Review of nonlinear vibration energy harvesting: Duffing, bi-stability, parametric, stochastic and others

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Abstract

Vibration energy harvesting (VEH) typically involves a mechanical oscillatory mechanism to accumulate ambient kinetic energy, prior to the conversion to electrical energy through a transducer. The convention is to employ a simple linear mass-spring-damper oscillator with its resonant frequency tuned towards that of the vibration source. In the past decade, there has been a rapid expansion in research of VEH into various nonlinear vibration principles such as Duffing nonlinearity, bi-stability, parametric oscillators, stochastic oscillators and other nonlinear mechanisms. The intended objectives for employing nonlinearity include broadening of frequency bandwidth, enhancement of power amplitude and improvement in responsiveness to non-sinusoidal noisy excitations. However, nonlinear VEH also comes with its own challenges and some of the research pursuits have been less than fruitful. Previous reviews in the literature have either focussed on bandwidth enhancement strategies or converged on select few nonlinear mechanisms. This paper reviews 8 major types of nonlinear VEH reported over the past decade; covering underlying principles, advantages and disadvantages, and application specific guidance for researchers and designers.

Keywords

vibration energy harvesting, MEMS, nonlinear, frequency bandwidth

Introduction

Context and underlying principles

Vibration is ubiquitously present in the ambient environment: from cars to airplanes, roads to tunnels, rail tracks to bridges, rolling stocks to lorries, engines to human motion, electrical transmission towers to wind turbines, and infrastructure construction systems to water and gas distribution networks. The research field of vibration energy harvesting (VEH) Priya and Inman (2009); Beeby and White (2010) is concerned with devising decentralised generators capable of converting the ambient vibration energy into useful electrical energy at the point of use.

Combined with the general trend of system miniaturisation, VEH has also been increasing integrated with MEMS (micro-electromechanical system) and IC (integrated circuit) technologies in order to realise self-powered microsystem Kazmierski and Beeby (2011). The applications of VEH can be found in powering autonomous and/or wireless sensors for condition monitoring and structural health monitoring; powering wearable, biomedical and implantable devices; and powering remote tracking and positioning systems Beeby et al. (2006); Mitcheson et al. (2008).

Some of the early examples of motional energy harvesting include the inductive kinetic watch Seiko (2011) or the dynamo powered battery-less flashlight Popular Science Monthly (1929). VEH research witnessed an emergence within the modern academia since the 1990s Antaki et al. (1995); Umeda et al. (1997); Williams et al. (1998); Xu et al. (1998); Meninger (1999). The convention relied upon employing a directly excited linear oscillator that can be modelled as a second order mass-spring-damper system

subjected to base point excitation as shown in figure 1 and the general form of the oscillator is given by equations 1 and 2.
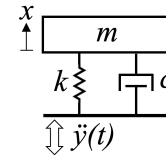

Figure 1. Model of a mass spring damper system subjected to base-point direct forcing with acceleration $\ddot{y}(t)$.

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{1}$$

where, m is the effective mass, c is the effective damping, k is the effective stiffness constant, F is the base point external forcing, x is the response displacement and t is the time domain.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = -\ddot{y}(t) \tag{2}$$

where, $\zeta = c/(2\sqrt{km})$ is the damping ratio, $\omega_n = \sqrt{k/m}$ is the angular natural frequency and y is the excitation displacement.

If the excitation is sinusoidal $y = Y\cos(\omega t)$, the response can be given by $x = |X|\cos(\omega t + \phi)$, with

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excitation frequency ω and phase angle ϕ. The mechanical energy accumulated in the oscillatory motion can then be converted to electrical energy via a mechanical-to-electrical transduction mechanism. This linear approach has been widely applied across various transducers, including: electromagnetic Williams and Yates (1996); Williams et al. (1996, 1998, 2001); Glynne-Jones et al. (2004); Beeby et al. (2007); Serre et al. (2008); Li et al. (2013), piezoelectric Roundy and Wright (2004); Kim and Priya (2008); Erturk and Inman (2008c,a,b); Elfrink et al. (2009); Erturk and Inman (2011b) and electrostatic Meninger (1999); Tashiro et al. (2000); Meninger et al. (2001); Roundy et al. (2002); Roundy (2003); Chu et al. (2005) and magnetostriction Huang et al. (2003); Bayrashev et al. (2004). Further more, there have also been derivates based on the electrostatic principle, such as electret Peano and Tambosso (2005); Suzuki et al. (2010); Renaud et al. (2013); Minakawa et al. (2013) and triboelectric Wang (2013); Wang et al. (2016).

By designing the oscillator to match the system resonant frequency ω_r (equation 3) towards the active frequency range of the ambient vibration for $\zeta < 1/\sqrt{2}$, fundamental mode resonance can thus be achieved to maximise the mechanical response (equation 4) and the resultant electrical power output. This resonant amplification is important to maximise the power output.

$$\omega_r = \frac{\omega_n}{\sqrt{1 - 2\zeta^2}} \tag{3}$$

$$|X_{max}| = \frac{Y}{2\zeta\sqrt{1 - \zeta^2}} \tag{4}$$

The half power bandwidth is usually taken as the practical operational frequency bandwidth for VEH systems, and the half power point can be found at: $|X| = X_{max}/\sqrt{2}$, for $\zeta << 1$.

Williams *et al.* Williams and Yates (1996); Williams et al. (1996) were amongst the first to establish an analytical model for a directly excited linear oscillator applied as VEH. The instantaneous power $P(t)$ coupled from the base-point vibration to the oscillator mass can be summarised by equation 5.

$$P(t) = -m\ddot{y}(t)[\dot{y}(t) + \dot{x}(t)] \tag{5}$$

For a given sinusoidal excitation, the theoretical power amplitude P recoverable from a linear VEH oscillator can be represented by equation 6 Williams et al. (1996).

$$P = \frac{m\zeta Y^2 (\frac{\omega}{\omega_n})^3 \omega^3}{[1 - (\frac{\omega}{\omega_n})^2]^2 + [\frac{2\zeta\omega}{\omega_n}]^2} \tag{6}$$

For a given mechanical damping ratio ζ_m and electrical damping ratio ζ_e from the transducer circuitry, the power generated is given by equation 7. The total damping ratio ζ is the sum of the damping in the mechanical and electrical domains.

$$P = \frac{mY^2\omega_n^3\zeta_e}{4(\zeta_e + \zeta_m)^2} \tag{7}$$

The quality factor Q of the oscillator can be measured from the limits of the frequency bandwidth ω_1 and ω_2, as

well as an inverse relationship with damping ratio: $Q = \omega_r/(\omega_1 - \omega_2) = 1/(2\zeta)$.

While linear damping is often used to include the electrical damping for electromagnetic transducers Williams and Yates (1996); Williams et al. (2001); often for piezoelectric transducers, electrical damping is not modelled just as part of linear damping alone Erturk and Inman (2008b, 2011b). For scenarios where the piezoelectric transducer damping is significant in relation to mechanical damping, equation 8 can be applied.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = -\ddot{y}(t) + \kappa V(t) \tag{8}$$

where, κ is the electromechanical coupling factor and V is the generated voltage.

By adjusting the overall quality factor Q (as outlined in figure 2), the VEH oscillator can be tuned to target towards either maximising power output or broadening frequency bandwidth; but both power and frequency bandwidth cannot be simultaneously enhanced. Q also represents the mechanical advantage of the resonating system to accumulate mechanical energy. However, in real world applications where vibration energy is likely to be broadband and varying, an ideal VEH system should be able to achieve both high power output and operate across a wide frequency spectrum.
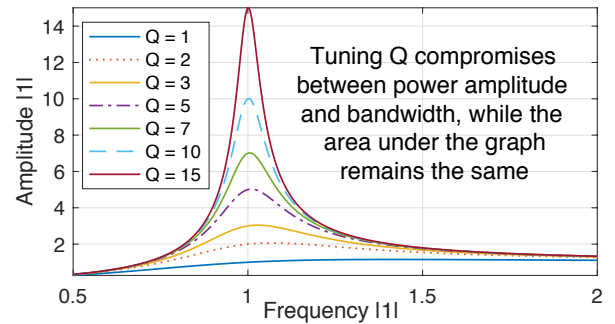

Figure 2. Frequency domain response for a linear mass-spring-damper system with varying quality factor Q. By tuning Q, the system compromises between peak power and frequency bandwidth.

Therefore, the conventional linear oscillator is only suited for specific VEH applications where the ambient vibration is that of a fixed single frequency source. When operated across a large bandwidth, a linear oscillator is fundamentally inefficient to accumulate mechanical energy. Over the past decade Tang et al. (2010); Zhu et al. (2010), in order to address the persisting challenges of inadequate power density and narrow operational frequency bandwidth, there has been an increasing variety of methods proposed, including: an array of coupled or uncoupled oscillators across various frequencies; frequency tuning mechanisms through mechanical or electrical means; as well as various nonlinear vibratory phenomena for VEH.

Often, the terms nonlinear vibration or nonlinear oscillator are narrowly used to describe a higher order polynomial nonlinear spring. However, vibratory nonlinearity and its uptake by the VEH community is both much broader and more nuanced. Nonlinear oscillatory systems are not a single class of vibratory systems, but rather an umbrella

definition used to encompass oscillators that are not linear. Various types of nonlinear oscillators can be as distinguished amongst themselves, as they are distinct from linear oscillators. Nonlinear oscillators can be broadly described by the general form shown in equation 9 Thomson (1998).

$$\ddot{x} + f(\dot{x}, x, t) = 0 \qquad (9)$$

where, $f(\dot{x}, x, t)$ is a function with at least one nonlinear term. As will be demonstrated throughout this paper, this function can take on various forms depending on the type of nonlinearity that is under investigation. The main distinction from linear oscillators is that simple superposition does not hold for the solutions of nonlinear oscillators.

Review of nonlinear VEH

A number of previous reviews exist on the use of nonlinear vibratory mechanisms for vibration energy harvesting. Most of these reviews have either targeted on enhancement bandwidth strategies Tang et al. (2010); Zhu et al. (2010) or primarily focussed on select few nonlinear principles such as Duffing and bi-stability Daqaq et al. (2014); Harne and Wang (2013). However, the motivation of employing nonlinear VEH extends beyond bandwidth enhancement, and it can be used to improve overall power density as well as responsiveness towards noisy or shock excitations. Furthermore, while Duffing and bi-stability are amongst the most well known nonlinear VEH, they are far from being the only nonlinear strategies.

This paper aims to add to the existing body of VEH review literature, by reviewing the underlying principles and advances within the VEH field that incorporates the various types of nonlinear vibration dynamics. Nonlinearity itself is an umbrella term, which can include vastly varying principles that are as distinct from each other as they are from linear systems. This paper reviews 8 major nonlinear VEH explored by the research community,

- Duffing nonlinearity and other higher order polynomial springs
- bi-stable and multi-stable oscillators
- parametric oscillators
- stochastic resonance
- mechanical frequency converters
- mechanical stoppers
- self-tuning mechanisms, and
- non-oscillatory mechanisms

Sometimes, combinations of these nonlinear mechanisms Jia and Seshia (2013) can be employed together along with a linear system. The research of these differing mechanisms by the literature aim to accomplish varying objectives: to enhance power peak, to broaden frequency bandwidth, to tune frequency bandwidth and to enhance responsiveness to broadband noise excitation. However, not all nonlinear mechanisms have been entirely successful in achieving these goals due to constraints either by practicality or by fundamental vibration principles.

Within the scope of this paper, only vibratory nonlinear interactions arising within the mechanical domain are reviewed. Frequency interaction mechanisms that employ active actuators or electrical damping Zhu et al. (2010)

are not covered herein. Transducers, such as piezoelectric and electrostatic, all exhibit nonlinearities in the electrical domain, which are not explored within this paper as well.

Duffing oscillators and higher order polynomial springs

Background of Duffing nonlinearity

One of the most well known nonlinear vibratory phenomenon explored in the VEH field is Duffing nonlinearity, named after Georg Duffing Kovacic and Brennan (2011). The oscillatory systems associated with this type of nonlinear dynamics are known as Duffing oscillators and the function used to describe the dynamics is known as the Duffing equation. Figure 3 summarises a generic model diagram for the Duffing type mass spring damper system.
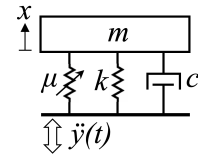

Figure 3. Model of a mass spring damper system with Duffing spring nonlinearity μ, subjected to base-point direct forcing with acceleration $\ddot{y}(t)$. The key element is the amplitude-dependent nonlinear spring term. This additional spring restoring term becomes increasingly significant at larger amplitudes.

The potential $U(x)$ term takes the form of $0.5kx^2 + 0.25\mu x^4$, where $\mu = 6m\beta$, and the Duffing equation is a differential equation with a cubic (third order polynomial) spring term as shown in equation 10.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x + \beta x^3 = -\ddot{y}(t) \qquad (10)$$

where $\beta x^3(t)$ is the amplitude dependent time function that describes the level of the spring nonlinearity present in the system. Therefore, with build up of oscillatory amplitude, spring hardening effect can be seen for $\beta > 0$ and spring softening effect can be observed for $\beta < 0$. Figures 4 and 5 illustrate numerically simulated examples of the restoring force and frequency domain characteristics of these nonlinear spring behaviour.
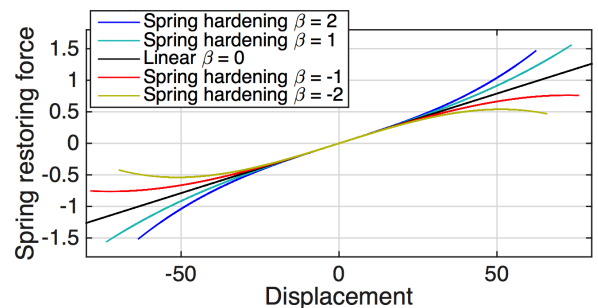

Figure 4. Numerically simulated amplitude dependent restoring force behaviour of the Duffing equation, illustrating spring hardening and softening effects. Axes dimensions are arbitrary.

Duffing nonlinearity for VEH

The core thesis of employing Duffing nonlinearity in VEH is to tap into the bending behaviour of the resonant peak
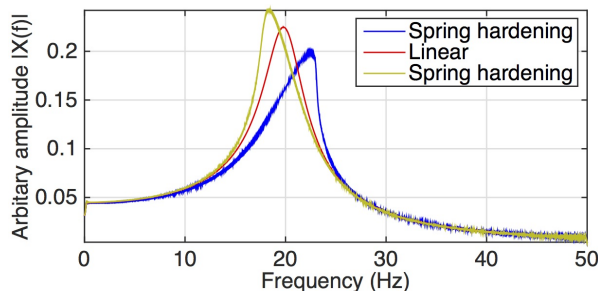
Figure 5. Numerically simulated frequency domain characteristics of the Duffing equation, illustrating resonant peak bending for spring hardening and softening effects. Axes dimensions are arbitrary.

as a means of capturing a modestly wider operational frequency bandwidth compared to the linear resonant peak Stanton et al. (2009); Erturk et al. (2009); Mann and Sims (2009); Barton et al. (2010); Sebald et al. (2011); Mallick et al. (2016). However, a key practical issue is the frequency hysteresis present in the frequency domain depending on the direction of the frequency sweep. For broadband excitations, the system might be trapped in the less desirable lower bifurcation point around the frequency vicinity of the resonant peak bend.

For most linear oscillators, when driven towards large oscillatory amplitude, a weakly nonlinear behaviour can onset as the spring deviates from its original geometry. This type of geometric nonlinearity is often defined by the Duffing equation with a relatively small β. However, strong spring nonlinearity, mathematically taking the form of a relatively large β, can be inherently present at small to modest oscillatory amplitude range as well.

This latter type is generally referred to as Duffing oscillators and the strong inherent nonlinearity can be achieved through a number of design approaches. This includes: coupling of a resonator end mass (ferromagnetic) with closely placed magnets to induce either spring softening or hardening effect depending on the magnetic coupling orientation and topology Stanton et al. (2009); Erturk et al. (2009); Burrow and Clare (2007); Burrow et al. (2008); the employment of magnetic springs instead of physical springs as the restoring force such as magnetically coupled rotary suspension Spreemann et al. (2006) and nonlinear magnetic levitation where a magnetic is suspended in a single axis tube (coils wound from outside) by two opposing magnetics on either ends Mann and Sims (2009); Daqaq (2010); and the application of axial pre-stress or pre-compliance to a spring in order to inherently induce hardening or softening Marinkovic and Koser (2009).

In real vibrational systems, truly linear resonators are quite rare and nonlinearity can be observed by simply driving an otherwise linear spring, such a simple cantilever, into high amplitudes Ramlan (2009); Ramlan et al. (2010). This is the weakly nonlinear type of the Duffing nonlinearity. The amplitude at which the Duffing term becomes dominant is dependent on the geometry and structural design of the spring. However, to effectively utilise the non-linear phenomenon for VEH, the aim is to achieve the non-linearity both at relatively low amplitudes and as few design complexities as possible.

Triplett *et al.* Triplett and Quinn (2009) demonstrated an intrinsically nonlinear piezoelectric composite plate due to the nonlinear coupling between the composite layers during curing. This enabled a nonlinear VEH without the addition of external magnets or magnetic springs. However, the device experimentally deviated from a classical non-linear model described by the Duffing equation and the behaviour could suggest the manifestation of an even higher order polynomial nonlinear spring, which has yet to be validated.

Through varying buckling configurations Leadenham and Erturk (2014); Huguet et al. (2019), it is possible to asymmetrically manipulate Duffing oscillators to enter harmonic regions in order to substantially expand the operational frequency of nonlinear resonant bend. However, the precariousness of these high energy solutions along the bifurcation branch still remain and can be a potential limitation when it comes to practical implementation.

Limits of Duffing nonlinearity for VEH

Barton *et al.* Barton et al. (2010) and Daqaq *et al.* Daqaq (2010) experimentally compared linear and Duffing-type nonlinear harvesters driven by broadband random vibration, and observed no noticeable improvements despite the increased device complexity and promising theoretical predications for limited band response. This could be a result of the presence of non-trivial boundary conditions required to access the potentially wider bandwidth of the non-linear regime. Therefore, the advantages of operating in the non-linear regime only becomes apparent when the various boundary conditions, dictated by the vibrational source, becomes favourable.

On the other hand, authors such as Erturk *et al.* Erturk et al. (2009), Ramlan *et al.* Ramlan (2009); Ramlan et al. (2010), Stanton *et al.* Stanton et al. (2009) and Cammarano *et al.* Cammarano et al. (2014a,b) all proposed that Duffing oscillators offer a broader bandwidth performance than their linear counterparts. This is backed up by both simulation and experimental results in the frequency domain. However, these results were achieving by sweeping the frequency response of the oscillators by a sinusoidal excitation. While a sinusoidal frequency sweep unveils useful frequency domain information, it is insufficient to describe the practical behaviour and broadband response of a nonlinear oscillator. This is because real world vibration is often noisy and random in nature, rather than sinusoidal.

Langley Langley (2014, 2015) established from fundamental principles: that all forms of oscillators with a given mass, both linear and nonlinear, will not demonstrate any overall difference in accumulated power under a broadband white noise excitation. Mass is the only dominant parameter that influences power responsiveness from a broadband excitation. This is backed up by independent experimental work from other groups Daqaq (2010); Barton et al. (2010) that subjected both comparable linear and nonlinear Duffing oscillators towards broadband excitations. While there might be incremental differences for narrow band excitations that approximate to sinusoidal forcing Nguyen and Halvorsen (2010); Quinn et al. (2010); Mallick et al. (2016), the previously perceived broadband promise Gammaitoni et al. (2009b) of Duffing oscillators over linear oscillators

is not backed up either fundamental vibration theory Langley (2014, 2015) or experimental validation Daqaq (2010); Barton et al. (2010).

Halvorsen *et al.* Nguyen and Halvorsen (2010); Tvedt et al. (2010); Nguyen and Halvorsen (2011); Halvorsen (2013) investigated Duffing oscillator implementations in MEMS designs. This is achieved either by electrostatic pre-stress or topological designs. A significant and complex nonlinearity onset was observed for the MEMS oscillators. While considerable bandwidth enhancement was initially observed from sinusoidal frequency sweeps Nguyen and Halvorsen (2010), later studies drew the same conclusions as Langley, Daqaq *et al.* and Barton *et al.*, namely: Duffing oscillators do not have a fundamental performance advantage over linear oscillators when subjected to broadband noise.

Sebald *et al.* Sebald et al. (2011) also suggested that under broadband excitation, even for coloured noise, the solution of a Duffing oscillator is typically almost always trapped in the lower bifurcation point. For a piezoelectric transducer, they proposed a fast voltage burst as a possible additional mechanism to take advantage of the potential higher bifurcation point for Duffing oscillations. Nonetheless, no improvement over linear oscillators was observed in both simulation and experiment.

Therefore, it can be concluded that while Duffing oscillators exhibit a seemingly broader frequency bandwidth under sinusoidal frequency sweeps, the average power response performance has no fundamental improvement over linear oscillators. Furthermore, the current characterisation method using sinusoidal frequency sweeps is insufficient to reveal the actual bandwidth performance towards realistic vibrational inputs. Characterisations should also incorporate broadband excitations on top of sinusoidal sweeps.

Higher order nonlinear spring oscillators

The nonlinear spring in discussion here is determined by the order of the spring function terms in the governing differential equation of the oscillatory system. Higher order polynomial spring term is not limited to the Duffing case. In fact, more generically, the source of this type of spring nonlinearity stems from the decision to truncate at a particular correction term of the Taylor series expansion of the potential energy function $U(x)$. For a linear spring, $U(x) = 0.5kx^2$ differentiates to the kx term in the differential equation of the harmonic oscillator. However, taking into account additional higher order terms, $U(x)$ can be expanded to equation 11 as an example.

$$U(x) = \frac{kx^2}{2!} + \frac{\lambda x^3}{3!} + \frac{\mu x^4}{4!} + \frac{\nu x^5}{5!} + \frac{\xi x^6}{6!} + \mathcal{O}(x^7) \quad (11)$$

where, $\mathcal{O}$ represents the error correction term from the indicated polynomial order onwards. Differentiating $U(x)$, and applying $\alpha = \lambda/2m$, $\beta = \mu/6m$, $\gamma = \nu/24m$ and $\delta = \xi/120$, the differential function (equation 12) governing the now anharmonic oscillator can be derived.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x + \alpha x^2 + \beta x^3 + \gamma x^4 + \delta x^5$$
$$+\mathcal{O}(x^6) = -\ddot{y} \quad (12)$$

For the case of Duffing oscillator, α and γ are equated to zero in order to eliminate the odd terms in $U(x)$, while ignoring the higher terms. In scenarios where odd terms in $U(x)$ are included, the $U(x)$ function is unstable and there can be scenarios where the solution does not converge and approach infinity: $U(x \to -\infty) = -\infty$ for $\lambda > 0$ and similarly for $\lambda < 0$ towards the other direction. Therefore, for total energy $E > 2k^3/3\lambda^2$, the oscillation can hop out of the local minimum and escape towards negative infinity.

This has been theoretically explored by Kurmann *et al.* (2015) Kurmann et al. (2015) for up to a 5th order nonlinear spring. Under certain boundary conditions and excitation conditions, the solutions can approach infinity and the system exhibit an ultra broadband frequency response in the nonlinear regime. However, the physical manifestation of such a system, and keeping the oscillatory system on the desired side of the bifurcation solution might not be as trivial in practical terms as the numerical simulation suggests. It has also been shown that magnetic spring is one potential source that can give rise to the higher order polynomial spring nonlinearity Kurmann et al. (2016a).

While many in the literature Mann and Sims (2009); Zhu and Beeby (2013b) assume a third order nonlinear restoring term for magnetic springs and often approximate the system to a Duffing oscillator, the approximation only holds for small oscillations Kurmann et al. (2016a). At large oscillations, the behaviour is much more complex and involves higher orders Jia et al. (2018c), leading to the theoretical potential of accessing the broadband behaviour described by the odd terms in $U(x)$. Work in this area within the VEH field is still scarce and further work is still needed to further assess the behaviour and feasibility of implementing such higher order spring systems.

Bi-stable and multi-stable oscillators

Background of bi-stability

Bi-stable systems employ negative stiffness as the restoring term, which typically involves an unstable equilibrium at the zero-displacement position. Figure 6 illustrates a generic model representation for such systems.
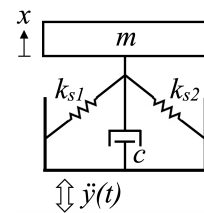
Figure 6. Model of a bistable mass spring damper system configured with negative spring restoring terms k_{s1} and k_{s2}, subjected to base-point direct forcing with acceleration $\ddot{y}(t)$.

For a linear spring operating within the elastic region, the potential energy function takes the form of $U(x) = -0.5kx^2$. The governing equation of the system can then take the form of equations 13 and 14.

$$m\ddot{x} + c\dot{x} - kx = F(t) \quad (13)$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} - \omega_n^2 x = -\ddot{y}(t) \quad (14)$$

Therefore, multiple intra system potential wells (figure 7) are created, which traps the resonators in one particular stable state (potential well). Energy can be harnessed through either the normal resonant behaviour within any of the stable states or from the instantaneous energy release when the system attains enough energy to hop from one potential well to another across a potential barrier. This inter-well hopping is also known as the snap-through state.
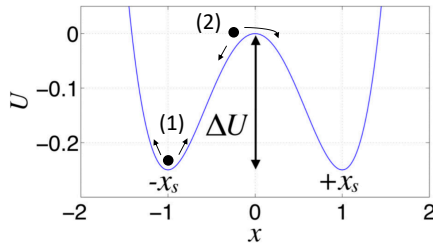

Figure 7. A bi-stable system, showing two potential wells separated by a potential wall of magnitude ΔU. The $x = 0$ position is now an unstable equilibrium, while $-x_s$ and x_s are two stable equilibria. Oscillation around position (1) represents intra-well oscillation, while position (2) illustrates inter-well hopping (snap through state) when the energy input exceeds ΔU.

Intra-well oscillation is similar to the Duffing oscillatory response where extra stiffness can be observed upon a given resonant response. On the other hand, the vibration required to induce the snap-through state is generally independent of the natural frequency of the system and only requires the forcing energy amplitude to be large enough to cross the potential barrier. Therefore, assuming ample amplitude, the snap-through mechanism is responsive across a wide range of frequencies, especially for low frequencies where displacement amplitude is high.

Bi-stability and multi-stability for VEH

Dogheche *et al.* in 2006 Dogheche et al. (2006) was amongst the earliest to suggest the possibility of implementing bi-stability in VEH through the demonstration of a bi-stable piezoelectric micro-machined ultrasonic transducer originally designed as an acoustic sensor. Ramlan Ramlan (2009) and Moehils *et al.* Moehils et al. (2009) investigated the higher power output available from snap-through motion over a linear resonator. Bi-stability can either be mechanically introduced through stressing or inherently configured into the system during manufacturing (figure 8). An example of the former is the use of axial loading to introduce a buckling force in a clamped-clamped beam Formosa et al. (2009). An example of the latter is the bistable carbon fibre piezoelectric composite plate by Arrieta *et al.* Arrieta et al. (2010), which was created from the residual pre-stress of non-aligned uni-directional fibre orientations in the composite stack.

Magnets are a popular choice to realise bi-stable VEH. Examples include cantilever with magnet end mass and an opposing magnet in the zero-displacement position to push the oscillatory proof mass magnet away Cottone et al. (2009); Stanton et al. (2010); Lin et al. (2010); the use of ferromagnetic end mass on a pendulum or a cantilever, and two attractive magnets placed on either sides to pull
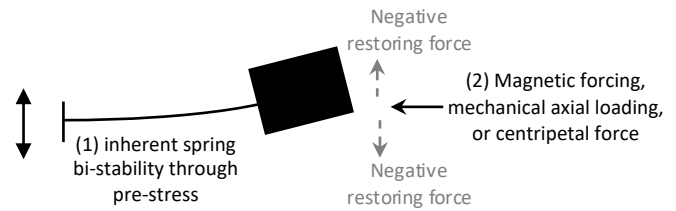

Figure 8. Generic schematic of inducing bi-stability in a resonator beam. (1) involves inducing inherent structural bi-stability by introducing significant residual stress from the manufacturing process, and (2) involves inducing bi-stability from external loading within the mechanism.

and nullify the stability of the zero-displacement position Erturk et al. (2009); Stanton et al. (2009); and two or more cantilevers with magnetically opposing end mass placed in close proximity to push each other into bi-stable Zhu and Beeby (2013b); Lin et al. (2010) or multi-stable Trigona et al. (2013) states. The bi-stable or multi-stable VEH thus created from magnetic coupling are not too unlike the use of magnetic coupling for enabling frequency tuning Challa et al. (2011), multi-coupled resonators Petropoulos et al. (2004); Challa et al. (2009) and non-linear resonators. Therefore, a system that is simultaneously all of the above can be realised to exploit multiple resonant and vibration regimes, along with the snap-through mechanism. Centripetal acceleration has also been used as a means for introducing bi-stability in a rotational system Horne et al. (2018).

The main advantage of bi-stability over linear systems is that the snap-through operation is amplitude dependent rather than frequency dependent. Therefore, for large amplitude vibratory applications, such as wide band aerospace vibration, bi-stability might be more favourable than narrow band linear oscillators. However, the snap-through response of the bistable system is still inherently non-resonant in nature. Therefore, while it benefits from broadband response, the same mechanical amplification from resonators are not enjoyed by snap through states.

Bi-stable oscillators can also exhibit resonance when the system is trapped within one of the potential intra-wells. However, the oscillatory response is typically of lower energy than the linear equivalent Zhu and Beeby (2013a); Jia and Seshia (2013), as it is constrained due to the higher pre-stress required to induce bi-stability. Furthermore, resonance within the potential well is equally confined in operational frequency bandwidth as linear oscillators.

Jia *et al.* Jia et al. (2019) reported a bi-stable system internally coupled with a subsidiary oscillator, which utilises the amplitude-dependent inter-well snap-through states to act as a defined internal frequency source to drive the subsidiary oscillator into resonance. Dubbed 'broadband resonance', this scheme can deterministically achieve resonant amplification from a broadband vibration irrespective of its excitation frequencies, as long as the average noise intensity is well defined.

Huguet *et al.* Huguet et al. (2017, 2018, 2019) studied the harmonic and subharmonic dynamics of bi-stable oscillators and suggested a broadening of operating frequency compared to pure first harmonic operation. These authors used an actuator to manipulate the buckling

level of bi-stable doubly clamped beams to control orbit jumps across various harmonic and subharmonic regimes. The expansion of phase portraits for the various higher harmonic and subharmonic bi-stable oscillatory orbits reveal a theoretical realm where bi-stable oscillators can exhibit a very broadband high energy response, akin to that of broadband resonance.

Once the high energy bi-stable inter-well snap-through states onset, the desirable large oscillatory amplitude is possible. However, the average power density has been experimentally shown to be equivalent to that of a linear oscillator counterparts Zhu and Beeby (2013a); Zhu et al. (2014). Furthermore, from a theoretical point of view, the power density will not over-perform that of linear oscillators Langley (2014); Barton et al. (2010) when subjected to broadband excitations. This is because while high energy inter-well oscillations are frequency-independent in nature, its also non-resonant in nature despite the large oscillatory response.

Therefore, the overall power density performance of bistable oscillators exposed to broadband noise are not necessarily found to be more advantageous than linear oscillators taking into account all these pros and cons Zhu et al. (2010); Tang et al. (2010). Nonetheless, for low frequency and large amplitude noisy applications, bistable systems can be more preferable over linear oscillators due to the amplitude-dependent and frequency-independent behaviour.

Parametric oscillators

Background of Mathieu instability

Parametric resonance is an inherently nonlinear vibratory phenomenon, which is distinct from directly excited oscillations due to the presence of a specific instability phenomenon sometimes dubbed the Mathieu instability. Instead of energy build-up from a forced response, parametric resonance involves a periodic modulation in system parameter. There could be two types of parametric resonance Minorsky (1974); Nayfeh and Mook (1979); Cartmell (1990): one is the externally induced heteroparametric resonance (which is generally known simply as parametric resonance) where a periodic modulation of system parameter is induced due to external forcing, and the other is the internally excited autoparametric resonance that arises from certain integer ratio relationships among the various natural frequencies of a multiple DOF system. Examples of the model representations for both are shown in figure 9.

In figure 9, a circular block is used to represent a parametric coupling term, where direct excitation/oscillation is translated into a periodic modulation on the spring stiffness, i.e., a parametric excitation. The physical manifestation of the parametric coupling term could involve a rotary mechanism such as that seen in a pendulum Nayfeh and Mook (1979), multi-axes leakage of base vibration Jia et al. (2014a) or movement of the vibrating base affecting the effective stiffness of the spring Jia et al. (2016). The direct excitation/oscillation components are intentionally drawn in the y direction and orthogonal to the parametric oscillatory response in the x direction, in order to illustrate that the parametric response does not onset from direct forcing in
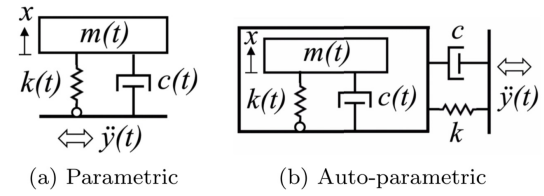

Figure 9. Model of mass spring damper system configured for (a) heteroparametric oscillation or (b) auto-parametric oscillation when subjected to external forcing with acceleration $\ddot{y}(t)$. The circle represents the parametric coupling term, where horizontal oscillations (here) in the y direction are translated into periodic modulation on the spring stiffness $k(t)$.

its response direction, but it instead arises from the system parameter modulation.

Mathematically, both types of parametric resonances reduce to an equation that contains a time coefficient in at least one of the system parameters, known as the Mathieu equation, named after the elliptical membrane problem studied by M.E. Mathieu Mathieu (1868). The canonical Mathieu equation Mathieu (1868); McLachlan (1947) is given by equation 15. The generic parametric oscillatory equation of motion Jia et al. (2013b) can be represented by equation 16 and the generic equations of motion for autoparametric oscillator Jia and Seshia (2014) can be shown by equations 17 and 18.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}z^2} + [\delta - 2\varepsilon \cos(2z)]y = 0 \qquad (15)$$

where, y and z are generic variables and δ and ε are generic parameters.

$$\ddot{x} + 2\zeta\omega_n\dot{x} + (\omega_n^2 - \alpha\ddot{y})x = 0 \qquad (16)$$

where, α is the heteroparametric coupling term relating the transfer of energy from the external excitation to the periodic modulation of the system parameter.

$$\ddot{z} + 2\zeta_1\omega_1\dot{z} + \omega_1^2 - \epsilon\mu(\dot{x}^2 + x\ddot{x}) = -\ddot{y}(t) \qquad (17)$$

$$\ddot{x} + 2\zeta_2\omega_x\dot{x} + (\omega_2^2 - \epsilon\ddot{z})x = 0 \qquad (18)$$

where, ϵ is the internal mode coupling coefficient between the directly excited degree-of-freedom and the parametric oscillator.

Traditionally, the study of parametric resonance has circled around its control and prevention in regards to structural failure of mechanical systems such as aircraft wings, marine crafts, civil structures, etc. Tondl et al. (2000); Fossen and Nijmeijer (2012); as oscillations, en route to chaos, could accumulate to significantly larger amplitudes than direct resonance. This implies that it could potentially act as a mechanical amplifier to maximise the energy conversion efficiency of a given transducer and drastically improve its output power density for a given forcing amplitude.

The wider transducer community has dubbed this as the 'parametric amplification effect' employed to achieve higher input-to-output sensitivity than direct resonance Carr et al. (2000); Oropeza-Ramos and Turner (2005); Requa and Turner (2006); Thompson and Horsely (2009). Parametric

resonance has the potential to simultaneously improve the power density and broaden the operational frequency bandwidth for a given drive acceleration amplitude.

Parametric resonance for VEH

Daqaq *et al.* in 2009 Daqaq et al. (2009) was amongst the first to report investigation into parametrically excited systems for the purpose of vibration energy harvesting (VEH) as well as deriving a mechanical based numerical model for such systems. While the parametric amplification effect has been previously reported to enhance sensing technologies Oropeza-Ramos and Turner (2005); Requa and Turner (2006), the implementation of parametric resonance for the power generating nature of VEH is fundamentally different as additional power to drive the parameter modulation is not readily available. In order to activate parametric resonance in practice, a critical threshold amplitude and strict frequency criterion needs to be attained Daqaq et al. (2009); Jia et al. (2014b), which gets increasingly more difficult for higher orders of parametric resonance or at higher damping situations Levy and Keller (1963); Turner et al. (1988).

Daqaq's 2009 study laid out the principles and behaviour of parametric resonance for VEH, noting that while parametric resonance is potentially beneficial, the conditions required to activate a heteroparametric system from external excitation might be too onerous for practical applications. This is further work was further developed by the same group Stabler (2010); Daqaq and Bode (2011) in an attempt to couple in both direct excitation to help activate parametric excitation. A compromise was hypothesised to exist between exciting a system at an angle, where both direct and parametric excitations can couple in.

In the meantime, Ma *et al.* Ma et al. (2010) looked at parametrically excited pendulum with a rotary generator coupled to the pendulum pivot. However, as electrical damping was applied, a similar critical threshold limitation can be observed. Jia *et al.* Jia et al. (2014b) coupled a parametrically excited pendulum to a second degree of freedom lever mechanism, and applied electrical damping on the second coupled mechanism. This enabled minimal damping on the parametrically excited oscillator, thus allowing the manifestation of parametric resonance at a relatively low acceleration amplitude.

Jia *et al.*'s work in both MEMS and macroscopic platforms Jia et al. (2012, 2013b, 2014a) laid out various passive design strategies for realising parametric resonance for VEH at relatively low initiation threshold amplitudes (summarised in figure 10). Jia *et al.* reported over an order of magnitude improvement in both power amplitude and frequency bandwidth for devices driven into parametric resonance in contrast to the same device operated in direct resonance. Additionally, unlikely direct resonators that have to compromise between amplitude and bandwidth as the quality factor changes, parametric resonators have been shown to demonstrate both power and bandwidth enhancements when damping decreases Jia et al. (2013a). This is because as damping decreases, the system operates deeper into the instability region, which is a fundamentally different underlying resonant principle than directly forced resonances.
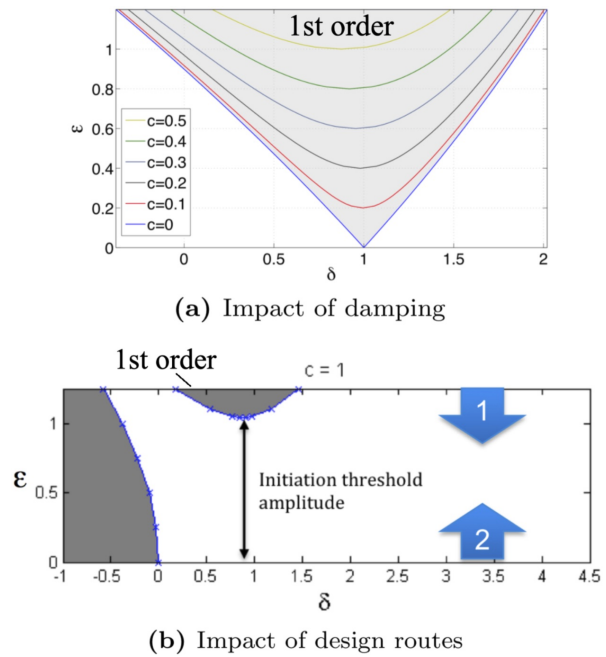

(a) Impact of damping

(b) Impact of design routes

Figure 10. Parametric resonance onsets within the shaded instability regions of the δ-ε parameter space from equation 15. This plot is known as the Strutt instability plot Minorsky (1974); Nayfeh and Mook (1979). (a) As damping (either mechanical or electrical) is introduced, an initiation threshold amplitude starts to build up, which needs to be attained before activating parametric resonance. (b) Passive design routes to minimise this initiation threshold for vibration energy harvesting include: (1) minimising damping and subjecting electrical damping to another degree-of-freedom, and (2) amplification of base excitation either directly or by auto-parametric means. Figure based on Jia (University of Cambridge).

Apart from hetero-parametric resonance (externally forced parameter modulation), the use of auto-parametric resonance has been recognised as an even more desirable method for vibration amplification at low amplitudes Jia and Seshia (2014) (illustrated in figure 11). This involves a strict internal frequency relationship between multiple degrees-of-freedom compound resonators, where one sub-resonator's frequency is twice that of another sub-resonator. Furthermore, a finely tuned auto-parametric resonator has been theoretically predicted to exhibit semi-resonant response towards broadband excitations Kurmann et al. (2016b), as the onset of parametric resonance is mainly reliant upon the intrinsic internal parameter ratio rather than excitation frequency. A similar broadband response is also theoretically predicated by Alveras *et al.* Alveras et al. (2017). However, the practical implementation of achieving such a system remains an open question.

Apart from the principal (first order) parametric resonance, higher orders of parametric resonance can also be used as additional operational frequency windows for VEH Jia et al. (2013b,a). The number of orders are determined by $\omega = 2\omega_n/n$, where ω is the excitation frequency and n is the order number. Therefore, higher orders are found in lower frequency submultiples. While realising higher order parametric resonance are relatively challenging at macroscopic scales, MEMS implementations can potentially more readily unveil these higher instability intervals Jia et al.
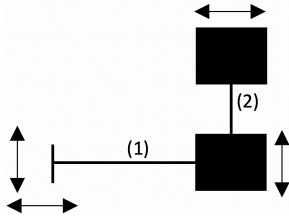
Figure 11. Generic schematic for a basic auto-parametric resonator, where the resonant frequencies of the first sub-resonator (1) and the second sub-resonator (2) are related to each other with a ratio of either $2:1$ Jia and Seshia (2014) or $1:2$ Jia et al. (2018b). This strict internal frequency f ratio enables the direct resonance onset of the $2f$ sub-resonator to drive the other $1f$ resonator into parametric resonance.

(2016, 2018a). However, the principal parametric resonance is always going to provide the highest energy spectrum and any yields from the higher order are likely to quickly diminish.

Zaghari *et al.* further undertook a series of studies for realising parametric resonance from a small actuating element as the source of the parametric modulation Zaghari et al. (2015) along with a base excitation, and characterising its nonlinear dynamic response Zaghari et al. (2016) while establishing a phase-dependent relationship to its activation Zaghari et al. (2018a). Zaghari *et al.* also built a numerical model Zaghari et al. (2018b) taking into account the electrical domain effects of an electromagnetic transducer.

Overall, the key challenge is to activate parametric resonance at a relatively more attainable set of frequency, amplitude and time-dependent boundary conditions, in order to practically realise its high power accumulating potentials. While parametric resonance holds the theoretical potential of achieving both higher power output and wider frequency bandwidth than directly excited counterparts due to the fundamentally distinct instability phenomenon, its design, implementations and operational considerations are far from trivial when compared to basic direct resonators such as simple cantilever beams.

Stochastic resonance

Background of Mathieu instability

Stochastic resonance Gammaitoni et al. (1998); Ando and Graziani (2000) is the vibrational phenomenon where an excitation (signal) that is usually too small to yield a meaningful result (to detect), becomes meaningful with the addition of noise. In sensing applications, it is popularly known as 'adding noise to remove noise', since the addition of either white noise or a specific coloured noise helps to boost the signal-to-noise ratio Droogendijk et al. (2013). This amplification effect has been observed in a number of areas such as electronics, neuroscience and climate change Gammaitoni et al. (1998); and have been exploited in communication and sensing applications Jung and Hanggi (1991); Ando and Graziani (2000). Though, it should be noted that not all noise are strictly stochastic excitations and a system designed to be responsive to noise/broadband vibration does not necessarily exhibit stochastic resonance.

An example of stochastic resonance that is particularly relevant to the VEH context is a bi-stable system (figure 12 and equation 19) experiencing a periodic forcing that is less than sufficient to cross the potential barrier. Although the system is trapped in a potential intra-well, when driven into resonance, it can still yield a meaningful power output. However, with the addition of noise (stochastic excitation) into this system, the extra energy in combination with the periodic forcing could activate the snap-through states by modulating the potential barrier at a specific frequency and realise a much larger power output.
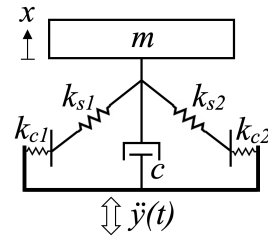

Figure 12. Model of a stochastically excited bistable mass spring damper system configured with negative spring terms k_{s1} and k_{s2}, and potential well modulating terms k_{c1} and k_{c2}, subjected to base-point direct forcing with acceleration $\ddot{y}(t)$. By modulating the potential barrier at a specific frequency, the probability of achieving bi-stable snap through state can be promoted.

$$\ddot{x} + 2\zeta\omega_n\dot{x} - \omega_n^2 x + \mu x^3 = \ddot{y} + N(t) \qquad (19)$$

where, N is the noise excitation. With the application of a sinusoidal periodic forcing with amplitude A as illustrated in equation 20, the bistable potential wells start to modulate as shown in figure 13.

$$U(x,t) = -\frac{1}{2}kx^2 + \frac{1}{4}\mu x^4 - Ax\sin(\omega t) \qquad (20)$$

The probability of the system hopping between the potential wells is defined by the Kramers' rate Kramers (1940); Gammaitoni et al. (1998); Su et al. (2013); Zheng et al. (2014) as defined in equation 21.

$$r_k = \frac{k}{\sqrt{2}\pi} \exp\left(-\frac{k^2}{4\mu D}\right) \qquad (21)$$

Stochastic resonance can be achieved with a potential barrier modulating frequency in the vicinity of $f = 0.5r_k$ as illustrated in equation 22.

$$\omega = \frac{k}{\sqrt{2}} \exp\left(-\frac{k^2}{4\mu D}\right) \qquad (22)$$

Stochastic resonance for VEH

As mentioned previously in section , the triggering of the bi-stable snap-through state is independent of the ambient frequency and is instead amplitude-dependent. This makes bi-stable systems more suitable for harvesting high intensity noise excitations. However, this is implies a high excitation amplitudes needs to be maintained for any meaningful output. Leveraging the essence of stochastic resonance, if a system is designed properly to experience stochastic resonance from a vibrational source that comprises of both
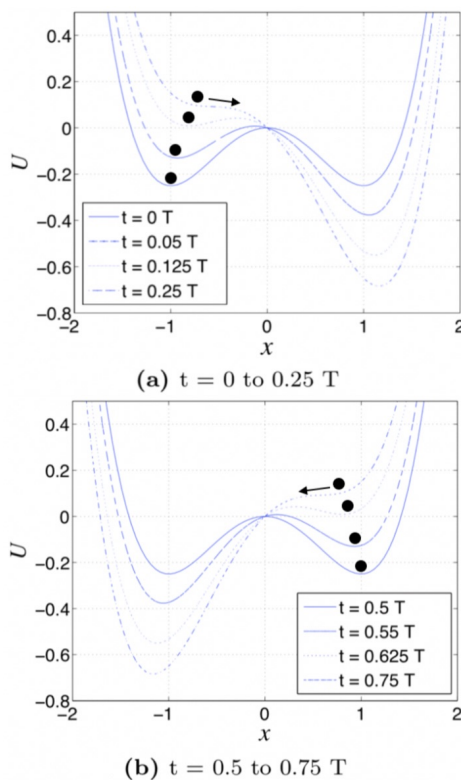
Figure 13. Modulation of the bistable potential well when subjected to a periodic forcing whose amplitude is just enough to match the potential barrier. T is the time period of the periodic forcing, showing that the system is positioned more favourably to cross the potential barrier every half cycle.

noise and periodic excitations, then such a system has potential to exhibit the above mentioned meaningful output even if either the noise or the periodic excitation does not meet the prerequisite amplitude criterion to cross the bi-stable potential barrier on its own McInnes et al. (2008); Gammaitoni et al. (2009b,a).

Therefore, the energy of the broadband noise can be harvested with the onset of appreciable mechanical response from the snap-through states in contrast to the confined response within the potential well. In essence, the combination of two individually insignificant energy sources (periodic and stochastic excitations) when fed to a carefully design stochastic resonant system, can yield a significant power response. Although stochastic resonance is not limited by bi-stable system, this snap-through working mechanism has been the core of the theoretical and experimental investigations within VEH thus far Moehils et al. (2009); Formosa et al. (2009); Zheng et al. (2014).

McInnes et al. in 2008 McInnes et al. (2008) were amongst the first to propose a theoretical framework to implement stochastic resonance to enhance vibration energy harvesting. McInnes et al. modelled the addition of a small periodic signal to a bistable oscillator, which was otherwise being exposed to an ambient broadband noise vibration signal that is insufficient to achieve snap-through states. At the right frequency conditions (meeting the Kramers' rate criterion), it was modelled that the bi-stable system starts to cross the potential barrier as stochastic resonance onsets. Formosa et al. Formosa et al. (2009) experimentally investigated a double bi-stable system in

an attempt to observe large amplitude amplification from stochastic excitations; however, it wasn't clear if stochastic resonance itself was achieved in their system. Cottone et al. Cottone et al. (2009) and Gammaitoni et al. Gammaitoni et al. (2009a) also investigated nonlinear VEH in relation to noise excitations. However, while the vibrational input might have been stochastic in nature, it also wasn't clear if stochastic resonance or any additional amplification was achieved.

The research led by K. Nakano and M.P. Cartmell Su et al. (2013); Zheng et al. (2014); Zhang et al. (2014) were the first group to definitively experimentally demonstrate stochastic resonance for VEH while showcasing its amplification effect from ambient broadband noise. Their work has shown the modulation of the bi-stable potential well Hu et al. (2012); Su et al. (2013, 2014), while also mapping out a comprehensive theoretical and experimental framework for stochastic resonance in a rotational VEH context Zheng et al. (2014). The rotational scenario has been experimented in automotive wheels Zhang et al. (2014, 2015, 2016), where the external ambient road vibration acts as the broadband noise excitation and wheel speed dependent vibration acts as the specific periodic signal. When the wheel operates within a certain velocity, the conditions becomes favourable for stochastic resonance to enhance the magnetic bi-stable snap-through states from the broadband road vibration. This is particularly useful because the system can be designed to be stochastically resonant for a specific vehicle speed. This design speed could potentially be the average urban vehicle speed or the national speed limit.

Therefore, in similar applications where an ambient noise excitation is present and a clear stable periodic signal is also available, the system can be designed so that equations 21 and 22 can be satisfied. This is particularly useful as many real world vibrations tend to be noisy, while having a few specific frequency information embedded within that broadband spectrum. However, designing a system to inherently match the application and system specific Kramers' rate within its system parameters is also by no means trivial.

Other principles and considerations

Frequency conversion

Miniaturisation of VEH is often desired to realise on-board system integration. However, at smaller dimensions, the natural frequency of the resonators are typically much higher than the frequency of the ambient vibration. Frequency up conversion is a design technique that can be incorporated in VEH to remedy this situation. Also, some transducers and interface electronics only attain optimal efficiency above a certain frequency. Additionally, the overall system can be responsive to multiple frequencies depending on the design.

A frequency up converter typically involves a primary resonator or dynamic system that is either linear Rastegar et al. (2006); Lee et al. (2008); Ferrari et al. (2013); Edwards et al. (2013) or rotational Priya (2005); Rastegar and Murray (2009) with potential to incorporate transmission gear trains. This primary resonator or dynamic system is typically designed to be responsive to a lower and/or wide band frequency to couple with the ambient source, which is

in turn coupled to a secondary resonating system that usually operates at a higher narrow band frequency with a higher quality factor.

As the larger primary resonator responds to vibration, energy internally flows towards the secondary resonator via a specific coupling mechanism, thus enabling the secondary resonator to oscillate and decay at its own natural frequency independent of the external vibrational frequency. This internal coupling mechanism could involve: physical interactions such as tapping or plucking, magnetic coupling or geared mechanism. The shock induced natural decay oscillatory response in the secondary resonator can then be harvested through a transducer.

Priya Priya (2005) and Rastegar *et al.* Rastegar et al. (2006); Rastegar and Murray (2009) demonstrated mechanical plucking of secondary piezoelectric elements, and Ferrari *et al.* Ferrari et al. (2013) investigated smaller piezoelectric cantilever beams driven by mechanical impact from the oscillation of a neighbouring large and parallel drive cantilever. On the other hand, Wickenheiser and Garcia Wickenheiser and Garcia (2010), and Kulah and Najafi Kulah and Najafi (2008) employed magnetic coupling to actuate the secondary resonators. These examples can be illustrated by the generic diagram shown in figure 14. Jung *et al.* Jung and Yun (2010b,a) reported a larger clamped-clamped beam that houses multiple smaller subsidiary cantilever beams as illustrated by the generic diagram in figure 14. This topology enables direct coupling of the primary and secondary resonators within the same structure rather than an external coupling mechanism that might dissipate additional energy.
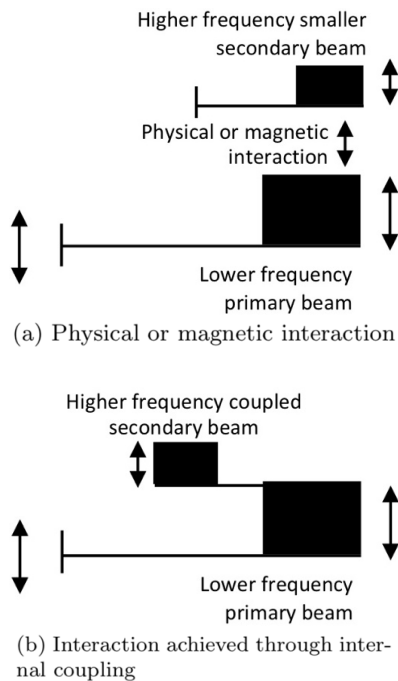

(a) Physical or magnetic interaction

(b) Interaction achieved through internal coupling

Figure 14. Generic schematic of frequency conversion using a lower frequency primary resonator to drive a higher frequency secondary resonator.

Frequency conversion mechanisms themselves don't necessarily improve bandwidth or power amplitude. In fact, the additional coupling mechanism required might be an extra source of nonlinear energy damping/dissipation.

However, the frequency conversion methods might be desirable the source vibration is far from (typically lower) the optimal operational range of a given harvester and the accompanying power conditioning electronics. Though, it should also be noted that the response from such systems, are rarely resonant in nature and rely more on oscillatory decay.

End stop effect

In an attempt to widen the operational frequency bandwidth, the use of mechanical stoppers to impose a physical amplitude limit has been studied by Soliman *et al.* Soliman et al. (2008, 2009). A general schematic of such a scheme is given in figure 15. The mechanical stoppers prevented the oscillating electromagnetic cantilever-based harvester to reach its potential peak; thus, achieving a flat and truncated peak from the resonant response. More than doubling of the operational frequency band was experimentally observed during a frequency upsweep with a peak power sacrifice of nearly one third. However, this broadening behaviour failed to manifest during downward frequency sweep as a hysteresis effect can be observed. Furthermore, double sided physical constraint demonstrated more immediate bandwidth broadening than a single sided physical constraint. Le and Halvorsen Le and Halvorsen (2012) also observed a very similar end stop behaviour in MEMS electrostatic energy harvesters, where anchors were designed to introduce a double sided physical constraint at a pre-determined displacement limit. In Le and Halvorsen's case, the bandwidth broadening from the end stop effect was as high as nearly 4 times than the linear counterpart.
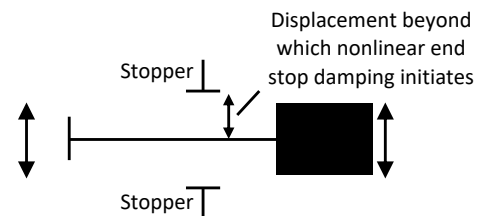

Figure 15. General schematic of a resonator with bidirectional end stops. Amplitude dependent end stop damping effect initiates as the resonator attains the end stop threshold. The end stop can either be mechanical or magnetic in nature.

In both groups' studies, the frequency broadening were not simply a truncation of what the original resonant peak, but there was a nonlinear resonant peak flattening and extension towards the higher frequency range as the physical constraint introduces an amplitude-dependent nonlinear damping effect. Beyond the amplitude of the physical constraint threshold, the quality factor of the resonator also lowers from this end stop damping, adding to the peak flattening. While the frequency broadening effect appears promising, such systems compromise the peak power achievable and the physical impacts from the mechanical stoppers promotes structural fatigue. However, it might be potentially possible to introduce magnetic or soft physical constraint to relief wear and tear from the continued physical impacts. For instance, squeeze film damping akin to air spring Chen et al. (2017) has been leveraged to act as a soft damper for MEMS oscillators that are likely to otherwise experience hard impacts.

Self-tuning

Literature has widely reported on VEH systems with active (drive power required) frequency tuning mechanisms that involves altering the spring stiffness or centre of mass Leland and Wright (2006); Hu et al. (2007); Challa et al. (2008); Garcia et al. (2010). This is typically achieved by an active actuator driving a physical or magnetic interaction with the spring-mass assembly. While tuning the resonant frequency towards the ambient frequency maximises power output, this also requires extra power investment to achieve the tuning. This is further problematic when the ambient frequency constantly varies. Roundy and Zhang in 2005 Roundy and Zhang (2005) and Zhu et al. in 2010 Zhu et al. (2010) have all analytically concluded that the power drain to continuously active frequency tuning will almost always exceed the net power recoverable. However, certain applications such as ferry engines operating with a fixed load for over an extended period of time has been shown to serve as a feasible candidate for intermittent active tuning Weddell et al. (2012).

On the other, there have been some researchers that have investigated various nonlinear dynamic behaviour that results in a passive (no extra energy required) self-tuning behaviour that matches the effective resonant frequency of the harvester towards that of the ambient source. The idea is for the device to dynamically adjust itself so its resonant frequency tend towards the source frequency.

For instance, Gu and Livermore Gu and Livermore (2010, 2011) have investigated the effect of centripetal acceleration on the proof mass of a cantilever beam inside a rotary system with radius r. As the rotational velocity ω increases, centripetal acceleration $a_c = -\omega^2 r$ also increases with a squared relationship. The induced centripetal force introduces the frequency shift behaviour. The proposed system can be a self-tuning mechanism by carefully designing the inherent frequency of the beam, the anticipated frequency shift from the changing rotational velocity and the anticipated change in ambient excitation frequency as a result of the rotational velocity variation. However, the implementation of such a solution is relatively niche as the frequency tuning effects from the change in rotational velocity needs to align with the drive frequency. This is therefore still unsuitable for wide-band excitation sources.

The collaborative studies amongst Imperial College London, University of California Berkeley and Vestfold University College Pillatsch et al. (2013); Miller et al. (2013); Gregg et al. (2014) investigated the peculiar behaviour of a doubly clamped beam with a sliding proof mass as illustrated by the generic schematic in figure 16. As the proof mass is positioned at various locations along the beam, the resonant frequency shifts. With the proof mass able to slide along the beam without any physical constraints apart from friction, the device self-tunes towards the source frequency as the mass self-slides along towards the most favourable position possible within the parameter range. The effect of friction from various material contact was seen to be a key influencing factor. The resulting frequency domain characteristics illustrate a bending resonant peak similar to spring hardening effect. While the device is still bandwidth limited, the operational bandwidth is significantly wider than

a linear counterpart. However, frictional energy loss as well as wear and tear from the sliding mechanism might be potential implementation concerns.
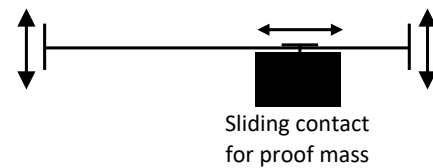

Figure 16. General schematic of the self-tuning resonator achieved through a sliding proof mass.

Non-oscillatory mechanisms

While any system driven out of resonance will qualify the non-resonant characteristic, there have been several studies that utilises some sort of non-restoring primary dynamic system to drive a secondary VEH resonator. In a sense, this is similar to the frequency converters, except the primary dynamic system here is neither a resonator nor does it possess a restoring term. Such a harvester will be independent of frequency but more sensitive to the absolute value of the excitation amplitude. Therefore, despite the broader frequency bandwidth, the profitability of such devices diminishes for higher frequency where the displacement amplitudes of the drive vibration are typically low and are insufficient to overcome the inertia.

Examples for these non-oscillatory harvesters include: cylindrical ferromagnetic roller in a chassis that closely houses an array of small piezoelectric cantilevers with magnetic end mass Pillatsch et al. (2011) or ferromagnetic/magnetic rolling balls that are coupled mechanically by impact Cavallier et al. (2005), which can all be either magnetically Ju et al. (2012, 2013) or electrostatically Bian et al. (2013) linked to a transducer. Roundy and Tola Roundy and Tola (2013) demonstrated a device with a single axis rolling ball inside a fixed tight container that mechanically pushes against a circular proof mass of a suspended piezoelectric cantilever embedded at the centre of the container above which the ball rolls across. From the experimental frequency analysis, it was found that the rolling ball is responsive to low frequencies and becomes increasingly energetic with increasing frequency over a wide range (1's Hz to ~25 Hz) until a specific higher velocity is attained where the ball abruptly becomes motionless due to the overwhelming coulomb friction.

Therefore, while the system is theoretically independent of frequency, it is actually practically unsuitable for high or even moderate frequency applications where the source displacement amplitude is unlikely to be substantial enough. On the other hand, the suitability towards low frequency and high displacement amplitude potentially align these mechanisms towards applications such as wearables or human/animal motions Pillatsch et al. (2011).

Metamaterial and micro-structural manipulation

As previously mentioned, Arrieta Arrieta et al. (2010) introduced composite material plates and beams with built in bi-stability by leveraging different alignment angles of composite fibres and pre-stress. Similarly, various forms of

nonlinearities can be inherently incorporated into materials and structures through various design and fabrication techniques. This is achieved through a class of artificial materials known as metamaterials that are specifically engineered at a microscopic level to exhibit unusual materials properties, including mechanical, acoustic, optical, electrical, dielectric, electromagnetic and other properties Kshetrimayum (2004).

Metamaterials themselves are not a different category of vibrational nonlinearity, but can be used to intrinsically exhibit nonlinearity. Within the context of kinetic energy harvesting, mechanical property manipulation is often the main focus. However, other properties can also be manipulated to help harvest other domains of energy such as acoustic, solar and electromagnetic Chen et al. (2014). Mechanical metamaterials can include auxetic materials and structures that have negative Poisson's ratio Ren et al. (2018) as well as materials with inherent bistable and self-locking properties derived from origami style cellular structural designs Kamrava et al. (2017).

Auxetics have already been demonstrated to help enhance the power efficiency for harnessing direct contact kinetic energy along with triboelectric Zhang et al. (2017) and piezoelectric Li et al. (2017) generators. However, its effectiveness to enhance more dynamic kinetic energy and its vibrational frequency behaviour are yet to be fully investigated. Thus far, the benefits of auxetics illustrated are essentially that of mechanical displacement and/or force amplifiers similar to the stacked amplifiers for piezoelectric transducers Zhou and Zuo (2013).

Furthermore, Hwang and Arrieta Hwang and Arrieta (2018) Hwang and Arrieta (2019) have proposed a bi-stable lattice design that essentially array microscopic bi-stable lattice cells together, where oscillatory wave of one snap-through state can propagate along and trigger subsequent snap-through states. This creates an inherently bi-stable metamaterial at the constituent lattice cell level and give rise to inherently frequency independent behaviour suitable for harnessing broadband vibration. While the propagating vibration waves can be triggered independent of input frequency, the oscillatory behaviour is still inherently non-resonant in nature. This new bi-stable lattice metamaterial maximises and embodies the advantages of conventional bi-stable structures and can potentially be used to capture low amplitude broadband noise vibration as well.

Solution co-existence and control of high energy orbits

Most nonlinear vibratory principles will have observe bifurcation and co-existence of solutions for a particular set of frequencies. Within the Duffing nonlinear resonant bending regime, there are both low and high energy output branches Daqaq (2010), which can manifest as apparent hysteresis that limit practical implementation. Within the context of bi-stable oscillators, the system with just about sufficient energy to overcome the potential barrier can either enter into the high energy inter-well snap-through states or be trapped within the relatively lower energy intra-well oscillations Daqaq et al. (2014). Furthermore, certain orders of parametric resonance, such as the second order ($n = 2$), is

also co-located in the excitation frequency domain with the fundamental mode direct resonance Jia et al. (2016), where only one of the two potential resonant solutions can manifest at a particular point in time.

Trying to sustain continued operation in high energy orbits within the Poincaré map is non-trivial and many studies have been devoted to understanding and controlling the dynamics of these more desired vibratory regimes. For spring nonlinearities, this includes employing impedance damping modulation from the electrical domain as a means of controlling the oscillatory trajectory in the high energy branch Mallick et al. (2016), using piezoelectric actuation impulses to perturb the oscillator into the high energy orbits Lan et al. (2017) and utilising mechanical impulses from a weakly coupled oscillator Remick et al. (2014) to achieve similar high energy results.

Furthermore, mechanical impacts Zhou et al. (2015) has been used to probabilistically increase the bi-stable snap-through states and a nonlinearly coupled bi-stable Duffing piezomagnetoelastic oscillator Erturk and Inman (2011a) has been shown to expand the orbit size of the phase portraits. In all these approaches, either through active control or careful design with known boundary conditions, high energy orbits can potentially be sustained and maximised.

Udani and Arrieta Udani and Arrieta (2017) proposed an active control strategy for bi-stable systems that results in basins of attraction to promote inter-well snap through states by introducing controlled phase shift perturbations via the primary transducer. Similarly, there have also been passive mechanical designs McInnes et al. (2008); Jia and Seshia (2013) that attempt to perturb the bi-stable potential barrier in order to encourage high energy inter-well oscillations.

While active control schemes will require an investment in energy, the objective is to harness more additional energy from the high energy branches compared to a passive system, taking into account the extra devoted control energy. As with most energy harvesting scenarios, the power profitability of any scheme will depend on the nature and condition of the vibrational source. However, the implementation feasibility and average power output comparisons of these schemes still requirement further assessment in order to practically evaluate the net profitability of using active control to sustain high energy orbits, especially when exposed to a fast varying, random and broadband real world vibrational source.

Discussion on challenges, recommendations and outlook

This paper has surveyed the research, theoretical background and advances in nonlinear vibration energy harvesting (VEH) to-date, from the angle of dynamic vibratory principles. The scope has primarily been limited to the mechanical domain. Many nonlinear vibratory principles have been borrowed from the more established vibration dynamics field. This included Duffing oscillators, bistable systems, parametric oscillators, stochastic resonance, mechanical frequency conversion, physical stoppers, self-tuning mechanisms and non-restoring mechanisms.

The use of nonlinearity in VEH has often been hailed as a means to broaden the operational frequency bandwidth. While this is certainly a key research motivation, various

Table 1. Summary of advantages and disadvantages of various nonlinear vibratory principles explored for vibration energy harvesting.

Vibratory principles	Advantage		Disadvantage	
Duffing oscillator and higher order nonlinear spring	-	Mildly wider bandwidth for band-limited vibrational excitations	-	No fundamental difference over linear oscillators when subjected to broadband noise excitations
	-	Weakly nonlinear oscillators require no additional design complexity	-	Strongly nonlinear oscillators require design complexity
	-	Higher order springs have the theoretical potential to achieve ultra wide-band	-	Hysteresis in frequency domain
Bi-stable oscillator	-	Frequency independent response	-	Non-resonant response
	-	Suitable for low frequency and large amplitude excitations	-	Inherent pre-stress bi-stability involves manufacturing complexity
	-	Suitable to respond to high amplitude broadband noise vibration	-	System induced bi-stability involves design and mechanism complexity
Parametric resonance	-	Provides both higher power and broader bandwidth than linear counterparts	-	Design and simulation complexity
	-	Power and bandwidth can increase simultaneously with lower damping	-	Assembly and tuning complexity to match internal frequency
	-	Auto-parametric oscillators can be semi-resonant towards broadband noise excitations at larger amplitudes	-	Hysteresis in frequency domain
			-	Zero response when Mathieu instability conditions not satisfied
			-	Complex impedance matching required
Stochastic resonance	-	Able to utilise broadband noise as part of the mechanism to output power	-	Design and simulation complexity
	-	Independent of the bandwidth of the noise component of the excitation	-	Assembly and tuning complexity to satisfy Kramers' rate
	-	Suitable for both low and high frequency vibration sources	-	A clear sinusoidal signal is still required, which needs to be matched
			-	Suitable only for large amplitude sources
Frequency conversion	-	Suitable for harvesting low frequency and large amplitude vibration	-	Wear and tear from impact mechanism
	-	Able to more efficiently interface with power conditioning electronics	-	Response is non-resonant in nature and relies on ring down oscillatory decay
End stop effect	-	Broader bandwidth than linear counterpart above a certain amplitude	-	Wear and tear from impact mechanism
	-	Suitable for harvesters that would otherwise provide excessive power	-	Energy/power loss from end stop damping effect
			-	Hysteresis in frequency domain
Self-tuning mechanism	-	Ability to match resonant frequency to ambient frequency within a small range without the need for drive power	-	Wear and tear for sliding mechanism
			-	Limited range for centripetal loading
Non-oscillatory mechanism	-	Frequency independent response	-	Not suitable for moderate to high frequency vibration sources
	-	Suitable for broadband low frequency excitations	-	Wear and tear expected from physical impacts and contacts

nonlinearities have also been employed to address a wider range of issues. Some of the main issues and challenges nonlinear VEH aims to address include,

- narrow and confined operational frequency bandwidth
- low or limited average power and power density
- slow time-varying frequencies, slowly de-tunes frequency-match
- fast time-varying frequencies, short transient states, cannot attain full resonance
- shocks in noise excitations, short decay states, no resonant build up
- broadband noise excitations, cannot achieve resonant amplification
- switching between low and high energy orbits within the Poincaré map

Some of these issues such as time-variation of excitation frequencies and broadband shocks are often neglected by researchers, as sinusoidal excitations are widely used in laboratory characterisations. However, real world vibration are rarely sinusoidal. Even for real world periodic excitations, the spectrum often includes noisy broadband signals. While Duffing has certainly enjoyed most of the fame within the arena of nonlinear VEH, its initial broadband promises have hardly fulfilled the earlier hype. The past decade saw a rapid emergence and evolution of research on these peculiar nonlinear strategies. A brief summary of the advantages and disadvantages for each type of nonlinear vibratory principles are presented in table 1.

Just like most engineering problems, there is rarely a one-size-fit-all solution. Almost always, selection of a most suitable method depends on the application specification and the vibration environment. Table 2 discusses a few

Table 2. General guidance, challenges and recommendations for various vibration scenarios. However, a more nuanced study is needed for any specific application case.

Vibration environment	Application examples	Potentially suitable mechanisms	Challenge comments and recommendation
Single fixed frequency	Fixed frequency motors, e.g. fixed speed compressor and pump motors. Single structures, e.g. bridge.	Auto-parametric, linear or Duffing oscillator	Challenge: to match ambient frequency. If well-matched, **auto-parametric oscillator** can provide the best power.
Single variable frequency	Variable frequency drives (VFD). E.g. VFD pumps and compressors. Motors for electric cars, boats & drones.	Duffing oscillator, End-stop effect, Self-tuning mechanism	Challenge: to follow frequency shift. **Self-tuning mechanism**, if designed to the right band, is most suitable.
Multiple fixed frequency	Fixed frequency motors with higher harmonics and chassis vibration.	Multiple array of linear, Duffing or auto-parametric oscillator. Frequency conversion.	Challenge: to capture all key vibration peaks in the band. **Frequency converter** can help to re-focus peaks.
Multiple variable frequency	Variable frequency drives/motors with higher harmonics.	Multiple array of Duffing oscillator, oscillator with end stop or self-tuning.	Challenge: to follow multiple frequency shifts. Can use **array of multiple self-tuning oscillators** for key peaks.
Multiple closely placed frequencies	Engineering structures and infrastructural assets with truss, tunnels & rail bridges.	Coupled array of linear, Duffing or auto-parametric oscillators. Oscillators with end stop or self-tuning. Frequency conversion.	Challenge: to recover the entire active spectrum. Can use **coupled array of finely tuned oscillators** targeting each frequency.
Narrow band-limited noise excitation (~0.1 kHz)	Engines in cars, motorbikes & small boats. Chassis and engine support structures.	Bi-stable oscillator. Coupled array of linear, Duffing or auto-parametric oscillators. Oscillators with end stop or self-tuning.	Challenge: fast time-varying & short transient state frequencies, difficulty to attain steady resonances. **Bi-stability** is more suitable.
Wide band-limited noise excitation (~1 kHz)	Aero-engine body for aircraft. Rocket body. Engine body for train or ship. Power plants and refineries.	Bi-stable oscillator. Bi-stable oscillator coupled to linear, Duffing or auto-parametric oscillator.	Challenge: same as above. Where amplitude is ample, can use **bi-stability with coupled oscillators**.
Wide band-limited noise excitation with defined frequency peaks	Aero-engines and aero-chassis for passenger planes, jets and helicopters. Engines and chassis for trains, ships or motorsports. Pump or compressor engine.	Stochastic resonator. Bi-stable oscillator. Bi-stable oscillator coupled to linear, Duffing or auto-parametric oscillator. Self-tuning mechanism.	Challenge: same as above. Can use *stochastic driven* **bi-stability coupled oscillators**. With noise, frequency peaks and a threshold, stochastic resonance can be used.
Wide band low-frequency noise/shocks (<< 100 Hz)	Footfall shocks, human or animal motion (few 1's Hz); Traffic induced vibration, car or bus body, wheel shocks (few 10's Hz).	Non-oscillatory mechanism, stochastic resonance, bi-stable oscillator, self-tuning mechanism.	Challenge: large and quick frequency variations, difficult to match or maintain resonance. **Non-oscillatory** is most suitable.
White noise excitation	Seismic noise, large structural bodies.	Bi-stable oscillator coupled to linear, Duffing or auto-parametric oscillator.	Challenge: very broadband and typically low amplitude. Resonance is implausible.

key vibration scenarios and their respective application examples, providing further comments on challenges and design recommendations.

This tabulated design guidance is derived from the experience and opinion of the Author based on this literature survey at this point in time and the Author's previous work on real world vibration assessments Jia et al. (2017); Du et al. (2017). Further future developments in respective nonlinear VEH mechanisms might evolve this design guidance. Also, the vibration environments categorised here are vastly generalised. In reality, real world vibration profiles are much more complex and may not neatly fit into a specific category.

Therefore, a more nuanced and case by case design process is needed by a learned expert in order to determine the most suitable vibratory mechanism to recover kinetic energy for a particular given real world vibration profile.

Furthermore, some mechanisms are easier to design, manufacture and calibrate than others, adding another engineering cost dimension to the overall optimisation process. For instance, given the presence of manufacturing tolerances, it can be more costly to finely calibrate individual auto-parametric oscillators to inherently possess the 2:1 internal frequency ratio compared to the frequency matching of linear oscillators to a given external frequency.

Therefore, even if the vibration environment is a suitable fit, the power density and bandwidth enhancements from potentially employing an auto-parametric oscillator over a linear oscillator should be balanced against the additional engineering cost in determining its worthwhileness.

In recent years, miniature, efficient and low power switch-mode power conditioning circuits Du et al. (2016b,a) have also proven to be significantly more efficient than traditional passive rectification circuits. For voltage amplitude range in excess of 1 voltage, the extra conditioned electrical power additionally recovered compared to that otherwise lost to passive rectification during each cycle (up to several folds in certain cases), typically exceeds the power consumption of these active circuits (1's μW to 10's μW). However, most of these active circuits are designed under sinusoidal or relatively periodic excitations. Some work does exist on assessing switch-mode circuit performance under real world noisy vibration profiles Du et al. (2017), but the harvester is still assumed to be linear in nature. Future work in energy harvesting power conditioning research should also account for mechanically nonlinear harvesters. A key difference in consideration is the relatively unpredictable and potentially sudden time period and range variations.

Nonlinear vibration energy harvesting is still a relatively niche but continuously evolving field, and each individual type of nonlinear phenomenon is as distinct from each other as they are from linear oscillators. While some of the previously promising advantageous attributes have since been fundamentally challenged (such as the broadband nature of Duffing oscillators), others have continued to fuel ongoing research motivations. No single vibratory principle can so far claim to be the ideal and generic solution for vibration energy harvesting.

While linear oscillators still offer the most simplistic design option, employing one or more nonlinear mechanisms can potentially help to address a particular set of technical challenges for specific target applications, depending on the nature of the vibration spectra. Future research could involve the nonlinear coupling and integration of various nonlinear mechanisms. Going forward, the ultimate goal for VEH to maximise power output while also being broadband responsive remains an open challenge for ongoing and future research.

Conclusion

Research into nonlinear vibration energy harvesting (VEH) has witnessed a rapid growth in the past decade. While Duffing is the most widely studied type of nonlinearity fir VEH, the numerous types of nonlinear VEH are as varied from each other as they are from linear systems. Adding onto previous reviews in the literature that focused on either the bandwidth issue or select few nonlinear approaches such as Duffing and bi-stability, this paper has reviewed 8 major types nonlinear vibratory principles and mechanism that have previously been reported in the VEH research field. This includes Duffing oscillators (and higher order nonlinear springs), bi-stability (and multi-stability), parametric resonance, stochastic resonance, mechanical frequency conversion, mechanical end stop effect, self-tuning mechanism and non-oscillatory mechanism. This paper has covered underlying principles, advances in the field, advantages and disadvantages, as well the application-orientated vibration environment suitability discussion for each of these vibratory principles. As often with any engineering systems, there is no one-size-fit all solution and the most appropriate approach almost certainly relies on the environmental and application-specific conditions. Therefore, researchers and designers attempting to develop nonlinear VEH can use the summary overview provided in this paper as an introductory guidance to navigate the varied field of nonlinear VEH.

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The Author declares that there is no conflict of interest.

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