The penetration resistance

of sands

by

Richard Gisbourne

Volume I

A thesis submitted to the University of Aston
in Birmingham for the degree of Doctor of Philosophy

December 1970

TEXT CLOSE TO THE EDGE OF THE PAGE IN THE ORIGINAL THESIS

- SYNOPSIS -

(Volumes I and II)

Preliminary experiments establish a criterion for the design of future apparatus and reveal behavioural patterns in sand which are explained in terms of compressibility and energy components.

Existing skin friction theories are found to be misleading but a comparison of experimental-theoretical penetration resistance results show good agreement at penetrations above full failure 'bulb' depths.

A simple sand pourer uses the 'rain' technique to prepare homogeneous sand samples. Displaced air is partly extracted during sand deposition with a variable suction and small sampling containers measure isolated porosities. The apparatus is found to give a high degree of homogeniety and reproducibility.

Miniature three-dimensional earth pressure cells are designed and manufactured, and calibrated over a wide range of stress systems. Results show consistent under-registration; matrices of correction factors are produced by analysing calibration results with a digital computer and the factors are adopted to convert electrical outputs into applied stresses.

Movement created by penetration into homogeneous sand masses are recorded in a 'half-section' apparatus. Incremental displacements are measured with an autographic plotter and strain-rate tensors are evaluated using a simple computer programme.

Existing equations of limiting equilibrium for plane strain conditions are expanded, using the Haar and Kármán hypothesis, to allow for axial-symmetry. Analysis of the resulting finite difference equations is carried out by digital computer. Boundary conditions are assumed and complete limiting stress fields are computed with soil parameters which represent the states of loose and dense sand.

Experimental results show the validity of the Haar and Kármán hypothesis and provide limited evidence from stress measurements in dense sand to support the Mohr-Coulomb failure criterion. Strain-rates illustrate the fallacy of a basic boundary condition assumption and provide the parameters which allow a complete determination of stress tensors.

Finally a different failure criterion is selected and strain-rate vectors are determined from the associated plastic flow rule.

To my wife, Diana, and children

Mark, Joanna and Sarah.

PREFACE

This work represents one of the earlier research investigations undertaken in the soil mechanics section of the Department of Civil Engineering at the University of Aston in Birmingham.

A general research programme was established in Autumn 1966 with a view to studying the behaviour of soil under conditions of loading analogous to those in the field; the Author's contribution to the overall project has been based primarily on a study of the behaviour of dry sand under one particular axially-symmetric condition of loading. The project was born out of a need for a radical approach to axially-symmetric problems in soil mechanics : problems always making their presence felt but having received very little direct attention in recent years.

Throughout the three year period of experimental and analytical work at Aston the Author was fortunate enough to have been sponsored by the Science Research Council as a full-time research student.

The research work was carried out without any form of collaboration. There are, however, many who deserve the Author's thanks for their invaluable help and encouragement while he was working in the Department of Civil Engineering.

The Author owes his deepest debt of gratitude to his supervisor, D. H. Bennett, B.Sc., C.Eng., M.I.C.E., M.I.E.(Aust)., F.G.S., for it is he who has directed the soil mechanics research work at Aston since its early days in 1966. His advice and criticism during this research has always been received gratefully and he has guided the Author well, not only as a supervisor but also as an enlightened Engineer. After working with D. H. Bennett for a few weeks it was realised that he expected high standards and hard work; the Author believes this thesis serves to illustrate the value of this simple philosophy.

The Author would like to express his thanks to Professor M. Holmes, Ph.D., B.Sc., C.Eng., F.I.C.E., F.I.Struct.E., F.I.Mun.E., the Head of the Department of Civil Engineering, for his friendly advice and timely encouragement, also K. Starzewski, PhD., B.Sc., C.Eng., M.I.C.E. for his constructive criticism and helpful suggestions.

The Author also wishes to thank his research student colleagues, particularly S.M. Dyson, Ph.D., BSc., A.M.I.C.E. and H.W. Sinclair-Jones, Ph.D., B.Sc., A.M.I.C.E., for providing the stimulating environment in which it was always a great pleasure to live and work.

The workshops and laboratories in the Department of Civil Engineering were under the close supervision of the chief technician, W. Parsons, and it is thanks to him that the Author was able to carry out his experimental work in ever-changing conditions. Despite an inefficacious air about his office, Mr. Parsons always managed to arrange the manufacture of apparatus and the delivery of equipment when it was most convenient.

Much of the more tedious laboratory work was carried out in the soil mechanics laboratory and the Author expresses his gratitude to M. J. Lyons, the soils laboratory technician, for his unfaltering assistance; when extreme patience was required he was particularly willing to co-operate.

Thanks are due to many other laboratory technicians, but particularly A. Garrett, R. Roberts and H. Woodward, for the excellent apparatus made in the departmental workshops. At times it was necessary for the Author to manufacture apparatus in the workshop and the guidance given by the above technical staff proved to be most helpful. It was during this stage of the work that the Author came to appreciate the value of good working drawings and the importance of designing apparatus which could be easily and accurately made.

The later chapters of this thesis were written while the Author was employed as a Soils Engineer by Geotechnical Engineering Limited and he would like to thank his employers and colleagues at Gloucester for their continual encouragement and assistance during the last year. The Author is also very grateful for the opportunity to reproduce the thesis on the Company's printing machine; this has resulted in consistent printing throughout the two volumes.

Finally the Author wishes to thank Miss A. Heaton for typing Volume I and Miss P. Sage, who was responsible for completing the figures in Volume II.

CONTENTS. VOLUME I.

C H A P T E R 1

Introduction

1. Introduction

The present investigation into "the penetration resistance of sand" was undertaken because it was felt that present-day in-situ penetration tests called for a comprehensive study which would help to explain what the tests are actually measuring. As far as the Author is aware such a study has never been completed.

The knowledge of overall behaviour during penetration into granular media has been rather obscure ever since in-situ penetration tests became a tool of the Soils Engineer some three or four decades ago. The present investigation, it was hoped, would establish fundamental behaviour patterns from which, firstly, suggestions could be made to improve the interpretation aspects of existing in-situ penetration tests and, secondly, further studies could proceed along the lines which showed most promise.

Throughout the following thesis it will be noticed that effort has, in particular, been directed towards obtaining a concise, but overall, picture of the behaviour of a granular mass during penetration. The type of penetration tool has, generally, been of secondary importance; the philosophy behind the project has been based on the objective of measuring the resistance of the sand, not necessarily the effects of a specific tool on the sand.

Nonetheless no investigation which is connected with the penetration resistance of sand would be complete without a brief review of the current techniques used in site investigations, hence such a review forms the next part of this chapter.

An appreciation of the advantages and disadvantages of in-situ penetration tests is also considered to be a prerequisite to understanding the reasons for carrying out this study, thus these are listed later. The list is by no means complete but it serves to illustrate many opportunities for improvement in the crude science (but not the art) of in-situ penetration testing used at present.

The two most widely accepted methods of in-situ penetration tests in today's field of soil mechanics and foundation engineering are:

i) The deep sounding (static) test, sometimes known as the cone penetrometer test, and

ii) The standard (dynamic) penetration test (SPT).

For reasons which will become obvious in ensuing paragraphs the static test has been adopted more in the lower European countries and the SPT has received a wider usage in North America, Gt. Britain and the Southern Continents.

The introduction of the deep sounding test seems to have been attributed to Barentsen (1936) in the early 1930's. His apparatus in its basic form consisted of a cone-shaped 60° point with a base area of 10 sq.cm., attached to thin connecting rods. An outer casing was used and the cone and casing were pushed alternately by hand into soft soils at an average rate of penetration, measuring cone resistance and shaft friction (and most probably end resistance of the casing) at ground surface as the resistance to pushing.

Progressive development of the apparatus in the Netherlands, firstly by the Delft Laboratory (1936) and later Vermeiden (1948) lead to the measurement of cone and sleeve resistance in sandy soils with the avoidance of end resistance of the casing (or sleeve). Plantema (1948), working along similar lines, also modified earlier penetrometers to measure point resistance and side friction. These apparatuses were capable of measuring much higher resistances than the hand sounding device and were normally anchored into surrounding ground by means of small screw-anchors to provide a suitable reaction.

Continuous research produced the friction cone (Begemann; 1953 and 1965) and more recently the cone penetrometer has been used with electrically excited stress transducers fitted into the cone base. One of the most up to date deep sounding tests was seen in operation by the Author in 1968; the penetrometer was being used by N.V. Fugro and had been connected into a 10 ton 6 wheel drive truck which normally provided the reaction to cone resistance. An automatic recording technique had been perfected and penetration resistance (cone and shaft resistance) was recorded as a continuous graph against depth.

Although development of the deep sounding apparatus had made rapid advances, the essential interpretation of results remained almost indefinite until several studies, more notably those of Platema (1948), Huizinga (1951), van der Veen (1953 and 1957) and van Weele (1957 and 1961), were published; these related the penetration resistance of the cone and the bearing characteristics of the soils. Because deep cone soundings are ideally suited to deep layers of relatively homogeneous loose or soft soils the empirical relationships were generally applied to the prediction of the ultimate bearing capacity of driven piles, this type of foundation being most appropriate in deep layers of loose soil.

The work of de Beer (1963) was perhaps the most significant theoretical contribution relating the characteristics of a granular soil (in this case a fine sand) to the results of deep sounding tests. The approach was unique in two respects: a hypothetical relationship was developed which predicted the mechanical properties of the sand from the cone resistance and: dimensional similitude (i.e. the scale effect) for a cone penetrometer and a full scale pile was investigated.

The studies which are referenced above (de Beer's being the exception) have related deep sounding test results and the ultimate bearing capacity of foundations. Since the static penetration test is considered to measure the ultimate strength of the soil the approach is entirely logical. The approach in the dynamic penetration test (SPT) is, however, somewhat different, as will be seen below.

In a section of their book 'Soil mechanics in engineering practice' which described methods of soil exploration, Terzaghi and Peck (1948) suggested, almost as though it were in passing, a 'simple method of obtaining at least some information concerning the degree of compactness of the soil in-situ'. The test which they went on to describe has become known universally in soil mechanics as the standard penetration test (SPT).

The apparatus consists of a 2in. outside diameter, 1.3/8in. internal diameter split sampling spoon which is attached to drill rods and lowered to the base of a borehole (normally cased). The test involves driving the split spoon 18in. below the base of the borehole using a 140 lb. drop weight, free falling 30in. The number of blows required to drive the spoon from 6in. to 18in. below the borehole base is taken as the SPT number, N.

Terzaghi and Peck (1948) suggested an empirical relationship between the SPT number and the relative density of sands. Because of the limitations of the test it was advised that for important jobs the SPT results should be supplemented by deep soundings. However, the SPT/relative density correlation was extended by Peck, Hanson and Thornburn (1953); the SPT number was related to the in-situ angle of internal shearing resistance (Ø) for cohesionless soils. The relationship between N, relative density, and Ø is given below:

N	Relative density	Ø°
0 - 4	Very loose	Under 30
4 - 10	Loose	30 - 35
10 - 30	Medium	35 - 40
30 - 50	Dense	40 - 45
Over 50	Very dense	Over 45

The SPT has been the subject of considerable research since its introduction, and the amount of literature on the test cannot be condensed here. Noteworthy contributions to a more effective interpretation of the SPT have, however, been provided by Gibbs and Holtz (1957), Palmer and Stuart (1957), Schultze and Mensenbach (1961) and Thornburn (1963).

Unlike the static cone penetration test very little development of the SPT has been undertaken since its introduction, but one modification has been suggested by Palmer and Stuart (1957), this being to use a conical-ended penetration tool in gravels. The number N for the cone penetrometer was found to be very similar to the SPT values in gravels and has been adopted quite generally in site investigation practices where gravels are encountered. In addition Palmer and Stuart recommended a modified test in submerged sands where upward movement of sand into a borehole was thought to be a likely occurrence; the modification meant driving the split spoon 30in. instead of 18in. and recording the number of blows over the last 12in. (i.e. from 18in. to 30in.).

Correlation between the static cone penetration test and the standard penetration test has been a topic for many studies, not least those of Trow (1952), Meyerhof (1956), Schultze and Knausenberger (1957), Rodin (1961) and Martins and Furtado (1963). In very general terms the above researchers found that an approximate relationship existed between the two tests, but different soil strata have indicated very different factors of proportionality.

Investigations which have attempted to correlate either the SPT or cone test with other soils tests and classifications are again quite numerous but the types of test and findings are not relevant to this particular study.

Having described the two most widely used in-situ penetration tests it remains to list the advantages and disadvantages of each method. However, it is first necessary to argue why the in-situ penetration tests are used at all.

In cohesionless materials it is difficult, and expensive, to recover 'undisturbed' samples. Even where 'undisturbed' samples are recovered the loosest of these are virtually impossible to test in the laboratory without creating disturbance during sample preparation. It is well known that disturbance to loose samples gives increased densities thus any laboratory results would indicate higher strengths than those in-situ; interpretation of laboratory tests would, therefore, give results on the 'unsafe' side.

In view of these problems it is not difficult to realise why in-situ penetration tests have been readily adopted in site investigation methods. Although the results are not fully understood and are used in an empirical manner, the in-situ penetration tests offer a more satisfactory means of assessing engineering characteristics of granular soils than do the complex sampling and laboratory testing techniques that are currently available. There are many other limitations in in-situ tests but these will be outlined by discussing the advantages and disadvantages of in-site penetration methods. Firstly, for the static penetration test the advantages are:

i) the cone penetrometer produces a continuous penetration resistance v. depth curve.

ii) separate recordings are obtained for point resistance and skin friction

iii) the static penetration can be carried out without trial boreholes

iv) the complete test is relatively cheap - between £20 and £40 per test to a depth of roughly 60 ft.

v) the penetration resistance is recorded as the stress on either pressure gauges or an autographic plotter; the need for an operator technique is partly eliminated.

Disadvantages of the static cone penetration test are as follows:

i) the test is practicable only where soils are loose, soft and fine grained, hence its suitability in the lower European countries where thicknesses of deltaic sands are to be found.

ii) the static penetrometer test does not recover a representative sample and boreholes must be sunk if samples are required for identification.

iii) the stresses along the sleeve and on the base of the penetrometer are measured using parts which move separately (Chapter 2.4.(iv)).

iv) the equipment is expensive and is easily damaged if obstructions are encountered. In extreme circumstances the penetrometer has been lost during a test.

v) a considerable reaction is required in soils other than loose-medium sands and firm clays.

Advantages which are normally associated with the standard penetration test are listed below:

i) in the original aplit spoon form the SPT is suitable for measuring N in sands and silts; in modified form the SPT can be used successfully in gravels.

ii) a representative sample is recovered in the split spoon during each SPT.

iii) the test is cheap, quick, simple and uses robust equipment which is attached to conventional drill rods.

iv) the split spoon (or Stuart and Palmer cone) is a single unit with no separately moving parts, therefore no redistribution of stresses occurs around the base during penetration.

v) the SPT can be carried out during a trial borehole investigation but need only be used when undisturbed samples cannot be recovered.

vi) the SPT relies upon the driving energy imparted by a drop hammer, thus no reaction to the penetration resistance is required at ground level.

Finally, disadvantages of the SPT are given below:

i) the dynamic in-situ penetration test is rather crude and lends itself to operational errors, particularly the variation in the free falling height of the drop hammer.

ii) the results of standard penetration tests are, contrary to the recommendations of Terzaghi and Peck (1948), not usually supplemented with additional results.

iii) SPT's do not produce a continuous record of sub-site conditions, in consequence thin looser layers (which are generally the less favourable conditions) may not be tested.

Several arguments have been used against in-situ penetration tests, the main one being that the results can only be interpreted empirically. Since in certain soils there is no reasonable alternative to in-situ penetration tests, the empirical correlation has, of necessity, been the subject of much research. Static and dynamic penetration tests can now be used fairly effectively to predict soil behaviour under load.

A second serious criticism levelled at the in-situ penetration test is that the parameters which the test measures have not been clearly defined. This of course is the basic disadvantage of the in-situ test and is one reason why the present investigation has been undertaken.

The amount of useful existing apparatus which was available to carry out the present investigation was rather limited and a number of new items of essential equipment were designed and developed by the Author. Since some of the new apparatus was to be designed from results obtained in initial experiments, carried out with existing apparatus, it was necessary to phase operations carefully over a period of about two years. The programme of experimental work was, therefore, separated into four phases, to allow development and use of existing equipment to run concurrently with the design and manufacture of new apparatus. The four phases of the programme are given in Figure 1.1.

The first phase covered the work in which existing apparatus was used, in conjunction with some new parts which were quickly designed and manufactured, to carry out preliminary experiments. The following two chapters, 2 and 3, give an account of these experiments, together with the modifications and calibration which formed an essential part of the experiments.

Chapter 2 describes the results of the preliminary investigations and includes a general resumé of previous work along similar lines. Also in Chapter 2 are listed the standard classifications for the soil used in the experimental studies and the laboratory tests which were thought to define the mechanical properties of this soil (a clean white fine quartzitic sand).

Chapter 3 describes the development and performance of the pouring apparatus, which was used to prepare homogeneous samples of the sand. Work on the pourer, in fact, constituted the larger part of the phase I. operations, taking approximately six months to modify and calibrate completely.

Phase 2 of the investigation programme includes the design, manufacture, calibration and performance of an earth pressure cell; the first part of Chapter 4 describes these stages. The manufacturing aspects of the cell were considered to be irrelevant to a proper understanding of the principle of the cell and its performance and has thus been described only very briefly.

As Figure 1.1. shows, phase 2 was carried out concurrently with phases 1 and 3. It is not possible to say exactly how long the pressure cell work took to complete, but the separate stages extended over a period of 18 months.

The third phase of the experimental programme was devoted to work on a 'half-section' container and the associated apparatus which was required. The latter sections of Chapter 4 describe the so called 'half-section' container and explain the reasoning behind its use; this is done by referring to previous research of an analogous nature.

The measurement of displacements, for which the apparatus was designed, is described in the final part of Chapter 4.

Chapter 5 describes the basic theory behind the prediction of stress fields in a cohesionless soil during quasi-static penetration. The overall condition is described by using cylindrical polar co-ordinates with axially-symmetric penetration.

The chapter can be read independently of other chapters and only in one instance is reference made to the experimental work. This was necessary because the assumed hypothesis did not specify which of the three principle stresses was intermediate in the regions of sand known as transition zones; it was, therefore, predicted from the results of stress measurement experiments.

Final computations, carried out using a digital computer, are represented by lines called limiting stress characteristics. Stress distributions along the sloping part of the straight (vertical) sided, conical ended penetrometer are obtained for various conditions along the vertical face; the distributions are plotted as the theoretical penetration resistance of sand.

Chapter 6 is the results chapter; it includes results from pressure cell and 'half-section' experiments, many of which are expanded to illustrate the behaviour of sand during penetration; the findings do not appear to have been realised before. In the final sections of Chapter 6 the experimental and theoretical results are compared and further behaviour patterns are revealed.

Chapter 7, the final chapter, summarises the results given in the preceding chapters and suggests lines along which future studies could proceed.

C H A P T E R 2

Preliminary investigations

2.1. Introduction

2.2. Fine white sand

2.3. Boundary effects

2.4. Penetration tests

2.5. Conclusions

2.1. Introduction.

This chapter lists the physical and mechanical properties of the material used in experimental studies and describes the experiments that form an introduction to the later studies of the penetration resistance of sands.

Earlier research workers in the field of soil mechanics were quite content to describe the materials they used in terms such as 'fine sand' and 'stiff clay' but with the rapid development of soil mechanics as an engineering science it became necessary to distinguish between different types of soil by using classifications that were readily applicable to a particular material and widely understood by researchers. For example, the various phases that soils can assume (in relation to air, water and solid) can be appreciated by using index properties that describe these phases. Specific gravity, moisture content and porosity are convenient index properties that relate the three components for a single grained structure such as sand; these are given in Section 2.2. The index properties are as precisely defined as present day techniques allow, and generally these are sufficient for a complete categorization of a soil within a wide range of soil types encountered in soil mechanics research.

If it is pointed out here that index properties of a particular soil (used for research purposes) not only serve to identify the soil for future reference but provide the basic properties that strength of soil is related to, then their importance is adequately emphasized. The methods of relating index and mechanical properties vary widely in different research establishments but it is this relation that forms the fundamental framework from which present day soil mechanics hangs. The techniques for determining the mechanical properties of soil also vary widely but an attempt to define such properties is a necessary pre-requisite for investigations such as those discussed in later sections of this chapter.

The limitations of existing soil testing equipment have long been recognised and in recent years equipment produced specifically for research studies has evolved (see for example Roscoe, 1953; Kirkpatrick, 1957) and if such equipment is available it is possible to obtain a more extensive picture of the soil behaviour under load. In the absence of sophisticated apparatus, existing apparatus must be used and results interpreted most effectively; the apparatus available at the start of the present series of experiments consisted of direct shear and triaxial test equipment (although work was in progress on the development of a unique true triaxial apparatus (Dyson, 1970)) and conventional direct shear tests were carried out on dry sand to give a range of values of the strength parameter ϕ for a range of initial porosities and confining pressures. The coefficient of friction (tan δ) between sand particles and brass, and glass were also determined using the direct shear box with sand occupying the upper section only. The discussion on direct shear tests (Section 2.2.) includes some interesting findings that do not appear to have been previously stated.

In laboratory experiments, where it is physically impossible to perform full scale experiments, the size of sample used must be related to the size of instrument used, for example, in penetration tests and model pile studies. It seems surprising that a large number of model investigations have been performed with inadequate apparatus; the Author considered the boundary effect investigations, described in Section 2.3. to be a useful exercise using simple, easily made apparatus. The limiting ratios of container to penetrometer diameter for restrictions below and around an instrument during advancement are discussed with reference to previous theory and experimental studies.

Section 2.4. describes experiments that were primarily designed to give separate results for total penetration resistance and the resistance of the advancing point of a penetrometer.

Much of the discussion readily falls back on the experiences, results and conclusions of other research workers and in this respect serves a second purpose, that of introducing relevant past research and comparing, where possible, the results with those obtained by the Author.

The discussion has deliberately avoided using conclusive evidence from later studies but has attempted to diverge from many existing lines of thought on the behaviour of sand during penetration. Results have been explained in a simple physical way; at this stage lack of results from more elaborate experiments demands this approach.

Substantial evidence to support the tentative suggestions and conclusions of this Section will be produced at a later stage.

2.2. Fine White Sand.

2.2.(i) Physical Properties.

Three tons of graded washed sand, referred to as fine white sand, was originally obtained from a local builders merchant (T.J. Graham and Co., Ltd.) The origin of the sand was unknown but it looked remarkably like a Leighton Buzzard sand that had been used in previous investigations in the Soil Mechanics Laboratory. The sand, when delivered, had an average moisture content of 17.8% but before being stored in hoppers it was passed through a drying machine to remove all moisture.

Sieve analyses (B.S. 1377: 1967) on the sand particles initially gave Curve 1 of Figure 2.1. but particles retained on No. 25 B.S. Sieve and passing No. 200 B.S. Sieve were removed by mechanical sieving before the experimental program was commenced. Sieve analyses were carried out throughout the penetration experiments in an attempt to detect changes in particle size caused by experimentation; there were no noticable changes in the distribution Curve of Figure 2.1. at any time. It must be emphasised that similar (or apparently identical) P.S.D. curves do not imply identical mechanical and physical properties of the sand but by initially ordering several times the amount of sand required, the rate of change of grain shape was at least controlled. The majority of experiments were done in a large materials testing laboratory and although care was taken to prevent excess contamination, the dusty atmosphere probably contributed more to changes in particle size than the transportation and usage of sand in experiments.

Maximum and minimum porosities of the fine white sand were determined using methods developed by the Author. Tests based on the findings of Kolbuszewski (1948) were suggested by Akroyd (1957), but the results obtained from these tests did not appear to represent the limiting porosities. Akroyd's minimum porosity test entailed compacting layers of sand in a Proctor mould with a Kango hammer; this was tried but the compaction effort seemed too severe. A sieve analysis of the sand after compaction indicated grain fracture and Figure 2.1. Curve 2 adequately indicates this phenomenon. The Author used a Proctor mould and compacted the sand with a mechanically operated drop-weight device. The principle of the apparatus was similar to that of the hammer used in the standard compaction test, but the compaction effort was distributed over a disc slightly less than 4" in diameter. Tests were done with a varying number of compaction drops on dry sand until the density became constant (at about 50 blows per sample) and after each test a P.S.D. analysis was done to detect crushing. No obvious grain crushing was discovered and the results were assumed to represent a minimum porosity.

Calibration experiments with small sampling containers (described in Chapter 3) gave a porosity of 47.16% but Akroyd's method for maximum porosity gave 45.95%. The porosity in the small containers was obtained by fixing a funnel $\frac{1}{4}$" above the upper perimeter and pouring sand into the funnel to form a cone in the container. A similar method using the Proctor mould and a funnel gave a comparable porosity (47.06%) and was obviously more realistic than that determined using Akroyd's method. Results of the limiting porosities are given in Table 2.1., together with the results from the recommended porosity tests given by Akroyd.

It is well known that certain sand dunes deposited by natural elements have significantly higher porosities than any maximum porosity that can be obtained in a Soil Mechanics Laboratory. The so-called maximum and minimum porosities, therefore, are useful only in that they are empirical index properties of the sand to which a particular state of packing can be related. The relation, known as relative porosity is comparable with the liquid and plastic limits of clays, and serves as an indication of the simpler mechanical (strength) properties of the sand.

Individual grains of sand were sub-rounded and appeared to consist mainly of quartz. Chemical analyses would have provided more information from the identification standpoint, but, this being the only purpose, the costs of such analyses were not really justified in the case of sands.

The roundness (Krumbein, 1941) and sphericity (Rittenhouse, 1943) were 0.40 and 0.81 respectively.

The specific gravity of sand grains was 2.649 (two tests gave 2.6488 and 2.6499).

Plate 2.1. shows a x20 magnification of the fine white sand.

2.2.(ii) Mechanical Properties.

A number of direct shear tests were carried out on the fine white sand described above. The upper section of the box was displaced relative to the lower part at 0.048 in./min. and shear stress - displacement curves for loose and dense sand specimens are shown in Figure 2.2.(a).

In the interpretation of results of shear tests on a fine molsand, de Beer (1965) related maximum shear stress (τmax.) and normal stress (σ_n) in any individual test by introducing the 'secant' angle ϕ'. The values of ϕ' computed from results of the present series of direct shear tests are shown against initial porosity of a specimen for constant normal stresses of between 2.3. and 17.3 p.s.i. (Figure 2.3.) The interesting (although not uncommon) feature of these results is the considerable increase of ϕ' with decrease of σ_n'. The type of experiment considered in this research program was characterized by relatively low stresses and it was decided to shear additional sand specimens at normal stresses of 0.16 p.s.i. Resulting values of ϕ' (59.9° for n = 45.0% and 65.9° for n = 38.8%) were very much higher than those obtained by de Beer using a molsand and interpolating results to obtain $\sigma' = 0$ (de Beer, 1965; Figure 3). The high 'secant' angle was explained by assuming that part of the weight of the upper half box was transferred to the 6 cm. square of shearing sand as additional normal load when the two parts were separated prior to shearing. At very low stresses the higher normal load reduced values of ϕ' to give Curve 6, Figure 2.3. Obviously at higher normal stresses additional normal load became less significant. Maximum shear stress versus normal stress for loose and dense sands are seen, in Figure 2.4., to possess a linear relationship with no decrease in ϕ for increasing σ'. The apex of the Mohr-Coulomb rupture envelopes in this figure intersected the σ' axis at -1.0 p.s.i. but imperfections in the apparatus were unlikely to give errors of this order and the curves were assumed to be correct; this gave a cohesion of 0.8 p.s.i. for dense sand and 0.4 p.s.i. for loose sand.

The direct shear tests have provided two parameters ϕ' and Coulomb ϕ, and it remains to select parameters that interpret the mechanical properties of the sand. Existing theories have usually been developed on the basis of the Coulomb ϕ parameter; the resulting penetration resistance or the like generally being independent of confining pressures that are experienced in a particular situation. At this stage, therefore, values of Coulomb ϕ will be assumed to represent the strength of a sand at a particular porosity. For the sake of convenience it is proposed to neglect values of cohesion in the ensuing analyses and treat the fine white sand as a cohesionless material. Values of ϕ are plotted against initial porosity in Figure 2.6. Also shown in Figure 2.6. are the corrected angles of friction introduced by Taylor

(1948) and Newland and Allely (1957).

Taylor's fundamental energy equation gave the frictional and dilation components of ϕ as:-

$$\underset{\text{(frictional)}}{\tan \phi_r} \quad + \quad \underset{\text{(dilation)}}{\tan \theta} \quad = \quad \underset{\text{(total)}}{\tan \phi} \qquad (2.1.)$$

where $\tan \theta = \delta y / \delta x$; δy and δx were respectively increments of vertical and horizontal displacement as obtained from shear results (Figure 2.2.(b)).

Newland and Allely developed the correction ϕ_f and differentiat total frictional energy in the form:-

$$\underset{\text{(total)}}{\phi} \quad - \quad \underset{\text{(dilation)}}{\theta} \quad = \quad \underset{\text{(frictional and remoulding)}}{\phi_f} \qquad (2.2.)$$

As in (2.1.) $\theta = \tan^{-1} \delta y / \delta x$.

The above equations in various modified forms have lead to much discussion related to energy equations, both in the direct shear test and triaxial test (for example Rowe, 1962; Rowe et al, 1964) but the Author has not been able to locate studies where the energy correction have been used in an analysis of general experiments. An attempt is made to correlate experimental results with the energy components in Section 2.4.

Direct shear tests for sand on a brass block produced a ratio for each experiment that will be referred to as δ'; i.e. τ_{max}/σ_n'. Results are plotted against initial porosity, as before, for a range of stresses (Figure 2.5.). Generally speaking it was found that $\delta' \simeq \phi'/2$ for corresponding normal stresses and porosities; this was a conclusion reached by Meyerhof (1950) when comparing δ' from direct shear and ϕ' from triaxial tests. Shear stress versus displacement for sand/brass are presented in Figure 2.2.(c). It can be seen that the peak shear stresses for sand moving on brass occurred at muc smaller displacements than those for direct shearing in sand. Another unusual phenomenon with the sand/brass tests was that peak stresses occurred in both loose and dense samples.

Two specimens in sand/brass direct shear tests were prepared by pouring and tamping respectively and by coincidence the porosities were very similar (43.17% and 42.91%). The τ max. versus σ' curves for the two samples, plotted in Figure 2.7., indicated an increase of about 24% in the angle of friction (δ max) for the compacted sand. The increase of strength with compaction has been discussed (Pellegrino, 1965) and the 'locked-in' stresses are well appreciated (Kolbuszewski and Jones, 1961) but it is not generally realised that compaction plays such a significant role in the strength of sands. The ultimate angle of friction in both poured and tamped specimens was the same (i.e. $\delta = 14.0^{\circ}$).

It has been shown above that the strength (parameters) of a particular sand depend on the porosity and the stress history and it seems feasible to suppose that the penetration resistance of the sand is also dependent upon the stress history. In the experiments described in the following Sections preparation was by the sand rain technique (Kolbuszewski and Jones, 1961) and 'locked-in' stresses were avoided but, in general, the method of sand placement plays a role in the resistance to penetration and cannot be ignored.

2.3. Boundary Effects.

2.3.(i) Test Procedure.

The general arrangement of apparatus for investigating the effects of a rigid boundary surrounding a sand mass is shown in Plate 2.2. The photograph, in fact, shows a penetration test with the brass penetrometer described in Section 2.4. but apart from strain recording equipment the apparatus was identical. Mild steel penetrometers with a 60° cone angle and varying in diameter from ½" to 1½", were pushed into a dense sand mass at a constant rate of 0.0528"/minute. The sand was 21.6" deep, weighed approximately 1080lb. and was contained within a 2'6" square rigid container. Each sand sample was prepared using a sand deposition apparatus described in Chapter 3. Total resistance to penetration was recorded against depth (i.e. recordings were taken every 10 minutes) on a high tensile steel proving ring. Results are summarized in Figure 2.8.

2.3. (ii) Experimental Results.

The effects of a rigid boundary below the penetrometer point became noticeable, except with the ½" diameter penetrometer, when a distinct increase in resistance was obtained as the boundary began to restrict normal distribution of stress during penetration. The depth below the point at which stress increase commenced, taken from Figure 2.8., is plotted against penetrometer diameter in Figure 2.9. The effect of a rigid boundary became noticeable when at a distance of approximately 4½ diameters below the penetrometer point.

The penetrometer resistance per unit area for the different size penetrometers at the same penetration /diameter ratios (6, 12 and 18) are also shown in Figure 2.9. The penetration resistance per unit area has been found to be appreciably constant (de Beer, 1958) provided there is no local grain crushing during penetration. Chaplin (1961) suggested that with major principle stresses less than 100 p.s.i. there was no local grain fracture in compression tests even on softer micaceous materials, and Thomas (1968) noticed that in laboratory deep sounding tests crushing did not occur when cone resistances were less than 130 p.s.i. The maximum stresses experienced in the present series of penetration resistance experiments were in the order of 130 p.s.i. and grain fracture seems an unlikely phenomenon; during investigations no grain crushing was observed. After many of the experiments with sand in a dense state, samples were observed under a x40 magnification but roundness and sphericity remained as originally, and sieve analysis did not reveal any apparent change in particle size. On this evidence grain crushings were ruled out and constant total stresses were then assumed. Figure 2.9. shows that the penetration resistance per unit area was not constant for penetrometer diameters greater than one inch. This implies that boundary effects became significant at container width to penetrometer diameter ratios greater than 30:1.

Experimental work by Yassin (1950) and Berezantzev et. al. (1961) indicated that boundary effects became significant at about 5 diameters below the instrument point. Berezantzev used a flat-ended punch in his studies but found the influence of a rigid boundary to be 6 diameters below its base. Yassin also used different sized containers and concluded that the width of influence was 36 times the diameter of wooden pins in a dense sand.

2.3.(iii) Comparison of Results with Existing Theories for Failure Zones.

In 1948 Jaky introduced a novel hypothesis that predicted the point bearing resistance at relatively large penetrations. He adopted an approach originally used by Terzaghi (1943) for footings on a soil surface and developed statically determined solutions with the limitations of certain kinematic conditions for two-dimensional models of a weightless material.

One of the main conclusions from this interesting conje cture was that point bearing resistance became constant at a certain penetration when a 'failure bulb' became completely developed (Figure 2.10(a)). Bearing in mind this conclusion it is worth referring back to Figure 2.8. Curves 4 and 5 gave results that were affected by rigid vertical boundaries (according to experimental findings) and they can therefore be usefully ignored, but Curve 3 had an apex point where two obviously different phenomenon merged together. If this is assumed to be the point of complete development then the 'bulb' of failure had a depth of approximately 12 diameters. Curves 1 and 2 also show change of slope at penetrations of about 12 times the penetrometer diameter.

Computations based on Jaky's slip surface (a surface of coinciding stress and velocity characteristics, incidentally) were carried out with a Coulomb ϕ value of 40.5° (direct shear tests gave $\phi = 40.5^{\circ}$ for dense sand with $n = 37.3\%$). Resulting plastically deforming regions had an overall width of 16 diameters and a total depth of 28 diameters. The failure regions extended to 2 diameters below the instrument base in the two-dimensional model (Figure 2.10(a)). These values compare with 30, 12 and 4½ obtained in the present three dimensional penetration experiments with a cone ended punch. Figure 2.10(b). serves as a visual comparison of the results of the theoretical and experimental studies.

By considering the weight of the material Meyerhof (1950) developed two separate failure 'bulbs' to include soil above the penetrometer point. The 'bulbs' were combined to produce a final failure surface with approximately the same width as that of Jaky, but with a smaller depth for complete penetration. The shape of Meyerof's regions of assumed plastic deformation were considerably more difficult to predict; they were analogous to the Terzaghi (1943) failure mechanisms for N_q and N_γ and although Meyerhof illustrated the failure 'bulbs' for sands, a simple mathematical solution that predicted the approximate shape is not available. Further discussion of these more complex studies is outside the scope of this text.

At this point it is perhaps relevant to introduce the argument for closely relating the rigid boundary of a container and the failure 'bulbs'. The 'bulb' is a plastically deforming region bounded by a slip surface or velocity discontinuity, and outside this region the material is rigid; the existence of an elastic-plastic material has not been considered and it is assumed that no deformations take place. Any boundary that the rigid sand mass comes into contact with will in no way affect the velocity characteristics within the plastically deforming regions and,provided the rigid boundaries of a sand container lie outside the dimensions of an ascending failure 'bulb' during penetration,the resistance, in theory, is the same as that expected if the boundaries were at infinity.

The difference in shape between Figures 2.10(a). and 2.10(b). becomes evident in the following section (2.4.) where it is shown that plastic deformation and compression contribute to the penetration resistance of the sand; i.e. an interaction of compression and dilation provided the resistance to penetration and a compressed failure 'bulbs' resulted from this interaction.

2.3.(iv) Size of Container.

The minimum width of a dense sand sample and the ratio of depth of influence to penetrometer diameter, as determined by the Author in the above experiments, are shown against penetrometer diameter in Figure 2.11. The findings are compared with dimensions of samples used by a number of researchers for investigating the behaviour of sands. In the case of surface footings (indicated by F in Figure 2.11.) a sand width to footing width ratio of 15 was obtained in medium to dense sands by using the solution of Terzaghi (1943) for shallow foundations. This suggested minimum ratio is also shown in Figure 2.11.

Restrictions on the size of sand samples for laboratory investigations are usually imposed by economic and practical considerations, and the size of any penetration equipment must be selected accordingly. From the wide range of points plotted in Figure 2.11. it appears that in a number of the experiments represented by these points a correct selection was made, and results that were indicative of the true behaviour of a particular sand during either surface loadings or penetrations could be expected.

The practical and economic problems, namely those of transportation of materials, available laboratory space and existing and new equipment, were carefully considered when selecting a suitable instrument for the present experimental program. It was decided to use a 1 in. diameter penetrometer in all experimental investigations; minimum dimensions of the sand sample were fixed at 30 in. square and 30 in. deep. In certain experiments, including those in Section 2.4. some existing apparatus was used in conjunction with the 1 in. penetrometer.

2.4. Penetration Tests - Components of Total Resistance.

2.4.(i) Test Procedure.

A 1" external diameter thick brass tube, having a load cell with a 60^{o} cone fitted as an integral part, was used to investigate the combination of skin friction over the vertical area and point bearing resistance on the conical point of a penetrometer. The design, construction and calibration of the load cell is described in Chapter 4.

The apparatus illustrated in Plate 2.2. consisted of a modified constant rate of strain C.B.R. apparatus (1) driving a 3 ft. long screw threaded shaft (2) with a specially adapted proving ring (3) attached to its lower end. The brass penetrometer was screwed to the lower proving ring pad and passed through a high precision stainless steel ball bushing (4). The penetrometer point and shaft were machined to 1.005" diameter then rubbed down with No. 180 carborundum cloth until the required fit within the bushing was achieved. The penetrometer could fall through the bushing under its own weight but had negligible lateral movement; this ensured a symetric penetration without the need to correct proving ring readings for shaft friction. A deflection gauge (5) was used to periodically check the rate of penetration. The average rate for a complete test was 0.05281"/minute and a range of $\pm$ 1.15 x 10^{-5}in./minute was obtained from the deflection gauge results. 6" by 3" channels supported the loading ring and these were fixed to a rigid container (6) through double corrugated rubber pads that were designed to eliminate vibrations (caused by the electric motor and gear box). A Peekel strain bridge and extension box (7) were used to record load cell measurements at 10 minute intervals during the penetration tests. Experiments were carried out on loose, loose-medium and dense sand samples, these being prepared using the pouring apparatus developed by the Author (see Chapter 3). Figures 2.12, 2.13. and 2.14. present the results as a series of resistance readings against penetration. Total penetration resistance, as recorded by the proving ring, was measured to the nearest 0.09lb. and results produced a smooth curve. On the other hand the load cell had a maximum expected error of $\pm$ 1.5lb. due to the limitations of the measuring equipment and consequently the recordings of end bearing resistance did not give smooth plots where total loads were relatively small (i.e. generally for loose samples at low penetrations). Negative proving ring readings during withdrawel gave the unloading curves in Figures 2.12, 13. and 14. and the skin friction was simply a difference between total resistance and point bearing resistance.

2.4.(ii) Experimental Results.

Figure 2.12. shows that in a loose sand (n = 44.6%) the skin friction accounted for approximately 14% of total penetration resistance during early penetration and increased to 20% at greater depths. The increase in point bearing resistance was virtually linear and 'pulling resistance' during withdrawel was roughly half the skin friction measured during penetration. In loose-medium sand (n = 43.45%) skin friction also increased linearly with depth of penetration (Figure 2.13.) but remained at 20% of total resistance over the range of penetrations considered. The point resistance and skin friction curves gave the same general pattern as those for loose sands and during withdrawal the skin friction was again approximately half that during penetration.

At penetrations of less than 4 diameters the distribution of skin friction in dense sands (n = 36.42%) was rather erratic (Figure 2.14.) but at this penetration it provided 8% of the total penetration resistance. At penetrations of 16 diameters and greater the skin friction represented 30% of total penetration resistance. Point bearing resistance increased with penetration until the penetrometer had reached a depth of 11 diameters, whereupon it remained appreciably constant. The resisting frictional force during withdrawal in the case of dense sands was initially ½ that for penetration but decreased as withdrawal continued and finally equalled the 'pulling resistance' measured in loose sand, a result of some significance in the following paragraphs.

2.4.(iii) Comparison of Results with Previous Studies.

The experiments reported in Section 2.3 (and Figure 2.14.) showed that boundary effects became significant in dense sand when the penetrometer was about 4½ diameters above the rigid boundary, but Figures 2.12. and 2.13. show that in loose and loose-medium specimens of sand the equivalent dimension is greater, being approximately 5½ and 5 for the two initial states of particle packing. Theoretical studies, such as those by Terzaghi (1943), Jaky (1948), Meyerhof (1950) and Berezantzev (1961), postulated that the region of plastic deformation was, in the main, a function of the Coulomb ϕ angle and it seemed quite reasonable, on this basis, to assume that much smaller regions of deformation would occur in loose granular materials. The resulting point bearing resistance curves of Figure 2.12. and 2.13. appear not to agree with the generally accepted modes of deformation but instead suggest compressive deformations below the penetrometer point. It is not proposed to discuss deformations during penetration in detail in this Section (this is done in Chapter 6) but a brief discussion, based on previous research experiments, may help to elucidate the above findings.

One of the many conclusions reached by Kerisel (1961) after performing an extensive series of penetration investigations was that changes in the shape of the stress-free horizontal surface (i.e. initially horizontal) were negligible in dense sands at penetrations greater than $4\frac{1}{4}$ times the penetrometer. It was difficult to assess the precise movement of the upheaving surface in the experiments described in this section but, as will be seen later, it was not radically different from that observed by Kerisel. From Kerisel's findings it follows that deformations around the advancing penetrometer would interact to produce an overall compression equal to the increase in volume of the penetrometer within the sand mass.

If at any stage of penetration below about 4 diameters an increment of penetration is considered, and it is argued that sand adjacent to the shank has sheared sufficiently to reach a critical state (Roscoe et.al., 1958) then deformations within the dense sand due to the incremental penetration can conveniently be separated into two types:

i) Those due to plastically deforming regions, based on modified Prandtl (1920) indentation solutions, and

ii) Compression of sand grains resulting from incremental penetration and corresponding sand displacement.

The existance of a zone of sand around the advancing penetrometer at a different porosity from the surrounding soil was realised by Robinsky and Morrison (1964). Their studies suggested that the zone was less dense than the overall sand mass and did not change in porosity after the base of the penetrometer had advanced beyond the particular zone. The assumption of a critical state, therefore, at present seems entirely acceptable and will go a long way in explaining the anomalies shown in Figures 2.12.and 2.13.

In dense granular media plastic deformations normally give rise to volume increases during shear; this dilation is restricted by surrounding dense sand in penetration tests at depth and higher interparticle stresses occur within and outside the plastic regions, with no overall expansion during shear. Compression (ii) again leads to increased stresses probably increasing grain contacts in accordance with the square root of compression law (Chaplin, 1961). This complex interaction of compression and dilation has been studied by Schultze and Moussa (1961) and Chaplin (1961 and 1965) but is not yet fully understood. Nonetheless it explains the problems of penetration into dense sands sufficiently, in terms of high interparticle stresses, to assume that a rigid mass of sand existed immediately below the penetrometer throughout its advancement. The collection of tightly packed particles, to a certain extent, resembled the rigid container base and it is not difficult to appreciate that this rigidity probably postponed the influence of a completely rigid container boundary. The penetration resistance of dense sand, derived from the extremely rigid layers of sand below the penetrometer, increased unnaturally when the container base and these layers coincided. The depth at which this coincidence occurred was less than the extent of plastic regions below the penetrometer point.

The simple physical model used above for dense sand, whereby all deformations took place below the shank, will suffice to explain compression in loose sand during penetration.

Regions of plastic deformation (i.e. failure 'bulbs') in loose sand extended to approximately $2\frac{1}{2}$ times the penetrometer diameter below its point (assuming $\phi = 32.0^{\circ}$ in Jaky's (1948) failure mechanisms) and are usually associated with constant volume or volume decrease. It is here assumed that there was no volume change during shear,* and as in the case of penetration into dense sand, there was a tendancy for overall compression to occur below the penetrometer point. In dense sand the tendency was translated into higher interparticle stresses and

* In the loose-medium sand, n = 43.49%, corresponding to an estimated final porosity in direct shear tests of 43.90% at equivalent stresses.

resulted in a more rigid sand mass without a great deal of compression. In loose sand, with fewer grain contacts, compression occurred at lower stresses (Chaplin,(1961) gives a detailed account of increase of compressibility with increasing porosity) and extended to depths immediately below the penetrometer that were greater than regions of plastic deformation in loose sand. Hence, although regions of plastic deformation were smaller in loose sands, the overall effect of compressibility was greater than in dense sand, and comparable depths affected by the penetrometer point were about five diameters in each case. The apparent premature increase in penetration resistance, shown in Figures 2.12. and 2.13. was the result not only of plastically deforming regions but vertical compression below the penetrometer point.

Any attempt to compare the results presented in Figures 2.12, 2.13. and 2.14. with previous experimental results must consider (i) the physical and mechanical properties of the material being penetrated and (ii) the techniques of measurement of results and penetration. A comparison of studies by Florentin et.al. (1948), Yassin (1950), Kerisel (1961) and Mohan et.al. (1963) adequately indicates the variation of results for what was assumed to be the same type of experiment. Results obtained by the Author, however, show acceptable agreement with those of Yassin for loose sands and those of Kerisel for dense sands in as much as the shape of load-penetration curves are similar. Further comparison of experimental studies seems irrelevant at this stage.

2.4. (iv) Frictional Component of Resistance - Experimental Results and Theory.

The usual equation for incremental skin friction (ΔSF) is:-

$$\Delta SF = \int_0^D Kf.\tan\delta.\,2\pi r.\,dz.\gamma. \quad (lbs.) \qquad (2.3.)$$

Kf = coefficient of earth pressure at sand-penetrometer boundary.

$\tan\delta$ = coefficient of friction between sand and penetrometer.

r = penetrometer radius (= B/2).

dz = vertical length of penetrometer over which the skin friction acts.

Integrating equation (2.3.) the total skin friction (SF) assumed to act along the full length (D) of the penetrometer becomes:-

$$SF = Kf.\,\tan\delta.\,\gamma D^2.\pi r. \quad (lbs.) \qquad (2.4.)$$

and a parabolic distribution of total skin friction is obtained, if Kf and $\tan\delta$ are assumed constant. From Figures 2.12, 2.13 and 2.14 it is obvious that either average Kf or $\tan\delta$ (or both) decrease in depth to give a linear, and not parabolic increase in skin friction. Substituting known values into equation (2.4.) a parameter Kf.$\tan\delta$ is computed for complete penetration in loose, loose-medium and dense sands (Figure 2.15(a)). In loose and loose-medium sands Kf.$\tan\delta$ decreases with depth to finally become asymtotic at values of 0.27 and 0.24 respectively. In dense sand Kf.$\tan\delta$ is still decreasing at the limiting penetrations of these particular experiments.

The arguement introduced above, that sand adjacent to the shank had sheared sufficiently to reach a critical state, may equally well apply in explaining the shape of Kf.$\tan\delta$ curves in Figure 2.15(a). The plastically deforming regions progress from shallow (Terzaghi, 1943) to deep (Jaky, 1948) mechanisms and during this transition equation (2.4.) has no real meaning, since stress distributions along the shank are associated with plastically deforming sand. Initial values of Kf.$\tan\delta$ greater than 2 ($\tan\delta$ was found from direct shear tests, to be 0.23. for loose sands, giving a value of Kf here of 8.7- unrealistically high for loose sands) for loose sands substantiates this conclusion. Equation 2.4. becomes meaningful along the shank only when a complete failure 'bulb' has developed and advanced below the surface (Figure 2.15(b)). As penetration depth increases the higher forces along the shank associated with plastic deformation have proportionally less effect (i.e. real skin friction increases with depth but these forces remain appreciably constant) and Kf.tanδ tends to a more realistic value. Since the sand adjacent to the penetrometer shank has reached a critical state, skin friction is a true frictional force between sand and brass and does not have a dilation component. The above explanation for decreasing Kf.tanδ will, therefore, hold for dense sand at relatively large penetrations. The contribution to skin friction provided by high stresses in plastic regions becomes less significant with penetration and only at considerable depths (much in excess of those considered in this work) will Kf.tanδ for loose and dense sands become similar. Extrapolation from Figure 2.15(a). seems to suggest this phenomenon. Real skin friction (defined by equation 2.4.) above the Jaky (1948) failure 'bulb' represents shear at critical state with Kf.tanδ for dense sands, equal to Kf. tanδ for loose sands.

Having discussed Curves 1 and 2 and part of 3 (Figure 2.15(a).) in general terms it remains to suggest possible sand deformations to explain Curve 3 at penetrations less than 8 diameters. The initial section of Curve 3 is in no way unique in its shape; Kerisel et.al. (1965) obtained similar curves for penetrometers of different diameters but did not elucidate the associated dense sand behaviour in their publication.

Values of Kf.tanδ for dense and loose sands at penetrations up to 3 diameters show very close agreement. The dense sand for small penetrations was allowed to dilate freely without restrictions. The rate of total work done in the plastically deforming regions, volume δv, in terms of friction and dilation is expressed as:-

$$W = \int (\sigma_{ij} . \dot{\epsilon}_{ij}) \delta v. \qquad (2.5.)$$
$$= \int (\sigma_{ij} . \dot{\epsilon}_{ij}) \text{ friction} . \delta v + \int (\sigma_{ij} . \dot{\epsilon}_{ij}) \text{ dilation} . \delta v.$$

For loose sand specimens the second term in the above equation was assumed to be zero, and work done was dissipated within the plastic regions as friction and remoulding. The two terms in parentheses in the R.H.S. of equation (2.5.) are functions of ϕ f and Θ (Newland and Allely, 1957) respectively and generally for granular materials δv is less for loose packings than dense (i.e. regions of plastic deformation are smaller). Combining values of δv and $(\sigma_{ij} . \dot{\epsilon}_{ij})$ the rates of work done in terms of frictional energy for loose and dense sands are not radically different and stress systems within the regions of plastic deformation would not have been greater in dense sands. Assuming that initially skin friction is dependent on the stress system within plastic regions the values of Kf.tan δ (i.e. skin friction) would have been comparable. Here a worth-while reiteration is that skin friction referred to above is not the true skin friction, given by equation 2.4., but the frictional forces along the penetrometer associated with stresses in the plastic regions that enveloped part of the shank. As restrictions on the expansion of dense sand were applied the stresses within plastic regions, and in particular along the shank, increased to give higher values of Kf. tan δ . This phenomenon happened between 4 and 7 diameters penetration. Values of Kf. tan δ began to decrease with penetration again when the assumed failure 'bulbs' (Figure 2.15(b)) developed and skin friction between penetrometer and sand corresponded to a critical state above the 'bulb' but below the sand surface. As stated before, negative skin friction measured during withdrawal for dense sand rapidly became equal to that for loose and loose-medium sands. This provides evidence to suggest a critical state along the penetrometor shank but at this stage it cannot be considered conclusive. It is interesting here to note that from the results of the withdrawal experiments the value of $Kf.\tan\delta$, computed from equation (2.4.) was found to be 0.13. Assuming a $\delta_{(cv)}$ of 14.0° a value of Kf of 0.52 was obtained. This value was applicable to 'withdrawal' skin friction but compares with the coefficient of active earth pressure (Ka) for a sand with an ultimate ϕ value of 32.2°; the results being respectively 0.52, from above, and 0.55 from theequation:-

$$Ka = \tan^2 (\pi/4 - \phi/2) \qquad (2.6.)$$

Figure 2.15(a) shows values of $Kf.\tan\delta$ computed from the results of penetration tests and the above paragraphs have given logical explanations for their shape. It is becoming more obvious that the interaction of skin friction, as it is known, and the point bearing resistance is a fundamental concept that needs to be carefully considered. in studies of the penetration resistance of sands. It is, therefore, not surprising to find that many researchers have attempted to measure skin friction during penetration tests.

Muller (1939) and Tschebsotariof and Palmer (1948) used basically identical penetrometers to measure skin friction only, but according to Mayerhof (1950) imperfections in apparatus gave erroneous results. Yassin (1950) and Meyerhof (1950) used a unique spring leaf device for measuring skin friction in terms of movement of an outer penetrometer tube. It is the Author's opinion that any attempt to measure skin friction and point resistance using a penetrometer with two or more separately moving parts affects the overall relationship between limiting stresses associated with plastically deforming regions and stresses along the shank. Separation at the joint between shank and point, no matter how small, obviously leads to a redistribution of stress in this region and a study of forces along the shank and over the point becomes a study of the affects of the separation. The penetrometer with a built in load cell, as used by the Author, eliminates separation and any interactions are recorded by the apparatus.

It is of historical interest only to note that from time to time the parameter Kf tan δ has been mathematically formulated. The more notable formulations are those by Benabenq (1911), Dorr (1922), Krey (1936), Kerisel (1939) and Caquot and Kerisel (1948). The values of Kf tan δ were, in all cases, constant with depth since Kf tan δ was a function of Coulomb ϕ and δ peak and these were assumed constant with depth for a given porosity (Figure 2.5.) Krey (1936) assumed that Kf equalled Kp (tan 2 ($\pi/4$ + $\phi/2$) and obtained the lowest values of Kf tan δ ; the values of this parameter were 0.75 for ϕ = 31.6° and δ = 13.1°, and 1.48 for ϕ = 38.0° and δ = 19.4°. The values of Kf in loose and dense sand were then 3¼ and 4¼ respectively whereas from Figure 2.15(a). corresponding results were 1 and 2.6.

2.4.(v) Penetration Resistance - Theoretical Solutions

Prandtl's (1920) and Reissner's (1925) solutions for the indentation of a die into metal provided a basic hypothesis from which many research workers were able to develop advanced theories for the bearing capacity of two-dimensional surface footings on soil. The mathematics, such as that appearing in the theories of Caquot (1934) and Buisman (1935), is of little interest in the present studies and must now be considered of historic value only. The contribution of Terzaghi (1943) on the other hand, was satisfactorily condensed into his well-known 'bearing capacity coefficients' and provided a solution to the problem of bearing capacity that was readily accessible to the Engineer. Terzaghi considered the weight of soil, the effects of surcharge and the strength parameters of the soil; the coefficients Nq and $N\gamma$ (for a cohesionless soil) were introduced to give the penetration resistance* (Qpd) as:-

* Terzaghi's problem was that of a soil and its ability to support load; the Author is concerned with penetration resistance of sands - the expression penetration resistance has more meaning than bearing capacity here

$$Qpd = A(\gamma . D . N_q + \gamma . B . N_\gamma / 2) \quad (2.7.)$$

On the basis of experiments Terzaghi and Peck (1948) derived equations for the penetration resistance of circular footings in the form:

$$Qpd = \pi . r^2 . (\gamma . D . N_q + 0.6 . \gamma . r . N_\gamma). \quad (2.8.)$$

Equation (2.8.) can be rearranged as:-

$$Qpd = (1 + C.D. / r) Qpo. \quad (2.9.)$$

$$\text{where } Qpo = \pi . r^2 . (0.6 . \gamma . r . N_\gamma) \quad (2.10)$$

$$\text{and } C = 1.67 N_q / N_\gamma . \quad (2.11)$$

The term in parenthesis in equation (2.9.) is a depth factor since it is a function of D only.

Equation (2.9.) refers to the penetration resistance of sand for a flat ended rough punch but corresponding values of Qpd for a cone ended punch were shown to be equal provided the cone angle was not less than 60° (Meyerhof 1961).

Finally Terzaghi used an empirical strength parameter ϕ' to allow for the high compressibility of loose sand below surface footings. ϕ' was given by:-

$$\tan \phi' = 2 \tan \phi / 3. \quad (2.12)$$

Penetration resistance curves, computed for loose, loose-medium and dense sands from equation (2.9.) are presented in Figure 2.16. and 2.17. Because Terzaghi based equation (2.5.) on a combination of conservative failure mechanisms for self weight, surcharge and shear strength he arrived at a conservative estimate of the penetration resistance; Figures 2.16. and 2.17. illustrate this conservatism.

Subsequent modifications to Terzaghi's fundamental failure mechanism for deeper penetrations are shown in Figure 2.18(a). A failure surface PP' was assumed and additional load was required to overcome the shearing resistance:-

$$T = K_f . \tan \phi . \gamma . D^2/2. \qquad (2.13)$$

of the soil along PP' (in three-dimensional penetration models PP' represents a cylindrical 'plug' of sand). A more comparable penetration resistance was obtained by combining equations (2.13.) and (2.9.) as shown in Figures 2.16. and 2.17. For loose sands ϕ' (equation 2.10) was used as a parameter in the evaluation of N_q, N_γ and T, and K_f for loose and dense states was taken as 0.67 and 2.60 respectively.

A more rational approach to the problem of failure mechanisms at considerable depth was used by Meyerhof (1950). The surcharge in equation (2.7.) was replaced by a stress acting along an equivalent free surface (PP'; Figure 2.18(b)) and the resulting equation for penetration resistance (Qpd) took the form:-

$$Q_{pd} = \pi . r^2 . \lambda . (q_o . N_q + \gamma . r . N_\gamma). \qquad (2.14.)$$

λ was an empirical shape factor deduced from experimental studies; q_o was a function of β (Figure 2.18(b)) and ϕ: N_q, N_γ and q_o for various values of ϕ, D/2r and β were condensed into graphical form.

Interpretation of Meyerhof's graphs for loose, loose-medium and dense sands give the theoretical curves for point penetration resistance versus penetration shown in Figure 2.16 and 2.17. The modified Terzaghi and Meyerhof theories were both considerably in error at penetrations greater than 12 diameters.

2.5. Conclusions.

The physical properties of the fine white sand have identified the cohesionless material in terms that the researching engineer in soil mechanics is familiar with.

Mechanical properties of the sand, determined in direct shear tests, provided the anticipated agreement with experimental results when applied in theoretical solutions.

Results of some direct shear tests suggested possible reasons for the wide range of results for penetration experiments that have been obtained in the past few decades. It is obvious that methods of packing are important factors in an assessment of the final strength of granular soils and a full description of sample preparation is relevant in the final analysis of results. Such a description forms a part of the following chapter (i.e. Chapter 3) in which an account of the development of a suitable sand preparation technique is given.

The shear strength parameters for the fine sand were given as a separation of energy components and these have proved useful in the ensuing discussion of results.

Some existing and much new equipment was developed to perform the series of useful preliminary studies of the penetration resistance of sands. Results from these studies, in general, showed good agreement with previous studies provided experimental techniques and sand properties were basically the same.

The study of boundary effects enabled a rational container - instrument size ratio to be selected and at the same time allowed a review of previous theories of failure zones in cohesionless soils.

Existing theories for distribution of skin friction have been found to be misleading and a logical interpretation of results has suggested reasons for a disagreement between theory and the present experiments. The more accessible solutions for penetration resistance have been reviewed and utilized as a comparison with experimental results. Although the theoretical solutions were developed to apply in foundation engineering a comparison with experimental results proved interesting; agreement was obtained for penetrations up to about ten times the penetrometer diameter and at this point modified Terzaghi and Meyerhof solutions tended to predict point resistances that were in excess of those recorded from experimentation.

It has been adequately shown that a knowledge of stresses within a failure 'bulb' during penetration, derived possibly from the displacements, partially measured independently, and computed from theoretical concepts, are essential ingredients for a comprehensive study of the problem of the penetration resistance of sand.

The logical step must be to investigate the distribution of stresses and strains more closely. Chapter 4 describes techniques for measuring stresses and strains within the sand during penetration. Chapter 5 gives an account of mathematical procedures for analysis and Chapter 6 deals with the rational interpretation of results.

C H A P T E R 3

Sand preparation

3.1. Introduction

It is well appreciated that samples of granular materials prepared for laboratory experiments generally do not correspond with beds of sands and gravels that have been deposited to form naturally occurring layers of soil. However, it is more convenient to prepare samples under controlled conditions in the laboratory and attempt to achieve uniform porosity throughout than to attempt to simulate natural conditions.

The preparation of homogeneous samples presents particular difficulties that, at first, may seem to be greater than those encountered when preparing and testing non-homogeneous, randomly placed samples of soils. In addition to the convenience of using homogeneous sand deposits in laboratory experiments there is a need to carry out tests on samples with the same porosities; it is essential to be able to reproduce (within experimental limits) identical sand beds as and when required. The requirement of consistent reproduction, together with that of homogeniety, introduces problems that are not easily overcome.

The many early methods of preparing homogeneous deposits of cohesionless materials probably originated from the work of Kolbuszewski (1948 a; 1948 b); this verified experimentally that a range of porosities could be obtained by varying the intensity and velocity of particles of sand falling through a medium to build up homogeneous layers at a specific porosity.

The procedures that have been adopted in previous research studies for obtaining samples of cohesionless materials were basically of a manual nature. Loose samples were usually prepared by pouring sand rapdily from as near the existing sand surface as was practically possible and dense samples were generally prepared by vibrating or compacting layers of placed sand either mechanically or manually.

Both methods relied upon a specific technique and it is highly probable that the resulting samples were not completely homogeneous. Good repeatability, which also depends on the placement technique, would not be automatically guaranteed and there would be a great necessity to carefully check the overall porosity of every sample that was prepared. Beds of sand prepared by hand have proved to be reasonably homogeneous (Hanna, 1960) but the time taken to form such samples becomes unreasonably long, particularly for larger scale laboratory experiments.

In recent years more sophisticated methods of preparing homogeneous sand samples have been employed with considerable success. A unique method that has particular relevance to the work on sand preparation carried out by the Author was "a variable aperture hopper" apparatus developed by Kolbuszewski and Jones (1961).

The hopper was essentially an open rectangular box with a $\frac{1}{4}$inch thick duralumium plate forming the base; this had an accurately drilled grid of $\frac{1}{2}$inch diameter holes. Two additional plates were drilled with identical patterns of holes and fixed below and above the base plate to form shutter and control plates respectively. The shutter was designed to move along steel runners and the control was fixed relative to the base position to give a particular aperture size; the size of each aperture, and therefore the intensity of sand falling from the hopper, controlled the final porosity of the sand falling into the receiver. The rains of sand were dispersed into a uniform rain by using a sieve mesh fixed between the hopper base and the sand receiver.

Using this apparatus Kolbuszewski and Jones (1961) were able to produce sand deposits over a wide range of porosities and a reproducibility of approximately 0.75% of the average individual porosities was achieved (i.e. no porosity was outside a range of $\pm$ 0.75% of the mean). In later tests with the apparatus it was found that a difference in height of fall did not significantly affect the final porosity and a fall of 4ft.2½in. above the sieve mesh was maintained for depositions at various aperature openings from 1/8 to one.

A different approach to the problem of forming uniform beds of sand was adopted by Walker and Whitaker (1967). Their apparatus was also designed to vary the intensity of deposition of the sand, and in order to complete a brief historical review of the apparatuses used to produce uniform deposits of sands, this method deserves a mention here.

The apparatus consisted of a sand filled hopper that moved over a cylindrical container at a fixed speed; below the open end of the hopper a roller was allowed to rotate and carry sand from the hopper on its curved surface. The roller was driven at various speeds and corresponding quantities of sand were drawn from the bottom of the hopper. The sand, after hitting an adjustable deflector, moved downwards to form uniform sand layers within the cylindrical receiver; the receiver was modified after initial testing to enable displaced air to escape through baffle plates and holes positioned diagonally opposite the starting point of the deposition operation. Ultimately it was possible to obtain porosities within a range of $\pm$ 0.3% of a mean porosity of each individual sample and a pre-determined porosity was repeatable to within $\pm$ 0.3% of the mean porosity of a large number of samples.

The height of fall of the sand was adjustable and deposits up to four feet deep were prepared with the apparatus.

The methods of forming beds of sand described above have been used successfully, and with both arrangements high degrees of reproducibility and homogeniety have been obtained. The apparatus developed by Walker and Whitaker gave less variations in porosity for individual and groups of samples than the 'variable aperture hopper' equipment, but the apparatus used by Kolbuszewski and Jones had obvious advantages because of its simplicity of design, manufacture and operation. These were important factors to consider when deciding the most feasible type of deposition apparatus to use in the present research project.

A simple pouring apparatus, similar to that used by Kolbuszewski and Jones had been used to a limited extent by Booth (1964) in the Civil Engineering Department and this obviously made the 'variable aperture hopper' method a logical choice for preparing homogeneous sand deposits. It appeared to be quite possible that some of the existing equipment could be adapted for use in the new sand deposition apparatus.

The problems caused by displaced air and air currents in any deposition apparatus where the sand grains are introduced into air space have been widely experienced. Walker and Whitaker (1967) measured variations in the sand porosity caused by air currents after a system of baffle plates had been fixed to the cylindrical receiver to direct displaced air through holes in the cylinder walls. Kolbuszewski and Jones (1961) suggested fitting a mesh base to the receiver to allow air to pass through the container, instead of moving upwards and creating currents through which the sand would fall.

In the design stages of the present sand deposition apparatus the problems of displaced air were carefully considered in the hope that disturbances, giving rise to non-uniform beds of sand, would be reduced or possibly eliminated.

With any apparatus that is used to prepare homogeneous beds of sand for laboratory experiments it is necessary, initially, to investigate the effectiveness of the apparatus in achieving uniformity. Sampling techniques such as probes and small calibrated containers have been used and more recently radiation techniques have been employed with small samples to investigate variations in porosity. A sampling technique adopted by the Author consisted of small cylindrical containers with gauze bases.

The small sampling containers, described in later sections of this chapter, were designed to represent the larger sand receiver that was used for the laboratory investigations into penetration resistance. The only incompatibility was then due to the relative sizes of the sampling containers and the receiver.

3.2. Sand deposition apparatus

3.2.(i) Design considerations

Previous methods of sand deposition, described above, relied on an overlap of falling sand to build up layers of the sample (Figure 3.1.(i)). Air currents were created and disturbances, usually peripheral, gave rise to slight non-uniformities. By using a hopper with plan dimensions equal to those of the receiver (Figure 3.1.(ii)), the disturbances could be reduced slightly and the concept of a mesh base to the receiver which was initially suggested by Kolbuszewski and Jones (1961) would allow some displaced air to escape without disturbing incoming sand (Figure 3.1.(iii)).

The apparatus finally developed by the Author had a rigid flume between the hopper base and receiver (Figure 3.1.(iv)). It was thought that increased pressure below the falling sand would cause air to move to equilibrium outside the receiver; the advantage of this arrangement was the elimination of internal air currents completely. Because a build-up of air pressure would occur at the base of the receiver the velocity of fall of sand could be expected to decrease. As a consequence of the decrease of velocity, both for loose and dense sands, lower porosities would be obtained (Kolbuszewski, 1948 a); i.e. the resulting samples would be more dense.

In practice this model did not completely solve the problem of air currents and the resulting layers of sand had pronounced edge disturbances. The transparent flume allowed the rain action to be seen; the resulting disturbances (Figure 3.1.(v)) accounted for the edge effects.

The higher air pressures in the lower part of the receiver could not dissipate sufficiently quickly and some air currents were introduced into the flume. A variable suction device was fitted to the base of the receiver in order to withdraw air at a rate equivalent to the quantity displaced by falling sand (Figure 3.1.(vi)).

The suction was applied to an existing receiver 2ft.6in. square but did not function as intended. A second receiver, designed specifically to incorporate the suction device was more successful; this will be discussed further in this chapter.

3.2.(ii) Description of apparatus

For the purpose of describing the deposition apparatus reference will be made to Plate 3.1. The various parts of the apparatus are numbered in Plate 3.1. according to the numbers of the descriptive paragraphs below.

1. The deposition apparatus consisted of three 4ft. by 4ft. duralumin plates drilled with identical patterns of ½in. diameter holes, as in Figure 3.2. The base plate was ½in. thick and supported a 1/16in. shutter and a 1/8in. control plate.

The shutter, which was operated by an eccentric cam lever (Plate 3.2.) was able to slide freely between two parallel steel guides. The guides were bolted between the control and base plates and the three thicknesses were, in turn, fixed to a rigid angle support frame. Figure 3.3. shows an elevation of the variable aperture section of the deposition apparatus.

The control plate was made with slotted holes and this provided a movement relative to the base plate of ½in. After a few trial operations, it was possible to fix the position of the control to within ± 0.0025in. of a required aperture size; this was achieved by using four adjustable knurled screws that forced against rigid steel stops at each end of the control plate (Plate 3.2.) The required dimensions at each stop position were measured by means of a micrometer depth gauge fitting flush with the steel stops.

2. The support frame was designed in two parts that fitted together to form one telescopic unit. The frame was built up from rigid angle (Handy Angle) and could be fixed in any position from 4ft. to 7ft.6in. above ground level. The variable height was obtained simply by supporting the upper section of the frame on wooden spacer blocks. Throughout the series of experiments, the adjustable frame was arranged so that sand fell through 24in. before arriving at the level of the particular receiver that was being used.

3. A rigid wooden frame was bolted, with the control plates, base plates and steel guides, to the support frame and this arrangement positioned a particular hopper on top of the control plate as shown in Plate 3.2. The rectangular hoppers, constructed from 1in. plywood sections, could be easily replaced and the wooden frame was designed to accommodate any size of hopper up to 4ft. square. The maximum size of receiver, incidentally, was 2ft.6in. square and therefore the largest hopper had these dimensions. The depth of hopper used by the Author was initially 3in. but this was reduced to $1\frac{1}{2}$in. in certain experiments.

4. A $\frac{1}{4}$in. thick perspex flume, 2ft. long and with the same internal dimensions as the receiver, was suspended from a lower wooden frame. The internal area of the flume came directly below the sand hopper (Figure 3.1.) and the upper perimeter provided a contact with the underside of the duralumin base plate. Thin rubber strips were glued along the lower edges of the flume to form a sealing flap

and a copper tube was fixed through one face and fed to a pressure recording gauge.

5. Heavy duty castors were attached to the lower support frame legs and gave the apparatus the mobility that was an inherent feature of its design. To prepare sand layers at a predetermined porosity the complete apparatus was moved into position over the receiver; the hopper, flume and receiver then formed a long rectangular hollow section interrupted only by the variable aperture base. The 1/8in. space between receiver and flume was sealed by forcing the rubber sealing strips flush with the internal faces of the rectangle by means of thin steel strips against the four sides of the flume. The resulting apparatus was then as depicted in Figure 3.1.(vi) and Plate 3.1.

A 2ft.6in. square wooden receiver, adequately stiffened with 3in. by 3in. angles, was available for preliminary tests with the sand deposition apparatus (see Chapter 2). The receiver, in its original form had a solid base, and this was replaced with an arrangement of inset channels connected to 1in. diameter outlet holes. A suction box was attached to the underside of the receiver and rubber connectors were used to allow air to pass into the box from the receiver. The suction box was connected to a manometer and suction apparatus.

6. For later investigations a rigid perspex receiver was used with the deposition apparatus. The base of the receiver was designed to be completely air tight, and consisted of a channelled perspex block covered with No. 200 B.S. sieve size mesh. Outlets were connected directly to the suction device and manometer.

7. The manometer, shown diagrammatically in Figure 3.4. was used to measure the difference in air pressure between points A and B (Figure 3.4.). The pressure difference will be referred to as the 'equivalent head of water' in the following account of the performance of the deposition apparatus.

The performance of the two receivers, in conjunction with the deposition apparatus of which they were an integral part, will be discussed in Section 3.4.

3.3. Sampling Containers

3.3.(i) Description

The concept of a receiver base that would allow displaced air to pass through the receiver was obviously applicable to small sampling containers and in this respect it suggested an improved method of investigating homogeneity within a large sand mass.

Twenty three containers were made from 1in. inside diameter copper tube, each being 1.3/8in. deep and having a knife edged upper perimeter. The lower ends of the tubes were fitted with No. 200 B.S. sieve mesh in order to prevent sand from passing through but at the same time allowing displaced air to pass through the container base to lower deposits of sand.

3.3.(ii) Measurement of Porosity

In initial deposition tests localized porosities were found by placing sampling containers at each level of sand in a receiver prior to deposition of further layers. A central area of the receiver was left free of containers and to recover them after sand preparation sand was dug out from the central area (by hand).

The sampling containers appeared when surrounding sand assumed its natural slope; further sand was removed carefully to reveal the complete container and it was then possible to level-off each container using a perspex screed. Because the sand used in the penetration experiments had been sieved to remove (as far as possible) particles finer than a No. 200 B.S. sieve mesh the sampling containers could be removed without loss of weight.

3.3.(iii) Calibration

The sieve mesh base fixed to each container was flexible and it was necessary to consider this flexibility when calculating the porosity of sand within a container.

A series of tests were carried out with the sampling containers on a sand bed (i.e. test series I) suspended in air (series II) and placed on a perfectly flat surface (series III). The experiments, at very loose and very dense states (almost equal to maximum and minimum porosities) enabled the sampling containers to be calibrated accurately.

Maximum deflections of the base inwards (δ_{mi}) and outwards (δ_{mo}) were measured using a micrometer depth gauge (Figure 3.5.). The outward deflection (δ_{wo}) caused by the weight of sand in the container was measured in the Series II tests. Since the inward deflection of the flexible base, due to the weight of the sampling container, could not be measured in Series I tests, it was assumed from the following ratio:

$$\delta_{wi} = (\delta_{wo} \cdot \delta_{mi}) / \delta_{mo} . \qquad (3.1.)$$

The weight of sand causing δ_{wo} was approximately 68 gm. and the weight of an 'average' sampling container was 44.5 gm., therefore applying a weight correction:

$$\delta_{wi} = (\delta_{wo} . \delta_{mi}) / \delta_{mo} . (44.5/68). \qquad (3.2.)$$

Values on the R.H.S. of equation (3.2.) are given in Table 3.1. δ_{wi} was found to be 0.051 cm.

The flexible base assumed a conical shape during Series II tests (also probably for Series I tests); the volume increase in both I and II was then given by:

$$\pi . d^2/4 . \delta_w /3 \quad c.c. \qquad (3.3.)$$

$\delta_w = \delta_{wo}$ for Series II tests.

$\delta_w = -\delta_{wi}$ for Series I tests.

A correction factor which was applied to the apparent porosities obtained with sampling containers was calculated as:
(Original Volume + Volume Increase) / Original Volume.
The base corrections gave adjusted porosities that were within the experimental limits for individual porosity measurement.

Although the Series II tests had no relevance in the calibration procedure, the results did verify that the assumptions used to arrive at base corrections gave sensible results. A final correction factor of 0.99438 was applied to all apparent porosities to allow for inward base deflections.

Porosities obtained with sampling containers during sand deposition, when compared with the average overall porosity of a sand sample, were generally 0.4% lower than the overall porosity. This agrees with the concept of different porosities caused by size of sampling container (Kolbuszewski, 1948 b).

3.4. Performance of apparatus

3.4.(i) Sand deposition without suction

The attempts to apply a variable suction at the base of the wooden receiver proved to be unsuccessful. Air leakage, particularly in the receiver base and connections to the suction box, prevented effective withdrawal of air from the apparatus and it was decided to perform preliminary tests without the application of a suction. In the rigid perspex receiver a satisfactory method of withdrawing displaced air was considered at the design stage and to devote additional time and effort to perfecting the suction method with the wooden receiver was, under these circumstances, unjustified. The receiver, however, was used with a modified base arrangement that allowed some displaced air to escape. It was fully appreciated that this condition (Figure 3.1.(iv)) produced slightly non-uniform peripheral deposits in the prepared sand mass but the samples were tested in such a way that the experimental results were not seriously influenced by such disturbances. The deposition experiments without application of suction, however, substantiated the assumptions mentioned in Section 3.2.(i).

Kolbuszewski and Jones (1961) carried out a number of experiments to determine the rate of pour through a single hole for a Leighton Buzzard sand. The rate of flow Qm (gm/sec) through a single circular hole, diameter D (in.) was given by the equation:

$$Qm = 278.\ D.^{2.73} \qquad (3.4.)$$

A further series of experiments carried out by Kolbuszewski and Jones determined the ratio of rate of flow through a single aperture (qm)(the size of the aperture was described by the coefficient y (Figure 3.2.)) to the rate of flow through a single circular hole, diameter D. The ratio, given in terms of the aperture coefficient, was found to be:

$$qm \;/\; Qm = y^{1.8} \qquad (3.5.)$$

Combining equations (3.4.) and (3.5.)

$$qm = 278.\, D^{2.73} \,.\, y^{1.8} \qquad (3.6.)$$

The results for qm.(gm/sec) computed from equation (3.6.) for D = ½in. and qm. obtained from deposition tests (i.e. the total weight of sand per pour per time per number of holes) with the apparatus without applied suction are given in Figure 3.6. and Table 3.2.

The curve obtained directly using the apparatus developed by the Author had the same slope up to about y = 0.55 but from there to y = 1.0 there was a significant decrease in the rate of flow for a given aperture size.

The overall porosities obtained at the aperture coefficients used in the above experiments are given in Figure 3.7. The second curve was obtained by Booth (1964) using a similar apparatus but without a flume and with a solid base; this apparatus gave lower porosities for all aperture coefficients. The difference in porosities was greatest at larger values of y (i.e. for loosest samples). Using the deposition apparatus, without applying a negative pressure to withdraw displaced air, a relative porosity range from 21.41% to 86.90% was obtained. The corresponding range achieved by Booth was from 30.08% to 86.88%.

The increase in porosity and decrease in intensity of pour, particularly above y = 0.50, indicated the effect of using a flume without adequate provision for the escape of displaced air. A build up of air pressure in the flume and receiver probably upset the natural raining phenomenon of the sand particles, causing the type of decrease in intensity, for a given aperture size, that is indicated in Figure 3.6. (Curve 1).

It is considered that the increase in air pressure in the lower section of the receiver and flume reduced the velocity of fall of the sand particles and hence decreased the final porosity of the sand samples (Curve 1, Figure 3.7.). The effects were particularly noticeable with loose deposits, where a high rate of air displacement was experienced, and consequently the permeable receiver base was of little benefit in this case, where the large volume of air ought to have been allowed to escape quickly.
At lower intensities of deposition excess air pressure was allowed to dissipate (although not completely, as will be shown in Section 3.4.(ii)) and the porosity differences depicted in Figure 3.7. were less than for very loose samples.

(ii) Sand deposition with suction

The operation of withdrawing displaced air from the deposition apparatus was, in all cases, carried out using the rigid perspex receiver. Three types of initial experiments were done to fully investigate the effects of the suction and these are described below. The investigations were repeated for seven different aperture coefficients.

I. An 18in. deep bed of sand was prepared in the receiver at the porosity corresponding to each particular aperture co-efficient. For each sample the 'variable aperture hopper' base was then fully opened, i.e. the setting was at y = 1.0, and the manometer difference was measured while a constant suction was applied at the base of the receiver (this particular test was analogous to the conventional constant head permeameter but it was not possible, in this work, to measure the quanitity of air passing through the sand sample per unit time).

II. The negative pressure at the base of the receiver was increased in 1cm. increments of equivalent head of water and for each increment a 3in. layer of sand was allowed to fall from the hopper. This test was repeated until an equivalent head of 5cm. had been added to the original suction; the porosity of each layer was determined using sampling containers.

III. The instantaneous increase in equivalent head of water was recorded (as accurately as possible) at increases in head of 1cm. and 5cm. during the sand pour. The results of the three types of test are shown in Table 3.3.

Figure 3.8. gives the results of the second series of tests. At high porosities (i.e. $0.75 < y < 1.00$) there was no significant difference between porosities at different equivalent heads of water, but as y decreased the increase in head gave appreciable decreases in porosity. It is here worth considering the three possible conditions during the application of a suction:

1) Ideal case - the rate of withdrawal of air from the receiver and flume equalled the rate of deposition of sand.

2) The rate of withdrawal of air was greater than the rate of deposition of sand - the negative pressure of air within the enclosed flume accelerated the sand particles resulting in a decreased porosity (Kolbuszewski, 1948a).

3) The rate of withdrawal of air was less than the rate of sand deposition - conditions were as they were without the application of suction and an increased porosity was obtained.

A condition similar to (3) existed for aperture coefficients greater than 0.75 (i.e. $0.75 < y < 1.00$). Condition (2) appeared to exist for $y < 0.75$, and if it is assumed that at $y = 0.75$ condition (1) applies (this may not be exactly so, but it will be seen later that the effect is not critical) the equation below can be used:

$$Q\,a\,l = A.\ ka.\ \Delta h\,/\,l. \qquad (3.7.)$$

where:

$Q\,a\,l$ = quantity of air leaving (gm/sec)

A = area of receiver (sq.cm.)

ka = air permeability of sand with respect to Δh (cm/sec)

Δh = equivalent head of water applied as suction (cm)

l = width of sand sample (cm)

At $y = 0.75$ $\quad Q\,a\,l = Q\,s\,e. \qquad (3.8.)$

where $Q\,s\,e$ = quantity of sand entering

but $Q\,s\,e$ = qm (no. of holes) / SG. (cc/sec) $\qquad$ (3.9.)

qm = 29.83 gm/sec for $y = 0.75$.

Therefore, equating (3.7.) and (3.9.)

$$ka = 78.6725\,/\,\Delta h. \qquad (3.10)$$

The first series of tests gave values of Δh corresponding to the various aperture coefficients. Using equation (3.10) the 'actual' permeability coefficient was computed; this coefficient was related to the initial porosity of the 18in. sand layer (Test I) and was completely independent of the value of ka deduced from the series III tests.

For the third series of tests, where instantaneous increases in equivalent head were recorded, the equation:

$$Q\,s\,e = A.\ ka.\ \Delta h\,/\,l. \qquad (3.11)$$

was applicable. In this case $Q\,s\,e$. was found using the intensity of deposition (gm) for each aperture coefficient. An 'apparent' coefficient of permeability was computed since all other variables in equation (3.11) were known. Values of 'actual' ka and 'apparent' ka for 1cm. and 5cm. equivalent heads are given in Figure 3.9.

If in equation 3.9., y had been given a different value the curve of ka 'actual' would have moved with respect to the ka axis only. The values of ka, therefore, are not necessarily accurate coefficients of air permeability in sand, but they do serve as a method of comparing the effects of application of suction to the receiver base.

The condition 3 obviously existed from y = 1.0 to y = 0.55 and y = 0.62 respectively for equivalent heads of water of 1cm. and 5cm. (Figure 3.9.). Although the variable suction device used was not used to apply suctions considerably greater than 5cm. it is feasible to assume that, provided the suction was great enough, values of 'apparent' ka would decrease to eventually coincide with the curve for 'actual' ka. Condition 1 applied to y = 0.55 and y = 0.62, where the curves for 'apparent' and 'actual' ka coincided. For lower values of y (i.e. low porosities) the suction had a considerable effect on the final state of deposition, where condition 2 existed. Particularly for an initial equivalent head of water of 5cm. the acceleration of sand particles was very noticeable.

The final decision on the use of the application of suction to deposit sand layers was to apply an equivalent head of 5cm. to obtain loose samples (i.e. 1.0 > y > 0.6) and 1cm. for dense samples (i.e. 0.6 > y > 0.06). A greater suction could have been applied during loose sample deposition but the above series of tests were used as a calibration of the deposition apparatus and, as such, were also used as a basis for the later experiments which required homogeneous samples.

The final range of porosities obtained using the deposition apparatus with a variable suction device was from 44.17 to 36.24; relative porosities were 23.91% and 92.63%.

(iii) Overall and mean individual porosities

The overall porosity, in addition to the mean individual porosity, was determined in 20 deposition tests. 12 tests were performed without the application of suction to the receiver base.

Overall porosities were completed knowing the total volume occupied by a sand sample and the weight of sand within that volume. The mean individual porosity was obtained from the results of 23 sampling containers.

Table 3.4. gives a summary of results of the experiments and Figure 3.10 shows the plot of results falling below a line representing equal porosities. For dense samples the average difference between overall and mean individual porosities was 1.92% and for loose samples was 0.57%; the considerable difference in size of samples produced this apparent difference in porosity (Kolbuszewski, 1948a).

The difference between overall and mean individual porosities for depositions without applied suction did not vary significantly from tests where a suction was employed.

(iv) Reproducibility

For a given aperture size, groups of tests indicated that it was possible to reproduce a sample with a porosity almost identical to previous samples. Without the application of a variable suction a variation from a mean of ± 0.23% was obtained for dense sand deposits. In all other tests the variation was considerably less; the three clusters of points on Figure 3.10 give a clear picture of the variation of porosity for loose, medium and dense sand samples.

(v) Uniformity

The results of the large number of porosity measurements using the small sampling containers are given in columns 2 and 3 of Table 3.3. For dense samples prepared without withdrawal of displaced air, the maximum variation was ± 0.52% (this is greater than the variation achieved by Walker and Whitaker (1967) but slightly less than values given by Kolbuszewski and Jones (1961)). The maximum variation of porosity when the suction apparatus was utilised was ± 0.16%; the number of tests was somewhat limited but the degree of uniformity has been adequately indicated.

3.5. Concluding Remarks

The 'variable aperture hopper' deposition apparatus developed to prepare homogeneous sand masses for laboratory experiments achieved a high degree of homogeneity and reproducibility. The deposition apparatus did not provide difficult manufacturing and assembly problems and proved to be relatively simple to adjust and operate.

The method of determining localised porosities with small sampling containers was both reliable and convenient and indicated that the samples prepared with the apparatus were within the limits set by the available measuring amd recording equipment. The sand samples were found to be quite homogeneous.

The use of a variable suction eliminated some of the problems that are generally associated with air currents, particularly when preparing dense samples of sand. It has been verified that individual samples prepared without using the suction had an acceptable degree of homogeneity and the variation from a mean porosity (expressed as the standard deviation) was not radically different from the variation within one sample that was obtained with the suction attachment.

Throughout this chapter emphasis has been placed on the porosity of a laboratory sand sample, in fact, the overall porosity of the sand. This parameter has been used consistently to describe the state of the fine white sand prior to performing a penetration experiment.

It has not been assumed that the resulting sand samples were isotropic, but in investigations with large sand samples this assumption of initial homogeneity facilitates much of the later work of analysis.

Since the method of preparation has been maintained for all samples, whether loose or dense, the porosity has more meaning than if the manual techniques (described earlier) had been employed. This in itself is an important reason for preparing samples of cohesionless soil with as little manual technique as possible.

Having established a suitable method of preparing homogeneous sand masses, it remains to describe certain techniques that were used to measure the stress and strain distribution in these sand masses during penetration. Chapter 4 includes such descriptions.

Chapter 5 lists the theoretical methods for predicting stress distribution in a homogeneous sand mass during a penetration test. It is here that the concept of initial homogeneity facilitates the procedure.

C H A P T E R 4

Stress and deformation measurement

4.1. Introduction.

The need to study the distribution of stress and strain in a particular laboratory experiment has long been a stimulus for research on more accurate methods of pressure and deformation measurement. For example, conventional soil testing apparatus, such as the triaxial and shear box tests, were introduced to measure the properties of soil but the limitations of conventional measuring techniques were only fully realised when research workers began to measure stress and strain distribution within soil samples during tests.

From early studies the techniques have evolved, particularly with regard to stresses, to measure the long-term conditions in civil engineering structures; this development has tended to direct efforts away from direct strain measurement (generally pressure measurement is based on some form of deformation but the interpretation of deformations in this case is associated with a pressure rather than a strain) until recent studies, notably at Cambridge University, have established strain measurement techniques as a useful addition to the accepted methods of investigating soil behaviour in laboratory experiments.

In this chapter an account is given of some of the available methods of measuring stress and strain fields in larger scale laboratory experiments; the account is not extensive but it enables the most suitable methods for the measurement of penetration resistance of sand to be appreciated.

It was accepted that, in certain cases, the methods that were finally selected could give results that were in error but from the experiences gained by past research studies these errors were believed to be small and could be allowed for.

Section 4.2. describes a small load cell that was used in preliminary experiments. The principle of the cell has recently been used to provide a measure of skin friction along a penetrometer shaft (Ebrahimi, 1969) but in the present studies it was used to measure the point resistance of a cone-ended penetrometer.

The two sections 4.3. and 4.4. describe a miniature three-dimensional pressure cell from design through to calibration. The cell, which was used to measure pressure distribution in a sand mass during penetration, was smaller than previously used pressure measuring devices and consequently took longer to manufacture and assemble; since this was the alternative to the larger laboratory apparatus that a larger pressure cell demanded, the additional time spent on constructing it was justified.

In the three dimensional pressure cell described in Section 4.3. it was possible to measure the stresses normal to a cell face. Each completed cell had three sensitized faces that formed part of a cube and stresses in the directions of the planes of the cube could be measured.

The resulting stress components formed the leading diagonal of a stress tensor at a point (the stress tensor at a point is defined fully in Section 5.2.(i)) and it can be shown that additional shear stress data was necessary for a full definition of the stress tensor; the additional data was, in fact, one shear stress (τ_{rz}) since axial symmetry eliminated other shear stresses in the stress tensor.

The experimental techniques introduced in the following paragraphs were used to provide a suitable method of determining the unknown data of the stress tensor; strain fields were obtained and these were contrasted with results from pressure cell experimental results.

Section 4.5. is concerned with methods of displacement measurement in the penetration experiments. Some past methods of displacement measurement are chronicled and more complex currently employed techniques form an introduction to the experimental procedures used.

An attempt was made to justify the basic methods used by the Author but certain limitations (such as insufficient time and available equipment to conclude the investigations) ruled out a satisfactory correlation between the well known radiography techniques and the less refined techniques eventually used.

Experiments that enabled displacements to be measured were, however, performed satisfactorily and the final section of this chapter describes the test procedure and the analytical techniques that were used.

It will be shown later that results obtained from pressure and displacement measurements were surprisingly effective considering the necessity for a certain degree of compromise between extreme accuracy and quantitive results.

4.2. Penetrometer load cell.

4.2.(i) Design and calibration.

In the experiments described in Section 2.4. a brass load cell was used to measure point bearing resistance during penetration. The cell (Figure 4.1.(a) and (b)) was sensitized by means of 4 No. Budd type C6 - 121 $\frac{1}{8}$ in. strain gauges fixed at 90° spacings along the inside wall; a dummy gauge was attached to the base of the thin walled section. A design criteria of one x 10^{-6} strain units per lb. load was selected and this gave a wall thickness of 0.0225 in. Micro-strain was measured on a Peekel type T200 strain recorder during penetration. The recording equipment was accurate to at least $\pm$ 0.5% and was graduated to read micro-strain within limits of $\pm$ 1.5. A more sensitive cell would have been desirable but the limitations of available recording equipment and the minimum practical thickness of the cell walls combined to give this degree of accuracy. As it was the cell had a working range of approximately 100 lb. and corresponding compression of the cell was estimated to be 50×10^{-6} in.

The brass point and sensitized cylinder was originally machined to 1.0005 in. diameter with a 60° cone; it was push fitted into a brass tube of similar diameter, fixed securely with Araldite adhesive and rubbed down with carborundum cloth to provide a flush fit, at 1 in. diameter. Lead wires passed through the hollow penetrometer and fed out through a small hole at its top. The lead wires were securely fixed to prevent damage to the gauges during penetration experiments.

Calibration of the penetrometer took the form of controlled strain and constant load cylic load-unload tests up to 100 lb. Strain was taken as the mean of the four gauge readings and typical load-strain curves for both types of test are presented in Figure 4.2. An assumed linear load-strain relation gave the calibration equation:-

Load = 1.0283 x Mean Strain + 0.2914.lb.,that was used in all experiments that measured point bearing resistance.

4.3. Pressure Cell - Design and Manufacture.

4.3.(i) Design Considerations.

The most commonly employed method of attempting to measure pressures within a sand mass, or at a soil-rigid body boundary, has been to measure the movement of either a rigid piston or flexible diaphragm. The piston or diaphragm generally fitted into a rigid walled cylinder and movement inwards was translated, either mechanically or electrically, to a remotely situated recording station. One inherent disadvantage with pressure measurement within a sand mass is that cables or tubes must provide a link between the sensing device, be it a resistance strain gauge such as that used by Redshaw (1954) or the vibrating wire with electro-magnetic exicitation (Whiffin and Smith, 1951), and recording equipment. The indication of a response due to application of pressure within a sand mass (The Author's primary concern was the measurement of pressure within a mass of sand and not pressures on a sand-rigid body boundary, hence the emphasis on pressure cells in a sand mass.) has presented relatively simple problems but an accurate interpretation of responses has proved either very difficult or impossible. There are no rigorous solutions for correct interpretation of output signals and only by using the experiences of other research workers can a suitable cell design be achieved.

An earth pressure cell that (i) has the same stiffness as the soil surrounding it, and (ii) has no extension cables or tubes, would present a true indication of the pressure that would exist at the location of the cell in the absence of the cell. As far as the Author is aware, no such cell exists, but an alternative to (i) was suggested by Monfore (1950), and has been accepted as a basic design criteria by a number of researchers, for example Peattie (1954), Allwood (1956) Lee and Brown (1957), Trollope and Currie (1960), Tory (1967) and Thomas and Ward (1969). Monfore's analysis showed that over registration (see Section 4.4.(i)) for an (overall) rigid pressure cell became constant at a modular ratio (E cell/ E soil) of about 50. Provided the rigidity of the cell was considerably greater than that of the surrounding soil, a constant over registration of approximately 10% was obtained. In the case of rigid metal pressure cells the modular ratio (whether by design or chance) was always much greater than 50.

The U.S. Waterways Experimental Station (1944) carried out a large number of investigations with flat cylindrical cells embedded in rigid boundaries and soil masses. The main findings (that are relevant to this study) were:-

(i) for cells embedded in a soil mass, the diameter - deflection ratio of a diaphragm should exceed 2000:1.

(ii) the overall diameter - thickness ratio for cells in a sand mass should be greater than 5.

Criterion (i) was based on a series of consistent results and was specifically applicable to the pressure cell used by the W.E.S.; this ratio may vary considerably with cells having different basic characteristics, and as such can only be taken as an approximate design criterion that may not apply to every earth pressure cell. The diameter - thickness ratio of 5 was suggested as that at which the effect of differential compressibility of cell and sand became constant (W.E.S. Figure 13). This criterion (ii) recommended by the W.E.S. did not consider the effects of lateral compressibility in their thin cylindrical cells (Allwood,(1956) discussed compressibility in detail and a development of Allwood's work, as used by Buck (1961) is introduced in a later section) , and as a consequence it cannot be considered as a rigid design criterion. Trollope and Currie (1960) carried out similar experiments to those of the W.E.S. using thin cylindrical cells with 'backing' plates that increased the thickness. For diameter - thickness ratios varying from 2.3. to 4.6. they found no significant difference in diaphragm registration; this was not in agreement with the findings of the W.E.S.

A valid conclusion is that no single rigid design criterion is applicable to all pressure cells and emphasis is again placed on the importance of referring to past experiences with pressure cell design.

The design of miniature pressure cells without considering the ratio of grain size (of the granular material) to diaphragm diameter is incomplete. Rowe (1954) calibrated a soil pressure cell using lead shot that gave a grain size - diameter ratio of 1:20, and found that at a diaphragm deflection - diameter ratio of 1:5000 there was no evidence of arching accross the pressure cell's rigid abutments. A uniform graded sand used with the same cell gave responses to sand pressure that were identical with fluid pressure responses provided the diaphragm diameter - deflection ratio was not less than 1500:1. The grain size - diaphragm diameter ratio for the calibration studies varied from 1:20 to 1:50.

Kallstenius and Bergau (1956) developed theories for the mechanism of diaphragm deflections under the action of sand pressure and concluded that a grain size - diaphragm diameter ratio of 5% was acceptable. Although Rowe, and Kallstenius and Bergau were concerned with the boundary pressure cells there is no reason to assume that the phenomenon of arching of sand grains is any different from that in earth pressure cells. The minimum ratio of diaphragm diameter - grain size, 20:1, will be used as a basic design criterion. A diameter - deflection ratio of 5000:1 corresponds with the criterion that prevents arching and consequently it will be accepted in the pressure cell design discussed in Section 4.3.(ii).

The earth pressure cells mentioned in this Section have assumed a flat cylindrical shape, with one or a pair of deflecting diaphragms supported by an annular separating section; the specific purpose of the cells have been to record pressures normal to the diaphragm(s). A novel three-dimensional pressure cell (Mackey, 1966) posessed the fundamental requirements of earlier cells with the additional attribute of being able to record pressures in three orthogonal directions. The frame used by Mackey was simply a steel cube with suitably sized holes drilled through appropriate faces. Obviously the lead wires were a necessity but the concept offered a pressure cell that would, in effect, take the place of three of the earlier type cells. Development of an earth pressure cell for use in penetration tests will use the concept of three-dimensional pressure recording introduced by Mackey.

The vibrating wire principle for measuring the movement of a deflecting diaphragm in an earth pressure cell has been employed successfully (Whiffin and Smith, 1951; Thomas 1966; Thomas and Ward, 1969); the resulting cell has 'long term stability and robustness' but there is a physical limitation of size. The resistance strain gauge method of recording a deflection can be employed in miniature earth pressure cells and, since long term stability and toughness are not particular requirements for the earth pressure cell considered here, the strain gauge technique will be employed in favour of the vibrating wire principle.

A three-dimensional pressure cell, using resistance strain gauges, will be designed. A diaphragm diameter - deflection ratio of 5000:1, coupled with a diaphragm diameter - grain size ratio of 20:1 has in past works been found to eliminate arching of sand grains across the cell abutments; these ratios will form the basic design criteria for the earth pressure cell. Other design considerations, such as methods of recording output and possible false registrations will be discussed in Sections 4.3.(ii) and 4.4.

4.3.(ii) Pressure Cell Design.

The average grain size of the fine white sand (Section 2.2.) was 0.22m.m., and using a 20:1 diaphragm diameter to grain size ratio, a minimum diaphragm diameter of 0.1732 in. was obtained. A $\frac{1}{4}$ in. cube with 3/16 in. diameter diaphragms provided a dimension of this order.

The problems of manufacture and assembly of such a cell were obviously more numerous than with currently available pressure measuring devices but its advantages lay in its size in relation to the sand sample (provided the criteria discussed in Section 4.3.(i) were satisfied).

The diaphragm diameter to deflection ratio of 5000:1 allowed a maximum deflection at the centre of the diaphragm of 3.75×10^{-6} in. In terms of elastic properties of the diaphragm, the dimensions and the applied pressure, the maximum diaphragm deflection was given by the equation:-

$$\delta = 3\,p\,r^4\,(1 - \mu^2) \;/\; (16\,E.t^3) \qquad (4.1.)$$

where: E = Young's modulus (30×10^6 p.s.i.)

μ = Poisson's ratio (0.25 for spring steel)

r = diaphragm radius (3/32 in.)

t = diaphragm thickness.

p = applied pressure.

The maximum estimated pressure within a sand mass during penetration at about one inch from the centre line of an advancing penetrometer was 10 p.s.i. This limiting value of applied pressure, with other parameters in equation (4.1.) gave a diaphragm thickness of 0.005 in. (i.e. $t^3 = 120 \times 10^{-9}$).

Theoretical radial strain distribution on the uniformly loaded circular diaphragm with clamped edges, is shown in Figure 4.3. A maximum strain occurred at the centre where:-

$$\epsilon r = 3\,r^2\,p\,(1 + \mu) \;/\; (8.t^2.E) = 55 \times 10^{-6}. \qquad (4.2.)$$

A maximum stress in the diaphragm at its centre (from equation 4.2.) was 1650 p.s.i; assuming a yield stress of 40,000 p.s.i. for spring steel this is well within the elastic range of the material. If equation (4.2.) is rewritten as:-

$$\epsilon r = CF.p. \qquad (4.3.)$$

the pressure applied to a diaphragm is related to the recorded pressure by CF. For a perfect pressure cell the factor CF = 5.49316×10^{-6};

that is to say if there were no false registrations the strain could be translated to applied stress using the factor CF.

The central quarter of the diaphragm was subjected to radial strain and any strain sensing device could only be fixed over the central half of the diaphragm diameter (the Redshaw (1954) gauge is an exception but it is not available at the size required here), limiting the length of gauge to a maximum of 0.09375 in.

The requirements of strain sensor length (0.09375 in.) and maximum strain range (0 to $55 \cdot 0 \epsilon \times 10^{-6}$), coupled with the availability of strain recording equipment ruled out the use of conventional strain gauges. For example, an accuracy of $\pm 3 \mu\epsilon$ would have been obtained using a Peekel strain recorder (Type 200) and a strain gauge with a gauge factor of 2. A recording oscillograph (Consolidated Electro-dynamics, Type 5-127) was available in the laboratory, and was capable of recording 1606 $\mu\epsilon$ on a 7 in. print-out paper when the most sensative galvanometer was used; an accuracy of 0.010 in. from scaling off the paper gave an accuracy of $\pm 2.29 \mu\epsilon$. A variation of this order for a maximum strain of $55 \mu\epsilon$ was hardly adequate in small scale pressure measurement, and a sensor with considerably higher sensitivity was sought.

A range of semi-conductor strain gauges manufactured by Kulite Semi-Conductor Products Inc. (New Jersey, U.S.A.) fulfilled the requirements of size and sensitivity that a $\frac{1}{4}$ in. pressure cell demanded. Gauges were available from a minimum length of 0.040 in. upwards, and gauge factors varied from 100 to 140. Although at first it seemed that the highest gauge factor was most suitable, the temperature coefficients associated with the gauge factor did not increase with gauge factor, and by careful selection, based on cost, length, gauge factor and heat dissipation properties a gauge type PEP - 350 - 090 was chosen. This particular gauge had a gauge factor of 130 $\pm$ 2%, the overall length was 0.090 in., cost per batch of 5 was £20 and recommended heat dissipation was 100 m.w. at a temperature of 70°F.

The gauges were delivered with extension leads attached but did not have the usual epoxy resin backing and it was necessary to develop a technique for assembling the diaphragms and semi-conductor gauges. To facilitate cell assembly special equipment was designed, and manufactured in the department workshop. The manufacture of cell diaphragms (and frames) demanded fine tolerance work and the equipment and methods described in the following section were used to ensure consistent accuracy.

4.3. (iii) Cell Manufacture.

An accurate drilling jig was designed to produce the mild steel frames for the $\frac{1}{4}$ in. pressure cell. An exploded view of the jig, with a pressure cell frame in position, is shown in Plate 4.1. The frames were originally milled from mild steel rod and then drilled completely in one direction and through one face in the two orthogonal directions. (Figure 4.4.) Each cell was located accurately within the jig and drilled through one face at a time; with some experience it was possible to manufacture one frame approximately every two hours. Dimensions of cell frames, measured using available workshop instruments, were identical for the 12 frames that were made.

Diaphragms were cut from $\frac{1}{4}$" spring steel strip and alternatively rubbed down by hand using No. 200 and No. 600 emery paper to give 0.25 $\pm$ 0.0005 in. squares. Making use of a x 20 magnification Vickers bench microscope the diaphragms were marked with a fine scribe (the point of a Wild compass) as shown in Figure 4.5.(a). The position of the central hole was marked on each diaphragm with metal strip that had been drilled with a 0.1875 in. diameter hole. End diaphragms were basically as other diaphragms but a 1/16 in. slot was filed out (Figure 4.5.(b)).

The cell frames and diaphragms were treated with carbon tetrachloride and 90% ammonia solution to degrease and prevent rusting, respectively.

4.3. (iv) Cell Assembly.

Cell assembly was conveniently separated into 3 operations:-

(i) Fixing gauges to the spring steel diaphragms.

(ii) Fixing diaphragms to the cell frame.

(iii) Connecting lead wires to gauge extension wires and sealing the cell and lead wires.

An assembly rig was designed and manufactured to carry out operations (i) and (ii) accurately. Plate 4.2. shows the rig during the process of fixing a cell diaphragm (i.e. operation (ii)). The stages (i), (ii) and (iii) can only be described in a meaningful way by including a full account of the numerous stages. A brief description of the more important techniques for cell assembly are listed below, but a detailed account of the many intricate details of assembly, such as methods of applying cement to gauges and diaphragms, methods of clamping gauges during the cement curing process etc., forms an unpublished section of the research project. A report giving instructions for complete cell assembly has been filed for future reference in the Soil Mechanics Laboratory of the Civil Engineering Department.

Strain gauges were fixed to the spring steel diaphragms, and the diaphragms were fixed to the cell frames using an 'Araldite' two compound cement: HY951 and MY753, that was recommended for use with steels. Whenever possible the cement was cured at 100°C. for 1½ hours in a laboratory drying oven. The gauge lead wires that protruded from within the cell were soldered to 8 ply P.V.C. covered extension lead wires and the exposed metal lengths of wire were insulated using the same 'Araldite' cement. The pressure cells were completely sealed using slotted end diaphragms and were rendered waterproof with layers of cement along the extension wires that passed through the end diaphragms. A x 20 magnification bench microscope was used as an aid to cell assembly and each cell took an average of twelve hours to complete from start to finish. Eleven cells were manufactured; an additional cell with two gauges affixed being constructed to provide 'dummy' gauges that were required when using one of the laboratory strain recording instruments.

Plate 4.3. shows a completed cell alongside the mild steel cube that was used as a frame for the pressure cell.

4.3.(v). Measurement of Electrical Output.

The series of calibration experiments (described in Section 4.4.) were performed in a constant temperature laboratory annex. A Consolidated Electrodynamics recording oscillograph (Type 5-127) was used to measure the electrical output from the sensitized cell diaphragms during the calibration procedures.

The advantages of this recording method were an output from the cells that produced a visible signal, a permanent record on sensitized print-out paper and (because manual recording operations were not necessary) uninterrupted testing. Disadvantages of the recording oscillograph for strain measurement were: the necessity of building in Wheatstone bridge circuits that were required for sensitive registrations: the time consuming task of scaling all results from a print-out and: a non-direct recording of strain.

A resistance bridge was built up from 330 ohm. high stability resistors and 50 ohm. carbon wound potentiometers (Figure 4.6.). It was found that the daily temperatures in the constant-temperature annex varied by less than $1^{o}F$, thereby eliminating the need for additional gauges for temperature comparison. A bridge was build for each diaphragm of one cell; each cell was calibrated individually and interchange of cells was by means of two 5-pole 3-way rotary switches connecting the ten pressure cells into a common bridge terminal.

It can be shown, with reference to Figure 4.6. that the current I_G through a galvanometer (G) due to a change in resistance (ΔR) of a strain gauge, is given by the equation:

$$I_G = Vs.c.\Delta R / \left\{(1+c)\ Rs\ (R1+R2+R_G(1+c))\right\} \qquad (4.5)$$

where:

Vs = system voltage (a constant voltage supplied by a Farnell control unit).

R1 = resistance of L.H. upper bridge arm (ohms.)

$R2 = (R2^1 + R_p + R4^1)\ R_o\ /\ (R1 + Rs)$.

Rs = strain gauge resistance ($360\,\Omega$).

R_G = internal resistance of galvanometer (ohms).

$C = R1/Rs$.

The gauge factor is given as:-

$$GF = \Delta R\ /\ (Rs.\epsilon_r). \qquad (4.6.)$$

and I_G in terms of galvanometer sensitivity (S) and trace movement on paper (TD) is :-

$$I_G = TD.S. \qquad (4.7.)$$

substituting (4.6.) and (4.7.) into (4.5.)

$$\epsilon_r = S.\ TD\ (\ R1+R2+R_G\ (1+C))\ /\ (Vs.\ c.\ GF). \qquad (4.8.)$$

The variables in equation (4.8.) are ϵ_r and TD, therefore rewriting with a constant A, the equation becomes:-

$$\epsilon_r = A.\ TD/VS.$$

For calibration tests the above equation gave the average central strain, as registered by a semi-conductor strain gauge, in terms of an electrical output that was recorded on a print-out paper.

The penetration experiments in which pressure cells were used were carried out in a large materials testing laboratory and for reasons discussed below it was decided to use a Peekel Type 200 strain recorder and extension box for these experiments.

The advantages and disadvantages of the Peekel recorder were virtually opposite to those of a recording oscillograph, but for penetration experiments, where many hundreds of readings were to be taken, direct reading equipment was more practical than a translation from print-out paper. At least the Peekel output was readily converted into a strain recording.

Modern methods of recording electrical output from a large number of strain gauges have been based on the use of high speed data loggers and these are usually integrated with line printers or punched tape output. Unfortunately the Dynamco logger that should have been available in the materials testing laboratory, in which the pressure cells were used, was inoperative. The Peekel strain recorder was considered the only feasible alternative.

The Peekel recorder had a built-in half Wheatstone Bridge (i.e. Rp, $R2^1$ and $R4^1$ in Figure 4.6.) and using a Peekel extension box one dummy gauge (R1) sufficed for 24 active gauges, (Rs). A dummy semi-conductor gauge, in conditions and surroundings analogous to those of the active gauge, was incorporated into the bridge circuit to provide a temperature compensation during experimentation in the materials laboratory. Autographic recordings of temperature over an average day gave variations of $\pm$ 3^{o}F; this suggested a real need for temperature compensation (see Section 4.4.(vi)) in pressure cell recordings in spite of the additional time and cost of providing one extra pressure cell.

Registrations from the Peekel equipment were converted to actual strain, and thence stress, as follows:-

ϵr (actual strain) = Registration. GFp/GFg. (4.10)

where:

GFp = gauge factor set on the Peekel T 200.

GFg = gauge factor of semi-conductor strain gauge.

Since GFp and GFg were virtually constant:

ϵr = 0.015769. Peekel registration.

Applied pressure for a diaphragm was obtained from equation (4.10) and the calibration equation (4.15). The calibration, however, is dealt with in some detail before arriving at equation (4.15.); this detail forming an introduction to the following section (4.4.) Before considering the calibration procedures it is necessary to justify using the recording oscillograph and Peekel recorder for calibration and experimentation respectively. This was done by performing an identical experiment with the two methods of recording output.

An individual calibration experiment (Section 4.4.(ii)) was carried out with cell No. 10, using the recording oscillograph and the Peekel strain recorder. The recorded strains for a 0 to 5 p.s.i. load cycle were (within the limits of pressure application to a diaphragm) the same. Since both measuring devices translated an electrical signal into a mechanical output similar strains (for identical stresses) were expected; the constants in equations (4.8) and (4.10) were accurate to, at most, $\pm$ 2% and consequently strain recordings for the two instruments were virtually identical.

4.4. Pressure Cell-Calibration and Use.

4.4.(i) Theory of Calibration.

Relevant existing theories that predict errors occurring in pressure cell measurement are briefly discussed and are combined to produce a general theory for calibration that is applicable to the pressure cell described in Section 4.3.

The registration errors that normally occur in pressure cell measurements were conveniently categorized by Taylor (1947) and Monfore (1950) as over-registration and under-registration, these being defined as follows:-

i) over-registration: a cell is more rigid than surrounding soil and for a given soil pressure the corresponding registration by the cell is higher than the true soil pressure.

ii) under-registration: the soil is more rigid than the cell that is placed within it and for a given soil pressure the registration by the cell is lower than the true soil pressure.

It is demonstrated below that these two phenomena are inseparable and, contrary to the theory of Taylor, the errors induced by differential rigidity are not clearly defined for the deflecting diaphragm type of pressure cell.

Taylor (1947) used a 'cell action factor' Ca to relate true soil pressure (P) and pressure (P+Pe) registered by a pressure cell. The diaphragm diameter (D) and cell half width (B) were considered and the constant Ca allowed for differential compressibility of soil and cell in the form:

$$Ca = (N/Es - N/Ec)/(1 + (B/D)(N/Ec)).$$

where:

Es, Ec are 'elastic modulus' for soil and cell respectively, and N is an elastic soil constant.

Taylor was concerned primarily with over-registration and used flat cylindrical cells of different diameters in his investigations. Since the fundamental action of cells used by Taylor involved an inwardly deflecting diaphragm the case of under-registration (i.e. case (ii) above) must to some extent apply for these cells with moving diaphragms. It is quite clear that the cases of over-registration and under-registration cannot easily be separated and it is necessary to consider a combination of registration errors, rather than employ rigid theories such as that of Taylor and obtain an overall 'cell action factor' that incorporates cases (i) and (ii) above.

Theoretical pressure distributions given by Allwood (1956) and Kallstenius and Bergau (1956) (Figure 4.7.) illustrate the combination of over and under registration. Trollope and Lee (1957) found that an overall under-registration occurred for cells that were basically the same as those used by Taylor. It can be concluded from these studies that over and under registration do not, as such, exist in cells with deflecting diaphragms, and theoretical studies serve to establish the necessary criteria for full calibration of the pressure cell.

A 'cell action factor' F introduced by Buck (1961) used the stress ratio σ_t/σ_m (see Figure 4.8.) as a main parameter. Buck used Redshaw (1954) type pressure cells and investigated the effects of stress ratio for cells placed at different inclinations inside a 4 in. by 8 in. triaxial sand sample. Experimental results gave increasing values of 'cell action factor' F (i.e. the ratio between cell response to a given pressure when the cell is embedded in a soil to the response when in a fluid at the same pressure) for decreasing stress ratio. F and σ_t/σ_m were also related empirically.

A feasible explanation for Buck's results can be found by considering stresses acting on the pressure cell (Figure 4.8.). Shear stress on the cell diaphragm, according to Rowe and Peaker (1965) have negligible effect on the cell performance and can be neglected. The uniform stress σt, tending to compress the anular separating ring, was translated as compressive stress in the two flexible circular diaphragms, giving rise to under-registration. Thomas and Ward (1969) found that, for pressure cells with only one sensitized face, twisting of the steel separating ring occurred even though it was $\frac{1}{4}$ in. thick; The Redshaw pressure cell had a $\frac{1}{4}$ in. thick plastic separating ring and the liklihood of compression seems to be much greater. This phenomenon confirms Buck's experimental studies in which greater lateral stresses gave lower 'cell action factors'. A valid conclusion that follows from these studies is that diaphragm compression in this type of pressure cell, due to lateral stresses, leads to an inherent under-registration characteristic. Only by extensive calibration of each pressure cell can false registrations be fully realised.

In the three dimensional pressure cell, described in previous sections of this chapter, σt represents stress on, say, diaphragms one and two and σm on diaphragm three. If 'cross sensitivity factors' α_{ij} , anologous to Buck's'cell action factor' F, are introduced the following calibration theory is developed.

σ_{ai} is the pressure applied to diaphragm i (i = 1, 2 or 3) and
ϵ_{rij} is the strain recorded on diaphragm i due to a pressure on diaphragm j
α_{ij} is a 'cross sensitivity factor' that converts a proportion of pressure on diaphragm j to a strain on diaphragm i. If $i \neq j$ α_{ij} has a negative value less than unity.

For a three dimensional pressure cell:

$$\epsilon_{r11} = \alpha_{11} \cdot \sigma_{a1} \quad ; \quad \epsilon_{r12} = +\alpha_{12} \cdot \sigma_{a2} \quad ; \quad \epsilon_{r13} = +\alpha_{13} \cdot \sigma_{a3} \, .$$

and combining these equations:

$$\epsilon_{r11} + \epsilon_{r12} + \epsilon_{r13} = \epsilon_{r1} = \alpha_{11}\sigma_{a1} + \alpha_{12}\sigma_{a2} + \alpha_{13}\sigma_{a3} \, . \qquad (4.11.)$$

Similar equations are obtained for diaphragm 2 and 3 and the resulting equation is obtained:

$$[\epsilon_{rij}] = [\epsilon_{ri}] = [\alpha_{ij}]\,[\sigma_{ai}] \qquad (4.12.)$$

If there were no false registrations the cross sensitivity factors would be:-

$$[\alpha_{ij}] = [\delta_{ij}] \,.\, \text{CF.} \qquad (4.13.)$$

where $\delta_{ij} = 0$ if $i \neq j$ and 1 if $i = j$, and CF is the conversion factor given in equation (4.3).

The 'cross sensitivity factors' were determined by calibrating each pressure cell with suitable combinations of applied pressure. The complete matrix $[\alpha_{ij}]$ was constructed by performing tests in which two applied pressures were zero and the third was increased to 5 p.s.i. and returned in 1 p.s.i. increments. Results were analysed by assuming: (i) a linear relation between applied pressure and recorded strain and, (ii) a calibration curve passing through the origin. For example, the test with $\sigma_{a1} \neq 0; \sigma_{a2} = \sigma_{a3} = 0$ gave curves of $\epsilon_{r1}; \epsilon_{r2}; \epsilon_{r3}$ versus σ_{a1} (Figure 4.9.) Regression theory was applied and factors were computed from the equations:

$$\alpha_{11} = \Sigma \sigma_{a1}.\epsilon_{r1} / \Sigma \sigma_{a1}^2.$$

$$\alpha_{12} = \Sigma \sigma_{a1}.\epsilon_{r2} / \Sigma \sigma_{a1}^2.$$

$$\alpha_{13} = \Sigma \sigma_{a1}.\epsilon_{r3} / \Sigma \sigma_{a1}^2.$$

The analysis was computerized for processing by an Elliot 803 computer and results for $[\alpha_{ij}]$ matrices for each cell were obtained. To check values of $[\alpha_{ij}]$, tests with $\sigma_{a1} \neq 0; \sigma_{a2} \neq 0; \sigma_{a3} = 0$ and hydrostatic tests were carried out. Substitution of measured values of σ_{a1} with $[\alpha_{ij}]$ into equation (4.12.) gave useful check values of ϵ_{ri}. The order of error obtained in these substitutions was $\pm$ 0.25%.

The factor Ca (Taylor, 1947) accounted for differential compressibility of sand and cell and since the factors $[\alpha_{ij}]$ only indicated stress ratio effects it was necessary to modify equation (4.12.) to include a combination of factors Ca and F (Buck, 1961). Equal all-round sand pressure tests provided values of Ca and these were incorporated into the leading diagonal of equation (4.12.)

The solution of this equation, using Cramer's rule, gives:-

$$\sigma_{a1} = D1/D \; ; \; \sigma_{a2} = D2/D \; ; \; \sigma_{a3} = D3/D \; . \qquad (4.14.)$$

where

$$D = \begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{vmatrix} \quad \text{and } D1 = \begin{vmatrix} \epsilon_{r1} & \alpha_{12} & \alpha_{13} \\ \epsilon_{r2} & \alpha_{22} & \alpha_{23} \\ \epsilon_{r3} & \alpha_{32} & \alpha_{33} \end{vmatrix} \qquad (4.15.)$$

D2 and D3 assume similar forms to D1. D and the minors of $\epsilon_{r1}, \epsilon_{r2}, \epsilon_{r3}$ were evaluated initially for each pressure cell and applied pressures were then found by simple substitution of recorded strains into these equations.

Final analysis was carried out on a Monroe Epic 2000 desk calculating machine.

4.4. (ii) Individual Diaphragm Calibration.

The cross-sensitivity factors $[\alpha_{ij}]$ for each pressure cell were found by applying a combination of hydraulic pressures to individual diaphragms through two $1\frac{1}{4}$ in. outside diameter by $\frac{1}{4}$ in. internal diameter brass limbs (Plate 4.4.) The limbs were connected independently to a constant pressure apparatus similar to the Bishop and Henkel (1962) arrangement shown in their Figure 27. The open end of each limb had a 45° chamfer on a $\frac{1}{2}$ in. outside diameter tube and a rubber membrane provided a cover over the tapered ends. The membrane was 0.04 in. thick with a $\frac{1}{4}$ in. diameter central area of 0.01 in. thickness; it was fitted over the brass limb with the thinner section coincident with the open end of the brass limb. Rubber 'o' rings provided a water-tight (and air-tight) seal between membrane and brass limb, and the limbs were screwed to a perspex base to form an exact right angle. The pressure cell was located on a central platform within the right angle such that no contact with sensitized diaphragms was possible except via the thinner rubber membrane of an hydraulic limb (Plate 4.4.) Outward deflection (if any) of unstressed diaphragms were permitted and stressed cell faces fitted flush with the area of thinner membrane through which pressure was translated. It was necessary to 'pack' the cell against brass limbs using 0.0025 in. shims slotted between the cell and its vertical support.

Pressures from 0 to 5 p.s.i. were applied via either brass limb by adjusting the height of the appropriate Bishop and Henkel compensating mercury pot device. A mercury manometer that was graduated in p.s.i. was fixed to the pressure control panel and ensured correct application of hydraulic pressure via the limbs; the constant pressure was accurate to approximately $\pm$ 3½% for each pressure increment of one p.s.i.

Six tests per cell were made with a repeat load-unload cycle for each test. A typical series of load-unload tests for cell No. 3 are presented in Figures 4.9(a), (b) and (c). Figures 4.9.(a) and (b) show one load-unload cycle with different stress combinations and Figure 4.9.(c) represents two load-unload cycles, (i) and (ii), with different constant pressures applied to diaphragm No. 2. for each cycle of loading.

In the case of cell No. 3 the values of 'cross-sensitivity factors' ranged from 2.75% to 4.24% of strain recorded on the stressed diaphragm. Figure 4.9(b) shows that registrations on diaphragms 2 and 3 were virtually identical and although this was not so with all pressure cell calibrations the differences were not great. Applied pressure - recorded strain relationships were linear within the limits of expected accuracy (pressure differences obtained with constant pressure apparatus varied slightly) and for a total of twelve tests per cell no hysteresis was experienced. Average values of 'cross-sensitivity factors' obtained from computerized analysis for cells No. 1 to 10 are given in Table 4.1.

The theoretical stress-strain curve was 21% less than experiments suggested but parameters that were used to obtain the theoretical diaphragm behaviour may have been in error. For example a diaphragm thickness of 4.5×10^{-3} in.(i.e. 5×10^{-4} in. less than the assumed thickness) would have given a theoretical strain only 4% lower than the experimental strain. Using approximate parameters, available workshop measuring instruments and equations for the central radial strain, the experimental results were of the same order as theoretically predicted strains.

4.4. (iii) Air and Water all-round pressure calibration.

(a) Air.

The perspex cell of a 4 in. diameter triaxial apparatus was used as an air pressure chamber for calibration of the pressure cells in air. The pressure cell lead wires were fed downwards through an enlarged p.w.p. hole in the base of the triaxial apparatus, passed through a Klinger tap and sealed by screwing a cup-shaped nut against the tap outlet. A rubber 'o' ring within the cup was squeezed against the lead wires. Complete air tightness was not automatically guaranteed and in some tests it was necessary to seal the connection with a liquid sealing compound (O-so-tite).

Air pressure was applied to the sealed perspex cell from an air compressor via a second Klinger tap; a control valve between the compressor and triaxial cell regulated pressure within the air chamber.

A third Klinger tap provided an outlet for a connection to the mercury manometer and by maintaining 1/16 in. water in the cell, air was prevented from moving into the pressure control apparatus.

The pressure cell was supported in the air chamber with two steel blocks sandwiching the extension leads, thus holding the cell above the water in the cell base.

Calibration tests were carried out for a range of applied pressure from 0 to 20 p.s.i.

(b) Water.

Application of pressure in the case of hydraulic calibration entailed filling the perspex cell completely with water and then applying pressure using the mercury pot pressure control device. The water used in the calibration experiments had been circulated in a de-airing apparatus that drew water from the mains and to ensure an appreciably constant temperature the cell, full of water, was left overnight. As with air pressure tests the range of applied pressures was from 0 to 20 p.s.i.

Figure 4.10. illustrates typical all-round pressure tests on cell No. 3 for both air and hydraulic load-unload cycles. Air calibrations generally gave slightly lower registrations than hydraulic pressures; this was thought to be due to better heat dissipation in air, although the manufacturers suggested a maximum continuous current of 20 mA . through a semi-conductor gauge with acceptable self-heating effects. A maximum continuous current of 7.25 mA. was passed through each gauge in calibration tests (i.e. a 5 volt system voltage was used to excite the 350 ohm. resistance gauges).

The hydraulic limb tests reported in Section 4.4.(ii) were carried out in open surroundings and since $[\alpha_{ij}]$ evaluation was based on these experiments, correction factors for the pressure cells represented behaviour in air and not water. The heat dissipation problem, therefore, was of little direct consequence in the calibration tests described here.

Obviously the performance of a pressure cell that is to be used to measure pressures in a sand mass must be investigated for the application of known sand pressures, and in this respect a series of experiments were performed with pressure cells in 4 in. diameter triaxial sand samples. Section 4.4.(iv) describes the sample preparation and the type of test, and Section 4.4.(v) discusses the results of the experiments.

4.4.(iv) Calibration in sand-sample preparation.

Each pressure cell was placed at the centre of a 4 in. by 8 in. high sand sample and calibrated under all-round pressure. The samples that contained pressure cells No. 2, 3 and 4 were subjected to drained triaxial tests with radial stresses of 5 p.s.i. and 10 p.s.i. applied to the vertical boundary of the sand cylinder.

To prepare the sand sample in a conventional triaxial apparatus the cell was initially suspended centrally at the mid-height of a three-section sand former. Cell lead wires were passed through the triaxial cell base and sealed at the outlet tap as described in the previous section. The pressure cell was thus fixed in a position with the top diaphragm (No. 3) horizontal and the lead wires from the face opposite diaphragm No. 3 extending vertically downwards into the triaxial cell base. Two $\frac{1}{4}$ in. wide strips of magnetic tape held the cell in its correct position and these were attached to a thin steel bar that rested on the perimeter of the sand former. Sand was poured into a 0.012 in. rubber membrane, that stretched over the former, until the underside of the pressure cell was just in contact with the sand surface; the magnetic strips were then carefully removed and the cell remained supported by the sand surface and lead wires. The sand pouring operation was completed and the sample was sealed with a light-alloy top cap and 'o' rings. An initial porosity of sand sample was found, for the series of ten experiments, to be approximately 40%, representing a medium-dense sand.

A negative pressure of 0.5 p.s.i. was applied to the sample, the former was removed and a triaxial sand specimen within a perspex cell was obtained. The specimen was left to reach equilibrium temperature under an all-round pressure of 1.7 p.s.i. (i.e. the difference in height between the de-aired water reservoir and the centre of the triaxial sample). The preparation of each sample for pressure cell calibration took approximately four hours and the test program was arranged so that prepared samples were left overnight in order to achieve the same temperature as the constant-temperature annex.

The equal all-round calibration tests took the form of load-unload cycles up to 5 p.s.i.; 8 cycles were carried out in all cases. The method of applying pressure was similar to that for all-round hydraulic calibration with pressure being recorded on a mercury manometer connected to the outer part of the perspex cell.

For the triaxial tests proving ring readings and deflections were recorded at fixed time intervals. Since the tests were conventional the measuring equipment gave load and compression at the top cap only. The recording oscillograph produced output from pressure cells on print-out paper and transverse lines on the print-out at every second simplified the interpretation of galvanometer deflection in terms of a recorded pressure at a specific time.

Discussion of the all-round and triaxial tests follows in Section 4.4.(v).

4.4.(v) Consolidation and drained triaxial tests.

Before the results of triaxial experiments are interpreted it is relevant to briefly discuss the distribution of strain (and stress) in a triaxial sand sample during consolidation and compression testing; this discussion follows.

It has been suggested that in triaxial sand samples the rigidity of end platens gives rise to anisotropy during consolidation under equal all-round cell pressure (Ladanyi, 1963). However, it is most likely that anisotropy is very slight and is predominent in the locality of the end platens, where restrictions on volume change are greatest. It is reasonable to assume that consolidation at the centre of a 4 in. diameter triaxial sand sample is isotropic; this assumption requires that pressures in the sand sample at the pressure cell position are the same as the all-round cell pressure and, as will be seen later, enables the consolidation tests to be used to calibrate pressure cells for use in sand.

During drained triaxial compression tests on sands strains at the centre of a cylindrical specimen are very different from average strains for the same apparent deviatoric stress (Roscoe et. al. 1963; Truesdale and Rusin, 1963; Rowe and Barden, 1964). Average strains are usually measured using a clock-type deflection gauge that records downward movement of the plunger and load cells or proof rings record load applied to the plunger, giving the deviatoric stresses on an assumed right cylindrical sample. The methods of localized strain measurement are varied and will not be discussed here.

According to Truesdale and Rusin axial strains at the centre of a 5.75 in. diameter sample were in some cases more than 100% greater than average strains (with a cell pressure of 9.75 p.s.i.) and the corresponding lateral strains were found to be 74% greater than average strains. Other results that have been reported show similar increases of axial and lateral strains at the centre of triaxial sand samples during drained testing.

It is now opportune to discuss the results of the consolidation and triaxial tests that were performed as a part of the pressure cell calibration program.

The results of consolidation tests for cell No. 3 are shown in Figure 4.10. along with stress-recorded strain relations for air, water and individual calibrations. Figure 4.10. indicates the false registrations during consolidation and, if the assumption that consolidation in itself does not lead to differences between triaxial cell pressure and sand pressure is adopted, the false registration represents an under-registration of 8½% for cell No. 3. For the complete batch of cells the under-registration varied from 7.9% to 8.8%.

The cause of under-registration was, in all probability, local arching of sand grains over the abutments of the pressure cell, this being in agreement with the theoretical distribution shown in Figure 4.7.

The under-registration phenomenon has played a retarding role in pressure measurement in sand, although the design criteria, in this case, based on the theory of Kallstenius and Bergau (1957) and the experimental studies of Rowe (1954), have usually been satisfied.

Figure 4.10. also shows the hysteresis during loading, and Figure 4.11. presents repeat load-unload cyclic tests. The under-registration of pressure within the sand was appreciably constant with increasing pressure and this is sufficient reason to assume that full calibration ensures that pressures existing in a sand mass can be measured provided such calibrations are undertaken. The final under-registration factor

for each pressure cell was determined from relationships similar to that shown in Figure 4.10.

After 8 load-unload consolidation tests to 5 p.s.i. the triaxial cell pressure (σr) was maintained constant and a drained triaxial test was carried out. Upon completion of the test the cell pressure was reduced to atmospheric pressure, then the sample was consolidated under 10 p.s.i. and a further triaxial test was performed. The results of radial and deviatoric stress versus axial compression obtained by conventional means and the pressure cells (i.e. 2, 3 and 4) are shown in Figures 4.12. and 4.13. The pressure cell recordings were interpreted from print-out from the recording oscillograph using equations (4.14.) and conventional results were obtained as explained in Section 4.4.(iv).

A rigorous analysis of these results is outside the scope of this investigation but relevant discussion follows.

The pressure cell output, interpreted as a pressure in the sand sample, differed by as much as 50% from average pressures measured by conventional methods, but the shape of deviator stress plots were similar. The obvious explanation for the differences illustrated in Figures 4.12. and 4.13. refers back to the discussion earlier in this section: during consolidation tests the pressure cell gave an under-registration of about 8% and using this difference the localized pressures at the centre of a 4 in. diameter triaxial sample were in the region of 62% higher than average stresses at the centre.

Without further discussion it suffices to point out that the pressure cell had proved its effectiveness in the measurement of sand pressures up to 300% of the designed maximum pressure and gave results that could be explained by means of the concept of non-uniformity during triaxial testing of dry sand. A valid conclusion is that the pressure cell can confidently be used, at best for loading, within the range of pressures expected during penetration resistance experiments.

The hysteresis phenomenon was indicated in Figure 4.11. and on first load-unload consolidation cycles this was quite noticeable. Unloading calibration must be treated more carefully and it is quite probable that interpretation of cell output will yield erroneous results.

In the triaxial tests the pressure cells were subjected to stresses much greater than they were originally designed to measure. There was no possibility of permanent damage to the pressure sensitive faces of the cell since the elastic range was 12x the design range, but even at the higher pressures the difference between cell readings and deviatoric stresses were appreciably constant. This implies that the effect of arching, (if arching did exist) is constant and can be allowed for in an extensive calibration sequence. The cell, in fact, could provide a small but precise instrument for the measurement of pressure in a sand mass during the penetration of the sand by a rigid, conical pointed penetrometer.

4.4.(vi) Temperature calibration.

Simply as a matter of interest pressure cell No. 10 was subjected to a range of temperatures from 18°C. to 40°C. The calibrations, in the form of air and water surround to the cell, indicated that the pressure cell was very sensitive to change in temperature. It was found that 1.67°C. gave an output (i.e. galvanometer trace distance) equivalent to the galvanometer deflection for an applied pressure of one p.s.i.; as expected an increase of temperature gave an increase of gauge output. The importance of either:-

(i) a true constant temperature enviroment, or

(ii) a dummy cell situated close to a recording cell,

was abundantly emphasized. In penetration tests the latter temperature correction was used and, as shown in final results, proved to be suitable in pressure cell measurement. Temperature correction (i) was achieved during the calibration experiments that formed a basis for assuming that the pressure cell could be used as intended.

4.4.(vii) Positioning of pressure cells for penetration tests.

In the penetration experiments with pressure cells the most difficult single operation was the placement of pressure cells within the mass of sand without undue disturbance to surrounding sand. The only satisfactory method of placing the cells was found to be placement by hand. The cells were placed in a radial plane, normal to the centre of the glass plate of a half section container (described in Section 4.5.(ii)); each cell position was obtained by using two theodolites; the first was set up at an accurately located point to the side of the container and the second was positioned to be in line with the central radial plane on which the cells were placed.

The precise position of a pressure cell was established by measuring horizontal and vertical angles and then using simple trigonometry; r and z co-ordinates (see Section 5.2.(i)) could be determined to the nearest 5×10^{-4} in. and θ (Section 5.2.(i)) was, of course, virtually constant.

The positioning of pressure cells in dense sand presented no problems but it was accepted that in loose and loose-medium sand samples the layers of sand immediately below the cells were disturbed. The sensitized diaphragms were covered with sand raining from above, hence the disturbances in the lower layer did not seriously affect cell performance. The disturbances caused by placing, however, affected the penetration resistance and probably altered the mode of failure in the loose sands. Since the area of deformation is small for loose sand samples, the effects of disturbances were not widespread and the investigations were not seriously in error.

The results from cell recordings, in fact, suggested that the pressure measurement technique was successful; Chapter 6 presents these results.

4.5. Measurement of displacements.

4.5.(i) Previous studies and radiography techniques.

Researchers in soil mechanics have long realised a need for measuring the displacements of particles during laboratory experiments on granular materials. The measurements in the past have been qualatitive but techniques have now evolved to enable strain fields in a granular mass to be accurately determined. A brief review of past methods and the state of present day techniques is not out of place here.

Krey (1936) was probably one of the earliest research workers to use a transparent boundary in laboratory experiments; a rigid glass plate, against which the penetrating tool and sand were in contact, formed one side of a large container. Displacements were recorded using a stationary camera technique; a course sand was used and results, in the main, were for surface footings. Similar experiments were performed by Meyerhof (1948); he used a smoked glass plate as the rigid boundary and obtained the pattern of overall deformations again for two-dimensional surface footings. Yassin (1950) dyed sand with methylene blue and placed the coloured sand as 0.1 in. layers alternating with naturally coloured sand. After penetration tests the layers were frozen by passing a weak solution of cement through the sand mass. Excavation of sand revealed the deformation patterns during penetration. Fleming (1957) used white and red sand in layers, each layer being mixed with 7% dental alabaster. The layers were frozen after penetration experiments, and then cut through vertical planes to give a picture of displacements in a solidified form. Rowe and Peaker (1965) placed columns of white sand in a brown test sand, the entire sand mass forming a body behind a model retaining wall. Upon completion of the experiments the sand was carefully excavated to reveal the amount of displacement of white sand columns in relation to the brown sand. There are other methods of strain measurement, whereby thin sheets of thermosetting resin are positioned within a sand and experiments are carried out at elevated temperatures. Subsequent cooling freezes the resin and permanent records of displacements are obtained. The scope of such methods is obviously limited and for experiments other than laboratory testing of soils the method seems to be impracticable.

More recent developments in strain (displacement) measurement in soils in laboratory models have used radiography techniques (Roscoe, Arthur and James, 1963). This method of recording displacements, without the effects of friction between a sand mass and transparent boundary, enabled Robinsky and Morrison (1964) to record the displacements around tapered and straight-sided model piles in loose and loose-medium sands at zero, 20 in. and in some cases 10 in. penetrations. The sand container used by Robinsky and Morrison was 20 in. by 28 in. in plan and penetration tools ranged from 0.8 in. to 1.48 in. diameter; these sizes obviously necessitated a more powerful source of radiation than had previously been used in soil mechanics research (e.g. see

Sirwan, 1965) but the technique itself was a logical development from the methods mentioned above. Future development along the lines adopted by Robinsky and Morrison is doubtful from a practical angle but the stage that was reached offered a more precise method of displacement studies.

Arthur and Roscoe (1965) investigated the displacements in a model earth pressure apparatus by using radiograph (i.e. X-ray 'photographs') and conventional photographic techniques. The experiments were confined to a two-dimensional study of strain fields in a 6 in. wide glass sided flume but a review of the methods and findings is worthwhile.

Lead shot placed in a central vertical plane normal to their rotating rigid boundary provided the necessary points for displacement measurement without 'edge effects' (i.e. the effects of frictional forces between the deforming sand and the rigid glass walls) and nylon hemispheres against the glass sides of the 6 in. wide flume moved with sand displacements to give the strain field with 'edge effects'. It was found that the distribution of maximum shear strain and the directions of major principle strain, obtained from analysis of radiographs and conventional photographs, showed good agreement. 'Edge effects' did not seriously affect the resulting displacements obtained from measurements against the glass sides of the model flume.

A second finding reported by Arthur and Roscoe was that the degree of rigidity of the glass side plates had a significant influence on earth pressure. As will be shown later, the rigidity of a glass plate used by the Author was adequate and in consequence the finding is only of slight importance.

In experimental studies such as those undertaken by the Author, where a knowledge of displacements is required, the transparent rigid boundary would appear to provide an acceptable series of results. Additional investigations similar to those of Robinsky and Morrison (1964) would add weight to the validity of experiments where 'edge effects' existed.

An attempt was made to measure strain fields using radiography techniques after the experiments (described in Section 4.5.(ii)) had been completed. A one curie source of Cobalt 60 gamma radiation was obtained (on a temporary basis) from the Physics Department at the University and was used to produce radiographs of the sand mass during penetration testing. Initial investigations of a theoretical nature had suggested that the one curie source of radiation would suffice provided certain basic criteria were satisfied.

It is not necessary to discuss the investigations in detail here, since this could only be done successfully by introducing the fundamental concepts associated with gamma radiation and developing these concepts to arrive at the final choice of geometric arrangement, exposures (in curie-hours), types of radiation film and expected film densities. A separate unpublished report on the radiography studies has been prepared and has been referenced in the Departmental Library. Much of the theory was interpreted from extensive studies by Rumyatsev (1965) and some computations were approximate because precise values of the narrow beam linear absorption coefficient of the fine white sand was not known; an approximate value (of 0.1086 cm^{-1}) at the average energy level of Cobalt 60 (i.e. 1.25 MeV) was assumed.

The final considerations that were to produce suitable radiographs were a source diameter of 0.25 in. Lead shot of diameter 0.12 in. was selected for the radiographic studies and was placed in a $\frac{3}{4}$ in. grid in the sand mass in a plane normal to the front glass plate of the 'half section' container. The container (described in Section 4.5.(ii)) was fitted with a vertical guide and a perspex lead shot placer was able to slide in a vertical plane that intersected the front glass plate at the centre line of a half section penetrometer (see Section 4.5.(ii)). The sand was deposited in $\frac{3}{4}$ in. layers and lead shot were placed on the surface of each layer by means of the perspex placer. An image size of 0.18 in. would be transmitted to the 15 in. by 18 in. film. 'Kodirex' X-ray film was selected from the Kodak Ltd., range; it was a coarse grain film but it gave the required contrast at an estimated film density of 1.0. Lead intensifying screens (0.004 in. front and 0.006 in. rear) sandwiched the film in a film cassette and the complete assembly was attached to the perspex side wall of the sand container opposite the radiation source. A lead backing sheet was clamped to the cassette to reduce scatter. Exposure times were estimated to be $8\frac{1}{2}$ hours for loose sand and 12 hours for dense sand samples and each exposure was carried out overnight; this arrangement meant that, provided adequate warnings were posted and safety precautions taken, there was no need for heavy protective shielding around the Cobalt 60 source. During the day the source was sealed in a lead container and cordonned off in the laboratory. A carefully worked out program of work ensured that, as far as was practicably possible, the laboratory was vibration free during one complete penetration test. The Cobalt 60 source and half-section container are shown in Figure 4.5.

The first radiograph experiment on loose sand did not reveal the lead shot markers on developed film, although the overall film density was about 1.3 (as measured with a densitometer). It was hoped that double exposure times would produce results but the resulting radiographs again showed no trace of lead shot.

The probable reason for a conflict between theoretically obtainable results and those actually obtained was scatter of radiation in the sand sample between the lead shot and film. The distance was probably such that extensive scatter had obliterated the trace of lead shot. The experiments were repeated with lead shot of 0.2 in. diameter but with double exposure times (i.e. 17 hours) the markers still did not show on developed film.

At this point a closer study of Robinsky and Morrison's work was thought necessary and it was found that they had, in fact, began experimentation with a one curie source of gamma radiation (Robinsky and Morrison, 1969) but eventually the experiments had demanded a radiation source of 135 curies (of Cobalt 60) before an arrangement similar to that of the Author produced suitable radiographs. The reasons given for gamma ray absorption that was vastly different from theoretically predicted behaviour were not clear but it appears that, with a material such as sand the voids between particles cause more scatter of gamma radiation from individual particles than is normally obtained in materials with similar specific gravities (e.g. concrete). It was difficult to appreciate that errors were of the order of 135x, but on the basis of the work done by Robinsky and Morrison (1969) it was considered impractical to continue the radiograph techniques at this stage of the research program.

A final experiment with the one curie Cobalt 60 gamma-ray source was carried out with a $\frac{1}{2}$ in. thick 'L' shaped lead block placed between the radiation source and sand container, adjacent to the perspex wall; with an exposure of approximately 9 hours the block was clearly revealed on the 'Kodirex' film and the object-film distance in this case was more than double what it was for the lead shot. This finding confirms the belief that radiation scatter was responsible for the unsuccessful attempts to produce radiographs of sand displacement in the container used by the Author.

The lack of useful results for measurement of displacements withc 'edge effects' must inevitably reduce the proposed method for investigating the movement of sand during penetration (described in the follc ing section) to a less quantitive analysis. Nonetheless the validity c the analysis still applies, and as shown later, provides an extensive picture of the strain fields during penetration into loose and dense sands.

4.5.(ii) Apparatus for deformation measurement.

A 1ft.3in. by 2ft.6in. by 3ft. deep container was designed and constructed in the department workshop, to allow the deformations against a rigid, transparent boundary to be determined during penetrat into a sand mass. Three stiffened $\frac{3}{4}$" thick perspex plates were bolted together through 2in. square rectangular hollow sections (R.H.S.) to form the sides and rear walls of the container. The R.H.S.'s were fixe to an inch thick mild steel base plate that rested on timber joists sandwiched between layers of anti-vibration rubber. A $\frac{3}{4}$in. thick glass plate formed the front face of the rigid container and this was seated on 1/8in. rubber strip fixed to a perspex base block; the perspex bloc fitted within the container and provided a seal with the stiffened walls. A vertical seal between the glass plate and perspex walls was achieved by clamping rubber strips between the two materials (Plate 4. The glass plate, clamped at its two vertical edges, provided a rigid and relatively smooth* boundary against which sand rested. Horizontal

* The coefficient of friction between sand grains and a glass plate, as determined in the direct shear box, was 0.17 for dense and 0.11 for loose sands respectively.

stiffeners were made as an accessory to the sand container to prevent lateral deflection of the glass plate. Rubber padded plates were screwed through two one inch square steel bars and applied pressure to the glass plate. The horizontal bars were designed to slide in a vertical plane in front of the plate to allow photographs to be taken without undue interference at any particular stage of penetration. In practice the horizontal stiffeners were found to be rather inconvenient to operate effectively and were discarded; the deflections of the glass plate were, however, considered to be negligible. A theoretical deflection at the bottom centre of the clamped glass plate was 4×10^{-4} in.; this was thought to be sufficiently small to be of no consequence in the penetration tests.

It was possible to dismantle the container in a few minutes, but re-assembly involved accurate alignment of the front plate using a theodolite and plumb-bob.

A semi-circular brass penetrometer was designed to move against the inside face of the glass wall with complete contact (see Plate 4.7.). The penetrometer was milled from a one inch diameter brass tube to form a semi-circular half-ring, a 60° hollow half cone was brazed to the flat base and an adapted ball bushing held the penetrometer flush against the inside glass surface. A brass half cylinder inside the bushing provided a surface against which the penetrometer was able to slide (Plate 4.6.) and the assembled bushing ensured verticality of penetration in planes normal and tangential to the glass boundary.

A grid of fine lines was drawn on the inside face of the glass using black Indian ink. The overall grid had $1\frac{1}{2}$ in. spacings but in a region closer to the centre line a $\frac{3}{4}$ in. grid was added. Each intersection point of horizontal and vertical lines represented the position of a marker in the sand adjacent to the plate. The $\frac{3}{4}$ in. grid was considered most suitable for penetration tests in loose sand samples and for dense tests the $1\frac{1}{2}$ in. grid was used. Small black perspex beads were chosen as markers, being approximately 0.11 in. in diameter, 0.08 in. thick and having a central hole 0.04 in. diameter. Originally small conical and hemispherical markers made of teflon were considered but these were not readily obtainable and proved expensive to manufacture in the departmental workshop (in terms of man hours). The plastic beads, on the other hand, were obtained from a local haberdasher and cost 2/6d. per gross. As it was the barrel shaped bead proved to be quite suitable, the central hole providing an accurate location point when movements were later measured from photographic negatives.

Sand samples were prepared in the 'half-section' container using the 'rain' technique (Kolbuszewski and Jones, 1961) and trial tests showed that the markers were best located by fixing them to the glass plate prior to its reassembly. The plate was laid horizontal, cleaned with trichloroethylene and the markers were fixed using Vaseline Petroleum Jelly. The technique for applying the Vaseline was to spread it thinly on a metal plate and (using tweezers) bring each marker into contact with the Vaseline spread. About $\frac{1}{4}$ c.c. of Vaseline was sufficient to coat at least 150 marker beads and the light coating was just sufficient to hold the marker in its correct position on the glass plate when it was lifted upright. In one earlier experiment the reassembled apparatus was left for 48 hours; during this time the laboratory temperature rose, a slight decrease in viscosity of the Vaseline resulted, and the markers slid about $\frac{1}{4}$ in. down the glass plate.

Provided the sand was deposited soon after the container had been reassembled the bond offered by the Vaseline was just enough to hold markers in position on the vertical face.

The markers remained in position throughout the preparation of dense sand samples (using the deposition apparatus described in Chapter 3) but during loose sample preparation a slight downward movement of the markers was noticed, the movement being confined to markers that were covered with sand. Although great care was taken to avoid knocking the loose samples during preparation it was thought that slight settlement of the sand has carried the markers along the glass plate. The maximum movement was estimated to be approximately 0.015 in. and appeared to be the same for all markers, but there was no way of verifying this without disturbing the loose sand further.

For a penetration test in the 'half-section' container the loading rig was lowered onto rubber pads, on the upper horizontal ends of the R.H.S's, using a fork lift truck. The penetrometer was aligned correctly with two theodolites to ensure a sand free contact between the penetrometer edge and the glass surface. The loading rig was then secured to the container with bolts.

The bushing and housing for the penetrometer were firmly secured and the penetrometer point was lowered to give zero penetration. Plate 4.8. illustrates the 'half-section' apparatus at this stage of an experiment. Plate 4.9. gives a front elevation of the container and shows the penetrometer after a full experiment.

Experiments were performed at a constant rate of penetration (see Section 2.4.) and photographic negatives were taken with a stationary camera at penetration intervals of 3 in., 1 in. and sometimes 0.1 in.

A Hasselblad 40010 camera was used and an object-film distance of 3 ft. 6 in. (approx.) was employed. The position of the camera was carefully checked before commencement of each test to ensure correct alignment in the plane of, and normal to, the glass plate.

Loose, loose-medium and dense sand samples were penetrated to depths of 15 in., 21 in. and 27 in. respectively, and a total of 24 displacement records per test were obtained.

The measurement of deformation would have been an impossible task but for the use of a precision instrument that was available in the Photogrammetry Section of the Civil Engineering Department. Methods of measuring the movement of plastic markers, discussed in the following Section, introduces the Wild A7 Autograph and briefly emphasizes its potential in the field of analysis of deformations in Civil Engineering studies.

4.5.(iii). Measurement of displacements.

A Wild A7 Autograph, with a co-ordinate plotter and printer, was used to measure displacements during a penetration test by recording the position of markers shown on a photographic negative.

A zero penetration negative was positioned on one of the two plotting cameras in the Autograph and a second negative showing deformations during penetration, was located on a glass plate grid of the opposing plotting camera. Stereoscopic coincidence was obtained with the two photographs by carefully adjusting their positions on the frame of plotting cameras and deformations were recorded from either negative.

The A7 Autograph mechanical co-ordinate measuring drums were connected to an electric typewriter through an integrated electronic circuit and co-ordinates of a small point measuring mark that was visible on the plotting camera plates were typed as required. By siting the measuring mark within the small hole of a bead marker (i.e. by moving the negatives in two orthogonal directions until a particular point coincided with the measuring mark) the position was recorded in terms of r and z co-ordinates, as shown in Plate 4.7. Initial experiments showed that, with experience, it was possible to locate the measuring mark within the hole of a marker bead to an accuracy of $\pm$ 0.01 m.m. on a negative 3.7 in. square. Since the negative represented an area of approximately 24 in. square on the glass plate, the corresponding accuracy, related to the size of the experiment, was $\pm$ 0.002 in.

The minimum number of points per negative was 280 and each point took approximately 30 secs. to locate and print. A greater degree of accuracy would have been possible with larger negatives but the present degree of accuracy was considered to be sufficient. Roscoe, Arthur and James (1963) were able to locate 2.5 m.m. diameter lead shot in a mass of sand to an accuracy of $\pm$ 0.001 in. with large radiographs, and with larger negatives the present analysis would have given comparable accuracies.

The type-written co-ordinates produced by the Wild A7 Autograph were converted into strain components using an analysis program for an I.C.L. 1905 computer. Originally a program had been developed for processing by a smaller computer, an Elliot 803, but this was phased out of operation and it was necessary to develop a modified program for use with the recently installed I.C.L. computer.

The theory of strain analysis is given in the following chapter, this being devoted mainly to a theoretical study of stress distribution in the sand mass during penetration.

4.6. Concluding remarks.

The development and calibration of a three dimensional miniature pressure cell has been described, and the extensive calibration procedures have shown that false registrations of the pressure cell did exist.

A general under-registration of approximately 8½% was determined from the results of calibration tests in sands, air and water and cross-sensitivity effects accounted for a 3½% (approx.) under-registration

The error factors have been combined into a suitable calibration matrix for each cell and this will subsequently be used to compute pressures in a sand mass during penetration experiments. It remains now to obtain values of stress that can reliably be used to help understand the penetration resistance of sands.

The measurement of strains, or rather the measurement of displacements, in the same sand mass was made possible by using the technique of recording movement of sand against a rigid transparent boundary. Previous work has suggested that the results from this technique gave reasonably accurate strain fields. The technique developed by the Author provided a convenient method of recording displacements; analysis was carried out quickly and accurately for many hundred displacements.

At this stage of the study of the penetration resistance of sands the separate projects have been described and they have been shown to be as successful as time and existing facilities allowed.

The results from the separate studies have, where possible, been reported as an integral part of a specific chapter and it is left to report and analyse the stress and strain results in Chapter 6. Since an important finding from the experimental studies arose as a result of a theoretical approach to the penetration resistance problem it is worth introducing the theoretical analysis as a continuation of the initial projects. The more relevant aspects of the theoretical development deserves a place in the following sections; this place is found in Chapter 5.

C H A P T E R 5

Theory of limiting stress

5.1. Introduction

It is not necessary to stress the importance of a viable sign convention in any continuum mechanics research investigation. Section 5.2 explains the convention which has been used in the following chapters; 5.2.(i) fully describes the convention for stresses at a point and 5.2.(ii) describes the associated sign convention for strains at a point.

Throughout the early studies which were reported in Chapter 2 it became apparent that the problemsof investigating the penetration resistance of sands could best be approached by studying the distribution of stress and strain around an instrument during its penetration into a mass of the material. The bulk of this chapter, Sections 5.3. and 5.4., relates the theoretical approach to the axially-symmetric penetration problem and explains selected results. The solutions are not exact, since they are entirely lower bound, upper bound solutions having been disregarded in the present programme of work. However, since it is usual to develop the statically admissible solution (i.e. the stress fields) first and then use the results to complete velocity fields, the approach employed by the Author is entirely logical.

The problem of axially-symmetric quasi-static penetration in sands appears to have received little attention in the last few decades, but nonetheless the state of development at present certainly suggested that such a solution was a probability. A brief review of the more significant of these developments is included as in introduction to the solution for determining stress distributions in the plastically deforming region.

Perhaps it is opportune here to discuss the major limitations of the solution since these are mainly problematical.

The numerical techniques which ensue are based on the development of a complete stress field from an initial vertical boundary, and a final inclined boundary, which are assumed to represent the shank and conical ended base respectively of a 1in. diameter brass penetrometer with a 60° cone base. The penetrometer has been adopted in the experimental programme, but has not given an indication of boundary conditions. The validity of the solution must therefore depend upon a number of assumptions, particularly concerning the properties of the sand at the sand/penetrometer interface. In fact, the validity of the solution to a certain extent rests upon a series of direct shear tests with sand on brass.

Because a complete stress field must be developed along the vertical boundary the limiting stress characteristic solution is only applicable to deep penetration; for loose sand conditions the minimum depth would probably be about 6in. and for dense conditions it would increase to about 10in. Since penetration resistance is not specifically associated with shallow depth conditions, this is not considered to be a serious limitation.

Chapter 5, beyond 5.2., is separated into two sections and a short conclusion. Section 5.3. describes the methods of obtaining the statically admissible solution, and includes the techniques such as mapping procedures and iteration methods which form a part of the solution. 5.4. is broken down into three parts which discuss different results of the limiting stress field computations. 5.4.(i) discusses boundary condition assumptions. A comparison of iteration methods and results for two and three dimensional studies is given in Section 5.4.(ii) and finally the limiting stress characteristics and stress distributions are discussed, and related to some earlier experiments, in section 5.4.(iii).

5.2. Stress and strain components.

5.2.(i) Stresses at a point

The study of the penetration resistance of sands, as with any mechanics of solids problem with axial-symmetry, requires the use of cylindrical polar co-ordinates in the analytical approach that is adopted. Figure 5.1.(a) shows the left-handed system of polar co-ordinates r, θ and z, as used in the present analysis.

If a small element of soil within a continuous soil body is subjected to some external axially-symmetric loading the stress components acting on the faces of the element (Figures 5.1.(a) and (b)) can be represented by the general stress tensor matrix:-

$$\begin{bmatrix} \sigma_r & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \sigma_\theta & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \sigma_z \end{bmatrix}$$

Effective and total stresses are equal (a consequence of studying the penetration resistance of dry sand) and the condition of axial-symmetry requires the shear stresses $\tau_{\theta r}$ and $\tau_{\theta z}$ to vanish. Since complementary shear stresses are equal, $\tau_{r\theta}$ and $\tau_{z\theta}$ also disappear and stress components σ_r , σ_θ , σ_z and τ_{rz} are independent of θ.

The sign convention that is universally adopted in soil mechanics gives the predominant stresses as positive; this convention, used throughout Chapter 5, is:-

(i) compressive stresses are positive.

(ii) shear stresses on faces of an element (such as that shown in Figure 5.1.(a)) adjacent to the axes are positive, in the directions of the axes.

Positive shear and normal stresses are shown in Figure 5.1.(c).

The principal stresses σ_i (i = 1, 2, 3) at a point can be obtained from the expansion of the determinant:-

$$\begin{vmatrix} \sigma_r & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \sigma_\theta & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \sigma_z \end{vmatrix} = 0 \qquad (5.1.)$$

The principal stresses are found from the expanded form of the determinant as follows*:-

$$(\sigma_\theta - \sigma_i)\left\{(\sigma_z - \sigma_i)\,.\,(\sigma_r - \sigma_i) - \tau_{rz}^2\right\} = 0 \qquad (5.2.)$$

Because of axial-symmetry the θ = constant plane is a principal plane and thus σ_θ is the intermediate principal stress; σ 2. Using this phenomenon equation (5.2.) can be rewritten in the form+ :-

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_r + \sigma_z}{2} \pm \left\{\frac{(\sigma_r - \sigma_z)^2}{4} + \tau_{rz}^2\right\}^{1/2}. \qquad (5.3)$$

$$\sigma_2 = \sigma_\theta.$$

It is relevant to note that σ_θ can be numerically equal to either σ 1 or σ 3 (i.e. σ 1 = σ 2 or σ 2 = σ 3) and as will be shown later the equalities in parenthesis form the basis of the numerical analysis methods outlined in this chapter.

The Mohr stress diagram will be seen to be important in the developments that ensue and it is useful here to specify the sign convention for stress representation on it, i.e.:-

(i) compressive stresses are positive

* Mohr's diagram of stress also gives the principal stress in terms of general stress components

\+ The principal stress suffix notation generally used in soil mechanics has been used here, this being σ 1 $\geqslant \sigma$ 2 $\geqslant \sigma$ 3.

(ii) shear stresses are positive if they are anti-clockwise about a point within the element of Figure 5.1.(c). Figure 5.1.(d) illustrates the stresses on a Mohr diagram.

Two parameters that will be used extensively in Section 5.3.(ii) are σ and ψ. σ is the mean value of stresses σ_r and σ_z (i.e. $\sigma = ((\sigma_r + \sigma_z)/2)$ and ψ gives the orientation of the major principal stress σ_1, with reference to the $r = 0$ axis, as shown in Figure 5.1.(c) and (d). An important property of ψ is its positive value with anti-clockwise rotation about the $r = 0$ axis.

5.2.(ii) Strains at a point

The general strain tensor matrix in terms of cylindrical polar co-ordinates is given as:-

$$\begin{bmatrix} \epsilon_r & \gamma_{r\theta}/2. & \gamma_{rz}/2. \\ \gamma_{\theta r}/2. & \epsilon_\theta & \gamma_{\theta z}/2. \\ \gamma_{zr}/2. & \gamma_{z\theta}/2. & \epsilon_z \end{bmatrix}$$

Symmetry about the z axis eliminates $\gamma_{\theta r}/2$ (= $\gamma_{r\theta}/2$) and $\gamma_{\theta z}/2$ = ($\gamma_{z\theta}/2$) and strain equations analogous to equations (5.3.) can be obtained:-

$$\left.\begin{matrix}\epsilon_1 \\ \epsilon_3\end{matrix}\right\} = \frac{\epsilon_r + \epsilon_z}{2.} \pm \left\{\frac{(\epsilon_r - \epsilon_z)^2}{4} + \frac{\gamma_{rz}^2}{4}\right\}^{1/2}. \qquad (5.4.)$$

$$\epsilon_2 = \epsilon_\theta.$$

A sign convention that is compatible with the convention for stresses must be established and in this respect the following has been adopted:-

(i) compressive strains are positive

(ii) shear strains are positive if the angle between the faces of an element adjacent to the positive axes increases

(iii) components of displacements in the direction of the axes are positive for the positive directions of the axes. The displacement components in the r, θ and z directions are u, v and w respectively and since the argument of axial-symmetry eliminates v, the strains are expressed in terms of the displacements u and w. From Figure 5.2.(a) and (b) it can be seen that strains at a point (at the centre of the infinitesimal element) are given as:-

$$\epsilon_\theta = u/r \; ; \; \epsilon_r = \partial u / \partial r \; ; \; \epsilon_z - \partial w / \partial z \, .$$
$$\gamma_{rz} = \alpha + \beta = \partial w / \partial r + \partial u / \partial z \, . \qquad (5.5.)$$

α and β are shown in Figure 5.2.(b). They are both shown as positive in this particular case.

Rotation of an element of soil in the θ = constant plane without the occurrence of shear strains is referred to as rigid body rotation w_θ ; in terms of partial derivatives of u and w :-

$$w_\theta = (\alpha - \beta)/2 = (\partial w / \partial r - \partial u / \partial z)/2 \qquad (5.5.(b))$$

The volumetric strain ϵ_v is given by the equation :-

$$\epsilon_v = \epsilon_r + \epsilon_z + \epsilon_\theta. \qquad (5.6.)$$

The sign convention for Mohr's diagram of strain gives compressive strains as positive and shear strains are positive for anti-clockwise directions about a point inside the soil element illustrated in Figure 5.2.(a).

The significant parameters in strain analysis are the maximum shear strain, γ_{rz} max., and the angle λ between the major principal strain and the r = 0 axis. The Mohr strain diagram (Figure 5.2.(c)), and Figure 5.2.(d) illustrate the parameters γ_{rz} max./2 and λ .

5.3. Axially-symmetric limiting stress fields

5.3.(i) Introduction

Sokolovski (1965) developed what were probably the most accessible series* of solutions for determining two-dimensional limiting stress

* i.e. accessible to the engineer

fields in the ideal soil mechanics material: a cohesive - frictional soil. The numerical techniques produced the statically admissible lower bound solution and completely neglected the kinematically admissible upper bound solutions that are generally associated with an understanding of strain fields. Nonetheless the procedures developed by Sokolovski have established a permanent position in present day soil mechanics, where in many problems the velocity field solutions are of lesser importance than the 'safer' limiting stress field solutions (see e.g. Drucker, Prager and Greenberg; 1951).

Other methods for the solution of limiting stress fields, such as the graphical solutions of Golushkevich (1957), de Jong (1959) and Bhattachrya (1960), have been developed but the numerical procedures offer the advantage of solution by high speed digital computer. For this reason the methods introduced by Sokolovski have been more widely accepted than the graphical methods; they will be used by the Author for the solution of the problem of penetration resistance of cohesionless materials.

Sokolovski's solutions for plane bearing capacity problems were volubly described by Harr (1966) but since the present problem is one of axial symmetry further development is necessary. In this respect a mention of relevant theoretical studies will not be out of context.

Cox, Eason and Hopkins (1961) produced a detailed theoretical analysis of axially-symmetric plastic deformation of soils. The generalized theory was applied in particular to the indentation of ' a weightless rigid, perfectly plastic soil' by a 'smooth, perfectly rigid, flat-ended punch'. Cox (1962) extended the work of Cox et. al. (1961) to include soil with weight; theoretical analyses for punch and die indentation (i.e. axially-symmetric and plane strain respectively) into an ideal ponderable soil was considered. The rigorous mathematical analyses relied upon the assumption of the existance of Haar and Kármán plastic régimes that were associated with the Mohr-Coulomb yield criterion for soils (see Section 5.3.(ii)).

A more recent theoretical analysis of limiting stress fields was reported by Graham (1968). A perfect, rigid-plastic cohesionless material was assumed and stress fields were determined for a two-dimensional deep strip footing; the numerical techniques, as developed by Sokolovski, were used in the analysis to compute bearing capacity co-efficients.

Although Graham initially assumed that failure took place at constant volume he postulated a narrow trapped zone of looser material along the vertical side of the foundation in order to obtain results that were in agreement with existing theories (e.g. Meyerhof, 1951) and experimental findings. Initial boundary conditions were deduced from the results of selected experimental studies carried out by other research workers.

The determination of limiting stress fields for axially-symmetric quasi-static penetration must consider:-

(i) the circumferential principal stresses, and

(ii) the limiting stress fields adjacent to a penetrometer in a cohesionless material.

In this respect the mathematical investigations of Cox et. al. (1961), Cox (1962) and Graham (1968) have provided a useful theoretical background to the problem considered by the Author. The final theoretical development, however, closely follows the method of solution given by Sokolovski (1965).

It must be emphasized that the numerical techniques developed in Sections 5.3.(ii) to (iv) are primarily directed towards the determination of limiting stress fields and does not attempt to consider the associated velocity fields for an axially-symmetric case. The kinematic solution (i.e. the determination of the velocity fields) for the penetration of sands will be studied with the aid of results from experimental measurement of displacements in Chapter 6.

The Mohr-Coulomb yield criterion for cohesionless soils * suggests that yield is independent of the intermediate principal stress, but by introducing the Hypothesis of Haar and Kármán the yield criterion will be demonstrated to combine suitably with the general equations of equilibrium for the case of cylindrical polar co-ordinates.

The following section describes the combination of yield criterion and equations of equilibrium and lists the equations necessary for the determination of the stress components σ_r , σ_z , σ_θ and τ_{rz}.

Section 5.3.(iii) concerns the development of the partial differential equations that govern the limiting stresses within plastically deforming regions of the material; further parts of the chapter list techniques that render the equations suitable for analysis by computer and discuss the results of the analysis.

5.3.(ii) Mohr-Coulomb yield criterion and equations of equilibrium

The solution of the problem of limiting stress fields, based on an acceptable yield criterion, demands a set of equations that control the distribution of stress within a soil mass; the equations for axially-symmetric conditions are developed, in cylindrical polar co-ordinates, from the well known Kotter equations of equilibrium, giving:-

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \qquad (5.7.a)$$

* described in Section 5.3.(ii)

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = \gamma_d. \qquad (5.7.b.)$$

γ_d is the dry density of the material considered.

The general form of Mohr's (1900) yield criterion can be expressed as:

$$(\sigma_1 - \sigma_3) = f \ (\sigma_1 + \sigma_3). \qquad (5.8.a.)$$

If the envelope of failure plotted from the above equation is assumed to be a straight line, then for an ideal cohesionless soil:

$$(\sigma_1 - \sigma_3) = (\sigma_1 + \sigma_3) \sin \phi \qquad (5.8.b.)$$

where σ_1 and σ_3 are the major and minor principal stresses and ϕ is the angle of inclination of the yield surface (Figure 5.3.(a)). A different form of the yield criterion (modified to apply for cohesionless soils):

$$\tau_f = \sigma_f. \tan \phi. \qquad (5.9.)$$

was proposed by Coulomb in 1776 and generally in soil mechanics ϕ in equation (5.8.b.) is referred to as Coulomb ϕ. The final yield criterion, known as the Mohr-Coulomb criterion, can be expressed in the general form:

$$\sigma_1 = \sigma_3. \tan^2 (\pi/4 + \phi/2) \qquad (5.10)$$

In Section 5.2.(i) the principal stress suffix notation:

$$\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$$

was established and this must obviously remain. However, the principal stresses σ_I, σ_{II} and σ_{III} need to be introduced to avoid confusion in the discussion that follows. The suffices I, II and III can assume any order of magnitude and contrary to general mechanics of solids convention, do not signify the major, intermediate and minor principal stresses. Substituting the possible combinations of σ_I, σ_{II} and σ_{III} into equation (5.10) it is possible to plot an irregular hexagonal pyramid with respect to the three axes that correspond to principal stress directions. The pyramid is the yield surface.

The resulting yield surface on a σ_{III} = constant plane is shown in Figure 5.4. and the conventional representation of the yield locus for the Mohr-Coulomb criterion is represented in Figure 5.5. Essentially Figure 5.5. represents Figure 5.4. with the σ_{III} axis (extending outwards normal to the plane of the paper in Figure 5.4.) tilted in the direction of the origin along the line DO, until the angles that σ_I, σ_{II} and σ_{III} make with the plane of the paper are all equal to $\cos^{-1}/\sqrt{3}$.

Figures 5.4. and 5.5. can be used to illustrate the concept of plastic régimes for a cohesionless soil.

The sides and points of the irregular hexagon fall into distinct plastic régimes (Cox et. al. 1961). Points A and D form the first group of 'singular edge régimes' in which $\sigma_{III} \geqslant \sigma_{II} = \sigma_I$, $\sigma_I = \sigma_{II} \geqslant \sigma_{III}$ respectively. Sides AF, AB, DE and DC form a second group of 'regular face' plastic régimes with the stress inequalities as shown in Figure 5.4. Points B, F, C and E comprise a third group of 'singular edge régimes', where $\sigma_I = \sigma_{III} \geqslant \sigma_{II}$, $\sigma_{III} = \sigma_{II} \geqslant \sigma_I$, $\sigma_I \geqslant \sigma_{III} = \sigma_{II}$ and $\sigma_{II} \geqslant \sigma_I = \sigma_{III}$ respectively. Sides FE and BC fall within a final group of régimes, these giving the principal stress inequalities illustrated in Figure 5.4.

From the four groups of plastic régimes it now becomes necessary to select the group(s) that offer a possible solution to the axially-symmetric problem of penetration resistance and in consequence reference is made to the detailed theoretical analysis given by Cox, Eason and Hopkins (1961).

In their analysis it was shown that, generally, no solution for stress fields was possible for group one; the application of group four solutions were very limited, hence groups two and three offered possible solutions in axially-symmetric problems. In group two stress fields were, however, indeterminate and the 'singular edge plastic régimes' that formed group three finally appeared to be most readily applicable in analysis with axially-symmetric problems.

The régimes of group three satisfy the Haar and Kármán (1909) hypothesis; stated simply - the magnitude of the intermediate principal stress equals that of one of the other principal stresses.

It is now opportune to dispense with the general principal stress notation by adopting the equivalents:

$$\sigma_1 = \sigma_I \; ; \; \sigma_3 = \sigma_{II} \; ; \; \sigma_2 = \sigma_{III} .$$

It can be seen from Figure 5.4. that the plastic régimes B, C, F and E are then represented as follows:

B: $\sigma_1 = \sigma_2 \geqslant \sigma_3$.

C: $\sigma_1 \geqslant \sigma_2 = \sigma_3$.

E: $\sigma_3 \geqslant \sigma_2 = \sigma_1$.

F: $\sigma_3 = \sigma_2 \geqslant \sigma_1$.

The expressions for régimes E and F violate the principal stress convention (i.e. $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$); recalling that the intermediate principal stress becomes the circumferential stress (σ_θ) for axial-symmetry, equations (5.3.) combine with the expressions representing B and C to give:

$$\sigma_\theta \; (= \sigma_2) = (\sigma_r + \sigma_z)/2 + k \; ((\sigma_r - \sigma_z)^2/4 + \tau_{rz}^2)^{\frac{1}{2}} \quad (5.11.)$$

K has values of $\pm$ 1 (see below).

The well known soil mechanics terms 'active' and 'passive' can now usefully be introduced, together with the concepts of 'maximal' and 'minimal' limiting stress fields. 'Active' and 'passive' are states of limiting equilibrium in which self weight of a soil respectively assists or resists incipient plastic flow.

By themselves these terms do not completely specify the type of problem that is to be solved by limiting stress methods and the additional terms 'maximal' and 'minimal' have been introduced to specify precisely the failure mechanism considered. Where the differential equations for stress distribution (Section 5.3.(iii)) are used to compute increasing limiting stress conditions (i.e. σ increasing) the mechanism is 'maximal' and similarly for decreasing limiting stress fields the mechanism is 'minimal'.

The parameter K in equation (5.11.) has values of +1 and -1 for active and passive limiting stress states respectively, giving $\sigma_3 = \sigma_\theta$ for the passive zone and $\sigma_1 = \sigma_\theta$ for the active regions.

If the parameters σ and ψ are recalled a rearrangement of equations (5.3.), (5.8.b) and (5.11) enables the stress components to be expressed in terms of σ , ψ and ϕ as follows:

$$\left.\begin{matrix}\sigma_r \\ \sigma_z\end{matrix}\right\} = \sigma\ (1 \pm \sin\phi \ \cos 2\psi\ ). \qquad (5.12)$$

$$\tau_{rz} = \sigma \sin\phi \sin 2\psi.$$

$$\sigma_\theta = \sigma\ (1 + k.\ \sin\phi).$$

In the discussion up to this point the rigorous mathematical approach to the statically admissible penetration problem has been maintained. The resulting equations specify stress conditions, in terms of σ and ψ , with particular plastic regions and the physical conditions are expressed by using the definitions given immediately above equation 5.12. The situation exists, therefore, where stresses can easily be related in the active and passive zones. Stress relationships within the transition zone (see Section 5.3.(iv)) now require elucidation.

From the experimental results, which are reported in the following chapter, the circumferential principal stress (σ_θ) appears to equal the minor principal stress in zones akin to the transition zone between active and passive regions. This realisation is, therefore, adopted for subsequent numerical analysis methods. The stress condition : $\sigma_\theta = \sigma_2 = \sigma_3$ is assumed to exist not only in the passive zone but also in the transitional zone. Reiterating, $\sigma_\theta = \sigma_1$, in the active zone, $\sigma_\theta = \sigma_2 = \sigma_3$ elsewhere.

By combining equations (5.12) with the equations of equilibrium (5.7.) it is now possible to obtain the basic differential equations that permit the axially-symmetric penetration problem to be solved.

A brief description of the procedures that finally produce the differential equations is given in the following section. Since the mathematical development does not differ widely from that of Sokolovski (1965) a full description is not included.

5.3.(iii) Equations of limiting stress characteristics.

Sokolovski suggested the dimensionless variables:

$$r^1 = r/1 \; ; \quad z^1 = z/1 \; ; \quad \sigma^1 = \sigma/s.$$

where 1 and s are arbitrary (characteristic) length and stress parameters. A substitution for parameter 1 is the diameter B of a penetrometer and for s a convenient substitution is γ d. B. In the equations below the primes signifying dimensionless variables have, for simplicity, been omitted.

An elaborate series of trigonometrical operations combine equations (5.7.) and (5.12) in the form:

$$\frac{\partial \sigma}{\partial z} + \frac{\partial \sigma}{\partial r}.\tan(\psi \mp \mu) \mp \frac{\partial \psi}{\partial r}.2\sigma.\tan\phi.\tan(\psi \mp \mu) \mp \frac{\partial \psi}{\partial z}.2\sigma.\tan\phi$$

$$= 2.\sigma.\tan\phi. \begin{pmatrix} a & + & c \\ b & & d \end{pmatrix}. \qquad (5.13)$$

The parameters a, b, c, d, are:

$$\begin{matrix} a \\ b \end{matrix} = \pm \frac{\gamma d.\ \sin(\psi \pm \mu)}{2.\sigma.\sin\phi.\cos(\psi + \mu)} \qquad \begin{matrix} (5.14.a.) \\ (b) \end{matrix}$$

$$\begin{matrix} c \\ d \end{matrix} = \mp \frac{\cos(\psi \mp \mu) - k\cos(\psi \pm \mu)}{2r.\cos(\psi + \mu)} \qquad \begin{matrix} (c) \\ (d) \end{matrix}$$

and μ (as shown in Figure 5.3.b.) equals $(\pi/4 - \phi/2)$. The angle μ represents the orientation of the limiting stress characteristics (sometimes known as slip lines) with respect to the direction of major principal stress.

Equations (5.13) can be reduced to Sokolovski type partial differential equations by introducing the new variables χ (xsi), η (eta) and ξ (chi). They are given as

$$
\begin{aligned}
\chi &= \cos\phi \;/\; (2.\log\sigma\;); \\
\xi &= \chi \quad + \psi\;; \\
\eta &= \chi \quad - \psi\;;
\end{aligned}
\qquad (5.15)
$$

Substitution of equations (5.15) into (5.13) gives, after further manipulation:

$$\frac{\partial \xi}{\partial\ z} + \tan(\psi + \mu).\ \frac{\partial \xi}{\partial\ r} = b + d. \qquad (5.16.a)$$

$$\frac{\partial \eta}{\partial\ z} + \tan(\psi - \mu).\ \frac{\partial \eta}{\partial\ r} = a + c. \qquad (b)$$

A comparison of equations (5.16) with Sokolovski's equations (1.15) and (1.16) indicates that the fundamental difference between them is reflected in the use of parameters d and c in equations (5.16). The difference is the direct result of assuming that the singular edge plastic régimes, discussed earlier, are the Haar and Kármán régimes for an axially-symmetric problem. By ignoring parameters d and c equations (5.16) revert back to the solution for a plane strain problem.

Equations (5.16) are non-linear partial differential hyperbolic type equations, the solution of which leads to a set of ordinary differential equations (first order):

$$dr/dz = \tan(\psi + \mu)\;;\quad d\xi/dz = (b + d) \qquad (5.17.a)$$

$$dr/dz = \tan(\psi - \mu)\;;\quad d\eta/dz = (a + c) \qquad (b)$$

These equations yield two distinct families of curves of limiting stress inclined throughout the stress field at angles of 2μ to each other. Equations (5.17.a) are those for members of curves known as ξ characteristics and the second set are those for η characteristics.

Figure 5.3.(b) shows stress characteristics with equations $dr = dz \tan(\psi \pm \mu)$ passing through a point P. Every point in a soil body is associated with such characteristics and thus it is possible to visualise an infinite number of ξ and η characteristics in limiting stress regions. The Sokolovski numerical procedure selects certain points in a stress field at finite distances apart and considers the characteristics particular to these points. Other characteristics can be plotted between the selected curves rather like the interpolation of intermediate height contours on a geographical map.

Initially the problem of determining the parameters σ and ψ (equations 5.13) in the physical plane r - z was presented and it now becomes a question of solving equations (5.17) to give distribution of ξ and η in a $\xi - \eta$ plane. The functions:

$$2\psi = \xi - \eta \; ; \; \sigma = \exp\left((\xi + \eta)\tan\phi\right). \qquad (5.18.a)(b)$$

allow a transformation from the ξ-η plane to the r - z plane. σ and ψ are determined at intersections of ξ and η characteristics from the transformation equations. The procedure for transforming intersections on to the physical plane, r - z, is known generally as a mapping technique.

Harr's (1966) extensive review of Sokolovski's procedures includes a concise description of the mapping technique and its subsequent usage in limiting stress field computations. It remains here to select the specific technique that is applicable in the case of stress fields for quasi-static penetration into sands and discuss it in detail.

Before the mapping technique can be fully explained it is necessary to specify boundary conditions in a convenient form. The general equations for boundary conditions are derived in the following section.

5.3.(iv) Boundary conditions

It is convenient to use the reduced stress pn. to represent the resultant of shear and normal stress components acting on a plane of an element. It is demonstrated below that boundary conditions can be expressed in terms of the reduced stress pn.

A new system of co-ordinate axes can be used in place of the r - z axes, in a θ = constant plane, to allow generality; the stress components related to the new co-ordinates n - t are shown in Figure 5.6. The directions of stress components in the figure coincide with those of the positive stresses shown in Figure 5.1.(c), hence they are positive. As shown in Figure 5.6, δ is positive in an anti-clockwise direction (a convention adopted for ψ, α and β) and it is seen that δ assumes the same sign as the shear stress component τ_{nt}. This phenomenon proves useful when the boundary variables are assessed prior to computation of the limiting stress fields.

It can be seen from Figure 5.6. that:

$$\sigma_n = p_n \cdot \cos\delta \; ; \; \tau_{nt} = p_n \cdot \sin\delta \; ; \; \tau_{nt} = \sigma_n \cdot \tan\delta \; . \qquad (5.19)$$

Transformation from the r - z to n - t system of co-ordinates gives equations analogous to equations (5.12), these being:

$$\left.\begin{matrix}\sigma_n \\ \sigma_t\end{matrix}\right\} = \sigma\ (1 \pm \sin \phi . \cos 2\beta\ ) \qquad (5.20.a)$$

(b)

$$\tau_{nt} = \sigma \sin \phi . \sin 2\psi . \qquad (c)$$

and $\sigma_\theta = \sigma\ (1 + k \sin \phi)$. as before (d)

Substititution of equations (5.20.a) and (c) into (5.19.c) and rearranging gives:

$$\sin \delta\ /\ \sin \phi\ = \sin (2\beta - \delta\ ). \qquad (5.21.)$$

Caquot and Kerisel (1956) replaced the ratio $\sin\delta / \sin \phi$ with the single parameter $\sin \Delta$ and introducing Δ into equation (5.21.) it eventually takes the form:

$$\beta = \Delta /2 + \delta /2 + n\pi \ . \text{ and} \qquad (5.22.a)$$

$$\beta = -\Delta/2 + \delta /2 + (2n - 1). \pi /2 \qquad (b)$$

n is any integer but in general has values of $\pm$ 1 or 0, depending upon whether β is positive, zero or negative.

From Figure 5.6. it can be seen that if $\delta = 0, \beta = 0$ or π, indicating that equation (5.22.a.) represents an active state immediately below the horizontal boundary, n = 0. Similar arguments apply for equation (5.22.b.) being associated with the passive state below the boundary. Hence the integer K (= $\pm$ 1) can be introduced and with the equation $\beta = \psi - \alpha$, it is possible to rewrite equation (5.22.) as:

$$\psi = \alpha + 1/2\ (\delta + k \Delta\ ) + (4n + k - 1)\pi/4. \qquad (5.23.)$$

If equation (5.20.c) is combined with (5.19.b) and the equality:

$$\sin\ (\Delta + k\delta\ )\ /\ \sin \Delta = \cos\delta + k \left(\sin^2 \phi - \sin^2\delta\right)^{\frac{1}{2}}.$$

is used, the stress parameter σ can be expressed in terms of pn, ϕ and δ as follows:

$$\sigma = pn\ /\ (\cos \delta + k \left(\sin^2 \phi - \sin^2 \delta\right)^{\frac{1}{2}}). \qquad (5.24.)$$

Equations (5.23.) and (5.24.) specify the important parameters σ and ψ as functions of variables that can be measured (or assumed) along the soil boundary from which limiting stress fields can extend. Provided the variables are known it is a simple matter to compute the complete set of the stress characteristics by using finite difference forms of equations (5.17.).*

The equations giving σ and ψ have been obtained by considering a series of equations; these equations (5.23.) and (5.24.) could have been obtained using the diagrammatic representation of the Mohr-Coulomb yield surface and incorporating the reduced stress pn., together with the parameter Δ introduced by Caquot and Kerisel. σ , as given in equation (5.24.), is a dimentionless stress; length and strength characteristics (B and γd) must be applied in order to render σ as a dimensionless parameter for use in subsequent computations.

For a maximal soil mechanics problem (such as the penetration resistance one considered here) the values of n in equation (5.23.) are zero throughout the stress field region. It can be seen that the value of α provides the correct orientation of the principal stress (σ_1) within the limiting stress fields.

Having specified the boundary conditions in general terms it remains to use these conditions in the following description of the mapping techniques for penetration resistance problems. Discussion of finite difference equations follow in Section 5.3.(vi).

5.3.(v) Mapping techniques

The two distinct families of stress characteristics sketched in Figure 5.7. are usually associated with the quasi-static penetration problem+ (e.g. see Graham, 1968). The characteristic curves are

* Finite difference equations are given in Section 5.3.(v)

+ The characteristics shown in Figure 5.7. do not represent a true distribution of characteristics in a granular material but they serve to illustrate a typical characteristic pattern.

bounded by an extreme characteristic or (using more common terminology) a line of discontinuity.

Stress fields have been shown to extend throughout the soil mass (i.e. to infinity) to give a complete stress field for surface indentation (Cox, Eason and Hopkins, 1961) but since additional stress fields in the non-plastic regions outside the zone sketched in Figure 5.7. would only prove the completeness of the solution, it suffices here to consider stress fields that extend only from the limits of the rigid boundary OD. These stress fields are adequate for the determination of boundary stresses along OD and hence the resistance to penetration of the sand in terms of the stress component on this rigid boundary.

The problem defined in Figure 5.7. can best be considered as a maximal case, with computations commencing along the vertical boundary OA and advancing through OB and OC to finally give the higher stresses along OD. It is interesting to note that Gorbunov-Passadov (1965) considered a minimal bearing capacity problem in which numerical procedures were taken from active to passive regions. Gorbunov-Passadov's techniques appear to have limited application since the method requires assumptions of stress distribution that other procedures set out to obtain; i.e. the load at failure is assumed in order to predict stress characteristics within the failing region.

Active and passive regions in Figure 5.7. are automatically specified from the definitions given earlier. Point O represents a singular point (Harr, 1966) and to demonstrate the mapping technique and related numberical procedures a continuous load distribution that increases with depth is adopted here for the initial boundary OA. For convenience the distribution will be assumed normal to OA, giving a value of ψ of 0 (equation(5.23) with $\delta = \Delta = 0; \alpha = \pi/2$; $n = 0$; $k = -1$). These assumptions have been adopted purely for the sake of simplicity in the description of mapping techniques; subsequent analysis will obviously be based on more realistic boundary conditions.

Recalling equation (5.18.a): $2\psi = \xi - \eta$, with $\psi = 0$ along OA gives the relation $\xi = \eta$. The value of σ increases along OA from A to O and equation (5.18.b) indicates that ξ and η must also increase from A to O. The above two conditions enable the boundary AO to be plotted in the $\xi - \eta$ plane as shown in Figure 5.8.

Sokolovski completed the transformation from the physical plane (r - z) to the $\xi - \eta$ plane by first neglecting the body forces within the material. This restriction merely implies constant values of ξ and η along respective characteristics; this enables the mapping from the r - z to $\xi - \eta$ planes to be explained more lucidly. At the end of this section it will be shown that transformation from the r - z to $\xi - \eta$ planes provides a figure from which computations can easily be arranged. If the restrictions of constant ξ and η along respective characteristics are lifted the result is a distorted figure in the plane; this in no way affects the arrangement of computations.

An equivalent mathematical procedure adopted here equates d and c with -b and -a respectively, to give the latter of the differential equations (5.17.) equal to zero. The equations then give constant ξ along AB, a ξ characteristic with slope $(\psi + \mu)$ (see Figure 5.7.) and constant η along OB, an η characteristic with slope $(\psi - \mu)$. Since η is constant along OB η must increase along AB to achieve the increase that movement from A to O necessitates. The ξ characteristic AB shown in Figure 5.7. then maps as the vertical line $\xi = \xi_0$ = constan

(Figure 5.8.). OB obviously maps to link C and B, since along OB $\eta = \eta_o$ = constant and ξ increases from B to O.

Completion of the triangle OAB is seen to represent the mapping of physical stress characteristics and the boundary OA onto the plane. Similar arguments will be used to complete the mapping process for the region ABCDO but it is here opportune to digress to consider the stress discontinuity concept for the point O and the region of transition, BOC.

It is first necessary to specify the parameter ψ along OD using the variables δ , n, α and k. Again for convenience the reduced stress along OD is assumed to be normal to the boundary and thus $\delta = \Delta = 0$; k = +1 and n = 0 for the active state and $\alpha = -2\pi/3$. Substituting variables in equation (5.23) ψ becomes $-2\pi/3$. Noting that $\psi = 0$ along OA (and in particular at a point O_1 immediately above O) and now having $\psi = -2\pi/3$ along OD commencing, for example, at a point O_2 just below O on OD, it is possible to see that the principal stress σ_1 rotates through an angle of $-2\pi/3$ from O_1 to O_2*. The point O (Figure 5.7.) represents a singular point (or degenerate ξ characteristic) from which η characteristics radiate and at which ψ is multi-valued. If at O, $\xi = \xi_o$ = constant, substitution into equations (5.18.a) and (5.18.b) produces the following equation:

$$\sigma = \exp \Big((\xi_o + \xi_o - 2\psi) \tan \phi \Big) = C.\exp (-2\psi \tan \phi) \quad (5.25$$

C is a constant dependant upon ξ_o .

If every intersection point of ξ and η characteristics shown in

* An astute method of explaining the rotation of the major principal stress and the concept of distinct stress discontinuities has been given by Schofield and Wroth (1968).

Figure 5.7. is referenced by the variable suffices i and j for ξ and η charactertistics respectively, it can be seen that, at O, j is a constant (i.e. along a degenerate ξ characteristic) and i varies, say, from i to (i + n). The parameters $(\sigma, \psi)_{ij}$ and $(\sigma, \psi)_{(i+n)j}$ apply for the i th and (i + n)th characteristics and using equation (5.25) they can be related as follows:

$$c = \sigma_{ij} \exp (2\psi_{ij} . \tan \phi) = \sigma_{(i+n)j} . \exp (2.\psi_{(i+n)j} . \tan \phi) \qquad (5.26.a)$$

rearranging, theequation becomes:

$$\sigma_{(i+n)j} = \sigma_{ij} . \exp \left((\psi_{ij} - \psi_{(i+n)j}) \, 2.\tan \phi \right) . \qquad (5.26.b)$$

If the suffices i and j represent the starting point of curve OB equation (5.26.) gives the distribution of σ around the point O from O_1 to O_2. It is noteworthy that the equation generally applies for the restrictions imposed on the ξ and η characteristics; the physical effect of this is to map η characteristics as a fan of straight lines in the r - z plane with a uniform distribution of stress (σ) along each line (a phenomenon analogous to Schofield and Wroth's concept of 'strong' discontinuities and uniform stress fields). The application of equations (5.17) over small finite lengths of characteristics reduces the effect of having $\eta = \eta_o$ = constant along a η characteristic as they radiate outwards from O; i.e. the effect of restrictions of constant η are quickly lost as computations emanate from O. ξ characteristics in the transition zone BO_1O_2C become a family of curves that map as logarithmic spirals in the r - z plane for the case of $\xi = \xi_o$ = constant (Harr, 1966). As with η characteristics these particular restrictions vanish at finite distances from O and the transition zone then represents the distribution of limiting stress for a general case.

Having established the concept of the point of singularity, and transition regions, it is possible to complete the mapping procedure by considering the variation of ψ from O_1 to O_2 and the constant values of ξ along individual ξ characteristics that join the active and passive regions (O_2CD and O_1AB) of the stress fields.

ψ decreases from O_1 to O_2 and equation (5.18.a) for the point O gives:

$$\eta = \xi_o - 2\psi .$$

Therefore, η increases from O_1 to O_2, mapping as the vertical plane O_1O_2 in the $\xi - \eta$ plane of Figure 5.8. The relation between ξ and η along OD is also obtained from equation (5.18.a), this being:

$$\xi - \eta = -4\pi/3. \tag{5.27}$$

Equation (5.27) plots parallel to OA with intercepts of $\mp 4\pi/3$ on the ξ and η axes respectively; the rectangle O_1O_2CB can then be mapped with O_2 at the interception of the vertical through O_1 and the line of equation (5.27.)

The remaining triangle O_2CD automatically assumes the position shown in Figure 5.8. To facilitate final computations Sokolovski suggested rotating the triangle O_2CD about the line O_2C to give the mapping of stress characteristics in the physical plane of Figure 5.7. as a trapezium $ABCD^1O_2O_1$ in the $\xi - \eta$ plane of Figure 5.8. (shown dashed).

It has been shown that the stress characteristics in the physical plane r - z can be mapped onto the $\xi - \eta$ plane as characteristics parallel to the pair of axes provided certain restrictions are applied. The reversal of the r - z to $\xi - \eta$

mapping, therefore, forms the basis of the numerical methods of determining the limiting stress fields by solving equations (5.17.)

In order to carry out mapping in a correct sequence the intersections of Figure 5.8. are interpreted as nodal points at which ξ, η, r and z can be found. The variables are known along AO_1O_2 and are partly known along O_2D^1, and existing techniques allow every intersection point on $ABCD^1O_2O_1$ to be evaluated. The techniques are included as a part of the following section.

The restrictions d = -b and c = -a were originally imposed to give constant ξ and η along respective characteristics and served only to obtain the trapezium $ABCDO_2O_1$. Having defined this region of $\xi - \eta$ mapping and, in particular, established a suitable computation sequence the restrictions can be removed without any drastic change in the shape of ξ and η characteristics; the elements within $ABCDO_2O_1$ that can be formed at the intersections of η and ξ (i and j) characteristics can still be formed even though the configuration may be distorted.

The method of computing the variables ξ, η , r, z and ψ for consecutive intersections of characteristics demands the use of finite difference methods in equations (5.17.).

5.3.(vi) Finite difference equations

Equations (5.17) can only be solved by using finite differences over small finite sections of the limiting stress fields. The differences replace the ordinary differentials and allow a group of variables (r, z, ψ and σ) to be propagated from an initial boundary, through a stress field, to a final boundary. Small regions of the $\xi - \eta$ and r - z planes shown in Figures 5.8. and 5.7. have been reproduced in Figures 5.9.(a) and 5.9.(b). The finite difference equations for a point Pij (Figure 5.9.) extend from P(i-1)j (along a ξ characteristic) and Pi(j+1)(along an η characteristic) and can be arranged as follows:

$$Zij = \frac{r_{(i-1)j} - r_{i(j+1)} + z_{i(j+1)}\tan(\psi_{i(j+1)} - \mu) - z_{(i-1)j}\tan(\psi_{(i-1)j} + \mu)}{\tan(\psi\ i(j+1) - \mu) - \tan(\psi\ (i-1)j + \mu)}. \qquad (5.28.a)$$

$$rij = r\ (i-1)j + (Zij - Z(i-1)j)\ \tan(\psi\ (i-1)j + \mu). \qquad (b)$$

$$\xi ij = \xi\ (i-1)j + (Zij - Z(i-1)j)\ (b + d)(i-1)j. \qquad (c)$$

$$\eta ij = \eta\ i(j+1) = (Zij - Zi\ (j+1))\ (a + c)\ i(j+1). \qquad (d)$$

$$\psi ij = (\xi ij - \eta\ ij)\ /2. \qquad (e)$$

$$\sigma\ ij = \exp.\ \left((\xi\ ij + \eta\ ij)\ \tan \phi\right). \qquad (f)$$

Equations (5.28) are fundamental to the determination of limiting stress fields for a bearing capacity type problem in cohesionless soil. Sokolovski recommended replacing ψ(i-1)j and ψ i(j+1) with corrected values to allow for the initial assumption of a series of straight lines between points Pij in the physical plane. The corrected parameters ψ^{c} are given as:

$$\psi^{c}(i-1)j = (\ \psi\ (i-1)j + \psi\ ij)\ /2 \qquad (5.29.a)$$

$$\psi^{c}i(j+1) = (\ \psi\ i(j+1) + \psi\ ij)\ /2 \qquad (5.29.b)$$

The corrected angles ψ^{c} are substituted into equations (5.28.a) and (b) to give z^{c}ij and r^{c}ij. at Pij (Figure 5.9.b)). Equations (5.28.c) and (d) use z^{c}ij and r^{c}ij to find new values of η and ξ, and hence σ and ψ. A further refinement using corrected parameters σ^{c} was adopted by Graham (1968). Similar equations to (5.29) were:

$$\sigma^{c}(i-1)j = (\sigma\ ij + \sigma\ (i-1)j)\ /\ 2 \qquad (5.30.a)$$

$$\sigma^{c}i(j+1) = (\sigma\ ij + \sigma\ i(j+1)\ /\ 2 \qquad (5.30.b)$$

Graham used an iteration procedure with predetermined convergency criterion for both σ and ψ (Graham, 1966) and substituted corrected values σ^{c} and ψ^{c} into equations analogous to (5.28.a) (b), (c) and (d).

Results given in Section 5.4. suggest that the iteration procedure adopted by Graham was incorrect; the procedure used by the author, carried out to third approximations, gave limiting stress fields with characteristics that avoided the 'excess curvature' condition obtained by Graham (1966).

In Figure 5.10. the elements within which the initial boundary conditions can be computed (using equations (5.23) and (5.24.)) are shown hatched in the lower triangle.

Equations (5.28) coupled with (5.29) can easily be applied to compute variables in the directions of arrows in Figure 5.10. to ultimately complete the solution for triangle O_1AB. The method of solution is known as the Cauchy problem.

The elements along O_1B of the central rectangle O_1O_2CB (Figure 5.10.) result from the Cauchy solution. Since O_1O_2 represents a singular point, r and z are constant for the many values of ψ, and ψ can conveniently be divided into equal increments of the total angle through which the major principal stress rotates at O. σ obtains from equation (5.26) for each value of ψ and thus the elements along O_1O_2 are completely evaluated.

The finite difference equations (5.28) allow computations to move, in effect, from the singular point O_1O_2 as shown in Figure 5.10, to give the complete rectangle O_1O_2CB. The solution for the rectangle presents the Goursat problem.

A combination of the Cauchy and Goursat problems, known as the 'mixed boundary value problem', is adopted for triangle O_2CD.

Along O_2C a complete knowledge of variables has been acquired from the Goursat solution and, along O_2D, ψ is computed using equation

(5.23)*. It can be shown that a relation between r and z along O_2D is a final requirement for a solution of the mixed boundary value problem. Figure 5.11. shows the physical plane at the singular point O and the relation between rij and zij from the figure is:

$$rij = B / 2 - \tan\rho \ (zij - D). \qquad (5.31)$$

Substituting equation (5.31) into the finite difference forms of (5.17.a)(since both equations give intersections on a ξ characteristic) and rearranging:

$$Zij = B/2 + D\tan\rho + Z(i-1)j \ \tan(\psi(i-1)j + \mu) - rij \tan(\psi(i-1)j + \mu) + \tan\alpha. \qquad (5.32.a)$$

$$rij = B/2 - \tan\rho \ (Zij - D). \qquad (5.32.b)$$

$$\xi ij = (Zij - Z(i-1)j)\ (b + d)\ (i-1)j + \xi\ (i-1)j \qquad (5.32.c)$$

$$\eta\ ij = \xi\ ij - 2\psi\ ij. \qquad (5.32.d)$$

$$\sigma\ ij = \exp.\ ((\xi + \eta)ij.\ \tan\phi). \qquad (5.32.e)$$

From equations (5.32.b) the variables at Pij are evaluated and the equations (5.28) can be re-employed to complete triangle O_2CD. The system of arrows in Figure 5.10 serves to indicate the order of numerical procedures that complete the final region of the limiting stress fields.

The relation between r and z along O_2D does not establish the position of D prior to the solution of limiting stress fields. It therefore becomes necessary to complete the characteristics on the basis of specific boundary conditions along O_1A and then adjust the limiting stress region such that the line of discontinuity (i.e. the extreme ξ characteristic) emanates from the tip of the conical ended penetrometer. All stress characteristics outside this

* It is, of course, necessary to assume δ along OD (Figure 5.7.) but interpretation of results from direct shear tests for sand on the penetrometer material (in this case, brass) will give acceptable values of δ.

extreme one are ignored (assuming that there are characteristics outside it); consequently the size of the plastic regions are adjusted to suit the final boundary OD.

The procedures that are required in the evaluation of limiting stress fields during quasi-static penetration into dry sand have been considered in this chapter and the finite difference equations have been arranged in a form suitable for analysis by an I.C.L. 1905 computer.

The complete series of operations, from the evaluation of initial boundary conditions to calculating the vertical stresses on the vertical face of the penetrometer (i.e. the point resistance) have been included in one computer programme.

The data required in the programme takes the form of constant parameters, to describe the problem geometrically and to establish the mechanism of failure, and the characteristics of the cohesionless material, in terms of γd, $\emptyset$ and δ .

Assumptions concerning the boundary conditions and the properties of the sand into which the penetrometer was driven are listed in Section 5.4., along with the results obtained from the theoretical analysis.

5.4. Computed limiting stress fields

5.4.(i) Boundary conditions

Section 5.3. has described, in some detail, the sequence of operations that ultimately lead to the statically admissible solutions for axially-symmetric penetration (quasi-static) in dry ponderable cohesionless soil. Each completed solution has produced a series of limiting stress characteristic intersection points, out of the infinite number which occur in the stress fields, together with values of the stress tensors $\sigma_z, \sigma_r, \sigma_\theta$ and τ_{rz} at each intersection point. The angle ψ, giving the orientation of the major principal stress (σ_1), and the mean stress σ have also been listed for the corresponding stress tensors. Finally the vertical stress, i.e. penetration resistance, has been computed along the final boundary at a number of characteristic intersection points, such as rij in Figure 5.11. It now remains to discuss the assumptions associated with the solutions and discuss some of the more relevant results.

The first step in the limiting stress field computations was to determine the initial boundary conditions. The computer programme which the Author developed was arranged so that these conditions, governed by the parameters n, k, α, ϕ, δ and pn, were determined by substitution into equations (5.23) and (5.24). The key parameters σ and ψ were thus calculated at certain pre-selected points along the initial boundary (for convenience spaced equal distances apart).

Figure 5.12 shows the system of stresses (assumed) acting on the boundary OA, also on OD, of the penetrometer. Values of n, k and α are included, since these were necessary to define the geometrical arrangement and the type of failure mechanism. The stress tensors in Figure 5.12 have been assumed, on the basis of downwards quasi-static penetration. Obviously the stress tensors are thus specified in relation to the vertical and inclined boundaries.

By comparing Figures 5.12 and 5.6. it can be seen that the established sign convention for stress tensors has given δ a negative value along the two boundaries. For reasons discussed later δ was assumed to be constant along OA and OD: -13.1° for the loose sand condition and -23.4° for the dense sand condition. Since ψ was shown to be dependent on δ and α (equation 5.23), the value of this angle was also constant along each boundary. The computed values of ψ along the initial boundary for loose and dense sand properties were 5.97^{o} and 7.38^{o} respectively. Along the final boundary respective values were -139.06^{o} and -150.78^{o}; rotations of the major principal stress were 145.03^{o} and 158.16^{o} for conditions of loose and dense sand.

Other properties of sand used in the computer solutions, namely ϕ and γd, were those determined and used extensively in Chapter 2.4. These are listed below:

	γ d.(lb/cu ft)	Coulomb ϕ^{o}
Loose	91.62	31.60
Dense	105.15	40.50

The remaining boundary condition parameter : pn, was calculated using the equation:

$$pn = Kf. \gamma d. z. /\cos \delta. \qquad (5.33)$$

Kf was the earth pressure coefficient synonymous with Kf. used in Chapter 2.4. and z was the depth from the stress-free surface (i.e. the original horizontal surface) to the point on the vertical boundary at which evaluation of pn was required. γd, as usual, was the dry density of the cohesionless soil.

From the known boundary conditions discussed above the complete solution was carried out according to the methods outlined in Section 5.3.(iv). A number of investigations were carried out, for loose and dense sand conditions, by varying Kf along the initial boundary OA. In all cases the parameters which defined the properties of the sand (i.e. ϕ, δ and γ d) were maintained constant and computations were completed for a 6 in. long vertical boundary with limiting stress characteristics initiating from quarter points (a total of 5 no.) along the boundary. Generally two geometrical conditions were adopted: one with the assumed zone of limiting stress characteristics extending from 6 in. to 12 in., giving a penetration of 12.866 in., and a second with the assumed zone of limiting stress extending from 12 in. to 18 in. (18.866 in. penetration). Results, including the iteration procedures and 2-D conditions, are discussed in the following section. Stress distributions within the limiting stress fields are represented in the form of figures in Section 6.4.

5.4.(ii) Iteration procedures and 2D-3D comparisons

During the development of the solution for limiting stress fields the Sokolovski ψ-iteration was combined with the σ-iteration suggested by Graham (1968). To reiterate, corrected values of ψ and σ from equations (5.29) and (5.30) were substituted into equations (5.28.a) and (b) and corrected values of r, z, σ and ψ were then used in the log-transform equations (5.28.c) and (d). A method in similar form to that just described was outlined and subsequently used by Graham (1966) but the condition known as 'excess curvature' was experienced in two dimensional solutions. The stress characteristics resulting from the iteration procedure which was used by the Author are plotted in Figure 5.13.(a). The 'excess curvature' phenomenon is clearly illustrated. The Author carried out the iteration procedure with a convergency criterion (either $\Delta\sigma/\sigma$ or $\Delta\psi/\psi$) of $\pm$ 0.0001 and by giving additional instructions for print-out during iteration cycles it was seen that the convergency limits were generally achieved after four iteration operations.

Obviously the 'excess curvature' condition implied a basic fault in the iteration method and the Author continued from these earlier results by carrying out an analysis without any form of iteration.

Figure 5.13(b) shows co-ordinates plotted for this condition with the same boundary data and sand properties as in the computations which gave Figure 5.13(a). The results shown in Figure 5.13 convincingly illustrated the need for a suitable iteration method.

It was realised, finally, that the substitution of corrected ψ and σ was necessary only in equations (5.28.a) and (b). The values of r and z obtained from these equations were substituted into equations (5.28.c) and (d) with original values of ψ and σ . The plot of stress characteristics which this method gave is shown in Figure 5.14.(a), with the vertical stress distribution along the inclined face plotted below the characteristic net. Purely for comparison purposes the limiting stres s characteristics for a plane strain condition are plotted in Figure 5.14.(b), also with the stress distribution shown below.

Because of the pre-determined initial boundary conditions whereby co-ordinates along the vertical face were decided prior to computations, the complete net of limiting stress characteristics was not obtained. It can be seen, however, that the plane strain limiting stress field is much more extensive than the axially-symmetric field. The complete field (i.e. a field with the slip or failure surface emanating from the penetrometer point) for the plane strain condition would cover a greater zone, both outwards and upwards from the base of the shank.

If the length above the inclined base over which limiting stress fields extend in the axially-symmetric example is las, and the similar length for plane strain is lps, the ratio lps/las = 1.89. Cox (1962) carried out punch and die identation studies in cohesionless soil in which the horizontal stress free boundary was a boundary analogous to the vertical boundary in the Author's penetration study.

It is interesting to note that the lps/las ratio obtained from the work by Cox varied from 1.26, for $\phi = 0^{\circ}$, to 1.85 for $\phi = 40^{\circ}$; this gives results of the same order as that quoted above. A similar ratio for the horizontal extent of limiting stress fields in the penetration problem was found to be 2.6. One result taken from the theoretical study by Cox (for G = 10) gave a ratio of 1.36.

Having evolved what appeared to be the correct iteration procedure and carried out a number of successful preliminary analyses, using dummy data, the logical continuation was an investigation of limiting stress characteristics for conditions which were thought to occur in laboratory tests. These conditions were simulated using the sand properties listed in Section 5.4.(i). Results, and the accompanying discussion, form the penultimate section of this chapter.

5.4.(iii) Stress fields and final stress distribution

The parameters which are variable in theoretical analysis of axially-symmetric quasi-static penetration into sand are quite numerous and for this reason a limit has been placed on the types of penetration problem that have been investigated as part of the theoretical study. Some of the parameters, such as those which specified the stateof compaction and the strength of the sand (i.e. ϕ, δ and γd), have already been introduced and discussed in Section 5.4.(i). It was finally decided to investigate penetrations into loose and dense sand, assuming Kf to be the only variable, at depths ranging from 20.866 in. (i.e. the shank extending to 20 in.) to 7.866 in. A total of 43 solutions were obtained for quasi-static penetration in loose sand (although only a selected number are reported here) and 16 were obtained for dense sand; the earlier results firmly established specific patterns for loose sand which were confirmed with a few selected dense sand computations. Some solutions were computed with constant σ (along the initial boundary) but these were obtained simply by adjusting Kf. at the pre-selected initial boundary points.

To illustrate the effect of varying Kf., the vertical stress acting on the inclined base of the penetrometer has been plotted against distance from the centre line on Figures 5.15 and 5.16. The plots are for loose and dense sand respectively and in both cases, but particularly with dense sand, extrapolation has been necessary to give the full stress distribution along the inclined base.

Several interesting findings have been revealed during the analysis of the computed results, these being listed below:

i) The computations with boundary conditions: constant σ, and : decreasing Kf with depth, which produced the curves 1, 4, 5 and 6 in Figure 5.15 gave virtually identical limiting stress fields, although the stress distributions along the final boundary varied by a factor as great as 5.

ii) This phenomenon was alo noticed in computations using dense sand properties. Curves 1, 2, 3 and 4 of Figures 5.16 show the stress distributions for the same limiting stress fields.

iii) Computations with the boundary condition : constant Kf, produced less extensive stress fields. A comparison of (a) and (b) in Figures 5.17 and 5.18 illustrates the different characteristic net shapes for the two boundary conditions. . Interestingly, curves 2 and 3, Figure 5.15, and curve 5, Figure 5.16., also showed stress distributions which were significantly different from those discussed in (i) and (ii). Curve 5, Figure 5.16, in particular does not follow the general pattern.

iv) Stress distributions along the final boundary varied in proportion to the variation of σ along the initial boundary. E.g. σ = 1.0 p.s.i. (constant) along the initial boundary gave a stress distribution 2/3 the final stress distribution for the σ = 1.5 p.s.i. (constant) example. Figure 5.19 is a plot of final boundary vertical stresses for different values of σ along the vertical face in loose sand; the proportionality is adequately illustrated.

v) A series of solutions were obtained for penetrations of 7.866 in. then 8.866 in. to 20.866 in. in incremental increases of 2 in. for loose sand conditions. The initial vertical boundary extended over 6 in. (e.g. from 14 in. to 20 in.) and σ along this boundary was assumed constant and equal to 1.0 p.s.i. The final boundary stress distribution (curve 1, Figure 5.15) was found to be identical for all depths; in fact, all stress distributions within each failure surface were identical. The limiting stress characteristic net was also the same for the set of 8 computations.

vi) The observations listed in (v) also applied in solutions with dense sand properties, with the same initial assumption (i.e. σ = 1.0 = constant). Curve 1, Figure 5.16., was obtained for 8 computations at the depths listed in (v).

vii) Having realised the phenomena discussed in (iv), (v) and (vi) it can now correctly be assumed that, provided an identical distribution of σ is maintained along the initial boundary, the penetration resistance will not vary with depth. This, of course, applies only when the full failure surface is below the stress free surface. It is, however, reasonable to suppose that σ is not constant but is a function of depth, since $\sigma \propto pn$ (equation 5.24) and $pn \propto z$ (equation 5.33). In practice the penetration resistance is expected to increase with depth.

In the review of Jaky's (1948) theoretical studies (Chapter 2.3.(i)) it was shown that his hypothesis predicted constant penetration resistance with depth after the full failure surface (Figure

2.15) had been developed. Meyerhof (1950) gave bearing capacity factors from which the Author computed the penetration resistance shown in Figure 2.16. Comparison with the Author's experimental results for penetration resistance indicates that Meyerhof's hypothesis gave high penetration resistance below about 10 times the penetrometer diameter. It appears that a suitable compromise between Jaky and Meyerhof results could be reached by using the limiting stress field solutions developed in this chapter. The assumption that σ distribution along the shank of a penetrometer, in the limiting stress fields, varies with depth could give realistic results for penetration resistance of sand.

The concluding solution in the present series of computed limiting stress fields was carried out using dense sand properties and a distribution along the initial boundary varying from 0.97 p.s.i. to 1.93 p.s.i. in roughly 0.12 p.s.i. increments. The initial boundary extended from 6 in. to 18 in., giving a penetration depth of 18.866 in. The stress characteristics which were obtained from the computation are plotted on Figure 5.20(a). and the distribution of vertical stress along the final boundary is shown in Figure 5.20(b). The overall shape of the characteristic net was not identical to the less extensive fields, e.g. Figure 5.18(a), but characteristics within the field initiating from the 12 in. boundary point were appreciably the same. Further comparative studies were not carried out because sufficient time was not available.

From the limiting stress fields shown in Figures 5.17, 5.18. and 5. 20, and the final stress distribution curves of Figures 5.15. and 5.16, the following conclusions were drawn:

i) In computations where loose sand properties have been used the length of the 'failure bulb' (see Chapter 2.3.(i)) was roughly 6 times the penetrometer diameter. The total width of the 'bulb' was again about 6 times the penetrometer diameter. The 'failure bulb' did not extend below the penetrometer point in the loose sand solutions.

The stress distribution curve no. 5, Figure 5.15, fell closest to the experimental value of penetration resistance in loose sand at 18.866 in. The experimental penetration resistance, taken from Figure 2.12, was approximately $17\frac{1}{4}$ p.s.i. and the average value determined from the broken line of Figure 5.15 was 17.6 p.s.i.

ii) Figure 5.20(a). illustrates the general type of 'failure bulb' for limiting stress analysis with dense sand properties. From the figure the relationships between overall length of 'bulb' and penetrometer diameter, and overall width of 'bulb' and penetrometer diameter, have been established. The overall length/diameter ratio was approximately 13 and the overall width/diameter ratio was 7.4. These compare with ratios of 28 and 16 for Jaky's (1948) assumed mode of failure for deep 2-dimensional foundations in dense sand. Sincethe 2D/3D ratio for limiting stress field dimensions is approximately 2 (Section 5.4.(ii) : 1.89 and 2.6) the Author's 3-dimensional and Jaky's 2-dimensional length and width ratios compare quite favourably.

Experimental penetration test results, plotted on Figure 2.14, gave a penetration resistance of 67.10 p.s.i. at a penetration of 18.866 in. into dense sand. Curve no. 4 in Figure 5.16. produced an average stress distribution of 69.0 p.s.i. Although this stress was closest to the experimental value it is worth noting that curve no. 3 of Figure 5.16 : the stress distribution on the final inclined boundary for constant (= 0.36) along the initial boundary, was used to compute an average stress of 63 p.s.i.

A rather important conclusion which arises from the comparisons of stress distribution in (i) and (ii) is that the σ = constant = 0.36 p.s.i. condition along the initial boundary gave an average stress distribution along the final boundary, for loose and dense sand conditions, that is not radically different from experimental results. The implication is that although sand properties are different, stress distributions along the shank are similar for dense and loose sand states.

Other aspects of the computer study of axially-symmetric quasi-static penetration of ponderable soils which were considered of interest are discussed in the following paragraphs.

The iteration procedure, in most cases, showed constant results after two cycles but where σ and ψ for the two points Pi (j + 1) and P (i - 1)j (see Figure 5.9.(b)) differed considerably the iteration procedure was completed after four or five cycles. These instances occurred particularly in the transition zone, close to the point of singularity, where the curvature of characteristics changed rapidly. Consequently σ and ψ showed greater variation than elsewhere and further cycles of iteration were necessary.

Computing time per solution varied with the number of solutions in one operation but generally the time taken to complete one solution and print the results was approximately 18 seconds on the I.C.L. 1905 computer. As a matter of interest, earlier computations were carried out on an Elliot 803 computer and the time taken then was roughly 45 minutes per solution. A series of 8 solutions, carried out independently but in one operation, took approximately 43 seconds using the I.C.L. 1905; this averaged about 5½ second per solution.

The computer language used throughout was Algol 803, with slight modifications to conform with the hardware facilities associated with the computer. It has already been stated that the theoretical investigations assumed constant values of ϕ and γd within each limiting stress field and constant values of δ along initial and final boundaries. These were adopted mainly because it was thought sensible to concentrate on one particular variable, this being Kf. The data which was required for each solution was much simplified by using constant sand properties; a total of 56 numbers were read instead of 280 for variable sand conditions with each individual solution.

5.5. Concluding remarks

Stress and strain sign conventions have been established in terms of cylindrical polar co-ordinates in Section 5.2. The most significant parameters : σ , ψ , γ rzmax and λ , have been defined and the conventional expressions relating stress tensors and strain tensors have been stated.

In Section 5.3. a method of evaluating limiting stress fields for quasi-static axially-symmetric penetration in a cohesionless ponderable soil has been developed. The method has combined the currently acceptable yield criterion for soils and equations of equilibrium for stress tensors in a cylindrical polar co-ordinate framework. Techniques, particularly the numerical ones outlined by Sokolovski, have been adopted in modified form to arrive at the basic simultaneous non-linear first order partial differential hyperbolic equations which define the series of intersecting limiting stress characteristics in a plastically failing medium. Again relying on methods attributed to Sokolovski the differential equations were expressed in finite difference form whence they were rendered suitable for solution by electronic digital computer.

In view of the difficulty of investigating three-dimensional problems the Haar and Kármán hypothesis has been accepted with one assumption; this was made possible by referring to experimental results obtained with three-dimensional pressure cells. It will be shown in the following chapter that the basic assumption had a certain amount of validity.

An iteration procedure which was used in previous limiting stress field computations for a two-dimensional quasi-static problem was proved to be incorrect. Satisfactory procedures were thus developed and have been used extensively to produce complete lower bound solutions for axially-symmetric penetration.

Finally the results from computations have been plotted for conditions representing loose and dense sand packings and the distributions of vertical stress along the final inclined boundary have been given for various boundary conditions. Subsequent discussion of results and comparison with the earliest experimental work of Chapter 2 had lead to interesting findings regarding stress distribution along the shank of a penetrometer.

The most logical continuation appears to be to compare the theoretical results, which have only been described briefly in this chapter, with the experimental results obtained using pressure cell and strain measurement techniques. The first sections of the next chapter, however, are devoted to discussion of the experimental results as separate entities.

CHAPTER 6

Results

6.1. Introduction

Three studies have been undertaken and each has yielded information on the behaviour of sand during penetration of a conical pointed circular brass penetrometer. The three projects : the measurement of stress fields during penetration : the measurement of strain fields during penetration and : the prediction of stress fields from a theoretical investigation, were all carried out independently but as integrated parts of one main project.

The stress fields were measured using miniature three-dimensional earth pressure cells; these were designed and manufactured in the initial phase of the investigation then calibrated under different environmental and stress conditions to produce correction factors. The factors, in the form of matrices, were used to convert pressure cell recordings to what were thought to be realistic in-situ stresses in the sand masses. The in-situ stresses are reported in the following section.

The results, plotted in Figures 6.1. to 6.3., were unique and it seemed judicious to attempt to establish their validity. There are several factors which provide evidence of the accuracy of experimental results, the first being consistency. Results which show consistency indicate a similar performance during experiments, although it must be remembered that experiments have been known to show consistent inaccuracies. The results obtained from the present series of pressure cell experiments at least showed reasonable consistency.

It should be possible to fit correct results to an accepted hypothesis. If a satisfactory fit is observed the inference is twofold: the pre-supposed hypothesis becomes a conclusion and : any doubt surrounding the validity of experimental results is dispelled, since it is more than just coincidence if experiments and an hypothesis show close agreement.

Measured stresses, which are included in Section 6.2., are represented in terms of the accepted hypothesis, i.e. the Haar and Kármán (1909) principle, and appear to agree almost exactly in certain instances.

Agreement with previous experimental results of an analogous nature is a factor which could help further to confirm the soundness of present results but as far as the Author is aware the only similar results were reported by Kerisel et. al. (1965). There was seen to be some similarity in the σ_z versus penetration plot but other stress distributions were not published.

Strain fields were measured by recording the displacements of small markers against a glass face in a 'half-section' axially-symmetric penetration experiment. The plate was not perfectly smooth, with small shear stresses in the sand immediately adjacent to, and parallel with, the glass plate. Studies which have verified that the strain fields are not significantly affected by these shear stresses have been discussed in Section 4.5.

A large quantity of strain results were processed, as described in Chapter 4, but certain difficulties prevented these from being analysed fully by using the available computer techniques*. Typical

* During the June-September period, 1969, the existing Elliot 803 computer was phased out of operation at the University Computer Centre and the replacement, the I.C.L. 1905, presented several software problems in addition to the need to rewrite the strain analysis programme and change all data from tape to cards.

results, however, were obtained and are plotted in Figures 6.11 to 6.15.

The next section of the chapter reports some of the results from the theoretical study described in Chapter 5. These are compared with experimental findings and outstanding differences in results are discussed.

The chapter ends with a section on combined stress and strain-rate fields. It was possible to show on a Mohr stress diagram the fully evaluated stress tensor during penetration into dense sand.

6.2. Experimental stress fields.

6.2.(i) Results

The miniature earth pressure cells were used to measure in-situ stresses in three types of penetration test, for convenience numbered 1, 2 and 3.

The first series of results (1) for penetration in a loose sand are plotted in Figures 6.1.(a) to (j). The cell positions within the sand mass are shown on the θ - constant plane in Figure 6.4.(a); Figure 6.4.(d) represents the plane as section A - A.

Pressure cells inserted in a sand mass with a loose-medium state of compaction (2) produced the stress results plotted in Figures 6.2.(a) to (j). Figure 6.4.(b) depicts these cell positions in the θ - constant plane. This particular sand condition was selected because it was assumed to resemble, as closely as possible, the state of sand for zero volume change during shear. The critical porosity* from direct shear tests was found to be 43.94% whereas the initial porosity for the loose-medium sand sample was measured, en masse, as 42.74%. The loose sand sample (1) had an initial porosity of 44.62% thus, in practice, the critical porosity lay between the porosities of tests (1) and (2).

Figures 6.3.(a) to (j) give pressure cell results during penetration into dense sand (3). Cell positions are shown on the θ - constant plane in Figure 6.4.(c).

The ordinates in Figures 6.1., 6.2. and 6.3. were taken as the depth of the penetrometer tip below the (originally) horizontal

* i.e. the porosity at which work done in shearing sand comprises frictional and remoulding components only; dilation is a zero component.

stress-free surface of the sand mass. To obtain this depth the penetrometer was held by switching off the electric drive motor at pre-determined intervals. Penetration was controlled with a stop-clock which was started at precisely zero penetration, stopped at each halt in penetration, and restarted simultaneously with the electric motor after pressure cell readings had been taken. This time method of controlling depth of penetration enabled a specific depth to be achieved with an accuracy estimated to fall within a range of ± 0.001 in.

The abscissae in Figures 6.1., 6.2. and 6.3. were the values of the measured stresses computed from pressure cell readings and corrected using the matrices given in Table 4.1. The scales along the abscissae vary from 1" ≡ 1 p.s.i. to 1" ≡ 2 p.s.i., since it was found necessary to adopt a scale most suitable for a particular test.

The three series of tests each used 9 cells*, these being placed by hand on the underlying layers of sand, deposited using the sand pourer described in Chapter 3. The dummy gauge which provided the temperature control was embedded in sand in a small aluminium (moisture content) tin and positioned in one corner of the sand container some 21 in. from the penetrometer. The depth of the control was approximately 6in. below the sand surface.

Because the control was completely enclosed by the aluminium tin, environmental conditions, such as short term fluctuations due to movement of air during sand pouring, and differential temperatures

* 10 cells and one dummy were manufactured but one cell, no. 4, was damaged beyond repair during the placement operations in the first penetration test (1).

within the sand (which required a longer period for stabilization) ruled out any possibility of recording stresses during sand preparation, since this would have meant extending the preparation time over several days. Strain gauge readings were, therefore, recorded only after pouring was complete and the sample had been left for at least twelve hours; this was considered sufficient time to achieve temperature stability throughout the sample.

Stress recordings after 12 hours gave results from which coefficients of earth pressure at rest (Ko) were calculated. The average values for the tests 1, 2 and 3 were found to be:

		K_o ($= \sigma_\theta / \sigma_z$)	K_o ($= \sigma_r / \sigma_z$)
1	Loose sand	0.99	0.91
2	Loose-medium sand	0.97	0.93
3	Dense sand	1.04	1.07

The penetration tests with pressure cell usage were carried out in the 'half-section' container with a half-section 1 in. diameter brass penetrometer (described in Chapter 4). While it was appreciated that the container was not the ideal apparatus, limitations of time and workshop facilities precluded a suitable alternative (i.e. a full section container and penetrometer).

In order to demonstrate the suitability of the 'half-section' apparatus several penetration tests were initially performed without pressure cells in place. The tests were continuous and the load required to drive the penetrometer into the sand sample was recorded using a conventional proving ring. The results appeared to be quite congruous and typical plots for penetrations into loose, loose-medium and dense sand are given in Figures 6.5., 6.6. and 6.7. (curve (a)). Curves (b) of the same figures show the estimated friction between the glass plate and the penetrometer for the above sand conditions (1), (2) and (3). This was obtained by assuming $\mu = 0.15$ between brass and glass and computing the shear stress over the penetrometer face for a horizontal/vertical stress ratio of unity. The third curve (c) shows the difference between (a) and (b) and curve (d) is half the total penetration resistance from the full section tests given in Figures 2.12., 2.13. and 2.14. The final curves (e) of Figures 6.5., 6.6. and 6.7. were obtained during penetration tests with stoppages, again by recording proving ring readings during advancement of the brass penetrometer.

The results presented in Figures 6.1., 6.2. and 6.3. have been plotted from a limited number of measurements, normally 21 per cell per diaphragm. Each row of points on one figure represents about 20 minutes work (i.e. reading 27 gauges) and one test lasted approximately 14 hours. True stress distribution between the wider spaced points may have showed a difference from the plotted distribution in Figures 6.1., 6.2. and 6.3. but the determination of stress at a greater number of points would have extended the test into a two day operation. Since one test, in its entirety, could be completed in one week the advantages of this arrangement out-weighed the benefits of additional results.

6.2.(ii) Discussion of results

If one group of figures, 6.1., is taken separately the results can be discussed individually before a comparative study is made. The stress plots are normally self explanatory and the following discussion will be in general, rather than specific, terms.

In Figures 6.1.(a) to (j) the stresses in polar co-ordinate directions, r, θ and z, did not show a significant difference from the initial stresses, for a particular pressure cell, until the penetrometer was some $3\frac{1}{2}$ in. (or $3\frac{1}{2}$ penetrometer diameters) above the upper face (i.e. the face in the z constant plane). Down to the point where penetrometer effects became noticeable the three stresses, σr, σ_θ and σ_z, were similar and computed values of the coefficient of earth pressure, K, agreed with the Ko values obtained initially for the loose sand.

In the following discussion the pressure cell results will be separated into three groups. The first group embraces cells no. 1, 3, 6 and 8, a second group includes cells no. 2, 5 and 7 and cells 9 and 0 form the remaining group. With the exception of cell no. 5., the first two groups differentiate between those pressure cells positioned less than about 1.1 in. from the z-axis and those with $r > 1.1$.in.

For cells 1, 3, 6 and 8 the vertical stress (σ_z) increased rapidly as the penetrometer advanced, reaching a maximum when the penetrometer tip was roughly at the same depth as the upper faces of the cells. Further penetration produced a reduction of σ_z to a constant stress slightly greater than the original overburden pressure (with the exception of cell no. 5). The depth below the maximum σ_z value at which the vertical stresses became constant varied between 4 in. and 8 in.

The horizontal radial stresses (σr) showed an increase of similar form to σ_z, although thecurves were displaced approximately 2 in. downwards from the vertical stress curves. The radial stresses reached a maximum when the penetrometer tip was about $1\frac{1}{2}$ in. below the cells, then decreased, but for the six cells remained higher than the vertical stress σ_z .

From these results it was apparent that the coefficient of earth pressure, originally Ko, had changed during penetration to a coefficient greater than unity. In conventional soil mechanics terminology the condition had transformed from an 'active' to a 'passive' state during penetration; this transformation shows a certain analogy to the surface bearing capacity problem which was solved by Terzaghi (1943) and formed the basis on which the limiting stress characteristics of Chapter 5 were developed.

The horizontal circumferential stresses (σ_θ) gave a more gradual increase during penetration for the cells listed above, reaching a maximum stress considerably lower than the horizontal radial stress, at a penetration between those for maximum σ_z and σ_r. As with the higher stresses, σ_θ decreased to become appreciably constant with depth. When constant, σ_θ was again less than the horizontal stress, σ_r.

Further discussion on the results in Figures 6.1.(a), (c), (e) and (g) will ensue in Section 6.2.(iii).

The results for the second group of pressure cells in loose sand, nos. 2, 5 and 7, were similar to the distributions discussed above for penetrations down to the cells' positions. Below this point, where the pentrometer tip was roughly in the same horizontal plane as the upper faces of the cells, the vertical stress showed a greater reduction than for cells 1, 3, 6 and 8, falling below the circumferential stress for about 2 in. of penetration then increasing to become the intermediate stress.

The difference in stress distribution between the groups of pressure cells can probably best be explained by considering the pressure cell in position in a loose sand mass. In loose sand the stresses have been shown to be of a low order, up to 4 p.s.i., but usually between $\frac{1}{2}$ p.s.i. and 2 p.s.i. The forces on pressure cell faces have therefore ranged from about 1/32 lbf. to 1/4 lbf., giving differential forces across opposing faces less than these. The pressure cells, in position, were connected to the recording equipment by several thin P.V.C. covered wires which, over the 2 in. to 3 in. length adjoining each pressure cell, were coated and made rigid with thermo-plastic cement (Araldite). It was considered that the resistance provided by the rigid 'wires' extending from the rear faces (i.e. the faces opposite those measuring σ_r) was greater than the forces imposed on the cells during penetration. In consequence it was thought that little or no rotation of the pressure cells was experienced in loose sand. Assuming this to be correct, it is not difficult to appreciate that with a fixed pressure cell an extensive redistribution of stresses would arise during movement of sand around the cell. Where the direction of movement tended to be away from active cell faces, or, where strains were predominantly negative in the directions of active faces, the measured stresses would be expected to be somewhat exaggerated, probably more so than with a cell which rotated during movement of sand.

Results for the two groups in loose sand, which have already been discussed, suggest regions where strains were certainly +ve (in the region $r < 1.1$ in) and where strains were tending to become -ve. It appears likely that in the regions where strains were -ve (or zero) pressure cell recordings could have been slightly in error.

The third group of pressure cells, nos. 9 and 0, show decreases in all three stresses below a penetration corresponding to the positions of the cells. In both cases the measured vertical stress became zero and in cell 0,σ_θ also became zero below 2 in. These irregularities are thought to be associated with -ve strain components in the directions of the cell faces. Further discussion would appear to be irrelevant.

Curve (e) of Figure 6.5., depicting penetration resistance in the loose sand sample containing pressure cells, indicates a denser pocket of sand at a depth between 6 in. and 8 in. The dense pocket coincides with the position of cell no. 6, which is positioned at 6.69 in. below the sand surface, and it is probable that the higher σ_z plot is related to this slightly denser zone. Since the pressure cell performed adequately in later tests the anomalies were not considered to be the result of an inherent fault in the pressure cell itself.

The pressure cell results for loose-medium sand were, in most respects, similar to those for loose sand; similar discussion, therefore, applies. Since the respective porosities differed by only 1.88% the alike results were anticipated.

The most significant findings are simply re-iterated: stress distributions again indicated a change from the 'active' to 'passive' conditions during penetration, the transformation occurring at a point where σ_r and σ_z were equal: the circumferential stress was, for pressure cells 1, 3 and 6 (i.e. cells within group 1), the minor of the three polar co-ordinate stresses σ_r, σ_θ, and σ_z : the point at which σ_θ was a maximum generally coincided with the point of σ_r and σ_z equality.

One major difference between the results shown in Figures 6.1. and 6.2. was the plot for the vertical stress, σ_z, for cells 5 and 7. During penetration in loose-medium sand the vertical stress became zero for both pressure cells at a penetrometer depth $2\frac{1}{2}$ in. below the upper faces of the cells. One possible explanation for the variation was the greater distance, r, from the z-axis for cells 5 and 7 in loose-medium sand. The value of r has already been shown to affect the σ_z distribution in loose sand (e.g. σ_z plots for cells 1 and 2, Figures 6.1.(a) and (b)) and a similar argument can be used to explain the disparity between the σ_z curves in Figures 6.1(f) and 6.2.(f), also 6.1(d) and 6.2(d).

One pressure cell test result which did not follow the regular pattern was that of cell no. 3. The final stress distribution shown in Figure 6.2.(c) gave the inequality $\sigma_\theta < \sigma_r < \sigma_z$, although all other results for cells in the first group (for loose and loose-medium sands) showed σ_r as the major stress.

Figure 6.6. (curve (e)) indicates a dense pocket in the loose-medium sand mass between 9 in. and 11 in., and since cell no. 3 has a co-ordinate z of 9.3 in. the dense sand pocket probably resulted during positioning of cell 3. The dense sand pocket, however, does not explain the anomaly $\sigma_z > \sigma_r$, unless it is a phenomenon associated with dense sand immediately adjacent to the penetrometer shank. Unfortunately pressure cells in dense sand

were not placed closer than about 1.3 in. from the z-axis* and further evidence to support or refute this as a conjecture is not available.

The stress distributions from pressure cell recordings taken during penetration into dense sand can also be conveniently discussed by separating results into distinct groups. The first group constitutes cells no. 1 and 5.

Figures 6.3.(a) and (d) depict the three stresses in polar co-ordinate directions; these remained appreciably constant until the penetrometer was roughly $5\frac{1}{2}$ in. above the upper faces of the two cells. The comparable distance for loose sand was $3\frac{1}{2}$ in. but it was found that boundary effects became significant in loose sand at a greater depth below the penetrometer than those in dense sand. The implication is that compression of sand is restricted to a narrow region immediately below the penetrometer point in loose sands.

Maximum σ_z occurred approximately where the penetrometer tip coincided with the upper faces of the cells. Maximum σ_r was reached at a further 2 in. penetration. The position of maximum σ_θ corresponded with the point of equality: $\sigma_r = \sigma_z$, in both sets of results. In fact, generally speaking, the discussion which applied to cells which were less than one inch from the z-axis ($r = 0$) in loose and loose-medium sands applies equally well to the stress distributions presented in Figures 6.3.(a) and (d). Differences in the respective plots, apart from the higher stresses obtained in dense sand, are the ratios σ_r/σ_θ and σ_r/σ_z. The penetration

* Stresses would have exceeded the design stress of 10 p.s.i. giving a reduced diaphragm diameter/deflection ratio.

tests in loose sand gave final ratios of 1.71 and 1.28 whereas the dense sand penetration tests produced final ratios of 2.43 and 1.28.

The second convenient group of cells in dense sand embraces nos. 2, 3, 6 and 8. Figures 6.3.(b), (c), (e) and (g) illustrate the pressure distributions. The horizontal radial stresses (σ r) increased to a maximum for a penetration equivalent to the depth to the upper faces of the cells (i.e. the σ_z face). With further penetration σ_z decreased, finally becoming constant after a further penetration of 8 in. or so. The vertical stresses decreased from the original overburden stresses to minimum values at the same penetration which gave maximum σ r. As the penetrometer advanced the vertical stresses increased to become constant, greater than the original overburden pressure but less than the constant radial stresses.

The circumferential stresses showed a similar distribution to σ_z. For cells 2, 6 and 8 these remained the minor stresses. For cell no. 3 the distribution $\sigma_z < \sigma_\theta < \sigma$ r was obtained, although the difference between σ_z and σ_θ was slight. The stresses assumed constant values which were similar to the original overburden pressures.

Before discussing the final group of pressure cell results it is worth considering Figure 6.8. The figure represents the surface of dense sand (on a much exaggerated vertical scale) during penetration. Sand movement was recorded during a 'half section' test but using the methods described in Chapter 4.5., a series of photographs were taken at specific penetrations and results were plotted with the aid of the Wild A7 Autograph plotter.

The sand surface continued to show an increase in the amount of heave up to 6 in. penetration. At 9 in. penetration the surface exhibited a fall from its maximum position. For penetrations below 9 in. the surface did not reveal any significant change.

The broken lines below the dense sand surface in Figure 6.8. are the conjectured lines delineating regions in which limiting stress conditions existed (i.e. they are slip lines). It is obvious that -ve vertical strains* prevailed within these regions until a penetration, somewhere between 6 in. and 9 in., had been reached. Figure 6.8., incidentally, compares with the result reported by Kerisel (1961); the maximum heave in dense sand was obtained for a penetration of about $4\frac{1}{2}$ times the diameter of the penetrometer instrument.

The stress distributions in Figures 6.3.(f), (h) and (j) suggest -ve strains in the form of a decrease in σ_z to a stress less than the original overburden pressures. Figures 6.3.(h) and (j) (cells no. 9 and 0) show a noticeable increase in stresses between 9 in. and 12 in. This agrees with the movement of the sand surface which is depicted in Figure 6.8.; the surface appeared to be falling at 9 in. penetration.

The two stresses σ_r and σ_θ exhibited similar distribution for cells 9 and 0 but the distributions for cell no. 7 resembled those of the first group (i.e. nos. 1 and 5).

By themselves the pressure cells have defined three of the four unknown components of the stress tensor at a point in a polar co-ordinate system; Figures 6.1., 6.2. and 6.3. present these

* i.e. strains in -ve directions of the z-axis

components. The following section (6.2.(iii)) presents these unknowns in a form which indicates the behaviour of the sand during penetration.

6.2.(iii) Mohr circles of stress

The Haar and Kármán hypothesis, adopted in Chapter 5 to predict stress characteristics and distributions, was used as the principle from which Mohr stress circles were plotted from pressure cell results. The circumferential (principal) stress at a particular penetration was assumed to be the minor principal stress and the two co-ordinate stresses σr and σz were expressed as the abscissa (σ) of the centre of a Mohr stress circle, i.e. $\sigma z + \sigma r = 2\sigma$. Typical results obtained from the stress distribution plots are Figures 6.9. 6.10 and 6.11., these being for experiments in loose, loose-medium and dense sand respectively.

The parameter ϕ_t will be introduced to define the angle of inclination of the lines drawn on Figures 6.9., 6.10 and 6.11. From the figures it can be seen that the lines are tangents from the origin to the stress circles with maximum $\tau f/\sigma n$ ratios, τf being the shear stress of the point on every circle at which a tangent from the origin would touch.

For 5 out of the 9 results plotted in Figures 6.9.(a) to (j) the values of ϕt corresponded with the value of Coulomb ϕ for the loose sand in its initial state of compaction. The angle ϕt for pressure cells 1, 2, 3, 7 and 9 were, at most, only a few degrees different from the Coulomb ϕ of 31.8° obtained from direct shear tests. Assuming the results are correct, the angle of internal shearing resistance ϕ for loose sand did not appear to increase during penetration into the sand mass, even though the stresses σ_r, σ_z and σ_θ, in all cases, increased significantly at penetrations approximately in line (horizontally) with the positions of the cells.

Mohr stress circles plotted from pressure distribution for cell no. 2 gave an inclination angle ϕt of 38.9° although from the observations discussed above a value in the order of 32° was expected. To illustrate the difficulties in establishing ϕt accurately an alternative, but feasible, σ_θ distribution between 9 in. and 12 in. penetration is shown by broken lines in Figure 6.1.(b). If this distribution had been used in place of the full line distribution, stress circles would have produced a ϕt of roughly 30°.

The values of ϕt shown in Figures 6.9.(a), (c), (d), (f) and (h) are, however, entirely acceptable; σ_θ distribution represented by the full line curves tended to be minimal, rather than maximal, thus alternative σ_θ distribution such as that shown in Figure 6.9.(b) would have given ϕt values much lower than these. Figure 2.6. indicates that Coulomb ϕ became constant at about 31.5° therefore values of ϕt considerably below this would have no real meaning.

Mohr circles of stress from measurements with cells 6 and 8 gave values of ϕt which were unrealistically high. Misalignment of the two cells may have given rise to such anomalies, since lower values of σ_θ * would have been obtained; with lower σ_θ a greater major/minor principal stress ratio obtains for the same major principal stress, giving a greater angle of inclination ϕt.

* Misaligned pressure cells would measure resolved components of σ_θ and these would be less than the true circumferential stress.

In the opinion of the Author misalignment was a most unlikely occurrence and it is considered that for these two cells sand movement during penetration produced misleading results. The abnormally high values of Øt were the result of displacement phenomena which the pressure cell was unable to accommodate, due to its fixed position in the loose sand mass.

With the exception of cell no. 8 and 9 and 0 for which results were rather limited, the line inclined at the angle Øt was tangential to the stress circles with the condition $\sigma r = \sigma z$. Also the stress circles with this condition gave maximum values of σ and the principal stress ratio $\sigma 1/\sigma 3$. The Mohr stress diagrams given in Figures 6.9. do not establish the complete stress tensor at a point because it has not been possible to determine the directions of the shear stresses, τ rz and τ zr, from the stress circles. The figures do, however, show that the condition $\sigma r = \sigma z$ is a limiting condition and the directions of major principal stress must be inclined at $\pm 45^{\circ}$ to the vertical. By using the results which are given in the following section the complete stress tensor can be determined during penetration into a loose sand mass.

In loose-medium sand three sets of Mohr stress circles gave values of Øt which were similar to the Coulomb Ø of the sand in its initial state of compaction. The pressure cells which produced these results were nos. 1, 5 and 8, giving values of Øt of 32.2°, 31.2° and 28° respectively.

Pressure cells 2, 3, 9 and 0 gave results from which higher, but realistic, Øt values were obtained; the angle varied from 36.9° to 39.6°.

Unrealistic results for Øt were deduced from Mohr circles of stress for cells 6 and 7. Øt from pressure cell no. 6 distributions was 57° and for cell 7: 48.5°.

If Figures 6.9. and 6.10 are compared it can be seen that there is no particular order to cells which give higher angles of inclination Øt, and this leads the Author to assume that a combination of misleading interpretations have produced certain unrealistic results. The first, mentioned above, was concerned with the displacement phenomena around a pressure cell during penetration. The second possible source of error lies in the stress distribution curves of Figures 6.1. and 6.2. Alternative interpretations of the same pressure cell results have already been demonstrated in Figure 6.1.(b) but a comparison of Figures 6.1.(c) and 6.2.(c) is more convincing.

Cell 3 in loose sand was positioned 0.53 in. from the r = 0 axis and in loose-medium sand r = 0.73 in. It is reasonable to assume that stress distributions from the two experiments would not be dissimilar but Figure 6.1.(c) shows a much lower distribution of stress. Interpolation between the results at 9 in. and 12 in. in Figure 6.1.(c) could have given, for example, a stress σ_z between about 2.2. p.s.i. and 5.5. p.s.i.; obviously this order of variation would be expected to affect the Mohr circles of stress and hence the inclination of the tangent to the circles.

It is reasonable to conclude that because of the limited number of pressure cell readings during penetration into loose and loose-medium sand the results serve to illustrate an overall pattern, rather than an exact picture of the sand behaviour.

Figures 6.11. show the Mohr circle of stresses from penetration tests in dense sand and it is proposed to report these as two distinct types of result. Into the first category fall cells 1, 2, 3 and 5 and the remainder fit into the second group.

Mohr circles of stress from cell 1 and 5 results are plotted in Figures 6.11.(a) and (d), both cells being at the same distance from the brass penetrometer : $r \doteq 1.3$ in. The results show ϕt values of 39.5° and 40° respectively, compared with a Coulomb ϕ of 40.8° from direct shear tests on dense sand. The stress circles show a distinct tangent line and it is obvious that the Haar and Kármán hypothesis has been confirmed for a range of stress distributions during axially-symmetric penetration. In addition it is felt that the validity of the Mohr-Coulomb yield criterion for dense sand has been proved.

It is interesting to note that the maximum values of σ_{θ} and σ produced the stress circle having the condition $\sigma_r = \sigma_z$; this offers a limiting stress condition, and the knowledge that the major principal stress was inclined at either 45° or -45° to the vertical.

Pressure cells 2 and 3 also gave results which, when plotted as Mohr stress circles, showed ϕt angles similar to the Coulomb ϕ for the dense sand condition: for cell 2, ϕt was 41.1° and $\phi t = 39.8^{\circ}$ from cell 3 results.

Figures 6.11.(e), (f), (g), (h) and (j) show the tangent angle for pressure cells 6 to 0. A range of results from 49.6° to 64.5° was obtained with the exception of cell 8 which gave $\phi t = 31.6^{\circ}$. Stress distributions for penetrations into dense sand at depths less than 6 in. must be associated with upward movement of the sand (as shown in Figure 6.8.) and the arguments used previously for loose and loose-medium sand experiments, relating large deformations with anomalies in stress distribution, are applied equally well here. Reiterating, where large deformations played a significant part in the behaviour of the sand the results appeared to be irregular. It is quite possible, therefore, that the Haar and Kármán hypothesis and/or the Mohr-Coulomb criterion could not be applied in such condition.

The angle Øt for pressure cells 1, 2, 3 and 5 in dense sand were similar, and comparable with Coulomb Ø and this likeness is considered to be more than just coincidence; it indicates that the pressure cell produced a true picture of stress distribution in stress conditions where (relatively) high compressive stresses were predominant.

The Mohr stress circles plotted in Figures 6.9., 6.10. and 6.11 showed a so-called 'elastic' state at the position of each cell for penetrations from the sand surface to a point a few inches above the cells' upper faces: that is, the state of stress at the position of pressure cells did not immediately reach the limiting condition (as defined by the Mohr-Coulomb criterion) during an initial increase of stress. As the penetrometer advanced towards the cells the sand reached a limiting condition, related to the Coulomb Ø of the particular sand mass into which the brass penetrometer was advancing. The sand remained at the limiting condition as the stress increased and finally further penetration beyond the level of the pressure cells brought the stress circles again into the'elastic' state, at values of Ø* less than the maximum Øt. With few exceptions this behaviour was seen for all cells but it

* Ø is defined as the maximum τ_n/σ_n ratio for any circle.

is best shown on Figures 6.11.(a) and (d). These two sets of results will be used, in particular, in later sections of this chapter to explain stress distribution during penetration.

Additional results which illustrate the behaviour of sand during penetration are related in Section 6.3., below. Although the results have been selected from a larger set of data they are used to complete the stress tensor by establishing directions of shear stress at points represented by pressure cells.

6.3. Experimental strain fields

6.3.(i) Selected results

The method of measuring displacements against a thick glass plate during axially-symmetric penetration into dry sand has been described in Chapter 4; the experiments have yielded a large quantity of co-ordinates which represented the relative positions of small markers on a square pattern grid. A computer programme was written so that the displacements on the glass plate could be analysed quickly and accurately and the computed results, namely $\dot{\epsilon}_r$, $\dot{\epsilon}_z$, $\dot{\epsilon}_\theta$, $\dot{\gamma}_{rz_{max}}$, $\dot{\epsilon}_1$, $\dot{\epsilon}_2$, and λ, were printed in a row for each point on the grid; computations for one increment of penetration thus formed a table of these results.

Initially it was anticipated that differences in deformations during 0.1 in. increments of penetration would be recorded and could subsequently be used to compute incremental strains for a large range of penetrations, giving, for example, incremental strains for 12 in. penetration by using displacements for penetrations of 11.9 in. and 12 in. In practice this arrangement proved to be unsuccessful and incremental strains were eventually computed for penetration increments of 2 in.

In Chapter 5 the strain tensor matrix has been defined in cylindrical polar co-ordinate form (Section 5.2.(ii)), the principal strains have been related to shear and normal strains in co-ordinate directions (equation 5.4.) and shear and normal strains have been equated with displacements (u and w) in the r and z directions (equations 5.5.). To permit a solution of equations 5.5. by computer methods it was necessary to rearrange them in finite difference form, these being listed in Appendix I of the thesis.

The computer programme was developed not only to analyse the deformations at a point and print results, but also to produce a graph plot of the directions of major principal strain-rate, λ , for each penetration increment. Several graph-plotter difficulties which arose throughout the period when the Author was using the I.C.L. 1905 computer precluded the use of this facility and eventually results were plotted by hand. Because each plot took roughly $1\frac{1}{2}$ hours to plot manually it was decided to select a few penetration depths and consider these to be representative of the typical distribution of major principal strain-rate directions.

Results for penetration into loose sand are shown in Figures 6.12. The figures, (a), (b), (c) and (d), illustrate the distribution of the angle of inclination λ for penetrations of 3 in., 6 in., 9 in. and 12 in. respectively. The horizontal spacings between sets of lines are 0.75 in. except those at r = 0 in. and r = 0.5 in., and rows of lines are again 0.75 in. apart. Figures 6.13 show the distribution of λ for penetration into dense sand, (a), (b), (c) and (d) being for 4 in., 9 in., 15 in., and 24 in.penetrations respectively. The grid was used for the dense sand experiments with a 1.5 in. spacing and a scale of approximately half that used in Figures 6.12 was adopted.

In Figure 6.14 the distribution of strain-rates $\dot{\epsilon}_r, \dot{\epsilon}_z$ and $\dot{\epsilon}_\theta$ have been plotted against depth of penetration for 6 points in a loose sand mass. The results were obtained by plotting the strain-rates for the same point from each complete strain-rate results table, to give the values shown on the curves.

Figures 6.15 illustrate the same type of results for penetration into dense sand, but for different points in the sand mass. Since there was a considerable difference in range of strain-rates, it was expedient to plot the results for loose and dense sand to different abscissa scales.

6.3.(ii) Discussion of results

The distribution of directions of major principal strain-rates in Figures 6.12 illustrates the change in direction of $\dot{\epsilon}_1$ during penetration. The direction of $\dot{\epsilon}_1$ became near-vertical beneath the instrument, and changed to near-horizontal alongside the shank, as the penetrometer advanced, giving a total change in direction of the major principal strain-rate of roughly 90°.

In dense sand the condition was very similar, with the direction of $\dot{\epsilon}_1$ varying as shown in Figures 6.13. The total angle through which the major principal strain-rate moved, however, was considerably less than for loose sand, the change being about 60°.

The effect of penetration was noticeable in regions to approximately 6 in. below the penetrometer point in loose sand (Figure 6.12(d)) and the random directions to the sides of the instrument, implying negligible strain rates in these regions, occurred at a distance of $r \approx 4$ in.

In Figures 6.13 the pattern was quite different. The depth below the penetrometer at which directions of $\dot{\epsilon}_1$ remained random was, at most, about 7½ in. (from Figure 6.13.b) and the zones in which values of λ appeared to be ordered extended 10 in. either side of the z-axis.

Displacements in sand on each side of the split penetrometer were measured and analysed separately but results were plotted on one figure. The two sets of 'half' results, when combined into one, adequately illustrate the axial-symmetry which was achieved with the 'half-section' container and penetrometer. Only where directions of major principal strain-rates was random did the two halves depict large differences in the angle λ ; this, of course, was to be expected.

Little would be gained from an extensive discussion of the results in Figures 6.14 and 6.15; they are, in the main, self-explanatory, as were stress plots for penetrations into the same sand mass.

Therefore, it is proposed to discuss only those phenomena which are particularly significant. The discussion, firstly, is centred on Figures 6.14.

In the loose sand condition, the circumferential strain-rates $\dot{\epsilon}_\theta$ were negative (tensile) at points more than 3 in. below the sand surface. For the same points the radial strain-rates were opposite in sign, and of roughly equal magnitude, and vertical strain-rates, in comparison, were small. However, $\dot{\epsilon}_z$ was initially greater than $\dot{\epsilon}_r$ but at a penetration somewhere between the level of the point and 2 in. below it, $\dot{\epsilon}_z$ became negative as the radial strain-rate reached its maximum value. Over the next 3 in. to 5 in. penetration,all strain-rates became near-zero and thereafter remained virtually constant. It is also interesting to note that the equality $\dot{\epsilon}_r = \dot{\epsilon}_z$ occurred when the penetrometer was some 1½ in. below the point for which strain-rates were plotted.

At points at 3 in. below the stress free surface the circumferential strain-rate initially assumed a positive value, rapidly became negative as the penetrometer tip reached the point, then tended to become zero again as the penetration depth increased. The vertical strain-rate $\dot{\epsilon}_z$ showed a similar distribution, although to a lesser extent, and for the point : $r = 1.25$ in., $z = 3.0$ in., the $\dot{\epsilon}_r$ distribution exactly opposed the $\dot{\epsilon}_\theta$ distribution.

Further close examination of strain-rate components in Figures 6.14 would serve only to emphasise the complex strain-rate patterns set up during installation of a penetrometer. The influence which these changes have on the properties of loose sand will become obvious when results are compared with predicted behaviour.

Referring back to Figures 6.1., the σ_r and σ_z stress distributions, for pressure cells at roughly the same positions as the points in the strain field, were of the same general form; σ_z reached a maximum before $\sigma_z = \sigma_r$ and this condition occurred before σ_r reached a maximum. A comparison of Figures 6.1.(a) and 6.14.(f) demonstrates the similarity, except that **strain-rates** became appreciably zero when the penetrometer point had passed, whereas stresses became constant at values higher than the original stresses; the higher stresses were, of course, the residual stresses created by penetration.

The volumetric strain-rate ($\dot{\epsilon}v$) shown in Figures 6.14. tended to be negative for the points with $r = 0.5$ in. and gave positive results for $r = 1.25$ in., suggesting expansion adjacent to the shank, coupled with compression in the loose sand away from the penetrometer.

The strain-rate results for dense sand are discussed briefly in two groups: one for a penetration of 3 in. and a second group for penetrations of 6 in. and 12 in.

Figures 6.15.(a) and (b) show, in particular, the negative distribution of the vertical strain-rate $\dot{\epsilon}_z$ and the inequality $\dot{\epsilon}_z < \dot{\epsilon}_\theta < \dot{\epsilon}_r$ for the two points r = 0.5 in. and r = 2.0 in. Since $\dot{\epsilon}_r$ and $\dot{\epsilon}_z$ lie on the largest of the three Mohr circles which represent the strain-rate tensors, $\dot{\epsilon}_\theta$ must be the intermediate (principal) strain-rate. These conditions do not fit the Haar and Kármán hypothesis although the sand is undergoing plastic deformation. In this particular case, where dilatation of dense sand is not supressed, the sand is probably furthest from the idealised rigid-plastic state on which the Haar and Kármán hypothesis depends. At shallow penetration in dense sand the principle appears to be inappropriate.

The outstanding feature of Figure 6.15.(b) is the positive $\dot{\epsilon}_\theta$ distribution; all other points in the sand mass have shown negative $\dot{\epsilon}_\theta$.

Figures 6.15.(c) and (e) gave very similar strain-rate distributions, apart from an uncharacteristic $\dot{\epsilon}_\theta$ plot at 21 in. in Figure 6.15.(e). Figures 6.15.(d) and (f) also compare favourably.

The general distribution patterns differ from results for loose sand but the results in Figures 6.15.(e) and (f) bear very close comparison with the stress distribution plots given in Figures 6.3.(a) and (b).

Volumetric strains for the two points at 3 in. below the dense sand surface showed a change from positive to negative at r = 0.5 in. and a positive distribution for r = 2.0 in. The distribution at 6 in. and 12 in. depths were entirely positive and entirely negative for r = 0.5 in. and r = 2.0 in. respectively, but negative values of $\dot{\epsilon}_v$ were generally much less than 1%.

For ten of the twelve points studied $\dot{\epsilon}_\theta$ was found to be either the maximum or minimum strain-rate and only at points r = 0.5 in., z = 3 in., and r = 2 in., z = 3 in., in dense sand was $\dot{\epsilon}_\theta$ an intermediate principal strain-rate.

The theoretical solution in Chapter 5 was developed from the two basic assumptions that: sand is a rigid plastic material and: boundary conditions for continuous penetration are the same as those in well known deep foundation bearing capacity problems (e.g. Meyerhof, 1951). The first assumption requires the soil parameters γ d and ϕ to remain constant throughout penetration, and the volumetric strains to be zero. Although this is an extreme idealisation, past research work has shown that answers obtained using rigid-plastic assumptions are not widely different from actual behaviour of sand.

The second assumption is thought to be unique and was used by the Author because nothing else was available. Results presented in this and the previous section, however, have suggested that this assumption was incorrect. Figures 6.1., 6.2. and 6.3. show the change in stress in sand from the 'as-poured' state to the residual state; the change is created by penetration and it is this residual state which should be considered in any mathematical model for continuous penetration. Figures 6.12 and 6.13 give the distribution of directions of major principal strain-rate and illustrate the effect that installation of a penetrometer has on a sand mass having no original strain state. These two sets of experimental results show boundary and internal stress conditions which are very different from those computed from the methods described in Chapter 5. Also experimental results do not agree with boundary conditions shown, for example, in Figure 6.21.; this is not surprising since the stresses here and those computed by the Author are closely related, because of the basic assumptions mentioned above. It is worth reiterating that Figure 6.21 shows an instrument (in this case a deep foundation) installed without disturbance to the surrounding soil, then loaded to failure with infinitesimal strain. It is now realised that this state is unlike that created when a penetrometer advances to a point from the surface of the sand.

Further evidence to point out the fallacy of the boundary condition assumptions is presented in the following sections, in which some results from Chapter 5 are arranged in a different form, and are discussed briefly.

6.4. Theoretical limiting stress fields

6.4.(i) Typical results

During the discussion in Chapter 5.4.(iii) it was shown that, provided the parameter σ was kept constant along the initial vertical boundary, the distribution of limiting stresses in the sand mass did not change during penetration*. The additional proviso: that the zone of limiting stress around the penetrometer was fully developed, was necessary to give the unchanging stress distribution with depth; the condition $\gamma \approx 0$ was also applied in all computations.

Stress distribution plots for the limiting stress conditions in loose sand, for points at positions r = 0.55 in. and r = 1.0 in., are illustrated in Figures 6.16(a) and (b); a penetrometer depth of 12 in. is shown but this could have been any depth greater than 6 in. and would still have produced the same distribution curves. The results were obtained by plotting the limiting stresses along the vertical planes defined by the above equations (r = 0.55 in. and r = 1.0 in.) for the stress characteristics shown in Figures 5.17.(a).

Directions of major principal stress in Figure 5.17.(a) are plotted on Figure 6.17.

Limiting stress distributions for the penetrometer in dense sand

* It was also assumed that the second parameter, ψ, which completed the specification of boundary conditions, was constant along the initial vertical boundary.

are plotted in Figures 6.18.(a) and (b). The curves for σ_θ , σ_z and and σ_r were obtained in the same way as those in Figure 6.16. but in this case the stresses are for a point 18 in. below the sand surface. Results are shown for stresses along the vertical planes r = 1.3 in. and r ; 2.6 in. in the characteristic net given in Figure 5.18.(b). The directions of major principal stresses plotted on Figure 6.19. have been taken from Figure 5.18.(b).

Figures 6.16. to 6.19. show results obtained using a mathematical solution for what was thought to be continuous axially-symmetric penetration into dry ponderable sand. It has been suggested already that the theoretical solutions do not relate to true axially-symmetric penetrations. Additional comparison between theoretical and experimental results is included in the following section, and reasons for the disagreement are put forward.

6.4.(ii) Differences between experimental and theoretical results

Figures 6.12. and 6.17. show the most obvious differences between results derived from experimental installation and results obtained using the theory of true quasi-static conditions considered in Chapter 5. The figures show strain-rate and stress directions but if the generally accepted conjecture: $\lambda \doteq \psi$ is assumed, that is, if it is supposed that directions of principal stress and strain-rate coincide, it is apparent that the two sets of results are incompatible. Of particular significance are the directions of major principal stress and strain-rate along the vertical boundary, these being respectively 6° and 80°.

Figures 6.13. and 6.19. show similar differences in plots for dense sand. Major principal strain-rates obtained experimentally are inclined at approximately 75° (i.e. λ = $+75^\circ$) along the vertical boundary, whereas predicted values of ψ are approximately 7°.

Marked differences between predicted and measured stresses at a fixed point in loose sand can be seen in Figures 6.16. and 6.1.(a). In particular the point at which $\sigma_r = \sigma_z$ occurs again suggests incompatibilities. In Figure 6.1.(a) the penetration giving $\sigma_r = \sigma_z$ coincided with the point of maximum σ_θ , also σ . In Figure 6.16. however the stress state $\sigma_r = \sigma_z$ occurs at a penetration where σ_θ is only 8% of the maximum value predicted.

Similar differences can be seen in the predicted and measured stresses for penetration in dense sand, shown respectively in Figures 6.18. and 6.3.(a).

The differences indicated above require at least a brief explanation, but it is expedient, first, to outline the steps taken to arrive at the basic mathematical model described in Chapter 5.

The distinct plastic zones in the axially-symmetric penetration problem were established by considering the well known bearing capacity problem, and the method of solution, developed by Terzaghi (1943). The active and passive zones in the bearing capacity problem and the direction of incipient movement are shown in Figure 6.20.(a). In this classic case the conventional 'active' and 'passive' terms and the associated stress conditions agree with the definitions used by the Author (p.125).

It is relevant here to point out that if a uniform stress normal to the boundary OA were introduced along this boundary, the overall pattern of characteristics* in Figure 6.20(a) would remain the same, provided the stress along OD still gave the condition of limiting equilibrium.

By turning Terzaghi's bearing capacity mechanism anti-clockwise through 90° and making slight modifications, the mechanism illustrated

* i.e. coincident stress and velocity characteristics

in Figure 6.20.(b) is obtained. Directions of incipient movement are the same in Figures 6.20.(a) and (b) and the active and passive conventions adopted in Chapter 5 are still applicable. The stress conditions are also the same, giving stresses parallel with the boundary which are greater than stresses normal to the boundary in the passive zone*. Conditions are of course exactly the opposite in the active zone.

The zones in Figure 6.20.(b), together with the boundary conditions shown in Figure 5.12., were used in all limiting stress computations, under the premise that they were correct for axially symmetric penetration into dry sand. It would appear possible to argue against such a simple approach, but as far as the Author is aware there are no results available which would offer a more logical method of deciding the stress systems in this problem.

Since experiments have shown that stress and strain-states created by the introduction of a penetrometer are radically different from those predicted, it can now be concluded that the assumptions made by the Author were not correct: boundary conditions for a penetrometer which is moving continuously into a mass of sand are not the same as those for an instrument which is installed in undisturbed sand, so that there are no residual stresses, then loaded to failure.

In the latter case, the major principal stress is vertical before load is applied and there is no strain state (i.e. the elements in the tensor on page 118 are all zero). Loading the instrument creates the boundary conditions shown in Figures 6.17 and 6.19. Figure 6.22.(a) has been produced from 6.17. to show the angle of rotation of σ_1, before its inclination, ψ , is -45°. Values of ψ marked with a single

* although this stress condition in Figure 6.20.(b) contradicts the conventional definition of a passive zone, in which the horizontal stress must be greater than the vertical stress, there is no conflict with the definition used by the Author.

asterisk in Table 6.1. show the initial and final inclinations of σ_1 and also the total rotation between the vertical boundary and the base. This rotation is roughly the same as the total rotation shown by the Author on Figure 6.21. The agreement is not really surprising since the stress characteristics in Figure 6.21. were a contributing factor in finally deciding that the zones in Figure 6.20.(b) were the ones to use in the theoretical solution. It can be seen that, if friction is introduced along the vertical boundary in Figure 6.20.(b), the principal stress directions become similar to directions shown in Figure 6.21.

The similarity between Figures 6.21. and, for example, 6.17., suggests that the assumptions and the analytical methods described in Chapter 5 have produced a workable solution for true quasi-static axially-symmetric behaviour of foundations such as deep circular piles or piers: the solution at present relates to foundations installed without setting-up residual stresses and strain states* in the surrounding homogeneous soil, then being loaded to failure with very little overall movement (i.e. settlement). The additional condition is one of ideal rigid-plastic sand behaviour.

In continuous axially-symmetric penetration it has been shown that preceding stages of penetration produce a residual state of stress, with an associated strain field. The residual stresses probably do not vary very much once the penetrometer point has passed, but in the theoretical studies in the previous chapter this stress state and any changes in the physical properties of the sand were ignored. This was not deliberate, but the full effects of installing the penetrometer were not realised. Experimental results reported in this chapter have

* this, of course, is physically impossible, but the Author's solutions have not been developed beyond these conditions yet.

presented part of the picture of the effects of installation, but not to the extent that a complete theory for continuous penetration can now be developed.

This section has introduced deep circular foundations which are installed in sand without disturbance, because the mathematical model was shown to represent this particular type of foundation. A type of pile which is more commonly used in sands is the driven pile. As a conclusion to this section the installation and loading of such a pile in sand are briefly discussed.

Installation of a circular penetrometer in homogeneous sand produces the patterns of rotation of major principal strain-rates shown in Figures 6.12 and 6.13. It will be proved in the following section that directions of major principal stress rotate similarly. Below the penetrometer values of ψ derived from experimental installation are predominantly vertical, but the angle changes to give a near-horizontal inclination to σ_1. In other words, loading after installation produces a stress system almost opposite to that shown in Figure 6.20.(b). Knowing that $\sigma_r/\sigma_z > 1$ in the zone adjacent to the vertical boundary AO, a close examination of Figure 5.7. shows that β changes sign. The integer n therefore changes sign and its substitution in equation 5.23. gives the values of ψ marked by double asterisks in Table 6.1., for the initial and final boundaries. These are the values of ψ obtained for a limiting stress condition after installation of the penetrometer; this sequence of operations is that experienced by a driven pile.

The initial rotation of σ_1 is plotted in Figure 6.22.(a), and the angle of rotation to reach an inclination of $+45^\circ$ is also shown. This figure is conjectural and is based on a number of simplifications, since there is no experimental evidence to support the above calculations, but it is put forward as a basic stress state for driven piles loaded to failure. This is in comparison with Figure 6.22.(a), which shows the predicted limiting stress condition for piles placed in homogeneous sand without disturbance, then loaded to reach the limiting stress condition. It can be seen that the directions of rotation plotted in Figure 6.22.(b) are not very different from the strain-rate plots in Figure 6.12.

In the following section selected results from experiments in dense sand will be extended to show the complete stress tensor during penetration. By introducing the concept of a moving pole point in a Mohr stress diagram, the phenomenon of σ_1 rotation will also be extended.

6.5. Complete stress tensor

Figure 6.11.(d) has been redrawn to a larger scale as Figure 6.23. Additional stress circles have been plotted.

Before proceeding it is necessary to discuss the possible pole points for a stress circle and explain how strain-rate results have been used to determine the complete stress tensor. The technique whereby the pole point position can be located is explained by referring to Figure 6.24. Here Mohr stress circles for penetrations of 6 in. and 8 in. into dense sand have been drawn from the previous figure. Each circle has two positions marked by solid circles where the pole point could occur. The positions are unique to every stress circle and are always found to lie on the vertical plane, $\sigma_n = \sigma_r$. Knowing that a line drawn from the real pole point to a particular stress (say σ_1), gives the inclination of the plane on which that stress acts, it is obvious that the directions of major principal strain-rates will establish which of the two points is the correct pole point. This procedure need only be done for a few selected stress circles since intermediate results will conform with the overall pattern. In fact, the plot of pole points in Figure 6.23. has illustrated this pattern and the need for a complete analysis of each stress circle has been eliminated.

Figure 6.23. also shows the stress σ_r for each stress circle and the $\sigma_r = \sigma_z$ condition is emphasised by the line drawn at 45° to the vertical, joining the pole point and σ_1. The heavy line requires no detailed explanation; it is the locus of pole points of every stress circle drawn on the figure.

Figure 6.25. shows the co-ordinate stresses, σ_r and σ_z, and the principal stresses for three penetration depths. The stress tensors can be evaluated easily from Figures 6.23. and 6.25., thus the redistribution of stresses during axially-symmetric penetration into a dense sand mass can be determined from these results.

6.6. Concluding Remarks

This chapter has reported the results of three sets of investigations, two experimental and one theoretical. The first investigation was carried out using the three-dimensional miniature earth pressure cell developed by the Author. The stress results were plotted as distributions of σ_θ, σ_r and σ_z versus the depth for continuous axially-symmetric penetration into loose and dense dry sand masses. The results, which are thought to be unique, were eventually used to demonstrate the mechanical behaviour of the sand during penetration into it.

The results of the stress measurement experiments were discussed in considerable detail in an attempt to explain some of the anomalies. The experimental work was then presented as Mohr stress diagrams. In certain cases the collection of stress results showed two striking phenomena. The first was the principle, originally expounded by Haar and Kármán in 1909, which stated that the circumferential (principal) stress was equal to one of the other (i.e. the major or minor) principal stresses, was proved to be correct. The verification was obtained particularly with pressure cells in dense sand at penetrations where overall displacements around the pressure cells were slight or negligible. The second conclusive phenomenon was realised when a line drawn as a tangent to Mohr circles of stress for dense sand gave an angle of inclination in agreement with the Coulomb ϕ measured in laboratory direct shear tests on sand at the same initial porosity. The angle remained constant for a range of stress circles, suggesting that the sand mass reached a limiting stress condition and became 'plastic' without a dramatic change in overall porosity, only a change in the stress tensor.

The second experimental investigation was undertaken with the intention of measuring displacements during axially-symmetric penetration, Selected results were interpreted in terms of strain-rates using computer techniques and these were presented in the third section of this chapter. The preliminary results, showing directions of major principal strain-rate, were plotted to illustrate the patterns of rotation of $\dot{\epsilon}_1$ during penetration. Later these results were used in a quantitative way to evaluate the elements of the stress tensor which were not measured directly by a pressure cell at the point in dense sand. Strain-rate versus depth distributions were also plotted to allow a comparison with stress distribution curves drawn earlier. The comparison showed a vague similarity of distribution and it was possible to conclude that apparent anomalies in stress measurement most probably were due to large displacements around the pressure cell (more noticeably in loose sand) rather than incorrect pressure cell recordings.

The third investigation, described in detail in Chapter 5, was of a mathematical nature. Several assumptions were made, these being based on a combination of past research work, the Author's intuition, and preliminary pressure cell results. It was realised only at a late stage in the research investigation that the theoretical solutions were not applicable in problems of continuous axially-symmetric penetration. Figures 6.9., 6.10. and 6.11. resulted from the study of Haar and Kármán's hypothesis, therefore the work in Chapter 5 at least has been of indirect value in the study of the penetration resistance of sand.

The theoretical study has produced a workable solution for axially-symmetric deep foundations, with slight deformations before collapse is assumed to have occurred. The potential for development offers a unique approach to the lower bound solution for the ultimate bearing capacity of single circular piles or piers. The basic numerical procedures are probably correct and a logical continuation of the present research work would be a re-analysis of limiting stress fields for the above foundation types.

At present the application of (Sokolovski's) limiting stress characteristics to continuous penetration is hampered by the lack of understanding of the stress and strain-rate changes which take place throughout penetration. Studies in this project have established conditions which exist within the mass of sand, but it still remains to investigate the boundary conditions during axially-symmetric penetration.

Finally, in Chapter 6 results from pressure cell and displacement measurement experiments have been combined to give the pattern of behaviour adjacent to the penetrometer during its movement into dense sand. It has been possible to complete the stress tensor matrix and also describe the change in direction of the major principal stress by plotting the pole point on the Mohr stress diagram for a stress result in dense sand. Apart from the axial-symmetry of this problem it is not unlike the bearing capacity problem illustrated in Figure 6.20.(a).

The interpretation and analysis of results has been taken as far as time allowed and a basic picture of the behaviour of sand during penetration has emerged. There are, however, many questions which remain unanswered and it is considered worthwhile continuing the present research. The final chapter of this work includes a few suggestions for future projects.

CHAPTER 7

Some conclusions and suggestions for future work

7. Some conclusions and suggestions for future work

There are an unlimited number of directions along which future research into "the penetration resistance of sand" could continue. Some of the more obvious are listed below:

i) a variation in the rate of penetration

ii) a variation in the shape of a penetrometer base (e.g. flat or pointed)

iii) a variation in the overall shape of a penetrometer (base and shank)

iv) a variation in the method of driving a penetrometer (e.g. drop hammer or constant rate of penetration)

The above are all centred around the study of a penetrometer and the resistance that it records while being pushed or driven into the sand. A study of any one, or a combination of these variations, would probably lead to an additional empirical relationship which could be used in the interpretation of in-situ penetration test results. The validity of the relationship would, of course, depend on the accuracy of the tests, the effects of model similitude, if relevant, and the many other parameters which are associated with the penetration resistance of sand for different methods of penetration.

In addition to the above suggestions for future work, there are numerous variations which could be adopted for the media into which the penetrometer would be forced. The use of sand in other than the dry condition is a possible subject for research; a well graded gravel-sand-silt soil could be used as a media in penetration resistance studies. In fact, the variations in media type are almost as extensive as the possible permutations from the four topics listed above. Experiments would produce empirical relationships and any form of interpretation would have to rely for its value on a large number of tests with different ranges of soils.

In this research work the solution to "the penetration resistance of soils" problem has been approached in a different way. The Author has adopted one type of penetrometer, one speed of penetration, a standard (and simple) instrument shape and dry fine white sand; as far as possible this has been similarly graded throughout the experiments. By maintaining the same basic equipment and soil type the number of parameters has been reduced considerably and it has been possible to make a detailed study of the behaviour of the material during axially-symmetric penetration. Results of the experimental work have been related specifically to phenomena within the sand; the penetrometer was designed simply as an instrument which would apply the axially-symmetric system of forces referred to as the penetration resistance of sand.

Many conclusions have been drawn as the work has been developed, and during the discussion of results. Some of these, particularly in Chapter 5, were proved later to be inappropriate to the problems being studied, simply because stress systems and boundary conditions for continuous penetration are not quasi-static. However, it remains to expand some important conclusions that are worthy of reiteration.

The most interesting and consequential conclusion is considered to be the proof of the Haar and Kármán hypothesis, stating that the intermediate (circumferential) principal stress is equal to one of the other principal stresses. The difficulty in obtaining true three-dimensional solutions has always been a stumbling block in soil mechanics research and, although less difficult, the axially-symmetric problem has received very little direct attention from research workers. The experimental results presented in Chapter 6 have clearly established the validity of the basic principle of two equal principal stresses. A more thorough investigation of the hypothesis obviously offers considerable scope for future work. One approach would be to use miniature pressure cells in large cylindrical soil samples. Research of this nature has been carried out in the past, using samples with conventional triaxial loading, but generally the cells have been one-dimensional and were placed along the vertical ($r = 0$) axis. In future work it would be judicious to place cells away from the vertical axis and measure the three principal stresses, i.e. the vertical (σ_z) stress and the two horizontal (σ_r and σ_θ) stresses, during true triaxial compression.

Proof of the Haar and Kármán hypothesis was obtained rather by accident after the principle had been adopted in the theoretical work described in Chapter 5. Unfortunately, there are very few conclusions to be drawn from the mathematical study which are particularly relevant in the present research study. This is not because the mathematical approach was incorrect, but because continuous penetration produces stress conditions around the instrument which are very different from those assumed to exist during penetration.

The limited conclusions from the theoretical study, however, do not detract from the value of the work carried out. Development of the mathematical procedures and the associated iteration techniques have suggested errors in the iteration method used by Graham (1966 and 1969). A useful continuation of the present research work would be a complete run of the computer programme with combinations of parameters such as Kf and tan δ , and γ d and ϕ. A wide range of values of the ultimate bearing capacity, and hence the factors N γ q (Meyerhof, 1950) could be obtained, and a comparison made with values already available.

In addition to providing evidence to support the Haar and Kármán hypothesis the results of certain pressure cell experiments have shown the Mohr-Coulomb failure criterion to apply during penetration into dense sand. By comparison, penetration experiments in loose and loose-medium sand did not show consistent agreement with the failure criterion and other results from experiments in dense sand were not meaningful when plotted on Mohr stress diagrams. The conclusion here is that the application of the parameter Coulomb ϕ is not appropriate, i.e. the Mohr-Coulomb yield criterion is not relevant.

A more advantageous method of representing the pressure cell results in the form of a failure criterion is shown in Figures 7.1., 7.2. and 7.3. The curves are plotted directly from σ_r and σ_z distributions in Figures 6.1., 6.2. and 6.3. and the arrows on each curve indicate the directions in which the penetration increases.

Before continuing it is necessary to introduce the concept of plastic potential. According to Mises (1928) the plastic strain-rate plots as a vector which is normal to the existing yield curve; in mathematical form the concept is usually written:

$$\dot{\epsilon}_{ij} = \lambda p. \frac{\partial \Gamma}{\partial \sigma_{ij}} . \qquad (7.1)$$

$\lambda p.$ is a +ve plastic flow-rate parameter and Γ is the yield function. If the Mises flow rule is now used in conjunction with the curves in Figures 7.1., 7.2. and 7.3. the directions of plastic strain-rates can be determined.

The integer against each curve denotes the number of the pressure cell and this can be related to numbers, hence positions, shown in Figures 6.4.

The curves in Figures 7.1., 7.2. and 7.3. are considered to be self-explanatory. By adopting the flow rule, the resultant strain-rate directions can be obtained as the normal to the yield function; strain-rates in the radial and vertical directions are simply components of these.

Although the exact extent of the plastic regions has not been determined, the curves over the greater part of their length probably represent limiting conditions. That is, where the circles touch the common tangent in, for example, Figures 6.3.(a) and (d), limiting stress conditions exist. A close examination of Figures 7.1., 7.2. and 7.3.

show clearly the reversal of direction of $\dot{\epsilon}_z$ in all cases, and although less obvious, the change of $\dot{\epsilon}_r$ from -ve to positive is indicated.

The experiments which measured deformations during axially-symmetric penetration have not been analysed completely but the distribution of directions of principal strain-rates and strain-rate components have been plotted from tests in loose and dense sand masses. One important result to emerge from the patterns of strain-rate directions deserves reiteration.

The major principal strain-rate, and presumably the major principal stress, was inclined at a near-horizontal direction (i.e. $\psi \not> 45^{\circ}$) to the penetrometer shank. This finding is contrary to the condition assumed in the quasi-static conditions, where the major principal stress is inclined at a near-vertical direction (i.e. $\psi \not< 45^{\circ}$). The obvious conclusion here is that boundary conditions, and most probably stress fields within the sand mass, in the case of continuous penetration are very different from those in true quasi-static cases.

It seems prudent, therefore, to undertake an extensive research project with a view to determining the distribution of stresses along the shank of the penetrometer. Work along these lines has constituted research projects in the past, but the Author has been unable to locate results which give convincing boundary conditions for an instrument continuously penetrating sand. Such a study would provide a more realistic picture of the boundary conditions along the penetrometer and could be used to develop a theoretical solution for the penetration resistance of sand.

Finally, it remains to emphasise that the theoretical study has introduced a novel approach to problems such as the ultimate bearing capacity of deep circular piles, piers, etc. Obviously there is considerable scope for further work in this direction.

It has been apparent throughout this research investigation that the penetration resistance of sand relates to the region represented by a failure 'bulb' similar to that shown in Figures 5.17 or 5.18. The Author does not consider it sensible to separate the sand behaviour above and below the base of a penetrometer shank and refer to one condition as 'skin friction' and the other the 'point resistance'. The penetration resistance is a concept which is complete only when the full stress characteristic net is considered and the stress fields are allowed to continue from the initial to the final boundary. By maintaining this condition the penetration resistance of sands is meaningful.

If currently used methods of in-situ testing are now examined, it is possible to discuss the mechanics of each test in relation to the complete failure 'bulb' concept.

The Dutch cone penetrometer most widely used in Holland today is the 'electrical' cone, so-called because resistance strain gauges record stresses during penetration. Unlike the earlier sleeve cone designed and developed by Begemann (1953 and 1965), which had a slightly tapered solid shank integral with the cone with a thin separating skirt, the 'electrical' cone has an end cone with a small length of shank (say 30 mm) and above this moves a separate longer shank. This basic shape produces unique stress and strain fields around the point.

Therefore, despite the use of detailed theoretical studies such as those of De Beer (1963) to interpret the cone results, the Dutch cone penetrometer used today must remain a purely empirical in-situ penetration test until the point resistance has been studied in detail.

In the Standard Penetration Test (SPT) the split spoon is a solid unit with no separately moving parts. The test is started when the penetrometer is 6 in. below the base of the borehole. For loose sands this arrangement is quite acceptable; the failure 'bulb' will have been developed fully and the true penetration resistance will be measured. During SPTs in dense sand the failure 'bulb' will not be fully developed and the condition is not one where true penetration resistance is recorded.

The above points may appear to be rather academic in view of the overall approach to in-situ testing, but the tests set out to measure the properties of the soil into which the penetrometer is driven. The present research study has indicated that the soil properties must be closely related to the natural stress and strain distributions in regions of plastic behaviour. It is believed this work has made a significant contribution to a fuller understanding of the stress and strain distributions for axially-symmetric penetration into dry sand.