On the Identifiability of Steady State Induction Machine Models using External Measurements

Ahmed M. Alturas, Shady M. Gadoue, Bashar Zahawi, *Senior member, IEEE*, and Mohammed A. Elgendy

***Abstract*—A common practice in induction machine parameter identification techniques is to use external measurements of voltage, current, speed, and/or torque. Using this approach, it has been shown that it is possible to obtain an infinite number of mathematical solutions representing the machine parameters. This paper examines the identifiability of two commonly used induction machine models, namely the T-model (the conventional per phase equivalent circuit) and the inverse Γ-model. A novel approach based on the Alternating Conditional Expectation (ACE) algorithm is employed here for the first time to study the identifiability of the two induction machine models. The results obtained from the proposed ACE algorithm show that the parameters of the commonly employed T-model are un-identifiable, unlike the parameters of the inverse Γ-model which are uniquely identifiable from external measurements. The identifiability analysis results are experimentally verified using the measured operating characteristics of a 1.1 kW three-phase induction machine in conjunction with the Levenberg-Marquardt (L-M) algorithm which is developed and applied here for this purpose.**

***Index Terms:* induction motor (IM), identifiability analysis, parameter identification.**

I. Introduction

DUE to their reliability and low cost, induction machines have been widely utilized in a large variety of industrial applications. Different induction machine models have been derived to represent the machine dynamic and steady-state behavior [1, 2]. One of the most commonly used steady-state models of the induction machine is the standard per-phase equivalent circuit (T-equivalent circuit) shown in Fig. 1. This model includes five electrical parameters: R_s, R_r, l_{ls}, l_{lr}, and L_m, where R_s is the stator resistance, R_r is the rotor resistance (referred to the stator), l_{ls} is the stator leakage inductance, l_{lr} is the rotor leakage inductance (referred to the stator), L_m is the magnetizing inductance and s is the slip given by $(n_s - n_r)/n_s$, where n_s is the speed of the stator field and n_r is the rotor mechanical speed.

A simple induction machine equivalent circuit that gives the same input impedance as the T-equivalent circuit, but with only two inductances, has also been proposed [3-5]. This equivalent circuit, referred to as the Inverse Γ-model, is shown in Fig. 2. For the same input voltage, the circuit produces the same output torque as the T-model. The relationship between the parameters of the T- and inverse Γ- models is given by the following equations:

$$\alpha = \frac{L_m}{L_r}, l_{ls}^{'} = L_s - \alpha L_m, L_m^{'} = \alpha L_m \text{ and } R_r^{'} = \alpha^2 R_r \qquad (1)$$

where L_s and L_r are the self-inductances of the stator and rotor given by $L_s = L_m + L_{ls}$and $L_r = L_m + L_{lr}$, respectively.
The parameters of the T-model and Inverse Γ-model may be identified by using the standard no-load, DC and locked rotor tests [6]. For the T-model, it is not possible to determine l_{ls} and l_{lr} from these tests, without making an additional assumption about the ratio l_{ls}/l_{lr}. This ratio may or may not be available in the machine datasheet, so l_{ls} and l_{lr} are often taken to be equal, or another ratio is assumed depending on motor classification. These assumptions are not always valid leading to inaccurate parameter estimation [7]. Similarly, motor parameters are often identified for the purpose of condition monitoring of a running motor that is coupled to a load. In such cases, it will not be possible to take the machine out of service in order to carry out the standard tests and an alternative approach is required. Several methods of parameter identification of the induction machine have been proposed in the literature [8-11]. These can be divided into two main categories; signal-based [12, 13] and model-based techniques where machine parameters are identified based on external measurements of voltage, current, speed, and/or torque [14-18]. In this case, different sets of parameter values may be obtained depending on whether the machine model is identifiable or not [4, 15, 19].

The concept of identifiability can be explained by comparing the two functions shown in Fig. 3. In Fig. 3a, there is only one combination of parameter values that results in the function having a global minimum. In contrast, an infinite number of combinations of parameter values can result in the same minimum value of the function shown in Fig 3b. The system represented in Fig. 3a is identifiable whereas that represented in Fig. 3b is non-identifiable.

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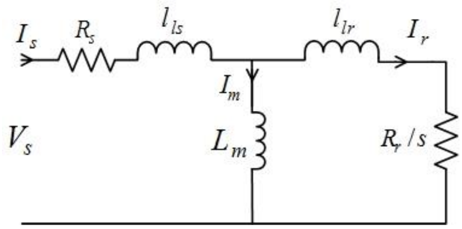


Fig. 1. Induction motor T-equivalent circuit.

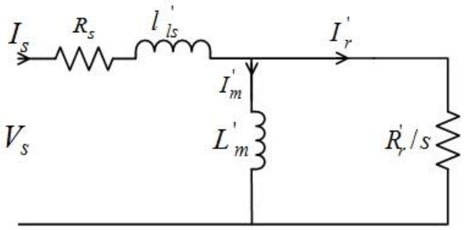
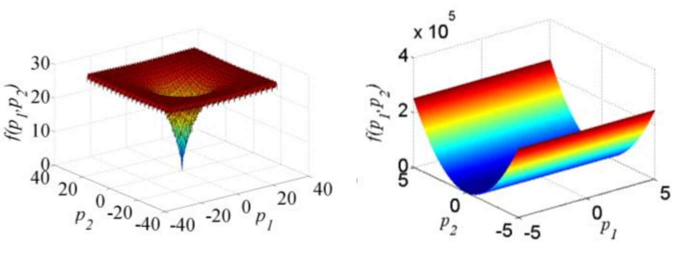


Fig. 2. Induction motor Inverse Γ- equivalent circuit.



(a) (b)

Fig. 3. Estimation of hypothetical parameters p_1 and p_2. (a) identifiable system, (b) non-identifiable system.

Several approaches for identifiability analysis have been proposed in the literature [20, 21]. More specifically, two approaches, the first based on a transfer function approach and the second on Bond graph techniques, have been applied to induction machine identifiability analysis [15, 22]. Using these techniques, it has been shown that the conventional machine T-model is non-identifiable. However, these approaches have not been employed to assess the identifiability of other machine models.

In this paper, a novel identifiability analysis approach is proposed in which the Alternating Conditional Expectation (ACE) algorithm [6, 7] is used for the first time to address induction machine model identifiability issues. The analysis is employed to examine the identifiability of both the T- and inverse Γ- models. Unlike previous approaches, the ACE is a fast algorithm that does not require a priori assumptions and can be employed for both linear and nonlinear systems. ACE results confirm the un-identifiability of the T-model parameters while demonstrating that the inverse-Γ model can be uniquely determined from externally measured machine waveforms. For verification purposes, an experimental model-based approach based on the use of the Levenberg-Marquardt (L-M) algorithm in conjunction with measured machine currents and voltages is developed to identify the parameters of a 1.1 kW three-phase induction machine, considering both the T- and the inverse Γ-models. In both cases, the parameter identification results confirm the identifiability analysis outcomes obtained from the ACE algorithm.

II. Alternating Conditional Expectation (ACE) Algorithm

Since the publication of the original paper on the subject in 1970 [2], the identifiability issue has received considerable attention in a number of fields including statics, economics, system engineering, and mathematical biology [19, 23-25]. The identifiability of induction machine model parameters is concerned with the unique association of the solution (identified model parameters) with the measured characteristics of the machine. If some parameters of a system model are not uniquely identifiable, there will be always several combinations of parameters that satisfy the solution. Identifiability analysis can be carried out either structurally or experimentally. A structural non-identifiability arises when there are redundant parameters in the model structure [26]. In experimental identifiability analysis, identifiability is tested by finding out if the measured information is enough to estimate the parameters reliably or not.

The ACE algorithm was initially developed in 1985 for the purpose of regression analysis [27]. It is a simulation-based approach that can be used to determine whether the model is identifiable or not. The power and usefulness of this algorithm lie in its ability to identify the effect of one or more independent variables (predictors) on a dependent variable (response) and reveal accurate relationships between them. In addition, ACE is a non-parametric approach that does not require any assumptions about the functional relationship between the dependent and independent variables [28, 29].

In the ACE approach, the problem of estimating a linear function of n-dimensional predictors $\mathbf{P} = (p_1, p_2, ..., p_n)$ and a response Y is replaced by estimating n separate one-dimensional functions of the predictors and a function of the response [29] as expressed by:

$$\theta(Y) = \sum_{i=1}^{n} \phi_i(p_i) + \varepsilon \tag{2}$$

where θ is a function of the response variable Y, $\emptyset_i$ is a function of the predictor p_i and ε is an independent normal random variable. These transformations are achieved through minimizing the variance of a linear relationship between the transformed response variable and the summation of transformed predictor variables. The normalized error variance (e^2) (for $\|\theta\|^2 = 1$) is given by:

$$e^2(\theta, \phi_1, \ldots \phi_n) = E\{[\theta(Y) - \sum_{i=1}^{n} \phi_i(p_i)]\}^2 \tag{3}$$

The minimization of the error is carried out through a series of individual function minimizations that result in the following expressions:

$$\phi_i(x_i) = E[\theta(Y) - \sum_{j \neq i} \phi_i(p_j) | p_i] \tag{4}$$

$$\theta(Y) = \frac{E[\sum_{i=1}^{n} \phi_i(p_i) | Y]}{\left\| E[\sum_{i=1}^{n} \phi_i(p_i) | Y] \right\|} \tag{5}$$

These two equations represent conditional expectation (E) and iterative minimization, from which the name of

Alternating Conditional Expectation is derived. Fig. 4 shows the operational steps of the ACE algorithm.

For a simple two dimensional case, considering two random variables p and y with zero expectation $E[p] = E[y] = 0$, the functions $\theta(y)$ and $\phi(p)$ are called optimal transformations if they satisfy:

$$\frac{E[\theta(y)-\phi(p)]}{E[\theta^2(y)]} = \min \tag{6}$$

This is equivalent to the maximization of the correlation coefficients between the transformed variables $\theta(y)$ and $\phi(p)$. ACE estimates the optimal transformations $\hat{\theta}(y)$ and $\hat{\phi}(p)$ which maximize the linear correlation R between $\hat{\theta}(y)$ and $\hat{\phi}(p)$ [30] non-parametrically (i.e. based on classification and ranking, not actual numbers):

$$\{\hat{\theta},\hat{\phi}\}_{y,p} = \sup_{\hat{\theta},\hat{\phi}} \left| R(\hat{\theta}(y),\hat{\phi}(p)) \right| \tag{7}$$

with a correlation coefficient of:

$$R(p,y) = \frac{E[py]-E[p]E[y]}{\sqrt{E[p^2]E[y^2]}} \tag{8}$$

where the goal is to minimize $\|\theta(y) - \phi(p)\|^2$ with $\|\theta\|^2$=1.

The maximum correlation coefficient R ($-1 \le R \le 1$) is used as a measure of the relationship between two variables p and y. R=0 if and only if p and y are independent. A large correlation coefficient, such as ±0.8, would suggest a strong relationship between parameters which may make a model not-identifiable. On the other hand, a small correlation coefficient, such as ±0.3, suggests weaker parameter dependence and an identifiable model. This concept can be extended to higher-dimensional problems with more than one predictor variable, i.e.

$$\frac{E[(\theta(y)-\sum_{i=0}^{n}\phi_i(p_i))^2]}{E[\theta^2(y)]} = \min \tag{9}$$

The calculation of (9) is carried out iteratively by the algorithm where new estimates of the transformation of the response serve as an inputs to new estimates of the transformation of the predictors and vice versa.

A simple example to demonstrate the use of the ACE is to consider a multivariate (multi-dimensional) case with three predictors (p_1, p_2, p_3) and a response y. Five hundred tuples of predictors are drawn independently and randomly from the interval $[0, 1]$ and the response is calculated for each tuple from (10), imitating 500 different observations.

$$y = 0.5p_1^3 + \tan^{-1}(p_2) \tag{10}$$

This was repeated three different times and, accordingly, three different matrices $\mathbf{k_i} = [y\ p_1\ \ p_2\ \ p_3]$ (i=1, 2 and 3) with dimension of 500×4 are obtained and serve as inputs for the ACE algorithm. Functionally related parameters provide quite stable optimal transformations from one sample to another and from the one matrix to another. If there is a relation between parameter, all matrices ($\mathbf{k_1}$, $\mathbf{k_2}$ and $\mathbf{k_3}$) render the same optimal transformations from one sample to another and vice versa.

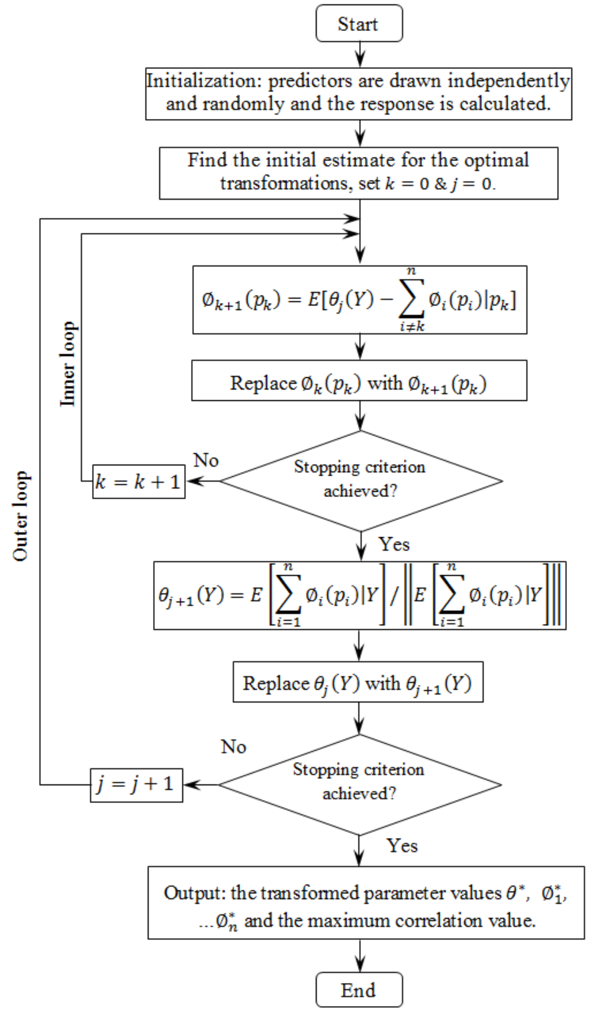


Fig. 4. ACE Algorithm description.

Fig. 5 shows a scatterplot of these data sets after applying ACE three times, where the three different colors illustrate the three estimates ($\mathbf{k_1}$, $\mathbf{k_2}$ and $\mathbf{k_3}$). One can see that, for each estimate (a row of the matrix $\mathbf{k}$), only the first three columns (y, p_1 and p_2) are functionally related (based on Equation 10) and the forth (p_3) is independent and, thus nearly linear transformations for all variables except p_3 exist. The transformations of the first three parameters (y, p_1 and p_2) remain stable from one sample to another and from one estimate to another, while the transformation of the fourth parameter (p_3) looks different. The estimated regression model of (10) from ACE transformed variables has a maximum correlation value of 0.99986 which is almost equal to 1. Such a high correlation coefficient between the parameters means that the model is not identifiable.

III. Identifiability Analysis of Induction Machine Models Using ACE

In this section, the ACE algorithm is used to assess the identifiability of induction machine T- and inverse Γ- models. To avoid complexity, skin effect, magnetic saturation and iron losses have been assumed to be negligible, a common assumption in parameter identification studies [31, 32].

A. T- Model analysis

The induction machine T-model (Fig. 1) is a multivariate model with a response (Z_{eq}) and five predictors (R_s, R_r, x_{ls}, x_{lr}, and x_m). Five hundred tuples of R_s, R_r, x_{ls}, x_{lr} and x_m are independently and randomly drawn from the interval $[0, 1]$ and Z_{eq} was calculated for each tuple. This was carried out three different times to obtain three different (500×6) matrices $\mathbf{k_i} = [Z_{eq}\ R_s\ R_r\ x_{ls}\ x_{lr}\ x_m]$ (i=1, 2 and 3) to serve as inputs to the ACE algorithm. The optimal transformations of T-model parameters are achieved through minimizing the variance between the transformed response variable $\theta(Z_{eq})$ and the summation of transformed predictor variables $\sum_{i=1}^{n} \emptyset_i(\mathbf{k}(p_i))$, where $\mathbf{P}$= $[R_s\ R_r\ x_{ls}\ x_{lr}\ x_m]$. The optimal transformations of the five predictors R_s, R_r, x_s, x_r, and x_m for the three different estimated matrices are shown in Fig. 6. It is difficult to draw the scatterplot for complex variables (Z_{eq}) because it would require four dimensions (for the real and imaginary parts of Z_{eq} and θ). Therefore, a scatterplot of $|Z_{eq}|$ is plotted to represent Z_{eq}.

For functionally related parameters, almost the same optimal transformations from one sample to another and from one estimate to another will be obtained. Nearly linear transformations are obtained for $|Z_{eq}|$, x_{ls}, x_{lr} and x_m. These transformations remained stable for all estimates and, thus the parameters are functionally related. However, different transformations are obtained for R_s and R_r. Optimal transformations of functionally related parameters are invariant under different estimates for each new drawn matrix $\mathbf{k}$. A non-identifiable model causes parameters to be functionally related. The maximum correlation between the response and the five predictors is 0.99583. Such a high correlation coefficient between the parameters means that there is a strong dependence between them which is a characteristic of a non-identifiable model.

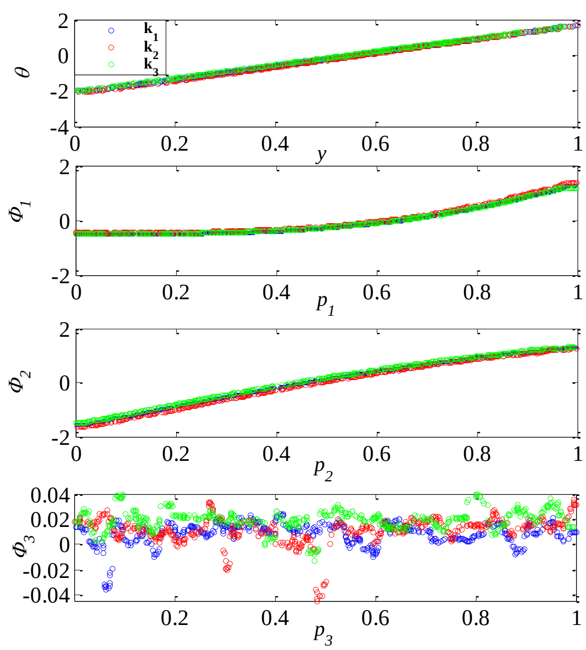


Fig. 5. ACE plot of Equation (10).

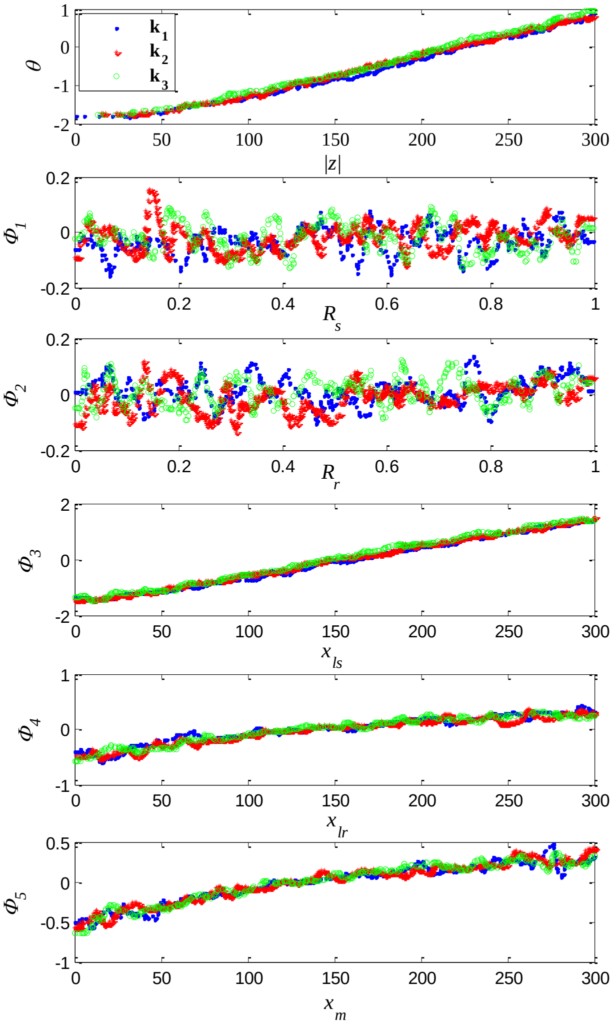


Fig. 6. ACE optimal transformations plot of the T-model parameters.

B. Inverse Γ-Model analysis

To assess the identifiability of the inverse Γ-model, the ACE algorithm is used to estimate the transformations of a response Z'_{eq} and a set of four predictor variables (R_s, R'_r, L'_{ls} and L'_m). R'_s, R'_r, x'_{ls}, and x'_m are independently drawn and the total impedance Z'_{eq} is calculated for each estimate. The optimal transformations for the response ($|Z'_{eq}|$), and the four predictors (R_s, R'_r, x'_{ls}, and x'_m) are shown in Fig. 7. The transformations look different from one estimate to another. This demonstrates the independence of the parameters and thus the identifiable nature of the model.

The maximum total correlation between the response Z'_{eq} and the four predictors was calculated at 0.0023038. This very low correlation coefficient between the inverse Γ-model impedance and the four electrical parameters means there is no dependence between the parameters. Thus, the parameters of the inverse Γ-model can be uniquely identified.

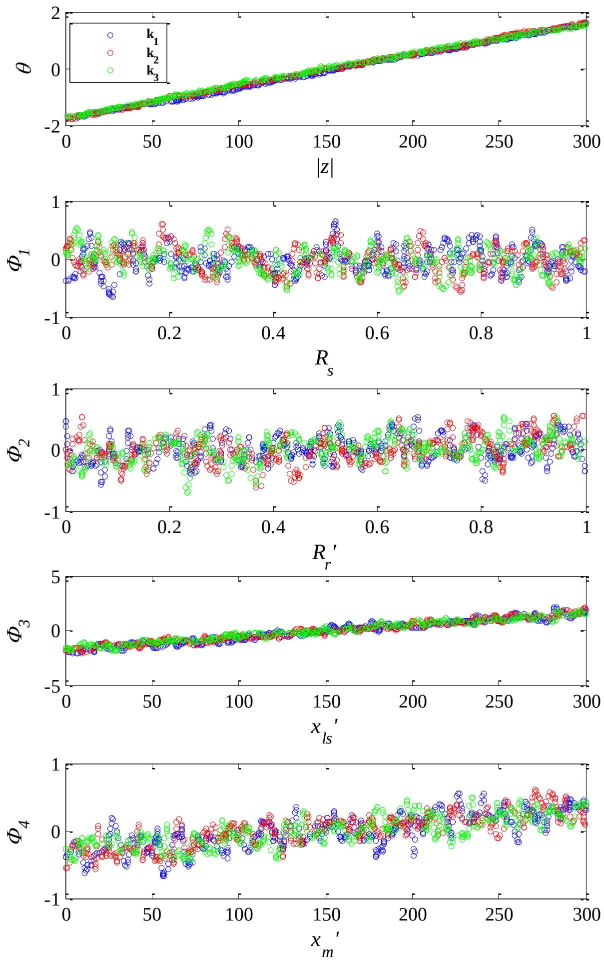


Fig. 7. ACE optimal transformations plot of the Inverse Γ-model parameters.

IV. Experimental Verification

In this section, the identifiability analysis presented in Sections II and III is verified using the Levenberg-Marquardt optimization tool in conjunction with measured time-domain data from a 1.1 kW, 50 Hz, 230/400 V, 4-poles three-phase induction machine operating under steady-state conditions. The Levenberg-Marquardt (L-M) [33] optimization tool is developed and employed to estimate the parameters of the T- and Inverse Γ-models of the induction machine. In the case of an identifiable model, the results should not be affected by the identification algorithm initialization. The algorithm will converge to the same solution (within acceptable limits) regardless of the initial conditions used to initialize the identification search. For a non-identifiable model, different parameter values will be obtained for different initial conditions (i.e. for different runs of the algorithm).

Herein, the L-M algorithm is used to find the best-fit machine parameters by minimizing an objective function, the weighted square errors between the measured data vector $\mathbf{Y_m}$ and the calculated data vector $\mathbf{Y_c}$. This is known as a chi-squared error function (χ^2) [33], given by:

$$\begin{aligned}\chi^2(\mathbf{P}) &= \sum_{i=1}^{q}\left[\frac{\mathbf{Y}_\mathrm{m}(t_i)-\mathbf{Y}_c(t_i,\mathbf{P})}{\omega_i}\right]^2 \\ &= (\mathbf{Y}_\mathrm{m}(t_i)-\mathbf{Y}_c(t_i,\mathbf{P}))^T\mathbf{W}(\mathbf{Y}_\mathrm{m}(t_i)-\mathbf{Y}_c(t_i,\mathbf{P})) \\ &= \mathbf{Y}_\mathrm{m}{}^T\mathbf{W}\mathbf{Y}_\mathrm{m}-2\mathbf{Y}_\mathrm{m}{}^T\mathbf{W}\mathbf{Y}_c+\mathbf{Y}_\mathrm{m}{}^T\mathbf{W}\mathbf{Y}_c\end{aligned} \tag{11}$$

where q is the number of data points, ω_i is a measure of the error in the measurement, $\mathbf{W}$ is a weighting matrix with $\mathbf{W}_{\mathrm{ii}} = 1/\omega_i^2$. The goal is to minimize χ^2 with respect to the parameters by finding the perturbation h to the parameter vector $\mathbf{P}$.

The L-M algorithm is an optimization technique that uses a combination of two methods; the Gauss-Newton method and the Gradient Descent method. The parameter values are updated in the opposite direction to the gradient of the objective function (error) and the error is reduced by assuming that the objective function is approximately quadratic near to the optimal solution.

Like many parameter estimation algorithms, especially for nonlinear models, the L-M algorithm is based on the minimization of an index (usually an error). The most commonly applied procedure is to search the best parameters set $\mathbf{P}^*$ in the search space S that minimize the error function *err*,

$$err = \min(E(\mathbf{P})) \tag{12}$$

The update relationships [33] are given by:

$$h = \alpha\mathbf{J}^T\mathbf{W}(\mathbf{Y}_\mathrm{m}-\mathbf{Y}_\mathrm{c}) \tag{13}$$

$$[\mathbf{J}^T\mathbf{W}\mathbf{J}+\lambda diag(\mathbf{J}^T\mathbf{W}\mathbf{J})]h = \mathbf{J}^T\mathbf{W}(\mathbf{Y}_\mathrm{m}-\mathbf{Y}_\mathrm{c}) \tag{14}$$

where α is a positive scalar which determines the length of the step in the steepest-descent direction, $\mathbf{J}$ is an $q \times n$ jacobian matrix $[\partial\mathbf{Y}_c/\partial p]$ represents the local sensitivity of $\mathbf{Y_c}$ to variation in parameters, h is the perturbation that moves the parameters in the direction of the steepest descent, and λ is the damping parameter.

For each step (iteration), if the present λ produces a smaller error, then the step is applied and λ is divided by a constant σ. In contrast, if the present λ produces a higher error, the step is discarded and λ is multiplied by σ. L-M acts in a similar way to the Gauss-Newton method when parameters are close to their optimum values (small values of λ) and similar to the Gradient Descent method at large values of λ. Fig. 8 shows the operational steps of L-M algorithm.

Steady-state experimental measurements of stator voltage (v_{Am}) and current (i_{Am}) were recorded with a digital oscilloscope using current and voltage probes. Motor speed (ω_r) was also measured using an encoder with a digital display unit.

The measured current is compared with calculated current with the model parameters adjusted by the L-M algorithm to minimize the error and to find the model parameters that give the best match between the two current sets. The block diagram of the identification process is shown in Fig. 9.

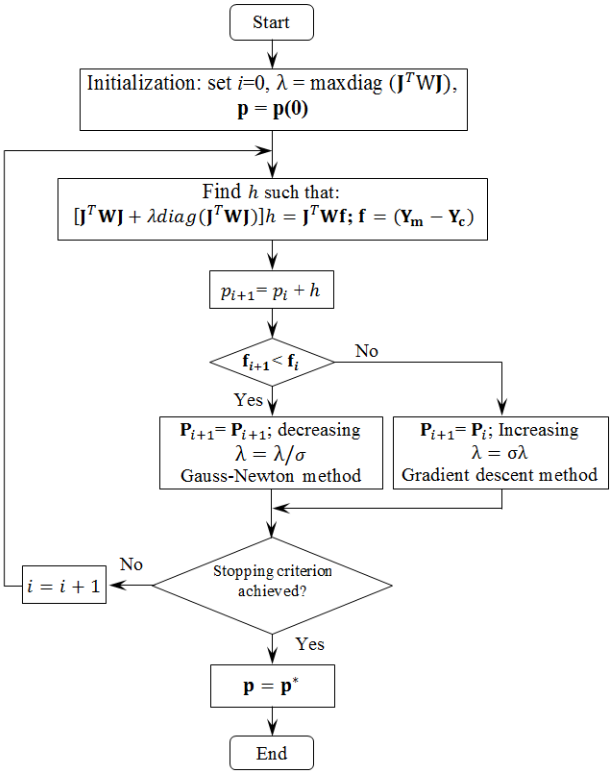


Fig. 8. L-M Algorithm description.

A. T- Model results

For the T-model, the L-M algorithm continuously updates the five parameter values (R_s, R_r, l_{ls}, l_{lr}, and L_m) and feed them to the system model (constructed in Matlab/Simulink) to calculate the phase current until a close agreement between the measured and calculated currents is achieved.

The process is then repeated for different initial conditions. Fig. 10 shows the convergence of the estimated parameters of the T-Model for different estimates. Fig. 11 shows the error function convergence for the 1st estimate. Table I shows three sets of L-M estimated parameter values (each obtained with different initial conditions). The total impedance corresponding to each estimate are also calculated and shown in the Table.

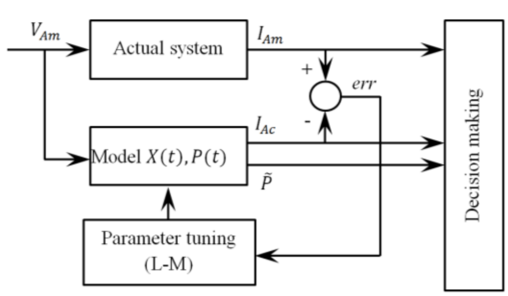


Fig. 9. General structure of the identification algorithm.

It can be observed from the results given in Table I that, completely different sets of parameters can be obtained depending on the initial conditions. The process is then repeated for different initial conditions. Table I shows three sets of L-M estimated parameter values (each obtained with different initial conditions). The total impedance corresponding to each estimate are also calculated and shown in the Table. Fig. 10 shows the convergence of the estimated parameters of the T-Model for different estimates. Fig. 11 shows the error function convergence for the 1st estimate. It can be observed from the results given in Table I that, completely different sets of parameters can be obtained depending on the initial conditions. Despite the significant differences between the three sets of parameters, the calculated current closely matches the measured current in each case. This confirms that the T-model is non-identifiable.

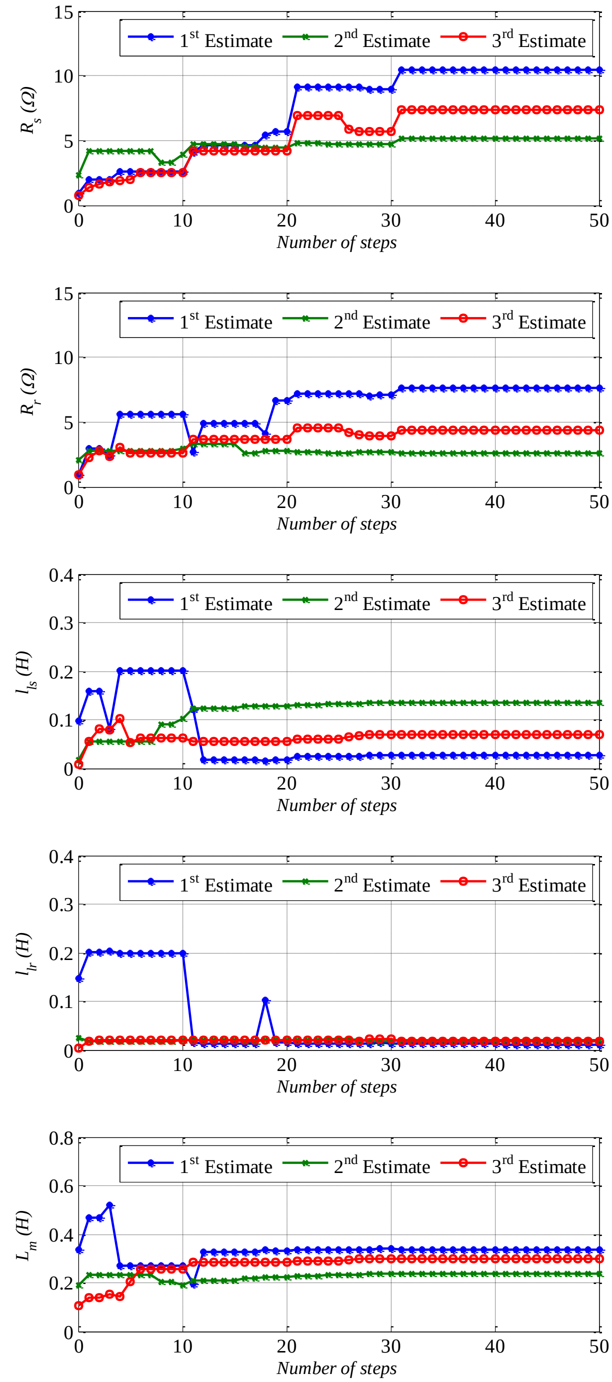


Fig. 10. Convergence of the estimated parameters of the T-Model for different estimates.

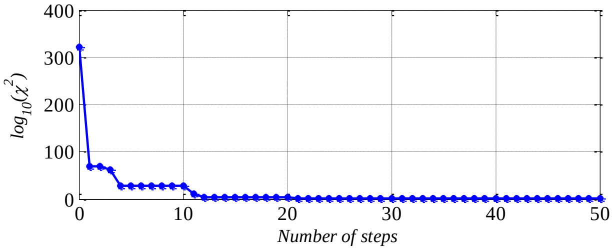


Fig. 11. The the **e**rror function convergence for the 1st estimate (T-model).

TABLE I. PARAMETER ESTIMATION OF T-MODEL

Pars	1st estimate	2nd estimate	3rd estimate
R_s	10.4824 Ω	5.1722 Ω	7.3494 Ω
R_r	7.6361 Ω	2.6230 Ω	4.3881 Ω
l_{ls}	0.0263 H	0.1346 H	0.0696 H
l_{lr}	0.0108 H	0.0199 H	0.0177 H
L_m	0.3387 H	0.2356 H	0.2975 H
$\lvert Z_{eq}\rvert$	115.58 Ω	115.54 Ω	115.37 Ω
$\angle Z_{eq}$	80.35 °	81.34 °	80.45 °

Fig. 12 shows the close agreement between the waveform of the measured current (I_{Am}) and that of the calculated current (I_{AC}) obtained from one of the parameter sets given in Table I (1st estimate) at a machine speed of 1491 rpm (slip of 0.006). Similar levels of agreement were obtained when using the other sets of the estimated parameters given in Table I. Fig 13 shows the squared error (χ^2) as a function of rotor and stator leakage inductances based on the measured data. As shown, infinite combinations of the two inductance values result in the same minimum value of squared error, i.e. there is no unique global minimum. This confirms that it is not possible to determine l_{ls} and l_{lr} uniquely using external measurements of voltage, current, and speed.

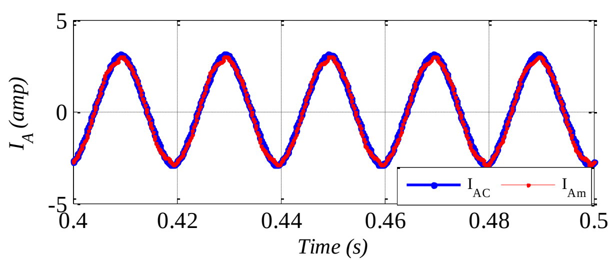


Fig. 12. Measured (I_{Am}) and calculated (I_{Ac}) stator currents waveforms coresponding to the optimal solution of the 1st estimate (T-Model).

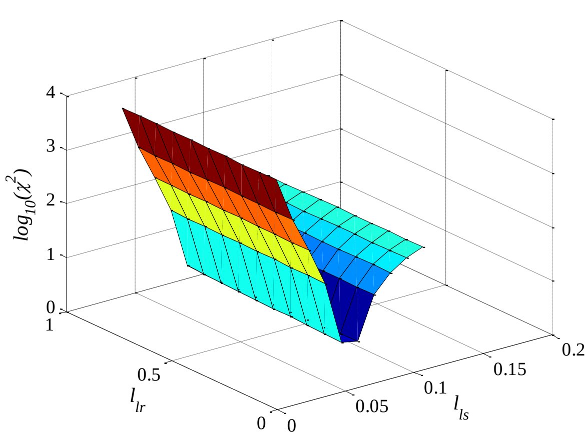


Fig. 13. The sum of the squared error as a function of l_{ls} and l_{lr} based on the measured data (T-Model).

B. *Inverse Γ- model results*

With the inverse Γ-model, the parameter vector **P** represents a set of the four parameters (R_s, R_r', l_{ls}', and L_m'). The same process described above was repeated, using the measured waveforms to estimate the parameters of the inverse Γ-model. Fig. 14 shows the convergence of the estimated parameters of the Inverse Γ-Model for the different estimates. Fig. 15 shows the error function convergence for the 1st estimate. Table II shows three sets of estimated parameters with different initial conditions. It is obvious that, regardless of initial conditions, the L-M algorithm can successfully estimate the same parameter vector of the inverse Γ-model (within acceptable limits).

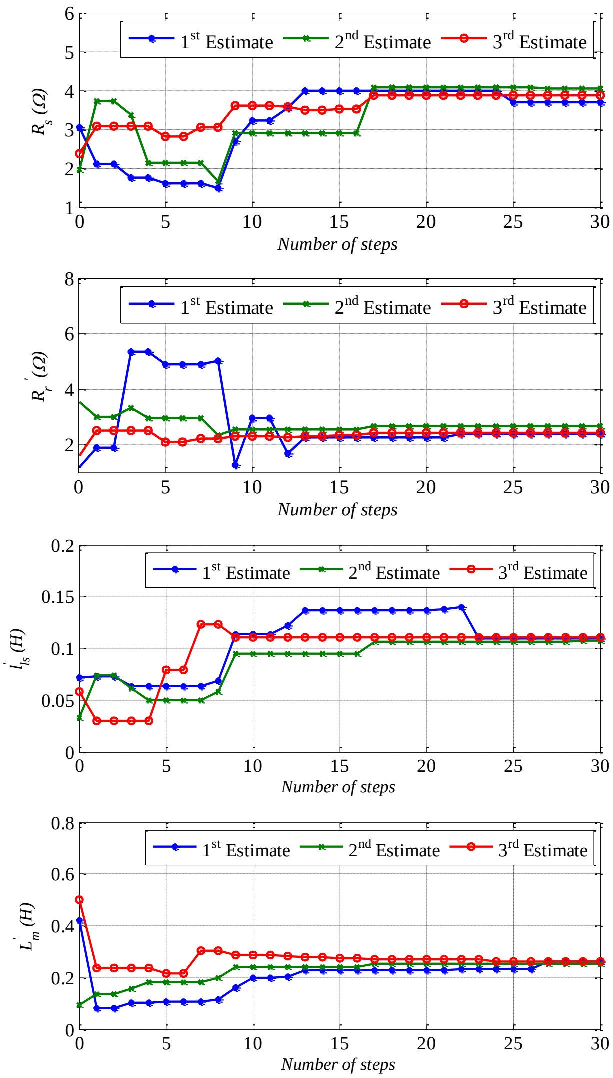


Fig. 14. Convergence of the estimated parameters of the Inverse Γ-Model for different estimates.

TABLE II. PARAMETER ESTIMATION OF INVERSE Γ -MODEL

Pars.	1st estimate	2nd estimate	3rd estimate
R_s	3.6848 Ω	4.0599 Ω	3.8638 Ω
R_r'	2.368 Ω	2.6858 Ω	2.4024 Ω
l_{ls}'	0.1098 H	0.1118 H	0.1101 H
L_m'	0.2627 H	0.2594 H	0.2616 H
$\lvert Z_{eq}\rvert$	115.4 Ω	115.53 Ω	115.29 Ω
$\angle Z_{eq}$	79.87°	80.79°	79.94°

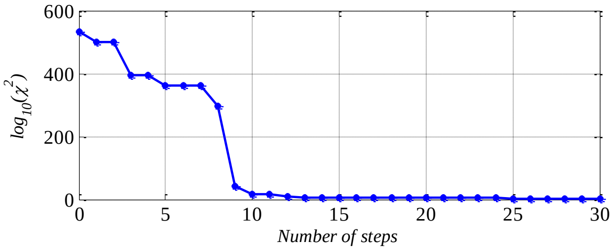


Fig. 15. The **e**rror function convergence for the 1st estimate (Inverse Γ-Model).

Fig. 16 shows the measured (I_{Am}) and the calculated current (I_{AC}) waveforms with one of the parameter sets given in Table II (1st estimate) at a machine speed of 1491 rpm (slip of 0.006). As shown, very good agreement between the measured and calculated current waveforms is realized. Similar agreement between current waveforms is obtained with the other sets of estimated parameters. The squared error (χ^2) as a function of the two inductances ($l_{ls}{}'$ and $L_m{}'$) based on the measured data is shown in Fig. 17. As illustrated, there is only one optimal combination of the two parameter values ($l_{ls}{}' \approx 0.11$ H, $L'_m \approx 0.26$ H) that satisfies the objective function and provides one global minimum.

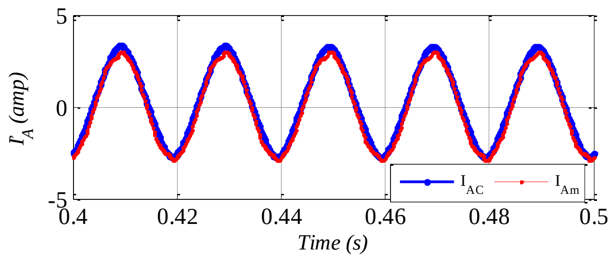


Fig. 16. Measured (I_{Am}) and calculated (I_{Ac}) stator currents waveforms coresponding to the optimal solution of the 1st estimate (Inverse Γ-Model).

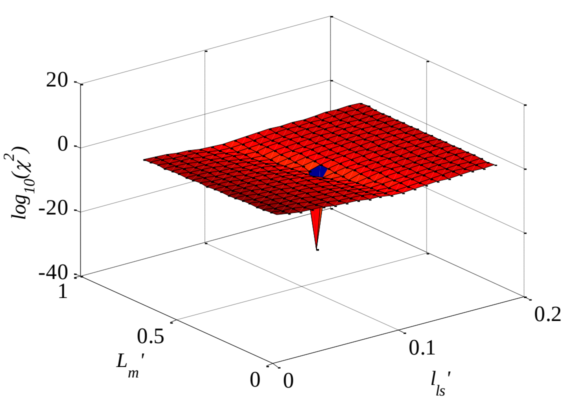


Fig. 17. The sum of the squared error as a function of of $l_{ls}{}'$ and $L_m{}'$ based on the measured data (Inverse Γ-model).

V. Conclusion

This paper presented a detailed study of the identifiability of the parameters of the T- and inverse Γ-equivalent circuits of the induction motor. The identifiability of both models is investigated using a novel approach based on the Alternative Conditional Expectation (ACE) algorithm. The analysis shows that the machine T-model is non-identifiable whilst the inverse T- model is. Results are experimentally verified using a 1.1 kW, 4-pole three-phase induction machine.

Using the ACE algorithm, a high correlation coefficient of about 0.996 between the parameters of the T-model is obtained suggesting that the parameters are dependent on each other and cannot be uniquely identified. On the other hand, ACE produces a small maximum correlation coefficient of only 0.0023 between the parameters of the inverse Γ-model suggesting that the parameters of the model are identifiable. These results are verified using measured machine waveforms in conjunction with the Levenberg-Marquardt (L-M) algorithm. When comparing measured and calculated current waveforms to minimize the sum of the squared errors, an infinite combination of parameter values produce the same input impedance of the T-model. In contrast, for inverse Γ-model, only one combination of parameter values provides the equivalent impedance and a single global minimum of the objective function is obtained.

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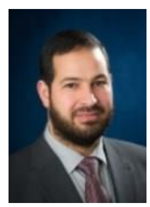

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