SOME ASPECTS OF DAMPING AND DYNAMIC CHARACTERISTICS OF MACHINE TOOL STRUCTURAL JOINTS

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A Thesis submitted for the degree of

DOCTOR OF PHILOSOPHY

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FEBRUARY 1976

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'ACKNOWLEDGEMENTS'

The author wishes to thank:-

Professor R.H.Thornley, M.Sc.,Ph.D., A.M.C.T., C.Eng., F.I.Mech.E., F.I.Prod.E., Mem.J.S.M.E. for permitting this work to be carried out in the Department of Production Engineering of the University of Aston in Birmingham. Also for his unflagging help, guidance and encouragement as supervisor throughout the entire project.

The University of Aston in Birmingham for providing the financial support.

Dr. J.L. Aston of the University of Aston in Birmingham for his valuable advice on the statistical methods; Associate Professor Y. Ito of T.I.T. for providing the information on practical levels of pressures on machine tool joints, and Mr. E. Langton of the University of Aston in Birmingham for his help with the instrumentation.

Messrs. S.Witherington, G.Yardley and J. Norman for their assistance in the manufacture of some of the parts for the apparatus.

SUMMARY

Machine tool joints not only provide the major source of energy dissipation but are also responsible in the main for a large proportion of overall deflections. They thus possess high potential as devices for optimizing directional orientations of modal flexibilities as well as improving the overall damping. The feasibility of these are first examined and then an improved 'approximate method' of optimization is proposed.

The main part of this work, however, is concerned with the design, manufacture and calibration of an apparatus for measuring the joints' parameters with universally applicable 'yardsticks'. Such 'yardsticks' could only be defined in terms of energies. A completely symmetric two degrees of freedom system results in orthogonal principal coordinates hence fulfilling this condition. Acting as a mechanical amplifier it dynamically isolates the rig from the surroundings whilst considerably reducing the size of the shakers. The first mode is employed for measuring the extraneous effects as well as calibrating the instruments whilst the second mode is used for the actual measurements.

Repeated tests are carried out on a number of 'classic joints' namely turned and ground to represent single and multi point tool cut surfaces. Equations are derived which relate stiffness and damping of turned surfaces to four independent variables of: area, roughness, pressure and viscosity of lubricant. It has been shown that the non-dimensional dependent variables employed here to describe the

effect of lubrication are almost independent of dry stiffness. This results in a more accurate and generalised form of information. Within the range of independent variables considered, area and roughness emerged as the main parameters influencing both loss and stiffness factors.

An improved distribution of potential energies can be achieved through maximising the quadrature component of the overall stiffness. This is believed to ensure the stiffness integrity whilst producing large increases in the overall damping. A jointed ensemble with lubricated and smooth but SPT cut surfaces is expected to experience by far the larger quadrature component of stiffness than the identical assembly with ground surfaces.

NOMENCLATURE

a_1, a_2	Acceleration of: upper, lower inertia block
A_2, B_2	Coefficients of coordinate transformation as defined in equation 5.6.2.
c	Spring-index as defined in equation 8.10
C.O.M	Coefficient of Merit - a measure of chatter resistance as indicated in Fig. 1.
d	Wire diameter of spring
D_o	Dissipated energy per cycle
D_{o1}, D_{o2}	Mean diameter of coil springs K_1 and K_2
E	Young's Modulus of elasticity
E.L.S.	Equivalent Length of Solid Steel
G	Shear modulus of elasticity
h	Hysteretic damping coefficient
k	Stiffness of elements in the uniform system of Fig. 18b.
$K+iC\omega$	Complex stiffness of 'jointed specimen column' of Fig.44b.
$K_{11}+iC_{11}\omega$	Complex stiffness of auxiliary system as indicated in Fig.24b.
$K_{12}+iC_{12}\omega$	Complex stiffness of auxiliary system as indicated in Fig.24b.
$K_{21}+iC_{21}\omega$	Complex stiffness of auxiliary system as indicated in Fig.24b.
K_1, K_2	Stiffness of auxiliary system of preloading-isolating, weight supporting spring (symmetric system)
$K_J+iC_J\omega$	Complex stiffness of joint
$K_m+iC_m\omega$	Complex stiffness of 'solid specimen column' of Fig. 44a.
$K_t+iC_t\omega$	Complex stiffness of the equivalent system of Fig. 15d. at second mode.
ℓ	Length of specimen column
ℓ_J	E.L.S. of joint
m	Mass of elements in the uniform system of Fig. 18b.
$\bar{m}_1, \bar{m}_2, \bar{m}$	Apparent mass vectors as defined in Table 8a.
M_1, M_2	Mass of upper, lower inertia block
M.P.T	Multi-Point-Tool
n	Number of active coils
N	Number of effective joints in 'specimen column'
NF_1, NF_2	Natural frequency of first, second mode of vibration
p_1, p_2	Principal coordinates
P_{max}	Maximum permissible load on spring
r_1, r_2	Modal shapes as defined in Table 8a.
R, R_{12}, R_{J2}	Stiffness rations as defined in Appendix 2.
R_1, R_2	Stiffness of preloading-isolating, weight supporting spring
R_J	Stiffness factor (lubricated inphase stiffness/dry inphase stiffness)
S_{max}	Maximum length M_1 can be raised or lowered.

Nomenclature continued

S.P.T.	Single-Point-Tool
V_o	Maximum potential energy stored during one cycle
w, W	Weight of specimen disc, inertia block

* * * *

δ	Log-decrement as defined in Appendix 2
Δ	Area of joint
ζ	Loss-factor as defined in Appendix 2
λ	Normal approach of jointed surfaces
$\partial, \partial_1, \partial_2$	Frequency ratios $\frac{\omega_2}{\omega_1}$, $\frac{\omega}{\omega_1}$, $\frac{\omega}{\omega_2}$
ξ	Damping ratio as defined in Appendix 2
σ	Joint Interface Pressure
τ	Permissible shear stress
ϕ	Phase angle between force and displacement
ψ	Specific damping capacity
ω_1, ω_2	Natural frequency of first, second mode of vibration
$\omega_n, \omega_d, \omega_R$	Undamped natural, damped natural, resonance frequency
Ω	Surface roughness: C.L.A.

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CHAPTER 1

INTRODUCTION

1.1 Efficiency of Machine Tool Structures

In modern performance testing of a machine tool, the criterion by which the machine is judged is "The ability of the machine to produce within a minimum time and at small operating costs, an end product with acceptable geometric accuracy and surface finish".

Geometric accuracy of the workpiece is determined by the static stiffness of the structure and the alignment of its sliding mechanisms; in turn the efficiency and accuracy of the slideways are dependent upon the static stiffness. Any deviation from optimum tool geometry and hence tool life, will also be influenced by the static stiffness.

Adequate surface finish would in most cases require a steady state operation. Excessive amplitudes in a steady state or a self-induced vibration, is the main factor limiting the machine output.

The stability of a cutting process is determined by the cutting conditions and some dynamic characteristics of the machine frame. Theories of chatter prediction rely on the quantitative values of response, namely operative receptance - the cross receptances of the structure in the direction of co-ordinates in line with P cutting force and Y normal to the cut surface (Fig.1). The maximum negative real component of this receptance limits the chip-size according to:

$$b_{lim} = \frac{1}{-2G_{min}\rho} \qquad 1.1$$

In the past, attempts have mainly been directed towards indiscriminate increase in stiffness/mass ratio of all the structural components. Whilst this would increase the overall dynamic rigidity and hence improve the chatter behaviour of the machines and on the whole is advantageous, not enough attention has been given to the funcational requirements of machines, the most important of them being the optimum orientation of decisive modes with respect to the cutting process. A very considerable increase (300 per cent) in the stability of a lathe, with the normal horizontal tool position, is reported to have resulted by a simple alteration in the cross-section of its tail stock(1)*. Another example of optimization can be found in reference (2) where much higher rigidity of a single column planer type milling machine in a decisive mode has been achieved through appropriate alteration made to its stiffness and mass distribution.

Furthermore, the increase in natural frequencies through increased stiffness/mass can never be large enough to avoid resonance conditions and hence damping ability of the system becomes increasingly significant as these frequencies are approached. Depending upon the ratio of fluctuating to steady cutting forces, increased dynamic rigidity or static stiffness becomes more effective respectively.

Provided adequate static stiffness is maintained, the optimum structure is therefore regarded as one with optimum directional orientations of

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* Figures in brackets refer to references given in Pages 230 to 235.

modal flexibilities with maximum inclusion of potential energies at highly damped regions of the structures.

Obviously, the prerequisite of any systematic methods of optimization is to be able to predict response from the drawing office stages of design.

1.2 Machine Tool Joints - Discrete and Major Sources of Energy Dissipation

Machine tool structures, unlike other engineering frames of highly stressed and usually dry constructions, normally undergo only small stresses within the members but, possess discrete regions of high damping properties, i.e. joints. Whilst the former case, due to almost proportional distribution of damping within the members, would permit uncoupling of the principal co-ordinates: in machine tool structures this should not be possible - the solution of eigenvalue problems would therefore involve complex values and hence increase the computational size to an impractical level.

Relative numbers of 1, 10 and 300 for the damping ability of a spindle, for unmounted and mounted conditions, give a good indication of the existence of such concentrated or discrete damping sources in a structure. The first two correspond to material damping of the spindle manufactured from low damping steel and high damping cast iron materials respectively. Whilst the last number refers to the same member mounted in bearings[3]. Loewenfeld[4] has shown how the level of damping is influenced by the joints of a lathe by measuring the log-decrements at its different stages of construction. (See Fig. 2.).

Damping ratios encountered in machine tool structures, despite presence of joints, however, very rarely attain values above 5%. This is because the joints are never allowed or in fact rarely can, experience significant proportions of the total potential energies stored in modes of vibration.

'Synthetic joints'* are increasingly being proved to show some advantages over the 'classic joints' in some areas[5, 6 & 7]. Their potentialities as high damping sources and/or devices for optimization of orientations of modal flexibilities or, damping, should be examined from practical viewpoints.

1.3 Machine Tool Joints - Significant Sources of Overall Deflections

The findings of Levina and Reshetov[8], though obviously not valid for all machines, due to the very high percentages reported (i.e. 85 - 90%), do however prove as a rule the significance of joints in determining the overall static behaviour of machine tool structures. Further evidence in support of this statement can be found in reference (9) where Taylor reports on an improvement of 39% in the stiffness of an open sided planing machine measured at the rear tool box, when the joint between the cross rail and the columns was assumed to be rigid instead of being assumed to have some reasonable value of stiffness. The work of Thornley and Khoyi on model structures of a plano miller has shown the percentage of overall deflection caused by the local deformations around the column-bed joints to be 60 - 75% when measured at the top of the columns[10].

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* 'Synthetic Joints' (as against 'Classic Joints') will refer to joints whose manufacture involves unconventional means or methods.

1.4 The Present Work

In this work 'joints' are under study, not only because they have always been considered as stumbling blocks towards response prediction but also appear to possess high potential as a device for optimizing and improving the overall damping capacity of machine tool structures, and consequently of practical value in the application of 'Modular Design'. The feasibility of the latter was first examined on a $\frac{1}{8}$ scale model of an arch type plano-miller and then an improved approximate method of optimization based upon the theory of 'energies at resonance' proposed.

The central issue however is that the stiffness and damping characteristics of all the structural elements, including joint layers (classic or synthetic) must be known quantitatively within ranges of interest, if guess-work on values of damping ratios (for approximate methods) or damping forces (for exact methods of optimization) is to be abandoned.

The measurement of stiffness and in particular damping of machine tool joints has suffered - more than its usual share - from problems commonly associated with such measurements, mainly because of the very high stiffnesses involved. The results were a small $\frac{S}{N}$ ratio, large extraneous effects due to various modes' coupling hence limited frequency range etc. Because of these difficulties and the fact that the mechanisms of damping within joints are not yet fully discovered no valid data reduction could be possible. It is mainly for these reasons that most of the previous works on the damping of joints must be regarded as of a qualitative nature.

The urgent need for a new technique of complex modulus measurement was therefore appreciated not only to overcome or reduce the above difficulties but also to facilitate measurement of damping with a universally plausible 'yardstick' best suited for response analysis and at the same time being independent of any particular regime of energy dissipation.

The bulk of the present work consisted of design, manufacture and calibration of such a measuring device which employed precisely this very undesirable feature (i.e. high transmissibility at joints) to create ideal test conditions (namely a highly isolated rig with a much reduced size of shakers), when operating at all levels of variables encountered in practice.

Due to the complexity of mechanisms involved, it was realised that extensive experiments had first to be carried out to measure the dependent variables and to observe the effect of independent variables quantitatively before any valid judgement could be made as to the cause of joints' behaviour. In the next stage of the work, therefore, experiments were carried out on a number of 'classic joints' (namely turned and ground to represent single and multi point tool cut surfaces) and equations relating stiffness and damping to the four independent parameters of: area, roughness, pressure and viscosity of lubricant derived.

Computer aided statistical methods were employed first to establish the effectiveness of the independent variables and their interactions (analysis of variance with replicates), and then to derive the equations of stiffness and damping (regression analysis).

CHAPTER 2

RESPONSE PREDICTIONS AND OPTIMIZATION OF MACHINE TOOL STRUCTURES WITH PARTICULAR REFERENCE TO JOINTS AND DAMPING

2.1 Introduction

Koenigsberger[11] in an article on trends in the design of metal cutting machine tools states:

'Another trend combining versatility with simplicity can be seen in the growing use of modular construction which becomes particularly attractive with group technology'

He adds

'The implementation of this concept requires much skill from designer, manufacturer and user alike. In particular the connecting faces between modules, fixed joints, guideways, the power transmission, drive combination, (mechanical or electrical), etc., will have to be quite different from those used in conventional machines.'

After a brief introduction to the methods of response prediction and their application in optimisation of machine tool structures, in this chapter, the potentialities of joints as a means of optimisation are first studied and then evidence from some specially designed experiments is given to check the feasibility of such an arrangement. Finally a method utilising joints (classic or synthetic) for this purpose is proposed.

2.2 Exact Methods

Theoretically, frequency response of a complex but linear system subject to any complex loading can be computed if stiffness, mass and damping distribution in the system are known, i.e. lumped constant method when the number of 'lumps' is taken to be sufficiently large[12, 13,14 and 15].

Practically however, there are limitations; difficulties arise not only from the fact that our knowledge of stiffness and in particular damping distribution is limited, but also computational size which makes the above mentioned technique, if not impossible, a very expensive process of computation.

The technique of 'synthesis of receptances' employed in (2) has been developed further in (16) to embrace not only lumped parameters of one dimensional model (i.e. beams) but also general finite elements and distributed mass and stiffness.

Hammill and Andrew[17] have developed receptances of lumped parameter systems containing only a small number of discrete sources relative to n the number of degrees of freedom necessary to define undamped characteristics and hence reduced computing size. Whilst this technique should prove valuable in the application of damping inserts or similar applications, until frictional forces at joints can be simulated, the method cannot be used in machine tool structures. Even if this were possible, due to the fact that damping ratios are relatively low, there is no reason to believe that exact methods will introduce less error than approximate methods.

2.3 Approximate Methods

Generally one of the major sources of uncertainties in theoretical simulation of machine tool structures is to be found around the regions of material discontinuity, i.e. joints which have in general inherently non-linear and frequency dependent characteristics. On the other hand, the permissible assumptions are extremely rewarding as far as approximate simplified solutions are concerned. The most important of them being:

(a) Low amplitude

A machine tool structure cutting in a steady state of operation will normally exhibit only small amplitudes hence permitting linearity assumptions, even though some inherently non-linear members such as joints are present (see Fig. 3.).

Therefore linear differential equations with effective values of viscous and spring constants could, with sufficient accuracy within the working range, describe the motion.

(b) Low damping

The damping ratios encountered within the frequency range of interest are normally sufficiently low (frequently less than 0.05) to permit solution of eigen-value problems to yield relatively accurate natural frequencies (or resonance frequencies in this case) and modal shape predictions. It is on the strength of this assumption that the following approximate methods of response prediction are based.

Method 1 - Viscous Damping with Estimated Values of Damping Ratios

The method presumes a damping ratio for each equivalent single degree of freedom system for each mode and from this the desired receptances are calculated as a factored sum of the modal flexibilities.

The method has successfully been employed in the past for both chatter prediction and design improvements.(Refs.13 and 1 respectively).

Method 2 - Energies at Resonance

In this method the structure is broken into a number of sub-systems such as beams, etc., specific damping capacity of which are known. The damping ratios for each mode is then computed from the modal shape.

This method is preferred to method 1 for two main reasons:

(a) it does not rely on purely guessed values for damping ratios

(b) energy methods offer the freedom from any particular mechanism of damping, e.g. stiffness and damping parameters can vary with frequency. This phenomenon could be important should the squeeze film effect become significant.

2.4 Significance of 'Ends' on the overall Behaviour of Machine Tool Structures

The term 'end effect' as referred to here would normally mean the 'joint effect' caused by any of the three forms of joints: sliding, fixed and sliding-fixed (clamped); but it is also meant to represent a general meaning. It not only implies any sort of connection between any parts of a machine, or machine and its foundation, but it implies any end condition a machine would take during a cutting operation, or different settings of tool and work-piece. For example, the slow moving parts of the feed drive system during a cutting operation,.or a different position of the overarm in a horizontal milling machine in different settings, would change the joint's configuration and hence the end conditions.

By changing the end conditions, not only the mass and/or stiffness distribution of the structure would change, but the damping capacity of the system, which is believed to be produced mainly through interactions within the interface of the joints. Therefore, by altering the end conditions one would expect some changes in natural frequencies, directional orientations of modes, dynamic rigidities and effective damping ratios.

If the first few modal shapes of any machine structure are examined to trace the weak areas, it will be apparent that joints are responsible for a large proportion of the total deformation, particularly at lower modes which are normally more likely to be excited and correspond to higher amplitudes (rocking modes). The extent to which joints influence the overall behaviour depends upon the type of machine. For example, in the case of a lathe, the frame and fixed joints are not as influential, as the other kinds of joints in the spindle and the machine workpiece interface. On the other hand, in radial drilling machines, the frame and the fixed joints play a major part in the stability of the machine. The modal shapes also reveal the directions of major flexibilities at the joints. For example in fixed joints it is mainly the rotational flexibility rather than shear which determines the joint behaviour.

In practice many apparently identical machines under identical cutting conditions have been found to perform differently under chatter conditions(18). The difference could only be attributed to the joints, especially in the groups of machines whose performance for chatter is more influenced by their spindle and its bearings and joints placed immediately in the cutting area rather than the whole frame.

Similar types of machines usually have roughly the same modal shapes. For example the first two modes of a radial drilling machine normally take the form of rocking, and that of a tuning fork. In some instances it may be possible to improve the stability of the machine by changing the end conditions. For example stiffness adjustment at the joints may place the modes in the most favourable position with respect to each other and thus prevent chatter or even increase the rigidity. It may even be possible in some cases that some structural sub-system could be made to act as a vibration absorber and/or damper at certain frequency bands. The addition of a pendant motor(19) was found to split the first principal mode of a horizontal boring/milling machine around 12 HZ into 11 and 15 HZ and resulted in 100% increase of tool point rigidity. Another possible application of adjustable end conditions could be that of shifting the natural frequencies to avoid chatter or dangerous resonances in forced vibration, e.g. in the case of unbalanced rotary mass, etc.

Experimental investigation into the effect of fixed joints and their orientation upon the static and dynamic characteristics of a model of an arch type plano-miller is given in Ref.(20). A significant difference was observed between the four configurations shown in Fig.(4). The overall distribution of mass and stiffness for all the machines were kept as close as possible except around the joint regions where the only difference was caused by the different orientations of the joints' planes.

Figures 5 and 6 show how the joints between the bed and the columns affected the damping - when the planes of the joints were altered from the vertical position to the horizontal position the damping ratio was found to increase about 100%. This was accompanied by a small rise in the resonance frequency indicating an increase in the actual damping capacity C at the joints of more than 100%.

An example of the effect on the modal flexibilities is also illustrated in Fig.6 where the joints between the columns and the top piece were found to be very influential with regard to the second principal mode of $B_B C_B T$. By simply changing the top piece from T_B to T_S a coupling/absorbing effect was produced between the two modes of the bed vibration and hence resulted in a further increase in rigidity. (Figs.6c,6d). A similar effect was observed in the normal direction. Figure 7 shows the frequency displacement plots together with the modal shapes. Again the increase in the tool point rigidity at the second mode was attributed to the bed vibration.

Further experiments were carried out on the $B_B C_B T_B$ configuration to examine the effect of the preload at the joints and therefore check the feasibility of using preload as a tool for optimisation purposes. Figure 8 shows the receptance plots in argand plane for two values of preloading of the cross-slide-column joint. Here a decrease in joint stiffness of the cross-slide has significantly increased the rigidity at the second principal mode without causing any decrease in the rigidity at the first.

In reference (20) it is also shown that for any of the four configurations, damping ratios, depending upon the modal shapes can vary grossly from mode to mode. These results suggest that method I of response prediction is indeed inaccurate, and it is essential to find damping ratios according to the method proposed below with the help of the modal shapes. (Method II).

2.5 Proposed Approximate Method of Response Prediction

The method described below will demonstrate the basic pattern of progress towards a response prediction of structures containing joints.

The method dispenses with damping forces for the reason mentioned earlier namely uncertainties with regard to validity of linear differential equations of motion and also the frequency dependence of the parameters.

It is basically an approximate energy method which would give the effective damping ratios whilst the natural frequencies and modal shapes are found by iteration technique. The assumption of low damping is retained and therefore all the points on the structure will move in-phase or out of phase.

It can be assumed that the energy dissipated during one cycle of a massless system or, a single degree of freedom system oscillating at resonance frequency, is proportional to the maximum potential energy stored in the systems. The factor of proportionality, being a measure of effective damping ability of the system, is called specific damping capacity (ψ). It can be shown that ψ is related to the damping ratio of the single degree of freedom system ξ by

$$\xi = \frac{\psi o}{4\pi} \qquad 2.1$$

where ξ is the damping ratio and ψ_o the specific damping capacity at resonance (see Appendix 1).

The assumption of low damping will then allow extension of this analysis to any complex system oscillating at the mth mode as follows. Assuming the system is broken into p sub-systems, each sub-system having q generalised co-ordinates, then the damping ratio at the mth mode is given by:

$$\xi_m = \frac{1}{4\pi} \cdot \frac{D_m}{V_m} = \frac{1}{4\pi} \cdot \psi_m = \frac{1}{4\pi} \cdot \sum_{i=1}^{p} \gamma_{im} \sum_{j=1}^{q} \gamma_{jm} \cdot \psi_{ij} \qquad 2.2$$

Distribution of potential energies in the system γ can be computed from the modal shapes (assuming that the potential energies are proportional to the amplitude squared). If the specific damping capacities of the sub-systems were known, then ξ_m could be computed for each mode and the desired receptances could then readily be found in the same way as in method 1, i.e. the factored sum of the modal flexibilities.

If in equation (2.2) a joint is taken as one sub-system and is then itself divided into a number of units, provided no interaction exists amongst units, the specific damping capacity of the joint can likewise be computed in terms of the parameters of the elements for each mode from:

$$\psi_{ij} = \sum_{1}^{e} \psi_{ej} \cdot \gamma_{ej} \qquad 2.3$$

Where, again ψ_{ej} is the specific damping capacity of a unit when it is oscillating in the j co-ordinate and γ_{ej} is the potential energy distribution at the joint.

Therefore, ψ_{ej} must be determined for each of 'q' generalised co-ordinates (q = 6). For most practical purposes 'q' for the joints can be reduced to two or even one. For example, in the case of flat joints these are normal and tangential directions to the interface. The boundaries must be identified and then the values of ψ_{ej} and k_{ej} modified for these conditions.

The solution of the eigen-value (undamped) could first be found assuming constant-static stiffness for the joint. Generally this is not the case for lubricated joints - any contaminated joint interface experiences higher stiffness during oscillation[21]. Because of this effect the variation of inphase component of stiffness K_{ω} with frequency at the joints must also be determined alongside that of specific damping capacity.

By establishing the variation of stiffness versus frequency, a second iteration of the eigen-value problem will yield more accurate values of both resonance frequencies and modal shapes, hence the pressure distribution,which in turn, would give new values of inphase component of stiffness for joints.

Repeating the above procedure a few times will ensure the final solution. The results can then be used in equation (2.1) to find damping ratios which would eventually render complete response.

2.6 Proposed Approximate Method of Optimization

As long as an exact method of response prediction is not fully developed, either due to the lack of necessary data or the large volume of computation necessary, optimum distribution of mass, spring and damping obviously can only be aproximated.

The following describes an approximate method of optimization, which when fully developed could be carried out with the aid of a computer. Even though the method is general, particular reference is given to joints as a tool for optimization.

Procedure: (See Figure 1)

(i) Study the most commonly occurring Y-P plane with respect to the V, U and W axes (if the Y-P variation is large, consider the worst position with respect to the decisive modes). From previous performance tests one can gain an insight into which modes play a major part in the stability of the cutting operation.

(ii) Take a number of different machine configurations, i.e. settings c_1, c_2..... c_n.

(iii) Assuming rigid joints, solve eigen value problems to give resonance frequencies and modal shapes.

(iv) In the frequency range of interest for the machine, consider m number of modes which have maximum projection on the Y-P plane. It will be found that not more than two or three modes are required.

(v) Calculate the distribution of potential and kinetic energies of each component (or sub-system) at these modes.

(vi) Find the tool point modal compliances and directions, using equation (1) and taking into account only material damping.

(vii) Calculate COM values (Coefficient of Merit) for each c_1, c_2 ..... c_n, using the method as used in reference (1).

(viii) By varying the positions of modal flexibilities with respect to Y and P, find the trend of α_J and α_K for COM improvements.

(ix) Consulting the modal shapes and the energy distributions, instigate a design change employing joints as new sub-systems connecting the olds as follows:

Using the equilibrium principle of mechanical energy at resonance determine the flexibilities at the joints which would give the optimum orientations of the modes. The equilibrium principle will ensure that the steady state cutting performance of the machine, if not improved, will not appreciably deteriorate. During optimization a component with the highest percentage of potential energy could be stiffened, or a component with negligible percentage of potential energy could be made more flexible.

In the appropriate directions of co-ordinates, where increased flexibility is required, discrete damping sources in the form of polymeric films or gaskets sandwiched between joints faces or cut-outs within appropriate members could be provided, and their effect on the dynamic behaviour of the machine re-examined. A few iterations will ensure the optimum structure.

2.7 Summary and Conclusions

The distribution of stiffness and mass in conventional machine tool structures could in some cases be corrected to concur with optimum directional orientations of modal flexibilities.

Sufficient evidence has now been presented to bring to light a new facade of 'joint' as an effective tool for the optimization.

In the light of the preceding observations, and also the increasing evidence for the advantages that could be gained by using some 'synthetic joints', it should not appear too unreasonable, for future optimization methods, particularly when applied to modular techniques, to think in terms of 'adjustable end conditions' - ends which could be adjusted during installations of the machine or even for different settings and cutting conditions to give the best possible performance of the machine.

The fact that near-resonance working conditions areby no means rare in practice indicates that the need for including the damping in the response analysis in a systematic and also economic way is much to be desired. Modal analysis of such structures which have non-proportional distribution of damping is not possible at present. Since the damping values are low, even if this were possible there is no reason to believe that this would introduce less error than the approximate energy methods described here.

Damping ratios could be calculated from the corresponding modal shapes (equation 2.2) which would in turn result in a more accurate response than other methods where these values are only estimated and as shown in this report could be grossly erroneous. Whenever possible attempts should be made to increase the potential energies associated with the highly damped region of the structure, such as joints, with respect to the total potential energies at the decisive modes of vibration. The principle of "Equilibrium of Potential Energies at Resonance", analogous to "Spring in Series" for a static loading, is to be employed in order to ensure that no significant drop in static stiffness occurs.

The damping terms preferred in the study were also in terms of energy; this dispensed with frictional forces and hence were not bound to any particular regime of energy dissipation - it could vary with frequency independently and also be valid for non-linear systems. In the same way as the mechanisms of static approach have, to a large extent, been unveiled, the quantitative values of complex modulus of the joints must therefore be measured (within the working range of interest and in terms of energy) before attempting a theoretical model.

CHAPTER 3

STIFFNESS AND DAMPING OF CONTACTING SURFACES

3.1 Introduction

The following is a resumé of the present state of the art. To avoid repetition the author has omitted detailed historical survey, as such a survey could indeed be found in many theses and reports presented up to the present time (e.g. see References 22 - 24).

3.2 Normal Approach

3.2.1 Mechanisms in Action

A great deal of work has, in the past 35 years, been directed towards understanding the mechanisms in action within interfaces. The beginning of the investigation, however, dates back to 1699 - Amonton's laws of friction; their verification by Coulomb and its later development by Bowden and Tabor[25].

As far as joints in machine tool structures are concerned the first known investigation into the relationship between load and deflection of mating surfaces was made by Votimov[26]. His work has been followed in more recent times by Levina, Reshetov and Ostrovskii[8,27] who present similar equations of approach. The most extensive work on the subject is however attributed to Thornley, Connolly and Schofield who set out to study the normal stiffness of mating surfaces and their effect on the overall behaviour of bolted structural elements[28 - 35].

Non-linear behaviour of contacting surfaces in the normal direction is attributed to the changes which take place during loading and unloading in the dimensions, numbers and the state of the load carrying elements, namely asperities.

Examination of the load-deflection graphs given in (29) for static approach of contacting surfaces show that if the load does not exceed the maximum in the history of the joints then subsequent unloading and loading curves overlap. That is to say the joint behaves elastically but non-linearity is still present. The stiffness is found to increase with load and the permanent approach being dependent on the maximum load. (See figure 9).

The progressive stiffness characteristic and also the permanent 'set' at zero load are explained by a number of possible mechanisms in action. They are all the direct consequence of the fact that the true area of contact is very small compared with that of the apparent. The increase in stiffness with load has been explained by:

a. Increased area of contact by Amonton's law, i.e. plastic flow at tip of asperities.
b. Coming into contact of new asperities as the load is increased. Approach of the surfaces is attributed to the deflection of asperities (plastic and/or elastic)
c. Work-hardening of asperities. This effect has been reported to be negligible[36].

The residual deflection or 'permanent set' is therefore the result of plastic deformation of asperities.

Depending upon joint conditions the degree of importance of any of the above mechanisms varies from joint to joint. Generally speaking, it is true to say that during loading of the joints combined elastic and plastic deformation of asperities takes place and gives rise to increased dimensions and numbers of the asperities in contact.

3.2.2 Real Area of Contact

Frictional forces between contacting surfaces are believed to be proportional to the real area of contact[25]. There is a good deal of evidence to suggest that the normal stiffness of joints if not proportional, is also closely related to the real area of contact.[22]

Amonton's laws of friction can readily be explained by the flow of material at the tips of asperities - proportionality between the area of contact and the load follows immediately. However the same result has been obtained by Archard and Greenwood [37,38] with purely elastic theories based on the Herzian method (Timoshenko & Goodier, 1951). The former uses model asperities of superimposed hemi-spheres (multiple contact) whilst the latter introduces statistical distribution of asperity heights.

3.3 Normal Static Stiffness

(i) Dry Condition

In the study of normal stiffness at joints it is helpful to classify the joints into two groups; those whose manufacture involves single point tools (SPT) and others involving multi point tools (MPT). The SPT cut surfaces possess regular geometric textures whilst the MPT cut surfaces display more or less random irregularities.

Reported investigations on normal stiffness of joints are substantial but mainly of qualitative nature. The reason is two fold: one being discrepancies attributed to flatness errors especially with large areas, and the other, difficulties in defining surface texture.

Surface roughness was initially thought to have a decisive effect upon the joint behaviour and work was directed towards analysis of some conventional machines surfaces. Schofield[30] has used idealized forms of enlarged asperities in calculating plastic and elastic components of deflection. Further development of the method can be found in references(34 and 35). However it was interesting to discover that rough surfaces with regular textures, e.g. SPT cut surfaces which could be simulated more readily by mathematical models tended to behave almost independently of surface roughness and only dependent on pressure[39].

Of all the relationships proposed for pressure σ and approach λ, the following two are most commonly used:

Refs.(29,32) $\lambda = \frac{1}{b} \log_e \sigma + C_o$ $8<\sigma<480$ KP/cm^2 ($100<\sigma<6800 PSI$) 3.1

Ref.(8) $\lambda = c\sigma^m$ $0.5<\sigma<50$ KP/cm^2 ($7<\sigma<700$ PSI) 3.2

The basic difference between the two equations is that the rate of increase of stiffness per unit area with pressure for 3.1 remains constant, whilst for 3.2 gradually decreases with pressure:

Ref. (29) $$K_J = \frac{d\sigma}{d\lambda} = b\sigma \qquad 3.3$$

Ref. (8) $$K_J = \frac{\sigma^{(1-m)}}{cm} \qquad 3.4$$

Values of coefficient 'b' for a range of machines surfaces produced from mild steel and those of 'c' and 'm' for a range of finishing processes are given in Tables 1 and 2 respectively.

The difference in practical terms appears to be due to the fact that in reference (8) the pressures looked at were very low, areas too large and surfaces too smooth (scraped and ground) which makes it more vulnerable to flatness errors hindering reliable stiffness prediction. On the other hand the values of 'b' given in reference (29) do not account for density of contact spots and also material properties such as hardness. In references (34 and 35) these aspects have been covered. The results take into account the density of contacting spots and material properties but they appear to be valid only for surfaces with regular geometry of asperities with amplitudes sufficiently large to reduce error due to flatness deviations. It can therefore be concluded that the results must be good for most surfaces machined with single point tools.

In references (39 and 40) evidence can be found in support of the above conclusion where stiffness of turned and shot-blasted joints are found to increase almost linearly with pressure.

(ii) Lubricated Condition

Some loss of stiffness (about 18%) was reported in reference (6) after lubricating the joint surface, compared with the dry condition.

However Levina[8] reports on a slight increase of static stiffness for rough and lubricated joints. Obviously the only conclusion which can be drawn is that the effect of oil on the static stiffness of joints can be regarded as negligible (see also reference (28)).

3.4 Normal Static Hysteresis

As mentioned previously there is almost only one single elastic curve representing the loading and unloading. Therefore no static damping is traceable. Hysteresis attributed to either external or internal friction within the interfaces is therefore negligible.

3.5 Normal Dynamic Behaviour

Measurements of stress and strain across joints' surfaces by many investigators has shown that the strain always lags behind the stress to a greater and lesser degree depending upon the joints' condition. In other words energy is being dissipated.

(i) Clean and Dry Joints

The expression clean and dry is only a relative definition for the joints' interface; air will always be present between asperities and will give rise to the air pumping effect in the case of oscillation which would consequently amount for some damping[41]. Corbach[42] states dry joints behave similar to equivalent Voigt Units with frequency independent parameters (see Fig.10d).

(ii) Contaminated Joints

When interfaces are said to be contaminated, it is meant that either lubricant or some kind of viscoelastic material is present between the surfaces. Contaminated joints, as far as their dynamic behaviour is concerned are superior to dry joints as they introduce a major source of energy dissipation, increase the in-phase stiffness and if properly employed they could even increase static stiffness of joints[5,6].

Some research work on dynamic behaviour of joints suggest that both the in-phase and quadrature components of joints stiffness increase with frequency[33,39].

However Reshetov and Levina's findings[21] are to the contrary, i.e. the effect of frequency is reported to be insignificant (and also that of boundary effect).

Theoretical works on squeeze film assumes bulk flow of contaminant[43-46]. It is not true to say that this is the case for all joints. A theoretical model based upon the squeeze film between long narrow landings has been tried out by Waring[40] for quantitative assessment of in-phase and quadrature component of stiffness for a large and small (1.0 in^2. and 0.6 in^2.) sizes of flat joints with surfaces generated by shot-blasting. The main conclusion drawn is as follows:

The normal dynamic stiffness of an oil-filled rough joint cannot be attributed only to the bulk flow of the lubricant. It has been suggested that the stiffnesses arise from the sum of the effects of individual asperites together with the bulk movement of the fluid. It is therefore added that: 'statistical analysis of surface topography is necessary to predict the dynamic stiffness of machined joints. Brown[47] has proposed a simplified model simulating hydrostatic thrust bearings by a piston and a cylinder which is connected to a constant pressure reservoir by a flow resisting orifice (see Fig.10a). He checked his experimental results upon this model and good agreement was found.

The results from some fixed-joints were also checked on this model and the authors suggest that such a model could also be regarded as representative of such joints[39]. Indeed the model proposed by Brown being an approximate one for hydrostatic bearings would apparently represent a more accurate model for fixed joints - the hydrostatic effect K_h being small compared with that of Maxwell unit was linearized in the original model and placed in parallel with it; as this simplified the analysis (Fig. 10b). For fixed joints not only the K_{st} is often larger or of the same order as that of the film but because it is actually in parallel with it the model should be more accurate.

The equivalent values of spring stiffness and viscous coefficients would therefore be as follows:

$$K_\omega = \frac{C^2\omega^2 K}{C^2\omega^2+K^2} + K_{st} \qquad 3.3$$

$$C_\omega = \frac{C\,K^2}{C^2\omega^2+K^2} \qquad 3.4$$

Variation of in-phase and quadrature component of stiffness with frequency in non-dimensional form can be seen in Fig.10c. For the purpose of comparison Fig. 10d shows the same only for a Voigt unit with frequency - independent parameters (e.g. a dry joint).

The above results indicate that the mechanisms in action within contaminated joint interfaces cannot be pin pointed with any degree of accuracy. It can only be concluded that hydro-elastic-frictional forces produced within surfaces result from both bulk flow as well as accumulated similar effects over much smaller areas around individual asperities. The significance of any of the above mechanisms varies with joint condition. It is likely that sliding-joints follow a bulk flow pattern whilst fixed-joints (with higher pressure) that of the latter.

3.6 Tangential loading

Information on stiffness properties of machine tool joints in the tangential direction is limited when compared to that of the normal. The reason is due not only to the fact that they account for only small flexibilities but also their constant deterioration caused by fretting and corrosion, and its irreversible behaviour which makes reliable predictions difficult. However, Kirsanova[48] has produced empirical formulae for stiffness of cast-iron joints implying that the elastic part of deformation is unaltered under repeated loadings. It must be remembered that two types of displacements (slips) are recognised within joint interfaces: micro-slip and gross-slip. Micro-slip is a continuous mode of deformation - it takes place even before the tangential load reaches the coulombian frictional force[51]. Micro-slip has been studied by Masuko et al. for some bolted joints who also investigated their damping properties [49-51]. Gross-slip on the other hand is expected to follow the coulombian laws and to occur at relatively low contact pressures. It can therefore be concluded that depending upon pressure distribution, surface texture, etc., regions of deformations over the interface distinct from one another is created where different combinations of elastic, micro or gross slip occurs[36]. Earle et al.[52,53], using the gross-slip have made an extensive study of frictional dampers. The theoretical analysis employed however does not take into account the elastic deformations of asperities and therefore the reported increase in coefficient of friction with slip amplitude still remains unexplained. In reference (53) evidence can be found as to the departure of experimental curves from purely coulombian damping and their approach to purely hysteretic damping (i.e.from semi circles to full circles in an argand plane). This observation is in agreement with K.Tanaka's opinion which was put forward in the discussion of reference (50). He suggested that damping within the jointed conti-beam was due to elasto-plastic deformation of asperities which is of the type produced within materials (i.e.structural hysteric) rather than distinct coulombian type. Masuko et al.[51] have subjected a bolted joint to repeated static loading in tangential direction. They report on two types of hysteresis loops. One having a repeatable loop (ground mild steel and brass) and the other a progressive loop having a gradually decreasing width (ground cast iron). Reshetov and Levina[21] have measured specific damping capacity of a conical and a flat joint, the former under combined normal and tangential and the latter under pure tangential oscillations. An interesting result of their work was that the specific damping capacity for lubricated joint under pure tangential oscillation was about 7 - 10 times that of the dry joint. It can therefore be concluded that hydro-elastic-frictional forces play a much greater part in determining the damping capacity of the system than Coulombian or material damping.

The above observations lead to the general conclusion that mechanisms of energy dissipation in tangential direction like those for the normal direction cannot be pin pointed or generalised. They vary with the joint condition. The major source of energy dissipation must first be recognised through quantitative measurement of specific damping capacity under practical conditions before attempting any elaborate analysis in search of a mathematical model.

3.7 Summary and Conclusions

The solution for the static stiffness of joints is now at its final stages. A great deal of work has revealed stiffness characteristics of contacting surfaces. The author has compiled some 28 results on the normal static stiffness of different joints obtained by some 20 researchers throughout the world and is represented in E.L.S.* in Fig.11a. An interesting conclusion which is drawn from these hystogrammes is that the E.L.S. of surfaces cut by single point tools remains almost constant for all at around 5 inches at 250 PSI preload (12.7 cms at 17.6 KP/cm^2). This result is in a very close agreement with the figures proposed by Andrew in reference (39). These quantitative results, when used together with appropriate pressure-deflection relationship, will provide enough information for an iterative method of deformation - pressure distribution to converge to an end solution of deflections. A similar method was employed in references (54 and 55) using power laws proposed by Levina and Reshetov, in dealing with some model structural joints and agreements between theory and experiments appears to be good. The technique is to make use of a computer in dealing with finite elements and it is thought that at a later stage the flatness errors could also be

* E.L.S. Equivalent Length of Solid Mild Steel.

included in the pressure distribution considerations and hence modify the end deflections.

Damping within joints could be attributed to both rate-dependent and rate-independent mechanisms of energy dissipation, i.e. dynamic and static hysteresis. The latter could be of non-linear Coulombian type due to rubbing and/or linear hysteresis caused by quasi elastic deformation of material itself - hysteretic damping, sometimes referred to as structural damping, is thought to be negligible compared with the other mechanisms of damping. The rate dependent is due to oil or any contaminant trapped within the surface asperities and joint surfaces which give rise to 'squeeze film' mechanism and hence energy dissipation.

Assuming a linear rheological network (i.e. combination of Voigt and Maxwell units) representing joints' layers attempts were made by the author to compile quantitative values of stiffness factor and specific damping capacity in the normal direction for SPT cut surfaces from the work of others, but it was soon realised that as far as quantitative assessment of these values was concerned no reliable conclusions could be drawn, and the picture was even more blurred for other types of cut surfaces (see Fig.11b and 11c).

The effect of size, shape or in other words that of boundary conditions is not yet fully understood. Reshetov and Levina[21] report only slight change of ψ with these factors. Further study of these effects over a wider range of joints' condition is most urgently needed to help unveil the major mechanisms in action within the interfaces.

The boundary condition must also be distinguished for dry joints under tangential loading where the major part of damping is believed to result from the plastic flow, micro and macro slip.

In general, therefore, both in-phase and quadrature components of joint stiffness should be considered as complex functions of frequency (and amplitude) and hence the only valid damping measure must be in energy terms such as Specific Damping Capacity or factors directly related to it (i.e. loss factor) which are not bound to any particular regime of damping and therefore remain valid under all conditions. In the same way as numerous testing of static stiffness revealed the mechanism of approach, the Specific Damping Capacity alongside oscillatory in-phase stiffness must first be measured quantitatively for various conditions, before attempting any theoretical model.

CHAPTER 4

SPECIAL FEATURES OF THE MEASURING SET-UP

4.1 Introduction

Of all the characteristics of a vibratory system, damping is the hardest to evaluate accurately. The actual dissipative mechanisms are rather complex and do not usually confine themselves to one type or another. Apart from the experimental difficulties encountered in the classical methods of damping measurement'joints' introduce problems of their own. These problems will unveil themselves step-by-step in the following discussion and by considering the practical ranges of joints'parameters. Finally an optimum method will emerge from these discussions.

In order to be able to choose a suitable technique of measurement, firstly the distinction between linear and non linear damping will briefly be examined.

The following classification embraces only mechanisms of energy dissipation which have been discovered during the history of research in this field. It should therefore not be regarded as exhaustive (see reference (57)).

4.1.1 Dynamic Hysteresis

This type of damping, as its name suggests, is a rate dependent mechanism of energy dissipation. Sometimes identified as viscoelastic, rheological hysteresis is best representative of materials such as polymers. But anelasticity first named by Zener, can also be observed to some extent in other materials as well, i.e. metals.

The relation between strain and stress is essentially linear, involving time derivatives:

$$\left(a_o + a_1 \frac{\partial}{\partial t} + a_2 \frac{\partial^2}{\partial t^2} + \ldots + a_n \frac{\partial^n}{\partial t^n}\right)\sigma = \left(b_o + b_1 \frac{\partial}{\partial t} + b_2 \frac{\partial^2}{\partial t^2} + \ldots\ldots + b_n \frac{\partial^n}{\partial t^n}\right)\varepsilon \qquad 4.1$$

The theoretical model can readily be built from the above formula, comprising of various configurations of springs and dashpots depending upon the mechanism, or in other words the actual values of a and b. This model can then be reduced to a single Voigt unit whose stiffness and damping coefficients are generally frequency dependent. (e.g. see Fig. 10c). For such a system therefore, in general, the damping measures derived from band-width methods would be invalid. The particular cases where the band-width method would still be valid are when the damping forces are proportional to velocity or displacement. The former describes the Voigt unit with frequency independent parameters - the most convenient model as far as theoretical analysis is concerned. The latter, often referred to as 'hysteretic or structural damping', best describes the behaviour of materials commonly used in structures. It is interesting to note that for these cases $n \leqslant 1$ in equation 4.1.

4.1.2 Static Hysteresis

In contrast with dynamic hysteresism static hysteresis involves stress-strain laws which are insensitive to time but are essentially non-linear; the shape of force-deflection loops under sinusoidal excitation are no longer elliptical, as is the case for linear damping, but become pointed at the ends (Fig. 12b).

It must be mentioned that not all static hysteresis should necessarily be of non-linear type; for a system to possess non-linear damping the dissipation energy per cycle deformation will be of the form

$$D = c\sigma^n \quad \text{where } n \neq 2 \qquad 4.2$$

As an example internal damping due to material at low stresses could, for all practical purposes, be assumed to have n = 2, i.e. linear - (For mild steel at stress levels less than 8000 PSI n ≃2.3).

The basic mechanism giving rise to static hysteresis could be said to be generally of Coulomb type friction only with smaller or larger dimensional scale, when it is describing internal-material damping it is said to be due to magnetoelasticity or microscopic elasto-plastic strain, and when it is describing external damping it is believed to be due to plastic shear due to micro- or gross-slip at contacting asperities. As far as the methods of measurement for this type of damping is concerned the energy loss and also the maximum potential energy stored in one cycle could be measured from the hysteresis loops obtained from static loading of the specimen(58). The only dynamic method which could give meaningful results is the treatment of the system to steady harmonic loading. Interpretation of results for non-harmonic loading of a non-Voigtian system is not yet developed theoretically. Furthermore it must be emphasized that the response to harmonic loading in this discussion is assumed not to be appreciably affected by the non-linearity of the damping.

4.1.3 Methods of Measurement

The following methods have frequently been used in the past for measuring damping capacity particularly that of metals and viscoelastic materials. (e.g. see references 59 - 63).

(i) Free vibration methods

(ii) Forced Vibration methods

a. Non Resonance

b. Resonance

(iii) Energy method - Free/forced vibration at resonance

(iv) Propogation methods

a. Continuous waves

b. Pulses

Variations in the 'experimental arrangements' using any of the above methods are virtually boundless. But on the other hand, practical difficulties and requirements limit the choice to a very large extent. Some of these limiting factors are, the size, shape, the mechanism or predominant mechanism of damping, the magnitude of damping capacity, the effect of conditions such as temperature, stress level and history, frequency and mode of vibration, the specific objective, and the most desirable form of data.

The literature reveals that important achievements have been attained with simple inexpensive damping test apparatus, whereas some elaborate and costly installations appear to have been unproductive.

For the purpose of the present work, method (iv) is obviously ruled out - its application is only to material damping in high frequency ranges (normally in the order of KHz for continuous and ultrasonic frequences for pulse method).

Either method (i) or (ii) could be considered when testing linear spring-dashpot system, i.e. Voigt unit with frequency independent parameters. If the system comprises of a number of springs and dashpots, i.e. viscoelastic or frequency dependent parameters, then method (ii) is more advantageous as the frequency effect can be examined without off-setting the set-up. The excitation must be harmonic in order to be able to determine the effective Voigtian parameters from the response of non-linear types.

One major practical consideration in the choice of free or forced vibration is based upon the level of damping which is to be measured: due to the problems associated.with accurate measurements of small phase angles the free vibration method is normally recommended for loss factors below 0.1 (corresponding to about 5.7° phase angle between strain and stress) whereas forced vibration methods are best suited for testing highly damped systems. For the purpose of the present work, therefore, it is desirable to be able to employ both free and forced vibration methods of measurement. (See Fig.11b for the range of loss factor likely to be encountered in the measurement of contaminated joints in machine tools.)

Methods (i) and (ii) would only furnish both the stiffness and the damping if M the equivalent mass of the set-up is either zero or known exactly (i.e. ideal system). If the system cannot be regarded as ideal then the inertia forces can cause error, particularly at high frequencies. This is of course due to the fact that in off-resonance condition for a single degree of freedom system the total energy stored (i.e. potential and kinetic) varies during one cycle and therefore $\psi = \frac{-D_o}{V_o}$ becomes ambiguous (see Fig. 12a). It is only at the natural frequency that the damping capacity of the whole system equals that of the specimen.

For non-ideal conditions the three system parameters K, M and C could be determined by 'Energy Method', method (iii) as follows:

Free vibration method will first yield ψ of the specimen: (see Appendix 1 and Table 7).

$$\psi = 2(\frac{\omega_d}{\omega_n}.\delta) \simeq 2\delta \qquad 4.3$$

The stiffness is then determined from the forced vibration at the natural frequency

$$K = \frac{2V_o}{X_o^2} \qquad 4.4$$

where

$$V_o = \frac{-D_o}{\psi} \qquad 4.5$$

$$D_o = -\pi C\omega \; X_o^2 \; = \; -\pi F X_o \mathrm{Sin}\phi = -\pi F X_o \quad \text{and} \qquad 4.6$$

$$M = \; K\omega_n^2 \qquad 4.7$$

Sin ϕ could be determined by measuring the phase angle between the force and the displacement or using a velocity pick up and multiplying the velocity by force and measuring the D.C. Component of the product. The latter technique would have the advantage of avoiding difficulties associated with accurate phase angle measurement of distorted signals which could be caused by non-linearity[62].

Off-resonant forced vibration methods would only yield accurate quantitative results if the extraneous inertia forces could accurately be determined at such frequencies.

The main problem encountered in measurement of damping is caused by the usually inevitable presence of extraneous modes introduced by auxiliary system containing the specimen. The problem of mode coupling becomes more acute for highly damped systems with close natural frequencies where the 'ideal system' can no longer be simulated - the coordinate phasors will no longer be completely in (or out of) phase and hence the above techniques break down. It is therefore essential that in such cases un-coupling of the modes must be realised through the only means possible, i.e. multi-point excitation; the number of excitation points being the same as the number of degrees of freedom.

4.2 Measuring 'Yardsticks' - Dependent Variables

After careful examination of the results from previous works (chapter 3) it was decided that the design of experimentation be based upon the supposition that the whole system would not only behave non-linearly but possess frequency dependent parameters.

Any meaningful measure of damping for such a system could only be defined in terms of energy, so as to be independent of any particular regime of energy dissipation. One such measure is 'Specific Damping Capacity' defined as the ratio of the energy dissipated to the maximum potential energy stored during one cycle. Depending upon whether the system can be simulated as ideal (massless) or not the 'Specific Damping Capacity' measurement would yield complete information regarding the specimen parameters at all the frequencies or only at the natural frequency of the system respectively. (see Appendix 1 and Fig. 12a). It also follows automatically that pure mode excitation becomes an essential requirement for energy methods, should the measuring arrangement exhibit multi-degree of freedom features.

The most desired parameter representing damping as far as ease in handling mathematical packages is concerned is the viscous coefficient which lends itself to simple mathematical analysis i.e. resulting in linear differential equations of motion. Such a parameter is associated with a'Voigt unit' with effective values at different frequencies (and amplitudes).

Effective values of specific damping capacity (ψ_ω) and oscillatory spring stiffness (K_ω) (or parameters directly related to them) if known alongside the frequency of excitation effective Voigtion parameters could readily be found. It was also decided that these parameters should be evaluated for unit area of joint in order to introduce 'area' as an independent variable in its own right.

4.3 Independent Variables

The variables known, or expected to affect the stiffness and damping of joints to a greater or lesser degree are listed over:

(i) Surface texture

(ii) Pressure

(iii) Area

(iv) Shape

(v) Properties and quantity of intermediate film

(vi) Properties of asperites

(vii) History of Joint and environmental factors

(viii) Frequency, and

(ix) Amplitude.

The levels of the variables for the present study were chosen by consulting the different test conditions and results from previous works (Fig.11) and the practical conditions in machine tool structures. The complete list of these levels used in actual testing can be found in chapter 14. As far as design of method of measurement and apparatus were concerned however, the following three parameters had to be determined in advance:

4.3.1 The Range of Area: $3 \leqslant \Delta \leqslant 13$ cm.^{2} ($.5 \leqslant \Delta \leqslant 2$ in.^{2})

In order to obtain, as nearly as possible, uniform pressure distribution, the area of joint layers must be taken as small as practically possible; as any method of machining or processing of surfaces will introduce random flatness or waviness errors, resulting in non-uniform distribution of pressure.

On the other hand if the area is taken too small, difficulties could arise in accurate phase angle or damping measurements. This may be due to the fact that the reduction in quadrature component of stiffness, by reducing joint area, is much sharper than that of the inphase. This problem was observed in reference (39) when testing small machined surface (3.2 cm^2). The theoretical model proposed by Brown[47] also predicts this to be the case in the frequency range of interest, i.e. near to the origin in Figure 10c.

Considering the evidence given in reference (39) that areas larger than 13Cm^2 would probably contain waviness errors, it seemed advisable that the range of areas to be considered for the present work to be $3 \leqslant \Delta \leqslant 13$ cm^2.

4.3.2 The Range of Preloading Pressure:
$0.1 \leqslant \sigma \leqslant 224.$ Kp/cm^2 $(1 \leqslant \sigma \leqslant 3185$ PSI)

The next step was to select the pressure range hence obtaining an estimate of stiffness which could in turn be used to help the design of an appropriate testing method.

(i) Fixed Joints

The maximum level of the apparent preloading pressure of the joints was determined by examination of some existing machine tool joints. Figure 13 shows the design of the bed-column fixation of a Köllman,A.G. plano-miller, a Billeter, A.G. and a Kinoshita Iron work planer , all of arch-type structure[64].

The preload Q produced by tightening a bolt up to maximum practical level is given by:

$$Q = 0.5\ Q_u \qquad 4.8$$

$$Q_u = A_b \cdot \sigma_u \qquad 4.9$$

where

A_b is the cross-sectional area of the bolt at its minor diameter

σ_u is the ultimate tensile strength of the material of the bolt.

$$\sigma_m = \frac{Q}{A_R} = \frac{0.5\ \sigma_u .\ A_b}{A_R}$$

where

σ_m is the maximum mean preloading pressure

A_R is the area on the interface bound by Rötscher's Cone.

Table 3 shows the variation of σ_m for the four different materials of the bolts for the above mentioned machines.

It is believed that the pressure values given in Table 3 are hard to achieve in practice unless some special means of tightening of the bolts is employed. On the other hand these values are only the mean values in the Rötscher's Cone not the maximum. It was therefore decided to use them as the maximum preloading pressures likely to be encountered in machine tool joints.

(ii) Sliding Joints

The following values for σ_{max} are compiled from the sources indicated. These figures are those of design data.

USSR NORM H49-2 (Ref.65)

σ_{max} on cast iron slideways

Lathe, Milling machine σ_{max} = 25-30 [KP/cm^2]

Planer, Shaper	$\sigma_{max} \simeq 8.0 [KP/cm^2]$
Large size machines operating at high sliding velocity	$\sigma_{max} < 4.0$
Large size machines operating at low sliding velocity	$\sigma_{max} < 10.0$
Special machine tools built for very heavy cutting conditions	$\sigma_{max} = 0.75$

σ_{max} for cast iron VS steel same as cast iron VS cast iron

σ_{max} for steel VS steel $\sigma_{max} = 1.2, 1.3$ times cast iron VS cast

The following data gives the <u>actual</u> values of contact pressure in the slideways.

(a) Presented by Mr.Saito in Yamaguchi University (66)

Small sized lathe	$\sigma_{max} \simeq 0.3 \sim 0.5 [KP/cm^2]$
Grinding machine	$\sigma_{max} = 0.4 \sim 0.65$
Hobbing machine	$\sigma_{max} = 0.2 \sim 0.3$

(b) Presented by one machine tool manufacturer in Japan (IKEGAI IRON WORKS)

8 feet Lathe (∿2.4 m)	non-cutting condition	0.1 ∿ 1.0 [KP/cm^2]
	max.cutting condition	1.3 ∿ 5.0
6 feet lathe (∿1.8 m)	non-cutting condition	0.07 ∿ 0.90
	max.cutting condition	0.7 ∿ 4.1

(c) According to Corbach's observations (67), 1 - 6 [KP/cm^2] has been shown to cover most cases under practical conditions.

4.3.3 <u>The Range of Frequency</u>: up to 2000 [Hz]

In previously reported research of this nature the main factor limiting the frequency band has been the mode coupling due to extraneous effects. In the works reported, the average frequency has rarely been above 150 [Hz]. Machine tools may of course be found to experience decisive modes at much higher frequencies. 0-500 [Hz] is thought to be the most susceptible range due to the fact that within this range most machines experience their lower principal modes which correspond to higher receptances. The frequency band of reliable testing will therefore be widened in the present study not only for the aforementioned reason but in order to gain a clear insight into the mechanism of energy dissipation.

4.4 Summary and Conclusions

The next stage of this work was therefore to instigate a method of Complex Modulus measurement, best suited for testing contacting surfaces in particular, but with the maximum possible versatility, i.e. with the maximum dynamic (and static) range of measurement enabling tests on the other materials and net-works, e.g. rubbers etc. The direction normal to the joint interface was considered as the principal loading direction but attempts were to be made in design to facilitate easy conversion to measurements in other directions such as tangential torsional, etc. and also other types of tests such as fatigue test etc.

Referring to Figure (11a); at about 17.6 KP/cm^2 (250 PSI) pressure for S.P.T. cut surfaces the E.L.S. could be taken as 12.7 cm. (5 inches). This would give an estimated stiffness of about 26,773 MN/m.(153x10^6 lbf/in) for a joint surface of 12.9 cm^2 (2 in^2) at the maximum pre-loading pressure of 224 KP/cm^2 (3185 PSI). This figure was obtained without taking into account the effect of squeeze film which would have resulted in even a larger value (see Fig. 11c).

Design of a test rig to contain such a small area with such a high stiffness would be difficult;even if measurement did not create much problem the separation or data analysis due to the inevitable presence of extraneous effects particularly that of damping would practically be impossible. It is therefore desirable to build in an isolator to reduce the transmissibility hence the extraneous effects and also to be able to measure them (i.e. pure mode excitation) in order that appropriate corrections could be made of the results. In addition the stiffness can be reduced by placing a number of discs in series to form a column. Providing a stack of identical discs would give a further advantage of statistical representation of surface texture. As the areas were small the effect of flatness error and out of parallelism on transmitting tangential loads which could be envisaged due to tilting of discs would be minimal (chapter 15). The corrections for the extraneous stiffness and damping within the jointed column is relatively simple and is dealt with in chapter 6. The question as to whether the inertia forces of discs would allow forced vibration data analysis on the assumption of an ideal system to give a reliable result is also studied in chapter 6 and answered in the affirmative.

From the foregoing discussions it is evident that the new measuring technique must include the following features:

(a) Both uncoupled free, and harmonic forced vibration should be possible.

It is therefore essential that all points on the structure should move completely in or out of phase relative to one another (i.e. pure mode excitation).

The above feature enables measurement of damping in terms of specific damping capacity ψ to be carried out at any two points within the structure.

(b) Due to high stiffness of the joints a built-in isolator is considered necessary to minimize the extraneous effects. For increased accuracy these extraneous effects, however small should be evaluated so that they can be taken into account.

The direct implication of the first feature (free-vibration) is the resonance condition. That of the second feature is the inclusion of some low stiffness spring,as isolators with associated mass block, which also determines the natural frequencies. The final system should therefore be of the type shown in Fig. 14a. It is basically a 2 degrees of freedom system. The isolating springs also act as preloading devices. The first mode will give values of stiffness and damping of extraneous elements which could then be taken into account in the second mode to find the joints' stiffness and dampings alone. The advantage of such a system is that the size of vibrators can be reduced considerably due to the amplifying effect of the mass blocks as resonance is approached (see Fig. 14b). The load amplification factor in the frequency range of $0 - \sqrt{2}.\omega_2$ is always greater than unity. Depending upon the damping ratio ξ_2, this factor tends to reduce from 1 at $\sqrt{2}.\omega_2$ to zero as the frequency is increased. The rate of reduction in the load amplification factor for undamped systems, i.e. the worst case, is 12db/Octave. Due to the fact that lower values of ω_2 correspond to lower values of joint preload and at these preloads the damping ratio is expected to be high, the drop in loading efficiency even at very high frequencies would consequently not be significant.

Similarly the transmissibility to the foundation is reduced by a factor of approximately $\frac{K_1}{2K} \approx (\frac{\omega_1}{\omega_2})^2$ compared with a situation where the specimens are in direct contact with the foundation (see Appendix 2).

Different frequencies of resonance could be achieved by altering the number of discs in the column. As far as forced vibration method is concerned the bandwidth of the system around the second natural frequency will determine the level of load amplification and its useful range of frequency, i.e. where high enough load magnification is still maintained. It is,therefore,possible to encounter a situation where for any value of preload only a certain frequency range can be examined.

It must be noted that when testing joints, as mentioned earlier, this condition is not likely to occur frequently because of high damping properties of joints. Furthermore it is expected that the floating direction of the ω_2 in the frequency domain if at all significant, would coincide with the direction of change (see reference (39) and Fig. 10c around the origin) hence creating a self regulating mechanism of load amplification.

CHAPTER 5

EVALUATION OF EFFECTIVE PARAMETERS OF A DAMPED TWO DEGREES OF FREEDOM SYSTEM

5.1 Introduction

The ultimate aim behind this analysis is to devise a simple technique of measuring the effective values of Voigtian parameters of the specimen C and K. (see Fig.15a)

The analysis is arranged in such a manner as to help master some allusive concepts such as orthogonality of principal coordinates, uncoupled free and forced vibration in actual physical terms so that the knowledge gained from it could be used in design of both the test rig and the measuring technique.

The system will be treated to harmonic loading so as to assist interpretation of results without having to pre-empt the Voigtian behaviour of the system. The analysis would therefore be valid for non-linear systems as long as the response is not appreciably distorted by the harmonics. (see Fig.3)

Furthermore, nowhere during the generalised analysis has reference been made to bandwidth or parameters related to it. It therefore remains valid for systems with frequency dependent parameters,

5.2 Differential Equations of Motion

Kinetic, potential and dissipative energies are:

$$2T = M_1\dot{x}_1^2 + M_2\dot{x}_2^2 \quad 5.1$$

$$2V = K_1x_1^2 + K(x_1 - x_2)^2 + K_2x_2^2 \quad 5.2$$

$$2D = C_1\dot{x}_1^2 + C(\dot{x}_1 - \dot{x}_2)^2 + C_2\dot{x}_2^2 \quad 5.3$$

Differential equations of motion are derived from the Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) - \frac{\partial T}{\partial x_i} + \frac{\partial V}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} = F_i \quad 5.4$$

$$[M_1\ddot{x}_1+(C+C_1)\dot{x}_1+(K+K_1)x_1] + [-C\dot{x}_2-Kx_2] = F_1 \quad 5.4.1$$

$$[M_2\ddot{x}_2+(C+C_2)\dot{x}_2+(K+K_2)x_2] + [-C\dot{x}_1-Kx_1] = F_2 \quad 5.4.2$$

Or in matrix form

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} C+C_1 & -C \\ -C & C+C_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K+K_1 & -K \\ -K & K+K_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad 5.4.3$$

(It can be shown that for any linear system the above matrices are always symmetric)

5.3 UnCoupled Free Vibration

5.3.1 Normal or Principal Modes and Coordinates

The differential equations of motion 5.4.1 and 5.4.2 are said to be coupled because x_1 and x_2 appear in both; x_1 and x_2 are hence called generalized coordinates. Appropriate coordinate transformation for any undamped system can be performed to uncouple these equations.

The new coordinates are referred to as normal or principal coordinates.

(It can be seen from 5.4.3 that for uncoupled motion the symmetric matrices will have the leading diagonals zero.)

In order that the system may have uncoupled free vibration [C] must satisfy certain requirements which would lead to the same and the only normal modes of the undamped system[68]. The eigenvalue problem is therefore solved first to find these normal modes and then a practical condition is instigated where de-coupling under damped conditions would also follow:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K+K_1 & -K \\ -K & K+K_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad 5.5$$

But

$$\begin{aligned} x_1 &= A_1 p_1 + B_1 p_2 \\ x_2 &= A_2 p_1 + B_2 p_2 \end{aligned} \qquad 5.6.0$$

or

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} \qquad 5.6.1$$

where p_1 and p_2 are the normal or principal coordinates. The above set of equations are found by linear transformation of coordinates which is valid for any linear system. Note that the above transformation is also valid for a damped system only this time 'x' and 'p' are in general phased relative to one another (see reference (69) P.P.441).

5.3.2 Undamped Free Vibration

x_1 and x_2 are the coordinates which can be measured directly; the coordinates are therefore normalized as follows:

let $x_1 = p_1$ when $p_2 = 0$ $\therefore A_1 = 1$

let $x_1 = p_2$ when $p_1 = 0$ $\therefore B_1 = 1$

$$\begin{cases} x_1 = p_1 + p_2 \\ x_2 = A_2 p_1 + B_2 p_2 \end{cases} \qquad 5.6.2$$

In order to have uncoupled vibration the cross products in the kinetic and potential energy equations must be zero.

$$2T = M_1\dot{x}_1^2 + M_2\dot{x}_2^2 = M_1(\dot{p}_1+\dot{p}_2)^2 + M_2(A_2\dot{p}_1+B_2\dot{p}_2)^2 \qquad 5.7$$

$$2V=K_1x_1^2+K(x_1-x_2)^2+K_2x_2^2 = K_1(p_1+p_2)^2+K[(1-A_2)p_1+(1-B_2)p_2]^2$$
$$+ K_2(A_2p_1+B_2p_2)^2 \qquad 5.8$$

the cross products are:

$2M_1+2M_2A_2B_2$ and $2K_1+2K_2A_2B_2$ $+2K[(1-A_2)(1-B_2)]$

therefore

$$\begin{cases} M_1 + M_2A_2B_2 = 0 \\ K_1 + K_2A_2B_2+K(1-A_2)(1-B_2) = 0 \end{cases} \qquad 5.9$$

from which A_2 and B_2 are found to be:

$$\begin{cases} A_2 = \frac{1}{2}\{[(1+b)-a(1+c)]+[[a(1+c)-(1+b)]^2+4a]^{-\frac{1}{2}}\} \\ B_2 = -\frac{a}{A_2} \end{cases} \qquad 5.10$$

where

$$\begin{cases} \dfrac{M_1}{M_2} = a \\ \dfrac{K_1}{K} = b \\ \dfrac{K_2}{K} = c \end{cases} \qquad 5.11$$

The eigenfunction or modal shapes are therefore found. The next step is to find the eigenvalues or the natural frequencies.

Equation 5.7 and 5.8 without the cross-products are as follows:

$$2T = (M_1+A_2^2M_2)\dot{p}_1^2 + (M_1+M_2B_2^2)\dot{p}_2^2 \qquad 5.12$$

$$2V = [K_1+A_2^2K_2+K(1-A_2)^2]p_1^2 + [K_1+K_2B_2^2+K(1-B_2)^2]p_2^2 \qquad 5.13$$

and from 5.4, the Lagrange's equation, the differential equation of motion are derived in terms of the principal coordinate as:

$$\begin{cases} (M_1+A_2^2M_2)\ddot{p}_1 + [K_1+A_2^2K_2+K(1-A_2)^2]p_1 = 0 & 5.14 \\ (M_1+B_2^2M_2)\ddot{p}_2 + [K_1+B_2^2K_2+K(1-B_2)^2]p_2 = 0 & 5.15 \end{cases}$$

i.e. the uncoupled homogeneous linear differential equations; hence the eigenvalues are:

$$\begin{cases} \omega_1 = \{[K_1+A_2^2K_2+K(1-A_2)^2]/(M_1+A_2^2M_2)\}^{\frac{1}{2}} & 5.16 \\ \omega_2 = \{[K_1+B_2^2K_2+K(1-B_2)^2]/(M_1+B_2^2M_2)\}^{\frac{1}{2}} & 5.17 \end{cases}$$

And the initial conditions required for free vibration in a pure mode, i.e. simple harmonic motion can be found from equations (5.6.2) after having found A_2 and B_2 from equations 5.10.

Note: The eigenvalue problem is normally solved with the aid of a computer using matrices in dealing with systems which possess a large number of degrees of freedom[6,7,70]. The method employed here was chosen not only because the system possessed only two degrees of freedom hence affording easy solution, but mainly because it provided easier understanding of normal modes and coordinates.

5.3.3 Damped Free-Vibration

Linear transformation which de-couples the equations of motion must also de-couple the damping term if single mode vibration in a state of free vibration is desired, i.e. the cross product in the dissipation function should become zero if A_2 and B_2 found from equations 5.10 are used in it.

$$2D=[C_1+C_2A_2^2+C(1-A_2)^2]\dot{p}_1^2+[(C_1+C_2B_2^2+C(1-B_2)^2]\dot{p}_2^2+[2C_1+2C_2A_2B_2+2C(1-A_2)(1-B_2)]\dot{p}_1\dot{p}_2 \tag{5.18}$$

$$\begin{cases} M_1 + A_2B_2M_2 = 0 \\ C_1+C_2A_2B_2+C(1-A_2)(1-B_2) = 0 \end{cases} \tag{5.19}$$

$$\begin{cases} B_2 = -a/A_2 \\ A_2 = \frac{1}{2}\{[(1+d)-a(1+e)] + [a(1+e)-(1+d)]^2 + 4d]^{\frac{1}{2}}\} \end{cases} \tag{5.20}$$

where $\frac{C_1}{C} = d$, $\frac{C_2}{C} = e$

Comparing equations 5.10 and 5.20 it follows that either

(i) $c = e$, $b = d$ i.e. $\frac{C_1}{K_1} = \frac{C_2}{K_2} = \frac{C}{K}$ Proportional distribution of damping, or: 5.21

(ii) $a = 1$, $b = c$ and $d = c$, i.e. complete symmetry.

For this case $A_2 = 1$, $B_2 = -1$ and $C_1 = C_2$ 5.22

Proportional distribution of damping is only possible if specimen damping is of a particular type (e.g.structural) which defeats the object. Therefore it is only the second condition which does not place such a demand and should be attempted, i.e. symmetry. In practice, however due to manufacturing error etc. perfect symmetry might not be achieved hence coupling due to damping in equation 5.19 might become significant hindering uncoupled free vibration. Forced vibration (double point excitation) would then be the only means by which single mode excitation is possible. Double point excitation at natural frequencies in any case is required to inject proper initial conditions should ω_1 and ω_2 be too close to one another or damping too high.

5.4 Uncoupled Forced-Vibration

The energy equations for the principal coordinates of any vibratory system can be written in the form:

$$2T = a_1\dot{p}_1^2 + a_2\dot{p}_2^2 + \dots\dots\dots\dots + a_n\dot{p}_n^2 \qquad 5.23$$

$$2V = c_1p_1^2 + c_2p_2^2 + \dots\dots\dots\dots + c_np_n^2 \qquad 5.24$$

$$2D = b_1\dot{p}_1^2 + b_2\dot{p}_2^2 + \dots\dots\dots\dots + b_n\dot{p}_n^2 + 2b_{12}\dot{p}_1\dot{p}_2 + \dots\dots \qquad 5.25$$

Differential equations of motions are derived using Lagrange's equation.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{p}_r}\right) - \frac{\partial T}{\partial p_r} + \frac{\partial V}{\partial p_r} + \frac{\partial D}{\partial \dot{p}_r} = P_r \qquad 5.26$$

where $$p_r = \sum_{i=1}^{n} F_i \phi_{ir} \qquad 5.27$$

and $$\phi_{ir} = \frac{\partial x_i}{\partial p_r} \qquad 5.28$$

For the system under study n = 2 and ϕ_{ir} is determined from equation 5.6

$$(a_1\ddot{p}_1 + b_1\dot{p}_1 + c_1p_1) + b_{12}\dot{p}_2 = F_1 + A_2F_2 \qquad 5.26.1$$

$$(a_2\ddot{p}_2 + b_2\dot{p}_2 + c_2p_2) + b_{21}\dot{p}_1 = F_1 + B_2F_2 \qquad 5.26.2$$

where

$$a_1 = M_1 + A_2^2M_2$$

$$b_1 = C_1 + A_2^2C_2 + C(1-A_2)^2$$

$$c_1 = K_1 + A_2^2K_2 + K(1-A_2)^2$$

$$b_{12} = C_1 + C_2A_2B_2 + C(1-A_2)(1-B_2)$$

and

$$a_2 = M_1 + B_2^2M_2$$

$$b_2 = C_1 + B_2^2C_2 + C(1-B_2)^2$$

$$c_2 = K_1 + B_2^2K_2 + K(1-B_2)^2$$

$$b_{21} = b_{12} \qquad 5.29$$

(see equations 5.12, 5.13 and 5.18)

For harmonic double point excitation with the same frequency, p_1 and p_2 will also be harmonic with the same frequency. In general they all will have complex amplitudes (phasors) each containing information concerning the amplitude and the phase angle as follows:

$$F_1 = \bar{F}_1 e^{i\omega t} \quad \text{where} \quad \bar{F}_1 = F_1 e^{i\phi_1}$$

$$F_2 = \bar{F}_2 e^{i\omega t} \quad \text{where} \quad \bar{F}_2 = F_2 e^{i\phi_2}$$

and

$$p_1 = \bar{p}_1 e^{i\omega t} \quad \text{where} \quad \bar{p}_1 = p_1 e^{i\phi_3}$$

$$p_2 = \bar{p}_2 e^{i\omega t} \quad \text{where} \quad \bar{p}_2 = p_2 e^{i\phi_4}$$

Equations 5.26 will then be of the form:

$$\begin{cases} (c_1 - a_1\omega^2 + i\omega b_1)\bar{p}_1 + i\omega b_{12}\bar{p}_2 = \bar{F}_1 + A_2\bar{F}_2 & 5.30.1 \\ i\omega b_{12}\bar{p}_1 + (c_2 - a_2\omega^2 + i\omega b_2)\bar{p}_2 = \bar{F}_1 + B_2\bar{F}_2 & 5.30.2 \end{cases}$$

[Solution of the above set of differential equations can be found from the following equation derived for a n degree of freedom system under forced vibration in the direction of x generalized co-ordinates and corresponding generalized forces F:

$$\bar{x} = \sum_{s=1}^{n} \alpha_{rs} \bar{F} \qquad 5.31$$

where

$$\text{receptance } \alpha_{rs} = (-1)^{r+s} \frac{\Delta_{rs}}{\Delta} \qquad 5.32$$

and Δ_{rs} is the characteristic determinant Δ with row r and column s omitted (see reference (69), pp.434)]

In order that only a single mode appear at any given frequency the amplitude and phase relation between the forces should be adjusted as follows:

$$\bar{p}_2 = 0 \begin{cases} [(c_1 - a_1\omega^2) + i\omega b_1]\bar{p}_1 = \bar{F}_1 + A_2\bar{F}_2 \\ (i\omega b_{12})\bar{p}_1 = \bar{F}_1 + B_2\bar{F}_2 \end{cases} \qquad 5.33$$

$$\bar{p}_1=0 \begin{cases} i\omega b_{12}\bar{p}_2 = \bar{F}'_1 + A_2\bar{F}'_2 \\ [(c_2 - a_2\omega^2) + i\omega b_2]\bar{p}_2 = \bar{F}'_1 + B_2\bar{F}'_2 \end{cases} \qquad 5.34$$

5.5 Evaluation of Effective Voigtian Parameters of Specimen K and C

5.5.1 Forced - Vibration Methods

(i) Asymmetry: $K_1 \neq K_2$ and/or $M_1 \neq M_2$, i.e. $A_2 \neq 1$, $B_2 \neq -1$.

From 5.6.2 the generalized coordinates $\bar{x}_1$ and $\bar{x}_2$ in terms of principal coordinates are:

$$\begin{cases} \bar{x}_1 = \bar{p}_1 + \bar{p}_2 \\ \bar{x}_2 = A_2\bar{p}_1 + B_2\bar{p}_2 \end{cases} \qquad 5.6.3$$

The first pure mode is realised when $\bar{p}_2 = 0$, or

$$\begin{cases} \bar{x}_1 = \bar{p}_1 \\ \bar{x}_2 = A_2\bar{p}_1 \end{cases} \qquad 5.35$$

take $\bar{x}_1 = \bar{p}_1 = X_1$ real

from equation 5.33

$$\begin{aligned} \omega b_1 X_1 &= F_{1I} + A_2 F_{2I} \\ (c_1 - a_1\omega^2)X_1 &= F_{1R} + A_2 F_{2R} \\ \omega b_2 X_1 &= F_{1I} + B_2 F_{2I} \\ 0 &= F_{1R} + B_2 F_{2R} \end{aligned} \qquad 5.36$$

The second pure mode is realised when $\bar{p}_1 = 0$ or

$$\begin{cases} \bar{x}_1' = \bar{p}_2 \\ \bar{x}_2' = B_2 \bar{p}_2 \end{cases} \qquad 5.37$$

take $\bar{x}_1' = \bar{p}_2 = X_1'$ real

from equation 5.34

$$\begin{aligned} \omega b_2 X_1' &= F_{1I}' + B_2 F_{2I}' \\ (c_2 - a_2\omega^2)X_1' &= F_{1R}' + B_2 F_{2R}' \\ \omega b_{12} X_1' &= F_{1I}' + A_2 F_{2I}' \\ 0 &= F_{1R}' + A_2 F_{2R}' \end{aligned} \qquad 5.38$$

For the purpose of visual representation of above equations figures 16 and 17 were constructed which were also to serve as a guide in actual testing, i.e. pure mode excitation.

Replacing the system parameters from 5.29 into equations 5.36 for the first pure mode:

$$C_1 + (1-A_2)^2 C + A_2^2 C_2 = \frac{F_{1I} + A_2 F_{2I}}{\dot{X}_1} \qquad 5.39$$

$$K_1 + (1-A_2)^2 K + A_2 K_2 - \omega^2 M_1 - A_2 \omega^2 M_2 = \frac{F_{1R} + A_2 F_{2R}}{X_1} \qquad 5.40$$

$$C_1 + (1-A_2)(1-B_2) C + A_2 B_2 C_2 = \frac{F_{1I} + B_2 F_{2I}}{\dot{X}_1} \qquad 5.41$$

and for the second pure mode into equation 5.38

$$C_1 + (1-B_2)^2C+B_2^2C_2 = \frac{F'_{1I} + B_2 F'_{2I}}{\dot{X}'_1} \qquad 5.42$$

$$K_1 + (1-B_2)^2K + B_2^2K_2-\omega^2M_1-B^2\omega^2M_2= \frac{F'_{1R} + B_2 F'_{2R}}{X'_1} \qquad 5.43$$

$$C_1 + (1-A_2)(1-B_2)C+A_2B_2C_2 = \frac{F'_{1I} + A_2 F'_{2I}}{\dot{X}'_1} \qquad 5.44$$

and from equations 5.9, the eigenvector equations

$$M_1 + A_2B_2M_2 = 0 \qquad 5.45$$

$$K_1 + (1-A_2)(1-B_2)K + A_2B_2K_2 = 0 \qquad 5.46$$

where

$$A_2 = \frac{X_2}{X_1} = -\frac{F'_{1R}}{F'_{2R}} \qquad 5.47$$

and

$$B_2 = \frac{X'_2}{X'_1} = -\frac{F_{1R}}{F_{2R}} \qquad 5.48$$

There are four equations in terms of K_1, K, K_2, M_1 and M_2 (equations 40, 43, 45 and 46) and also four equations in terms of C_1, C and C_2 (equations 39, 41, 42 and 44). Equations 41 and 44 are in fact one, as the coupling effect for a linear system would be identical at either mode, i.e. $b_{12} = b_{21}$.

Theoretically, therefore, the first set of simultaneous equations in terms of stiffness and mass, would, under forced vibration, be indeterminate but the second set of equations in terms of dashpot constants could be solved provided the system is forced to vibrate separately at the first and the second mode.

Practically, however, it could be said that both the above sets are indeterminate equation 41 (or 44) is expected to be of little practical use as it represents a small difference which would be hard to measure accurately. The small difference would be due to proportional distribution of damping and/or near symmetry. Hence one more equation is required for each set to render them determinate. This can be achieved by having one further measurement of force at the specimen column, i.e. $\bar{F}$ measurement (see Fig. 15b). Such a measurement is generally not considered permissible during the actual tests as the size and shape of the specimen under test might not allow it. However, it can be carried out under a specially designed calibration arrangement for evaluating the auxiliary parameters K_1, M_1,C_1 and K_2,M_2,C_2 at different frequencies (and amplitudes). This information could then be used, under actual testing of specimens, at the second pure mode to determine C and K from equations 42 and 43.

The following set of five simultaneous equations is to be used when F measurements are carried out. Such a measurement produces the three equations 49, 50 and 51, in place of 40 and 43 which consequently become redundant. Equations 49, 50 and 51 together with the two equations 45 and 46 (Eigenvector equations derived as 5.9) will constitute five equations in terms of the five unknown parameters of stiffness and mass, As for damping factors equations 52, 53 and 54 will directly give C_1, C and C_2.

It is interesting to note that when employing F measurement only a single mode need be excited. This is of great practical value when the two natural frequencies NF_1 and NF_2 are too far apart; under such a condition, excitation of a mode at frequencies close to the NF of the other mode would require very large and accurately balanced loads.

• From 5.9

$$M_1 + A_2B_2M_2 = 0 \qquad 5.45$$

$$K_1 + (1-A_2)(1-B_2)K + A_2B_2K_2 = 0 \qquad 5.46$$

and from Fig. 15b.

$$K_1-M_1\omega^2 = \frac{F_{1R}+F_R}{X_1} \qquad 5.49$$

$$K = \frac{F_R}{X_2-X_1} \qquad 5.50$$

$$K_2-M_2\omega^2 = \frac{F_{2R}-F_R}{X_2} \qquad 5.51$$

and also

$$C_1 = \frac{F_{1I}+F_I}{\dot{X}_1} \qquad 5.52$$

$$C = \frac{F_I}{\dot{X}_2-\dot{X}_1} \qquad 5.53$$

$$C_2 = \frac{F_{2I}-F_I}{\dot{X}_2^2} \qquad 5.54$$

F is positive in tension and negative in compression.

<u>Note:</u> For proportionally distributed damping where the coupling term (equations 5.41 or 5.44) is zero and $\frac{C_1}{K_1} = \frac{C_2}{K_2} = \frac{C}{K}$ a further equation in terms of stiffnesses can be derived from equations 5.39 and 5.542, hence F measurement becomes redundant.

(ii) <u>Stiffness and Mass Symmetry: $K_1 = K_2$, $M_1 = M_2$ but $C_1 \neq C_2$</u>

From equations 5.45 and 5.46 it follows that $A_2 = 1$ and $B_2 = -1$; Consequently $X_2 = X_1$, $\bar{F}_{2R} = \bar{F}_{1R}$ at the first and $X_2' = -X_1'$, $\bar{F}_{2R}' = -\bar{F}_{1R}'$ at the second mode (equations 5.47 and 5.48).

The most important feature of this condition is that K_1 and M_1 need not be known separately but it is sufficient to evaluate $(K_1 - M_1\omega^2)$. This implies that F measurement becomes redundant which itself results in a further advantageous condition where $(K_1 - M_1\omega^2)$ is found under the actual test condition. Hence it need not be known in advance (unless excitation of the first mode becomes unpractical, e.g. at very high frequencies where for the required amplitude of vibration the shaker unit capacity is exceeded) or conversely if the auxiliary parameters are known in advance it is possible to calibrate the transducers quite simply at the first mode before proceeding to the actual tests at the second. (See Chapter 11).

The following sets of equations reduced from equations 39, 40 and 41, are used to evaluate the effects due to the auxiliary parameters (or for calibration of transducers):

$$C_1 = \frac{F_{1I}}{\dot{X}_1} \qquad 5.55$$

$$K_1 - M_1\omega^2 = \frac{F_{1R}}{X_1} = \frac{F_{2R}}{X_1} \qquad (\text{as } \bar{F}_{1R} = \bar{F}_{2R}) \qquad 5.56$$

$$C_2 = \frac{F_{2I}}{\dot{X}_1} \qquad 5.57$$

The following equations reduced from 42 and 43 would furnish the specimen parameters:

$$C_1 + 4C + C_2 = \frac{F'_{1I} - F'_{2I}}{\dot{X}'_1} \qquad 5.58$$

$$(K_1 - M_1\omega^2) + 2K = \frac{F'_{1R}}{X'_1} = \frac{-F'_{2R}}{X'_1} \text{ (as } F'_{1R} = -F'_{2R}) \qquad 5.59$$

Equation 44 is similarly reduced to

$$C_1 - C_2 = \frac{F'_{1I} + F'_{2I}}{\dot{X}'_1} \qquad 5.60$$

which represents small differences of relatively large values hence not of great practical value.

(iii) 'Complete Symmetry': $K_1 = K_2$, $M_1 = M_2$ and $C_1 = C_2$ (see Fig.15c)

From equations 5.45 and 5.46 it follows that: $A_2 = 1$, $B_2 = -1$ and $b_{12} = 0$; consequently $X_2 = X_1$, $\bar{F}_2 = \bar{F}_1$ at the first and $X'_2 = -X'_1$, $\bar{F}'_2 = -\bar{F}'_1$ at the second mode (equations 5.47, 5.48, 5.41 and 5.44).

The extra feature of this condition, compared to that of the stiffness and mass symmetry is its capability to undergo uncoupled free vibrations which is an important requirement in accurate measurement of parameters from the decay curves; provided K_1 and M_1 are known separately.

As it will be shown in the next few pages, the separation of K_1 and M_1 from $(K_1 - M_1\omega^2)$ is in fact possible without the use of F measurement but under a combined forced/free vibration at the natural frequencies of the system (The Energy Method).

Equations 5.39, 40 and 41 are now reduced to

$$C_1 = C_2 = \frac{F_{1I}}{\dot{X}_1} = \frac{F_{2I}}{\dot{X}_1} \qquad 5.61$$

$$K_1 - M_1\omega^2 = \frac{F_{1R}}{X_1} = \frac{F_{2R}}{X_1} \qquad 5.62$$

which when used in the following equations (reduced from equations 5.42 and 5.43) would furnish the specimen parameters at the second mode:

$$2C + C_1 = \frac{F'_{1I}}{\dot{X}'_1} = \frac{-F'_{2I}}{\dot{X}'_1} \qquad 5.63$$

$$(K_1 - M_1\omega^2) + 2K = \frac{F'_{1R}}{X'_1} = \frac{-F'_{2R}}{X'_1} \qquad 5.64$$

5.5.2 Free-Vibration Method

For the purpose of accuracy in extracting information from the decay curves, this method requires uncoupled free-vibration. This condition is realised when the coupling term due to damping is eliminated, i.e. $b_{12} = b_{21} = 0$. Equations 5.21 and 5.22 indicate that the above requirement would be satisfied if the dampings within the system were proportionally distributed according to the stiffnesses, or the system were completely symmetric.

(i) Complete Symmetry: $K_1 = K_2$, $M_1 = M_2$, $C_1 = C_2$, i.e. $A_2 = 1$, $B_2 = -1$, $b_{12} = 0$

The coefficients in the differential equations of motion in terms of the principal coordinates as given in equations 5.29 are modified for this condition as follows:

$$a_2 = 2M_1$$

$$b_1 = 2C_1$$

$$c_1 = 2K_1$$

$$b_{12} = 0 \qquad 5.65$$

and

$$a_2 = 2M_1$$

$$b_2 = 2C_1 + 4C$$

$$c_2 = 2K_1 + 4K$$

From equations 5.26.1 and 5.26.2

$$\begin{cases} M_1\ddot{p}_1 + C_1\dot{p}_1 + K_1p_1 = \frac{1}{2}(F_1+F_2) \\ M_1\ddot{p}_2 + (C+2C)\dot{p}_2 + (K_1+2K)p_2 = \frac{1}{2}(F_1-F_2) \end{cases} \qquad 5.66$$

The normalized coordinates in equation 5.6.2 are further simplified as follows:

$$\begin{cases} \bar{x}_1 = \bar{p}_1 + \bar{p}_2 \\ \bar{x}_2 = \bar{p}_1 - \bar{p}_2 \end{cases} \qquad 5.67$$

The first pure mode is excited when $\bar{p}_2 = 0$, therefore $\bar{x}_1 = \bar{p}_1$ and also from equations 5.39 and 5.48: $F_1 = F_2$ or:

$$M_1\ddot{x}_1 + C_1\dot{x}_1 + K_1x_1 = F_1 \qquad 5.68$$

The second pure mode is excited when $\bar{p}_1 = 0$, therefore $\bar{x}_1 = \bar{p}_2$ and also from equations 5.44 and 5.47: $F_1' = -F_2'$, or

$$M_1\ddot{x}_1' + (C_1+2C)\dot{x}_1' + (K_1+2K)x_1' = F_1' \qquad 5.69$$

Equations 5.68 and 5.69 represent two equivalent single degrees of freedom as shown in Fig. 15d. The treatment of S.D.F. system is given in Appendix 1 for the purpose of reference.

Generally speaking the free vibration methods on their own will not render sufficient information to determine the system parameters. However if the auxiliary parameters K_1, C_1 and M_1 were determined in advance with the aid of forced/free vibration at the first pure mode, C and K could readily be found from the free vibration method at the second pure mode:

$$\omega_2^2 = (K_1+2K)/M_1 \qquad K = \frac{M_1\omega_2^2-K_1}{2} \tag{5.70}$$

$$C_1+2C = \zeta_2\omega_2M_1 \qquad C = \frac{1}{2}(\zeta_2\omega_2M_1-C_1) \tag{5.71}$$

where from Table 7

$$\zeta_2 = \frac{\omega_d}{\omega_2} \cdot \frac{\delta_2}{\pi} \simeq \frac{\delta_2}{\pi}$$

One particular case when the auxiliary parameters are frequency independent has much practical use:

$$K = \frac{M_1}{2}(\omega_2^2-\omega_1^2) \tag{5.72}$$

$$C = \frac{M_1}{2}(\zeta_2\omega_2-\xi_1\omega_1) \tag{5.73}$$

Evaluation of the Auxiliary Parameters K_1, M_1 and C_1

In contrast to the forced-vibration method where only $(K_1-M_1\omega^2)$ and C_1 would be sufficient to account for the auxiliary paremeters effect, infree-vibration method K_1 and M_1 should be known individually alongside C_1.(See equations 5.70 and 5.71)

Frequency Independent Auxiliary Parameters

(a) The Energy Method

A combined forced/free vibration method at the first undamped/damped natural frequency of the system would render K_1, M_1 and C_1 as follows:

Free-decay curve will yield ψ_1 by measuring the logarithmic decrement δ_1. (see Appendix 1 and Table 7);

$$\psi_1 = \frac{\omega_d}{\omega_1}.\delta_1.2 \simeq 2\delta_1 \qquad 5.74$$

K_1 and M_1 are determined from the forced vibration at the first 'undamped-natural frequency' ω_1:

$$K_1 = \frac{2V_o}{X_o^2} \qquad 5.75$$

where

$$V_o = \frac{-D_o}{\psi_1} = \frac{\pi F_o X_o}{\psi_1} \qquad 5.76$$

and

$$M_1 = \frac{K_1}{\omega_1^2} \qquad 5.77$$

C_1 is determined from

$$C_1 = \psi_1 . \frac{\sqrt{K_1 M_1}}{2\pi} = \psi_1 . \frac{M_1 \omega_1}{2\pi} = \psi_1 . M_1 . f_1 \qquad 5.78$$

The accuracy of the above technique is largely dependent upon that of input energy measurement. If $C \to 0$ as indeed is the desired and expected case, D_o measurement might prove impractical.

(b) Band-Width Method

This method requires exact location and measurement of the first natural frequency and two frequencies at either side of the natural frequency where amplitude remains at a constant level relative to it (The exciting forces are kept constant). At -3db level, the damping ratio is gound from:

$$\xi_1 = \frac{\Delta\omega}{\omega_1} \qquad 5.79$$

K_1 and M_1 are determined from equation 5.62.

If $C_1 \to 0$ this method could also prove impractical. In this case the free-vibration method should be employed:

$$C_1 = \zeta_1 \omega_1 M_1 \qquad 5.80$$

where

$$\zeta_1 = \frac{\omega_d}{\omega_1} \cdot \frac{\delta_1}{\pi} \simeq \frac{\delta_1}{\pi}$$

Frequency Dependent Auxiliary Parameters

Generally speaking, if the effect of frequency were significant the only method of measurement which would be valid and render all the auxiliary parameters is the 'Energy Method' at the first natural frequency of the system. The fact that the 'Energy Method' requires natural frequency excitation it places a limitation on the investigation of the frequency effect. However, it is theoretically feasible to employ the second natural frequency at which M_1 is determined whilst C_1 and K_1 are evaluated at the first mode from equations 5.61 and 5.62 respectively:

From 5.63 and 5.71

$$2C + C_1 = \frac{F'_{1I}}{\dot{X}_1} = -\frac{F'_{2I}}{\dot{X}'_1} \qquad 5.63$$

$$2C + C_1 = \zeta_2\omega_2 M_1 \qquad 5.71$$

$$\therefore \; M_1 = \frac{2C+C_1}{\zeta_2\omega_2}$$

where ζ_2 is determined from free-decay curve.

$$C_1 = \frac{F_{1I}}{\dot{X}_1} = \frac{F_{2I}}{\dot{X}_1} \qquad 5.61$$

$$K_1 - M_1\omega^2 = \frac{F_{1R}}{X_1} = \frac{F_{2R}}{X_1} \qquad 5.62$$

The above technique implies the use of specimens with varied stiffness in order to cover the required range of frequency.

(ii) <u>Proportionally Distributed Damping</u>

$K_1 \neq K_2$ and/or $M_1 \neq M_2$, but $\frac{C_1}{K_1} = \frac{C_2}{K_2} = \frac{C}{K}$

The above condition results in $A_2 \neq 1$, $B_2 \neq -1$, but $b_{12} = b_{21} = 0$. It can be seen from equations 5.26.1 and 5.29 that unlike the system with complete symmetry, the auxiliary parameters cannot be determined from the 'energy method' at the first natural frequency alone. The 'energy method' at the second natural frequency must also be employed to furnish three more equations which together with the two eigenvector equations of 5.9 will render all parameters determinate (i.e. eight equations and eight unknowns). This procedure, of course, assumes that the parameters are frequency independent. In the general case where the parameters are frequency dependent the forced vibration method should be employed (see the 'Note' for asymmetry).

5.6 Summary and Conclusions

5.6.1 Orthogonality of Principal Coordinates-Uncoupled Free Vibration

Equations 5.30.1 and 5.30.2 indicate that unless b_{12}(coupling term due to damping) is zero neither $\bar{p}_1$ nor $\bar{p}_2$ could, under free vibration, exist on their own. In other words if $\bar{F}_1 = \bar{F}_2 = 0$ and say $\bar{p}_1 = 0$; $\bar{p}_2$ must also be zero as $b_{12} \neq 0$ (trivial solution). The free-vibration would therefore in this case consist of two harmonic motions of different frequencies ($\sim \omega_1$ and ω_2); that is to say a decaying periodic rather than harmonic motion hence making the analysis of results from the decay curves inaccurate.

On the other hand if $b_{12} = 0$, $\bar{p}_1$ and $\bar{p}_2$ would literally have no 'projection' upon one another hence referred to as 'orthogonal'; therefore $\bar{p}_1$ and $\bar{p}_2$ can exist independently implying that $\bar{p}_1$ or $\bar{p}_2$ can be set to zero by injecting appropriate initial conditions given in equations 5.6.2.

The prerequisite of free-vibration exhibiting simple harmonic motion is therefore that no 'coupling' due to damping exists within the system. This condition is satisfied only if the principal coordinates are orthogonal, i.e:

a) either the damping is proportionally distributed, or

b) the system is completely symmetric.

5.6.2 Multi-Point Excitation

Equation 5.30 also indicates that single point excitation will in general excite both modes unless:

(i) principal coordinates are orthogonal, i.e. $b_{12} = 0$, and

(ii) the loading occurs in the direction of a principal coordinate, i.e. $\phi_{ir} = 0$ (equation 5.28).

An example of such an arrangement is the axial loading of the joint column, a 'uniform system' theoretically shown to possess uncoupled equations of motion along the axis of the column (See Fig. 18b). In practice however normal coordinates rarely manifest themselves, therefore single point loading will have projections upon the other normal coordinates hence causing impurity - the coordinate phasors will be phased relative to one another hindering the quantitative analysis of the results.

Double forcing along the two generalized coordinates x_1 and x_2 is therefore necessary in order to excite the pure modes which would not only create appropriate initial conditions for free-vibration method but also make it possible to account for the auxiliary system effects in exact quantitative terms.

As a general guide and quick reference to methods of testing and data reduction Tables 8a and 8b were constructed which are in fact the summary of the preceding analyses and therefore no further elaborations will be necessary. It will, however, be noted that equations 5.39 - 5.54 are now represented in terms of vectors $\bar{m}_i$ and r, where:

$$\bar{m}_i = \frac{\bar{F}_i}{\bar{a}_1} \text{ is the 'apparent mass vector'} \qquad 5.63$$

and

$$r = \frac{\bar{a}_2}{\bar{a}_1} \text{ is the modal shape} \qquad 5.64$$

This style of representation was adopted, not only for its obvious attractive feature, i.e. simplicity, but mainly for its direct practical application when using accelerometers and a measuring device with inbuilt vector-dividing facility.

5.6.3 The optimum system -'Complete Symmetry'

All three effective frequency dependent parameters of a single degree of freedom system (K, M and C) can only be determined from the 'Energy Method' at its natural frequency, i.e. a combined forced and free vibration method. Similarly the eight parameters of a two degrees of freedom system are determinate only if the system can be simulated by two equivalent single degree of freedom systems, i.e. pure mode excitation with no coupling due to damping. The method of measurement must again be the 'Energy Method' but at the two natural frequencies of the system ω_1 and ω_2; each furnishing three equations which together with the two equations from the eigenvector equations will render the parameters determinate. This technique does however assume that the parameters remain unchanged at ω_1 and ω_2. It therefore follows that in general the two degrees of freedom system with frequency dependent parameters is indeterminate as far as evaluation of system parameters is concerned.

Forced vibration method although does not require the orthogonality of principal coordinates i.e. zero coupling due to damping, however still remains indeterminate: it provides seven equations in all (two of which are again the eigenvector equations 5.4) four in terms of the mass and stiffness parameters and three in terms of damping coefficients. Hence damping coefficients will be determinate whilst the stiffness and mass equations need one more equation. This extra equation can be obtained if the damping within the system is of the structural type, i.e. proportionally distributed which also results in orthogonality of the principal coordinates. This assumption (as F measurement) cannot be accepted in general as they defeat the object: no limitation on the behaviour of the specimen parameters K and C (or their shape and size) is allowed.

However, if the system possesses the stiffness and mass symmetry, it has been shown in the analysis that the first mode becomes completely independent of the specimen (i.e. $A_2 = 1$, $B_2 = -1$). This feature of the symmetric system enables it to be calibrated for the auxiliary parameters'effect under the exact test conditions and without the necessity of determining the individual values of K_1 and M_1 but merely that of $(K_1 - M_1\omega^2)$i.e. the inphase component of stiffness at the first mode (equation 5.56). The specimen parameters are then evaluated from equations 5.58 and 5.59.

One further refinement of the system was realised by making it completely symmetric; $K_1 = K_2$, $M_1 = M_2$ and $C_1 = C_2$ which resulted in orthogonal principal coordinates, hence uncoupled free vibrations. Free-vibration method at relatively low frequencies $(\omega_1 < \omega < 4.5\omega_1)$ would still require the individual values of K_1 and M_1 which could be predetermined at ω_1 and ω_2 using a range of calibration specimens to induce varied ω_2 (the energy method). However, it is unlikely that the auxiliary parameters would be frequency dependent hence the 'Energy Method' or similar methods around ω_1 are expected to be sufficient in establishing the values of the auxiliary parameters.

CHAPTER 6

'THEORETICAL MODEL OF THE JOINT ASSEMBLY'

6.1 Rheological Representation of Joint Layers

Exact mathematical analysis of any vibratory system is only possible if the frictional forces are viscous (or hysteretic) and the springs too are linear.

The theoretical model of a joint layer will therefore be simulated by a Voigt unit having 'equivalent' values of viscous and spring coefficients. These values are in general frequency dependent - depending upon the distribution of Voight and Maxwell units the mechanism of dynamic approach can vary according to:

$$(a_o+a_1 \frac{\partial}{\partial t} + a_2 \frac{\partial^2}{\partial t^2} + \ldots + a_n \frac{\partial^n}{\partial t^n}) F = (b_o + b_1 \frac{\partial}{\partial t} + b_2 \frac{\partial^2}{\partial t^2} + \ldots + b_n \frac{\partial^n}{\partial t^n}) X \qquad 6.1$$

For sinusoidal excitation $F = \bar{F} e^{i\omega t}$

$$(a_o+ia_1\omega-a_2\omega^2+ \ldots a_n\omega^n) \bar{F} = (b_o+ib_1\omega-b_2\omega^2 + \ldots b_n\omega^n)\bar{X} \qquad 6.2$$

(See reference (61)).

6.2 The Ideal System - Forced Vibration

The total deflection across an array of Voigt Units connected in series is the sum of the deflections of the individual units:

$$\bar{X}_t = \Sigma \bar{X}_r \tag{6.3}$$

Because there is no inertia forces present each unit transmits exactly the instantaneous force applied to it from the neighbouring unit - all units are subject to the same load at all times:

$$\bar{F} = K_t^* \bar{X}_t = K_r^* \bar{X}_r \tag{6.4}$$

where $K^* = K(1+i\zeta)$ denotes the 'complex stiffness'.

Similarly, for a number of Voigt Units connected in parallel so that they all experience the same deflection at all times, the total force applied to the system is always equal to the sum of the forces exerted by the individual springs.

$$\bar{X}_t = \bar{X}_r \tag{6.5}$$

and

$$\bar{F} = K_t^* \bar{X}_t = \Sigma K_r^* \bar{X}_r \tag{6.6}$$

Referring to Appendix (1) the energy loss per cycle of the assemblies in terms of the individual units can be written as follows:

$$D_t = \Sigma D_t = -\Sigma\pi C_r \omega |\bar{X}_r|^2 = -\pi\Sigma\zeta_r . K_r |\bar{X}_r|^2 \tag{6.7}$$

but

$$D_t = -\pi C_t \omega |\bar{X}_t|^2 = -\pi\zeta_t . K_t |\bar{X}_t|^2 \tag{6.8}$$

$\therefore$

$$\zeta_t = \frac{\Sigma\xi_r K_r |\bar{X}_r|^2}{K_t |\bar{X}_t|^2} \tag{6.9}$$

Equations 6.3 and 6.4 for the series ensemble, and 6.5 and 6.6 for the parallel ensemble when multiplied would yield the energy expression:

$$(K_t^* \bar{X}_t)\bar{X}_t = (K_r^* \bar{X}_r)\Sigma\bar{X}_r = \Sigma(K_r^*\bar{X}_r)\bar{X}_r \qquad 6.10$$

$$(K_t^* \bar{X}_t)\bar{X}_t = \bar{X}_t \, \Sigma \, K_r^* \bar{X}_r = \Sigma(K_r^*\bar{X}_r)\bar{X}_r \qquad 6.11$$

Therefore, it can be concluded that for any series ensemble or for any parallel ensemble and hence also for any combination of series parallel ensemble the energy expression can be written as:

$$(K_t^* \bar{X}_t)\bar{X}_t = \Sigma(K_r^*\bar{X}_r)\bar{X}_r \qquad 6.12$$

From equation 9 in Appendix (1).

$$\tan\phi_r = \frac{C_r\omega}{K_r} = \zeta_r \qquad 6.13$$

if $\frac{C_r}{K_r}$ = constant, i.e. proportional distribution of damping it is evident from equation (6.13) that all points of the structure will be inphase. Therefore the real part of the energy expression in equation (6.12) can be written as follows:

$$K_t|\bar{X}_t|^2 = \Sigma K_r|\bar{X}_r|^2 \qquad 6.14$$

The above equation when used in equation 6.9 will yield the loss factor of the assembly:

$$\zeta_t = \frac{\Sigma\zeta_r K_r|\bar{X}_r|^2}{\Sigma K_r|\bar{X}_r|^2} = \zeta_r \qquad 6.15$$

If a joint assembly is comprised of identical units (Fig.18a) the loss factor of the assembly will therefore be equal to that of a single unit:

$$\zeta_t = \zeta_1 = \zeta_2 \;.....= \zeta_r \qquad 6.16$$

and the equivalent values for coefficients of spring and damping of a joint unit could be found from the respective values for the whole assembly at the exciting frequency as follows:

$$K_r = \frac{N}{P} K_t \qquad 6.17$$

$$C_r = \frac{N}{P} C_t \qquad 6.18$$

where

$$C_t = \frac{K_t}{\omega} \zeta_t \qquad 6.19$$

N is the number of units in series and P the number of units in parallel (see reference (71)).

6.3 The Real System

In practice the ideal system cannot be simulated over an indefinite range of frequency. As the frequency is increased the extraneous inertia forces become significant taking up a considerable proportion of the elastic restoring forces and causing mode coupling.

The following analysis is made in order to establish the effect of the inertia forces due to the mass of the jointed column.

6.3.1 Eigenvalue and Eigenfunctions of a Uniform System

The system shown in Fig. 18b represents a stack of discs making up the jointed column with no damping.

If m and k are the mass and stiffness of the discs and the joints respectively, the equation for dynamic equilibrium for (r+1)th disc can be written as:

$$m\ddot{U}_{r+1} + k(2U_{r+1} - U_r - U_{r+2}) = 0 \qquad 6.20$$

The general solution of the above equation, assuming the system possesses n modes of vibration is:

$$U_r = \sum_n U_{r,n}\,(C_n \mathrm{Cos}\,\omega_n t + D_n \mathrm{Sin}\,\omega_n t) \qquad 6.21$$

for a pure mode n,

$$\ddot{U}_{r+1} = -\omega_n^2\, U_{r+1} \qquad 6.22$$

Equation 6.20 can now be written as

$$U_{r+2,n} + 2\left(\frac{m\omega_n^2}{2k} - 1\right) U_{r+1,n} + U_{r,n} = 0 \qquad 6.23$$

A solution for the above equation is:

$$U_{r,n} = B_n\,\alpha_n^r \qquad 6.24$$

or

$$(\alpha_n^2 + 2\beta_n\alpha_n + 1)B_n\alpha_n^r = 0 \qquad 6.25$$

where

$$\beta_n = \frac{m\omega_n^2}{2k} - 1 \qquad 6.26$$

but B_n and $\alpha_n \neq 0$, otherwise there would be no vibration.

$\therefore \quad \alpha_n^2 + 2\beta_n\alpha_n + 1 = 0$

or

$$\alpha_n = -\beta_n \pm \sqrt{\beta^2 - 1} = \exp.(\pm\, \theta_n) \qquad 6.27$$

where $\quad \text{Cosh}\theta_n = -\beta_n \qquad 6.28$

$$\therefore \quad U_{r,n} = {}_1B_n \exp.(r\theta_n) + {}_2B_n \exp.(-r\theta_n) \qquad 6.29$$

or $\quad U_{r,n} = G_n \text{Sinh}(r\theta_n) + H_n \text{Cosh}(r\theta_n) \qquad 6.30$

The values of G_n and H_n are determined by the boundary conditions.

The boundary conditions are satisfied by equating the strain at both ends of the system with zero. Using the central finite difference representation the strains at the ends are:

$$\frac{U_{1,n} - U_{-1,n}}{2} = 0 \qquad 6.31$$

$$\frac{U_{N+1,n} - U_{N-1,n}}{2} = 0 \qquad 6.32$$

From equation 6.30

$$U_{1,n} = G_n \text{Sinh}\theta_n + H_n \text{Cosh}\theta_n$$

$$U_{-1,n} = G_n \text{Sinh}(-\theta_n) + H_n \text{Cosh}(-\theta_n)$$

or

$$2G_n \text{Sinh}(\theta_n) = 0 \qquad 6.33$$

but $\text{Sinh}\theta_n \neq 0$, otherwise there would be no vibration

$\therefore \quad G_n = 0$

or

$$U_{r,n} = H_n \text{Cosh}(r\theta_n) \qquad 6.34$$

and from 6.32 for $r = N+1$

$$H_n \text{Cosh}(N\theta_n + \theta_n) - H_n \text{Cosh}(N\theta_n - \theta_n) = 0$$

or

$$\text{Sinh}N\theta_n \text{Sinh}\theta = 0 \qquad 6.35$$

as $\quad \text{Sinh}\theta_n \neq 0 \quad \therefore \quad \text{Sinh}N\theta_n = 0$

but

$$N \text{ and } \theta \neq 0 \quad \therefore \quad N\theta_n = in\pi \qquad 6.36$$

or

$$\theta_n = i\,\frac{n}{N}\,\pi$$

θ_n from above is substituted in equation 6.34 to give the eigen-function or the modal shape:

$$U_{r,n} = H_n \text{Cosh}(i\,\frac{n}{N}\,\pi.r)$$

or

$$U_{r,n} = H_n \text{Cos}(\frac{n}{N}\,\pi.r) \qquad 6.37$$

The eigenvalues or the natural frequencies are in turn found from equations 6.26 and 6.28:

$$-\beta_n = +\text{Cosh}\theta_n = +\text{Cos}\,\frac{n}{N}\pi = 1 - \frac{m\omega_n^2}{2k}$$

or

$$\omega_n^2 = \frac{2k}{m}\,[1 - \text{Cos}(\frac{n}{N}\pi)] \qquad 6.38$$

$$= 4\,\frac{k}{m}\,\text{Sin}^2\,\frac{n\pi}{2N}$$

$$\omega_n = 2\sqrt{\frac{k}{m}}\cdot\text{Sin}\,\frac{n\pi}{2N} \qquad 6.39$$

or for $\quad \frac{n\pi}{2N} \rightarrow 0$

$$\omega_n \simeq \sqrt{\frac{k}{m}}\cdot\frac{n\pi}{N} \qquad 6.40$$

Obviously the discrete mass approximation in this way does not predict the natural frequencies for higher than the N th mode ($1 \leq n \leq N$). The first natural frequency of a solid column given by the discrete mass approximation underestimates the true natural frequency ($\omega_n = \sqrt{\frac{k}{m}} \cdot \frac{n\pi}{N}$). Equations 6.39 and 6.40 show that for n = 1 and N=4 this error is only about 2.5 per cent.

6.3.2 Forced Vibration of a Uniform System

Central finite difference approximation to the second derivative at (r+1)th disc is:

$$\frac{\partial^2 U_{r+1}}{\partial r^2} = + U_{r+2} - 2U_{r+1} + U_r \qquad 6.41$$

Equation 6.20 can now be approximated to:

$$m\ddot{U}_{r+1} - k \frac{\partial^2 U_{r+1}}{\partial r^2} = 0$$

or

$$\frac{\partial^2 U_{(r,t)}}{\partial r^2} - \frac{m}{k} \frac{\partial^2 U_{(r,t)}}{\partial t^2} = 0 \qquad 6.42$$

which has a steady state solution of the form:

$$U(_{(r,t)} = U(r) \; [C \; \mathrm{Sin}\omega t + D \; \mathrm{Cos}\omega t] \qquad 6.43$$

Equation 6.42 is therefore reduced to:

$$\frac{\partial^2 U_{(r)}}{\partial r^2} + \frac{m}{k} \omega^2 \; U_{(r)} = 0 \qquad 6.44$$

which itself has the solution:

$$U_{(r)} = C_1 \text{Sin}\alpha r + D_1 \text{Cos}\alpha r \qquad 6.45$$

where

$$\alpha^2 = \frac{m}{k}\omega^2 \qquad 6.46$$

$$\therefore \quad U_{(r,t)} = (C_1 \text{Sin}\alpha r + D_1 \text{Cos}\alpha r)(C\text{Sin}\omega t + D\text{Cos}\omega t) \qquad 6.47$$

to satisfy the end conditions, at r = 0 and r = N the strains are constant and directly proportional to the loads. The constant of proportionality being k:

$$+ k \left[\frac{\partial U_{(r,t)}}{d_r} \right]_{r=N} = \bar{F}_N \text{ Cos}\omega t \qquad \text{tension} \qquad 6.48$$

$$- k \left[\frac{\partial U_{(r,t)}}{\partial r} \right]_{r=0} = \bar{F}_o \text{ Cos}\omega t \qquad \text{compression} \qquad 6.49$$

from 6.47

$$\frac{\partial U_{(r,t)}}{\partial r} = (C_1 \alpha \text{Cos}\alpha r - D_1 \alpha \text{Sin}\alpha r)(C\text{Sin}\omega t + D\text{Cos}\omega t) \qquad 6.50$$

At r = 0, assume $\bar{F}_o = F_o$, real

from equation 6.49 for r = 0,

$$-k\alpha(C_1 - 0)(C\text{Sin}\omega t + D\text{Cos}\omega t) = F_o \text{Cos}\omega t$$

$$\therefore \quad C = 0 \qquad 6.51$$

let $\quad C_1 D = C_3 \qquad 6.52$

or $\quad -k\alpha C_3 = F_o$

$$\therefore \quad C_3 = \frac{-F_o}{k\alpha} \qquad 6.53$$

and at r = N, from equation 6.48:

$$k\alpha(C_1\text{Cos}\alpha N - D_1\text{Sin}\alpha N)D\ \text{Cos}\omega t = \bar{F}_N\text{Cos}\omega t$$

$$k\alpha(C_3\text{Cos}\alpha N - D_1 D\text{Sin}\alpha N) = \bar{F}_N$$

let $$DD_1 = D_3 \qquad 6.54$$

$$k\alpha(C_3\text{Cos}\alpha N - D_3\text{Sin}\alpha N) = \bar{F}_N$$

$$\therefore \quad D_3 = \frac{-1}{k\alpha}\ (F_o\text{Cot}\alpha N + \bar{F}_N\text{Cosec.}\alpha N) \qquad 6.55$$

Equation 6.47 can now be written in terms of F_o and $\bar{F}_N$ and system parameters as follows:

$$U_{(r,t)} = (C_3\text{Sin}\alpha r + D_3\text{Cos}\alpha r)\text{Cos}\omega t$$

$$= \frac{-1}{k\alpha}\ [F_o\text{Sin}\alpha r + (F_o\text{Cot}\alpha N + \bar{F}_N\text{Cosec.}\alpha N)\text{Cos}\alpha r]\text{Cos}\omega t$$

or

$$U_{(r,t)} = -\frac{1}{k\alpha}.C.\text{Sin}(\alpha r + \phi)\text{Cos}\omega t \qquad 6.57$$

where

$$C = [F_o^2 + (F_o\text{Cot}\alpha N + F_N\text{Cosec}\alpha N)^2]^{\frac{1}{2}} \qquad 6.58$$

and

$$\tan\phi = \frac{F_o\text{Cos}\alpha N + F_N}{F_o\text{Sin}\alpha N}. \qquad 6.59$$

For the symmetric system shown in Fig. 16d, at the second mode:

$$F_N = -F_o$$

therefore

$$C = F_o\text{Sec.}\ \frac{\alpha N}{2}, \qquad \text{and}$$

$$\tan\phi = -\tan\frac{\alpha N}{2} \qquad \phi \simeq -\frac{\alpha N}{2}$$

or $$U_{(r,t)} = \frac{-F_o}{K\alpha \cos\frac{\alpha N}{2}} \sin\alpha(r - \frac{N}{2})\cos\omega t \qquad 6.60$$

But:

$$\alpha_{max} = \frac{\omega_{max}}{\sqrt{\frac{k}{m}}} \qquad \text{(see equation 6.46)}$$

where

$$\omega_{max} = \sqrt{2}\omega_2 = \sqrt{2}\,\frac{\sqrt{2K+K_1}}{\sqrt{M_1}} \simeq 2\sqrt{\frac{K}{M_1}} \simeq 2\,\frac{\sqrt{k}}{\sqrt{M_1(N+1)}}$$

(see Fig. 15d)

or

$$\alpha_{max} = \frac{2}{\sqrt{N+1}}\sqrt{\frac{m}{M_1}} = \frac{2}{\sqrt{N+1}} \sqrt{\frac{w}{W}} \qquad 6.61$$

For small values of $\frac{\alpha N}{2}$, Equation 6.60 is reduced to:

$$U_{(r,t)} \simeq \frac{F_o}{k}(\frac{N}{2} - r)\cos\omega t \qquad 6.62$$

which represents a linear distribution of deflection along the system under ideal (massless) conditions. (see Fig.18c).

Figure 19 shows the variation of $\frac{\alpha_{max.}N}{2}$ with N and ^{w} for the different sizes of specimens (see also Fig. 22). Equation 6.62 can therefore be used instead of equation 6.60 in the following procedure in estimating the inertia forces in terms of the percentages of the total elastic restoring forces along the jointed column.

$$F_r = F_o - m\omega^2 \sum_{r=o}^{r} U_{(r)} \qquad 6.63$$

The transmitted load along the jointed column is minimum at the centre disc:

$$F_{N/2} = F_o - m\omega^2 \sum_{r=o}^{N/2} U_{(r)}$$

$$\approx F_o - m\omega^2 \frac{N}{4} U_o \qquad \text{(see Fig. 18c)} \qquad 6.64$$

but

$$F_o = 2KU_o \quad \text{for the ideal system.}$$

Therefore the maximum percentage error due to the inertia of the discs is:

$$(E_i)_{max} \approx \frac{m\omega^2_{max.} N}{8 K} \times 100 \qquad 6.65$$

where

$$\omega_{max} = \alpha_{max.} \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{m}} \cdot \frac{2}{\sqrt{N+1}} \sqrt{\frac{w}{W}}$$

and $K = \frac{k}{N+1}$

or

$$(E_i)_{max} \approx \frac{N}{2} \cdot \frac{w}{W} \times 100$$

Figure 19also shows the variation of this error with N and w It is evident from this figure that the percentage of the elastic force contributed to inertia of disc increases with size and number of discs. However it is not expected to exceed about 2.5 per cent in the worst case (E270 type specimens and 36 number of discs), hence it is negligible.

6.4 The Effect of Stiffness and Damping of the Discs Pseudo - Material Effect

The inertia effect of the discs as proved in 6.3 is insignificant, however the stiffness and damping effect due to the material of the discs must be extracted from the results in order to determine those of the joint alone. This will be referred to here as the Pseudo-material effect in order to indicate also other extraneous effects which happen to be present e.g. that of the transducer, etc.

The Pseudo-material effect can be determined from a separate test of equivalent solid column shown in Fig.44a.

The model representing the jointed specimens is one of a Voigt unit which itself is comprised of two Voigt units connected in series. (see Fig. 18d). The receptance of the assembly being the sum of the receptances of the two units gives two sets of equations relating the stiffness and damping of the assembly to those of the pseudo-material and the joints as follows:

$$\frac{1}{K+iC\omega} = \frac{1}{K_m+iC_m\omega} + \frac{N}{K_J+iC_J\omega} \qquad 6.67$$

where N is the effective number of joints. Equation 6.67 can be written in two separate equations representing the real and quadrature components:

$$\frac{NK_J}{K_J^2+C_J^2\omega^2} = \frac{K}{K^2+C^2\omega^2} - \frac{K_m}{K_m^2+C_m^2\omega^2} \qquad 6.68$$

and

$$\frac{NC_J\omega}{K_J^2+C_J^2\omega^2} = \frac{C\omega}{K^2+C^2\omega^2} - \frac{C_m\omega}{K_m^2+C_m^2\omega^2} \qquad 6.69$$

K_J and C_J could therefore be found from the above equations. Alternatively these equations can be written in terms of stiffness and loss-factors as shown below:

$$\frac{N}{K_J(1+\zeta_J^2)} = \frac{1}{K(1+\zeta^2)} - \frac{1}{K_m(1+\zeta_m^2)} \qquad 6.70$$

and

$$\frac{N\zeta_J}{K_J(1+\zeta_J^2)} = \frac{\zeta}{K(1+\zeta^2)} - \frac{\zeta_m}{K_m(1+\zeta_m^2)} \qquad 6.71$$

or

$$\zeta_J = \frac{\text{Equation 6.71}}{\text{Equation 6.70}} \qquad 6.75$$

and

$$K_J = \frac{N}{(1+\zeta_J^2)\ \text{Equation 6.70}} \qquad 6.73$$

6.5 Summary and Conclusions

The distribution of deflection along the jointed column is linear for ideal and sinusoidal for real system. The latter is due to the inertia of the discs. The wavelength depends upon the frequency of excitation and the system parameters:

$$\lambda = \frac{2\pi}{\alpha}$$

where

$$\alpha = \omega \sqrt{\frac{m}{k}}$$

The natural frequencies of the system are struck when

$$\omega_n = \frac{n\pi}{N} \sqrt{\frac{k}{m}}$$

Therefore $\lambda = \dfrac{2N}{n}$

The maximum frequency of excitation ($\sqrt{2}.\omega_2$) with the largest specimen size (E270 type) and the maximum number of discs (36), i.e. the worst case, is only about 13 per cent of the first natural frequency of the jointed column. This corresponds to about 12° at the ends of the column of full 180° half wave length representing a nearly linear distribution of deflection. The maximum inertia force due to the mass of the discs is only about 2.5% of the elastic restoring force. For all practical purposes therefore it can be concluded that the effect of inertia of discs is negligible and the system can be regarded as ideal. (see Fig. 18 and 19).

The parameters of a single joint unit (together with the stiffness and damping due to the discs' material) can be found from those of the jointed column if the joint units are identical which results in completly in-or-out of phase movement of the discs, hence

$$\zeta_r = \zeta$$

and

$$K_r = (N+1)K$$

In order to account for the effect of material it is suggested that an extra test on an identical but solid column be carried out (Fig.44a) and the results be used together with those from the jointed column in equations 6.72 and 6.73 to determine the parameters of the joint alone.

CHAPTER 7

DESIGN OF SPECIMENS

7.1 Surface Texture

The only variable as far as surface texture in this work was concerned was 'roughness' or micro-irregularities. Attempts were therefore directed towards minimizing large wavelength irregularities, i.e. waviness (caused by vibration, tool wear, sudden entry of tool into the material in the case of non homogenous material) and irregularities with even larger wavelengths (caused by slideway misalignment, tool wear etc) which together constitute flatness errors.

Two types of joints, namely turned and ground, were selected to represent the basic types of single and multi point cut surfaces respectively.

The grades of surface finish to be produced were chosen after consulting Fig. 20 and accounting for practical limitations as far as manufacture and roughness measurement were concerned at the time. The target values of roughness for turned and ground surfaces were as follows:

	TND1	CLA	Average	60μ" (1.52μm)
	TND2	CLA	Average	180μ" (4.57μm)
	TND3	CLA	Average	380μ" (9.65μm)
and	GND	CLA	Average	8μ" (0.20μm)

7.2 Form

Circular plan form was chosen for the following reasons:

1. SPT machining can be simulated by turning, which is best suited for holding small size specimens.
2. It is also best suited for eliminating flatness errors as the machine configuration remains unaltered in the cutting direction (unlike e.g. shaping or milling operations) but only in the feed direction. Measures were to be taken to minimize the errors due to lack of parallelism and to flatness in the feed direction, using a magnetic chuck.
3. The advantages of polar symmetry which facilitate testing and analysis in particular for torsional tests.
4. Mass production of specimens on a capstan lathe.

7.3 Shape

The shape factor was expected to have a significant effect on the behaviour of joints should the squeeze film be pronounced. It was therefore decided to define a shape factor for circular plan forms. This parameter is sometimes referred to as the Aspect Ratio.

$$A.R = \frac{(OD-ID)}{\pi(OD+ID)} \qquad 7.1$$

The two extremes will therefore be

$$ID = 0.0 \qquad A.R. = \frac{1}{\pi} = 0.318\ , \qquad \text{and}$$

$$ID = OD \qquad A.R. = 0.0$$

which represents a solid disc and an infinitely thin ring respectively. Fig. 21, constructed with the aid of a computer, shows the variation of O.D. with A.R. for different values of the joint area.

It was decided to study the effect of shape over an area 0.9 in^2 (5.8 Cm^2) as this was thought to be small enough to have minimal flatness error at the same time being large enough to give significant values of quadrature stiffness components. (see Fig. 22).

Consideration was given to minimize the cutting time by using standard 'hollow bars' and also the ease in use of quartz load washers which were available on the market. Table (4) shows the range of shape-factors together with the corresponding sizes and part numbers of standard hollow bars and type numbers of the load washers which were to be used.

7.4 Size

It was decided to study the effect of area for four convenient sizes of specimens shown in Fig. 22 with A.R. = $\frac{1}{10}$ small enough to allow one dimensional squeeze film analysis to be valid (for reasons of simplicity). For A.R. $>$ 0.1 as seen from Fig. 21 the variation of O.D. is minimal therefore any major alteration of standard hollow bars would be carried out on the inside diameter. This was considered advantageous because the bore diameters of the hollow bars were found to be eccentric (about 2.5 mm on the average) and therefore single alteration of the inner diameter would result in both correcting this defect whilst bringing its size to the desired level.

Reduced flatness error was achieved by having no greater an area than 2.7 $in.^2$ (17.4 $Cm.^2$). Consideration was given to minimising cutting time by using suitable standard hollow bars as shown in Table (5): only A45 specimens were manufactured from a steel due to their small size. The choice of material was made by consulting the chemical composition of the hollow bars and their yield point - the latter was considered the main possible effective parameter in the joint's behaviour as far as its material was concerned.

7.5 Method of Manufacture

All groups of specimens (except E270) were machined in a Capstan lathe (Herbert.A.) which made it possible to complete all the operations; turning, boring and parting-off in a single setting of appropriate tools and stops on the turret and tool holder. The operation proved very efficient, i.e. a great number of specimens could be produced in a relatively short time. The next operations were as follows:

1. Numbering and marking for angular positioning.
2. Relieving stress in the case of work hardening during cutting operation.
3. With a magnetic chuck assembled on the spindle of a lathe a thin sheet of steel was placed on it and a very small cut was then taken to correct for untrue positioning of the chuck.

The specimens were secured on this surface and very small cuts were taken with appropriate speed and varied feed in order to produce different grades of surface finish. This operational procedure ensures minimum error due to both flatness and lack of parallelism. Because of the very small depth of cut and abundant use of coolant the work-hardening was minimal.

7.6 Cutting Conditions for Turned Surfaces

Due to the complexity of roughness formation during the cutting operation, exact cutting conditions for any given roughness were not expected to be realized; However, it was possible to obtain relatively good estimates from published results on this area (i.e. references (73), (74) and (75)).

7.6.1 Speed

Estimates of speeds were made for each of the specimen sizes when using a High Speed Steel Tool[76]. For this purpose hardness measurements had to be made. This measurement was also informative in so far as indicating whether significant work-hardening due to cutting at the first stage had occurred. For this purpose a batch of specimens were stress relieved and tested.

It was found that the average values of hardness remained constant at about 50 Rockwell B scale roughly equivalent to 83 BH number. The results indicated that no tangible work-hardening had occurred under the first stage cutting or subsequent grinding operation. This was attributed mainly to the abundant use of coolant. Heat treatment was therefore not considered necessary. Further information gained from hardness tests was that En 5A which was used for A and BS specimens possessed very similar yield point to that of the standard hollow tubes used for the B, C and E type specimens.

$$\text{RPM} \simeq \frac{V}{\pi \frac{OD+ID}{2}} \simeq \frac{48.8}{\pi \frac{OD+ID}{2}} \qquad 7.2$$

48.8 m/min (160 ft/min) is the recommended speed given in reference (76).

7.6.2 Feed

For S.P.T. cut surfaces a number of empirical relations were considered; they relate roughness to feed rate and nose radius of the cutting tool. One such equation is given below, whilst the other is shown in Fig. 23 (reference (73)).

(Ref.(74)) $$\Omega = \frac{s^2}{18\sqrt{3}r} \quad \text{[CLA]} \qquad 7.3$$

where

s is in [m/rev.] and r in [m].

7.6.3 Depth of Cut

The depths of cut had to be kept to a minimum in order to reduce the flatness errors and also possible indetectable work-hardening of asperities at about 25-50 μm (1-2 thou).

7.6.4 Established Cutting Conditions

The cutting conditions were finally established after some process of trial and error and they are given in Table (6). The tip radius was 1.0 mm.

Twelve discs were turned for each group of specimens from which after roughness measurements four surfaces with the highest error of roughness were first marked and then ground to form the ground end surfaces (see Fig. 44b).

CHAPTER 8

DESIGN OF TEST RIG

8.1 Introduction

The ideal system, as far as dynamic behaviour was concerned, was to be realised by reducing the stiffness of the preloading-isolating springs K_1 and also making the system symmetrical as far as practically possible. Both these features would help result in reduced mode coupling, i.e. approaching a semi-definite (proportionally damped) and completely symmetric system and therefore satisfying the condition given in equations 5.21 partially and that given in 5.2 2. completely. Reduced K_1 would also have the advantage of increased isolation property and also extended useful frequency range at the lower end of the spectrum.

It was imperative that some sort of arrangement be made for supporting the weight of M_1, the loading platform and the K_1 springs, which had to be capable of adjustment in the normal direction to accommodate different sizes of specimens. This was achieved by using another set of springs K_2. Ideally these springs were to be added to the system of Fig. 14a symmetrically as shown in Fig. 24a. This was not considered practical and indeed proved to be unnecessary:

K_2 springs were employed only under the top inertia block M_1. (Fig. 24b). By having K_2 springs much more flexible than K_1 springs not only was deviation from symmetry negligible but also a very high static loading efficiency $\frac{F_J}{F_{11}}$ was reached (see Appendix 2, equation 13). Attempts were therefore made in the design to maximize R_{J2} and R_{12}, the stiffness ratios defined in Appendix 2.

In order to ensure that the natural frequencies lay within the frequency range of interest (i.e. NF 15 - 1500 Hz), firstly estimates were made of quantitative values of joint stiffnesses, likely to be encountered during the tests. The maximum stiffness of the jointed column and the higher end of the frequency spectrum determined the mass which had to be placed at its end. The minimum stiffness of the jointed column and the lower end of the frequency spectrum on the other hand determined the stiffness of the preloading springs.

Resonance conditions at any particular preloading were to be achieved by varying the number of discs placed in series within the jointed column. The maximum number of discs or the maximum length of the jointed column had of course to be limited to a practical level lest : not only the system would be prone to unwanted modes of vibration but also become tedious and time consuming during the actual tests.

In the following preliminary calculations, the above considerations were taken into account in sizing-up the overall dimensions of the rig: an essential starting-off platform for the detailed design of parts which followed.

Because most British manufacturers still turned out standard parts in the Imperial System of units, this system was employed principally in this section as a matter of expediency. Nevertheless, wherever appropriate the Metric equivalents are also given.

8.2 Preliminary Calculations

8.2.1 E.L.S. of Joint Surfaces

The following calculations assume that the joint surfaces are manufactured by single point tools. Quantitative values of stiffness in terms of 'Equivalent Length of Solid Steel' for these types of joints found by sifferent researchers as seen in Fig. 11a, are in good agreement and therefore considered to result in reliable estimates. The length determined from these references was about 5.0 inches at 250 PSI joints preload. Linear relationship between stiffness and preload was also assumed(29,32).

$$\ell_J = 5'' \times \frac{250}{\sigma} \qquad 8.1$$

for

$$100 < \sigma < 6800 \text{ [PSI]}$$

The range of preloading pressure was chosen by consulting the values given in Table 3 for fixed joints and in Chapter 4.5.2 for sliding joints respectively. Maximum pressure can be as high as 3185 PSI and minimum as low as 1.0 PSI. For preloads as low as this there could be no reliable estimate of stiffness at this stage.

The increase in the in-phase component of stiffness under dynamic loading was taken to be 1.5 times its static stiffness, as a fair estimate (see Fig. 11c). This effect was thought to increase as the preload was reduced.

The maximum stiffness of joint surface likely to be encountered in practice was therefore presented by:

$$(\ell_j)_{min} = \frac{5x250}{1.5x3185} = .25 \text{ [in.]} \qquad 8.2$$

8.2.2 E.L.S. of Jointed Column

If ℓ_c is the E.L.S. of the jointed column, t the thickness and N the number of discs, then

$$\ell_c = (N + 1)\ell_j + N\,t \qquad 8.3$$

Four discs were chosen as a minimum. This number was thought to be sufficiently large to be statistically representative of the surface roughness. Therefore the maximum stiffness of jointed column was:

$$K_{max} = \frac{\Delta_{max}.E}{(\ell_c)_{min}} = \frac{2.7 \text{ x } 30 \text{ x } 10^6}{5x.25+4x.375} = 29.45 \text{ x } 10^6 \text{ [lbf/in.]} \qquad 8.4$$

(see Fig. 22)

8.2.3 The Maximum Length of Jointed Column

A frequency band of about three octaves was considered sufficiently large to indicate the effect of frequency, should the resonance vibration become indispensible. The maximum number of discs making up the jointed column would therefore be about 30 - 36 bringing its maximum length to about one foot (∿ 30.Cm).

8.2.4 Shape and Weight of Inertia Blocks

A cylindrical shape for the mass blocks was chosen, not only for its obvious ease of manufacture but because the design of the test rig should be such that at a later date the rig can be converted to a torsional test rig.

The mass of the inertia block was found from the following formula:

$$M = \frac{2\,K_{max}}{(\omega_2)^2_{max}} \qquad 8.5$$

where

$K_{max} = 29.45 \times 10^6$ [lbf/in.], and

$(\omega_2)_{max} = 2\pi(N.F_2)_{max} = 2\pi \times 1500 = 3000\pi$ [rad/sec.]

$$M \approx 256.0 \text{ [lb.]} = 116.0 \text{ [Kg.]}$$

8.2.5 Dimensions of Inertia Blocks (and K_1 Springs)

From equation 8.5 and for cylindrical inertia blocks made of steel it follows:

$$D \sqrt{H} = 34 \qquad 8.6$$

Where, D and H are the diameter and the length of the blocks in inches.

The values of D and H were determined after considering the following points:

(i) number and stiffness of preloading springs K_1

(ii) overall height of the test rig.

These springs were required to have minimum stiffness in order to (a) increase the isolation properties of preloading springs and (b) place the two resonance frequencies as far apart as possible to reduce the mode coupling. This implied a maximum spring index $\frac{D_o}{d}$ (D_o is the mean diameter of spring coil and d is the diameter of the spring wire) and also a minimum number of springs whilst producing the maximum preloading required. It can be shown that if this is to be achieved by N_1 number of identical springs in parallel, the overall stiffness would be proportional to $N_1 \frac{d_1}{n_1 D_{o1}^2}$:

$$K_1 \propto N_1 \frac{d_1}{n_1 D_{o1}^2} \qquad 8.7$$

where n_1 is the number of active coils per spring.

To reduce d_1 it was therefore decided to use a high duty spring bar of EN 45 material which had the maximum permissible stress of 55.0 [tons/in^2.]. To increase D_{o1} on the other hand meant maximum use of the space available on the interia blocks. A single central spring was not considered practical because of the difficulty in access to the centre of the mass blocks for connecting them to the shakers. The best alternative therefore, was to arrange three springs at 120 degrees pitch around the centre of the mass block. The outside diameter of the springs relative to D the diameter of the mass blocks was so chosen that if required extra springs could be added to increase the preloading capacity whilst making the best use of the space available (see Fig. 25).

$$D_1 = 3(D_{o1} + d_1) \qquad 8.8$$

or

$$D_1 = 3d_1(c_1 + 1) \qquad 8.9$$

where

$$c_1 = \frac{D_{o1}}{d_1} \qquad 8.10$$

is the 'spring index'.

As the stiffness of the jointed column tends to zero the second natural frequency of the system approaches to that of the first; in the limit

$$(\omega_2)^2_{min} = (\omega_1)^2 = \frac{3R_1}{M_1} \qquad 8.11$$

where

$$(\omega_2)_{min} = 2\pi(N.F_2)_{min} = 2\pi \times 15 = 30\pi \text{ [rad/sec.]}$$

and $R_1 = \frac{K_1}{3}$, the spring rate (stiffness)

therefore from 8.5 and 8.11

$$\underline{R_1 = 1970 \quad \text{[lbf/in.]}}$$

The following formulae for helical springs together with an expression for the overall height of the rig were used in finding another expression between D and h (equation 8.17). This relationship in conjunction with equation 8.6 roughly determined the dimensions of the inertia blocks. The maximum load capacity of a helical spring P_{max} is determined from:

$$P_{max} = \frac{\pi d^3 . \tau_{max}}{8D_o . \nu} = \frac{\pi d^2 . \tau_{max}}{8c . \nu} \qquad 8.12$$

where ν is the Wahl factor found from:

$$\nu = \frac{c + .2}{c - 1} \qquad 8.13$$

which corrects for variation of shear stress within the cross-section of wire due to the coils curvature. τ_{max} is the maximum permissible shear stress of the wire material: 55 [Ton/in^2.].

but $$P_{max} = \frac{\sigma_{max.} \Delta_{max.}}{3} \times 1.1 \qquad 8.14$$

$$= \frac{3185 \times 2.7}{3} \times 1.1 = 3153 \quad \text{[lbf.]}$$

(The factor 1.1 was introduced in the calculations of both springs in order to avoid the non-linear part due to the gradual and random reduction in the number of active coils at the near closing state of the springs.)

The active number of coils n and the free length of the spring were determined from equations

$$n = \frac{Gd^4}{8D_o^3R} = \frac{Gd}{8\,c^3R} \qquad 8.15$$

and

$$L = (n + 1)d + \frac{P_{max}}{3R} \qquad 8.16$$

where

G is the shear modulus of the wire material:
11.5×10^6 [lbf/in.]

One further equation was required and this was obtained by introducing a constraint equation relating the lengths of springs, mass blocks the jointed column and the platform together with associated locking nuts (estimated at 6 inches) as follows:

$$2L_1 + 2H + 12 + 6 = 42$$

or

$$H = 12-L_1 \qquad 8.17$$

The 42 inches was the height between the cross-beams made up of two standard universal beams making the overall length of 5',4" (1.62m). This height was considered appropriate permitting easy access to all parts of the rig.

The procedure was as follows:

For different values of c_1, d_1 and n_1 were found from equations 8.12 and 8.15 respectively which were then used in equation 8.16 to give the free length of the springs. From equation 8.17 and 8.9 respective values of h and D were evaluated. Figure 25 shows both the curves found by this method and from equation 8.6. The point of intersection of two curves determined the optimum dimensions of the inertia blocks and the K_1 springs as:

$h = 7.4''$, $D = 12.6''$, and

$d_1 = .66''$, $D_{o1} = 3.52''$, $L_1 = 4.6''$ and $n_1 = 3.25$.

The above calculations served as a sketch in bringing to light the overall outlines of the rig and subsequently avoiding not only unattractive features but also practical problems such as difficulties in access to all parts of the rig, etc. The final decision on design parameters was then followed with the aim of simplicity and reduced cost, e.g. use of standard parts, etc.,after detailed examination of possibilities which are reported in the rest of this chapter.

8.3 Design of Inertia Blocks and K_1 Springs (Part Nos.5 & 24) *

A spring is completely defined by four parameters of: wire diameter d, coil diameter D_o, free length L and number of active coils n.

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* The 'General Assembly' drawing of the final system and the 'List of Items' are given in Figures 29.1 and 29.7 respectively. Figures 29.2 - 29.6 inclusive show the detailed drawings of the parts.

The nearest standard size of spring bar to the optimum size found from Fig. 25 was d = .6875". In order to be able to use the springs to their full capacity - for larger areas of joints or higher preloads of joints, it was decided to increase the number of active coils. The extra length was to be compensated by introducing appropriate counter sinks in the mass blocks. This would, in addition have the advantage of bringing the centre of gravity of the mass blocks closer to the load bearing surfaces hence reducing the effect of possible unbalanced loading in the lateral directions. On the other hand compensation also had to be made for the reduction of weight of the mass blocks. The correction was made by a slight increase in the dimensions of the mass blocks and a reduction in diameter of the springs. The inside diameter of the springs was reduced from 2.85" to 2.375" corresponding to the outer diameter of an appropriate standard ball-bushing of 1.5" bearing diameter (Part No.26). Equations 8.12, 8.15 and 8.16 were then used to give n_1 and L_1 (n_1 = 5.67, L_1 = 6.23").

The final decision was made on the dimension of mass blocks and spring parameters as follows:

$$\underline{h = 8.0'' \qquad D = 13.0''}$$

$$\underline{d_1 = 0.6875'' \;,\; D_{01} = 3.0625'' \;,\; L_1 = 6.0'',\; n_1 = 5.25}$$

the ground spring bars after manufacture of springs were to be shot-peened to increase the fatigue resistance (see Fig.29.6).

8.4 Design of the Rig Frame (Parts No.1, 2 & 3)

8.4.1 Shape:

A type of arched structure was chosen to contain the double spring mass system with facilities at its top end to accommodate a platform driven by a set of three hydraulic cylinders which provided preloading of specimens through the springs K_1. The lateral movement of the blocks was prevented by using three ball-bushings housed at 60° to the hydraulic cylinders within the blocks. The latter contained three pillars which connected the top and bottom cross beams and the weight supporting springs K_2 were arranged around these pillars (see G.A.Fig.29.1).

8.4.2 Form:

A rectangular shape was chosen to reduce the machining cost by fabricating standard RSJ's, universal beams and plates.

8.4.3 Size:

The overall height was not to be greater than 5'.4" (1.62m) to provide easy access to all parts.

The final dimensions were established as shown in Fig. 29.2. from the standard R.S.J's and universal beams available on the market and after determining the sizes of the hydraulic cylinders, the pillars and the shakers (Chapter 9.6.2).

8.5 Choice of Hydraulic Cylinders (Part No.29)

8.5.1 Introduction

In order to be able to lock the platform (Part No.6) for downward movement, the arms of the cylinders (Part No.13) had to be capable of being locked in this direction. Hollow cylinders were set upright between the plates of the upper sandwich (Part No.2) locked to it at both ends using the threaded rings (Part No.7). The arms were then locked to the assembly through locking nuts at the upper end of the cylinder where they protruded sufficiently not only for this purpose but also for that of safety in acting as a second support for the vibrator (Part No.21).

8.5.2 Stroke and load carrying capacity

The maximum stroke of the cylinder was first estimated from equations 8, 11 and 13 in Appendix 2 as follows:

$$(x_{11})_{max} \simeq \frac{(F_{11})_{max}}{Ke} \simeq \frac{2(F_J)_{max}}{K_1} \qquad 8.18$$

but

$$(F_J)_{max} = \sigma_{max} \cdot \Delta_{max} = 3185 \times 2.7 = 8599.5 \text{ [lbf.]}$$

and from equation 8.15, R_1 was found to be 2130 [lbf.in.], therefore

$$K_1 = 3R_1 = 3 \times 2130 = 6390.0 \quad \text{[lbf/in.]}$$

$$(x_{11}) \simeq \frac{2 \times 8599.5}{6390.0} = 2.69 \quad \text{[in.]}$$

the least required load carrying capacity F_{11} was then determined from equation 13 in Appendix 2 as:

$$\frac{1}{3}(F_{11})_{max} \approx \frac{1}{3}(F_J)_{max}$$

$$= \frac{8599.5}{3 \times 2240} = 1.28 \quad [tonf.] \qquad 8.19$$

The range of strokes and load capacities of commercially available hollow cylinders are rather limited, normally having higher load capacity and lower strokes than those found for the present work. The choice was made by searching for maximum stroke and suitable dimensions. The dimensions considered suitable were those of the height and diameter of a cylinder which could be contained in the upper sandwich using standard size RSJ's, leaving enough space both at the ends to take the locking rings (Part No.7) and also on the sandwich to take the pillars' locking nuts.

The decision made was for cylinders each with 2.0" stroke and 20 [tonf] capacity. These cylinders were threaded only on one end and therefore the other end had to be cut. This simple alteration was the only preparatory work which had to be done on the cylindets. The 0.9 [inch] remainder of stroke was to be achieved by locking the platform to the pillars and then lowering the extended arms of the cylinders (Part No.13) for a second stage of loading. Locking the platform to the pillars was considered necessary, not only for this purpose but also because the rams' stiffness in the upward direction was not considered adequate. The stiffness at the maximum stroke was estimated from:

$$\frac{dF_{11}}{dx_{11}} \approx 3A^2 \frac{B}{V} \qquad 8.20$$

B, bulk modulus of oil ≈ 200,000 [lbf/in^2.]

A, effective area ofpressure = 4.725 [in^2.]

V, volume of oil at maximum = 10.0 [in^3.]

or $$\frac{dF_{11}}{dx_{11}} \approx 1.34 \times 10^6 \quad [\text{lbf/in.}]$$

8.6 Design of K_2 Springs (Part No.25)

8.6.1 The Outer Diameter

As seen from Equation 4 of the Appendix 2 for large values of R_{12} the difference between the ideal symmetric and the system with a single set of K_2 springs is negligible as far as the preloading efficiency is concerned (see also Fig. 26a). For large values of R_{12} and R_{J2} the system with a single set of K_2 springs (Fig. 24b) will also approach the ideal symmetric system (Fig. 24a) dynamically, i.e. reducing mode coupling in free vibration, since they will experience only small proportions of F_{11} and hence the additional elastic and dissipative forces will be minimized. In view of the fact that a single set spring system, apart from the above consideration, would result in a much simpler system, it was decided that this system should be adopted and attempts be made to reduce the stiffness of the K_2 springs to the minimum level possible. This meant increasing its index $\frac{D_{o2}}{d_2}$ to the highest possible level. Reduced d_2 was achieved by having a high duty ground bar. Maximum D_{o2} was obtained by making use of all the space available between the pillars and the counter-sinks on the blocks. The outer diameter of springs were therefore found to be 2.75" at its maximum, giving about 5/16" clearance between the nuts upon which these springs rest (Part Nos. 11 & 12) and the counter-sink on the M_1 (Part No.5). This clearance was considered the minimum required in order to be able to employ a purpose-built spanner in altering the position of the nuts on the pillars for very small lengths of test specimens.

$$(OD)_2 = 2.75 \quad [\text{in.}]$$

The optimum wire diameter, free length and number of coils were determined by considering further limitations of space; strength and resistance to lateral loading or buckling as explained in the following sections:

8.6.2 The Constraint Equations

If 'S' denotes the displacement of M_1 (Part No.5) in order to accommodate the specimen and,

L_1 free length of K_1 springs (Part No.24)

L_2 free length of K_2 springs (Part No.25)

ℓ_{min} minimum length of specimen

t_p thickness of the platform (Part No.6)

x_w displacement of M_1 and K_2 springs due to the weight of M_1 and those of the platform and $3K_1$ springs

${}_tx_1$ total displacement of M_1 (including displacement due to loading)

x_{11} displacement of platform due to loading alone

Four constraint relationships amongst these lengths could be written as follows:

a - threaded parts of the pillars which support the springs K_2 should not enter the bearing on the M_1:

$$S \leqslant L_2 - ({}_tx_1)_{max} \qquad 8.21$$

b - the locking nuts of the platform should not enter the bearing area on the pillars:

$$S \leqslant L_1 + t_p - (x_{11})_{max} \qquad 8.22$$

c - the overall length between the cross-beam being 42" is distributed between different parts as shown in Fig.29 gives the third equation:

$$S \leqslant 15.0 - (t_p + \ell_{min}) \qquad 8.23$$

d - the minimum distance between the mass blocks ℓ_{min} is determined by dimensions of parts 5,11 & 12 and .75" clearance :

$$\ell_{min} = L_2 - (x_w + 2.75) \qquad 8.24$$

but from equations 8 and 9 of Appendix 2, the maximum displacements of M_1 and the platform are found to be:

$$({}_t x_1)_{max} = x_w + \frac{(F_{11})_{max}}{K_1} \frac{R}{1+R} = x_w + 1.7 \frac{R}{1+R} \qquad 8.25$$

and

$$(x_{11})_{max} \approx \frac{(F_{11})_{max}}{K_1} \frac{1+2R}{1+R} = 1.7 \frac{1+2R}{1+R} \qquad 8.18$$

(see Fig.29.6)

NOTE: All lengths are in inches.

R_{J2} was taken infinity and R is replaced for R_{12} for simplicity The objective functions are 'S' which is to be maximized and 'K_2' which is to be minimized. S_{max} can directly be found from the above constraint relationships:

$$S_{max} = L_2 - (x_w + 1.7 \frac{R}{1+R}) \qquad 8.26$$

$$S_{max} = 6 + t_p - 1.7 \frac{1+2R}{1+R} \qquad 8.27$$

$$S_{max} = 15.0 - (t_p + \ell_{min}) \qquad 8.28$$

$$0 = -L_2 + x_w + \ell_{min} + 2.75 \qquad 8.29$$

Adding the above equations together and dividing by 3 gives S_{max}:

$$S_{max} = 7.9 - 1.7 \frac{1+3R}{3(1+R)} \tag{8.30}$$

or in other words the optimum stroke is independent of all parameters but R:

$$(S_{max})_{R\to\infty} \simeq 7.9 - 1.7 = 6.2''$$

And adding equations 8.26 and 8.29, it follows that:

$$\ell_{min} = 5.1 - \frac{1.7}{3(1+R)} \tag{8.31}$$

or

$$(\ell_{min})_{R\to\infty} \simeq 5.1''$$

and from equation 8.27 and 8.29 t_p and $L_2 - x_w$ are found as:

$$t_p = 1.9 + \frac{1.7}{3}\,\frac{2+3R}{1+R} \tag{8.32}$$

or

$$(t_p)_{R\to\infty} \simeq 3.6''$$

$$L_2 - x_w = 7.8 - \frac{1.7}{3(R+1)} \tag{8.33}$$

or

$$(L_2 - x_w)_{R\to\infty} \simeq 7.8''$$

It can be shown that for $R > 20$ these values remain almost constant, e.g. maximum difference between the values found for $R = 20$ and $R = 50$ is less than 5%.

8.6.3 Range of Wire Diameter d_2

If W_t were the total weight, and

W_S the weight of K_1 spring

W_P the weight of platform

the total force exerted on K_2 spring is found from equations 6 and 7 of Appendix 2:

$$F_{21} = \frac{F_{11}}{1+R} + W_t \qquad 8.34$$

$$(F_{21})_{R\to\infty} = W_t = \frac{W_B+3W_S+W_P}{3} = \frac{251+3x6.8+112}{3} = 128 \text{ [lbf]} \qquad 8.35$$

would give the lower end of the range. Knowing the thickness of the platform its weight W_P was found and added to those of the M_1 and K_1 springs to give the overall load which was to be taken up by $3K_2$ springs when $R\to\infty$.

W_P = 112 [lb] is the weight of 3.6" thick disc of 13" diameter with one central hole of 4" diameter; and 3 holes of $1\,\frac{7}{8}$", 3 holes of $1\,\frac{1}{4}$" diameter through which the pillars and preloading arms pass respectively.

W_B = 251 and W_S = 3 x 6.8 [lb] were calculated from Figs.29.3 & 29.6 respectively.

$$\therefore \quad F_{21} = \frac{F_{11}}{1+R} + 128 \qquad 8.36$$

Equation 8.12 gives $(d_2)_{min}$ = 0.24" for P_{max} = 1.1x128 = 141 [lbf]

τ_{max}, permissible shear stress for high duty ground spring bars of this size was taken 34 [ton/in^2.].

The upper limit of d_2 was estimated at 0.31" - giving 1/8" clearance between the inner diameter of spring and the outside diameter of the threaded pillar round which the springs were to be located.

$$0.24 < d_2 < 0.31 \text{ [in.]}$$

Figure 26b was constructed by calculating the critical values of the stiffness ratio R, for different values of d_2 within the above range for strength, space and stability limitations. The procedure is outlined in the following three sections.

8.6.4 Strength Limitation R_{min}; Curve 1

For different values of d_2 between the above range R_{min} is found from the following formula:

$$(F_{21})_{max} = W_t + \frac{(F_{11})_{max}}{R_{min}+1} \qquad 8.37$$

where

$(F_{21})_{max} = P_{max}$ of K_2 found from equation 8.12

$W_t = 128$ [lbf.]

$(F_{11})_{max} = 1.7 \times 2130 = 362$ [lbf.] (see Fig. 29.6)

Curve 1 of Fig. 26b shows the variation of R_{min} with the wire diameter. It can be shown that values of R_{min} found by this method are also valid for any number of K_1 springs placed in parallel.

8.6.5 Space Limitation R_{max}; Curve 2

The next step was to find the maximum number of coils which could be accommodated in the length given by equation 8.33 (in order to increase R to its maximum possible level as far as space permitted). To this end an iteration technique was developed which was performed for each wire diameter as follows:

1. Take R_{min} from Curve 1 of Fig. 26b.
2. Find x_w from

$$x_w = \frac{W_t}{K_2} = \frac{128}{2130} \cdot R \qquad 8.38$$

3. Find L_2 from equation 8.33

$$L_2 = 7.8 + x_w$$

4. Find stiffness per coil R_C from equation 8.15

$$R_C = \frac{G.d}{8c^3} \qquad 8.39$$

5. Find deflection per coil δ_C from

$$\delta_C = \frac{P_{max}}{R_C} \qquad 8.40$$

where P_{max} is given by equation 8.12

6. Find the maximum number of coils from

$$n_2 = \frac{L_2 - 2d_2}{\delta_C + d_2} \qquad 8.41$$

7. Find R_2, the stiffness of K_2 springs from

$$R_2 = \frac{R_C}{n_2} \qquad 8.42$$

8. Find R, the stiffness ratio from

$$R = \frac{R_1}{R_2} = \frac{2130}{R_2} \qquad 8.42$$

9. Compare R with the original R_{min}. If different the above procedure was repeated with the new value of R until the final value of R_{max} was established. These values of R_{max} are represented by Curve 2 in Fig. 26b.

8.6.6 Stability Limitation: Curve 3

A spring's resistance to buckling depends upon the ratio of maximum deflection to its free length, the type of supports and its coil diameter.

The following procedure was adopted for different values of R, and wire diameters:

1. Find the deflection due to the weight alone from 8.38

$$x_w = \frac{W_t}{R_2} = \frac{128}{2130} \cdot R$$

2. Find the total deflection of the spring due to the weight and preloading at its maximum from 8.25.

$$({}_t x_1) = x_w + 1.7$$

3. Find the free length of the springs from equation 8.33.

$$L_2 = 7.8 + x_w$$

4. Construct Curve 1 (Figure 27) of $\frac{x_1}{L_2}$ against R.

5. Determine $\frac{C_e D_o}{L_2}$. (See figures 27 and 28)

where C_e depending on end-supports varies from 1.0 to 2.0. For K_2 springs take C_e = 1.5 (i.e. the top end pivoted and the lower end fixed).

and

$$D_o = 2.75 - d_2$$

6. Referring to figure 28 determine $(\frac{x_1}{L_2})_{cr}$.

7. Construct $(\frac{x_1}{L_2})_{cr}$ set of curves numbered 2 in Fig.27.

8. Find the points of intersection of Curves 2 and curve 1, for each wire diameter, in Fig.27.

9. Transfer these points to Fig. 26b making the Curve 3; R_{cr}.

8.6.7 The Choice of Wire Diameter d_2

The ranges of permissible R and d_2 were determined by the boundary of the shaded area shown in Fig. 26b, i.e.

$$18 < R < 52 \qquad 8.43$$

and $$0.265'' < d_2 < 0.310'' \qquad 8.44$$

The wire diameter of 0.265" was the optimum for loading efficiency, i.e. resulting in the maximum permissible R of 52. On the other hand this point represented the critical values for both strength and stability. The sensitivity of R_{cr} (Curve 3) to the way in which the spring is supported is too great. This effect is well illustrated in Fig.27 where R_{cr} for different end supports (C value) are shown to differ grossly.

The above consideration and noting the loading efficiency (Fig.26a) led to a choice of wire diameter of 0.294" (standard size) which left a safe clearance of about 9/64" from the pillars.

$$d_2 = .294"$$

The spring specifications were determined as follows:
A 10% increase in R_{min} corresponding to $d_2 = 0.294$ was allowed for safety reasons.

$R = 1.1 \times 30 = 33$, therefore

$$R_2 = \frac{R_1}{R} = \frac{2130}{33} = 64.5 \text{ [lbf/in]}$$

From 8.15 the number of active coils was determined as

$$n_2 = \frac{Gd_2{}^4}{8D_{o2}^3 \cdot R_2} = 11.5$$

and the free length of the spring was obtained from 8.33.

$$L_2 = x_w + 7/8 - \frac{1.7}{3(1+R)}$$

where

$$x_w = \frac{W_t}{R_2} = \frac{128}{64.5} = 1.98 \quad \text{[in]}$$

$$L_2 = 9.80"$$

(see Fig. 29.6)

The extra capacity of the spring (at its solid length, 356 lbf.) can be employed in reducing ℓ_{min} by loading to the required amount for shorter length specimens. A small portion of this extra capacity could also be employed to exert a preload on the arms of the cylinders to help overcome back lashes, the frictional and gravitational forces within the system and return the pistons to their original 'flush' positions. Care must be taken, however not to exceed the limit specified in equation 8.21 in order to avoid any damage to the M_1 bearings.

8.7 Supplementary Design and Final Observations

The constraint equations 8.21, 8.22, 8.23, together with the known dimensions of the rig frame, the inertia blocks and the springs provided enough information to design the pillars (Part No.4), the platform (Part No. 6) and the remaining auxiliary parts. Details of all parts can be found in the drawings given in figures 29.2 - 29.6.

The following charts indicate the final values obtained after manufacturing the parts. A slight deviation from the optimum or designed values of some parts were either deliberate as matters of expediency or due to some manufacturing errors. It is noted that these differences as seen below are small and have no significant bearing upon the overall behaviour of the apparatus.

Name	Part No.	Parameter	Unit	Optimum	Designed	Manufactured
M_1	5	weight	pounds	256	251	245.7
M_2	5	weight	pounds	256	251	244.5
K_1 springs	24	stiffness	pounds/in.	1970	2130	2337
Platform	6	thickness	inches	3.6	3.25	3.25
Platform	6	weight	pounds	112	103	103
K_2 spring	25	length	inches	9.8	9.8	9.57
K_2 spring	25	Active coils	number	11.5	11.5	11.75
K_2 spring	25	Stiffness	pounds/in.	64.5	64.5	64.0

which resulted in slight changes of some parameters as follows:

From equation 8.42	R	-	-	33	35
From equation 8.38	x_W	inches	2.06	1.88	1.90
From equation 8.29	ℓ_{min}	inches	4.99	5.17	4.92
From equation 8.27	S_{max}	inches	6.25	5.90	5.90
From equation 8.11	NF_1	Hz	15.0	15.8	16.6

For the purposes of the present work only about 80% of the full preloading capacity of the apparatus was needed (see equation 8.14 and Fig. 29.6). Consequently S_{max} could be raised to the following figure if required

$$(S_{max})_{max} = L_1 + t_p - \frac{(F_{11})_{max}}{K_1} \; \frac{1 + 2R}{1 + R} \quad \text{(equations 8.22 and 8.18)}$$

or

$$(S_{max})_{max.} = 6.55 \text{ [in.]}$$

For the purpose of quick reference a summary of specification of the apparatus is presented in Table (9) which was produced with the help of the results from Chapters 10 and 12 (static and dynamic calibration of the rig).

CHAPTER 9

INSTRUMENTATION

9.1 The Features

The basic requirements of the measuring apparatus were as follows:

(a) Accurate measurements of phase and gain to realize pure mode excitation; point-to-point measurement and vector division was most desirable for the purpose of increased efficiency.

(b) Double point excitation with variable phase and gain in order to sustain pure mode.

(c) Recording of decay curves, and

(d) Accurate measurement of frequencies.

(See Table 8)

The following procedures investigate and fulfil every one of the above requirements leading to the final choice of instruments.

9.2 Frequency Response Analysis

Until recently manufacturers of transfer function or frequency response analysers have been concerned mainly with electrical servo systems. The results in the majority of cases were (a) the frequency band was too high and (b) point-to-point measurement, which is of particular interest to vibration or control engineers was not possible or extremely tedious.

Low cost phase-meters, designed principally for use in power systems and transmission lines, work on the principal of 'zero crossing' and hence are too sensitive to noise and harmonics. More refined and accurate variations of such instruments (sometimes accompanied by a gain meter) do exist but apart from the above basic shortcomings there is always a limitation on the minimum input signal or the ratio of the two input signals. Furthermore such instruments are within or very close to the price bracket of Transfer Function Analysers. Again most Transfer Function Analysers are equipped to be used only for 'single-point-measurement', i.e. the response measured would include the effect of the shaker and its associated power amplifier or other intermediary network plus that of the 'Black Box' under study.

The process of cleaning the response signal of noise and distortion (e.g. harmonics caused by non-linearity) is an integral part of the frequency response analysis as far as gain is concerned. This process also becomes an essential part of any accurate phase angle measurement. Depending upon the principle of the cleaning operation these systems are divided into two main groups :

1. Sine wave correlation
2. Filtering.

9.2.1 <u>Sine Wave Correlation</u>

Inphase and quadrature signals from the built-in generator driving the system, are multiplied by the two (point-to-point; Solartron) or one (single point; S.E.LAB.) response signals and are integrated over a finite number of cycles depending upon the accuracy required. The spurious signals having frequencies other than that of the excitation will result in a zero A.C. and only D.C. components proportional to inphase and quadrature components will be the outcome. In some systems a 'mechanical reference synchroniser' is provided to lock the generator frequency to some external signal. Such a facility is of great value in the case of external excitation. This option will be doubly beneficial in single-point-analysers as it makes point-to-point measurement also possible.

9.2.2 Filtering Techniques

There are two basic variations of these instruments distinguished by the method with which the centre frequency of band pass filter and frequency of excitation are brought together.

(i) Programmable Active Filters

The centre frequency of the filter is continuously programmed by a D.C. voltage until it is on tune with the input signal. The feedback voltage is produced within a phase sensitive detector by comparing the input signal with the quadrature component of the filter output. When on tune, the resonant circuit (the filter) produces two signals which are inphase and quadrature with the fundamental frequency of the input signal.

These filters when combined with other phase sensitive detectors can form a 'tracking filter' (AIM).

An error in phase angle measurement of up to 10° has been reported when using active filters within the frequency range of 2 Hz - 2KHz.

(ii) Single and Double Conversion Heterodyne

The principle of operation is similar to that of some radio receivers. The tuning is achieved by shifting the input signal along the frequency domain by frequency modulation. The operational technique here is the reverse of that used in (i): it employs passive filters.

A carrier converter combines the tuning signal frequency with a relatively high but constant frequency which is produced by a crystal oscillator. The next stage of operation is the mixing of this signal with the input signal in a 'balanced modulator' to give the sum and the difference of frequencies. Now if a crystal filter with the same centre frequency as the crystal oscillator is used to attenuate alien frequency signals, then this type of analyser is called single conversion heterodyne. Derritron, Spectral Dynamics and Ad-Yu systems all use analysers of the single conversion heterodyne type. If instead of a crystal filter low-pass filters are employed then the analyser has to be of the Double Conversion heterodyne type. The reason is that the tuning frequency has to be lowered further to a practical frequency spectrum which can be handled with low pass filters. Bruel & Kjaer and Quantech both used the double conversion technique in their analyser.

It is true to say that the filtering techniques are only suitable for electrical servo systems. Point-to-point measurement is not possible, and if a sweep-frequency plot is required the only possible way is to have two 'identical' analysers in order to obtain transfer functions plots of two points, e.g. force and displacement and then normalise the plots. However to have identical filters with identical frequency dependency is almost impossible in practice. A further disadvantage of the filtering technique is the limitation imposed by the band width. In reference (82) a semi-automatic method of correction for such errors has been employed using a computer.

For the purpose of this work, however, frequency sweep plots were not required and therefore considering the evidence given above the most suitable type of analyser was that employing sine wave correlation or integrating techniques which afford point-to-point and more accurate phase measurement. The choice had to be made between two products, (a) Solartron 1179 series, and (b) SESM 200 series. The former not only offered point-to-point measurement but also vector division, auto ranging, frequency sweep, automatic D.C. rejection and a frequency range of 0.001 Hz - 10 KHz. The latter was about 10% cheaper but lacked the following features: auto ranging, D.C. rejection and frequency sweep facilities. Also, it could only measure single points whilst Solartron afforded both direct and indirect point-to-point measurement in the case of excitation within. The Solartron was not equipped as yet for external excitation and hence constituted the major disadvantage. The S.E. analyser, on the other hand, did provide the 'mechanical reference synchroniser' but under practical conditions it was found that in order to lock the frequencies the input to synchroniser had to be clean (and relatively of high levels-min.150 mv RMS) hence defeating the object of the exercise.

9.3 Double-Point Excitation and the Choice of Analyser

This requirement was satisfied by having two sets of power amplifier-shakers. The decision however had to be taken on whether to use the generator output of the analyser with a phase control unit, or a separate variable phase oscillator and a mechanical reference synchroniser in order to lock the frequency of the analyser to that of the oscillator (e.g. S.E.LAB). The limitations imposed by the former systems were two-fold.

(a) Should the system possess perfect symmetry the analyser could be redundant when only free-vibration methods are to be employed (Fig. 30). To achieve pure mode excitation the loads are equal in magnitude, in or out of phase at all frequencies. Such a condition could be ensured by a simple variable phase oscillator and a simple monitoring device, e.g. oscilloscope. (see Figs. 17b and 30)

(b) As the generators within the analysers produce sinusoidal signals by 'bits', distortion is rather high (≈ 1%) compared with conventional L.C. analogue oscillators (distortion ≈ 0.1%) but they have a much higher stability (crystal controlled) which makes them ideal for locating the natural frequencies.

On the other hand the alternative, i.e. use of a separate variable phase oscillator seemed to limit the choice of analyser to 'S.E.LAB' which was, unlike the Solartron, equipped with an external synchronising unit.

As both methods were similar in cost, attempts were made to modify a variable phase oscillator so that it could be slaved to an external generator and thus not only make the best of both worlds but still retain the option on the analysers. As will be explained in the following section, this was achieved using Philips' variable phase oscillator (PM 5161) for which a very simple synchroniser unit was built and added.

The decision to acquire the Solartron (1172) analyser was finally reached; in short, the most powerful feature of 1172 which tipped the balance in its favour was its capability of point-to-point measurement which was considered of great value not only for the present work but for any similar work which might be undertaken in the future. Fig. 31 shows the complete block diagram of instrumentation. Information regarding individual blocks are given below.

9.4 External Synchroniser for Oscillator (PM 5161)

The following briefly describes the additional circuitry which enabled the Philips PM 5161 oscillator to latch on to the analyser's frequency. The specification of the network was supplied by the Company and it will be given here for future reference (see Fig. 32).

The collector of BC 108 transistor was connected to the collector of TS 302 within the oscillator. When in use the oscillator was set up to approximately the frequency of excitation. The analyser sine output was connected to the base of BC 108 and the output was increased until BC 108 collector was pulled down which in turn pulled down the TS 302 forcing the oscillator to lock into synchronism.

The 'lock', once established enabled the frequency from the analyser to be varied over about 3 dbs without affecting stability or increasing distortion. The signals thus obtained, possessed the high frequency stability of the crystal oscillator and also the low distortion of classic oscillators.

9.5 Triggering Control Unit

The unit was built as shown in Fig. 33 to perform the following functions:

(a) Cut-off drive to shaker units and simultaneously trigger the oscilloscope to record and store the damped oscillation which ensued.

(b) Attenuate equally the input voltages to the shaker units.

(c) Switch individual shaker units independently.

The first requirement was achieved employing a three pole changeover relay. Two of its poles were used to switch the inputs to the shaker units and the third to send a pulse to trigger the oscilloscope. The contacts 'normally closed' when the relay is de-energised, routes the oscillator outputs to the inputs of the power amplifiers (P.A.1 and P.A.2 using RL1 and RL2).

With the relay energised, these poles cut off the input to the power amplifier from the oscillator outputs and short-circuited the inputs of the power amplifiers. The high inherent damping of the power amplifiers enabled this approach to be used successfully with no high pulses reflecting on the shakers.

As a 12 volt D.C. supply was available to actuate the relay this supply was employed to feed a resistance potential divider to give a tapping point at approximately +2.5 volts for connection to the 'open contact' of the third pole of the relay (RL3). When the relay was de-energised (Trigg. Switch off) the 100 pf capacitor was connected to the relay's normally closed contact and was discharged by Z_{in} (100 KΩ) the input impedance of the external trigger circuit of the oscilloscope. When the relay was operated the third pole connected this 100 pf capacitor to the 'open contact' which was at 2.5 volts. The resultant spike triggered the oscilloscope which had previously been set up for the external trigger mode. The transient signal applied to the y axes of the oscilloscope were thus stored on the screen.

The second requirement was achieved employing a dual ganged potentiometer placed in series and preceding the two poles of RL1 and RL2. This enabled simultaneous attenuation of the inputs to the shaker units up to about 2 dbs for fine adjustments.

The third requirement was achieved by simply placing two toggle switches (switches 1 and 2 in Fig. 33) at the inputs to the respective power amplifiers.

9.6 The Transducers, the Associated Amplifiers and the Shaker Systems

Force and displacement, under both static and dynamic test conditions are normally measured using inductive or capacitive detectors with associated amplifiers, basically consisting of high frequency carrier and filtering techniques. It is therefore evident that, as far as dynamic measurements are concerned they suffer from the same disadvantages mentioned for filtering methods of analysers, namely phase changes through the amplifiers which make phase measurements extremely tedious. Difficulties are also experienced with 'setting' as the sensitivity of such transducers normally depend upon the 'gap' or positioning of probes. Furthermore for measuring very small displacements which is the case when testing joints (mostly fractions of 1 μm) sufficient sensitivity is often not available with such transducers.

The latest development in piezo-electric transducers and charge amplifiers has made possible a faithful reproduction of loading regime under both static and dynamic loading. That is to say that apart from very good linearity the output will be representative of actual timing or phase of the load. Obviously displacement measurement is not possible under static conditions as the crystals have to be strained to sustain electric charges. Piezo-electric accelerometers were therefore developed to remedy this shortcoming in measurement of motion. They employ seismic weights against crystals with relatively high natural frequencies. At sufficiently low frequencies the following equation:

$$F = - ma = m\omega^2 x \qquad 9.1$$

gives acceleration or displacement. The upper and lower limits of frequency are therefore determined by the natural frequency of the seismic assembly or transducer connection to the vibrating body whichever is lower, and the time constant of the range circuitry of charge amplifiers.

A decision was therefore made to use piezo-electric transducers with associated charge amplifiers for both force (F_1, F_2, F) and acceleration (a_1 and a_2) measurements.

The make, type and sensitivity of the transducers with associated amplifiers were specified after the following preliminary calculations which roughly determined the range of forces and accelerations likely to be encountered during the tests. These calculations also determined the size and wattage of the shakers and power amplifiers respectively.

Apart from the assumption made in the design, i.e. E.L.S. of the joint being found from:

$$\ell_J = 5'' \times \frac{250}{\sigma}, \quad 100 < \sigma < 6800 \qquad [PSI] \qquad 9.2$$

One further assumption concerning the quantitative behaviour of joints i.e. the range of damping values likely to be encountered was made. This range was estimated from Fig. 11b as:

$$\zeta_j = 0.1 - 1.0 \qquad \text{(at 250 PSI)} \qquad 9.3$$

In order to simplify the calculations, perfect symmetry of the system, zero extraneous effects and frequency independency of parameters were also assumed.

9.6.1 $(a_i)_{min}$: The Choice of Accelerometers and Charge Amplifiers

In an attempt to gain an insight into the order of amplitude (and frequency range) in machine tool joints; it is reported in reference (82) that displacement of 2μ"(.05 μm) were typical values found at the tool holder joints of a lathe.

The displacement of the inertia blocks, at its minimum was therefore estimated as:

$$(x_1')_{min} = \frac{1}{2} (N+1)_{min} x_J \qquad 9.4$$

where

$$(N+1)_{min} = 5$$

and $$x_J = 2 \times 10^6 [in.] = 0.05 \ [\mu m] \qquad 9.5$$

The minimum frequency was determined by referring to Fig. 43: for all practical purposes the frequency at which the second mode can be excited, without the possibility of significant deviation from the 'symmetry' and hence minimal mode coupling, is about 30 [Hz]. The minimum acceleration was determined from:

$$(a_1)_{min} = \omega^2_{min} (x_1')_{min} \qquad 9.6$$

$$= 0.178 \ [in/sec^2.] = 0.46 \times 10^{-4} [g]$$

The resolution and accuracy of the analyser (Solartron 1172) is 10[μv] and ±0.3 per cent ± digit respectively. The highest conditioning gain available on charge amplifiers is normally 10[v/g] when used with accelerometers with sensitivities of at least 10[Pc/g].

The minimum size of accelerometers (to increase the useful range of its frequency) and the associated amplifiers which would produce signals of the order of [mv] for $(a_1)_{min}$ were then determined as follows:

2 x charge amplifiers B & K 2626 max.grain 10 [V/g]

2 x accelerometer B & K 4343 sensitivity 75 [Pc/g]

one standard accelerometer B & K 8305 sensitivity 1.247 [Pc/g] for calibration purposes (see Chapter 11).

This arrangement would produce a 4.6 [mv] signal representing a 5 [μ"] displacement of the blocks at 30 [Hz], a minimum possible signal likely to be experienced during the testing of joints.

The sensitivity thus obtained was about thirty times of that achieved in reference (39) and seven times of that achieved in reference (40). Both used inductive type displacement transducers but the latter was specially tailored for maximum sensitivity by replacing the copper core with a ferrite rod. It is therefore evident that for small dynamic displacements such as those experienced in testing joints the acceleration measurements not only offer much simpler method but also much higher sensitivities.

9.6.2 $(F_1)_{max}$: The choice of the Shaker System

From equation 15 of Appendix (2), the excitation force in terms of stiffness of specimen column K and deflection of the blocks x_1', at the second mode, was estimated at;

$$\left|F_1\right| = 2Kx_1' \left[(1-\partial_2^2)^2 + \partial_2^2\zeta_2^2\right]^{\frac{1}{2}} \qquad 9.7$$

at the second natural frequency of the system NF_2, $\partial_2 = 1$:

$$\left|F_1\right| = 2Kx_1'\zeta_2 \qquad 9.8$$

Assuming material damping to be zero, from equation 6.71

$$\zeta_2 = \frac{NK}{K_J} \frac{1 + \zeta^2}{1 + \zeta^2} \zeta_J \qquad 9.9$$

and

$$x_1' = \frac{1}{2} \frac{F}{K^*} \qquad 9.10$$

but

$$F = x_J . K_J^* \qquad 9.11$$

$$\therefore \quad x_1' \simeq \frac{1}{2} x_j . \frac{K_J^*}{K^*} = \frac{1}{2} x_J \frac{K_J}{K} \frac{(1 + \zeta_J^2)^{\frac{1}{2}}}{(1 + \zeta_2^2)^{\frac{1}{2}}} \qquad 9.12$$

Equations 9.9 and 9.12 were inserted for ζ_2 and x_1' in equation 9.8, yielding $|F_1|$:

$$|F_1| = NKx_J \frac{1 + \zeta_2^2}{1 + \zeta_J^2} \zeta_J \qquad 9.13$$

but

$$NK = N \frac{\Delta E}{\ell} \simeq \frac{\Delta E}{\ell_J + t} \qquad 9.14$$

$$\zeta_J > \zeta_2 \qquad (\zeta_m \to 0) \qquad 9.15$$

therefore

$$|F_1|_{max} = E \frac{\Delta x_J \zeta_J}{\ell_J + t} \qquad 9.16$$

at 250 PSI, ℓ_J = 5 inches, equation 9.16 was reduced to:

$$|F_1|_{max} = 11.16 \quad \Delta\zeta_J \quad [lbf.] \qquad 9.17$$

but $0.45 < \Delta < 2.7[in^2]$ and $0.1 < \zeta_J < 1.$, therefore:

$$0.5 < |F_1|_{max} < 30.1 \quad [lbf]$$
$$0.2 < |F_1|_{max} < 13.6 \quad [KP] \qquad 9.18$$

The above figure indicated roughly the size and power of the shaker and its amplifier: it was reasonable to assume that variation of $\frac{\zeta_J}{\ell_J + t}$ with pressure would be small and hence the above estimate would be tenable for other pressures too. The following system satisfied the above requirements as well as having suitable dimensions:

2 x Shaker Derritron VP4B load capacity 50 $[lbf.P_K]$ = 22.7 $[KP.P_K]$

2 x Power Amplifier Derriton 300 wattage 300 [W]

9.6.3 The Load-Washers, the Force-links and the Associated Charge-Amplifiers

(i) F measurement

Appropriate load-washers were selected after referring to Fig. (22) for dimension and to Table (9) for the minimum load carrying capacity, i.e. 5.0 [Tons].

	Make	Type	Max.load	Sensitivity
1 x Load-washer	Kistler	903A	6 [Tons]	4.4 [PC/N]
1 x Load-washer	Kistler	906B	20 [Tons]	21 [PC/KP]
1 x Charge amplifier	"	5001	-	-

(see Tables 4 and 5)

(ii) F_i measurement

As the force-links were to carry only dynamic loads of relatively low levels (23 KP.max.) the criteria for selection were only the suitable dimensions and high sensitivities:

2 x Force-links	B & K	8200	500 [KP]	∿ 4.0 [PC/N]
2 x Charge amplifier	B & K	2626	-	-

9.7 Fast Routing of Recovered Signals and 'Earthing'

(see Figure 31)

A switch-box was built to satisfy the following requirements:

(a) To be able to route any of the five recovered signals (F_1, F_2, F, a_1 and a_2) speedily to any measuring/monitoring device.

(b) To avoid 'earth loop ' or similar disturbing effects on the recovered signals.

Inter-unit wiring was carefully arranged and run to the switch box using screened co-axial cables and B.N.C. connectors. The switches not only directed desired signals to the measuring devices but also simultaneously short circuited the screenings (braids) of the input/output lines. The input/output B.N.C. sockets were similarly floated on an insulating panel. To prevent 'cross-talk' between the signals only alternate switch positions were used for signal/screen connections, the interleaving switch positions being earthed.

The earth point on the 'measuring system' was made at one of the conditioning charge amplifiers (Kistler). The rest of the instrument terminals were floating but for the power amplifiers which were also earthed to the rack. As the power amplifiers were isolated from the rig no alternative routes existed for the recovered signals but through the 'common earth point',i.e.the rig, thus no earth loops were possible (See Fig.35).

9.8 Recording of Decay Curves

Two double beam storage oscilloscopes (Telequipment DM 64) together with a specially designed triggering mechanism (Fig. 33) provided for both on the spot examination and photography of decay curves.

9.9 Frequency Counting

The analyser had a digital frequency display - however should the system possess perfect symmetry in the absence of the analyser (Fig.30) a frequency counter would be required to measure the natural frequencies (e.g. Racal 9835).

9.10 Summary

All the objectives listed in section 9.1 were realised in addition to the following advantageous features:

(i) The System of Excitation

The high frequency stability of the crystal oscillators were retained together with the low distortion feature of the L.C. oscillators. This was achieved employing the crystal controlled generator of an analyser to drive a variable phase oscillator which in turn produced the two signals required for the double point excitation.

The use of an 'external mechanical reference synchronizer' and a 'phase control unit' was obviated by a simple modification of the oscillator (see Figs. 31 and 32).

A further advantage gained by retaining the variable oscillator was that it could be used under its own steam in the absence of the analyser;

e.g. whenever the system exhibited 'perfect symmetry' (Free-vibration method. Fig. 30).

(ii) The System of Measurement

The use of piezo-electric transducers and charge amplifiers resulted in accurate and direct phase angle (and gain) measurements between any two points within the system (Point-to-point measurement).

The use of accelerometers apart from the above advantages resulted in high sensitivity under dynamic measurements of very low displacements, hence they were ideal for testing 'joints' ($a = - x\omega^2$): Sensitivity at 30 [Hz] was about 40 [mv/μm] or 1 [mv/μ"], being seven and thirty times those reported in references (39) and (40) respectively, where inductive transducers were employed.

Direct measurement of the 'Apparent Mass Vector' $\bar{m}$ and the 'Modal Shape' $\bar{r}$ (see Table 8) was possible by a turn of a switch; thanks to the vector dividing facility on the analyser and a switch box which was built for this purpose.

Recording of decay curves and on-the-spot measurement of stiffness and damping was possible under free-vibration methods by employing the storage facility on the oscilloscope and a specially designed triggering mechanism (see Figs. 30 and 33).

CHAPTER 10

TEST RIG ASSEMBLY AND ITS STATIC CALIBRATION

10.1 Static Calibration of Preloading Springs K_1 (24)*

A Denison machine was used to test the springs with a view to optimising the stiffness (and mass) distribution to achieve as close a symmetry as possible when assembled. Furthermore as shown in Chapter 10.3 the best method of determining the preload on the specimen was by measuring the deflection of block M_2; therefore their stiffness had to be measured accurately.

The error for the certified range (grade A1) according to BS 1610: 1964 were as follows:

$0 - \frac{1}{5}$ full scale ±0.1% of full scale

$\frac{1}{5}$ - full scale ±0.5% of verification load

All the springs were numbered and then tested measuring the deflection across the flats using a 10^{-3} [inch] dial gauge. The results are shown in Fig. 34. The 'Best Lines' in Fig. 34 were determined with the aid of a computer and represented those resulting in minimum sums of squared deviations from the observed values.

* The number in brackets in line with the text refers to part numbers given in the general assembly drawing; Figure 29.1.

There were two regions of non-linearity in the force-deflection graphs as expected. The reduction in stiffness at the initial stages of loading was due to the end coils. Increased stiffness at high loads on the other hand was due to the gradual reduction in the number of active coils.

The springs' height was measured and they were also tested for squareness. Errors in height were found to be about 20 - 30 'thou' (0.5-0.8 mm) and in squareness they fell within the range of 5 - 10 thou' taken over the diameter. Errors of squareness were always found to be in the same direction hence thought to have been caused by mis-alignment in the slideway mechanisms of the grinding machine.

The selection of K_{11} and K_{12} sets of springs were made by considering the following points:

(a) In order to minimize the possibility of damage caused by bending moments at bearing (26) and the effect of interfering extraneous modes of vibration, the stiffnesses of the springs of each set should be as close as possible to one another.

(b) The set with higher stiffness was chosen for K_{12} so as to counteract the effect of K_{21} hence resulting in closer symmetry (M_1 = 111.4, M_2 = 110.9 Kg, assumed equal).

Examination of Fig. 34 resulted in the following grouping.

Spring Nos. 1, 3, 4 for K_{12} , and

2, 5, 6 for K_{11}

The difference in stiffness between the groups was about 3% hence compensating almost fully for the effect of K_{21} springs.

Two more attempts at further refinement were followed by partially compensating for the errors in length and squareness of the springs. As an example, the springs in group K_{12} were positioned on the lower sandwich (3) in such a manner that the errors in the length were in the opposite direction to that of the 'level' of the sandwich. As for squareness errors it was considered favourable to have identical angular position relative to the centre to reduce bending at the bearings.

10.2 Rig Assembly

The following briefly describe the assembly of different parts in the order which they took part in the ensemble. (see Fig.29.1 and 29.7).

10.2.1 Supporting Plate (20)

It was important that the base plate upon which all the other parts rest be level for ease of alignment in gravitational directions and to reduce bending moments particularly on the pillars (4). The level of the concrete platform and position of the rag bolts were adjusted using a dummy wooden base plate. The accuracy of this method was not too great due to non-homogeneity of the concrete mixture - the level changed somewhat after the mixture dried. The final adjustments had to be made, therefore, using shims under appropriate supporting RSJ's (20) after the lower sandwich (3) had been fixed to them at its lower plate.

10.2.2 Supporting R.S.J's (20) and Lower Sandwich (3)

Assembly of these parts was accompanied by accurately levelling the lower sandwich using aluminium shims which were placed under appropriate supports. They were rings placed round the bolts connecting the R.S.J's to the base plate. The thickness of the shims was determined by a high accuracy spirit level (.0005" over 10") about .022" (.56 mm). The resultant level accuracy of the lower sandwich in all directions was ± two divisions of the spirit level which was considered to be quite adequate.

Alignment of the centres of the lower sandwich and the base plate was carried out using a plumb-line before tightening the bolts. This was necessary in order to ensure alignment of the centres of the lower shaker and mass block M_2(5).

10.2.3 Columns (1)

The columns were then placed in position, the bolts slightly tightened allowing slippage if knocked by a wooden hammer.

10.2.4 Pillars (4), K_{12} Springs (24) and their Seatings (10), Inertia block M_2 (5), K_2 Springs' seatings and locking Discs (11, 12).

M_2, with its bearings (26) placed in position using the appropriate retainers (17), was suspended at its centre with the hardened end facing upwards. The pillars (4) were then inserted through the bearings and where they emerged appropriate K_{12} springs were placed and their seatings (10) were screwed on. It proved convenient to have the K_2 seatings and the locking discs positioned at their lowest position on the pillars so that they would hold the pillars by resting against M_2. The K_1 spring seatings were then screwed all the way up until they came against the springs. The whole unit was lowered into position using a plumb-line to align the centres of M_2 and the supporting plate (20). The pillars were then locked to the lower sandwich to retain the alignment using the locking nuts (33) and the associated retaining rings (27).

10.2.5 K_2 Springs (25), Inertia Block M_1 (5), K_{11} Springs (24) and Platform (6)

The distance between the blocks was adjusted to the desired level by screwing the K_2 seatings away from M_2 and locking them in position before inserting the K_2 springs. Because these springs possessed very low stiffness (64 lbf/in) their dimensional accuracy and also their positioning accuracy on the pillars did not need to be great. The next steps were to lower M_1 (with its hardened surface facing downwards) onto the K_2 springs, to insert the K_{11} springs round the pillars, to lower the platform in position and finally screw in the locking nuts (31) and (32) respectively.

10.2.6 Upper Sandwich (2), Hydraulic Hollow Cylinders (29) with their Sleeve (19), and mounting Rings (7), Strengthening Studs and Associated Retaining Rings (28), Retaining Rings for the Pillar (27) and Extended Arms for the Cylinders (13)

Using a plumb-line, the centres of upper sandwich and M_1 (5)

were aligned using a wooden hammer knocking in position the columns (1) and the upper sandwich, before tightening the nuts (28, 32, 37). The mounting rings (7) were tightened after screwing in position the extended arms for the cylinders (13) and aligning them with centres of correspon-ding holes in the platform (6). Before finally locking the pillars to the upper sandwich by means of the nuts resting against the lower face of the upper sandwich a small preload of about 8 [KP] was applied to K_{11} and K_2 springs by pumping some oil to the cylinders. This had two beneficial effects. Firstly it helped the rams' fast return when un-loaded and secondly it eliminated any backlash within the system, res-ulting in good permanent grip of the springs K_{11} and K_2.

The final rig complete with the appropriate adaptors (8, 9, 14, 15, 16, 30, 34) for shakers (21) together with the measuring instruments is shown in Fig. 35.

10.3 Measurement of Preload on Specimens

In order to determine the preload on specimens there were three possibilities:

(i) Measuring pressure at the cylinder (Part No.29):

The efficiency of this method was undermined by

(a) Correction which had to be made due to the flexibility of the specimen itself:

$$F_J = \frac{R_{J2} \cdot R_{12} \cdot F_{11}}{R_{J2}(1+R_{12}) + R_{12}} = \frac{35R_{J2}}{36R_{J2} + 35} \cdot F_{11} \qquad 10.1$$

(equation 4 Appendix 2)

For flexible specimens ($R_{J2} \rightarrow 0$), F_J must be determined from equation 10.1.

(b) The initial pressures, e.g. that for returning the rams and overcoming the back-laches, had to be taken into account particularly at low levels of preload.

(c) Sufficiently high resolution (which was required for testing at very low levels) on a pressure gauge which would cover the whole range (∿ up to 777 PSI = ∿ 54 KP/cm^2.) could not be achieved.

(ii) To measure the load directly across the specimen using load-washers in pseudo-static mode.

This was considered to have the following disadvantages:

(a) For accurate measurement of D.C. output a digital voltmeter was considered necessary.

(b) Step-by-Step loading or unloading would not be possible.

(c) To avoid 'drift', the leads' connection had to be protected against any foreign agents at all times thus rendering it impractical.

(d) It was not considered acceptable to assume that shape and size of specimens would, as a matter of course, be accommodating as far as the load washers were concerned.

(iii) To measure deflection of M_2 (5);

This meant accurate calibration of M_2 deflection against the joint load. The calibration was to be carried out

(iii) continued

using the load washers and measuring the deflection by a 10^{-3} [inch] dial gauges. It was of course necessary to calibrate the load washers beforehand.

The above method proved to be the most practical of the three for the present work. However a pressure gauge (35) was considered necessary for safety if not for measurement.

10.4 Static Calibration of Load Washer

The Denison testing machine was also used to calibrate the load washers. The output from the charge amplifier was measured separately using a digital voltmeter for each setting of the range capacitors. Checks were made on drift by fast and slow loading and unloading. The difference was not significant.

Examples of results can be found in Fig.36 where output voltages of the charge amplifier at certain settings of amplification are shown.

10.5 Static Calibration of the Test Rig

Static calibration of the test rig was carried out using one of the load washers (906B Kistler) with the charge amplifier which was set at the same amplification factor as in Fig. 36. The set-up was as shown in figure 44a for testing equivalent length of solid of jointed columns of C180 specimens.

The oil was pumped into the cylinders (29) taking note of:

(i) Oil pressure [PSI]

(ii) Reading on D.V.M. [volts]

(iii) Deflections of M_2 (5) against the lower sandwich (3) at 3 equi-spaced points close to the edges of the block to check it for rotation. Dial gauges with magnetic bases were used for this purpose. [10^{-3} inches].

Differences between the three deflections measured at maximum load (about 4000 KP) were never greater than 0.002" (∿ 0.05 mm) indicating that no significant rotation or asymmetry was present as far as static behaviour of the system was concerned. A further check on this was made by comparing the results of (ii) and (iii) above. The latter was found by relating the measured deflections to the corresponding loads at K_{12} spring given in Fig. 34.

The results are shown in figures 37 - Figure 37b was used when testing at low preloads (e.g. A45 specimens) as figure 37a lacked the sufficient resolution at such preloads.

The 'best-curves' of degrees one and two determined by the computer, representing the mean of the results (ii) and (iii), within the range 0 - 4000 [KP], were as follows:

$$F_J = -59.8 + 127.5\, X_2 \qquad SSD \simeq 5024 \qquad 10.2$$

or

$$F_J = -21.4 + 119.8\, X_2 + 0.25\, X_2^2 \qquad SSD \simeq 470 \qquad 10.3$$

(F_J in KP and X_2 in mm.)

The slight deviation from linearity was again attributed to the end coils'/closing coils' effect at the lower and the higher ends of the range respectively.

10.6 Summary

The K_1 springs were calibrated with better than 1% accuracy using a 'grade A1' Denison Testing machine (Fig.34). There were two regions of non-linearity in the force-deflection graphs attributed to the effects of 'end coils' and 'closing coils', at the initial and the final stages of loading respectively. The 'best-lines' were determined using a computer representing those which resulted in minimum sums of squared deviations from the observed values. The maximum difference found within the rates was 7%.

In order to minimize 'asymmetry', appropriate K_1 springs were chosen to form the preloading-isolating springs K_{11} and K_{12} (Fig. 24b). The difference between the rates of the sets were about 3% thus compensating almost fully for the effect of the weight-supporting springs K_{21}.

The assembly of the rig was then commenced in a systematic manner set out in this chapter.

The next stage of operation consisted of static calibration of the rig. The measurement of the load on the specimens was made by (i) a pressure gauge (ii) a load washer (which was in turn calibrated with the Denison testing machine and found to be accurate within 5%. Fig.36) and (iii) relating the deflections of the interia block to the corresponding loads given for K_{12} springs in Fig. 34. The results of this calibration are shown in Figs. 37 & 38. The curves (ii) and (iii) in Fig. 37a are congruent to within 4%, indicating a good alignment of the mechanisms. For increased precision, therefore, the mean of these results was used in deriving the 'best-curves': equation 10.2 (degree 1) and equation 10.3 (degree 2). The slight deviation from linearity was again attributed to the end coils'/closing coils' effect at the lower and higher ends of the range respectively. These effects are responsible for about 8% drop in the 'best rates' for the 0-4000 [KP] and 0-600 [KP] ranges as shown in Figs. 37a and 37b.

For safety and quick reference Fig. 38 was constructed from Fig. 37a relating the pressure gauge readings (i) to the preload on the specimens.

It is noted that depending upon the test conditions and accuracy required any of the three methods (i), (ii) or (iii) could be used. For the purpose of the present work method (iii) was preferred for its better resolution and greater accuracy.

CHAPTER 11

DYNAMIC CALIBRATION OF INSTRUMENTS

11.1 Introduction

11.1.1 General

There are basically two distinct methods of calibration. The first method determines the transfer function of each individual unit whilst the other calibrates the whole of the measuring system for the desired mechanical parameter and under the exact simulated test conditions. The latter method sometimes referred to as 'mechanical calibration' is obviously preferable as it increases efficiency.

For linear systems the calibration technique can be refined still further. The ratio of output to input in transfer function analysis of a composite system is sufficient to define the system without the need for accurate measurement of its individual absolute values. In impedance measurement for example, if the ratio of acceleration to force is found over the frequency range of interest the system can be defined in terms of 'Inertiance'; or inversely in terms of 'Apparent Mass'. The calibration procedure is therefore to apply a force to a solid block of known mass and to measure both the input force and the resulting acceleration signals. The ratio of these two signals is directly proportional to the mass of the calibration block which may be independently measured to a high degree of accuracy.

$$f = Ma \text{ or } \frac{f}{a} = M \text{ which gives } \frac{f}{aM} \text{ in [db/Kg] or } \left[\frac{\text{signal ratio}}{\text{Kg}} \right]$$

Practical difficulty arising from such a method is the reduced dynamic range caused by the following two limiting factors. First the link between mass-transducer-shaker which can be rarely made sufficiently stiff to preclude the elastic forces' effect at high frequencies. The other end of the frequency spectrum is marked by the fact that the stroke of the shaker table is limited. Fortunately the piezo-electric transducers possess very good linear characteristics(82); therefore calibration can be carried out at low levels affording the use of the small size M hence not only extending the frequency spectrum but also simplifying the calibration.

11.1.2 Calibration for $\bar{m}_i$ and $\bar{r}$ Signals - Relative Calibration Method

As can be seen from the equations in Table 8a all the measured quantities ($\bar{m}_i$ and $\bar{r}$) are relative to $\bar{a}_1$. As far as calibration of force links is concerned it consists of finding the ratio per unit mass and that of accelerometers relative to $\bar{a}_1$ (for different settings of charge amplifiers). The Solartron analyser (type 1172) is most suitable for such measurements as it provides 'Vector Division' therefore both these quantities can be read either in direct ratios or in dbs. The phase angles relative to a_1 should also be examined. It should be 180°/0° and 0° for the calibration of force links and accelerometers respectively. The error if significant can be taken into account during the actual tests.

The input to the X channel of the analyser is a_1 and the input to the Y channel is F_i or a_2 depending upon whether calibrating for $\bar{m}_i$ or $\bar{r}$ respectively. The mode of measurement being Y/x, $(\theta_y - \theta_x)$ the ratio per unit vectors or in other words the relative sensitivities and phase errors are determined from

$$\text{for } m \quad \frac{Y/x}{M} \text{ [db/Kg] or } \left[\frac{\text{signal ratio}}{\text{Kg}} \right] \qquad 11.1$$

$$\text{and} \quad \text{phase error} = (\theta_y - \theta_x) + 180 \text{ [Degrees]} \qquad 11.2$$

$$\text{for } r \quad Y/x \text{ [db] or [signal ratio]} \qquad 11.3$$

$$\text{and} \quad \text{phase error} = (\theta_y - \theta_x) \text{ [Degrees]} \qquad 11.4$$

11.1.3 <u>Absolute Calibration Method</u>

Whilst the 'Relative and Mechanical Method' of calibration remains undoubtedly the most efficient tool for dealing with familiar and linear systems, nevertheless the 'Absolute calibration' method was desirable at the initial stages of this work for the following reasons:

(a) Should the rig or the measuring system behave non-linearly then the whole of the foregoing analysis would not be valid, e.g. the force measured across the specimen cannot be readily related to the input forces F_1 and F_2 exerted to the inertia blocks. In such cases the absolute values of force and acceleration need to be measured.

(b) Interchangeability, i.e., matching transducers with charge amplifiers will be possible in order to reduce errors after having found the errors on each unit separately.

(c) The level of amplification or gain settings of the charge amplifiers generally cannot be determined in advance. All possible combination of gain settings is too numerous (16 for m_1, m_2, r and 48 for m) for relative mechanical calibration method to be practical at the initial stages.

11.2 Design of the Method of Calibration

The 'chatter-ball' technique for absolute calibration of acceleration was not favoured mainly for its high error which is normally greater than 5%[83]. Instead a 'Standard Accelerometer' a purpose built calibrator which was calibrated with laser interferometry resulting in 0.001 [PC] resolution was to be used. An estimated error of sensitivity given by the manufacturer, of the SA used in this work was only 0.5%.

Such an accelerometer seemed ideal for a_1 measurement as it afforded conversion of relative calibration to the absolute once the error of the charge amplifiers were taken into account. Unfortunately for maximum gain and phase accuracy over a wide frequency band, such calibrators must possess a high natural frequency hence they are of small size and consequently of very low sensitivity, e.g. of the order of 1 [PC/g]. This sensitivity was not considered high enough for the level of accelerations envisaged in this work (see Chapter 9) so the idea was abandoned and it was only used for calibration purposes.

The accelerometer under calibration was secured on top of the 'Standard Accelerometer'. There is a 180° phase shift between the two signals hence the word 'Back-to-Back' is sometimes used to describe such a method.

An arrangement whereby both absolute and relative calibration could be carried out simultaneously by the Back-to-Back method and by the aid of vector-dividing facility on the analyser is shown in Fig. 39a. The mass block was manufactured from mild steel to roughly 4x2 times diameter of the force-link. Before being assembled its weight together with the two accelerometers was measured to within less than a gramme accuracy. This was to be replaced with the inertia blocks on the rig once their effective values were established from Chapter 12 (see Fig. 45).

11.3 Error in Gain and Phase of accelerometers

With the analyser in Y/x, $(\theta_y - \theta_x)$ mode, measurements were taken at different frequencies when feeding the a_i signal into the X and the a_S into the Y channels. The errors were calculated from

$$E_{ai} = 100 - [\ E_{ci} + \frac{Y}{x} \cdot \frac{G_i}{G_f}\ (100 - E_{cf})\ ] \text{ per cent} \qquad 11.5$$

$$\Delta\theta_{ai} = (\theta_y - \theta_x) - (\Delta\theta_{ci} - \Delta\theta_{cf}) - 180 \quad \text{Degrees} \qquad 11.6$$

(180° phase change due to back-to-back effect)

G_i and G_f are the gain settings on the charge amplifiers, i.e., volts per unit of acceleration [volts/g]. (see Fig. 39a).

11.4 Error in Gain of force-link-transducers

The force signal from the transducer under calibration was fed via a charge amplifier Cf_i (named after the same force number) into the X channel and the signal from the standard accelerometer via cf into the Y channel. The error was calculated from:

$$E_{Fi} = 100 - [E_{ci} - Mg \ \frac{Y}{x} \cdot \frac{G_i}{G_f} \ (100 - E_{cf})] \ \text{per cent} \qquad 11.7$$

$$\Delta\theta_{Fi} = 0 \ \text{Degrees} \qquad 11.8$$

as the elastic signal produced by the crystal under load is instantaneous.

G_i in [volts/N] and G_f in [volts/g] are gain settings on respective charge amplifiers, M in [Kg] and g, the gravitational acceleration, in [m/sec^2], (see Fig. 39a).

As seen from equations 11.5, 11.6 and 11.7 the first requirement is that the charge amplifiers be calibrated.

11.5 Calibration of Charge Amplifiers

As only one of the charge amplifiers, namely the Kistler (5001) was equipped with calibration facilities other charge amplifiers (B&K 2626) had to be calibrated against it.

(i) Calibration of the Kistler Charge Amplifier

The calibration facility of this charge amplifier consisted of a 1000 [PF] capacitance shunted across the input circuitry converting every mV into a PC, i.e.

$$Q = C.V = 1000 \times 10^{-9} \times 10^{-3} = 10^{-9} \ \text{Coulomb} = 1 \ [PC]$$

The main consideration throughout calibration (see Fig. 39b) was to ensure that the maximum and minimum output limit of about 5 [volts] and 5 [mV] were never violated. The former limit was to sustain good phase accuracy and the latter to reduce the noise effect.

The results of this calibration can be found in Fig. 40. The effect of 'Transducer Sensitivity' and 'Sensitivity Range Settings' on the response was proved insignificant. The error was caused only by the tolerances on the 'Range Capacitor' or in other words the 'Gain Settings'.

(ii) Calibration of B&K charge amplifiers

The input impedance of 5001 being sufficiently high (10^8 MΩ) as compared to that of the accelerometers ($\sim 10^5 - 10^6$ MΩ), it could safely be driven without overloading the accelerometers. The arrangement was the same as shown in Fig.39a. Only the Kistler amplifier was used in conjunction with the standard accelerometer to give the true value of acceleration. The same signal, i.e., from the S.A. was then fed into the other charge amplifiers in turn and their output measured. The errors in gain and phase were then determined by comparing these vectors with the true acceleration vector. (Fig. 40).

11.6 The Error of Calibration

One fringe benefit of the above method was to check the accuracy of calibration itself. This was done by finding the true value of acceleration at different 'gain settings' of the 5001 charge amplifier. The maximum difference for the first four high gain positions was not greater than ·3% and ·3° for gain and phase respectively, - well within the errors of the measuring equipment.

11.7 Calibration of Transducers and the Relative Sensitivity Errors

The results from the calibration of the charge amplifiers (Fig.40) were used in equations 11,1, 11.2 and 11.3 to determine the sensitivity errors of the transducers (deviation from the nominal values) which are shown in Fig. 41.

The results of all the above calibrations were then used in correcting the nominal sensitivities of the transducers and their pairing off with the charge amplifiers which were to be unaltered throughout this work. (The Force-link F_2 was corrected for + 6%). The corrected sensitivity error vectors relative to $\bar{a}_1$ at their maximum were constructed from the absolute calibration results and are shown in Fig.42 for $\bar{m}_i$ and $\bar{r}$.

11.8 Calibration of Load Washers

The load-washers had to be calibrated for both static and dynamic loading. Whilst the former had already been carried out on a 'Denison Testing Machine' (Fig.36), a new technique had to be devised for calibration under both static and dynamic loads simultaneously, i.e.,when a dynamic load is superimposed upon a static load. This condition was achieved by using the test rig under simulated symmetric condition where $\bar{F}$ measurement became redundant (See Table 8b). The method of calibration is briefly described below.

The Kistler charge amplifier was used with different sizes of the Kistler load washers placed within the appropriate specimens given in Table 4: the symmetric condition was to be realized by using the higher stiffness specimens, i.e., the solid columns as shown in Fig.44a. $\bar{F}$, the output from the charge amplifier was measured on the Y channel of the analyser. The relative and absolute sensitivities together with the phase error of the 'm' method were determined at the second mode, from the following equations:

$$\text{Relative Sensitivity} = \frac{|Y/X|}{\sqrt{m_R^2 + m_I^2}} \quad [\text{db/Kg}] \text{ or } [\text{Signal ratio/Kg}] \qquad 11.9$$

$$\text{Absolute Sensitivity} = \left|\frac{Y}{F}\right| = \left|\frac{Y}{ma_1}\right| = \frac{G_f}{g\sqrt{m_R^2+m_I^2}} \left|\frac{Y}{X}\right| \quad [\text{volts/N}] \qquad 11.10$$

and

$$\text{Phase error} = (\theta_y - \theta_x) - \tan^{-1}\frac{m_I}{m_R} \qquad 11.11$$

X and Y in [volts], G_f in [volts/g] and m in [Kg], g the gravitational acceleration [m/sec^2]. m_R and m_I were determined from equations 12 and 15 in which the values of K and C had already been measured from 'm_i' or free-vibration methods.

Practical problems associated with this method were : firstly to determine $\tan^{-1}\frac{m_I}{m_R}$ with any degree of accuracy and secondly the fact that the second mode could only be excited at high and over a very narrow band of frequency with effective load amplification due to the high stiffness and low damping of the solid steel specimens. Therefore, a range of specimens had to be used for both increased damping and also frequency range.

For these reasons it was decided that this check/calibration should be made simultaneously with the actual tests on the joint specimens in order to check firstly the reliability of 'm' method and secondly, if the results were affirmative, to calibrate against a much wider range of frequency and damping values.

The result of this check/calibration is presented in Chapter 14.5.

11.9 Summary

A method was devised whereby both 'absolute' and 'relative' calibration of instruments and mechanical units could be carried out simultaneously. The mechanical parameters of particular interest were the 'apparent mass vector $\bar{m}_i$' and the 'modal shape vector $\bar{F}$' (see Fig.39a Table 8a and equations 11.1 - 11.4).

It is intended that when the effective values of the auxiliary parameters, i.e. K_1, M_1, K_2, M_2 are determined (Chapter 12.3); the calibrations are to be carried out directly on the apparatus, thus resulting in the maximum possible efficiency.

The results of the 'absolute' calibration of the charge amplifiers and the transducers are shown in Figs. (40) and (41). The accuracy of calibration itself was checked by measuring a constant level of acceleration at different given settings of a calibrated charge amplifier. The maximum error of gain and phase were only ·3% and ·3° respectively well within the errors of the measuring equipment. The gain and phase errors of the charge amplifiers and the transducers were found to be within ± 4% and±4.0° respectively (the frequency range 15-2000 Hz). The error in phase through the charge amplifiers tended to change direction as the frequency was increased. A similar effect was observed for the gain error of the transducers: only the change was in an opposite direction. These effects were due to the influence of an electrical and a mechanical resonant circuit within the charge amplifier and the assembly of Fig. 39a respectively.

The corrected relative sensitivity error vectors for $\bar{m}$ and $\bar{r}$ measurement are shown in Fig.42. The envelopes were determined by combinations of gain settings of the charge amplifiers for F_i and a_i measurement which resulted in maximum possible errors. It can be seen from Fig.42 that the errors confined to the following limits:

for $\bar{m}_i$ measurement

$-5 <$ gain error $< +2$ [per cent]

$-3 <$ phase error < 0.5 [degrees]

and for $\bar{r}$ measurement

$-2 <$ gain error $< + 1.5$ [per cent]

$-1 <$ phase error $< + 1$ [degrees]

A method of calibration for $\bar{F}$ measurement (calibration of the load washers and associated instruments under combined static and dynamic loading) is also given using the test rig under simulated 'symmetric' condition where $\bar{m}$ measurement becomes redundant (see Table 8b). The result of this check/calibration is presented in Chapter 14.5.

CHAPTER 12

PILOT TESTS - DYNAMIC CALIBRATION OF TEST RIG

12.1 Introduction

Tables 8a and 8b were used as a guide for calibration of auxiliary parameters. It must be noted that the term 'symmetry' referred to in this table represents a relative term. As shown in the following discussion the sensitivity of the system to asymmetric parameters becomes more pronounced as the stiffness of specimen K is reduced. Table 8b assumes $\bar{\bar{F}}$ measurement (i.e. direct measurement of the force on the specimen) is not possible during the actual tests as is the general case when the specimens change shape and size. As seen from this table such a measurement appears only for calibrating the asymmetric conditions: $\bar{\bar{F}}$ measurement, in dealing with linear systems, is only necessary when calibrating for highly damped asymmetric auxiliary parameters. However, asymmetry was not expected to arise in testing high stiffness specimens such as joints. As far as asymmetric condition is concerned $\bar{\bar{F}}$ measurement could prove necessary if checks are required to be made at high frequencies and amplitudes where excitation of the first pure mode lies beyond the capacity of the forcing system, e.g. boxes III and XIX.

The notion of frequency dependent auxiliary parameters was not likely to materialise in practice unless external coupling due to misalignment or poor isolation proved significant. Every possible care had been taken therefore to avoid this, nevertheless the concept is dealt with theoretically mainly to indicate that the arrangement could afford such a check if the situation arose. In the absence of $\bar{F}$ measurement, for symmetric configuration the calibration may be carried out under individual test conditions provided both modes are excited, (i.e. boxes IV, V and XX, XXI). The practical implications of this will be discussed in the next section under the heading 'Pilot Tests'.

The frequency spectrum is divided into 3 bands marked by the following limiting factors:

a. $f_1 = 1.5$ [Hz] limitation inposed by the shakers

b. $f_2 = 4.5\ NF_1 \simeq 75$ [Hz] This is the frequency corresponding to the elastic forces generated by K_1 and K_2 springs being 5 per cent of the total inphase forces which include elastic and inertial forces. For frequencies higher than f_2 the effect of K_1 and K_2 will either be ignored (as in table 8b) or assumed frequency independent depending upon the accuracy of testing which will be determined from the pilot tests.

c. $f_3 = \frac{1}{2\pi}\left[\frac{F_{1\,max}}{M_1 x_1}\right]^{\frac{1}{2}}$ [Hz] limitation imposed by the forcing system in exciting the first mode with the required amplitude x_1. For frequencies above f_3 the second mode has to be employed for calibration purposes, i.e. $\bar{F}$ measurement.

With regard to the limitation on the amplitudes of vibrations;it was determined by the maximum stroke of the shakers at 1 Cm. [PK]. This level of amplitude is far greater than any possible level required for testing 'joints'; however it may be a limiting factor in testing other units such as rubber.

As can be seen from Table (8b) the most favourable conditions are those corresponding to perfect symmetry as it would not only permit free-uncoupled vibration but also obviate a need for previous calibration, i.e. empty boxes of I, II and XVII, XVIII. In case the effective auxiliary parameters proved frequency independent and damping linear, measurement of ω_2 and ζ_2 will give C and K directly, i.e. boxes XII and XVI. For a non-linearly and/or highly damped system where free vibration methods fail, forced vibration techniques would be used; C.f.box VIII.

In order to predict the conditions under which one is likely to operate, i.e. to determine the effect of varied stiffness and mass distribution upon the 'symmetry' a computer programme was written to solve the eigenfunctions for all the possible combinations of K, K_1, K_2, M_1 and M_2 (EIGENZWEI1). Later, this programme was amended to include the solution for eigenvalues, i.e. the natural frequencies so that the print out could also be used in the analysis of test results (EIGENZWEI2); the programme together with a sample print-out is given in Appendix 3. Equations 5.10, 5.16 and 5.17 were used in computing the modal shapes and the natural frequencies respectively.

Figure (43) was constructed with the aid of 'EIGENZWEI2'. The envelopes were estimated by allowing the tolerances, given in B.S. 1726[79], on the nominal rates of the springs. (see Fig. 29.6).

$$(K_1)_{\substack{max.\\min.}} = (K_{11}+K_{21})\pm^{7.5}_{5.0} \text{ per cent.}$$

$$1095 < K_1 < 1239 \qquad [KN/m)$$

$$(K_2)_{\substack{max.\\min.}} = (K_{12})\pm^{7.5}_{5.0} \text{ per cent}$$

$$1063 < K_2 < 1203 \qquad [KN/m]$$

and $M_1 = 111.4$ [Kg] weighed

$M_2 = 110.9$ [Kg] weighed

As seen from this figure for K > 5000 [lbf/ in.] = .876 [MN/m.] the error in 'symmetry', for the worst possible combination, is less than 10%. This would mean x_1 and x_2 (or F_1 and F_2) need to be measured with far less than 5% error if asymmetry is to be detected above 5000 [lbf/in] stiffness of specimen. As for natural frequencies there is only .1[Hz] difference at 1000 [lb/in.] stiffness of specimen between the extreme cases.

The 10% overall level of error was considered to be the maximum acceptable. The bulk of this error was expected to be due to extraneous modes' coupling, e.g. rotational; (the error due to measuring instruments as shown in Fig.42 is minimal).

Fig. 43 also clearly demonstrates the expected fact that when the two natural frequencies approach one another, the coupling term (due to damping) is intensified:

$$b_{12} = C_1 + A_2B_2C_2 + C(1 - A_2)(1 - B_2)$$

For all practical purposes, therefore, it is expected that the system containing joint specimens would behave in a very closely symmetric fashion as far as stiffness and mass distribution is concerned; as for damping the coupling term b_{12} would be consequently minimized (i.e. $A_2 = 1$, $B_2 = -1$).

If the auxiliary parameters proved to be dependent upon frequency and/or preloading the above statement would still be valid, as changes on either side of the plane of symmetry would be very nearly identical.

12.2 Pilot Tests

12.2.1 Introduction

The validity of the previous analyses was checked by a series of pilot-tests specially designed for this purpose employing the first mode. The aim was to devise an efficient method of evaluating the values of the auxiliary parameters at different levels of preload, frequency (and amplitude).

12.2.2 Procedural Planning

A specimen with high stiffness but low damping, e.g. solid specimen of equivalent dimensions and material to those of the joint column, resulted in the highest possible relative symmetry and hence reduced the coupling term b_{12}. Furthermore damping being mainly of the structural (material) type, it would be proportionally distributed resulting in further decoupling; $b_{12} \to 0$ hence the whole system could be regarded as undamped. One direct implication of such an arrangement would be inphase movement of M_1 and M_2 provided $\bar{F}_1$ and $\bar{F}_2$ were inphase (see equations 5.26.1, 5.26.2 and 5.29). Small adjustments of relative gain between $\bar{F}_1$ and $\bar{F}_2$ were then to be made to ensure $\bar{F}$ assumes zero value (i.e. $\bar{a}_1 = \bar{a}_2$). For such a condition the rig parameters could be evaluated independently in either side of the plane of symmetry. Equations 1 and 2 of Table (8a) could be then broken up into:

$$C_1 = -\omega m_{1I} \ \cdots\cdots \qquad 12.1$$

$$C_2 = -\omega m_{2I} \cdots\cdots\cdots \qquad 12.2$$

and

$$K_1 - M_1\omega^2 = -\omega^2 m_{1R} \ \cdots\cdots\cdots \qquad 12.3$$

$$K_2 - M_2\omega^2 = -\omega^2 m_{2R} \ \cdots\cdots\cdots \qquad 12.4$$

The accuracy of these tests depended upon the assumption that no significant degree of coupling existed within the system. This was determined by measuring $\bar{F}$: If m_R or m_I did not rise above 5 per cent of m_{iR} or m_{iI} (see Fig.42) the method was regarded as reliable - In the presence of higher values of coupling force $\bar{F}$, calibration procedures of Table (8b) were to be employed, i.e. pure mode excitation (Boxes XXV and XXVI.

Pure mode excitations were to be realized by forcing M_1 and M_2 to move completely in or out of phase by adjusting the phase and amplitude of $\bar{F}_2$ relative to $\bar{F}_1$. If $\bar{a}_2$ and $\bar{a}_1$ could not be made inphase by mere adjustment of $\bar{F}_2$ amplitude it was deduced that coupling due to damping (b_{12}) was present.

This coupling could be caused either by $C_1 \neq C_2$ whilst stiffness and mass symmetry holding, or in the general case of asymmetry where all the damping sources contribute i.e. C, C_1 and C_2.

The former case is dealt with in the boxes XX and XXI (and XIX for lower amplitudes):

$$\bar{F} = 0 \quad \text{as } \bar{r}_1 = 1$$

$$m_{1R} = m_{2R} \text{ and } m_{1I} = \frac{-C_1}{\omega}, \quad m_{2I} = \frac{-C_2}{\omega}$$

practically this means that slight adjustment of phase (and of amplitude) would be required.

As for the general case of asymmetry where $\bar{r}_1 \neq 1$, $\bar{F}$ can never be made zero, hence it had to be measured if the parameters were to be determined from the experiment as indicated in boxes XXV and XXVI of Table (8b).

With regard to the forced-vibration method there was no need to separate K_1 and M_1 in $(K_1-M_1\omega^2)$ or K_2 and M_2 in $(K_2-M_2\omega^2)$, but for 'complete symmetry' where uncoupled free-vibration would be possible, and only below 75 [Hz] (see Boxes I and IX). In this case, as indicated in the table both forced and free vibration methods could be employed at NF_1 for this purpose (Energy Methods: Box IX). However Box IX is heavily dependent upon the accurate measurement of C_1 and ζ_1 from forced and free vibration respectively. Equation 31 could prove impractical as indeed C_1 was expected to be small. This on the other hand would imply very narrow band-width permitting evaluation of K_1

and M_1 prior to C_1 at near resonance frequencies; equation 20 and/or frequency/frequencies close to it would furnish two equations whereby M_1 and K_1 could be determined. Free decay curves would then give C_1 from equation 33 (e.g. see Box XIV).

12.2.3 Experimental Procedure

E type solid specimens (Fig. 22) were used to give maximum K. The set up is shown in Fig. 44a. The assembly was loaded to the maximum load (∿ 1.2" deflection of $M_2 \equiv 4000$ KP). At this load the 'first mode' was excited at different frequencies and amplitudes; measurements of $\bar{m}_1$ and $\bar{m}_2$ were made at minimum $\bar{m}$ experienced by fine adjustment of $\bar{F}_1$ and $\bar{F}_2$. (originally $\bar{F}_1 = \bar{F}_2$).

Measurements were repeated at different preloads as the system was being unloaded.

12.2.4 Results

It was observed that stiffness and mass symmetry held extremely good at around the first natural frequency, i.e. $r_1 = 1 \pm 3\%$ and $\theta_y - \theta_x = \pm 0.5°$. $\bar{F}_1 = \bar{F}_2$ further indicated that no effective mode coupling due to damping existed. It was also realized that the measurement of m_{iI} due to the small values of damping within the auxiliary parameters could not, however, be made accurately.

As the frequency was increased the symmetry deteriorated slightly which was attributed to very slight extraneous mode coupling around 200 and 400 [Hz] caused by floor and frame vibration respectively. This was expected and in no way was it detrimental to accuracy as they could easily be avoided.

As expected the effect of preload and amplitude upon symmetry was not detectable, the changes, if any, were identical on either side of the plane of symmetry.

It was therefore concluded that the assumption of 'complete symmetry' could be regarded as a valid one. Hence free-vibration would be uncoupled and therefore could be used to evaluate the auxiliary parameters.

12.3 Dynamic Calibration of the Test-Rig

12.3.1 Method

Due to the low damping hence narrow band-width at the first mode, it was possible to determine K_1 and M_1 at two very close frequencies around NF_1., i.e. equation 20. This provided two equations required to find K_1 and M_1 whilst logarithmic decrement from free decay curves (equation 33) yielded C_1 (Box XIV). These values were considered valid only at low frequencies ($NF_1 \simeq 16$[Hz]):Measurements were carried out at higher frequencies to check the variation of M_1, K_1 and C_1 with frequency (Box V).

12.3.2 Results and Discussions

Equation 20 was used above 75 [Hz] to evaluate $(M_1 - \frac{K_1}{\omega^2})$ and showed that its maximum variation with frequency never exceeded the experimental error (Fig. 42). It was concluded that M_1 was frequency independent and as for K_1, even if it varied with frequency, could not have any influence upon the measurement.

Forced vibration techniques in measuring C_1 (equation 31) as expected ran into some difficulties mainly because of the very low value of damping. Accurate measurement of continuously decreasing m_{1I} with frequency proved impractical as higher frequencies were approached. However the evidence from the free-decay curves, i.e. very nearly exponential envelopes of decay indicated that the damping was mainly of the linear type hence mechanism of energy dissipation was either viscous or hysteretic (structural). A slight non-linearity in damping was manifested by deviation from exponential to linear of the decay envelopes at lower amplitudes. The Coulomb friction at the bearing was thought to be responsible for this and also for the slight increase in general of the effective values of C_1 and K_1 at very low speeds. The case for distinguishing between the two linear mechanisms, i.e. viscous and hysteretic could not be established firmly. However the evidence from forced vibration tests seemed to point towards 'viscous' as being the main mechanism of damping. This was realized by considering m_{1I} values as it reduced with frequency - its variation with frequency was closer to $\frac{h}{\omega}$ than $\frac{h}{\omega^2}$.

The effect of preloading upon the auxiliary parameters unlike that of frequency, was significant in the sense that measuring technique could detect it; their effect upon actual measurement of joint specimen however as will be shown in later chapters remained insignificant. The results are shown in Figures 45 and 46. As can be seen from these figures stiffness and damping increased with preload.

The 'best-curves' given below were determined by the aid of a computer representing those which resulted in the minimum sums of the squared deviations:

$K_1 = 1187 + 10.9\ X_2$ SSD ≃ 1990 12.5

K_1 in [KN/m]

X_2 in [mm.]

$M_1 = 118.5$ [Kg] Mean 12.6

$C_1 = 601 + 13.7\ X_2$ SSD ≃ 18311 12.7

$C_1 = 659 - 1.1\ X_2 + 0.46\ X_2^2$ SSD ≃ 3654 12.8

C_1 in [N/m/Sec.]

X_2 in [mm.]

$\zeta_1 = 0.046 + 0.001\ X_2$ SSD = 0.00033 12.9

$\zeta_1 = 0.05365 - 0.00064\ X_2 + 0.00005\ X_2^2$ SSD = 0.00013 12.10

X_2 in (mm.]

The increase in the effective oscillating inphase stiffness of K_1, from the nominal static stiffness, was about 30% at 4000 [KP] as seen from Fig. 45. The rate of increase in the dynamic stiffness reached a constant value after the initial stages were due to the 'end coils' effect larger rates of increase were experienced. The rate of increase given in equation 12.5 was about twice the rate implied by equation 10.2 obtained under static loading. The doubling of the rate indicated the existence of some joints' effects. Continuous micro slip and/or 'squeeze film' within the bearings gave rise to some additional elastic forces thus increasing the effective oscillatory inphase stiffness (see Chapter 3, and reference (21)).

The increase in damping started with a small drop at low preloads and then a continuous rise began(with almost linearly increasing rate) which at its maximum reached a value of about 80% higher than the minimum (Fig. 46). The causes of this considerable rise in damping were attributed to the increased frictional forces at the bearings, also to the continuous rise in the shear stress within the springs' wires hence resulting in higher material damping.

The effective value of M_1 as indicated in Fig. 45 was about 6 per cent higher than that obtained by weighing. This was attributed to the inertial effects of other moving parts mainly that of springs.

The pilot-tests have proved that the system possesses an excellent degree of symmetry it also indicated that $\bar{m}$ measurement was basically redundant. Such a measurement, was on the other hand, an essential tool in checking that symmetry did in fact exist. As far as actual tests were concerned it was decided that $\bar{m}$ measurements be retained for the following reasons:

(a) The accuracy of m method, i.e. direct measurement could be checked against those of m_i and free vibration methods : m measurement was expected to contain an element of error attributed to the non-axial loads caused by almost certain existence of out of parallelism of specimens. This information was considered useful for future reference.

(b) As referred to in the pilot-tests non-linearity within the auxiliary system did in fact exist, though to a very small extent, and therefore if it was desirable to check the auxiliary parameters at a required level of amplitude and at frequencies above f_3 and under exact test conditions, $\bar{m}$ measurement at the second mode would become imperative (Boxes III and XIX).

(c) If $\omega < f_3$ but still $\gg \omega_1$, e.g. (Box V), excitation of the first mode could involve large values of F_1 and F_2 which consequently increase the effect of extraneous modes, e.g. those of the floor vibration etc., at certain frequencies. It is therefore advantageous to be able to use the second mode, i.e. m method for such instances.

12.4 Summary and Conclusions

Depending upon the values of stiffness of the specimen under test, the effective distribution of stiffness, mass and damping within the auxiliary system and the frequency of excitation, the testing procedures might involve measurement of force at three, two, one point(s) or none at all. The ideal condition being that of 'symmetric' results in uncoupled free vibration and also greater testing efficiency (Table 8).

In order to be able to predict the conditions under which one was likely to operate a computer programme was written to solve the eigenvalue problem. Fig. 43 was constructed with the aid of this programme. As seen from this figure for $K \gtrsim .9$ [MN/m] the error in 'symmetry' is confined to better than 10% for the worst possible distribution of stiffness and mass likely to be encountered.

Prior to calibration proper, a series of pilot tests were carried out under a simulated condition of symmetry (when $\bar{F}$ measurement is redundant) to check the validity of the above prediction, and to realize a suitable technique of calibration. The results of these tests indicated that: (a) stiffness and mass symmetry held extremely good at around the first natural frequency with only ± 3% and ± .5° error, (b) $\bar{F}_1 = \bar{F}_2$ further indicated that no effective mode coupling due to the damping existed and (c) due to very small values of damping within the auxiliary system only the free-vibration method would prove viable in the evaluation of these parameters.

Preload had a decisive effect upon the stiffness and the damping in particular. At 4000 [KP] of preload the oscillatory inphase stiffness K_1 was about 30% higher than the nominal static stiffness and the damping experienced an 80% rise from a minimum, which was reached at an initial stage of loading. The 'best-curves' to represent the experimental points in Figs.(45) and (46) were determined using a computer which minimized the sums of the squared deviations (equations 12.5 - 12.10). The rate of increase of inphase stiffness was about twice the rate under static loading. This indicated the existence of some joints' effects, for which continuous micro-slip and/or 'squeeze film' within the bearings were responsible. The causes of the considerable rise in damping were attributed to both the joints' effect as well as the increased material damping within the springs' wires.

The effect of amplitude and frequency upon the Voigtian auxiliary system parameters proved insignificant : A slight non-linearity at very low amplitudes (in order of μm) was, however, detectable which was attributed to the Coulombian friction. At higher amplitudes, a case for viscous or hysteretic (i.e. linear) damping could not be established firmly as such a check required forced-vibration methods which proved impractical due to the small values of damping. However evidence was found from such tests to suggest that the viscous effect was predominant : variation of m_{iI} with frequency was closer to $\frac{h}{\omega}$ than to $\frac{h}{\omega^2}$. The mechanism of energy dissipation within the auxiliary system, in the next chapter (13.3), will therefore be considered as viscous.

CHAPTER 13

METHODS OF TESTING AND DATA REDUCTION

13.1 Introduction

The theoretical methods developed in Chapter 5 and summarized in Tables 8a and 8b are further developed in this chapter from a practical viewpoint considering the results of the pilot-tests and the dynamic calibration presented in Chapter 12. Detailed experimental procedure together with some recommended techniques of data reductions are given here to determine the desired parameters of specimens from the Voigtian parameters of 'equivalent single degree of freedom systems' shown in Fig. 15d. It therefore assumes 'complete symmetry': The results will be accurate within the experimental error if the stiffness of the specimen column K is above 0.9 [MN/m], e.g. testing of joints (see Fig. 43). One further result from Chapter 12, namely the frequency independency of auxiliary parameters, form the second basis for the following techniques.

Although particular reference in this chapter is given for testing joints, the methods are intended to be used for general 'Complex-Modulus' measurement whenever the symmetry holds within an accepted overall experimental error.

13.2 Experimental Procedures

13.2.1 General

The following procedure was employed for the 'jointed specimen' and 'equivalent solid' tests of Fig. 44.

Firstly, for future reference, adherence to one particular measure of safety must be strongly emphasised and that is:

Before loading specimens, the Connector 2 (Part No.16) must sufficiently be retracted into the Sleeve 2 (Part No.9) to avoid loading and consequently damaging the armature of the lower vibrator (Part No.21)

The desired level of preload was established by determining the corresponding deflection of the lower inertia block M_2 (Part No. 5) from Fig. 37 which was then realised using a dial gauge with a magnetic base.

Whilst the above method was employed in testing 'joints', it must be mentioned that for other applications of the apparatus, alternatives given in Chapter (10.3) should also be considered.

After the preload was applied to the specimens, the platform (Part No.6) was securely locked in position. The vibrators were then connected to M_1 and M_2. As a matter of expediency the outputs of the variable phase oscillator were so assigned as to conform with the convention used in the theory: M_1 was driven by the 'reference' and M_2 by the 'variable phase' outputs respectively (see Fig. 31.)

For safety the toggle switches 1 and 2 on the triggering/control unit were initially set to the 'off' position thus short circuiting the input terminals of the power amplifier (Fig. 33).

Measurements were taken under both forced vibration ('m_i' and 'm' methods) and also free vibration. The complete pattern of testing procedure will emerge in the following sections where each of these methods is explained in detail.

The initial parameters found from the second pure mode (for 'm_i' and free vibration methods) represented those of the 'equivalent single degree of freedom system'. They contained two extraneous parameters namely those of auxiliary systems and the pseudo-material within the jointed column. These effects were found from Chapter (12.3) and 'equivalent-solid' tests respectively and later subtracted from the results of 'jointed specimens' to give the parameters of joint alone. The initial parameters were

$$K_t = 2K + K_1 \qquad 13.1$$

$$C_t = 2C + C_1 \qquad 13.2$$

and

$$\zeta_2 = \frac{C_t \omega}{K_t} = \frac{(2C+C_1)\omega}{2K + K_1} \qquad 13.3$$

13.2.2 Free-Vibration Method

(a) Search for NF_2

The free vibration method, as far as damping measurement was concerned proved practical only when the log decrement of the system (at the second mode) remained below 0.69(loss factor of about 0.2), i.e.

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it applied when the damping was low enough to sustain at least two complete cycles of vibration across the screen of the oscilloscope whilst the amplitude reduced from the full screen (4 cm) to the smallest graduated level (1 cm.):

$$\delta_t = \frac{1}{n} \text{Ln} \frac{X_n}{X_1} = \frac{1}{2} \text{Ln} \left(\frac{4}{1}\right) \simeq 0.69 \qquad 13.4$$

or

$$\zeta_t = \frac{1}{\pi} \cdot \delta_t \simeq 0.2 \qquad 13.5$$

The procedure was as follows:

First it was made sure that the output from the analyser was zero. Employing the 'variable phase oscillator' under its own steam, a single force, e.g. $\bar{F}_1$ with a very low amplitude was applied in search of the second natural frequency. This preliminary test was simply to help locate the natural frequency in the actual test without having to adjust the frequency on the oscillator to sustain synchronisation during the frequency sweep; hence increase efficiency (see Chapter 9.4).

(b) Check the 'Symmetry'

In order to avoid excessive amplitudes, the output levels on the oscillator were kept to a low level before switching on the other load. The generator of the analyser was then made to drive the oscillator by programming it as follows:

Output level	2 volts RMS
Frequency	close to NF_2
Δf frequency increment of sweep	$\sim$ 0.1 - 1 [Hz]
and X input	$\bar{F}_1$ or $\bar{a}_1$
Y input	$\bar{F}_2$ or $\bar{a}_2$

Pure Mode Excitation

The inputs to the analyser were initially F_1 into X and F_2 into Y channel with the measuring mode of the instrument set for direct ratio, i.e. $\left|\frac{Y}{X}\right|$ and $(\theta_y - \theta_x)$. The variable phase output on the oscillator was initially set at 0° to result in $\bar{F}_1 \simeq -\bar{F}_2$. Fine adjustment of phase and amplitudes were then made to ensure $\left|\frac{Y}{x}\right| = 1$ and $\theta_y - \theta_n = 0$. Due to high stiffness of the jointed column and hence 'complete symmetry' of the system this procedure resulted invariably in $\bar{r}_2 = 1,180°$. However the general method was to adjust $\bar{F}_2$ for phase and amplitude to obtain complete out-of-phase movement of M_1 and M_2, i.e. pure second mode of the system; note the acceleration ratio $\bar{r}_2 = \frac{\bar{a}_2}{\bar{a}_1}$ and then proceed with forced vibration methods (see Boxes XXVII and XXX in Table 8b). $\bar{r}_2$ was measured by switching into the X and Y channels of the analyser, $\bar{a}_1$ and $\bar{a}_2$ signals respectively.

(c) Hit NF_2 and Calculate Stiffness K_t

The next stage of the tests was to search for an exact value of the frequency where the inertial and elastic forces counteract one another. This was achieved by switching $\bar{F}_1$ and $\bar{F}_2$ signals in turn into the Y channel of the analyser (and $\bar{a}_1$ into X channel) and measuring the phase $(\theta_y - \theta_x)$ of $\bar{m}_i$ vectors whilst sweeping the frequency with small increments (Δf). When $\bar{m}_1$ and $\bar{m}_2$ assumed phase values of 90° and 270° respectively (see Fig. 17b), the frequency of the balance between the inertial and elastic forces was struck (NF_2). This frequency rendered the stiffness K_t from equation 30 of Table 8a:

$$K_t = M_1 . \omega_2^2 = 118.5 \ (2\pi)^2 \ NF_2^2 \simeq 4678 \ (NF_2)^2 \quad [N/m] \qquad 13.6$$

(d) Store Free-Decay Curve and Calculate Damping C_t or ζ_t

The free-decay curves were obtained by cutting off the inputs to the shakers and at the same time triggering the storage oscilloscope. The procedure for this part of the operation was as follows:

The signal $\bar{a}_1$ (or $\bar{a}_2$) was projected on the screen; the full length of the Y axis was deployed to contain the $\bar{a}_1$ signal. The 'Ext. Trigg' push button on the oscilloscope was then pressed and the 'Trigg. level turned down till the trace disappeared. The next step was to change into 'store' mode by pressing the appropriate button and turning on the 'Trigg. Switch' in Fig. 33. The decay curves then appeared and were stored on the screen.

The above procedure was repeated so that a sufficient number of complete cycles appeared on the time base of the screen for the amplitude to decay from the full to the minimum measurable level on the screen (i.e. 4/1). n, the number of cycles was then counted to give damping at the second mode from the equation 34 of Table (8a):

$$C_t = \zeta_t . \ \omega_2 . \ M_1 \qquad 13.7$$

but $$\zeta_t = \frac{1}{n\pi} . \ \mathrm{Ln} \ \frac{a_o}{a_n} \simeq \frac{1}{n\pi} . \ \mathrm{Ln} \ \frac{4}{1} = \frac{0.44}{n} \qquad 13.8$$

or $$C_t = (\frac{0.44}{n}) \ (2\pi(NF_2)) \ (118.5) = 327.6 \ \frac{(NF_2)}{n} \ [N/m./Sec.] \qquad 13.9$$

13.2.3 Forced Vibration

(i) 'm_i' method

(a) 'Resonance Test'

With the frequency still set at NF_2 and the inputs to the analyser switched to measure m_i, the display mode of the analyser was altered from the direct ratio and $(\theta_y - \theta_n)$ into real and imaginary (a,b mode). Such an arrangement rendered directly the m'_{iI} values which were needed to evaluate the damping at the second mode from equations 32 of Table (8a):

$$C_t = -\omega m'_{iI} \qquad 13.10$$

and $$\zeta_t = \frac{C_t . \omega}{K_t}$$

but from equation 13.6

$$K_t = M_1 \omega_2^2$$

$$\therefore \quad \zeta_t = \frac{-m'_{iI}}{118.5} \qquad 13.11$$

(b) 'Off-Resonance Test'

Frequency was then altered by appropriate programming of the analyser's generator and checks were made on the oscilloscope (or on a frequency counter) of the oscillator output to ensure the synchronisation of the oscillator frequency with that of the generator. If the oscillator was thrown out of synchronisation a simple adjustment of its frequency, i.e. by setting it to a frequency closer to that of the generator, the synchronisation was restored.

The 'parameters' were calculated from equation 23 and 32 of Table (8a):

$$K_t = \omega^2(M_1 - m'_{iR}) = \omega^2(118.5 - m'_{iR}) \qquad 13.12$$

$$C_t = -\omega m'_{iI}$$

$$\therefore \zeta_t = \frac{C_t . \omega}{K_t} = \frac{-m'_{iI}}{M_1 - m'_{iR}} = \frac{-m'_{iI}}{118.5 - m'_{iR}} \qquad 13.13$$

As in 'complete symmetry' $\bar{m}'_2 = -\bar{m}'_1$, therefore for increased accuracy their mean values were used for the place of m'_i :

$$K_t = \omega^2(M_1 + \frac{m'_{1R}-m'_{2R}}{2}) = \omega^2(118.5 + \frac{m'_{1R}-m'_{2R}}{2}) \qquad 13.14$$

$$C_t = \omega(\frac{m'_{2I}-m'_{1I}}{2}) \qquad 13.15$$

$$\therefore \zeta_t = \frac{C_t . \omega}{K_t} = \frac{m'_{2I}-m'_{1I}}{2M_1+(m'_{2R}-m'_{1R})} = \frac{m'_{2I}-m'_{1I}}{237+(m'_{2R}-m'_{1R})} \qquad 13.16$$

It must be noted that the $\bar{m}_i$ values given above are in kilogrammes, hence in converting the electrical signals into [Kg] appropriate factors must be evaluated by considering the gain settings on the charge amplifiers, which includes the sign convention, e.g. $\bar{m}_1$ has to be multiplied by -1 as the positive direction of $\bar{m}_1$ used in the analysis results in electric signals for $\bar{F}_1$ and $\bar{a}_1$ of opposite directions.

(ii) 'm' method

The Y input to the analyser was switched to measure $\bar{F}$. The measuring and display modes were left at their previous settings,the stiffness and damping of the specimen column were determined from equations 12 and 15 of Table (8a).

$$K = \frac{\omega^2}{2} m_R \qquad 13.17$$

$$C = \frac{\omega}{2} m_I \qquad 13.18$$

$$\therefore \zeta = \frac{C\omega}{K} = \frac{m_I}{m_R} \qquad 13.19$$

$\bar{F}$ and $\bar{a}_1$ produced signals with the directions in accordance with the convention used in the analysis hence no sign change was needed; e.g. when M_1 was moving in the positive direction from the equilibrium position the specimens were under compression which resulted in negative signals for both $\bar{F}$ and $\bar{a}_1$.

13.2.4 The Effect of Amplitudes

The effect of amplitude was studied mainly at NF_2 as at this frequency the efficiency of load amplification is maximum (almost) hence a greater range of amplitudes could be excited. (see Fig. 14b).

13.2.5 The Effect of Frequency

Forced vibration methods at off-resonance frequencies were initially employed to study this effect. The limitation imposed upon this method was mainly due to the reduction in force amplification factor by moving away from NF_2; the determining factor being the band width at the second mode. (Fig.14a). However the frequency band of one actave (6db) at $x_1 \simeq 0.25$ μm, which proved possible under all conditions, was considered large enough to indicate any effect of the frequency. For later studies, if this frequency band needed to be enlarged, provision had been made in the design to accommodate a varied number of discs (or sizes of specimen) thereby moving the NF_2 in the frequency domain for another 10 dbs maximum (assuming no pseudo-material effect of specimen column).

13.3 Data Reduction

The purpose of the following analyses was to determine the parameters of the joints alone from the results of the above tests and those of Chapter (12.3).

13.3.1 Correction for the Effect of Auxiliary Parameters the Correction-Factors

The values of the parameters determined at the second pure mode for the 'equivalent single degree of freedom' (using free-vibration or 'm_i' method) were first corrected for the effect of auxiliary parameters to give those of the 'jointed specimen column' alone, i.e. K, C (and ζ). The values of auxiliary parameters had already been determined from Chapter 12.3. (see Figs. 45 and 46). From equations 13.1, 13.2 and 13.13.

$$K = \frac{1}{2}(K_t - K_1)$$

$$C = \frac{1}{2}(C_t - C_1)$$

and

$$\zeta = \frac{C\omega}{K} = \zeta_2 + \frac{K_1}{2K}(\zeta_2 - \zeta_1 \frac{\omega}{\omega_1}) \qquad 13.20$$

The corrections due to auxiliary parameters amounted to very small values, particularly for stiffness (normally less than 1%).

because of the high stiffnesses at the specimen column. In Figs. 47 and 48 correction-factors R_K and R_ζ for stiffness and loss factors are given for free or forced resonance test to demonstrate this point.

The correction-factors were defined as follows: when they are multiplied by half the Voigtian parameters of the equivalent single degree of freedom system of Fig. 15d, they will render those of the specimen column, i.e.:

$$K = R_K \left(\frac{K_t}{2}\right) \qquad 13.21$$

$$C = R_C \left(\frac{C_t}{2}\right) \qquad 13.22$$

$$\therefore \quad R_\zeta = R_C . R_K \qquad 13.23$$

At the natural frequency of the system, i.e. at ω_2 the correction factors for K, C and ζ can be determined from:

$$R_k = 1 - \frac{K_1}{K_t} = 1 - \frac{1}{\partial^2} \qquad 13.24$$

$$R_C = 1 - \frac{C_1}{C_t} \qquad 13.25$$

and

$$R_\zeta = \frac{1}{R_K} \left(1 - \frac{\zeta_1}{\zeta_2} . \frac{1}{\partial}\right) \qquad 13.26$$

where $\partial = \dfrac{\omega_2}{\omega_1}$

and $\omega_1^2 = \dfrac{K_1}{M_1}, \quad \omega_2^2 = \dfrac{2K+K_1}{M_1}$

$\frac{\omega_2}{\omega_1}$ of 10→ 100 and $\frac{\zeta_2}{\zeta_1}$ of 1 → 10 were the ranges most commonly occurring during the testing of joints. As can be seen from Figs.(47) and (48) the correction-factors approach unity as ω_2 is increased. In the worst possible case, i.e. $\frac{\omega_2}{\omega_1} = 10$ and $\frac{\zeta_1}{\zeta_2} = 1$ the correction factors were at $R_K \simeq -0.1$ and $R_\zeta \simeq -0.7$ dbs indicated that the maximum errors caused by ignoring the effects of auxiliary parameters were only about 1 and 9% for stiffness and loss factor respectively.

13.3.2 Correction for Pseudo-Material Effect

The last stage in data reduction was to extract the effect of the pseudo-material, i.e. that of the equivalent solid (Fig.44a) from the results obtained for jointed specimens (Fig. 44b).

Theoretical treatment of this effect can be found in Chapter 6.4. The correction procedure is therefore as follows:

(a) Calculate

(i) $$\frac{1}{K(1+\zeta^2)} - \frac{1}{K_m(1+\zeta_m^2)} \qquad 13.27$$

(ii) $$\frac{\zeta}{K(1+\zeta^2)} - \frac{\zeta_m}{K_m(1+\zeta_m^2)} \qquad 13.28$$

(b) Calculate ζ_J from

$$\zeta_J = \frac{(ii)}{(i)} \qquad 13.29$$

(c) Calculate K_J from

$$K_J = \frac{N}{(1+\zeta_J^2)\ (i)} \qquad 13.30$$

and if required

(d) Calculate C_J from

$$C_J = \frac{\zeta_J \; K_J}{\omega} \qquad 13.31$$

where, N is the effective number of joints

K_m and ζ_m are the pseudo-material stiffness and loss factor, and

K_J and ζ_J are the joint stiffness and loss factor respectively.

13.4 Summary

Step-by-step and clear procedures for various 'Complex Modulus Testing' and 'Data Reduction' techniques are set out in this chapter. The techniques make the most efficient use of the equipment in employing equations which are in terms of 'directly measurable quantities' (equations 13.6 - 13.19).

Data reduction techniques involve extracting the effects of the auxiliary system and/or that of the pseudo-material from the results obtained using 'free' and 'm_i'/or 'm' method. The former methods yield the effective parameters of the 'equivalent single degree of freedom system' shown in Fig. (15d) and the latter method gives directly those of the 'jointed specimen columns'.

Correction factors have been introduced for the auxiliary system's effects and shown to be only marginal when testing highly stiff and damped specimens, e.g. joints: For the worst possible case envisaged, i.e. when $\frac{\omega_2}{\omega_1} = 10$ and $\frac{\zeta_2}{\zeta_1} = 1$, the maximum errors caused by ignoring these effects are only about 1 and 9% for stiffness and loss-factor respectively (see Figs.47 and 48).

CHAPTER 14

DESIGN OF TESTS ON JOINTS

14.1 The Variables

The tests fell into two sections which examined two types of surfaces representing 'single' and 'multi' point tool cut surfaces, i.e. 'turned' and 'ground' respectively. (see Chapter 7).

The independent variables under study were: pressure, area (and shape), roughness, viscosity of lubricant (and frequency, amplitude). The dependent variables were initially chosen as the effective values of Voigtian parameters (K_J & C_J) together with stiffness factor (R_J) representing the effect of lubrication on the inphase components of stiffness; all per unit area of joint.

14.2 The Levels of Independent Variables

14.2.1 Pressure

In selecting the pressure levels the following relationships between stiffness K_J and pressure σ were assumed in order to obtain approximately equal increments of stiffness with pressure:

$K_J \propto \sigma$ From Table 1 for turned surfaces 14.1

$K_J \propto \sigma^{0.5}$ From Table 2 for ground surfaces 14.2

For simplicity in the analysis of results it was decided to use only one of the above relationships for both types of surfaces. Equation 14.2 proved to result in more uniform stiffness increments and hence was adopted.

Referring to Table 3 for maximum pressure (i.e. 224 KP/cm^2) the following pressure levels were calculated from equation 14.2:

224, 99, 25, 10 [KP/cm^2]

14.2.2 Area (and shape)

The following values of area were established from Chapter 7 as:

A45	0.45 [in^2]	= 2.9	[cm^2]
B90 (& S90, R90)	0.90	= 5.81	
C180	1.80	= 11.61	
E270	2.70	= 17.42	

(see Figure 49).

14.2.3 Roughness

The following mean values of roughness were established from Chapter 7 as:

	TND1	60 [μ in.]	= 1.52	[μm]	CLA
	TND2	180	= 4.57		"
	TND3	380	= 9.65		"
and	GND	8	= 0.20		"

14.2.4 Lubricant, Interface Condition

Apart from the 'dry condition' two types of common industrial lubricants representing the extreme viscosities were employed to simulate the 'lubricated conditions'. These lubricants were:
Shell Tellus 23 and 41 with kinematic viscosity at 38° centigrade and in atmospheric pressure of 25.2 and 112 [CS] respectively.

The test conditions for lubricated joints were realised by saturating the surfaces with oil whilst 'dry conditions' were achieved through abundant application of 'Carbon-tetrachloride' as a powerful degreasing agent.

14.2.5 Frequency · The Number of Discs

Initially it was decided to use a small number of discs to reduce manufacturing time and also increase the frequency range of load amplification. (see Fig. 14b). On the other hand this number had to be sufficiently large to lower the effective stiffness attributed to the joints to at least to the same order to that of pseudo-material stiffness. The effective number of jointed surfaces chosen was to be ten, thus satisfying the above conditions and also simplifying the calculations. (see Fig. 44b). Furthermore this number of discs resulted in NF_2 values within the frequency range frequently encountered in machine tool structures (∿ 100 - 1000 [Hz]).

14.2.6 Amplitudes

The effect of amplitude was studied at levels 1:4; level four corresponding to 0.25 [μm] R.M.S. deflection of the blocks resulting in a typical order of amplitude level per joint (of about 0.05 μm) reported to have been measured at the joints of a lathe's tool holder[82].

14.3 Test-procedure

The appropriate discs were placed on M_2 and on top of one another in a manner so that:

Firstly the ends faced the same direction (e.g. upwards); secondly they were concentric with M_2 (a number of concentric grooves on M_2 were specially made to help achieve this); and thirdly they maintained a predetermined angular position relative to one another (grooves along the axis of the specimens were used for this purpose).(see Fig. 50).

The load-washers were always placed at the middle of the specimen columns, i.e. at the nodal point, resulting in a more accurate measurement of F and also a closer symmetry of the system.

The pressure levels established in section 14.2.1 were related to the corresponding loads for each type of specimen. The loads in turn gave the corresponding deflections of M_2 which were then realised on the rig using a [10^{-3} in.] dial gauge.

In order to eliminate or reduce the possibility of flatness error and also to increase the efficiency, only B90/C180/S90 specimens were to be tested initially. Furthermore the study was to concentrate on single point tool-cut surfaces, i.e. turned surfaces.

The specimens were first loaded to the maximum pressure level at which the first set of measurements were carried out. They were then unloaded to the lower pressure stop and measurements again taken. This procedure was repeated to incorporate the rest of the pressure levels.

For each specimen size identical measurements were carried out using 'equivalent-solid' length of specimen columns.

Detailed methods of measurement and data reduction is given in Chapter 13.2.

14.4 Calibration/Check for the 'm' Method

Calibration of the Load-Washers under Combined Static and Dynamic Loading

The load washers had been calibrated under static load by the aid of a 'Denison Testing Machine' (see Fig. 36). The errors were always less than 5% for both sizes of the load washers used (about 3 per cent in average). The accuracy of 'm' method was however checked under dynamic loadings and this was achieved by comparing the results of the 'm' method with those of 'm_1'' which were accurate within ±5% for stiffness and ±0.05% for the loss factor corresponding to ±3° phase error (see Fig. 42). The error due to 'asymmetry' or extraneous effects was not allowed to exceed these levels throughout the tests in order to sustain the same level of accuracy - whenever a 'purely symmetric' motion was no longer possible with ±5% gain and 3° phase accuracy, the result of that test was discarded.

The independent variables for this check were chosen as frequency and preloads. The method was to calculate the percentage errors of stiffness and errors in the loss factor of 'specimen columns' found from 'm' relative to 'm_i' method; the 'm_i' results considered,were first corrected for the auxiliary parameters' effect. The frequency ranges of 300 - 450 [Hz] and 600 - 900 [Hz] constituted the low and high levels and as for preloads, 4 levels corresponding to the 4 pressures used in the study of joints were considered. For each level of preload and frequency 3 replicate results were considered. These results were themselves the average of results obtained for each level of preload and at different frequencies within each level of frequency.

The analysis of variance was performed using a computer - the significant levels being determined by 'F tests'. The results of these tests when using 906B load washer can be seen in Tables 10 and 11.

It is interesting to note that the precision of forced-vibration methods is indicated by the error variance : being the sum of the variances of the two methods. The corresponding values of standard deviations for loss-factor and stiffness were estimated at .032 and 3.2%. These figures are in fact the maximum standard deviations which can be expected due to random errors for 'm' or 'm_i' method.

The low values of SD also prove the high precision of the calibration itself furthermore indicating that either non-axial loading of the washer does not occur , or its effect is insignificant.

In evaluating the standard deviation due to random error for the loss-factor-phase, the variance around the Grand mean was considered; as apparently neither frequency nor static load had a significant effect upon the accuracy (i.e. SD = $(0.00105)^{\frac{1}{2}}$ = 0.032 rad). In other words there was no reliable evidence as to the validity of 'F' values, they may well have happened by chance. As for stiffness - Gain error, there was clear evidence that the two parameters did affect the accuracy. Fig. 51a illustrates this and also confirms that no interaction exists between frequency and static load. The points on the graph were determined by taking the average of the 3 replicates.

It was therefore reasonable to assume that Fig. 51a represented merely the fixed-Gain error of the 906B load washer/5001 charge amplifier.

Generally speaking the gain appears to drop with preload after reaching a maximum at some relatively low preload. On the other hand the effect of frequency on gain, seems to be opposite, it increases with frequency.

A similar test was carried out on the smaller load-washer 903A; the results are presented in the Summary Tables 12,13 and Fig. 51b.

14.5 The Effect of Frequency and the 'Yardstick' for Measurement of Damping.

The checks were made using the forced vibration results. For increased precision the mean of the two results were considered. To eliminate the possible effect of amplitude, tests were carried out with deflection of blocks of around 0.25 μm at all frequencies.

The variation of stiffness and loss-factor with frequency did not exceed experimental error in almost all the cases. Only at the lowest pressure of 10 [KP/cm^2] and for smooth and lubricated surfaces the stiffness tended to increase with frequency, whilst the loss-factor still remained unchanged.

It was at this stage that the decision on the type of parameter to represent damping throughout this work was made:

The loss-factor was retained as the most suitable 'yardstick'. The pseudo-hysteretic damping coefficient 'h_J', if required, could be determined directly from $h_J = K_J \zeta_J$. The latter would represent the quadrature component of stiffness in determining the damping forces in solution of linear differential equations of motion whilst the former could directly be employed in the approximate methods of response prediction.

14.6 The Effect of Amplitude (Linearity)

No significant variations were observed when amplitudes were reduced by [four fold]$^{-1}$, hence it was considered reasonable to assume that the free-vibration method would render as accurate results as 'm' and 'm_i' methods of forced vibration.

14.7 Improved Precision of Measurement

It was decided to do a statistical test upon the results obtained for the turned B90 and C180 specimens by the three methods of 'm', 'm_i' and free-vibration in order to establish an estimate of random error due to the choice of any particular method. This was achieved by using the results of the three methods (loss factor and inphase stiffness per unit area of joints) as 'replicates' in the analysis of variance (basically designed for testing the effects of the independent variables and used in the next chapter) whose summary can be found in Tables 14 and 15. The Standard Deviations with 96 degrees of freedom (144 number of observations) were estimated at .036 and 112. [$MN/m/cm^2$] for the loss-factor and the stiffness measurements respectively.

For increased overall precision, therefore, in the actual analyses the mean of these three methods will be considered. This would result in 'Standard Errors' (The SD of the mean) of only 0.02 and 65 [$MN/m/cm^2$] in the measurement of the loss-factor and stiffness respectively.

14.8 The Effect of Time

Some increase in both stiffness and damping of joints was observed under longer periods of testing. Such an effect, however, was only detectable for fine surfaces and at lower pressures. It was, on the whole, true to say that no significant loss of oil occurred, if any at all, and the mechanism was one of pumping-in-and-out, sustaining the same amount of oil within the surfaces at all times.

14.9 Summary and Conclusions

Recognition of sources of error and its evaluation are of paramount importance in establishing confidence in realisation of effective variables in qualitative study and accuracy in the eventual quantitative correlation amongst them.

The parallel terms for accuracy and precision are significant by fixed and random errors. Corrections for the fixed errors were carried through calibration of instruments in Chapter 11, and the random error due to unknown, uncontrolled or controllable variables will be estimated through repeated testing (Chapter 15).

Dynamic calibration of piezo-electric load-washers have shown that the gain drops with preload after reaching a maximum at some relatively low preload. On the other hand it rises with frequency. No significant effect of preload or frequency could be observed on the 'phase' characteristics of the load washers.

The tests have shown that neither 'm' nor 'm_1' method of forced vibration is expected to have standard deviation for random errors greater then 0.03 and 5.2% (for the loss-factor and stiffness measurements respectively).

For increased precision of measurements, therefore, it was decided to take the mean of the results obtained from the three methods: 'm', 'm_i' and free-vibration. It was possible to pool the results of the free-vibrations with those of the forced as the effect of frequency and amplitude proved to be minimal. The 'Standard Errors' thus obtained were computed at only 0.02 and 65 [MN/m/cm^2] for the loss factor and the inphase component of stiffness of joints per unit area, after 144 observations.

Since the effect of frequency proved to be insignificant (for turned surfaces and the ranges of independent variables considered). The most suitable 'yardstick' for measurement of damping was the loss-factor. The loss-factor alongside K_J, the inphase stiffness, would render $\dot{h}_J$ the pseudo-hysteretic damping coefficient and thus furnishing all the information required for the exact or the appropriate methods of response prediction: without any need to refer to the frequencies of excitation.

Although, the effect of frequency for turned surfaces proved to be insignificant, nevertheless it could not be generalised: it was therefore decided that off-resonant tests be carried out within as wide a frequency range as possible under all test conditions and the mean of the results be considered for the analyses.

Statical methods, i.e. variance and subsequent correlation and regression analyses <u>with replicates</u> were employed and are reported in the next chapter. In dealing with the results, a computer package programme (UASTATSXDS3) was used.

CHAPTER 15

RESULTS OF TESTS ON JOINTS

15.1 Turned Surfaces

15.1.1 General

The results of experiments on specimens B90 and C180 are shown in a compact format in figures 52 - 65. Each figure contains loss factor and stiffness per unit area for both dry and lubricated conditions. Due to the multi-dimensional nature of the problem these figures exhibit only some aspects of joint behaviour - the picture is by no means complete. In order to bring to light all aspects of the problem and eventually to 'measure' them, a systematic approach was employed. The following is the result of such an attempt.

15.1.2 The Effect of Flatness Errors - Stiffness under dry conditions

Mean	Minimum	Maximum $[MN/m/cm^2]$	Variance $[MN/m/cm^2]^2$
604	85	2351	321539

It was essential, at the outset of the experiments, that the 'flatness errors' if present, be detached. For this purpose stiffness per unit area of the specimens was measured under dry conditions. The results of these tests can be seen in Fig. 52.

The results of rougher surfaces, i.e. TND2 and TND3 are in close agreement with the previous works both qualitatively and quantitatively (29,39). That is to say that firstly the effect of roughness and area is insignificant, secondly the stiffness at around 17.6 [KP/cm^2] (250 PSI) does in fact correspond to 12.7 cm. (5.0") of E.L.S. (see Fig.11a).

The results of the finest surface, i.e. TND1 however seemed to suggest the possibility of flatness error. To investigate this point further, the tests were repeated with deliberate change in relative angular position of discs and results were compared with those of the previous tests. Repeatability was excellent (Fig.53). (Similar tests were obtained using A45, E270 and S90 specimens; Fig. 68). It was concluded that: (a) flatness errors were obviated by the special technique of cutting, i.e. use of the magnetic chuck (see Chapter 7.5) and therefore (b) the effect of area (and shape) appeared to be significant. It was not clear exactly what caused this phenomenon. The effects of roughness and/or material could not be ruled out but were believed to be insignificant. (The values of roughness given in Fig. (68) are the average of values measured over twenty surfaces and at four quadrangles for each set of specimens. For information on material of the discs see Chapter 7. and/or Fig.22). It must be mentioned that at the pressure levels under study the clean and fine surfaces tended to 'stick' or 'ring' to one another. (Note the sharp drop in stiffness when the pressure is reduced to 99 [KP/cm^2]).

With a view to reducing the possibility of prejudicing the lubricated joints' results, due to any unknown effect such as that mentioned above, a non-dimensional parameter 'stiffness factor' (defined as the ratio of lubricated inphase stiffness to that of dry) was chosen in place of inphase stiffness. The validity of this statement was checked by computer. The correlation factors for the non-dimensional parameters, inphase and quadrature component of stiffness with the stiffness under dry conditions, were as follows:

	Loss factor	Stiffness factor	Inphase Stiffness	Quadrature Stiffness
Dry Stiffness	- 0.17	- 0.12	+ 0.955	+ 0.54

The choice of stiffness factor was considered advantageous not only for the reason mentioned above but also because of its generalized, i.e. non-dimensional nature, it could be used in conjunction with the results of other works on dry joints to give estimates of a joints' stiffness under lubricated conditions.

15.1.3 Four Factor Analyses of Variance with Replication

Analyses of variance were performed with one set of replicates for both loss factor and stiffness factor: After having established a measure of repeatability, namely the error variance, the F values were determined as a ratio of variances due to respective independent variables (and their interactions) to the error variance. The significance levels were then determined from the 'F tables'. The results are summarized in Tables 16 and 17 for the loss-factor and stiffness factor respectively. By referring to these tables the following points emerged:

Loss Factor, ζ_J (see Table 16)

	Mean	Minimum	Maximum	Variance
	0.274	0.018	1.629	0.115

A measure of overall precision is half the error variance, i.e. 0.0065 which is only about 6% of the total variance. (Note that small difference between the values of total variance given here and that indicated in Table 10 is due to the more accurate levels of independent variables which were used in deriving the above value)

The difference between the loss factors at different levels of roughness is highly significant, so too is the effect of area. The only interaction term having a highly marked effect on the loss factor occurs between roughness and area. Most of the other effects, although significant are responsible for comparatively lower changes in the loss factor within the ranges considered. For example pressure and viscocity have 'F' values lower than that of roughness by a factor of approximately seven and fifteen respectively.

As a guide for the following regression analysis figures 60, 61 and 62 were constructed by taking means of loss factors for lubricated joints over the whole ranges of independent variables. For example each point on figure 60 Represents the mean of 16 values; 2 lubricants x 4 pressures x 2 replicates. Figure 60 suggests a sharp reduction in loss factor with roughness for smoother surfaces; similarly for TND1 and TND2 these values seem roughly to double with the doubling joint area whilst at very rough joints no significant differences were apparent. These points are signified by high values of 'F' for interaction between area and roughness (about 34).

The effect of roughness, viscosity and pressure on the loss factor are demonstrated in Figs. 61 and 62. The finest surface, i.e. TNDI resulted in a much higher level of energy dissipation and at the same time being more sensitive to pressure. This was in fact expected as the flow resistance through micro-macro-scopic apertures within the interface would be greater for a fine surface. However the level of roughness at which this effect became sensitive to pressure was much lower than that envisaged during the experimental design. The effect of viscosity on the loss factor, as can be seen from Fig.62, is relatively small. This could be explained by the fact that although the flow resistance increased with viscosity it resulted in a reduced volume of flow hence levelling off the overall effect (see also Reference (40)).

Stiffness factor, R_J (see Table 17)

Mean	Minimum	Maximum	Variance
1.376	1.006	3.000	0.227

A measure of overall precision is half the error variance, i.e.0.017, which is only about 8%of the total variance.

Again the effect of roughness and area is predominant. The viscosity effect is now almost completely diminished and that of pressure reduced considerably. The major interaction term remaining is that between roughness and area although it is somewhat more pronounced.

The above results could also be appreciated by referring to Figs. 63, 64 and 65 which were primarily constructed to help in appropriate transformations required to improve correlation in the multiple regression which followed.

15.1.4 Multiple Regression

The 'STATSXDS3' was employed in an attempt to formulate the above qualitative results in multiple regression equations.

The model employed in the package was of the following linear form:

$$Y = A(i) + B(J) + C(k) + D(\ell) + AB(ij) + AC(ik) + AD\ (k\ell) + BC(jk) + BD(j\ell) + CD\ (k\ell) + ABC\ (ijk) + ABD(ij\ell) + ACD(ik\ell) + BCD(jk\ell) + ABCD(ijk\ell) + E(ijk\ell m) \qquad 15.1$$

It was therefore obvious that appropriate transformations had to be performed on variables in order to mould the observed points to this model as closely as possible.

Generally speaking there is no systematic method with which optimum transformations could be found. However the following considerations led the search eventually to a new 'field' at which optimisation could in fact be performed. This was possible mainly due to the fact that the relationships were closely of the form:

$$Y \simeq A^a B^b C^c D^d \qquad 15.2$$

The procedure was as follows:

The values of a_1, b_1, c_1 and d_1 were to be estimated initially by referring to the previous works on this subject as a rough guide. As such relationships were rare or nonexistent, the information was extracted from sets of figures 60 - 62 and 63 - 65 through varying a, b, c or d and replotting Y with non-linear scales on abscissae as the curves straightened, approximating to lines. The last part of optimization procedure was one of iteration through transforming the variables into their logarithms. By so doing the above equation was transformed into linear form whose coefficients were determined by computer:

$$\log Y \simeq a_2 \log A + b_2 \log B + c_2 \log C + d_2 \log D \qquad 15.3$$

If a_2, b_2, c_2 and d_2 were different from the values previously chosen, the new values thus found were inserted in the original equation 15.2 and regression analysis repeated as before; the independent variable being $\log(A^{a_2} B^{b_2} C^{c_2} D^{d_2})$. This procedure was repeated until multiple correlation was maximized. Whilst there was no definite evidence as to the validity of this procedure to result in the 'best fit', because of the high correlation factors thus obtained the method was considered as a powerful tool. (Correlation factors always greater than .93). The following equations represent the results obtained for dry stiffness, stiffness factor and loss factor using the above technique:

DRY STIFFNESS 'K_J'

$$K_J = 25 \ \sigma^{0.788} \ \Omega^{-0.251} \qquad 15.4$$

with multiple correlation at 1% significance level of .983.

STIFFNESS FACTOR 'R_J'

$$R_J = 1 + 0.0628\ \sigma^{-0.256}\ \Omega^{-1.219}\ \Delta^{1.740}\ -0.02 \qquad 15.5$$

with multiple correlation at 1% significance level of .978

LOSS FACTOR 'ζ_J'

$$\zeta_J = 0.319\ \sigma^{-0.33}\ \Omega^{-1.00}\ \Delta^{1.00}\ +0.001\mu\ -0.12 \qquad 15.6$$

with multiple correlation at 1% significance level of .961

or alternatively

$$\zeta_J = 0.803\ \sigma^{-0.39}\ \Omega^{-1.13}\ \Delta^{0.67} \qquad 15.7$$

with multiple correlation at 1% significance level of .933

NOTE: K_J in [MN/m./cm^2.]

σ Pressure in [KP/cm.^{2}]

Ω Roughness in [μm] CLA

Δ Area in [cm.^{2}]

15.1.5 Further Experiments on the Fine Turned Surfaces

As the results indicate, area (and roughness) play a major part in determining the behaviour of lubricated joints (see Equations 15.5, 15.6 and 15.7). Further experiments were carried out on fine surfaces (TND1 : 1.2 micron CLA) in order to confirm the validity of previous findings over an extended range of variables and to study the effect of shape factor. The choice of fine surface was made since such a surface possessed a much higher loss factor and stiffness factor and on the whole was more interesting.

Experiments were carried out on specimens A, S and E. The effect of viscosity having proved minimal only one lubricant was used, namely Shell Oil Tellus 41.

All the tests were repeated and the mean of the two results considered in the following presentation.

The effect of the area on the loss factor and stiffness factor can be seen in Figure 66 where each point of the graph represents the mean of four values measured at the four pressure levels. The effect of the shape factor at 5.8 [cm^2] area is also indicated in this figure to be insignificant.

Figure 67 shows the effect of pressure on the loss factor and the stiffness factor. Each point on the graph constitutes the mean of five values measured for specimens A, B, BS, C and E.

15.2 Tests on 'Ground' Specimens

Due to the very high stiffness of ground joints under lubricated conditions, quantitative study of such joints resulted in large errors. The source of this error was in the evaluation of small differences of large values, i.e. K and K_m, in the process of correction for the material effect (see equations 13.29 and 13.30).

To demonstrate this point figure 69 was constructed where stiffness of solids and those of dry columns are shown for the different sizes of specimens. As can be seen from this figure the values of stiffnesses are high and the differences between respective solids and jointed columns are small, particularly at high pressures. Due to the hydro-elastic forces, this effect was even more pronounced for lubricated joints only with reversed pressure influence: that is to say at low pressures where 'squeeze film' was more effective, stiffness of jointed columns rose more sharply than at higher pressures. Figure 72 shows this point where for the purpose of clarity the mean of all tests obtained from the fine specimen groups are presented. Figure 74 presents a similar set of results for the fine turned surface TND1 for comparison.

15.2.1 Dry Conditions

The results of tests on dry ground joints are shown in figure 70. Variation of stiffness per unit area with area and shape factor appeared to become progressively significant as pressure was increased. However no conclusive evidence could be found as to the causes of such phenomena. The error in calculating these values could well have masked any effects due to other sources (i.e. flatness error, roughness error, area and shape factor effect).

Figure 71 was constructed for the purpose of comparison with turned surfaces. For increased precisionthe mean values of stiffness for B and C specimens were taken as the best estimate. As seen from this figure the ground joints exhibit a rate of increase with pressure higher than those of the turned surfaces by factors of approximately five and eight respectively.

15.2.2 Lubricated Conditions

For the reason mentioned above, any analysis of ground joints under lubricated conditions could only be carried out on the results obtained for the jointed columns.

15.3 Comparative Study

Figures 72 - 75 show how a jointed column made of ground joints would compare with the same length of solid and the jointed column of single point tool cut surface (i.e. turned) at different pressure levels. The points on the graphs are means of results obtained for the five specimen groups (i.e. A, B, 'S, C and E).

Figure 76 shows the result in a still more compact form by taking the average of the above results for B90 and C180 specimens at three pressure levels of 24, 99 and 217 [KP/cm^2]. The term used to represent the elastic effect of the columns is the 'Equivalent Length of Solid Steel'; as for damping apart from the loss-factor a further measure namely energy loss/cycles/unit amplitude has also been introduced in order to represent the quadrature component of stiffness (in this case also the hysteretic damping coefficient).

The flexibility due to the 'pseudo-material' and the 'joints', within the specimen columns, can be estimated directly from Fig. 71a. Fig. 71b, however, contains the information regarding a combined effect of stiffness and damping distribution within the jointed column according to equation 6.71. These figures illustrate clearly the guide line towards the optimization, i.e. 'the principle of equilibrium of the potential energy distribution'. Adherence to this principle could ensure the stiffness integrity of the system whilst resulting in enormous increases in damping. Fig. 76c was employed in determining the optimum distribution of stiffness and damping as it represents a measure of $K \times \zeta = h$. A clear picture emerging from this figure is that fine and lubricated but turned surfaces offer the optimum distribution where both high stiffness and damping is achieved. The optimum distribution under dry conditions, on the other hand, is achieved using ground surfaces.

By referring to equations 6.16 and 6.17 it is concluded that similar hystrogrammes would be obtained for a single jointed member, the member having E.L.S. of approximately 15 [mm.] and placed in series with the joint. The larger the E.L.S. the more pronounced will be the difference between the SPT and MPT behaviour outlined above.

15.4 Summary and Conclusions

Single Point Tool Cut Surfaces - Under dry Conditions

The results for rougher surfaces, i.e. TND2 and TND3 are in good agreement with the predicted results from Fig. 11a both qualitatively and quantitatively. The 'Equivalent Length of Solid Steel' representing the joints' flexibility, at lower pressure levels, can therefore be estimated from:

$$E.L.S. \simeq 223\ \sigma^{-1} \quad [cm.]$$

The behaviour of the finer surface, i.e. TND1 under dry conditions, however, could not be explained in clear terms. The possibility of flatness error was ruled out when results under repeated but deliberately altered angular position of discs were found to be concurrent. It was possible that the 'clean and fine' surfaces experienced a 'molecular adhesion' because of the relatively high pressures used in this study. Further work is needed to establish the exact cause of these effects.

The repeatability of the dry tests' results also indicated the effectiveness of the special technique of cutting which was designed to eliminate or reduce the flatness errors.

The following equation for dry stiffness resulted in a multiple correlation factor of about 0.98 at 1% significance level:

$$K_J = 25\ \sigma^{0.79}\ \Omega^{-0.25} \qquad [MN/m/cm^2]$$

Single Point Tool Cut Surfaces - Under Lubricated Conditions

With a view to reducing the possibility of prejudicing the lubricated joints' results due to any unknown effect such as the one mentioned above, the non-dimensional parameters R_J (and ζ_J) were used in the regression analysis in preference to the inphase (and quadrature) components of stiffness: the correlation factors found by computer for these non-dimensional parameters with 'dry stiffness' were approximately $\frac{1}{8}$ (and $\frac{1}{3}$) of those for the inphase (and quadrature) components of stiffness respectively.

The overall repeatability of individual duplicate tests on turned surfaces were estimated at 0.006 and 0.017 being half the error variance for loss and stiffness facror respectively. These values were only about 6 and 8% of the total values of variance.

For the range of the independent variables considered area and roughness emerged as the main parameters influencing both R_J and ζ_J. The effect of pressure was reduced drastically when compared with dry stiffness and that of viscosity was non-existent upon R_J and only marginal upon ζ_J.

The following equations resulted in multiple-correlation-factors, at 1% significance level, of .98 and .93 for stiffness factor and loss factor respectively:

$$R_J \simeq 1 + 0.063\ \sigma^{-0.26}\ \Omega^{-1.22}\ \Delta^{1.74}$$

$$\zeta_J \simeq 0.80\ \sigma^{-0.39}\ \Omega^{-1.13}\ \Delta^{0.67}$$

Further Experiments on Fine-Turned Surfaces

Further experiments on fine-turned surfaces verified the above findings and also showed that the effect of 'shape' was not significant. The latter result together with the high interaction which was observed between area and roughness indicated that a combined effect due to 'micro' and 'macro' flow of the lubricant within the interfaces occurred. It is therefore concluded that fundamental modifications to the theories of 'squeeze film' are needed if any analytical study of joints is to result in building a realistic theoretical model.

Multi-Point-Tool Cut Surfaces.qualitative study

The ground joints exhibited a rate of increase in stiffness with pressure of about 5 - 8 times those of the turned surfaces, under dry conditions. Due to the very high stiffness of ground joints, and hence overwhelming pseudo-material effect, accurate quantitative study of these joints, despite the very high precision of measurement, could not be made, particularly under lubricated conditions. Nevertheless, comparative studies *were* possible using the results of the 'jointed' columns.

A Method of optimization is proposed for the stiffness and damping distribution of a jointed assembly. It is based upon maximising the 'quadrature component' of the overall stiffness. This would result in a more balanced distribution of potential energies within the assembly, which, whilst retaining overall stiffness integrity produces a large amount of damping.

Joints produced by single point tools with the minimum amount of roughness are shown to be far superior to the ground joints as they result in maximum $K \cdot \zeta = h$, averaging approximately three times that of the ground joints.

A reverse of this was observed for dry joints where ground joints resulted in larger values of 'h' as compared with the turned surfaces.

CHAPTER 16

FUTURE WORK

The following recommendations could prove useful in future studies.

16.1 Response Prediction and Optimization of Machine Tool Structures

A programme of work is outlined here for investigating the use of joints (classic or synthetic) as a tool for optimization purposes of structural components. The research could find a direct relevance and great importance in the design of machine tool structures particularly when applied to the 'Modular techniques'.

1. Testing for practical application of the equations derived in this work for joint stiffness and damping for predicting the overall behaviour of actual or model structures with the aid of equation 2.2 (approximate method).

2. Optimizing for improved overall damping capacity of a jointed assembly as proposed in Chapter 15.3.

3. Optimizing for directional orientation of modal flexibilities as proposed in Chapter 2.6 (approximate method).

4. Area and surface texture should be given special attention as design parameters, in the optimization processes.

5. Employing the outcome of the above studies and of further quantitative measurement of the joints' behaviour in other directions an efficient 'exact method' of response prediction should be developed to include the elastic and dissipative forces generated within the interfaces.

16.2 The Apparatus

Application of the complex modulus measuring apparatus is virtually boundless. Some possible extension to its use and capability could be envisaged as follows:

6. C.M. measurement of materials and components such as rubbers, joints between a tool and a work piece, etc.

7. Fatigue testing.

8. Extend its capability by introducing a mechanism placed between the inertia blocks to alter the dynamic loading direction and/or its mode from normal to tangential and/or torsional.

9. Further improve the isolation property of the apparatus by isolating the lower shaker from the base-plate.

10. Calibrate for static and dynamic loading of the rig above the 4000 [KP] level of preload.

11. Extend the Pilot-Tests in order to establish the variation in errors of 'symmetry' with reduced stiffness of specimen columns.

16.3 Joints

Extend the study of joints with the recommended levels of independent variables and introducing new variables as listed below:

12. Lower the pressure levels.

13. Lower the roughness levels.

14. Extend the levels of area used in the regression analyses to embody specimens sizes A45, E270 as well as B90 and C180.

15. Study the effect of 'shape factor' at different levels of area, preferably at A45, B90 and C180.

16. Examine the effect of quantity of lubricants especially for a solid lubricant such as grease.

17. Increase the number of discs in the specimen columns in order to widen the frequency range in the lower end of the spectrum.

18. Improve the accuracy of data reduction in accounting for pseudo-material effects (particularly when testing joints with high stiffness, as in the case of ground surfaces) by reducing the thickness of the discs as far as possible and avoiding the use of load-washers.

19. Test a more varied range of surface texture particularly for MPT cut surfaces, e.g. scraped, lapped, etc.

20. Investigate the effect of material of joints particularly for rougher surfaces where such an effect is expected to be more noticeable.

21. Study and test the potentialities of 'synthetic joints' employing various materials as intermediary agents between the interfaces, e.g. viscoelastic materials particularly composites of polyisobutylene, lead or rubber foils, bonded joints, etc.

22. Further experiments under dry conditions should be carried out to establish the causes of the phenomena observed in this work for fine surfaces.

23. After an extensive quantitative study of the joints' parameters, attempts should be directed towards building theoretical models representing the actual mechanisms within the interfaces. The assumption of micro-macro flow of the intermediary agent within spherical model asperities could prove valuable as a first attempt.

CHAPTER 17

CONCLUSIONS

17.1 Response Prediction and Optimization of Machine Tool Structures

1. Distribution of stiffness, mass (and damping) in conventional machine tool structures could in some cases be corrected to concur with the optimum directional orientations of modal flexibilities (and also improve the overall damping capacity of the structure).

2. The first step towards 'exact methods' of response prediction is taken in this work by quantitative measurement of stiffness and damping to represent elastic and damping forces produced in the normal direction within the interfaces. Because of the non-proportional distribution of damping within machine tool structures (i.e. because of joints) these methods could prove impractical from the point of view of the size of computation.

3. The results of the tests on the model structures have shown that the use of joints as a device for optimization is feasible.

4. An improved approximate method of optimization is envisaged whereby damping ratios are estimated from the modal shapes (equation 2.2) and joints are employed as a tool in optimization for both directional orien tation of modal flexibility and also improvement in the overall damping capacity of the structure.

5. A method of optimization proposed here for the improvement in the overall damping of a jointed assembly is based upon the principle of the equilibrium of the potential energy distribution through maximising the

quadrature component of the overall stiffness. This is believed to ensure the stiffness integrity whilst producing large increases in the overall damping.

17.2 The Apparatus

6. For testing 'joints', the ideal conditions of 'massless and proportionally damped ensemble' could not be realized in the past, as the high transmissibility due to the high stiffness of joints invariably resulted in modes' coupling hence rendering the data reduction impractical or limited only to very small frequency bands.

7. The 'Energy Method', introduced in this work, is shown to be the only method capable of evaluating the frequency dependent parameters of a 'real system'. (a) The evaluation of the energies is only possible if the 'ensemble' is forced into a pseudo-ideal condition where all the points on the system are in-or-out of phase with one another, i.e. when the system undergoes uncoupled vibration, (b) The prerequisite of free vibration exhibiting simple harmonic motion is also that no coupling due to damping exists within the system. This condition is satisfied only if the principal coordinates are orthogonal, i.e.

(i) either the damping is proportionally distributed, or

(ii) the system is completely symmetric.

(c) To create the above conditions 'multi-point excitation' is, in general, necessary; the number of excitation points being equal to the number of degrees of freedom.

8. An ideal arrangement for complex modulus measurement of highly stiff specimens such as joints is a 'completely symmetric' two degrees of freedom

system. The main features of such an apparatus are as follows:

(a) Uncoupled free (and forced) vibration.

(b) Minimal transmission of load to the surroundings, hence minimal extraneous effects. These effects, nevertheless, are determined accurately at the first mode.

(c) Reduced size of shakers.

(d) The provision of uncoupled free-vibration not only enables the 'Energy Method' of measurement to be carried out but also extends the capability of the apparatus in measurements of damping at very low levels where forced vibration methods fail in accuracy.

9. The introduction of inertia blocks as load amplifiers and inducers of the free-vibration resulted also in the following advantageous features which were never achieved collectively in the past:

(a) A higher sensitivity and greater simplicity of measurement by employing piezo-electric accelerometers.

(b) Uniformly distributed static and dynamic loading of specimens.

(c) On-the-spot and efficient calibration of instrumentation.

(d) Direct measurement of force on specimens (i.e. 'm' method) can be obviated if the shape or size of specimen did not allow such a measurement. This would also have the advantage of reduced pseudo-material flexibility hence a greater accuracy of data reduction when testing highly stiff specimens.

(e) A higher precision of measurement achieved through pooling the results of three methods namely the forced (m_i and m) and the free-vibration methods.

(f) An extended preloading capacity [5 tons].

10. Introduction of 'apparent mass' and 'modal shape' vectors ($\bar{m}$ and $\bar{r}$) resulted in an efficient method of calibration, testing and data reduction.

11. The systems of excitation and measurement have the following main features:

(a) The frequency stability of a crystal oscillator (10^{-4}%) and the low distortion of an LC oscillator (10^{-1}%) are retained in the reference and variable phase signals of the excitation.

(b) A considerably simpler system of excitation and measurement is provided under free-vibration where the analyser becomes redundant.

(c) Point-to-point measurement and vector dividing facility on the analyser result in direct measurement of $\bar{m}$ and $\bar{r}$.

12. A method for calibration of load-washers, under combined static and dynamic loading, has been developed using the symmetric apparatus where F measurement is redundant. The results of these tests suggest that the gain drops with preload after reaching a maximum at some relatively low preload. On the other hand it rises with frequency. No significant effect of these factors could be observed on the 'Phase' characteristics of the load washers.

13. Although provisions *have* been made in the design of the rig to achieve varied second natural frequencies (through varied number of discs in the specimen column) nevertheless due to the inevitable presence of the pseudo-material effect, it is expected that the target value of three octave frequency band could never be achieved in practice (particularly when testing highly stiff joints such as ground).

14. The results of pilot tests indicated that 'complete symmetry' of the rig is realised. Furthermore only a very small amount of damping within the auxiliary system is present.

15. Extraneous modes' coupling proved insignificant when testing joints. However, some rotational vibration of the inertia blocks is observed when $K \to 0$.

16. The preload proved to have a decisive effect upon the stiffness and damping of the auxiliary system. Equations (12.5 - 12.10) represent the variation of these parameters with the preload.

17. Correction factors have been introduced for the auxiliary parameters' effects and are shown to be only marginal when testing highly stiff and damped specimens, e.g. joints: For $\frac{\omega_2}{\omega_1} = 10$ and $\frac{\zeta_2}{\zeta_1} = 1$, the maximum errors caused by ignoring these effects are only 1 and 9% for stiffness and loss factor respectively.

17.3 Joints

18. The production of specimen 'blanks' using standard Hollow Bars on a Capstan lathe proved most efficient. The use of a magnetic chuck, in the final operation of roughness formation on the specimens, in reducing or eliminating the flatness errors, also appears to have been effective.

19. The distribution of deflection along the jointed column is linear for ideal and sinusoidal for a real system. The 'wavelength' depends upon the frequency of excitation and the system parameters. For the worst possible case in this study the maximum frequency of excitation was only about 13% of the first natural frequency of the jointed column.

This corresponds to about 7% of the first wavelength representing a nearly linear distribution within the column. The maximum inertia force due to the mass of the discs was only about 2.5% of the elastic restoring force. For all practical purposes, therefore, it can be concluded that the effect of inertia of the discs is negligible and the system can be regarded as ideal.

20. The correction for pseudo-material effect within the specimen column can prove erroneous for highly stiff joints. Ideally these effects should, therefore, be minimized. On the other hand, it must be mentioned that some flexibility due to the pseudo-material is beneficial when testing highly damped joints under free vibration.

21. The 'yardsticks' employed in the quantitative study of joints' behaviour were: inphase stiffness under dry conditions, inphase stiffness factor (lubricated/dry) and loss factor under lubricated conditions. The non-dimensional parameters were preferred to their dimensional counterparts (inphase and quadrature components of stiffness) because they proved to be almost independent of dry stiffness hence resulting in a more accurate and also generalized form of information.

22. Quantitative analyses of the results were achieved through the highly accurate and precise methods of measurement afforded by the measuring apparatus. The fixed errors determined from calibration of the instruments were found to be minimal for identical gain settings of charge amplifiers. The overall repeatability of individual duplicate tests on turned surfaces were estimated at 0.006 and 0.017 being half the error variance for loss and stiffness factor respectively. These values were only about 6 & 8% of the total values of variance.

23. Within the ranges of independent variables considered area and roughness emerged as the main parameters influencing both the loss and stiffness factor.

24. There appeared also high interactions between area and roughness whilst the effect of 'shape' and viscosity of lubricant proved minimal. These contradictory results indicated a combined effect due to micro-macro, rather than just a macro flow, of the lubricants within the interfaces.

25. Stiffness under dry conditions was mainly dependent upon pressure rather than roughness; however, for finer surfaces a combined effect of area, roughness and shape factor signified the presence of phenomena which could not be explained clearly. Surface molecular effects could be responsible for this phenomena in the same way as they invalidate the classic sinkage or bulk flow theories (c.f.24).

26. The rate of increase in stiffness of joints with pressure tends to zero as pressure is increased. This process is considerably faster for lubricated than it is for dry joints.

27. For rough surfaces under dry conditions the level of pressure at which this occurs approaches infinity. These surfaces behave almost independently of roughness and area but depend primarily on pressure.

28. The ground joints under dry conditions exhibited a rate of increase in stiffness with pressure of about 5-8 times those of the rough and the fine turned surfaces respectively.

29. The rate of increase of stiffness factor is an increasing function of area whilst that of the loss-factor is a constant or a slowly reducing function of area.

30. The rate of increase in stiffness and loss factor with reduced roughness is almost independent of roughness (or only very slowly increasing functions of it).

31. The rate of increase in stiffness factor with reduced pressure increases somewhat faster than it does for the loss factor.

32. A linear increase of loss factor with the viscosity of lubricant was observed although this effect proved to be small. Viscosity had no influence upon the stiffness factor.

33. The effect of frequency and amplitude proved insignificant for the ranges of independent variables considered.

34. The following equations represent the results of the regression analyses, performed by the aid of a computer, upon: dry stiffness, stiffness factor and loss factor of the turned surfaces, all per unit area of joint.

$$K_J \simeq 25\sigma^{+0.79} . \Omega^{-0.25}$$

$$R_J \simeq 1 + 0.063\sigma^{-0.26} . \Omega^{-1.22} . \Delta^{+1.74}$$

$$\zeta_J \simeq 0.319\sigma^{-0.33} . \Omega^{-1.00} . \Delta^{+1.00} + 0.001\mu - 0.12 \quad \text{, or}$$

$$\zeta_J \simeq 0.803\sigma^{-0.39} . \Omega^{-1.13} . \Delta^{+0.67}$$

35. Due to the very high stiffness of ground joints, and hence the overwhelming pseudo-material effect of the discs, accurate quantitative study of these joints, despite the very high precision of measurement, could not be made under lubricated conditions. Nevertheless comparative studies *were* possible using the results of the 'jointed columns'.

36. A jointed assembly with lubricated and smooth but SPT cut surfaces is expected to exhibit by far the larger quadrature component of stiffness than the identical assembly with ground surfaces. The convernse of this statement holds true for the dry conditions but to a lesser degree.

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APPENDIX 1

RESPONSE TO HARMONIC EXCITATION OF LINEAR 'COMPLEX SPRINGS' AND OF SINGLE DEGREE OF FREEDOM SYSTEMS. VARIOUS DEFINITIONS FOR DAMPING

DEFINITIONS OF DAMPING COEFFICIENTS

(i) Viscous Damping Coefficient C

is the frictional force opposing the motion at unit velocity:

$$F_I = -C\dot{X} \quad \text{or} \quad C = -\frac{F_I}{\omega X} \qquad \ldots 1$$

(ii) Hysteretic Damping Coefficient h

is the frictional force opposing the motion at unit displacement.

$$F_I = -ihX \quad \text{or} \quad h = -\frac{F_I}{X} \qquad \ldots 2$$

Comparing equation 1 with 2 it follows:

$$C \equiv \frac{h}{\omega} \qquad \ldots 3$$

Response to Harmonic Excitation of 'Complex Springs'

(i) Viscous

The differential equation of motion is

$$C\dot{X} + KX = F_o e^{i\omega t} \qquad \ldots 4$$

The solution is of the form

$$X = \bar{X}\, e^{i\omega t} \qquad \ldots 5$$

where $\bar{X}$ is the displacement phasor

$$\therefore \quad (K + iC\omega)\ \bar{X} = F_o \qquad \ldots 6$$

or

$$\bar{X} = \frac{\frac{F_o}{K}}{\sqrt{1+\zeta^2}} \quad e^{-i\phi} \qquad \ldots 7$$

$$X = \frac{X_{St}}{\sqrt{1+\zeta^2}} \quad e^{i(\omega t-\phi)} \qquad \ldots 8$$

where

$$\zeta = \frac{C\omega}{K} = \tan\phi \qquad \ldots 9$$

assuming $F_o e^{i\omega t}$ is to represent $F_o \mathrm{Sin}\omega t$ equation 8 can now be written as

$$X = \frac{X_{St}}{\sqrt{1+\zeta^2}} \mathrm{Sin}(\omega t-\phi) \qquad \ldots 10$$

(ii) Hysteretic

The above procedure can be reprated for springs with hysteretic damping only C is replaced by $\frac{h}{\omega}$, where h is the coefficient of hysteretic damping.

The equation of motion is:

$$\frac{h}{\omega}\dot{X} + KX = F_o e^{i\omega t} \qquad \ldots 11$$

which has the solution

$$X = \frac{X_{St}}{\sqrt{1+\zeta^2}} \ \mathrm{Sin}\ (\omega t - \phi) \qquad \ldots 12$$

where

$$\zeta = \frac{h}{K} = \tan\phi \qquad \ldots 13$$

Definition of Damping Capacity for 'Complex Springs'

Specific Damping Capacity ψ

(i) Viscous

It is a non dimensional measure of damping defined as the ratio of the energy dissipated to the maximum energy (potential) stored per cycle of oscillation:

$$D_o = \int_0^T F_I \dot{X}\, dt \qquad \ldots 14$$

from equation 1

$$F_I = -C\dot{X}$$

$$\therefore \quad D_o = \int_0^T -C\dot{X}^2\, dt$$

for $X = X_o \text{Sin}(\omega t - \phi)$

The solution of the integral is:

$$D_o = -\pi C\omega X_o^2 \qquad \ldots 15$$

Alternatively,

$$D_o = -\int_0^T F\dot{X}dt = -\int_0^T F_o \text{Sin}\omega t \cdot \omega X_o \text{Cos}(\omega t - \phi)dt \qquad \ldots 16$$

$$D_o = -\pi F_o X_o \text{Sin}\phi \qquad \ldots 17$$

$$\therefore \quad D_o = -\pi C\omega X_o^2 = -\pi F_o X_o \text{Sin}\phi \qquad \ldots 18$$

$$V_o = \int_0^{X_o} F_R . dX = \int_0^{X_o} KX.dX = \frac{1}{2} KX_o^2 \qquad \ldots 19$$

$$\psi = \frac{-D_o}{V_o} = \frac{\pi C\omega X_o^2}{\frac{1}{2} KX_o^2} = 2\pi \frac{C\omega}{K} = 2\pi \tan\phi \qquad \ldots 20$$

Alternatively

$$\psi = \frac{-D_o}{V_o} = \frac{\pi F_o X_o \text{Sin}\phi}{\frac{1}{2} K X_o^2} = 2\pi \frac{F_o}{K X_o} \text{Sin}\phi \qquad \text{...21}$$

(ii) Hysteretic

$$\psi = \frac{-D_o}{V_o} = 2\pi \frac{h}{K} = 2\pi \tan\phi \qquad \text{...22}$$

Alternatively

$$\psi = \frac{-D_o}{V_o} = 2\pi \frac{F_o}{K X_o} \text{Sin}\phi \qquad \text{...23}$$

$$\text{where } \phi = \tan^{-1} \frac{h}{K} \qquad \text{...24}$$

Loss Factor ζ

(i) Viscous

It is a non-dimensional measure of damping defined as the ratio of the quadrature to the inphase component of stiffness:

$$\zeta = \frac{C\omega}{K} = \tan\phi \qquad \text{...25}$$

(ii) Hysteretic

$$\zeta = \frac{h}{K} = \tan\phi \qquad \text{...26}$$

NOTE: The above analysis remains valid for all the rheological models where the parameters are frequency dependent. The equivalent Voigtian parameters, for such cases, could be defined at the frequency of excitation from equation 6.2.

Response to Harmonic Excitation of Single Degree of Freedom Systems with Frequency Independent Parameters

The response of a linear single degree of freedom system with viscous damping is found from the following differential equation of motion:

$$M\ddot{X} + C\dot{X} + KX = F_o e^{i\omega t} \qquad \ldots 27$$

(i) Steady Response - Forced Vibration

$$X = \bar{X}e^{i\omega t}$$

$$(K-M\omega^2)\bar{X} + iC\omega\bar{X} = F_o \qquad \ldots 28$$

or

$$\bar{X} = \frac{\frac{F_o}{K}}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \; e^{-i\phi} \qquad \ldots 29$$

$$X = \frac{\frac{F_o}{K}}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \; e^{i(\omega t-\phi)} \qquad \ldots 30$$

assuming $F_o e^{i\omega t}$ to represent $F_o \mathrm{Sin}\omega t$, equation 30 can now be written as

$$X = \frac{X_{St}}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \; \mathrm{Sin}(\omega t-\phi) \qquad \ldots 31$$

where

$$\partial = \frac{\omega}{\omega_n} \qquad \ldots 32$$

$$\omega_n = \sqrt{\frac{K}{M}} \qquad \ldots 34$$

$$\xi = \frac{C}{2\sqrt{KM}} \qquad \ldots 34$$

and

$$\tan\phi = \frac{2\partial\xi}{1-\partial^2} \qquad \ldots 35$$

(ii) Transient Response - Free Vibration

The transient response of the system is found by adding two particular solutions of the form e^{dt} which is found to satisfy the homogeneous equation of motion:

$$M\ddot{X} + C\dot{X} + KX = 0 \qquad \ldots 36$$

$$(Md^2 + Cd + K)X = 0 \qquad \ldots 37$$

X = 0 trivial solution

$$\therefore \quad Md^2 + Cd + K = 0 \qquad \ldots 38$$

or

$$d^2 + \frac{C}{M} d + \frac{K}{M} = 0$$

but from equations 33 and 34

$$\frac{K}{M} = \omega_n^2 \text{ and } \frac{C}{M} = 2\xi\omega_n$$

$$d = -\xi\omega_1 \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2} = -\xi\omega_n \pm i\omega_n\sqrt{1-\xi^2} \qquad \ldots 39$$

$$\text{take } \omega_d = \omega_n\sqrt{1-\xi^2} \qquad \ldots 40$$

$$X(t) = Ae^{(-\xi\omega_n+i\omega_d)t} + Be^{(-\xi\omega_n-i\omega_d)t} \qquad \ldots 41$$

$$X(t) = e^{-\xi\omega_n t}(Ae^{i\omega_d t} + Be^{-i\omega_d t})$$

or

$$X(t) = De^{-\xi\omega_n t}\,\text{Sin}\,(\omega_d t - \alpha) \qquad \ldots 42$$

where the values of D and α are determined from the initial conditions. Equation 42 represents an exponentially decaying harmonic motion.

(iii) The overall response

The overall response is determined as the sum of the steady and transient responses given in equations 31 and 42.

$$X(t) = \frac{X_{St}}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \text{Sin}(\omega t-\phi) + De^{-\xi\omega_n t} \text{Sin}(\omega_d t-\alpha) \quad \ldots 43$$

Definitions of Damping Capacity for an S.D.F.System with Frequency Independent Parameters

Damping Ratio ξ

It is a non-dimensional measure of damping within the system defined as the ratio of the viscous coefficient to that of the critical and is given in equation 34:

$$\xi = \frac{C}{2\sqrt{KM}} = \frac{C}{C_c} \quad \ldots 44$$

or

$$\xi = \frac{C}{2M\omega_n}.$$

If $C = C_c = 2\sqrt{KM}$ the system is said to be critically damped; from equation 40 ω_d for $\xi = 1$ is zero, i.e. no oscillatory motion is possible under transient conditions.

Band-width $\Delta\omega$

If n is the amplitude ratio obtained under a constant elevel of input at ω and ω_n, it is deduced from equation 31 that there are two values of ω which would satisfy this condition:

$$\omega^2 = \omega_n^2 \, [(1-2\xi^2) \pm \sqrt{(1-2\xi^2)^2 - (1 - \frac{4\xi^2}{n^2})}] \quad \ldots 45$$

or

$$\omega^2 = \omega_R^2 \left[1 \pm \frac{2\xi\sqrt{\xi^2 + (\frac{1}{n^2} - 1)}}{1 - 2\xi^2} \right] \qquad \ldots 46$$

where

$\omega_R^2 = \omega_n^2 \, (1-2\xi^2)$ is found by maximizing for x in equation 31.

Practical application of the above equation is realised only when $\xi \to o$ and $n \to 1$, i.e. for systems with a small amount of damping and at frequencies close to ω_n where $\Delta\omega$ at either side of ω_n is equal and proportional to ξ:

$$\omega \simeq \omega_R \, (1 \pm \xi \, \sqrt{\frac{1}{n^2} - 1})$$

or

$$\Delta\omega \simeq \omega_R . \xi . \sqrt{\frac{1}{n^2} - 1} \qquad \ldots 47$$

Band-widths are frequently given at -3db points, i.e. at frequencies where $n = \frac{\sqrt{2}}{2}$ taken relative to the amplitude at ω_R (note that for $\xi \to o$ $\omega_R = \omega_n$):

$$\Delta\omega \simeq \xi . \omega_R$$

or

$$\xi \simeq \frac{\Delta\omega}{\omega_R} = \frac{\omega_2 - \omega_1}{2\omega_R} \qquad \ldots 48$$

Logarithmic Decrement δ

It is a non-dimensional measure of damping defined as the natural logarithm of the ratio of the displacement levels of two successive cycles of the oscillatory decay curve; or to express the average for n cycles,

$$\delta = \frac{1}{n} \operatorname{Ln} \frac{X_1}{X_n} \qquad \ldots 49$$

δ is found from equations 42 and 40 :

$$\delta = \frac{1}{n} \operatorname{Ln} \frac{X_1}{X_n} = \frac{2\pi}{n} \cdot \frac{\xi}{\sqrt{1-\xi^2}} \approx \frac{2\pi}{n} \xi \qquad \ldots 50$$

Specific Damping Capacity ψ

It is defined as the ratio of the energy dissipated to the maximum available (potential and kinetic) energy per cycle. It can be shown that the latter energy equals to the maximum potential energy stored in one cycle, i.e. from equation 19:

$$V_o = \frac{1}{2} KX_o^2$$

but from equation 15

$$D_o = - \pi C\omega X_o^2$$

$$\therefore \quad \psi = \frac{-D_o}{V_o} = 2\pi \frac{C\omega}{K} \qquad \ldots 51$$

or alternatively from equation 17

$$\psi = \frac{-D_o}{V_o} = \frac{\pi F_o X_o \operatorname{Sin}\phi}{\frac{1}{2} KX_o^2} = 2\pi \frac{F_o}{KX_o} \operatorname{Sin}\phi \qquad \ldots 52$$

where ϕ is the phase angle between $\overline{F}$ and $\overline{X}$. ...53

It is important to recognize that equations 51 and 52 found for S.D.F. systems are identical to those found for the Voigt unit, i.e. equations 20 and 21 only F and ϕ are now being measured at the mass block. Equations 51 and 52 attain a particular significance at the natural frequency of the system where they also represent the damping capacity of the ensemble:

$$\psi_n = 2\pi \frac{C\omega_n}{K} = 4\pi\xi = 4\pi \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \simeq 2\delta \qquad \ldots 54$$

It is also interesting to note that at the natural frequency of the system, the sum of the kinetic and potential energies does not vary with time. This is a direct consequence of the balance which is struck at this frequency between the elastic and the inertial forces (see Fig. 12a).

The practical implication of the above statement is that it is possible to determine the effective values of C, K and M at the natural frequency of the system as follows (the energy method):

(1) from decay curves the log-degrament is found using equation 50:

$$\delta = \frac{1}{n} \operatorname{Ln} \frac{X_1}{X_2}$$

but from equation 54

$$\psi_n = 4\pi \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$$

which gives ψ_n

(2) from equation 15

$$\sim -D_o = \pi F_o X_o \qquad (\operatorname{Sin}\phi = 1)$$

(3) but

$$\psi_n = \frac{-D_o}{V_o} \qquad \therefore \quad V_o = \frac{-D_o}{\psi_n}$$

(4) From equation 19

$$V_o = \frac{1}{2} KX_o^2$$

$$\therefore \quad K = \frac{2V_o}{X_o^2} \qquad ...55$$

(5) From equation 54

$$\psi_n = 2\pi \frac{C\omega_n}{K}$$

$$\therefore \quad C = \frac{\psi_n . K}{2\pi\omega_n} \qquad ...56$$

(6) From equation 33

$$K = M\omega_n^2$$

$$\therefore \quad M = \frac{K}{\omega_n^2} \qquad ...57$$

NOTE:

In measurement of frequency dependent parameters of 'complex springs' if the ideal massless condition can not be simulated; then valid measurements can only be made at the natural frequency of the system. That is to say the free vibration and/or the energy method should be employed. Table 7 was constructed for the purpose of reference and shows the relationships amongst different measures of damping at the natural frequency of a single degree of freedom system.

Amplification Factor A

It is defined as the ratio of the dynamic to the static deflection which is also a measure of damping but only at the natural frequency of the system:

From equation 29

$$\bar{A} = \frac{\bar{X}}{X_{St}} = \frac{1}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \quad e^{-i\phi} \qquad ...58$$

<u>Quality factor Q</u>

It is $|\bar{A}|$ only At the natural frequency of the system, i.e. when $\partial = 1$:

$$Q = |A_n| = \frac{1}{\sqrt{4\xi^2}} = \frac{1}{2\xi} = \frac{1}{\zeta} \qquad ...59$$

<u>Transmissibility T</u>

It is defined as the ratio of the transmitted load to the foundation through the 'complex spring', to the load applied to the mass. It can be shown easily that this ratio is the same as that of the amplitude of the mass to the amplitude of foundation when the vibration is transmitted to the system through the foundation.

$$\bar{F}_t = (K + iC\omega)\bar{X} \qquad ...60$$

F_t is the transmitted load to the foundation and from equation 29:

$$\bar{X} = \frac{\bar{F}/K}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \quad e^{-i\phi}$$

$$\therefore \quad F_t = \frac{\frac{\bar{F}}{K}\cdot K(1 + \frac{C^2\omega^2}{K^2})^{\frac{1}{2}}}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \quad e^{i\alpha}\cdot e^{-i\phi} \qquad ...61$$

$$\alpha = \tan^{-1}\frac{C\omega}{K} = 2\xi\partial \qquad ...62$$

or

$$\bar{T} = \frac{\bar{F}_t}{\bar{F}} = \frac{\sqrt{1+4\partial^2\xi^2}}{\sqrt{(1-\partial^2)^2+4\xi^2\partial^2}} \; e^{i(\alpha-\phi)} \qquad \ldots 63$$

At the natural frequency of the system i.e. when $\partial = 1$, T is a measure of damping:

$$|\bar{T}| = \left|\frac{\bar{F}_t}{\bar{F}}\right| = \sqrt{\frac{1+4\xi^2}{4\xi^2}} = \sqrt{\frac{1+\zeta^2}{\zeta^2}} \simeq \frac{1}{\zeta} \qquad \ldots 64$$

APPENDIX 2

THE STATIC AND DYNAMIC RESPONSE OF THE SYSTEM

(i) Static Loading

The distribution of load within the specimen and the springs, together with the associated deflections for the ideal and the real system, are derived after referring to the respective models shown in Figs. 24a and 24b.

The Ideal System	The Real System	
$K_{11} + K_{12} = K_1, \; K_{21} = K_{22} = K_2$ and	$K_{11} = K_{12} = K_1, \; K_{21} = K_2, \; K_{22} = 0$ and	
$R_{12} = \frac{K_1}{K_2}, \; R_{J2} = \frac{K_J}{K_2}$	$R_{12} = \frac{K_1}{K_2}, \; R_{J2} = \frac{K_J}{K_2}$	

The effective stiffness at the platform K_e is found as:

The Ideal System	The Real System	
$K_e = [\frac{R_{J2}(R_{12}+2)+(R_{12}+1)}{2R_{J2}(1+R_{12})+(R_{12}+1)^2}] K_1$	$K_e = [\frac{R_{J2}(R_{12}+1)+R_{12}}{R_{J2}(2R_{12}+1)+R_{12}(R_{12}+1)}] K_1$	... 1

and

The Ideal System	The Real System	
$\frac{F_J}{F_{21}} = \frac{R_{J2}(R_{12}+1)}{R_{J2}+(R_{12}+1)}$	$\frac{F_J}{F_{21}} = \frac{R_{J2} \cdot R_{12}}{R_{J2}+R_{12}}$	... 2
$F_J = F_{12} + F_{22}$	$F_J = F_{12}$	... 3

the static loading efficiency E is

The Ideal System	The Real System	
$E = \frac{F_J}{F_{11}} = \frac{R_{J2}(R_{12}+1)}{R_{J2}(R_{12}+2)+(R_{12}+1)}$	$E = \frac{F_J}{F_{11}} = \frac{R_{J2} \cdot R_{12}}{R_{J2}(R_{12}+1)+R_{12}}$	... 4

For $R_{J2} \rightarrow \infty$ and $R_{12} = R$

The Ideal System	The Real System	
$K_e = \frac{R+2}{2(R+1)} \cdot K_1$	$K_e = \frac{R+1}{2R+1} \cdot K_1$	... 5
$\frac{F_J}{F_{21}} = R+1$	$\frac{F_J}{F_{21}} = R$	... 6
$E = \frac{F_J}{F_{11}} = \frac{R+1}{R+2}$	$E = \frac{F_J}{F_{11}} = \frac{R}{R+1}$	... 7

and therefore

The Ideal System	The Real System	
$x_{11} = \frac{F_{11}}{K_e} = \frac{F_{11}}{K_1} \frac{2(R+1)}{R+2}$	$x_{11} = \frac{F_{11}}{K_e} = \frac{F_{11}}{K_1} (\frac{2R+1}{R+1})$	... 8
$x_1 = \frac{F_{21}}{K_2} = \frac{F_J}{K_2(R+1)} = \frac{EF_{11}}{K_2(1+R)} = \frac{F_{11}}{K_1} \frac{R}{2R+1}$	$x_1 = \frac{F_{21}}{K_2} = \frac{F_J}{RK_2} = \frac{EF_{11}}{RK_2} = \frac{F_{11}}{K_1} \cdot \frac{R}{R+1}$	... 9
$x_2 = \frac{F_{12}}{K_1} = \frac{F_J}{K_1} = \frac{EF_{11}}{K_1} = \frac{F_{11}}{K_1} \frac{R+1}{R+2}$	$x_2 = \frac{F_{12}}{K_1} = \frac{F_J}{K_1} = \frac{EF_{11}}{K_1} = \frac{F_{11}}{K_1} \frac{R}{R+1}$	... 10

For R_{J2} and $R_{12} \rightarrow \infty$ and $R_{12} = R$

The Ideal System	The Real System	
$K_e = \frac{K_1}{2}$	$K_e = \frac{K_1}{2}$	... 11
$\frac{F_J}{F_{21}} = R$	$\frac{F_J}{F_{21}} = R$	... 12
$E = \frac{F_J}{F_{11}} = 1$	$E = \frac{F_J}{F_{11}} = 1$	... 13

(ii) Dynamic Loading

The load amplification and transmissibility are determined by making the following assumptions:

a - The system is completely symmetric (i.e. ideal)

b - The parameters are frequency independent, and

c - The excitation is harmonic.

The following derivations are made from the equivalent single degree of freedom system at the second mode shown in Fig. 15d.

Load Amplification

Load amplification is defined here as the ratio of the load produced on the specimen F to the load applied to the inertia blocks by the shakers F_1.

$$\bar{F} = (2K + i2C\omega)\bar{X}_1 \qquad \ldots 14$$

but from equation 29 of Appendix 1

$$\bar{X}_1 = \frac{F_1}{(2K+K_1)\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} \, e^{-i\phi_2} \qquad \ldots 15$$

(Assuming $\bar{F}_1 = F_1$, real)

$$\bar{F} = \frac{2K(1+i\,\frac{2C\omega}{2K}).\,F_1}{(2K+K_1)\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} \, e^{-i\phi_2} \qquad \ldots 16$$

$$\frac{\bar{F}}{F_1} = \frac{2K}{2K+K_1} \, \frac{\sqrt{1+\zeta^2}}{\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} \, e^{i(\phi-\phi_2)} \qquad \ldots 17$$

or

$$\left|\frac{\bar{F}}{F_1}\right| = R_K \frac{\sqrt{1+(R_\zeta\zeta_2)^2}}{\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} \quad \ldots 18$$

where R_K and R_ζ are the correction factors due to the auxiliary parameters. (see Chapter 13.3.1).

for $2K \gg K_1$ and $2C \gg C_1$, R_K and $R_\zeta \to 1$

equation 18 is reduced to:

$$\left|\frac{\bar{F}}{F_1}\right| = \frac{\sqrt{1+4\partial_2^2\xi_2^2}}{\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} = |\bar{T}_2|$$

(see Equation 63, Appendix 1) ... 19

Variation of $\left|\frac{\bar{F}}{F_1}\right|$ with frequency and for different values of ξ_2 can be seen in Fig. 14b.

Transmissibility

Transmissibility is defined here as the ratio of the load transmitted to the foundation (or the platform - Part No.6) F_t to the load applied to the inertia blocks by the shakers F_1.

$$\bar{F}_t = (K_1 + iC_1\omega)\bar{X}_1 \quad \ldots 20$$

or

$$\bar{F}_t = \frac{K_1(1+i\frac{\omega C_1}{K_1}) \cdot F_1}{(2K+K_1)\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} e^{-i\phi_2} \quad \ldots 21$$

$$\frac{\bar{F}_t}{F_1} = \frac{K_1}{2K+K_1} \cdot \frac{\sqrt{1+\zeta_1^2}}{\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} e^{i(\phi_1-\phi_2)} \quad \ldots 22$$

But $\frac{K_1}{2K+K_1} = (\frac{\omega_1}{\omega_2})^2 = (1-R_K)$ and $\zeta_1 = 2\xi_1\partial_1$

$$\therefore \quad \left|\frac{\bar{F}_t}{F_1}\right| = (1-R_K) \frac{\sqrt{1+4\partial_1^2\xi_1^2}}{\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}} \qquad \ldots 23$$

for $\xi_1 \to 0$

$$\left|\frac{\bar{F}_t}{F_1}\right| \simeq (\frac{\omega_1}{\omega_2})^2 \frac{1}{\sqrt{(1-\partial_2^2)^2+4\partial_2^2\xi_2^2}}$$

$$\simeq (\frac{\omega_1}{\omega_2})^2 \left|\bar{A}_2\right| \text{ (see Eqn.58 Appendix 1)}$$

... 24

and for ξ_1 & $\xi_2 \to 0$

from equations 19 and 24 it follows that:

$$\left|\frac{\bar{F}_t}{F_1}\right| \simeq (\frac{\omega_1}{\omega_2})^2 \left|\frac{\bar{F}}{F_1}\right| \qquad \ldots 25$$

or

$$\left|\bar{F}_t\right| \simeq (\frac{\omega_1}{\omega_2})^2 \left|\bar{F}\right| \simeq (1-R_K)\left|\bar{F}\right| \qquad \ldots 26$$

$$\left|\bar{F}_t\right| \simeq \frac{K_1}{2K} \left|\bar{F}\right| \qquad \ldots 27$$

APPENDIX 3

A STUDY ON VARIATION OF MODAL SHAPES AND NATURAL FREQUENCIES OF A TWO DEGREES OF FREEDOM SYSTEM

The following style of computation, in dealing with the eigenvalue problem, was adopted in order to illustrate (a) the effect of stiffness and mass distribution upon the modal shapes and (b) the effect of stiffness and mass quantities upon the natural frequencies.

The solutions are derived for all combinations of values assigned to the parameters of the system shown in Fig.15a. A sample print out is given below. The levels of the auxiliary parameters K_1, M_1, M_2 and K_2 are reflected from Chapter 12.1 but in Imperial units. The programme together with the input data is given in the next page. Note that S, A, B and C denote K, a, b and c of Chapter 5.3.2 respectively.

K	K1	K2	M1	M2	A	B	C	A2	B2	NF1	NF2
0.100E 01	7075.00	6870.00	265.75	244.50	1.005	.707E 04	.687E 04	169.87810	-0.00592	16.6	16.8
0.100E 01	7075.00	6070.00	265.75	244.50	1.005	.707E 04	.607E 04	975.96520	-0.00103	15.6	16.8
0.100E 02	7075.00	6870.00	265.75	244.50	1.005	.708E 03	.687E 03	17.04160	-0.05868	16.6	16.8
0.100E 02	7075.00	6070.00	265.75	244.50	1.005	.708E 03	.607E 03	97.60195	-0.01052	15.6	16.8
0.100E 03	7075.00	6870.00	265.75	244.50	1.005	.707E 02	.687E 02	2.15917	-0.66551	16.6	17.0
0.100E 03	7075.00	6070.00	265.75	244.50	1.005	.707E 02	.607E 02	9.83676	-0.10218	15.7	16.9
0.100E 04	7075.00	6870.00	265.75	244.50	1.005	.708E 01	.687E 01	1.08831	-0.92555	16.7	18.9
0.100E 04	7075.00	6070.00	265.75	244.50	1.005	.706E 01	.607E 01	1.59788	-0.62905	16.1	18.6
0.100E 05	7075.00	6870.00	265.75	244.50	1.005	.707E 00	.687E 00	1.00851	-0.99665	16.7	32.8
0.100E 05	7075.00	6070.00	265.75	244.50	1.005	.707E 00	.607E 00	1.04976	-0.95767	16.2	32.6
0.100E 06	7075.00	6870.00	265.75	244.50	1.005	.707E-01	.687E-01	1.00085	-1.00426	16.7	90.9
0.100E 06	7075.00	6070.00	265.75	244.50	1.005	.707E-01	.607E-01	1.00487	-1.00026	16.2	90.8
0.100E 07	7075.00	6870.00	265.75	244.50	1.005	.707E-02	.687E-02	1.00008	-1.00505	16.7	282.9
0.100E 07	7075.00	6070.00	265.75	244.50	1.005	.707E-02	.607E-02	1.00049	-1.00462	16.2	282.9
0.500E 07	7075.00	6870.00	265.75	244.50	1.005	.141E-02	.137E-02	1.00002	-1.00510	16.7	631.8
0.500E 07	7075.00	6070.00	265.75	244.50	1.005	.141E-02	.121E-02	1.00010	-1.00501	16.2	631.8
0.100E 08	7075.00	6870.00	265.75	244.50	1.005	.707E-03	.687E-03	1.00001	-1.00510	16.7	893.3
0.100E 08	7075.00	6070.00	265.75	244.50	1.005	.707E-03	.607E-03	1.00005	-1.00506	16.2	893.3
0.100E 09	7075.00	6870.00	265.75	244.50	1.005	.707E-04	.687E-04	1.00000	-1.00511	16.7	2824.5
0.100E 09	7075.00	6070.00	265.75	244.50	1.005	.707E-04	.607E-04	1.00000	-1.00511	16.2	2824.5
0.100E 01	6253.00	6870.00	265.75	244.50	1.005	.625E 04	.687E 04	0.00156	-652.15030	15.8	16.6
0.100E 01	6253.00	6070.00	265.75	244.50	1.005	.625E 04	.607E 04	151.96878	-0.00661	15.6	15.8
0.100E 02	6253.00	6870.00	265.75	244.50	1.005	.625E 03	.687E 03	0.01561	-65.23279	15.8	16.6
0.100E 02	6253.00	6070.00	265.75	244.50	1.005	.625E 03	.607E 03	15.25769	-0.06588	15.6	15.8
0.100E 03	6253.00	6870.00	265.75	244.50	1.005	.625E 02	.687E 02	0.15056	-6.67688	15.9	16.7
0.100E 03	6253.00	6070.00	265.75	244.50	1.005	.625E 02	.607E 02	2.01370	-0.49916	15.6	16.0
0.100E 04	6253.00	6870.00	265.75	244.50	1.005	.625E 01	.687E 01	0.72442	-1.38365	15.1	18.5
0.100E 04	6253.00	6070.00	265.75	244.50	1.005	.625E 01	.607E 01	1.07867	-0.95181	15.7	18.0
0.100E 05	6253.00	6870.00	265.75	244.50	1.005	.625E 00	.687E 00	0.96801	-1.03833	16.2	32.6
0.100E 05	6253.00	6070.00	265.75	244.50	1.005	.625E 00	.607E 00	1.00761	-0.99752	15.7	32.3
0.100E 06	6253.00	6870.00	265.75	244.50	1.005	.625E-01	.687E-01	0.99675	-1.00639	16.2	90.8
0.100E 06	6253.00	6070.00	265.75	244.50	1.005	.625E-01	.607E-01	1.00076	-1.00633	15.7	90.7
0.100E 07	6253.00	6870.00	265.75	244.50	1.005	.625E-02	.687E-02	0.99967	-1.00546	16.2	282.9
0.100E 07	6253.00	6070.00	265.75	244.50	1.005	.625E-02	.607E-02	1.00008	-1.00504	15.7	282.9
0.500E 07	6253.00	6870.00	265.75	244.50	1.005	.125E-02	.137E-02	0.99993	-1.00518	16.2	631.8
0.500E 07	6253.00	6070.00	265.75	244.50	1.005	.125E-02	.121E-02	1.00002	-1.00510	15.7	631.8
0.100E 08	6253.00	6870.00	265.75	244.50	1.005	.625E-03	.687E-03	0.99997	-1.00515	16.2	893.3
0.100E 08	6253.00	6070.00	265.75	244.50	1.005	.625E-03	.607E-03	1.00001	-1.00510	15.7	893.3
0.100E 09	6253.00	6870.00	265.75	244.50	1.005	.625E-04	.687E-04	1.00000	-1.00512	16.2	2824.5
0.100E 09	6253.00	6070.00	265.75	244.50	1.005	.625E-04	.607E-04	1.00000	-1.00511	15.7	2824.5

```
C     NORMALIZED EIGENFUNCTIONS AND EIGENVALUES FOR A 2DFS.
C     SEE CHAPTER 5.3.2
      DIMENSION S(500),A(100),B(2500),C(2500)
      REAL K1(100),K2(100),M1(100),M2(100)
      READ(1,81) IX,IY,IZ,IM,IN
 81   FORMAT(I0)
      IK=IM*IN
      IL=IX*IY
      IJ=IX*IZ
      CALL SUBREZA(IX,IY,IZ,IM,IN,IK,IL,IJ,S,M1,M2,A,B,C,K1,K2)
      STOP
      END
      SUBROUTINE SUBREZA(IX,IY,IZ,IM,IN,IK,IL,IJ,S,M1,M2,A,B,C,K1,K2)
      DIMENSION S(IX),A(IK),B(IL),C(IJ)
      REAL K1(IY),K2(IZ),M1(IM),M2(IN)
      READ(1,11) (S(I),I=1,IX),(K1(I),I=1,IY),(K2(I),I=1,IZ),(M1(I),I
     1=1,IM),(M2(I),I=1,IN)
 11   FORMAT(F0.0)
      L=0
      DO14 JM=1,IM
      DO14 JN=1,IN
      L=L+1
      A(L)=M1(JM)/M2(JN)
 14   CONTINUE
      M=0
      DO 22 J=1,IY
      DO 22 I=1,IX
      M=M+1
      B(M)=K1(J)/S(I)
 22   CONTINUE
      N=0
      DO 23 I=1,IX
      DO 23 K=1,IZ
      N=N+1
      C(N)=K2(K)/S(I)
 23   CONTINUE
      WRITE(2,51)
 51   FORMAT(///,7X,1HK,        8X,2HK1,7X,2HK2,6X,2HM1,5X,2HM2,5X,1HA,
     16X,1HB,8X,1HC,12X,2HA2,9X,2HB2,9X,3HNF1,6X,3HNF2)
      DO 24 L=1,IK
      JM=(L-1)/IN+1
      JN=L-(L/IN)*IN
      IF(JN.EQ.0)JN=IN
      DO24 M=1,IL
      KX=(M-1)/IX
      KY=KX*IX
      KZ=M-KY
      DO25 N=(IZ*(KZ-1)+1),(IZ*KZ)
      A2=0.5*((1.+B(M))-A(L)*(1.+C(N))+SQRT((A(L)*(1.+C(N))-(1.+B(M)))
     1**2+4.0*A(L)))
      B2=-A(L)/A2
      J=KY/IX+1
      KZZ=N-(N/IZ)*IZ
      IF(KZZ.EQ.0) KZZ=IZ
      PIE=3.141592
      C1=K1(J)+A2*A2*K2(KZZ)+S(KZ)*(1-A2)*(1-A2)
      C2=K1(J)+B2*B2*K2(KZZ)+S(KZ)*(1-B2)*(1-B2)
      A11=(M1(JM)+M2(JN)*A2*A2)/386.
      A22=(M1(JM)+M2(JN)*B2*B2)/386.
      FN1=(SQRT(C1/A11))/(2*PIE)
      FN2=(SQRT(C2/A22))/(2*PIE)
 25   WRITE(2,26)S(KZ),K1(J),K2(KZZ),M1(JM),M2(JN),A(L),B(M),C(N),A2,B2
     1,FN1,FN2
 26   FORMAT    (5X,E9.3,1X,F8.2,1X,F8.2,1X,F6.2,1X,F6.2,1X,F5.3,1X,
     1E8.3,1X,F8.5,4X,F9.5,2X,F10.5,3X,F6.1,2X,F6.1)
 24   CONTINUE
      RETURN
      END
      FINISH
10
2
2
1
1
1.
10.
100.
1000.
10000.
100000.
1000000.
5000000.
10000000.
100000000.
7075.
6253.
6870.
6070.
245.75
244.5
```

FINISHED SPECIMENS						STANDARD BAR			LOAD WASHER KISTLER
NAME	AREA in^2	AREA cm^2	ASPECT RATIO	I.D. mm.	O.D. mm.	I.D.	O.D.	MATERIAL	TYPE NO.
R90	0.90	5.8	$33x10^{-3}$	34.0	42.7	32.0	45.0	451*	906B
B90	0.90	5.8	$99x10^{-3}$	16.8	32.0	16.0	32.0	325*	903A
S90	0.90	5.8	$318x10^{3}$	0.0	27.2	0.0	25.4	E_n5A	903A

TABLE 4. Specimens for study of 'aspect ratio'

FINISHED SPECIMENS						STANDARD BAR			LOAD WASHER KISTLER
NAME	AREA in^2	AREA cm^2	ASPECT RATIO	I.D. mm	O.D. mm	I.D. mm	O.D. mm	MATERIAL	TYPE NO.
A45	0.45	2.9	.099	11.9	22.6	0.0	25.4	E_n5A	903A
B90	0.90	5.8	.099	16.8	32.0	16.0	32.0	325*	903A
C180	1.80	11.6	.100	23.4	45.0	20.0	45.0	455*	906B
E270	2.70	17.4	.099	29.1	55.4	28.0	56.0	565*	906B

TABLE 5. Specimens for study of 'area'.*Desford hollow bar TI52.

	CLA [µm] \ Specimen	A	BS	B	C,R	E
SPEED R.P.M.	1.52	580	580	399	291	291
	4.57	580	580	399	291	291
	9.65	580	580	399	291	291
FEED µm/Rev.	1.52	60	60	47	40	32
	4.57	240	240	210	147	120
	9.65	550	550	640	550	550

TABLE 6. The cutting conditions.

To obtain / Multiply	C	h	ζ	ξ	$\frac{\omega_d}{\omega_n}\delta$ *	ψ
C	BY 1	$\frac{K}{M}$	$\frac{1}{\sqrt{KM}}$	$\frac{1}{2\sqrt{KM}}$	$\frac{\pi}{\sqrt{KM}}$	$\frac{2\pi}{\sqrt{KM}}$
h	$\frac{M}{K}$	1	$\frac{1}{K}$	$\frac{1}{2K}$	$\frac{\pi}{K}$	$\frac{2\pi}{K}$
ζ	$\sqrt{KM}$	K	1	$\frac{1}{2}$	π	2π
ξ	$2\sqrt{KM}$	$2K$	2	1	2π	4π
$\frac{\omega_d}{\omega_n}\delta$	$\frac{\sqrt{KM}}{\pi}$	$\frac{K}{\pi}$	$\frac{1}{\pi}$	$\frac{1}{2\pi}$	1	2
ψ	$\frac{\sqrt{KM}}{2\pi}$	$\frac{K}{2\pi}$	$\frac{1}{2\pi}$	$\frac{1}{4\pi}$	$\frac{1}{2}$	1

TABLE 7. Relationships amongst different measures of damping at the natural frequency of an S.D.F. system.

* $\omega_d = \omega_n \sqrt{1 - \xi^2}$ and $\omega_n = \sqrt{\frac{K}{M}}$

Length of specimen (zero preload)	$81.3<\ell<317.5$ [mm]	:	$3.2<\ell<12.5$ [in.]
Length of specimen (max. preload)	$124.5<\ell<274.3$ [mm]	:	$4.9<\ell<10.8$ [in.]
Maximum diameter of specimen	116.8 [mm]	:	4.6 [in.]

* * * * * *

Maximum preloading capacity	5 [ton.f.metric]	:	4.92 [ton.f.,U.K.]
Maximum pump pressure	5.360 [MPa]	:	777 [PSI]
Maximum deflection of lower block	43.2 [mm]	:	1.7 [in.]
Average rate of preloading - Isolating spring	127.5 [KP/mm]	:	7138 [lbf/in.]

* * * * * *

Effective mass of inertia blocks 118.5 [kg] : 261 [lb.]

Effective stiffness of aux.system $K_1 = 121 + 1.114\,X_2$ [KP/mm], X_2 in [mm.]

Effective loss-factor of aux.system. $\zeta_1 = 0.046 + 0.001\,X_2$, X_2 in [mm.]

Average first natural frequency. 16.3 [Hz]

* * * * * *

Correction Factors due to auxiliary parameters.

$$R_K = \frac{\partial^2 - 1}{\partial^2} \qquad R_\zeta = \frac{\partial^2}{\partial^2 - 1}\left(1 - \frac{\zeta_1}{\partial \zeta_2}\right) \quad \text{where } \partial = \frac{\omega_2}{\omega_1}$$

* * * * * *

Maximum load capacity of shaker system 22.7 [KP] : 50 [lbf]

Maximum load amplification at ω_2

$$F = \frac{1}{\zeta_2} \cdot R_K \sqrt{1 + R_\zeta^2 \zeta_2^2} \quad \left(\simeq \frac{1}{\zeta_2}\right)$$

* * * * * *

Overall size = 1,676 x 1,118 x 737 [mm] : 5'6" x 3'8" x 2'5"

* * * * * *

Overall weight ≃ 2500 [lb] : 1134 [Kg]

TABLE 9 - SPECIFICATION OF THE RIG

Source	S.S.	DF	VAR.Estimate	F	Significance
Frequency	0.00244	1	0.00244	3.047	NS
Static Load	0.00399	3	0.00133	1.661	NS
Interaction	0.00483	3	0.00161	2.010	NS
Error	0.01281	16	0.00080		
Total	0.02407	23	0.00105	SD = $(.00105)^{\frac{1}{2}}$=0.032[rad.]	

TABLE 10 - PHASE ERROR

Source	S.S.	DF	VAR.Estimate	F	Significance
Frequency	357.3	1	357.3	34.3	< 0.1%
Static Load	199.5	3	66.5	6.4	< 1%
Interaction	6.8	3	2.3	0.2	NS
Error	166.8	16	10.4	SD = $(10.4)^{\frac{1}{2}}$= 3.2[%]	
Total	730.4	23	31.7		

TABLE 11 - GAIN ERROR

Summary tables for two factor analysis of variance with three-fold replication.

The significance of the effect of:

FREQUENCY and

STATIC LOAD

upon the accuracy (and precision) of the 'm' method of measurement when using Kistler 906B/5001 load washer and charge amplifier.

Source	S.S.	DF	VAR.Estimate	F	Significance
Frequency	0.00044	1	0.00044	1.537	NS
Static Load	0.00071	3	0.00024	0.829	NS
Interaction	0.00005	3	0.00002	0.058	NS
Error	0.0046	16	0.00029		
Total	0.00581	23	0.00025	SD = $(.00025)^{\frac{1}{2}}$= 0.016[rad.]	

TABLE 12 - PHASE ERROR

Source	S.S.	DF	VAR.Estimate	F	Significance
Frequency	334.5	1	334.5	12.4	< 1%
Static Load	101.4	3	33.8	1.2	NS
Interaction	10.9	3	3.6	0.1	NS
Error	431.7	16	26.9	SD = $(26.9)^{\frac{1}{2}}$= 5.2 [%]	
Total	878.5	23	38.2		

TABLE 13 - GAIN ERROR

Summary tables for two factor analysis of variance with three-fold replication.

The significance of the effect of:

FREQUENCY and

STATIC LOAD

upon the accuracy (and precision) of the 'm'method of measurement when using Kistler 903A/5001 load washer and charge amplifier.

Source	S.S.	DF	VAR.Estimate	F	Significance
A = Area	0.935	1	0.935	716.86	<0.1%
B = Pressure	0.602	3	0.209	154.03	<0.1%
C = Viscosity	0.110	1	0.110	84.11	<0.1%
D = Roughness	7.150	2	3.375	2741.84	<0.1%
AB	0.335	3	0.112	85.63	<0.1%
AC	0.015	1	0.015	11.35	< 1.0%
AD	1.190	2	0.595	456.25	<0.1%
BC	0.088	3	0.029	22.52	<0.1%
BD	0.469	6	0.078	59.98	<0.1%
CD	0.087	2	0.043	33.27	<0.1%
ABC	0.042	3	0.014	10.81	<0.1%
ABD	0.771	6	0.128	98.52	<0.1%
ACD	0.004	2	0.002	1.57	NS
BCD	0.142	6	0.024	18.12	<0.1%
ABCD	0.034	6	0.006	4.32	< 1.0%
Error	0.1252	96	0.0013	$SE = (\frac{.0013}{3})^{\frac{1}{2}}$	= 0.021
Total	12.0993	143	0.08461		

TABLE 14 - LOSS FACTOR PER UNIT AREA OF JOINT

Summary table for four factor analysis of variance on the results of the three methods of measurement(considered as 'replicates'). The precision of the methodsof measurement and the standard deviation of the mean error.

Source	S.S.	DF	VAR.Estimate	F	Significance
A	257,218	1	257,218	20.37	< 0.1%
B	56,473,900	3	18,824,600	1490.78	< 0.1%
C	110,224	1	110,224	8.73	< 1.0%
D	14,918.000	2	7,458,980	590.70	< 0.1%
AB	57,694	3	19,321	1.53	NS
AC	21,073	1	21,073	1.67	NS
AD	18,822	2	9,411	0.74	NS
BC	38,319	3	12,773	1.01	NS
BD	11,064,700	6	1,844,120	146.04	< 0.1%
CD	90,298	2	45,148	3.57	< 5%
ABC	147,751	3	49,250	3.90	< 2.5%
ABD	334,511	6	55,751	4.41	< 0.1%
ACD	29,546	2	14,773	1.17	NS
BCD	56,335	6	9,389	0.744	NS
ABCD	151,621	6	25,270	2.001	NS
Error	1,212,230	96	12,627	$SE = \left(\frac{12627}{3}\right)^{\frac{1}{2}} = 64.9$	[MN/m/cm^2]
Total	84,982,500	143	594,284		

TABLE 15 - INPHASE STIFFNESS PER UNIT AREA OF JOINT

Summary table for four factor analysis of variance on the results of the three methods of measurement (considered as 'replicates').

The precision of the methods of measurement and the standard deviation of the mean error.

Source	S.S.	DF	VAR.Estimate	F	Significance
A	0.737	1	0.737	56.9	<0.1%
B	1.176	3	0.392	30.3	<0.1%
C	0.187	1	0.187	14.4	<0.1%
D	5.803	2	2.902	224.0	<0.1%
AB	0.241	3	0.080	6.2	< 1.0%
AC	0.002	1	0.002	0.1	NS
AD	0.088	2	0.044	33.8	<0.1%
BC	0.056	3	0.019	1.4	NS
BD	0.953	6	0.159	12.3	<0.1%
CD	0.239	2	0.119	9.2	<0.1%
ABC	0.022	3	0.007	0.6	NS
ABD	0.457	6	0.076	5.9	<0.1%
ACD	0.004	2	0.002	0.1	NS
BCD	0.065	6	0.011	0.8	NS
ABCD	0.027	6	0.004	0.3	NS
Error	0.6218	48	0.01295		
Total	11.4689	95	0.12073		

TABLE 16

Summary table for four factor analysis of variance with two-fold replication.

The significance of the effects of:

A = AREA
B = PRESSURE
C = VISCOSITY OF LUBRICANT, and
D = ROUGHNESS

upon the loss-factor per unit area of turned surfaces.

Source	S.S.	DF	VAR.Estimate	F	Significance
A	3.466	1	3.466	99.3	<0.1%
B	1.710	3	0.570	16.3	<0.1%
C	0.027	1	0.027	0.8	NS
D	10.581	2	5.290	151.6	<0.1%
AB	0.559	3	0.186	5.3	< 1.0%
AC	0.005	1	0.005	0.1	NS
AD	2.983	2	1.492	42.7	<0.1%
BC	0.098	3	0.032	0.9	NS
BD	1.148	6	0.191	5.5	<0.1%
CD	0.092	2	0.046	1.3	NS
ABC	0.055	3	0.018	0.5	NS
ABD	0.045	6	0.074	2.1	NS
ACD	0.077	2	0.038	1.1	NS
BCD	0.066	6	0.011	0.3	NS
ABCD	0.048	6	0.008	0.2	NS
Error	1.6752	48	0.0349		
Total	23.0368	95	0.242		

TABLE 17

Summary table for four factor analysis of variance with two-fold replication.
The significance of the effects of:

A = AREA
B = PRESSURE
C = VISCOSITY OF LUBRICANT and
D = ROUGHNESS

upon the stiffness-factor per unit area of turned surfaces.