A Preference-guided Multiobjective Evolutionary Algorithm based on Decomposition

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Abstract. *Multiobjective evolutionary algorithms based on decomposition (MOEA/Ds) represent a class of widely employed problem solvers for multi-criteria optimization problems. In this work we investigate the adaptation of these methods for incorporating preference information prior to the optimization, so that the search process can be biased towards a Pareto-optimal region that better satisfies the aspirations of a decision-making entity. The incorporation of the Preference-based Adaptive Region-of-interest (PAR) framework into the MOEA/D requires only the modification of the reference points used within the scalarization function, which in principle allows a straightforward use in more sophisticated versions of the base algorithm. Experimental results using the UF benchmark set suggest gains in diversity within the region of interest, without significant losses in convergence.*

1. Introduction

Decomposition-based algorithms represent a widely used class of problem solvers for multiobjective optimization problems (MOPs). In particular, methods based on the Multiobjective Evolutionary Algorithm based on Decomposition (MOEA/D) framework have been adopted for a variety of applications since its introduction in 2007 [Zhang and Li 2007], as evidenced by a recent comprehensive survey of the literature [Trivedi et al. 2016].[1]

Similarly to dominance and indicator-based approaches, MOEA/Ds have been extensively developed and used in the context of *a posteriori* techniques to the solution of MOPs. This approach consists of first trying to obtain an extensive coverage of the Pareto-optimal front - which is generally seen as converging to a Pareto-optimal set of points homogeneously distributed in the space of objectives - and then delivering this set of options to a decision-making entity, which is tasked with the function of selecting a single solution for implementation.

Recent works have called this *a posteriori* approach into question, pointing out that in many cases one is not interested in actually mapping out the full Pareto front in detail, which can lead to wasteful optimization approaches that spend most of their computational budgets refining solutions that are clearly of no interest to the end user.

[1] Throughout this text we use the term “MOEA/Ds” to refer to the class of algorithms based on decomposition for multiobjective optimization, i.e., the original MOEA/D and its many variants found in the literature.

Goulart and Campelo [Goulart and Campelo 2016] summarize this position (in the context of many-objective optimization), by declaring that “finding Pareto-optimal points does not necessarily mean that one has solved the practical many-objective optimization problem” [Goulart and Campelo 2016]. Indeed, even discounting the computational budget issue, it can be argued that returning hundreds of Pareto-optimal points for a decision-making entity to sort out and deal with poses an unnecessarily complex problem, which can be addressed by incorporating the preferences within the multiobjective optimization algorithm itself.

This work presents an adaptation of the approach originally proposed by Goulart and Campelo [Goulart and Campelo 2016] for generating preference-guided multiobjective evolutionary algorithms. In that work, the authors presented a framework for incorporating information regarding the stated preferences of a decision-making entity in the form of a reference point, but only in the context of dominance- and indicator-based algorithms. In the present work we extend those concepts to decomposition-based algorithms, which have been shown to perform well in the very classes of problems for which preference incorporation seems to be most critical [Goulart and Campelo 2016]: many-objective optimization problems [Asafuddoula et al. 2015, Li et al. 2015] and applied engineering problems (where preference information is often available) [Trivedi et al. 2016].

The remainder of this paper is organized as follows: Section 2 provides a quick introduction to the MOEA/D, and Section 3 reviews a few works dealing with preference incorporation within the MOEA/D structure. Section 4 discusses the concepts of the Preference-based Adaptive Region-of-interest (PAR) framework. The proposed PAR-MOEA/D is described in Section 5, and the results of computational experiments are presented in 6. Finally, Section 7 closes the paper with discussions, conclusions and perspectives.

2. Multiobjective Evolutionary Algorithms based on Decomposition

For the purposes of this work we deal with continuous MOPs subject only to box constraints, as defined as in (1):

$$\begin{aligned}&\min_{\mathbf{x}}\ \mathbf{f}\left(\mathbf{x}\right) = \big(f_1(\mathbf{x}), \ldots, f_{n_f}(\mathbf{x})\big)\\ &\text{subject to: } \mathbf{x} \in \mathbb{R}^{n_v} \mid x_i^{min} \leq x_i \leq x_i^{max},\ \forall i \in \{1, \ldots, n_v\}\end{aligned} \tag{1}$$

where n_f is the number of objectives, n_v is the number of decision variables, $\mathbf{x} \in \mathbb{R}^{n_v}$ represents a candidate solution, $\mathbf{f}(\cdot) : \mathbb{R}^{n_v} \mapsto \mathbb{R}^{n_f}$ is a vector of objective functions, and the feasible decision space (Ω) is defined by the real space $\mathbb{R}$ bound by x_i^{min} and x_i^{max}, the lower and upper limits allowed for each variable. The image of the set Ω, $\mathbf{f}(\Omega)$, defines the set of attainable objective values [Miettinen 1999, Hidden and Hidden 2017].

MOEA/Ds decompose a MOP into a finite number of scalar optimization subproblems, each of which is defined by a weight vector and a scalar aggregation function. As discussed in [Hidden and Hidden 2017], a MOEA/D can be fully characterized by the design choices made for each of its components. For the algorithm considered in this work, these choices are:

- Decomposition strategy (which generates the weight vectors $\boldsymbol{\lambda}_i$ used for defining each subproblem): simplex-lattice design (SLD) [Chan 2000, Zhang and Li 2007];

- Scalar aggregation function (which provides the scalar subproblems to be minimized): as defined in Section 5;
- Neighborhood strategy (which regulates the locality of the exchange of information between subproblems): fixed neighborhood based on the distances between the weight vectors [Zhang and Li 2007];
- Objective function scaling: simple scaling [Hidden and Hidden 2017];
- Variation operators: differential mutation, binomial recombination, and polynomial mutation [Li and Zhang 2009];
- Update strategy (which selects which candidate solutions will compose the population at the end of each iteration): standard greedy selection [Zhang and Li 2007];
- Stop criterion: number of candidate solution evaluations.

The general structure of the MOEA/D is provided in Algorithm 1 [Hidden and Hidden 2017]. For more information on each module of the algorithm please refer to the references provided, particularly the original MOEA/D [Zhang and Li 2007] and the MOEA/D-DE [Li and Zhang 2009]. Details on the scalarizing functions will be given in Section 5.

Algorithm 1 MOEA/D structure

```
$t \leftarrow 0$
Generate weights $\boldsymbol{\lambda}_i$
Generate initial population $\mathbf{X}^{(t)}$
Define neighborhoods $\boldsymbol{b}_i$
while $t \leq t_{max}$ do
    $\mathbf{X}'^{(t)} \leftarrow$ DifferentialMutation$\left(\mathbf{X}^{(t)} \mid F\right)$
    $\mathbf{X}'^{(t)} \leftarrow$ BinomialRecombination$\left(\mathbf{X}'^{(t)} \mid CR\right)$
    $\mathbf{X}'^{(t)} \leftarrow$ PolynomialMutation$\left(\mathbf{X}'^{(t)} \mid \eta_M, p_M\right)$
    Evaluate solutions in $\mathbf{X}^{(t)}$ and $\mathbf{X}'^{(t)}$ using scalarizing function;
    Update population using standard greedy selection;
    $t \leftarrow t + 1$
end while
```

3. Preference-based MOEA/Ds

The incorporation of preference information into the structure of multiobjective evolutionary algorithms has been explored for some time (an extensive review of dominance- and indicator-based methods can be found in the references provided [Goulart and Campelo 2016, Goulart 2015]). In the present work three works incorporating preferences within the specific MOEA/D framework are briefly discussed.

The first reference dealing with this particular aspect in the context of decomposition-based approaches dates from 2011, in the form of a paper by Gong *et al.* [Gong et al. 2011]. That work proposes an interactive version of the MOEA/D, in which a human decision-maker (DM) is required, once every few iterations, to select its most preferred candidate solution from a subset of the current population. The main drawback of this approach is the need for iteratively querying the decision maker, which puts a heavy burden on the human operator. This can also make solutions subject to priming or anchoring effects [Tversky and Kahneman 1975], wherein the judgment of the DM becomes subjectively biased by the solutions presented, and deviates from her rational utility model.

A second strategy was presented by Mohammadi, Omidvar and Li [Mohammadi et al. 2012], employing the concept of predefined reference points to bias the search towards template solutions provided by the DM. Their approach starts by running a common MOEA/D for a few iterations, and then transitions to a preference-guided scheme in which the weight vectors associated with the closest solution to each reference point are determined, and a new set of weight vectors is generated around each of these vectors. More recent versions of this method [Mohammadi et al. 2014] maintain the same general idea. While this eliminates the DM from the algorithmic loop, the method still relies on several additional external parameters that must be provided by the user - in general, it more than doubles the number of required user-defined parameters.

Finally, a third method was presented in 2015 by Pilát and Neruda [Pilát and Neruda 2015], which incorporates preferences via a coevolutionary approach in which the weights undergo an iterative refinement similar to that of the population. Preferences are initially expressed in the form of a binary judgment associated with each candidate solution as either preferred or non-preferred. The preference value associated to each point is then refined, and the preference function that emerges from this process is then used at each iteration as a quality value to compare the weight vectors, which are subject to a mutation-based variation. Despite presenting very interesting results for preference-guided multiobjective optimization, this approach also suffers from the same problem of an explosion in the number of free parameters. The algorithm structure also seems overly complex, which may possibly be an unavoidable price for incorporating arbitrarily complex preference structures within the framework of a multiobjective optimization approach, but can also be a result of the common practice of assembling an algorithm without much regard to the contribution of individual components, as suggested in the literature on component-based design and analysis of algorithms [Bezerra et al. 2016, Bezerra et al. 2015, Hidden and Hidden 2017].

Regardless, we argue that more parsimonious approaches are possible in the case of simple preference structures, which emerge in several engineering problems. The PAR framework [Goulart and Campelo 2016] represents a straightforward way to incorporate preferences within a MOEA, without the need for interactive querying or the addition of new parameters. While its current definition accepts only a single reference point as a template solution to express the preferences of the DM, its simplicity and easy incorporation into dominance- and indicator-based MOEAs motivated the current proposal of using it within the MOEA/D structure. The following section describes the adaptation of the MOEA/D for using the PAR framework for incorporating preferences into its optimization procedure.

4. The PAR framework

Assume the DM possesses aspiration levels z_i^r for the i-th objective, which collectively compose a *preference point*[2] $\mathbf{z}^r \in \mathbb{R}^{n_f}$. A straightforward way of satisfying the DM is

[2] The original term is *reference point* [Wierzbicki 1982]. However, we opted for *preference point* to prevent confusions with the actual reference point employed by the MOEA/D to generate the subproblems, and to emphasize its character of representing the DM's preferences.

by computing a solution that is closest to $\mathbf{z}^r$ according to some indicator, as performed in techniques such as goal programming and weighted metrics [Miettinen 1999]. However, minimizing the distance to $\mathbf{z}^r$ by means of a regular norm does not guarantee Pareto-optimality [Wierzbicki 1982], which motivated the use of *achievement scalarizing functions* (ASFs) [Wierzbicki 1982], such as the *augmented Tchebycheff function*:

$$s(\mathbf{f}'(\mathbf{x}), \mathbf{z}^{r\prime}) = \max_i \{f_i'(\mathbf{x}) - z_i^{r\prime}\} + \epsilon_a \sum_{i=1}^{n_f} (f_i'(\mathbf{x}) - z_i^{r\prime}) \tag{2}$$

where $\epsilon_a \in \mathbb{R}_{>0}$ is a small positive multiplier for the augmentation term. This formulation assumes that the objectives and the preference point are properly scaled [Goulart 2015], which can be guaranteed by defining:

$$\begin{aligned} f_i'(\mathbf{x}) &= \frac{f_i(\mathbf{x}) - f_{i,min}^{(t)}}{f_{i,max}^{(t)} - f_{i,min}^{(t)}}, \ \forall i \in \{1, \dots, n_f\} \\ z_i^{r\prime} &= \frac{z_i^r - f_{i,min}^{(t)}}{f_{i,max}^{(t)} - f_{i,min}^{(t)}}, \ \forall i \in \{1, \dots, n_f\} \end{aligned} \tag{3}$$

where $f_{i,min}^{(t)}$ and $f_{i,max}^{(t)}$ denote the smallest and largest values of the ith objective function at the iteration within which the evaluation is being performed, t.

If the underlying algorithm already possesses a mechanism to handle multiple solutions, as is the case in the MOEA/D, we can approximate a *region of interest* (ROI) that is close to the preference point. The underlying technique to define the ROI is dependent on each method. The Preference-guided Adaptive Region of interest (PAR) approach [Goulart and Campelo 2016] has the advantage of not requiring any additional parameters to be set by the user. Its general structure can be described as follows:

1. Given a set of available solutions $\mathbf{X} = \{\mathbf{x}_1, \dots, \mathbf{x}_\mu\}$, compute the ASF value of each point with relation to the preference point $\mathbf{z}^r$ using (2). Denote the smallest ASF value in the set as $s_{\min}$;
2. Compute n_f auxiliary points by adding $s_{\min}$ to each coordinate of $\mathbf{z}^r$:

$$\mathbf{z}_{aux,i}^r = \mathbf{z}^r + \mathbf{e}_i s_{\min}, \forall i \in \{1, \dots, n_f\} \tag{4}$$

   where $\mathbf{e}_i \in \mathbb{R}^{n_f}$ is the unit vector with the i-th coordinate set to unity and all others to zero.
3. Compute the ASF of each point of $\mathbf{X}$ in relation to each $\mathbf{z}_{aux,i}^r$, and find the points with the smallest value for each auxiliary point. Denote these n_f points as $\mathbf{x}_{\star,i}$;
4. The points belonging to the ROI are then defined as those for which $f_i(\mathbf{x}) \leq f_i(\mathbf{x}_{\star,i})$ for any $i \in \{1, \dots, n_f\}$.

Notice that the size of the ROI is dependent only on the distance of the preference point $\mathbf{z}^r$ to the image of the population in the space of objectives. This allows for an adaptive process that reduces the ROI size as the population converges, without having to adjust any additional parameters. More details are given in [Goulart and Campelo 2016].

5. PAR-MOEA/D

Incorporating preference information within the MOEA/D using the PAR framework is quite straightforward, requiring only a small modification in the way the scalarized value is calculated for each candidate solution. The procedure is contained within the evaluation step of the MOEA/D (Algorithm 1: line 9), and can be described as follows:

1. Follow the PAR structure to determine the points $\mathbf{x}_{\star,i}$ (Section 4, step 3);
2. Based on the points $\mathbf{x}_{\star,i}$, generate a new reference point $\hat{\mathbf{z}}$ with elements calculated as:
$$\hat{z}_j = \min_i f'_j(\mathbf{x}_{\star,i}), \forall j \in \{1, \ldots, n_f\} \tag{5}$$
where $f'_j(\cdot)$ is the value of the j-th objective function at the point, scaled according to (3).
3. For each point in the population, calculate the performance value using the MOEA/D scalarization strategy as defined below.

By following the procedure above, we are essentially adopting the idea of the PAR framework, namely that of adaptively modifying a region of interest, based on the stated preferences of the decision-making entity and the current state of the population. All aspects of the MOEA/D remain unchanged, with the exception of the scalarization procedure and reference point used therein. This means that, at least in principle, this adaptation can be easily adopted within existing implementations without much effort, regardless of specific variation operators, constraint handling approaches, decomposition methods, or any other aspects of the method.

5.1. Scalarization functions

In this work we investigate the use of several different scalarization functions within the proposed PAR-MOEA/D. Notice that in all cases the reference point used is $\hat{\mathbf{z}}$, calculated as in (5), and not the estimated ideal point used in regular, non-preference MOEA/Ds. In the definitions below, let $\odot$ denote the Hadamard product; $\|\cdot\|_\infty$ the Tchebycheff norm; $\epsilon_0 \in \mathbb{R}_{>0}$ a small positive constant used to prevent divisions by zero; and $\epsilon_a \in \mathbb{R}_{>0}$ the small positive multiplier for the augmentation factor in the scalarization strategies that adapt the ASF.[3]

1. Weighted Tchebycheff (WT) [Miettinen 1999, Zhang and Li 2007]:
$$f^{agg}(\mathbf{x} \mid \boldsymbol{\lambda}, \hat{\mathbf{z}}) = \|\boldsymbol{\lambda} \odot (\mathbf{f}(\mathbf{x}) - \hat{\mathbf{z}})\|_\infty \tag{6}$$
2. Adjusted Weighted Tchebycheff (AWT) [Qi et al. 2014, Wang et al. 2013]:
$$f^{agg}(\mathbf{x} \mid \boldsymbol{\lambda}, \hat{\mathbf{z}}, \epsilon_0) = \|\boldsymbol{\rho} \odot (\mathbf{f}(\mathbf{x}) - \hat{\mathbf{z}})\|_\infty \tag{7}$$
where $\boldsymbol{\rho} \in \mathbb{R}^{n_f}$ has elements given as:
$$\rho_j = \frac{(\lambda_j + \epsilon_0)^{-1}}{\sum_{j=1}^{n_f} (\lambda_j + \epsilon_0)^{-1}}, \ \forall j \in \{1, \ldots, n_f\}$$

[3] In this work we set $\epsilon_0 = \epsilon_a = 10^{-6}$.

3. WT with augmentation term from ASF (ASF-WT), an extension of the WT approach which incorporates the augmentation term from the ASF formulation in Equation (2):

$$f^{agg}\left(\mathbf{x} \mid \boldsymbol{\lambda}, \hat{\mathbf{z}}\right) = \left\|\boldsymbol{\lambda} \odot \left(\mathbf{f}\left(\mathbf{x}\right) - \hat{\mathbf{z}}\right)\right\|_{\infty} + \epsilon_a \sum_{i=1}^{n_f} \left(f_i(\mathbf{x}) - \hat{z}_i\right) \tag{8}$$

4. AWT with augmentation term from ASF (ASF-AWT), an extension of the AWT approach which incorporates the augmentation term:

$$f^{agg}\left(\mathbf{x} \mid \boldsymbol{\lambda}, \hat{\mathbf{z}}\right) = \left\|\boldsymbol{\rho} \odot \left(\mathbf{f}\left(\mathbf{x}\right) - \hat{\mathbf{z}}\right)\right\|_{\infty} + \epsilon_a \sum_{i=1}^{n_f} \left(f_i(\mathbf{x}) - \hat{z}_i\right) \tag{9}$$

It should be mentioned at this point that there is one existing method that employs scalarization by weighted ASF and a weight determination formula similar to the ASF-AWT in the context of preference-guided multiobjective optimization. This method, known as WASF-GA [Ruiz et al. 2014], defines a region of interest by means of a reference point, and employs a scalarization analogous to (9) to assign utility values to each point in a population. To our knowledge, there is no precedent for either ASF-WT or ASF-AWT in the MOEA/D literature, even though the use of analogous scalarization strategies in different contexts can be found in earlier works [Wierzbicki 1982, Miettinen 1999].

6. Experimental validation

For this preliminary testing of the PAR-MOEA/D, we employed all 2-objective problems of the UF benchmark set[4], UF1-UF7, as test functions. The preference points were artificially created in the following way: given the true ideal point of a given test problem, $\tilde{\mathbf{z}}$, and a randomly selected efficient solution $\mathbf{f}(\mathbf{x}^*)$ of that problem, we generated the $\mathbf{z}^r$ points over the line segment between these two points:

$$\mathbf{z}^r = \alpha\mathbf{f}(\mathbf{x}^*) + (1-\alpha)\tilde{\mathbf{z}} - \epsilon_r \tag{10}$$

with $\alpha \in \{0, 0.25, 0.50, 0.75, 1\}$, to investigate the performance of the algorithms tested for preference points ranging between the ideal point and a region close to the true Pareto-optimal front. A small constant $\epsilon_r = 0.025$ is used to prevent the ROI from collapsing into a single solution when $\alpha = 1$ and $\mathbf{z}^r$ lies exactly on the efficient front.

6.1. Algorithms

In this work we compare the PAR-MOEA/D variants described in the previous section against the two original PAR approaches, namely PAR-DEMO(nds) and PAR-DEMO(ind) [Goulart and Campelo 2016]. Both algorithms equip the PAR approach within the structure of the DEMO algorithm [Robič and Filipič 2005], the first (*nds*) using nondominated sorting as its main selection approach, and the second (ind) using the additive ϵ indicator instead.

Both PAR-DEMO approaches were configured as described in [Goulart and Campelo 2016], while the PAR-MOEA/D used the following configurations (arbitrarily set based on values commonly practiced in the MOEA/D literature, without any specific tuning effort):

[4] `http://dces.essex.ac.uk/staff/qzhang/moeacompetition09.htm`

- Differential mutation: $F = 0.5$;
- Binomial recombination: $CR = 0.4$;
- Polynomial mutation: $\eta = 20$ and $p_m = 1/30$;
- Neighborhood size: $T = 20$;

All algorithms have a population size of $\mu = 100$. The stopping condition is set to interrupt the execution after 30,100 function evaluations, which corresponds to 300 iterations.[5] All algorithms were run $n = 30$ times on each (problem, preference point) pair.

6.2. Quality indicators

The outcome of each algorithm was evaluated according to three quality indicators: convergence, pertinence and diversity [Goulart and Campelo 2016, Goulart 2015]. The specific definition of these three indicators is provided below.

For *convergence*, we employed the inverted generational distance (IGD) indicator [Zitzler et al. 2003], which is defined as follows: let $\mathbf{X}_f$ denote the set of nondominated points returned at the end of a given run of an algorithm on a problem, and $\mathbf{P}^*$ be a set of uniformly distributed points belonging to the Pareto-optimal front of the problem. Then:

$$IGD\left(\mathbf{X}, \mathbf{P}^*\right) = \frac{\sum\limits_{\forall \mathbf{p}^* \in \mathbf{P}^*} \left[\min\limits_{\mathbf{x}_i \in \mathbf{X}_f} \left\{ \|\mathbf{p}^* - \mathbf{x}_i\|_2 \right\}\right]}{|\mathbf{P}^*|} \tag{11}$$

Given that the aim of the algorithms under comparison is to approximate only points within their ROI, only Pareto-optimal points belonging to the exact ROI of each method (which can be easily obtained by following the procedure from Section 4 using the full set $\mathbf{P}^*$) were used for calculating this indicator.

For *diversity*, we used the Hierarchical Cluster Count (HCC) indicator, as proposed in [Guimarães et al. 2009]. This indicator, which is able to measure both the spread and the uniformity of a set of points and does not require any external parameter, is calculated as follows:

1. Set $k = 0$, $\tau_k = 0$ and $r_k = 0$;
2. Let each point $\mathbf{x}_i \in \mathbf{X}_f$ define a cluster, C_i, with itself as the single element;
3. Find the clusters $\mathcal{C}_{i1}$ and $\mathcal{C}_{i2}$ with the smallest value of separation distance, using complete linkage [Guimarães et al. 2009]. Assign this minimal value of separation distance to r_{k+1} and join these clusters;
4. Set $\tau_{k+1} = \tau_k + (r_{k+1} - r_k)(|\mathcal{P}| - k + 2)$ and $k \leftarrow k + 1$;
5. Go back to step 3 until there is only one cluster left. Return τ_k as a value of the diversity of the set $\mathbf{X}_f$.

For the calculation of this indicator, the final population of each algorithm was standardized to the interval $[0, 1]^{n_f}$ in the *objective space*, and the diversity was computed for these transformed data.

Finally, the *pertinence* indicator, which measures the degree of satisfaction of the preferences expressed by the DM, was calculated as the smallest ASF in relation to the

[5] The initial population requires μ evaluations before the iterations start to be counted.

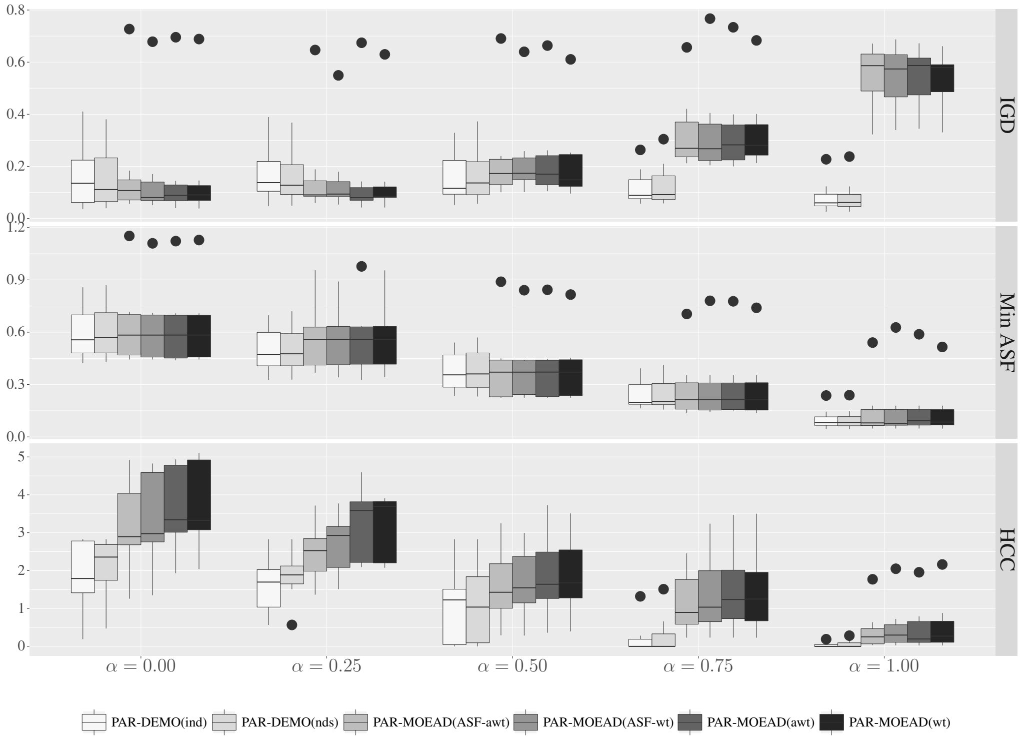

Figure 1. Box plots of the performance of each algorithm for each value of preference point distance, regarding the three quality indicators. Each box is composed by the averages of the performance of an algorithm on all problems for that value of α.

preference point $\mathbf{z}^r$, as suggested by [Goulart and Campelo 2016]:

$$s_{min}(\mathbf{X}_f \mid \mathbf{z}^r) = \min_{\mathbf{x_i} \in \mathbf{X}_f} \{s(\mathbf{f}(\mathbf{x}_i), \mathbf{z}^r)\} \tag{12}$$

The scaling when computing this indicator was performed as in (3), but employing the true ideal and Nadir solutions of each test problem.

Before proceeding, it is important to highlight that IGD and pertinence are indicators for which *smaller is better*, whereas for the HCC *larger is better*.

6.3. Results

The results obtained for the 30 replicates, summarized by means within each (problem instance, preference point distance, indicator) tuple, are illustrated in Figure 1. From the box plots, a few trends seem to be apparent, as discussed below.

First, IGD values seem to be slightly better for the PAR-MOEA/D approaches in lower values of α (i.e., for preference points farther from the true Pareto-optimal front), with comparable average values but smaller variability. These approaches start losing to the PAR-DEMO methods as the preference points approach the attainable region of the space of objectives.

With respect to diversity, the PAR-MOEA/D approaches appear to return superior (i.e., larger) values of the HCC indicator across all values of α, when compared to the

PAR-DEMO ones. The absolute HCC values of all methods is reduced as α increases, which is consistent it being harder to maintain good diversity in a smaller ROI.

Finally, regarding the pertinence indicator, which measures the degree of satisfaction of the stated preferences, the results also seem to be comparable across all values of α, with the PAR-DEMO approaches seemingly slightly better for preference points closer to the true Pareto-optimal front.

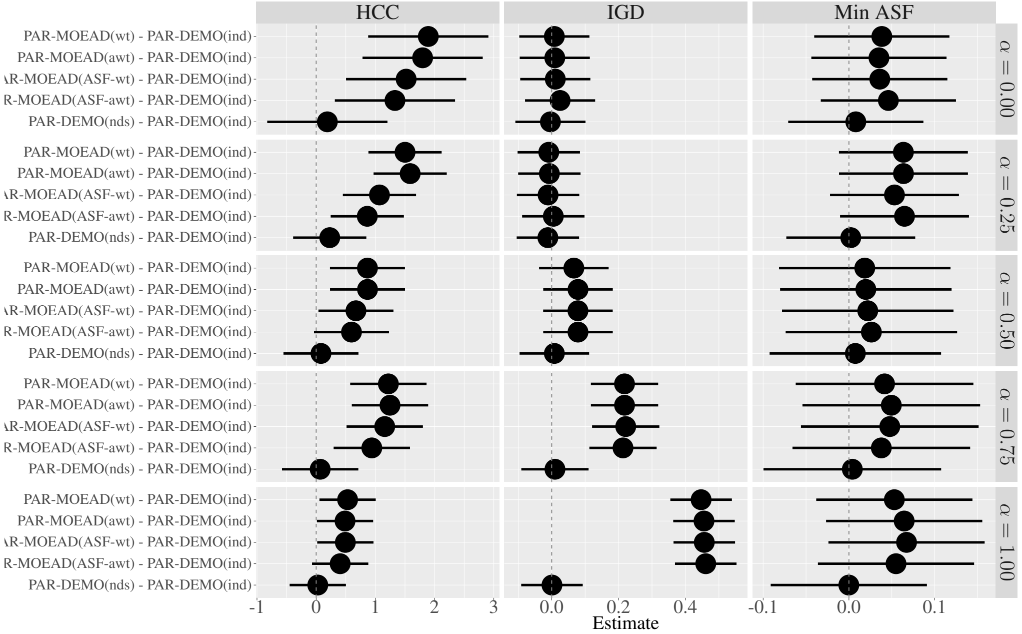

Figure 2. Simultaneous 95% confidence intervals for differences between each algorithm and PAR-DEMO(ind) in each quality indicator.

To reinforce these qualitative considerations, an RCBD ANOVA model [Montgomery 2012] was fit for the data within each value of α, with *algorithm* as the experimental factor and *problem instance* as a blocking variable. After fitting the model, the PAR-DEMO(ind) was selected as a reference method (since its performance is similar to the other PAR-DEMO approach for all cases) and all-vs-one paired comparisons were performed using Dunnett contrasts [Crawley 2007]. The results of these comparisons are illustrated in the form of simultaneous 95% confidence intervals in Figure 2, and corroborate the observations derived from examining the box plots.

7. Conclusion

In this work the Preference-guided Adaptive Region-of-interest (PAR) framework was included into the MOEA/D structure, allowing this method to incorporate preference information in its search procedure. While only a simple MOEA/D was used as the base algorithm in this work, the introduction of the PAR approach is straightforward, requiring only a modification in the procedure for calculating the scalarized utility of each point, which allows for its use with more sophisticated MOEA/D variants without much effort.

Results obtained using the UF benchmark set for different preference point specifications suggest that the PAR-MOEA/D versions tend to compare favorably to the PAR-DEMO approaches in terms of diversity of the nondominated set returned, while maintaining the same overall convergence and pertinence characteristics. Since the original PAR work [Goulart and Campelo 2016] already shows that the PAR-DEMO approaches used here as the comparison baseline tend to outperform other preference-based MOEAs (such as the R-DEMO and PBEA) in terms of both convergence and pertinence, but lose in terms of diversity, the use of MOEA/D as the basis for the PAR framework (which improves diversity over the PAR-DEMO) seems like an interesting step in the direction of improving the performance of PAR-based methods in the treatment of multiobjective optimization problems for which preference information is available.

Future works include the investigation of more sophisticated MOEA/D variants as base algorithms for the PAR framework, the incorporation of methods for treating nonlinear constraints, and tests in many-objective scenarios.

Acknowledgment

This work has been partially funded by XXXXXXXXXXXXX.

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