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## **Modelling Crack Propagation in Bituminous Binders under a Rotational Shear Fatigue Load Using Pseudo J-integral Paris' Law<sup>1</sup>**

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**ABSTRACT**

Fatigue resistance of bituminous binders plays a critical role in determining the fatigue performance of asphalt pavements. It was reported in the literature that, under a rotational shear fatigue load like a dynamic shear rheometer (DSR) test, the crack grows in the cylindrical bitumen sample as a circumferential crack that initiates at the periphery of the sample and propagates toward the centre of the sample. This study aims to model this crack propagation in bituminous binders under the rotational shear fatigue load in a time sweep (TS) fatigue test using the DSR. The crack length in the TS test is determined using a damage mechanics-based DSR-C model which is a function of the shear moduli and phase angles under undamaged and damaged conditions. The crack evolution is modelled by a pseudo J-integral based Paris' law. A virgin bitumen 40/60 and a polymer-modified bitumen X-70 under unaged and aged conditions are tested by the TS tests at different temperatures, frequencies and strain levels. Results show that the pseudo J-integral Paris' law is able to accurately predict the crack propagation in bituminous binders under the rotational shear fatigue load. The crack grows faster for the aged bitumen or at lower temperatures. The Paris' law model parameters ( $A$  and  $n$ ) are independent of loading frequency or load amplitude. They are fundamental material properties and can be determined at one loading frequency and amplitude, then can be implemented to predict the crack growth of the bituminous binders at different loading frequencies or amplitudes.

**Keywords:** Bituminous binders, Fatigue, Crack propagation, Paris' law, Pseudo J-integral

## INTRODUCTION

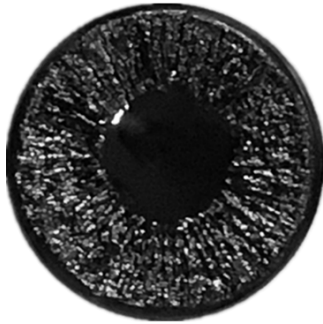
Fatigue property of bituminous binders under repeated loading has a critical impact on the resistance of asphalt mixtures to fatigue cracking. Some researchers have made efforts to characterise the fatigue properties of bituminous binders at intermediate temperatures. In Strategic Highway Research Program (SHRP), a fatigue parameter,  $|G^*| \cdot \sin\delta$ , was proposed to quantify the fatigue resistance of the bituminous binders (1). However, a number of studies have demonstrated that  $|G^*| \cdot \sin\delta$  is incapable of correlating to the asphalt mixture's fatigue life, especially for polymer-modified binders which can sustain a much higher strain before fatigue cracks occur (2-4). To improve the fatigue characterisation of bituminous binders, fatigue parameters based on the dissipated energy concept were proposed to define the fatigue failure, including the dissipated energy ratio (DER) (5-8) and the ratio of dissipated energy change (RDEC) (9-11). While these studies have produced important results for fatigue evaluation, the fatigue mechanisms in bituminous binders, particularly the prediction of fatigue crack evaluation, are not clear yet.

In order to better understand the fatigue mechanisms, the internal damage (i.e., cracking) in bituminous binders under shear fatigue loading was investigated by using photographic techniques (12-14). A cylindrical bituminous binder specimen was prepared and the time sweep (TS) test was performed by the dynamic shear rheometer (DSR). After the test, the upper portion of the sample adhering to the top plate was detached from the lower part of the sample adhering to the bottom plate. The fracture surface of sample was photographed and the imaging analysis was conducted to study the fatigue cracking of bituminous binders. A typical image of fracture surface in a bitumen sample is shown in **Figure 1(a)**. This type of cracking morphology was referred to as 'factory roof' (15). It was found that the fatigue crack of cylindrical bitumen specimens is an 'edge crack' under a rotational shear fatigue load. This edge crack is a circumferential crack, which starts at the periphery of the sample and propagates inward to generate a rough fracture surface with the radial peaks and valleys. The circumferential cracking reduced the effective radius of cylindrical sample. **Figure 1(b)** shows a schematic side view of cylindrical sample with a circumferential crack. Hintz and Bahia (13) predicted the crack length in bituminous binder based on the decrease of the measured torque ( $T$ ) with the number of load cycles during the DSR TS testing. The results showed that the predicted crack length was comparable to the measured values at a small number of load cycle but become greater at a larger number of load cycle. To better predict the crack length in bituminous binders under a rotational shear fatigue load, Zhang and Gao (14) developed a DSR-based cracking (DSR-C) model based on damage mechanics. This DSR-C model was derived using torque and dissipated strain energy (DSE) equilibrium principles. In the DSR-C model, the crack length was predicted using the shear moduli and phase angles of bitumen in the undamaged and damaged conditions, as shown in **Equation 1**. By comparing the DSR-C predictions with the measured crack length using the photographic method, it validates that the developed DSR-C model is able to accurately predict the crack length in bituminous binders during DSR fatigue testing such as a time sweep test.

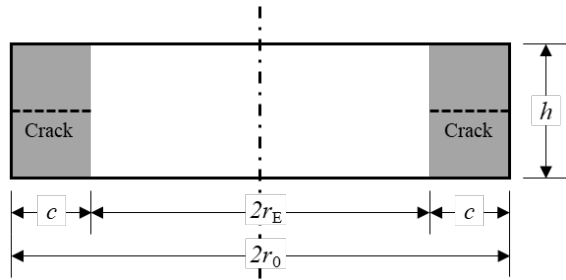
$$c = r_0 - r_E = \left[ 1 - \left( \frac{|G_N^*| / \sin(\delta_N)}{|G_0^*| / \sin(\delta_0)} \right)^{\frac{1}{4}} \right] r_0 \quad (1)$$

where

- $c$  = the crack length in the cylindrical bitumen sample;
- $|G_0^*|$  and  $\delta_0$  = the shear modulus and the phase angle at the undamaged condition, respectively;
- $|G_N^*|$  and  $\delta_N$  = the shear modulus and the phase angle at the Nth load cycle at the damaged condition, respectively; and
- $r_0$  and  $r_E$  = the original radius and the effective (uncracked) radius in the bitumen sample shown in **Figure 1(b)**, respectively.



(a) A ‘factory roof’ morphology of cracking surface



(b) A schematic side view of a cylindrical sample with a circumferential crack

**Figure 1 Fatigue cracking in bituminous binders under a rotational shear fatigue load (14)**

Once the crack length was obtained by the computational models, the crack growth with respect to the number of load cycles can be investigated in bituminous binders under a rotational shear fatigue load. Hintz and Bahia (13) analysed the crack growth and developed a failure criterion for estimating the fatigue resistance of bituminous binders. Nuñez et al. (16) commented that this criterion needs further development to account for the load and strain level associated with the failure crack length. They investigated the crack growth of modified bituminous binders using the finite element model and presented a new criterion to estimate their fatigue characteristics. Shan et al. (17) studied the crack growth in bituminous binders under the controlled-stress mode and controlled-strain mode. These studies on crack growth were all based on the torque predictions of crack length. However, it has been proved that the calculated crack lengths based on the torque prediction were inaccurate when the dynamic modulus ( $G^*$ ) value of the material falls to 10% of its initial value (17). To better understand the crack propagation, there is a need to fundamentally investigate the crack growth rate for the bituminous binders under a rotational shear fatigue load.

The objective of this study is to develop a mechanics-based crack propagation model to predict the crack propagation in bituminous binders under a rotational shear fatigue load. The DSR-C model was employed to calculate the crack length  $c$  in the cylindrical bitumen samples and then the crack growth rate ( $dc/dN$ ) was obtained. The energy release rate ( $\Delta J_R$ ) was derived based on dissipated pseudo-strain energy (DPSE) that is used for crack propagation. The crack evolution was modelled by a pseudo J-integral based Paris' law. One virgin bitumen and one polymer-modified bitumen under unaged and aged conditions are tested by the TS fatigue tests at the different temperatures (15, 20, 25 °C), frequencies (5, 10, 20 Hz) and strain levels (3%, 5%, 7%) using the DSR. The Paris' law coefficients ( $A$  and  $n$ ) are analysed under different temperature, loading frequency and strain level conditions.

## CRACK PROPAGATION MODELS FOR BITUMEN

### Viscoelastic Properties of Bituminous Binders

Bituminous binders are a kind of viscoelastic material. When bitumen is subjected to a rotational shear fatigue load, the stress versus strain curve follows a hysteresis loop during the loading and unloading processes in each cycle. The area inside the loop is the dissipated strain energy (DSE). The DSE in the undamaged viscoelastic state is the energy consumed for viscoelastic relaxation of the materials. When the bitumen sample is damaged, the DSE is the total energy expended for the viscous and damage effects of the materials.

In order to remove the viscous effect of a viscoelastic material, the pseudo-strain was defined in the elastic-viscoelastic correspondence principle by Schapery (18) as below

$$\gamma^R(t) = \frac{1}{G_R} \int_{0^-}^t G(t-s) \frac{d\gamma(s)}{ds} ds \quad (2)$$

where

- $\gamma^R$  = the pseudo-strain;
- $\gamma(s)$  = the measured total strain;
- $s$  = the time before current time  $t$ ;
- $G(t)$  = the shear relaxation modulus; and
- $G_R$  = the shear reference modulus.

Pseudo-strain has been used in the damage characterization of asphalt mixtures (19-22). After introducing pseudo-strain, the stress versus pseudo-strain curve in the linear viscoelastic state becomes a straight line with no enclosed area. In the nonlinear viscoelastic state, the stress versus pseudo-strain curve becomes a loop. The enclosed area in the loop represents the energy consumed for the nonlinearity of bituminous binders, defined as the dissipated pseudo-strain energy (DPSE).

### Dissipated Pseudo-Strain Energy (DPSE) for Crack Growth

To determine the dissipated pseudo-strain energy (DPSE) in the Nth load cycle of bituminous binders, the pseudo-strain at the damaged condition is first further derived. In a strain-controlled rotational shear fatigue test, the shear strain at the damaged condition is expressed as

$$\gamma_d(t) = \gamma_d \sin(\omega t) \quad (3)$$

where

$\gamma_d$  = the shear strain amplitude; and  
 $\omega$  = the loading frequency.

The strain amplitude at a given radial position in a cylindrical bitumen sample is defined as

$$\gamma_d(r) = \frac{\theta_0}{h} \cdot r \text{ with } 0 \leq r \leq r_0 \quad (4)$$

where

$\theta_0$  = the amplitude of rotational angle that is controlled by DSR directly; and  
 $h$  = the height of the cylindrical sample.

Taking into account **Equation 3** in **Equation 2**, the pseudo-strain  $\gamma_d^R(t)$  at the damaged condition in a strain-controlled cyclic torsional test can be then derived as

$$\gamma_d^R(t) = \frac{\gamma_d}{G_R} |G_0^*| \sin(\omega t + \delta_0) \quad (5)$$

As previously explained, the dissipated pseudo-strain energy density  $DPSED_N$  in the Nth load cycle is defined as the energy corresponding to the enclosed area in the loop. The  $DPSED_N$  at a given radial position of the cylindrical bitumen sample at the damaged condition can be calculated as

$$DPSED_N(r) = \int_{(N-1)T}^{NT} \tau_N(t) d\gamma_d^R(t) \quad (6)$$

where

$N$  = the number of loading cycles; and  
 $T = \frac{2\pi}{\omega}$  = the loading period.

Based on the controlled shear strain  $\gamma_d(t)$  shown in **Equation 3**, the measured shear stress at the damaged condition is expressed as

$$\tau_N(t) = \tau_N \sin(\omega t + \delta_N) \quad (7)$$

where

$\tau_N$  = the shear stress amplitude.

According to the linear viscoelastic stress-strain law, the stress amplitude at a given radial position in a cylindrical bitumen sample can be written as

$$\tau_N(r) = |G_N^*| \gamma_d(r) \text{ with } 0 \leq r \leq r_0 \quad (8)$$

Substituting **Equations 5, 7** and **4** into **Equation 6** gives

$$DPSED_N(r) = \pi \frac{|G_0^*|}{G_R} |G_N^*| \left( \frac{r\theta_0}{h} \right)^2 \sin(\delta_N - \delta_0) \quad (9)$$

Thus, the  $DPSED_N$  at a given radial position of the cylindrical bitumen sample under the damaged condition can be determined by **Equation 9**.

To determine the energy consumed in one load cycle due to the crack growth in the bituminous binders, the dissipated pseudo-strain energy  $DPSE_N$  in the Nth load cycle is calculated by integrating the  $DPSED_N$  over the volume in which the material dissipated the energy for cracking growth.

$$DPSE_N = \iiint_{V_{\Delta c}} DPSED_N(r) dV_{\Delta c} \approx DPSED_N(r) \cdot V_{\Delta c} \quad (10)$$

where

$V_{\Delta c}$  = the volume where the material has dissipated energy for the crack increment ( $\Delta c$ ) occurring in the Nth load cycle.

This volume can be expressed as

$$V_{\Delta c} = \pi(r_E + \Delta c)^2 h - \pi(r_E)^2 h = \pi h(\Delta c^2 + 2r_E \Delta c) \quad (11)$$

where

$\Delta c$  = the crack increment in Nth load cycle.

Based on **Equations 9** and **11**, **Equation 10** can be further written as

$$DPSE_N = \pi^2 \frac{|G_0^*|}{G_R} |G_N^*| \frac{(r_E^{N-1} \theta_0)^2}{h} \cdot (\Delta c^2 + 2r_E \Delta c) \cdot \sin(\delta_N - \delta_0) \quad (12)$$

### Crack Propagation Model Based on Pseudo J-integral Paris' Law

Paris' law proposed by Paris and Erdogan (23) is the most widely employed fracture mechanics tool to describe the crack growth in engineering materials due to fatigue. It establishes a power relationship between the crack growth rate and the stress intensity factor, as shown in **Equation 13**.

$$\frac{dc}{dN} = A(\Delta K)^n \quad (13)$$

where

- $c$  = the crack size;
- $dc/dN$  = the crack growth rate;
- $\Delta K$  = the stress intensity factor per load cycle; and
- $A$  and  $n$  = the fracture coefficients.

$K$  quantifies the stress state near the crack tip. Paris' law is normally used under the linear elastic fracture conditions. To characterise crack propagation of the viscoelastic materials, Schapery (18) suggested replacing the stress intensity factor with the pseudo J-integral. The Paris's law is then modified as

$$\frac{dc}{dN} = A(\Delta J_R)^n \quad (14)$$

where

$\Delta J_R$  = the incremental pseudo J-integral defined as the pseudo-strain energy release rate.

The pseudo J-integral quantifies the change rate of the dissipated energy due to the crack growth in the viscoelastic materials and eliminates the viscous effect on the crack growth. The pseudo J-integral Paris' law has been widely used to characterise the fatigue cracking damage of the asphalt mixtures in tension and compression and is able to capture the fracture properties of the materials (20-21, 24-27). The pseudo J-integral Paris' law is also applied in this study to model the crack evolution in bituminous binders under a rotational shear fatigue load. It must be noted that the Paris' law is applicable only for the steady crack propagation in a fatigue test. Thus the pseudo J-integral Paris' law will be used to model the steady crack growth in the propagation stage in this study.

To compute Paris' law coefficients  $A$  and  $n$  in **Equation 14**, the crack length  $c$  and the incremental pseudo J-integral  $\Delta J_R$  should be firstly determined. In this study, the crack length  $c$  is obtained by the DSR-C model shown in **Equation 1**. It is noted that the pseudo J-integral equals the energy release rate resulting from cracking damage. This means the pseudo J-integral represents the rate of the dissipated energy that is solely consumed for crack growth, rather than for other physics such as viscoelastic relaxation or heat loss. The incremental pseudo J-integral  $\Delta J_R$  is defined as the dissipated pseudo-strain energy (DPSE) to create a unit area of crack in the viscoelastic materials, as expressed in **Equation 15**.

$$\Delta J_R = \frac{\partial DPSE}{\partial CSA} = \frac{\partial DPSE / \partial N}{\partial CSA / \partial N} = \frac{DPSE_N}{CSA_N} \quad (15)$$

where

$DPSE$  = the total dissipated pseudo-strain energy which is accumulated from the first load cycle;

$CSA$  = the crack surface areas including the upper and lower crack surfaces that is accumulated from the first load cycle; and

$DPSE_N$  and  $CSA_N$  = the dissipated pseudo-strain energy (defined by **Equation 12**) and the crack surface area, which are the incremental values in the Nth load cycle, respectively.

The  $CSA_N$  is calculated by

$$CSA_N = 2 \left[ \pi (r_E + \Delta c)^2 - \pi (r_E)^2 \right] = 2\pi (\Delta c^2 + 2r_E \Delta c) \quad (16)$$

Substituting **Equations 12** and **16** into **Equation 15**, the energy release rate per unit of crack surface area can be determined by the following **Equation 17**.

$$\Delta J_R = \frac{\pi}{2} \frac{|G_0^*|}{G_R} \left| G_N^* \right| \frac{(r_E^{N-1} \theta)^2}{h} \sin(\delta_N - \delta_0) \quad (17)$$

where the shear reference modulus  $G_R$  is assigned to the shear modulus  $|G_0^*|$  of bituminous binders at the undamaged condition according to a previous study by Zhang et al. (19). Note that



a number of existing studies followed Schapery's suggestion and used a unit (1 Pa) for the reference modulus. However, this has effectively normalized the pseudo strain when modelling the stress-pseudo strain relations, leading to an unclear physical meaning of the pseudo strain. The previous study by the authors (19) proved that, when assigning the modulus at the undamaged conditions (elastic or dynamic modulus) to the reference modulus, the pseudo strain earns a physical meaning which is the difference between the total strain and the viscous strain. By this way, it has enabled the use of pseudo strain in a constitutive modelling of the viscoelastic materials.

Once the crack growth rate ( $dc/dN$ ) and the energy release rate ( $\Delta J_R$ ) were obtained, the crack evolution in bituminous binders under the rotational shear fatigue load can be modelled by the Paris' law using the **Equation 14**. The model parameters  $A$  and  $n$  can also be determined by regressing the data of crack growth rate ( $dc/dN$ ) against the energy release rate ( $\Delta J_R$ ).

## MATERIALS AND LABORATORY TESTS

Two kinds of bituminous binders were employed in this study, including an unmodified binder with 40/60 penetration grade (40/60 bitumen) and a polymer-modified binder with 45/80 penetration grade (X-70 bitumen). Both binders were tested before and after Thin-Film Oven Test (TFOT) laboratory aging. Bitumen samples were prepared using the silicone mould by following the standard method of DSR test (AASHTO T 315). A Kinexus DSR from Malvern Panalytical with an 8 mm diameter parallel plate geometry (2 mm gap) was used to conduct the laboratory tests. The bitumen sample was heated to the testing temperature and remained 5 minutes for stability before it was subjected to the testing load. The frequency sweep (FS) test and the time sweep (TS) test were performed to determine the model parameters and evaluate the crack propagation of bituminous binders. At each testing condition, two replicates were tested and their average value was used for data analysis. One more replicate was required if the deviation between the two measured results was greater than 10%.

In order to obtain the shear modulus ( $|G_0^*|$ ) and the phase angle ( $\delta_0$ ) of the binders at the undamaged condition, FS tests were conducted at temperatures of 15, 20 and 25 °C and frequencies between 1 and 25 Hz under 0.5% shear strain level. TS fatigue damage tests were performed to obtain the shear modulus ( $|G_N^*|$ ) and the phase angle ( $\delta_N$ ) of the binders with the number of load cycles at the damaged condition. The strain levels of 3%, 5% and 7%, corresponding to the rotation amplitudes of 0.015, 0.025 and 0.035 rad, respectively, were selected for the TS fatigue damage tests. Based on the results from Linear Amplitude Sweep (LAS) tests, it was found that the applied strain levels are sufficiently high to generate fatigue cracking damage for all tested samples (14). The three strain levels have been used in the TS tests to study the fatigue damage response of bituminous binders in the literature (28-29). To investigate the effects of temperature and loading frequency on crack evolution, TS tests were performed at different experimental conditions, as shown in **Table 1**.

In the FS and TS tests for unaged X-70 bitumen, aged X-70 bitumen and unaged 40/60 bitumen, the three test temperatures (15, 20 and 25 °C) were selected within the intermediate temperature range at which a fatigue cracking commonly happens. For aged 40/60 bitumen, the test temperatures of 20, 25 and 30 °C were used due to its high stiffness (shear modulus). The three test frequencies (5, 10 and 15 Hz) were employed to produce fatigue cracking within the reasonable load cycles.

**TABLE 1 Laboratory Test Plan**

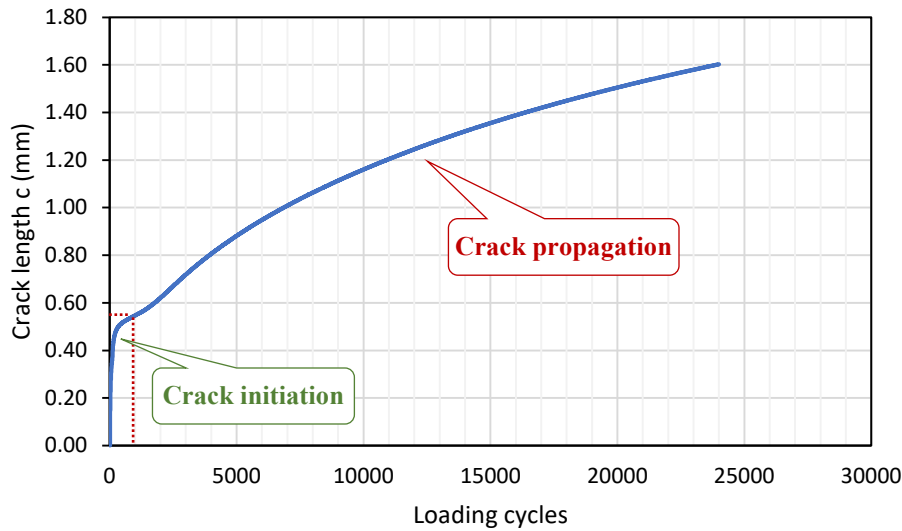
Tests	Bituminous binders	Temperature (°C)	Frequency (Hz)	Strain levels	Load cycles
Frequency Sweep	Unaged X-70 Unaged 40/60 Aged X-70	15, 20, 25	1-25	0.5%	NA
	Aged 40/60	20, 25, 30			
Time Sweep	Unaged X-70 Unaged 40/60 Aged X-70	15, 20, 25	5, 10, 20	3%, 5%, 7%	24000
	Aged 40/60	20, 25, 30			

## RESULTS AND DISCUSSIONS

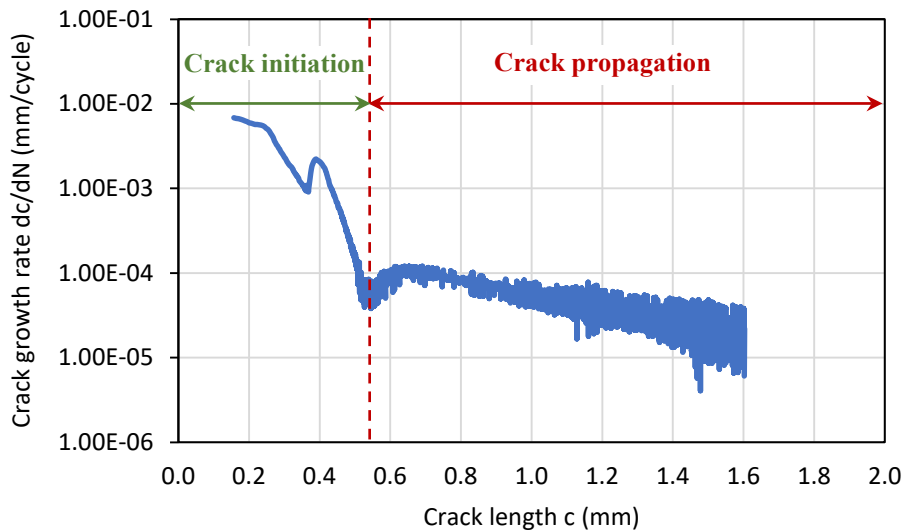
### Analysis of Crack Growth Stages

Crack growth in bitumen under a rotational shear fatigue load demonstrates different cracking stages (14). Using the DSR-C model in **Equation 1**, the crack length was calculated and the crack growth stages were analysed. A typical crack length curve with the number of load cycles is shown in **Figure 2(a)**. Based on the calculated crack length, the crack growth rate ( $dc/dN$ ) is depicted in **Figure 2(b)**. It can be seen that during the initial portion of crack growth (before the 1000<sup>th</sup> load cycles or the crack length of 0.55 mm), the crack length grows sharply (**Figure 2(a)**) with a significant fluctuation in the crack growth rate (**Figure 2(b)**). This initial unsteady portion is defined as the crack initiation stage. After its lowest point, the crack growth rate slightly increased and then steadily decreased with increasing crack length, which is defined as the crack propagation stage. Therefore, the crack growth is divided into crack initiation and crack propagation, as shown in **Figures 2(a)** and **(b)**.

Crack initiation is a short and unsteady cracking stage due to complex geometry dependence of initiation. At this stage, different mechanisms are used to explain the cracking within the bituminous binder depending on the length scale. When the external cyclic load is applied, the bitumen shows a rapid change in material properties (modulus and phase angle). At a molecular scale, such phenomenon is attributed to the rearrangement of different molecular species or components in the material (30,31), which is commonly referred to as the thixotropic effect. After this molecular reorganization, at a slightly larger length scale, separate and unconnected microcracks occur and grow at the edge of cylindrical bitumen sample. Once these microcracks are interconnected, a circumferential crack can be generated and then propagates steadily toward the centre of the sample.



(a) Crack length (obtained by DSR-C model using frequency sweep and time sweep test results) vs. the number of load cycles



(b) Crack growth rate vs. crack length indicating different stages of crack initiation and crack propagation

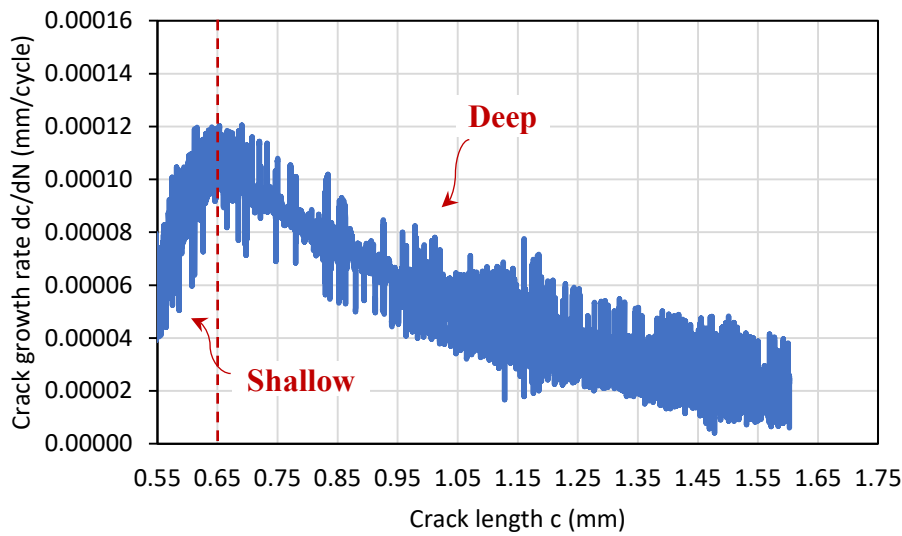
Figure 2 A typical crack growth trend in a cylindrical bitumen sample under a rotational shear fatigue load

During the crack propagation, it is found from **Figure 3(a)** that the crack growth rate increases as crack length increases, followed by a decrease in crack growth rate with increasing crack length. The crack propagation trend was referred to as ‘shallow’ and ‘deep’ crack growth (32). The shallow crack growth is driven by the strain energy from the material in the immediate vicinity of the crack. As the crack propagates inward, the more volume of material near the sample edge starts to supply the energy, resulting in an increase in the crack growth rate. However, this energy source would only last a short time and then be limited by the physical top

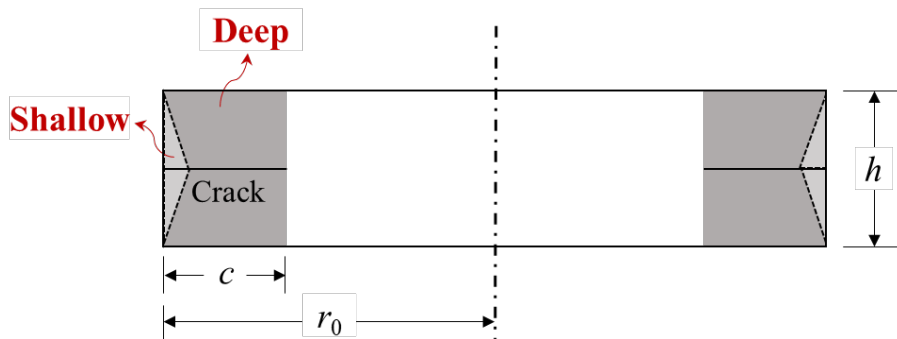
and bottom boundaries of the sample (e.g., the top and bottom plates in the DSR test), as shown in **Figure 3(b)**. At this point, the crack propagation becomes the deep crack growth, which is driven by the strain energy from the material in the interior of the sample. According to the strain distribution in the sample, the amount of energy available decreases and consequently crack growth rate decreases with the increasing crack length. The deep crack growth is observed as a steady cracking stage during the crack propagation.

In summary, the crack growth can be regarded as a process including (1) crack initiation that initiates in the form of molecular reorganization followed by the formation and interconnection of microcracks at the edge of the sample and (2) crack propagation that propagates in the form of a circumferential crack with the ‘shallow’ and ‘deep’ crack growth.

Based on the above crack growth analysis, the crack propagation model shown in **Equation 14** based on pseudo J-integral Paris’ law is used during the deep crack growth in this study as the Paris’ law is only applicable to stable crack growth (33).



(a) Crack growth rate vs. crack length during crack propagation

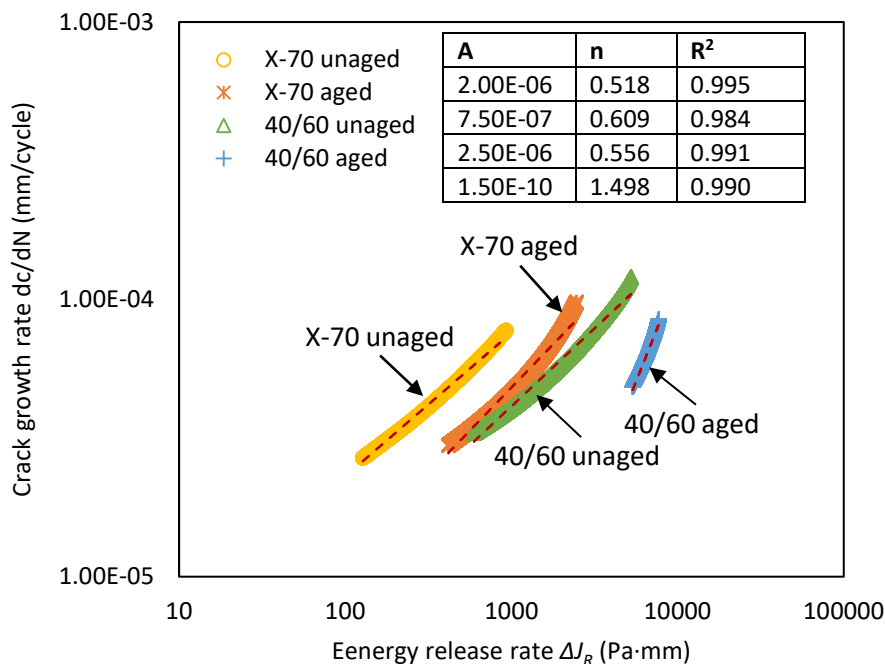


(b) A schematic side view of a cylindrical sample during crack propagation

Figure 3 Crack propagation including ‘shallow’ crack growth and ‘deep’ crack growth

Paris’ Law Coefficients  $A$  and  $n$

Crack growth rate  $dc/dN$  as a function of energy release rate  $\Delta J_R$  during the deep (steady) crack propagation is plotted in **Figure 4** for the four binders tested at 20 °C and 10 Hz under 5% strain level. **Figure 4** demonstrates that crack growth in bituminous binders under the rotational shear fatigue load generally follows a power law relationship between crack growth rate and energy release rate. Based on this result, the developed crack propagation model is used to fit the  $dc/dN$ - $\Delta J_R$  curve from the time sweep (TS) testing to determine the Paris' law coefficients  $A$  and  $n$ , which are also shown in **Figure 4** for the four tested binders. It can be seen that high coefficients of determination were obtained for all four binder ( $R^2 > 0.98$ ), proving that the crack propagation model based on pseudo J-integral Paris' law is able to accurately characterise the crack evolution in different bituminous binders under a rotational shear fatigue load.



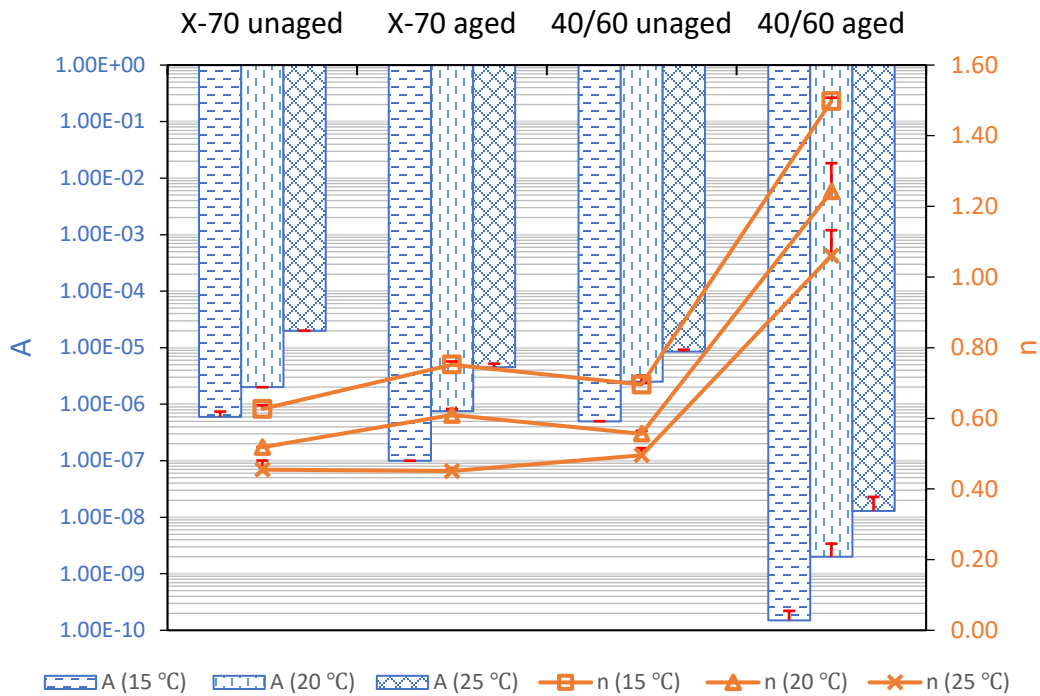
**Figure 4 Relationship between crack growth rate ( $dc/dN$ ) and energy release rate ( $\Delta J_R$ ) during the deep (steady) crack growth at 20 °C and 10 Hz under 5% strain level**

**Figure 5** shows the Paris' law coefficients  $A$  and  $n$  for the four different binders determined from the time sweep (TS) tests at the different temperatures and aging conditions. It is found from **Figures 5** that, when the bituminous binders become stiffer due to laboratory aging, the Paris' law coefficient  $A$  decreases and the Paris' law exponent  $n$  increases. Compared to the polymer modified X-70, the unmodified 40/60 bitumen introduces bigger changes in the Paris' law coefficients. For instance, when the binders became aged, the  $n$  value increases by 169.17% for the unmodified 40/60 bitumen but increases by only 17.56% for the polymer modified X-70 at all testing temperatures. This indicates that the aging has a more significant impact on the crack evolution of the unmodified bitumen than that of the polymer modified bitumen.

More importantly, **Figure 5** demonstrates that the temperature has a significant effect on the Paris' law coefficients  $A$  and  $n$  for the four types of bituminous binders. The Paris' law

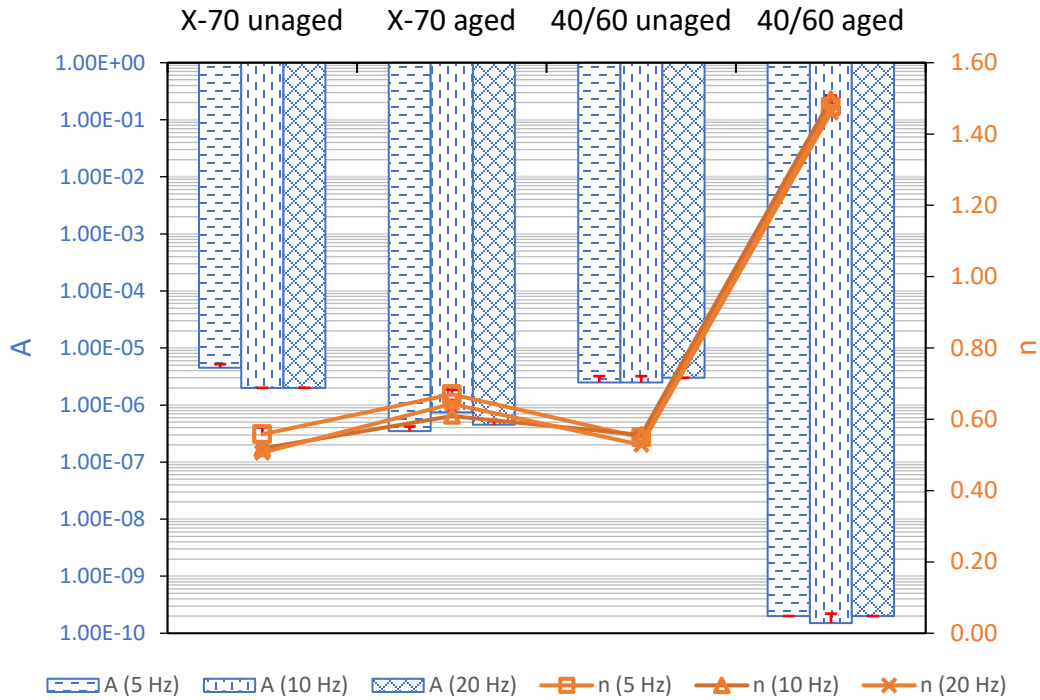
coefficient  $A$  increases and the Paris' law exponent  $n$  decreases with an increasing temperature due to the viscoelastic nature of the bituminous binders. This phenomenon has also been found in asphalt mixtures, as reported in the literature (34,35).

In conclusion, a stiffer bituminous binder due to laboratory aging or decreasing temperature tends to have a smaller Paris' law coefficient  $A$  and a larger Paris' law exponent  $n$ . A higher  $n$  value leads to a quicker crack propagation. This means the crack grows faster for the aged bitumen or at lower temperatures.

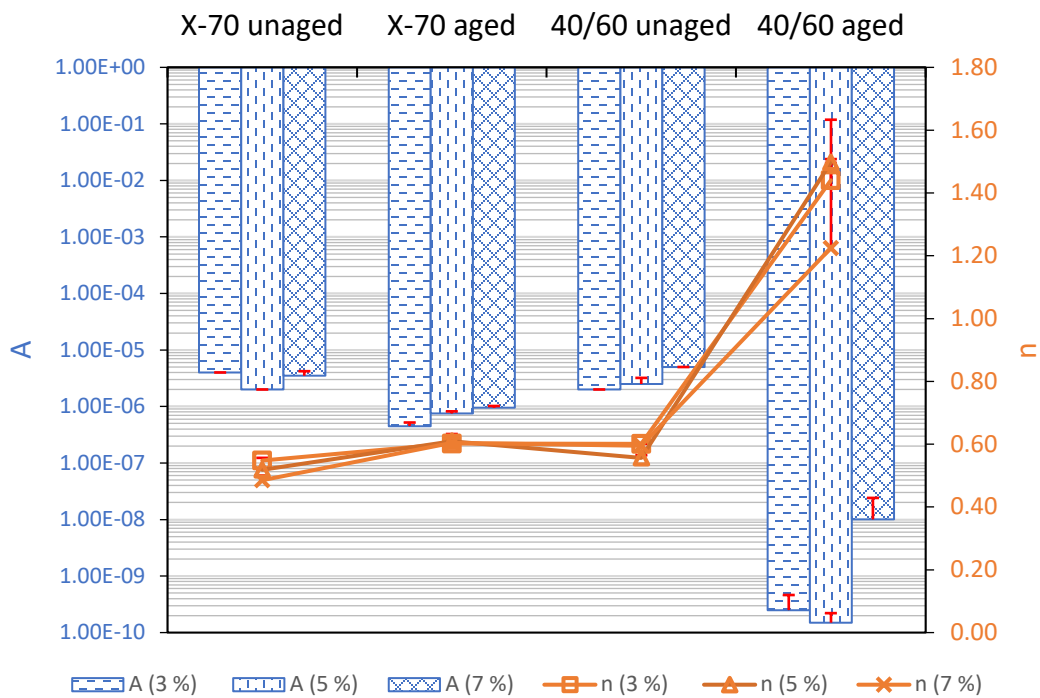


**Figure 5 Paris' law coefficients at different temperatures and aging conditions (@ 10 Hz and 5% strain level)**

**Figure 6** presents the Paris' law coefficients  $A$  and  $n$  of all binders at different loading frequencies and strain levels. **Figure 6** indicates that the Paris' law coefficients  $A$  and  $n$  of a specific bituminous binder do not vary significantly with either loading frequencies or loading amplitudes (strain levels). Therefore, it can be concluded that the Paris' law coefficients  $A$  and  $n$  are fundamental material parameters and independent of loading frequency or loading amplitude. The independence of the Paris' law coefficients of loading rate or loading mode has been verified for asphalt mixtures (26). Thus the Paris' law coefficients  $A$  and  $n$  can be determined at one loading frequency and amplitude, then be implemented to predict the crack growth of the bituminous binders at different loading frequencies or amplitudes.



(a) Paris' law coefficients at different loading frequencies (@ 20 °C and 5% strain level)



(b) Paris' law coefficients at different strain levels (@ 20 °C and 10 Hz)

Figure 6 Paris' law coefficients at different loading frequencies and strain levels for different binders

It should be noted that the variability of the results was also analysed in this study using two replicates at each testing condition. The standard deviation for all the tested samples under different testing conditions was shown in **Figures 5 and 6** by red error bar. It can be seen that the standard deviation for Paris' law coefficients  $A$  and  $n$  is very low, which indicates that the variability of the results is much less significant. To avoid high variability, the testing conditions shown in the section of MATERIALS AND LABORATORY TESTS were recommended. These conditions have been tested and successfully used in the authors' previous study (14).

## SUMMARY AND CONCLUSIONS

Crack propagation in bituminous binders under a rotational shear fatigue load was investigated by a time sweep (TS) fatigue test using the DSR. Using a damage mechanics-based DSR-C model, the crack length in the bituminous binders is derived as a function of the shear moduli and phase angles under undamaged and damaged conditions. The crack evolution is modelled by a pseudo J-integral based Paris' law. TS tests were conducted for the virgin bitumen 40/60 and the polymer-modified bitumen X-70 before and after laboratory aging. These fatigue tests were performed at different temperatures, loading frequencies and loading amplitudes. Conclusions were drawn from the research as below:

- The crack propagation model based on pseudo J-integral Paris' law is able to accurately characterise the crack propagation in bituminous binders under a rotational shear fatigue load.
- A stiffer bituminous binder due to laboratory aging or decreasing temperature tends to have a smaller Paris' law coefficient  $A$  and a larger Paris' law exponent  $n$ . This proves that crack grows faster for the aged bitumen or at lower temperatures.
- Aging has a more significant impact on the crack propagation of the unmodified bitumen than that of the polymer modified bitumen.
- The Paris' law coefficients  $A$  and  $n$  are expected to be independent of loading frequency or loading amplitude, which indicate that they are fundamental material properties.

Based on the above conclusions, the Paris' law coefficients  $A$  and  $n$  can be determined at one loading frequency and amplitude, then be implemented to predict the crack growth of the bituminous binders at different loading frequencies or amplitudes. In future study, more bituminous binders from different sources will be tested to further validate the developed model. The Paris' law coefficients  $A$  and  $n$  will serve as a direct evaluation parameter of the fatigue resistance to quantify and differentiate the fatigue performances of various bituminous binders.

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## AUTHOR CONTRIBUTIONS

The authors confirm the contribution to the paper as follows: study conception and design: Yuqing Zhang; data collection: Yangming Gao; analysis and interpretation of results: Yangming Gao, Linglin Li, and Yuqing Zhang; draft manuscript preparation: Yangming Gao. All authors reviewed the results and approved the final version of the manuscript.



## REFERENCES

1. Anderson, D. A., and T. Kennedy. Development of SHRP Binder Specification. *Journal of Association of Asphalt Paving Technologists*, 1993. 62: 481-507.
2. Bahia, H. U., H. Zhai, M. Zeng, Y. Hu, and P. Turner. Development of Binder Specification Parameters Based on Characterization of Damage Behavior. *Journal of Association of Asphalt Paving Technologists*, 2001. 70: 442-470.
3. Planche, J. P., D. A. Anderson, G. Gauthier, Y. M. L. Hir, and D. Martin. Evaluation of Fatigue Properties of Bituminous Binders. *Materials and Structures*, 2004. 37: 356-359.
4. Zhou, F., W. Mogawer, H. Li, A. Andriescu, and A. Copeland. Evaluation of Fatigue Tests for Characterizing Asphalt Binders. *ASCE Journal of Materials in Civil Engineering*, 2013. 25: 610-617.
5. Ghuzlan, K., and S. Carpenter. Energy-Derived, Damage-Based Failure Criterion for Fatigue Testing. *Transportation Research Record: Journal of the Transportation Research Board*, 2000. 1723: 141-149.
6. Anderson, D. A., Y. Hir, M. Marasteanu, J. P. Planche, D. Martin, and G. Gauthier. Evaluation of Fatigue Criteria for Asphalt Binders. *Transportation Research Record: Journal of the Transportation Research Board*, 2001. 1766: 48-56.
7. Bonnetti, K.S., K. Nam, and H. U. Bahia. Measuring and Defining Fatigue Behavior of Asphalt Binders. *Transportation Research Record: Journal of the Transportation Research Board*, 2002. 1810:33-43.
8. Martono, W., H. U. Bahia, and J. D. Angelo. Effect of Testing Geometry on Measuring Fatigue of Asphalt. *ASCE Journal of Materials in Civil Engineering*, 2007.19: 746-752.
9. Shen, S., and S. Carpenter. Application of the Dissipated Energy Concept in Fatigue Endurance Limit Testing. *Transportation Research Record: Journal of the Transportation Research Board*, 2005.1929: 165-173.
10. Shen, S., G. D. Airey, S. H. Carpenter, and H. Huang. A Dissipated Energy Approach to Fatigue Evaluation. *Road Materials and Pavement Design*, 2006. 7 (1): 47-69.
11. Shen, S., H. M. Chiu, and H. Huang. Characterization of Fatigue and Healing in Asphalt Binders. *ASCE Journal of Materials in Civil Engineering*, 2010. 22(9): 846-852.
12. Tan, Y., L. Shan, Y. R. Kim, and B. S. Underwood. Healing Characteristics of Asphalt Binder. *Construction and Building Materials*, 2012. 27: 570-577.
13. Hintz, C., and H. Bahia. Understanding Mechanisms Leading to Asphalt Binder Fatigue in the Dynamic Shear Rheometer. *Road Materials and Pavement Design*, 2013. 14: 231-251.

14. Zhang, Y., and Y. Gao. Predicting Crack Growth in Viscoelastic Bitumen under a Rotational Shear Fatigue Load. *Road Materials and Pavement Design*, 2019. DOI:10.1080/14680629.2019.1635516
15. Tschegg, E., R. Ritchie, and F. McClintock. On the Influence of Rubbing Fracture Surface on Fatigue Crack Propagation in Mode III. *International Journal of Fatigue*, 1983. 5(1): 29-35.
16. Nuñez, J. Y. M., E. D. Leonel, and A. L. Faxina. Fatigue Characteristics of Modified Asphalt Binders Using Fracture Mechanics. *Engineering Fracture Mechanics*, 2016. 154: 1-11.
17. Shan, L., S. Tian, H. He, and N. Ren. Internal Crack Growth of Asphalt Binders During Shear Fatigue Process. *Fuel*, 2017.189: 293-300.
18. Schapery, R. A. Correspondence Principles and a Generalized J Integral for Large Deformation and Fracture Analysis of Viscoelastic Media. *International Journal of Fracture*, 1984. 25(3): 195-223.
19. Zhang, Y., R. Luo, and R. L. Lytton. Characterizing Permanent Deformation and Fracture of Asphalt Mixtures by Using Compressive Dynamic Modulus Tests. *Journal of Materials in Civil Engineering*, 2012. 24(7): 898-906. DOI:10.1061/(ASCE)MT.1943-5533.0000471
20. Zhang, Y., R. Luo, and R. L. Lytton. Mechanistic Modelling of Fracture in Asphalt Mixtures under Compressive Loading. *Journal of Materials in Civil Engineering*, 2013. 25(9): 1189-1197. DOI:10.1061/(asce)mt.1943-5533.0000667
21. Zhang, Y., R. Luo, and R. L. Lytton. Anisotropic Characterization of Crack Growth in the Tertiary Flow of Asphalt Mixtures in Compression. *Journal of Engineering Mechanics*, 2014. 140(6) 04014032. DOI:10.1061/(ASCE)EM.1943-7889.0000745
22. Zhang, Y., X. Luo, R. Luo, and R. L. Lytton. Crack Initiation in Asphalt Mixtures under External Compressive Loads. *Construction and Building Materials*, 2014. 72: 94-103.
23. Paris, P. C., and F. Erdogan. A Critical Analysis of Crack Propagation Laws. *Journal of Basic Engineering*, 1963. 85(4): 528-533.
24. Luo, X., R. Luo, and R. L. Lytton. Modified Paris's Law to Predict Entire Crack Growth in Asphalt Mixtures. *Transportation Research Record: Journal of the Transportation Research Board*, 2013. 2373 (1): 54-62.
25. Luo, X., Y. Zhang, and R. L. Lytton. Implementation of pseudo J-integral based Paris' law for fatigue cracking in asphalt mixtures and pavements. *Materials and Structures*, 2016. 49(9): 3713-3732.
26. Zhang, Y., F. Gu, B. Birgisson, and R. L. Lytton. Modelling Cracking Damage of Asphalt Mixtures under Compressive Monotonic and Repeated Loads Using Pseudo J-Integral Paris' Law. *Road Materials and Pavement Design*, 2017. DOI:10.1080/14680629.2018.1418706

27. Lytton, R. L., Y. Zhang, F. Gu, and X. Luo. Characteristics of Damaged Asphalt Mixtures in Tension and Compression. *International Journal of Pavement Engineering*, 2018.19 (3): 292-306.
28. Wang, C., C. Castorena, J. Zhang, and Y. R. Kim. Unified Failure Criterion for Asphalt Binder under Cyclic Fatigue Loading. *Road Materials and Pavement Design*, 2015. 16(sup2): 125-148. DOI:10.1080/14680629.2015.1077010
29. Wang, C., Z. Han, C. Castorena, J. Zhang, and Y. R. Kim. Identifying Fatigue Failure in Asphalt Binder Time Sweep Tests. *Construction and Building Materials*, 2016. 121: 535-546.
30. Schmets, A.J.M., N. Kringos, A. Scarpas, C. Duif, G. Schitter, and T. Pauli. First-Principles Investigation of the Multiple Phases in Bituminous Materials: the Case of Asphaltene Stacking. *Advanced Testing and Characterization of Bituminous Materials*, 2009. 1: 143-150.
31. Jahangir, R., D. Little, and A. Bhasin Evolution of Asphalt Binder Microstructure Due to Tensile Loading Determined Using AFM and Image Analysis Techniques. *International Journal of Pavement Engineering*, 2015. 16(4): 337-349. DOI: 10.1080/10298436.2014.942863
32. Gent, A. N., and O. H. Yeoh. Crack Growth in Twisted Rubber Disks. Part 3. Effects of Crack Depth and Location. *Rubber Chemistry and Technology*, 2003. 76(5): 91-107.
33. Anderson, T.L. *Fracture Mechanics: Fundamentals and Applications*. Boca Raton, FL: CRC Press, 2005.
34. Jacobs, M. M. J. Crack Growth in Asphaltic Mixes. TU Delft: Delft University of Technology, 1995.
35. Zhou, F., S. Hu, T. Scullion, D.H. Chen, X. Qi, and G. Claros. Development and Verification of the Overlay Tester Based Fatigue Cracking Prediction Approach (With Discussion). *Journal of the Association of Asphalt Paving Technologists*, 2007. 76: 627-662.