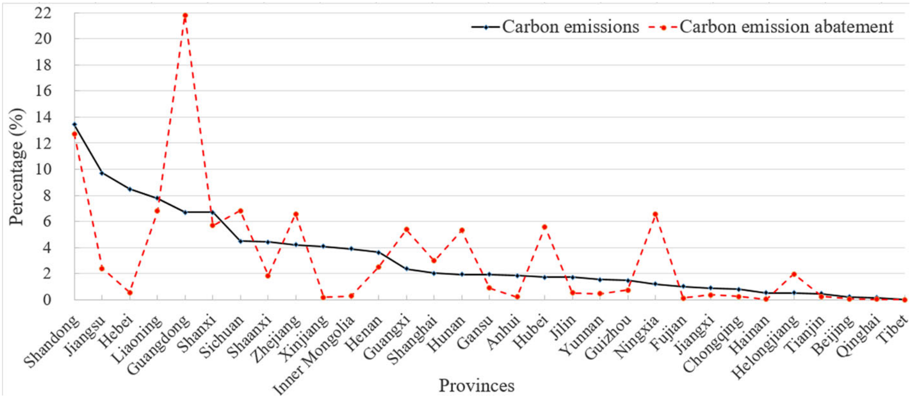

Figure 3. Percentage of carbon emissions and abatement quota for 31 provinces.

Machinery, DMU_{28}) and 5.5532% (DMU_{13}, Processing of Petroleum, Coking and Processing of Nuclear Fuel), respectively. Note in addition that the reduction goal (602,781.9745) is 14.5786% of the total CO_2 emissions (4,134,711.6380) in Chinese manufacturing industries, 13 out of 17 sub-industries have shown a CO_2 reduction percentage exceeds the national average reduction rate. Through a correlation test we find that the reduction percentage is a little weakly negatively correlated with the percentage of current CO_2 emission and correlated to the percentage of allocated carbon abatement quota, as the correlation coefficients (−0.3454 and −0.1590) are located in the interval [−0.4, −0.2] and [−0.2, 0], respectively. On the contrary, the correlation coefficient between the current CO_2 emission percentage and the percentage of allocated carbon abatement quota reaches 0.7093, implying a relatively high relevance. Based on above observations, it might be concluded that the game-based carbon emissions abatement quota allocation mechanism is implicitly similar with the proportional sharing method, and we have a carbon emissions abatement quota allocation plan that is highly consistent with each unit's carbon emissions proportion.

5.3. Provincial allocation

In this subsection, we further allocate the reduction quota for each sub-industry that is obtained in Section 5.2 into different provinces. The summary statistics of 31 provinces for 31 sub-industries is given in Table 9. For the provincial allocation, we take each province as a DMU and provinces with the same sub-industry as a sample.

By repeatedly solving the cooperative game DEA approaches proposed previously, we obtain the allocation results listed in Tables 10–12. In such a way, we allocate the national carbon emissions abatement quota into a two-layer framework. For details, the Manufacture of Non-metallic Mineral Products (I_{18}) in Guangdong province, Processing of Petroleum, Coking and Processing of Nuclear Fuel (I_{13}) in Shandong province and Smelting and Pressing of Ferrous Metals (I_{19}) in Liaoning province are top three provinces that receive the most carbon emissions abatement quota across 31sub-level manufacturing industries in 31 provinces, reaching 74,471.5030, 49,499.2709 and 37,107.5793 (in thousand tons), respectively. In addition, these three regions of different industries are allocated more than 5% of the total carbon emission abatement quota. Also, the Manufacture of Raw Chemical Materials and Chemical Products (I_{14}) in Sichuan province is another region that accounts for more than 5% of the total carbon emission abatement quota, reaching 35,739.9897. On the other hand, Tibet has no need to reduce its CO_2 emissions in many sub-level industries. This phenomenon can be mainly caused by the undeveloped economy, as Tibet is a backward province where there aren't most sub-industries and corresponding CO_2 emissions.

To sum the allocated reduction amounts in various sub-industries, we can also obtain the reduction responsibility for each province, as given in Table 13. Guangdong, Shandong, Sichuan, Liaoning, and Zhejiang rank the top five provinces that are allocated the most CO_2 reduction quota, and Tibet, Hainan, Qinghai, Beijing, and Fujian rank another least five provinces that are allocated the smallest CO2 reduction quota among 31 provinces. The carbon emissions abatement quota allocated to Guangdong is 131,473.9251 (thousand tons), which accounts 21.8112% of the national abatement commitment. Shandong is another province that is responsible for more than ten percentage of the national abatement commitment, with 76,669.5284 (thousand tons) for 12.7193%. In addition, the carbon emissions abatement quota allocated to the top five provinces are 329,808.8273 (thousand tons), which accounts for 54.7144% of the total national carbon emissions abatement goal, and the percentage reaches 83.2386% and 91.1493% when it covers the top 10 and top 13 provinces, respectively. This result shows that the corresponding provinces are key focus to realize the national carbon emissions abatement commitment and allocate the total carbon emissions abatement quota.

Additionally, we present the tendency of carbon emissions percentage and corresponding allocated carbon emissions abatement quota percentage among all provinces. It can be learned from Figure 3 that although there will be some variations between the carbon emissions percentage and corresponding allocated carbon emissions abatement quota, the main tendency is a little similar. We present the carbon emissions percentage in descending order in Figure 3, and the percentage of allocated carbon emission abatement quota shows also a downward tendency. In fact the correlation coefficient between the carbon emissions percentage and corresponding allocated carbon emissions abatement quota percentage is 0.5444, implying a positive association. This result demonstrates again that larger shares of the total carbon emissions abatement quota are more likely to be allocated to units with larger carbon emissions percentages.

5.4. Discussion

In this subsection, we will discuss (1) briefly the impact on the results of using alternative solution concepts for the cooperative game, and (2) some practical implications and indications of how these allocated carbon emission abatement targets might be used in practice.

5.4.1. Impacts of alternative game solutions

Note that this article uses only the nucleolus solution and there exists alternative solution concepts for the cooperative game. Although it is a little difficult to calculate all kinds of solutions for the carbon emission abatement quota allocation problem, we can present a brief discussion on the impacts of alternative solution concepts, for instances, Nash bargaining solution, core (least core, weak least core, and proportional least core) and Shapley value.

Nash bargaining solution is an imputation that maximizes the satisfaction degree of all DMUs. Acknowledging that the excess value in this article is a measure of dissatisfaction degree, the Nash bargaining solution based allocation plan would minimize the excess values for n individual DMUs. That is to say, the Nash bargaining solution based allocation plan just focuses on n singleton coalitions instead of all nonempty coalitions of n DMUs, and it might be considered as a simplified version of nucleolus based allocation plan (it likes model (17)). Besides, the nucleolus based allocation plan minimizes the excess values by lexicographical order, while the Nash bargaining solution based allocation plan adopts a product of individual utilities due to technical device (Osborne & Rubinstein, 1994).

The core solution is an imputation that also minimizes the excess values for all nonempty coalitions of DMUs, and the core solution can be divided into least core, weak least core, and proportional least core according to which parameter is added to the characteristic function in computing the excess values. It is clear that the core based allocation plan just focuses on the most unsatisfied coalition (i.e. the least excess value), neglecting the second unsatisfied coalition, third unsatisfied coalition, and so on. It is acknowledged that any cooperative game will always have a unique nucleolus and the nucleolus must be one of least cores if the game has nonempty cores. Put it differently, the core based allocation plan just realizes the first least excess value, and it would be changed into the nucleolus based allocation plan if it minimizes the excess values to the most by lexicographical order.

The Shapley value is an imputation that represents the average contribution of DMUs to the guarantee level, namely, characteristic function, thus the Shapley value based allocation plan involves fair concern and would be much fairer given DMUs' input-output measures. The Shapley value based allocation plan is a good choice for carbon emission abatement quota allocation in Chinese manufacturing industries, but its computation depends on the characteristic function for each coalition, which is impractical when the number of coalitions is very big (in this article the number of nonempty coalitions is $2^{31}-1$).

All in all, using alternative solution concepts for the cooperative game would generate very similar allocation plans, which might be strongly positive correlated with each other (Li, 2008). The Nash bargaining solution based allocation plan is much closer to traditional proportional sharing method, while the nucleolus based allocation plan favors much for vulnerable groups, and the cores based allocation plan would waver between the Nash bargaining solution based allocation plan and the nucleolus based allocation plan.

5.4.2. Practical implications and indications

Note in particular that the previous results are obtained based on the implicit assumption that any inefficient DMU can move to the efficient frontier by adjusting its GIOV and carbon emissions. However, it is not the case in the real world as a certain DMU cannot change its production bundle and production structure remarkably, and rapid change would confront huge resistance. The carbon emission abatement may cause unemployment and reduction in energy consumption in the short term, but it is no doubt that the technological change has a solid effect on the carbon dioxide emissions in the long run (Grimaud & Rouge, 2008; Smulders & Di Maria, 2012). Moreover, technology change is supposed as the most essential factor for reduce the carbon dioxide emissions (Li et al., 2017a; Li & Qu, 2012), thus much attention should be paid to technology development and adoption and progress for the sake of implementing these carbon abatement allocation targets in practice.

Some efforts can be taken to promote the technology change and further implement these reduction targets under low-carbon technology progress in the long run. First of all, the Chinese government should develop and improve the mechanism of low carbon technology progress, which will promote the development of low carbon technology. The key point is to continue to implement energy-saving emission reduction technology special action plan, and solve the key and common difficulties of low-carbon technology. It is clear that the R&D investment should be increased. Then, the Chinese government should take a tendentious policy to invest in low-carbon technology and innovation across different industries and regions. The western and central areas must learn the best practice and low-carbon technology from eastern area, and speed up its technological progress. At the same time, industries with high carbon intensity must learn the best practice and low-carbon technology from peers. Last, the Chinese government can promote the optimization and upgrading of industrial structure, which means to decrease and even limit the heavy polluting industries. In the long run, the Chinese government is supposed to develop a new industrialization pattern with minimal energy consumption and pollution emission, which can support both the economy and public through achievable and sustainable development goals.

Some direct measures can be also useful for implementing these carbon abatement allocation targets in the short term. For instance, all levels of the Chinese government can actively optimize its energy consumption structure across regions and industries, accelerate the speed in closing down backward production facilities (Wu, Lv, Sun, & Ji, 2015). It is also very useful to pay much attention to labor efficiency, as the labor efficiency is significantly connected with the substitution of labor for energy and improving the labor efficiency will accelerate the substitution of labor for energy, which will further enhance industrial production efficiency and reduce environment pollution. In addition, the carbon trade market can be of strategic importance. The carbon trade market makes it possible for these determined carbon abatement allocation targets be re-adjusted considering certain local situations.

6. Conclusions and perspectives

This article focuses on a real problem on how to allocate the national carbon emission abatement quota in Chinese manufacturing industries. It adopts a decentralized perspective to address the carbon emission abatement quota allocation problem, which is different from most studies in the literature that solve the studied problem from a centralized view and seek after the collective objective. To this end, this article integrates data envelopment analysis and game theory to propose a cooperative game DEA approach, considering the cooperation and competition relationship among units simultaneously. In addition, a practical computation procedure based on constraint generation mechanism is developed to calculate the nucleolus solution and the nucleolus-based solution is taken as the final carbon emission abatement quota allocation scheme. In the empirical study, we present a two-layer way to decompose the national carbon emission abatement quota into different sub-industries and further into different provinces. The results show that five sub-industries (Processing of petroleum, coking and processing of nuclear fuel; Smelting and pressing of ferrous metals; Manufacture of non-metallic mineral products; Manufacture of raw chemical materials and chemical product; Smelting and pressing of non-ferrous metals) and two provinces (Guangdong and Shandong) will be allocated more than 10% of the total national carbon emissions abatement goal.

The carbon dioxide emission abatement quota allocation is supposed to be the key issue for China to respond to global warming. This article has suggested a feasible and rational method to allocate the carbon emission abatement quota into different industries and provinces. However, the carbon emission abatement allocation still needs much more work in the future. On one hand, the difference among the industries and provinces is not well considered. The Chinese government has made a lot of policies to guide the industrial development and regional development, which is supposed to separate these industries and provinces apart. On the other hand, a significant feature of this current article is that it presents a two-layer allocation framework, where the total carbon emission abatement quota is first allocated to different sub-industries and further to different provinces within sub-industries. This practice is similar with a network perspective. It is acknowledged that a network DEA approach would provide new insights in allocation results (Li et al., 2019; Yu, Chen, & Hsiao, 2016), and accordingly making the allocation mechanism more reasonable and the allocation results more acceptable. However, the game would be more complex under network DEA environment and the computation would be also much more complex, so future work can try to extend the proposed approach to situations considering internal network structures. Besides, the political implications for this game DEA approach are not so explicit, future research can also work on this aspect deeply.

Endnotes

1. http://www.theguardian.com/environment/2015/jun/30/china-carbon-emissions-2030-premier-li-keqiang-un-paris-climate-change-summit.

Acknowledgements

The authors thank the Editor of Journal of the Operational Research Society and three anonymous reviewers for their kind work and valuable comments, they have done a significant work for us to improve this article. The authors also acknowledge Dr. Xiaoyu Han for her assistance on numerical calculation.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This research was financially supported by the Fundamental Research Funds for the Central Universities (No. JBK1901013 and JBK190916) for Feng, the National Science Foundation of China (No. 71671181) for Guoliang, the National Science Foundation of China (No. 71671172 and 71631006) and the Youth Innovation promotion Association of Chinese Academy of Sciences (CX2040160004) for Yongjun, and the Fundamental Research Funds for the Central Universities (WK2040160028).

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Appendix

Appendix A

Theorem 1. $R_j^{\max} = b_j \, (j = 1, \ldots, n)$.

Proof. It is easy to know that $R_j^{\max} \le b_j \, (j = 1, \ldots, n)$, thus we just need to show that $R_j^{\max}$ can reach b_j. Consider a solution $(\hat{\lambda}_{jo}, \hat{\xi}_{jo}, \hat{\bar{y}}_{ro}, \hat{R}_o)$ when $\text{DMU}_o \, (o = 1, \ldots, n)$ is under consideration in model (8), $\hat{\xi}_{oo} = 1$, $\hat{\xi}_{jo} = 0 \, (j \ne o)$, $\hat{\lambda}_{jo} = 0 \, (j = 1, \ldots, n)$, $\hat{\bar{y}}_{ro} = -y_{ro} \, (r = 1, \ldots, s)$ and $\hat{R}_o = b_o$. It is clear that $(\hat{\lambda}_{jo}, \hat{\xi}_{jo}, \hat{\bar{y}}_{ro}, \hat{R}_o)$ is a feasible solution of model (8), as it can satisfy all constraints such that

$$\sum\nolimits_{j=1}^{n} \left(\hat{\lambda}_{jo} + \hat{\xi}_{jo}\right) x_{ij} = x_{io} \le x_{io}, i = 1, \ldots, m, \tag{A1}$$

$$\sum\nolimits_{j=1}^{n} \hat{\lambda}_{jo} y_{rj} = 0 \ge y_{ro} - y_{ro} = y_{ro} + \hat{\bar{y}}_{ro}, r = 1, \ldots, s, \tag{A2}$$

$$\sum\nolimits_{j=1}^{n} \hat{\lambda}_{jo} b_j = 0 = b_o - b_o = b_o - \hat{R}_o, \tag{A3}$$

$$\sum\nolimits_{j=1}^{n} \left(\hat{\lambda}_{jo} + \hat{\xi}_{jo}\right) = 1. \tag{A4}$$

Therefore, the optimal objective function of model (8) is no less than that with the solution $(\hat{\lambda}_{jo}, \hat{\xi}_{jo}, \hat{\bar{x}}_{io}, \hat{\bar{y}}_{ro}, \hat{R}_o)$, namely, $R_o^{\max} \ge R_o^{\max}(\hat{\lambda}_{jo}, \hat{\xi}_{jo}, \hat{\bar{x}}_{io}, \hat{\bar{y}}_{ro}, \hat{R}_o) = b_o$, implying that when $\text{DMU}_o \, (o = 1, \ldots, n)$ is under consideration the objective function model (8) has reached b_o.

Note in addition that the considered DMU_o is chosen randomly, therefore, it can be concluded that for any $\text{DMU}_j \, (j = 1, \ldots, n)$, it holds $R_j^{\max} = b_j$. This completes the proof of Theorem 1.

Appendix B

Theorem 2. *The optimal objective function of model (10) is always zero.*

Proof. First, acknowledging that the objective function of model (10) is no less than zero, which is demonstrated by the fact that the first constraint $\sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r y_{ro} - \sum_{r=1}^{s} \bar{Y}_{ro} + \omega_1 b_o - T_{1o} - \omega_2 b_o + T_{2o} + u_0 \ge 0$ is held.

Then we show that the objective function of model (10) can reach zero. To this end, let $b_{\min} = \min_{j=1,\ldots,n}\{b_j\}$ and $y_{\max} = \max_{j=1,\ldots,n}\{y_{sj}\}$. Then we consider a solution $\xi' = (v'_i, u'_r, T'_{1j}, T'_{2j}, \omega'_1, \omega'_2, u'_0, \bar{Y}'_{ro}, \forall i, r, j)$, where $v'_i = 0 \, (i = 1, \ldots, m)$, $u'_s = 1/y_{\max}$, $u'_r = 0 \, (r \ne s)$, $\omega'_1 = 1/b_{\min}$, $\omega'_2 = 0$, $u'_0 = 0$, $T'_{2j} = 0 \, (j = 1, \ldots, n)$, $T'_{1j} = Rb_j/b_{\min} \sum_{j=1}^{n} b_j$, $\bar{Y}'_{so} = -y_{so}/y_{\max} + b_o/b_{\min} - Rb_o/b_{\min} \sum_{j=1}^{n} b_j$ and $\bar{Y}'_{rj} = 0 (r \ne s)$. Then ξ' is a feasible solution to model (10), as it satisfies all conditions of model (10) such that

$$\begin{aligned}
&\sum\nolimits_{i=1}^{m} v'_i x_{io} - \sum\nolimits_{r=1}^{s} u'_r y_{ro} - \sum\nolimits_{r=1}^{s} \bar{Y}'_{ro} + \omega'_1 b_o - T'_{1o} - \omega'_2 b_o + T'_{2o} + u'_0 \\
&= -u'_s y_{so} - \bar{Y}'_{so} + \omega'_1 b_o - T'_{1o} \\
&= -y_{so}/y_{\max} - \left(-y_{so}/y_{\max} + b_o/b_{\min} - Rb_o/b_{\min} \sum\nolimits_{j=1}^{n} b_j\right) \\
&\quad + b_o/b_{\min} - Rb_o/b_{\min} \sum\nolimits_{j=1}^{n} b_j = 0
\end{aligned} \tag{B1}$$

$$\begin{aligned}
\sum\nolimits_{i=1}^{m} v'_i x_{ij} - \sum\nolimits_{r=1}^{s} u'_r y_{rj} + \omega'_1 b_j - \omega'_2 b_j + u'_0 &= -u'_s y_{sj} + \omega'_1 b_j \\
&= -y_{sj}/y_{\max} + b_j/b_{\min} \ge 0, \; j = 1, \ldots, n
\end{aligned} \tag{B2}$$

$$\sum\nolimits_{i=1}^{m} v'_i x_{ij} + u'_0 = 0 \ge 0, \; j = 1, \ldots, n \tag{B3}$$

$$\sum\nolimits_{r=1}^{s} u'_r y_{ro} + \omega'_1 b_o - \omega'_2 b_o = y_{so}/y_{\max} + b_o/b_{\min} \ge b_o/b_{\min} \ge 1 \tag{B4}$$

$$0 \le T'_{1j} = Rb_j/b_{\min} \sum\nolimits_{j=1}^{n} b_j \le b_j/b_{\min} = \omega'_1 b_j, \; j = 1, \ldots, n \tag{B5}$$

$$0 \le T'_{2j} = 0 \le \omega'_2 b_j, \; j = 1, \ldots, n \tag{B6}$$

$$\sum\nolimits_{j=1}^{n} T'_{1j} = \sum\nolimits_{j=1}^{n} \left(Rb_j/b_{\min} \sum\nolimits_{j=1}^{n} b_j\right) = R/b_{\min} = \omega'_1 R \tag{B7}$$

$$\sum\nolimits_{j=1}^{n} T'_{2j} = 0 = \omega'_2 R. \tag{B8}$$

Additionally, it is easy to verify that $v'_i, u'_r, \omega'_2 \ge 0 \, (\forall i, r)$ and $\omega'_1 > 0$. Therefore, the optimal objective function of model (10) is no less than the objective function with solution ξ', namely, $\hat{\psi}_o^* = \hat{\psi}_o^*(\xi') = 0$. So, it demonstrates that the objective function of model (10) can reach zero.

Based on above discussion, it is clear that the objective function of model (10) would be zero. Note in addition that the subscript s is labeled randomly among all desirable outputs, so we can conclude that the optimal objective function of model (10) is always zero. This completes the proof of Theorem 2.

Appendix C

Theorem 3. *All DMUs can be simultaneously efficient with a certain carbon emission abatement allocation scheme under a set of common weights.*

Proof. Return to the proof process of Theorem 2 and the considered solution ξ', it is demonstrated by Theorem 2 that the efficiency score of DMU_o is unity, as

$$e_o = 1 - \left(\sum_{i=1}^{m} v_i' x_{io} - \sum_{r=1}^{s} u_r' y_{ro} - \sum_{r=1}^{s} \bar{Y}_{ro}' + \omega_1' b_o - T_{1o}' - \omega_2' b_o + T_{2o}' + u_0' \right) = 1. \quad \text{(B1)}$$

Also, using ξ' and $\bar{Y}_{sj}' = -y_{sj}/y_{\max} + b_j/b_{\min} - Rb_j/b_{\min} \sum_{j=1}^{n} b_j \; (j = 1, \ldots, n)$, the constraints of model (10) can be satisfied for any $\text{DMU}_j \; (j = 1, \ldots, n)$ under consideration. In addition, it is also possible to determine the efficiency score of other $\text{DMU}_j \; (j = 1, \ldots, n)$ to be one, as

$$\begin{aligned} e_j &= 1 - \left(\sum_{i=1}^{m} v_i' x_{ij} - \sum_{r=1}^{s} u_r' y_{rj} - \sum_{r=1}^{s} \bar{Y}_{rj}' + \omega_1' b_j - T_{1j}' - \omega_2' b_j + T_{2j}' + u_0' \right) \\ &= 1 + u_s' y_{sj} + \bar{Y}_{sj}' - \omega_1' b_j + T_{1j}' \\ &= 1 + y_{sj}/y_{\max} + \left(-y_{sj}/y_{\max} + b_j/b_{\min} - Rb_j/b_{\min} \sum_{j=1}^{n} b_j \right) - b_j/b_{\min} + Rb_j/b_{\min} \sum_{j=1}^{n} b_j = 1. \end{aligned} \quad \text{(B2)}$$

To sum up, it is concluded that the solution $(v_i', u_r', T_{1j}', T_{2j}', \omega_1', \omega_2', u_0', \bar{Y}_{rj}', \forall i, r, j)$ can make all DMUs simultaneously efficient. This completes the proof that all DMUs can be simultaneously efficient with a certain carbon emission abatement allocation scheme under a set of common weights.

Appendix D

Corollary 1. *The efficient carbon emission abatement allocation scheme can be denoted as following System (11) under a set of common weights:*

$$\begin{aligned} &\sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} + b_j - R_j - \omega b_j + T_j + \mu_0 = 0, j = 1, \ldots, n \\ &\sum_{j=1}^{n} R_j = R \\ &\sum_{j=1}^{n} T_j = \omega R \\ &0 \le R_j \le b_j, j = 1, \ldots, n \\ &\upsilon_i, \mu_r, \omega, T_j \ge 0, i = 1, \ldots, m; r = 1, \ldots, s; j = 1, \ldots, n; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.} \end{aligned} \quad (11)$$

Proof. According to Theorem 2 and Theorem 3 we can learn that all DMUs can be simultaneously efficient with a certain carbon emissions abatement allocation scheme under a set of common weights, implying

$$e_j = 1 - \left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} \bar{Y}_{rj} + \omega_1 b_j - T_{1j} - \omega_2 b_j + T_{2j} + u_0 \right) = 1, \quad \text{(D1)}$$

Therefore, it must be

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} \bar{Y}_{rj} + \omega_1 b_j - T_{1j} - \omega_2 b_j + T_{2j} + u_0 = 0. \quad \text{(D2)}$$

Note in particular that $T_{1j} = \omega_1 R_j \; (j = 1, \ldots, n)$ and $T_{2j} = \omega_2 R_j \; (j = 1, \ldots, n)$, then by substituting $v_i = \omega_1 \upsilon_i \; (\forall i)$, $u_r = \omega_1 \mu_r \; (\forall r), \bar{Y}_{rj} = \omega_1 \tilde{y}_{rj} \; (\forall r, j)$, $T_{2j} = \omega_1 (\omega_2/\omega_1) R_j \; (\forall j)$ and $u_0 = \omega_1 \mu_0$ into (D2), we have

$$\sum_{i=1}^{m} \omega_1 \upsilon_i x_{ij} - \sum_{r=1}^{s} \omega_1 \mu_r y_{rj} - \sum_{r=1}^{s} \omega_1 \tilde{y}_{rj} + \omega_1 b_j - \omega_1 R_j - \omega_1 (\omega_2/\omega_1) b_j + \omega_1 (\omega_2/\omega_1) R_j + \omega_1 \mu_0 = 0. \quad \text{(D3)}$$

Then divide Equation (D3) by $\omega_1 > 0$, we have

$$\sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} + b_j - R_j - \omega b_j + \omega R_j + \mu_0 = 0,$$

where $\omega = \omega_2/\omega_1$.

Together with the full allocation requirement $\sum_{j=1}^{n} R_j = R$, non-negative weight requirement $\upsilon_i \ge 0 \; (i = 1, \ldots, m)$ and $\mu_r \ge 0 (r = 1, \ldots, s)$, and feasible reduction requirement $0 \le R_j \le b_j \; (j = 1, \ldots, n)$, we have the following formula (D4).

$$\begin{aligned} &\sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} + b_j - R_j - \omega b_j + \omega R_j + \mu_0 = 0, j = 1, \ldots, n \\ &\sum_{j=1}^{n} R_j = R \\ &0 \le R_j \le b_j, j = 1, \ldots, n \\ &\upsilon_i, \mu_r, \omega \ge 0, i = 1, \ldots, m; r = 1, \ldots, s; \tilde{y}_{rj} \text{ and } \mu_0 \text{ free.} \end{aligned} \quad \text{(D4)}$$

(D4) can be changed into a linear formula as given in the following (D5).

$$\begin{aligned} &\sum_{i=1}^{m} \upsilon_i x_{ij} - \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{r=1}^{s} \tilde{y}_{rj} + b_j - R_j - \omega b_j + T_j + \mu_0 = 0, j = 1, \ldots, n \\ &\sum_{j=1}^{n} R_j = R \\ &\sum_{j=1}^{n} T_j = \omega R \\ &0 \le R_j \le b_j, j = 1, \ldots, n \\ &\upsilon_i, \mu_r, \omega, T_j \ge 0, i = 1, \ldots, m; r = 1, \ldots, s; j = 1, \ldots, n; \\ &\tilde{y}_{rj} \text{ and } \mu_0 \text{ free.} \end{aligned} \quad \text{(D5)}$$

(D5) is the same as System (11), where a carbon emission abatement allocation scheme $(R_1, \ldots, R_n)$ can make all DMUs simultaneously efficient under a set of common weights $(\upsilon_i, i = 1, \ldots m; \mu_r, r = 1, \ldots, s; T_j, j = 1, \ldots, n; \mu_0; \omega)$. Therefore, this completes the proof of Corollary 1.

Appendix E

Theorem 4. *The characteristic function $V(K)$ satisfies super-additive, that is, for any two coalitions $K, L \subseteq N = \{1, \ldots, n\}$ and $K \cap L = \emptyset$, it holds $V(K) + V(L) \le V(K \cup L)$.*

Proof. $V(K) + V(L) = \sum_{j \epsilon K} \vec{R}_j - \vec{R}_K + \sum_{j \epsilon L} \vec{R}_j - \vec{R}_L = \left(\sum_{j \epsilon K} \vec{R}_j + \sum_{j \epsilon L} \vec{R}_j \right) - \left(\vec{R}_K + \vec{R}_L \right) = \sum_{j \epsilon K \cup L} \vec{R}_j - \left(\vec{R}_K + \vec{R}_L \right)$.

Reconsider model (13) we can easily verify that $\vec{R}_K + \vec{R}_L \ge \vec{R}_{K \cup L}$.

Therefore, we have $V(K) + V(L) \le \sum_{j \epsilon K \cup L} \vec{R}_j - \vec{R}_{K \cup L} = V(K \cup L)$.

It completes the proof of super-additive property. □

Appendix F

Table F1. The codes for two-digit Chinese manufacturing industries

DMUs	Sub-level manufacturing industries
DMU_1	Processing of Food from Agricultural Products
DMU_2	Manufacture of Foods
DMU_3	Manufacture of Liquor, Beverages and Refined Tea
DMU_4	Manufacture of Tobacco
DMU_5	Manufacture of Textile
DMU_6	Manufacture of Textile, Wearing Apparel and Accessories
DMU_7	Manufacture of Leather, Fur, Feather and Related Products and Footwear
DMU_8	Processing of Timber, Manufacture of Wood, Bamboo, Rattan, Palm and Straw Products
DMU_9	Manufacture of Furniture
DMU_{10}	Manufacture of Paper and Paper Products
DMU_{11}	Printing and Reproduction of Recording Media
DMU_{12}	Manufacture of Articles for Culture, Education, Arts and Crafts, Sport and Entertainment Activities
DMU_{13}	Processing of Petroleum, Coking and Processing of Nuclear Fuel
DMU_{14}	Manufacture of Raw Chemical Materials and Chemical Products
DMU_{15}	Manufacture of Medicines
DMU_{16}	Manufacture of Chemical Fibres
DMU_{17}	Manufacture of Rubber and Plastics Products
DMU_{18}	Manufacture of Non-metallic Mineral Products
DMU_{19}	Smelting and Pressing of Ferrous Metals
DMU_{20}	Smelting and Pressing of Non-ferrous Metals
DMU_{21}	Manufacture of Metal Products
DMU_{22}	Manufacture of General Purpose Machinery
DMU_{23}	Manufacture of Special Purpose Machinery
DMU_{24}	Manufacture of Automobiles
DMU_{25}	Manufacture of Railway, Ship, Aerospace and Other Transport Equipment
DMU_{26}	Manufacture of Electrical Machinery and Apparatus
DMU_{27}	Manufacture of Computers, Communication and Other Electronic Equipment
DMU_{28}	Manufacture of Measuring Instruments and Machinery
DMU_{29}	Other Manufacture
DMU_{30}	Utilization of Waste Resources
DMU_{31}	Repair Service of Metal Products, Machinery and Equipment