THE BEHAVIOUR OF STEEL PLATE CONNEXIONS IN PRECAST CONCRETE CONSTRUCTION

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SUMMARY

This thesis describes the work which was carried out to investigate the behaviour of a joint used to connect precast beams to columns. The system consists of a steel plate cast vertically into a concrete column to which are bolted similar plates which protrude from the ends of concrete beams. The scope of the investigations is limited to a study of the behaviour of the column plate and the stresses in the column produced by the loads on the plate.

A theoretical study of these stresses, which makes use of both a finite element method and a continuum approach is described and the two methods are compared. The results of the theoretical investigation are compared with those obtained from tests in which short lengths of column incorporating a single vertical plate were tested to failure. It is shown how the ultimate strength of columns loaded in this way varies with the cube strength of the concrete and with the cross-sectional dimensions of the columns.

Various ways of increasing the strength of the joint are described; these include the use of a horizontal ledge attached to the edge of the column plate and the inclusion of transverse reinforsement

in the column, which is in contact with the plate.

The effects of loading a column in two directions is considered and an experimental investigation of the factors involved when a column is submitted to unsymmetrical loading is described.

Due to an increased understanding of the major factors which contribute to the mechanism of the transfer of loads from steel plates to columns resulting from these investigations, it was possible to propose more rational design procedures than have been used up to the present time.

For Pauline

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CONTENTS

NOTATION

A	Area of a beam or a column.
b	Width of loading strip or ledge.
D, D_1, D_2	Horizontal dimensions of a prism, a column or a cylinder.
D'	Distance from the upper or loaded surface of a cube or prism.
d	Depth of cast in column plate.
E	Modulus of elasticity.
e_1, e_2	The distances of the points of application of loads applied to a cast in column plate from the faces of the column.
f_c	Experimental compressive strength of concrete.
f_f	Experimental value of frictional stress.
f_{ldg}	The maximum load divided by the cross-sectional area that a cube or a column can sustain when loaded through a horizontal ledge attached to the lower edge of a vertical plate.
f_o	The maximum load divided by the cross-sectional area that a cube or a column can sustain when loaded through a narrow line load or a vertical plate.
f_t	Experimental strength of concrete.
H	Shear flow.
h	Thickness of plate element.
L	The horizontal distance between the two loading points of a plate used to apply load to a cube or a column.

M	Moment.
P	The load applied to a cube or a column.
P_b	That component of the ultimate load that can be sustained by a plain column which can be accounted for by pure bearing, calculated from the formula: $f_t = \frac{2P_b}{\pi D D_1}$
P_c	The load to which a column is pre-compressed.
P_f	That component of the ultimate load that a column can sustain that can be accounted for by friction.
P_{ldg}	The maximum load that a cube or a column can sustain when loaded through a horizontal ledge attached to the lower edge of a vertical plate.
P_m	The maximum load applied to a column when the cast in plate is subjected to unsymmetrical loading.
P_o	The maximum load applied to a cube or a column through a narrow line load or plate.
P_s	The load at which spalling was first observed.
P_1, P_2	Eccentrically applied loads.
T	Thickness of vertical cast in column plate.
T_1, T_2	Direct stress flows.
x, y	Cartesian rectangular coordinates.
α, β, γ	Constants multiplying D.
δ_1, δ_2 ...	Translational displacements.
ε	Direct strains

λ, $k\lambda$	Side lengths of framework model.
$r\lambda$	Diagonal length of framework model.
μ	Coefficient of friction.
ν	Poisson's ratio.
σ	Direct stress.

Statistical Notation

b	Regression coefficient.
N	Number in sample.
r	Correlation coefficient.
s	Standard error of sample mean.
$\hat{\sigma}$	Corrected standard deviation of population.
$\hat{\sigma}_W$	Estimate of the standard error of the difference of two samples.
Σ	Summation.

CHAPTER 1

CHAPTER 1

INTRODUCTION

1.1 CONTEXT OF THE PRESENT INVESTIGATIONS

1.1.1 Precast Concrete Construction

Precast concrete construction as opposed to in situ construction can be defined as that method of construction which makes use of components which are manufactured elsewhere than in their final location in the structure in which they are used. This technique has become accepted in most industrialised countries over the past fifteen years or so as an indispensible method of construction contributing towards the achievement of an adequate rate of growth of the building industry required to cope with the ever increasing demand for factories, schools, hospitals, housing and the rest.

This has come about for several reasons. The cost of building by more conventional methods has increased mainly as a result of the rapidly increasing labour costs involved in building and erecting the often complicated shuttering required for in situ concrete; in manually bending and fixing the steel reinforcing bars at site and generally in paying for all types of skilled and semi-skilled operatives who necessarily have to be kept employed for considerable periods of time. Also when building is carried out using in situ concrete considerable delay can be caused by the necessity for adequate curing time to be provided between the casting of one part of the structure and related parts. Very careful planning of site operations is required to ensure that work is dovetailed so that untoward delays do not in fact occur. This of course is true of all building and civil engineering operations but there is no doubt that the use of precast components not only allows construction to procede as fast as the members can be assembled together but also allows considerably more flexibility in the long term planning of the entire operation.

However, the major advantage that precasting has over the more conventional methods is undoubtedly due to the fact that modern methods of manufacture and quality control can be applied in factory conditions at competitive costs provided there is a sufficient degree of repetition. If this is the case then the cost of shuttering, which itself can be manufactured to a high surface finish and high degree of dimensional accuracy becomes almost negligible. Steel reinforcement can be bent and even assembled automatically and placed in the moulds with little or no manual intervention. Concrete can be made in factory conditions to high quality and casting and curing procedures can be controlled easily without there being any need to rely on suitable weather conditions. Hence it is possible to ensure that many of the volumetric changes which concrete members undergo in the early part of their life occur before they are incorporated in buildings. Of course movements due to changes in the temperature and the humidity can easily be taken care of in precast concrete construction because of the articulated nature of the structure, at least in the early stages of construction.

Because of the high degree of control and the more refined manufacturing techniques which can be applied in the enviroment of a permanent factory it is possible not only to make better use of skilled labour but also to economise in the use of materials, clearly a benefit in present day conditions.

1.1.2 Types of Precast Concrete Construction

It is convenient to consider buildings constructed from precast units as falling into two distinct but necessarily closely related types. In the first category are buildings which for the most part are assembled from load bearing slabs which form the roof, the floors and the walls. Individual columns and beams are either absent or only occur at certain special locations. This type of construction has been shown over the last few years to be eminently suitable for high rise flats where excessive concentrated live loads, excepting wind loads, are likely to be rare or absent altogether.

The second category comprises buildings which are formed from small individual units, columns and beams, which are connected together before the floors and walls are added. This second type of construction has some obvious advantages. Because walls need not be capable of transmitting loads, floor plans can be altered, within broad limits, at any time during the life of the building without affecting the ability of the latter to resist load. Moreover construction procedures can be somewhat simplified and made more flexible compared with the first category when a framework is available from the outset on which to locate both precast or in situ wall and floor slabs. Indeed most buildings which require large unobstructed floor areas on more than one storey usually require a primary skeletal structure.

1.2 OBJECT OF THE PRESENT WORK

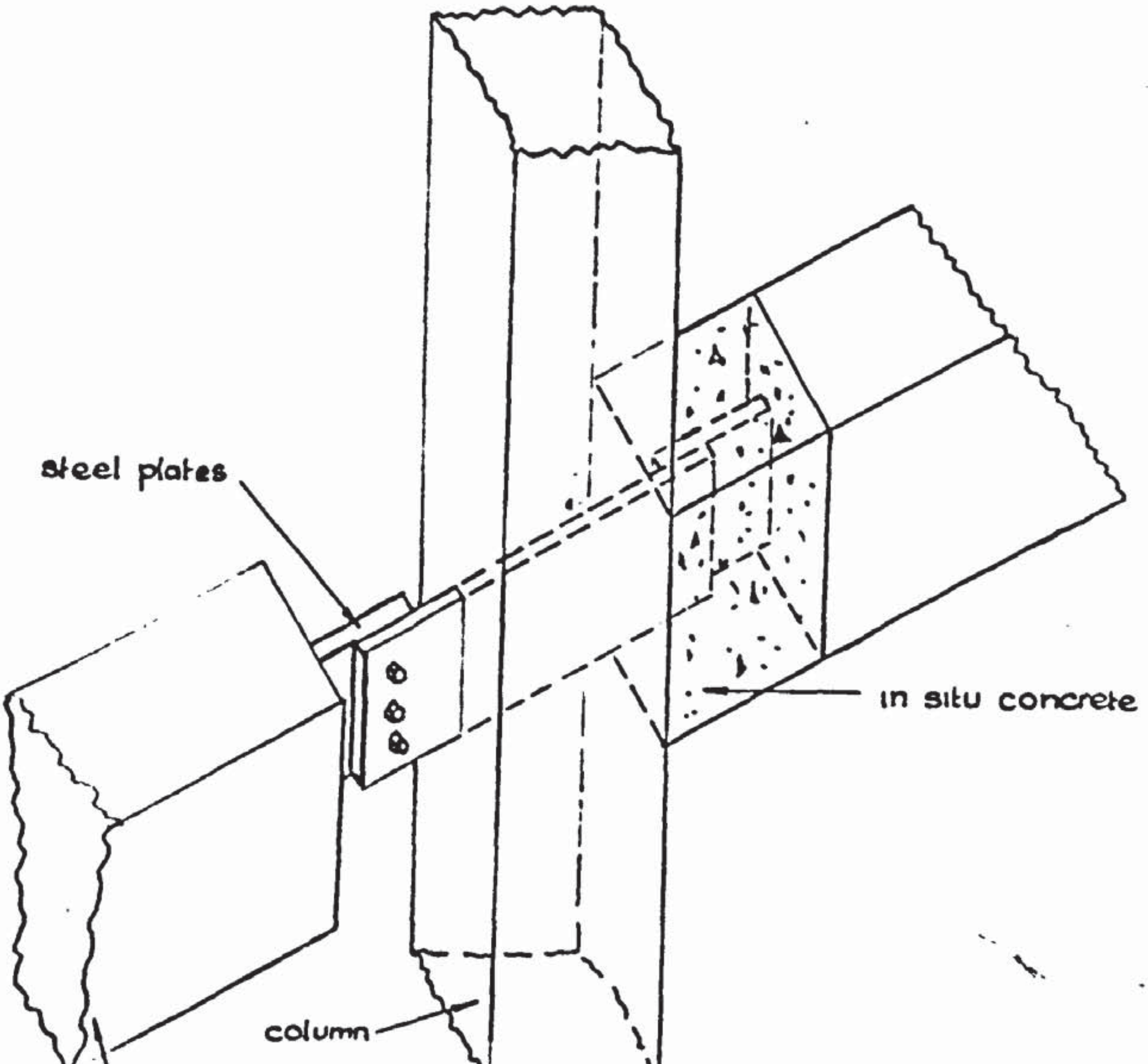
In both of the categories referred to in the previous section it is necessary to make some kind of structural connexion to join the units together and it often happens that these joints are necessarily made where forces due to bending or shearing or both are relatively large. Unless a considerable amount of care is taken in the method of execution of the connexions they may well form points of weakness which would clearly cancel out all of the advantages of this type of construction. Of all the joints that would generally be required in a building, either beam to beam, floor to wall or column to column it has been said (1) that the most difficult joint to form is the one between a beam and a column: it is an investigation into this last type of joint which forms the subject of this thesis.

1.3 NEED FOR RESEARCH

It has become apparent in view of the vast expansion in the use of precast concrete construction that many different types of connexions have been devised but that there has not always been enough time for fundamental knowledge of the strength and behaviour of these connexions to catch up with the techniques themselves. Consequently designers have often been left to deal with problems of considerable importance to which the established principles of steel and reinforced concrete design are only partly applicable. Even when tests on particular connexions have been carried out they have as often as not been of an ad hoc nature and of very limited applicability and it is really only within the last five years that serious attempts by research workers have been made to obtain, by means of comprehensive testing and analysis, more fundamental knowledge about this subject than was the course hitherto. It has been noticed that even in the case of the type of connexion which forms the subject of the present investigation, and in spite of the fact that it has been incorporated in a considerable number of buildings designed by different firms, the rule of thumb method of design bore no relation whatsoever to the actual mechanism of the connexion and there is no doubt that by far the most common cause of failure of precast concrete structures, whether during erection or after completion, has been joint failure, rather than member failure.

1.4 DESCRIPTION OF CONNEXION

The connexion is the bolted steel plate type the simplest form of which consists of a rectangular steel plate cast vertically into a precast concrete column so that one or both ends of the plate protrude from the faces of the column. A similar steel plate fixed to the reinforcement of a beam and protruding from its end is bolted to the column plate so that a gap of a few inches is left between the end of the beam and the face of the column. The arrangement is shown in Fig.1.1. In this way a shear connexion is made between beam and column which depends for its strength on the behaviour of the concrete in the column when subjected to the load transferred to it by the plate. Of course it is possible with a column of a square or a rectangular cross-section to have as many as four beams to connect to the column and this is easily facilitated. One form of the connexion known to have been used in practice involves the use of two beam plates side by side, separated by a small gap so that they can be slotted over the column plate and bolted through. This procedure could be useful to releive



STEEL PLATE CONNEXION

Figure 1.1

shearing stresses in the belts but it is doubtful whether, in the light of the investigations reported in this thesis, any great advantage is gained from the point of view of the bearing capacity of the column.

1.5 <u>ADVANTAGES OF THE TYPE OF CONNEXION</u>

Once the connexion has been made between the ends of the beam and the columns by belting the plates together, the assembly can take lead. In other words, the process is essentially the same as it would be if a steel beam was being located and fixed between two steel stanchions and the time taken for assembly in the two cases is probably about the same. It then only remains to make up the gaps left between the ends of the beams and the faces of the columns with lean mix concrete to satisfy the regulations concerning fire protection and also architectural considerations but as the load carrying capacity of the connexion is not dependant on the strength of the infilling this can be added at a later stage in the construction process. Provided the joint is strong enough to resist the imposed loads in the first place there is no doubt that it is a very useful way of making a beam and a column because its main advantage lies in the fact that the speed of erection of the units is not dependant on any other factor than the rate of working of the erection gangs. This is an important point and is vital to the whole concept of the precast concrete construction technique. This gives it, in the view of the writer, a distinct advantage over other types of connexion which depend for example on dowling action between steel reinforcement and concrete. With this type of connexion it is necessary to wait for adequate curing of the necessary concrete grouting and infilling. This added material will inevitably have a tendency to shrink away from the relatively more mature concrete as moisture is drawn from it into the air and into the surrounding concrete rendering a most unsatisfactory joint.

The use of the bolted connexion also obviates the need to use complicated shuttering which would be required if for example the beam to column connexion is formed by resting the ends of the beams on brackets projecting from the column, a details which, from an architectural point of view does not add to the aesthetic quality of many types of building, particularly when they are used for housing. Moreover the use of complicated shuttering required for the construction of this type of connexion increases costs on or off the site and there is no reason why, provided care is taken, columns containing cast in steel plates should not be cast in situ, particularly if the designer wishes to save the cost of transporting columns forty or fifty feet long to the site.

Another advantage of the use of the steel plate is that there is no interference with the column reinforcement steel which so often occurs when a dowled joint is used. In the case of the latter,it is often difficult to place the concrete with an adequate degree of compaction because of the congestion of the steel at the location of the joint. The steel plate connexion also does away with the need for site welding which, particularly if it is done in inclement weather, is an expensive and often a somewhat ineffectual practice requiring a high degree of expertise and supervision. The only welding that would have to be done, if it were considered to be essential, would be to join the steel plate on the end of the beam to perhaps two of the steel reinforcing bars and this would in any case be carried out in the factory before the beam was cast.

1.6 CONTENTS OF THIS THESIS

This thesis describes the work which was carried out to investigate the behaviour of the joint described in the forgoing paragraphs with particular reference to the way in which the simple steel column plate transfers loads from the beams to the column. The emphasis was naturally on experimental work in view of the fact that it was necessary to determine the factors affecting the mechanism by which the plate transfers load to the concrete: these were thought to be dependant to some extent on the particular characteristics of the concrete in the column. It was inevitable that a statistical approach would be required to correlate the test results in view of the inherent variability of concrete and consequently it has only been possible in some cases to make tentative design recommendations. Nevertheless an attempt was made to determine the stresses set up in the column in an idealised system and this is described in the first part of this thesis.

The connexion works by primarily transferring shear forces from the ends of beams to the concrete in the column and although it is common design practice to regard the beams of precast structures, at least under dead load, as "pin-jointed" it is clear that moment could be carried from the end of one beam, via the column plate to a beam on the opposite side of the column in one of two ways. Either the tolerances on the bolt holes can be kept to the absolute minimum so that unrestricted rotation of the beams is kept as small as possible or, and more probably, friction grip bolts can be employed to transfer rotation from the beam plate to the column plate. If the movements at each end of opposite beams were equal clearly the column would still only be required to resist a resultant shear force but if an out of balance moment condition was developed between the beams then the column itself would be required to take moment. Some tests are described which were used to investigate the behaviour of the joint when out of balance moments were applied.

The latter part of the thesis deals with investigations that were carried out to determine how the strength could be increased above that obtained when the simple plate alone was used to form the connexion. These include the use of a horizontal ledge attached to the lower edge of the column plate and the use of transverse stirrup reinforcement in the column which is attached to the lower edge of the plate. It will be shown how a possible design method was evolved to enable the width of the ledge and its thickness to be determined.

The effect of loading in columns in two directions is considered and the influence of the shape of the edge of the simple plate itself were investigated.

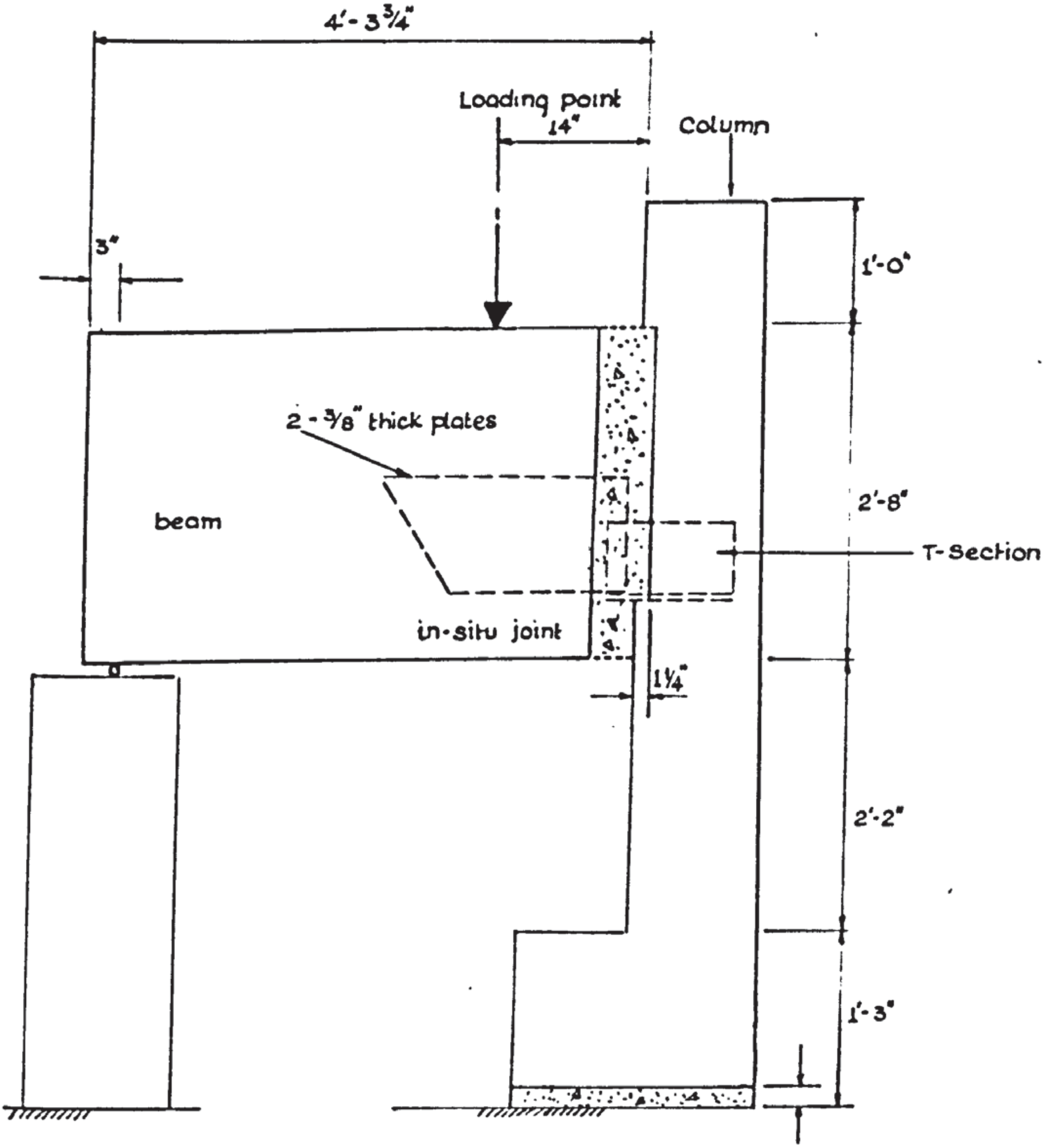
1.7 HISTORICAL SURVEY

This survey does not cover all types of beam to column connexion as they have been adequately dealt with elsewhere, in particular by Somerville and Burhouse[(2)] in their informative paper on the subject of joints between precast concrete members. The few references that have been found concerned with connexions incorporating steel plates are discussed below.

The earliest of these dates from 1960 in which Gifford[(3)] describes some tests that he carried out. The connexion was formed by a rolled steel T - section which projected from the column with its web vertical and two plates which project vertically from the end of a beam, the beam plates sitting directly on the flange of the T - section. The column was normally recessed $1\frac{1}{4}$in. above the flange of the T - section and a gap was left between the column and the precast beam, the gap being 4in. above the table of the T - section and $2\frac{3}{4}$in. below. A sketch of the arrangement is given in Fig.1.2.

The T - section and the plates projected $2\frac{1}{4}$in. and $3\frac{1}{2}$ in. respectively giving $\frac{1}{2}$in. tolerance. The contact surfaces were welded together so that load could be transmitted. A stirrup was placed round the connexion and the whole was encased with in situ concrete. Bending and sheer was produced in both the T - section and the plates. Bearing stresses in the column were assumed to be uniformly distributed over the flange of the T - section and were given a nominal value of 2000 lb/sq.in., apparently for all the specimens tested, whatever the cube strength of the concrete.

Three tests only were made on units containing different sizes of plates and T - section. Failure occured by vertical hair cracks forming in the joint between the beam and the in situ concrete which was followed by the beam parting at the soffit from the joint. Hair cracks also appeared on the inside face of the column (through which the T - section protruded) underneath the flange of the T - section, spreading diagonally downward to the corners of the column. The cracks subsequently continued diagonally upwards on the two sides of the column

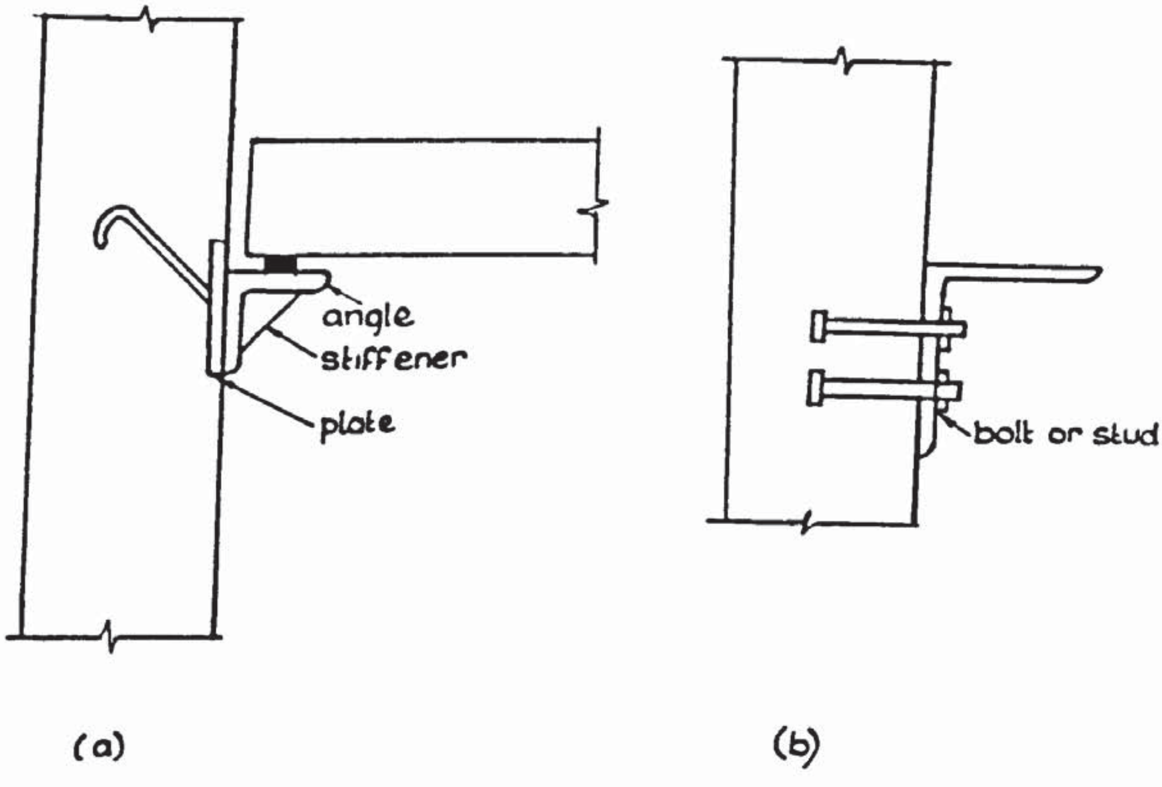


GIFFORD'S TEST ARRANGEMENT
Figure 1.2

to cause ultimately complete failure of the column. Gifford developed an approximate theory to account for the bearing, shearing and bending forces and found that it was quite conservative.

The only other report on tests on steel plate connexions was that produced by Holmes and Bond[4] in 1963 which is dealt with fully in Chapter 8. The arrangement they used was basically the same as that described in section 1.4.

Descriptive papers on joints similar to the ones shown in Figs.1.3 and 1.1 respectively were produced by Birkeland[5] and Burrough[6] but no evidence of testing was given. This is also true, unfortunately, of the report of the A.C.I - A.S.C.E. Committee[7] which gives only vague recommendations as to how the "sheer-plate" type of connexion should be designed and is of little real use. Examples of two joint details given in the report regarded as good design are shown in Fig.1.3. The report states that no reliance should be placed in the bearing capacity of the embedded vertical plate of Fig. 1.3(a) and all shear forces should be taken by the attached reinforcement. They also show the type of joint which forms the subject of this thesis but again are not specific about design methods.



COMMON CONNECTION DESIGNS

Figure 1.3

CHAPTER 2

CHAPTER 2

CONTINUUM METHODS OF STRESS ANALYSIS

2.1 INTRODUCTION

2.1.1 Purpose of Work

In order that as complete an understanding as possible could be gained of the manner in which load is transferred from a symmetrically loaded plate set vertically on an axial plane of a column to the material of that column it was clearly desireable to make some theoretical determination of the stress distributions which result from this type of loading. If it could be shown experimentally that a particular theoretical analysis was valid at working load and, moreover, that the same distributions of stresses existed at loads close to the ultimate then it would be possible to use a knowledge of those distributions to estimate theoretically the load which caused failure of the column. Moreover, it was of considerable interest to obtain a viable theoretical analysis in order to determine whether a column loaded in the manner described above could be simulated by a cube loaded through a central narrow strip. If this could be shown to be justifiable theoretically, the advantages arising out of the simplified nature of such tests would be considerable.

The tests which were reported by Holmes and Bond[4] showed that when an unreinforced column was loaded through a vertical cast in plate the mode of failure was characterised by the formation of a tensile crack forming in the axial plane containing the plate which appeared to occur almost instantaneously at maximum load and resulted in the column being divided into two equal portions. Clearly tensile stresses at right angles to the plane of the plate had been induced by the system of loading. It was inferred from the apparent brittle behaviour of the material of the column and its mode of failure that an elastic approach could be used to enable an estimate of the major component of the failure load to be made and it is described how use is made of the analogy between the mode of failure of the cylinder used in the indirect tensile test for concrete, which is described in Chapter 5, and the mode of failure of their tests or columns. This question of estimating the failing load of a column is considered in more detail in Chapter 8 but it is sufficient to state here that no fundamental approach to the problem of estimating the failing loads of columns and of obtaining a valid stress analysis had been attempted until the work reported in this thesis was initiated.

2.1.2 Assumptions

The question of whether the material that is being considered behaves as a linearly elastic homogenous solid is a serious one but without making these assumptions any attempt at a completely theoretical solution of the state of stress of a prism under concentrated loading becomes totally intractable. In the case of concrete in particular, it is known that the manner in which a member is loaded can affect the behaviour of the material and the values of the limiting stresses and strains which occur when the member is loaded to failure. Moreover it is implied in these conditions that the values of the elastic moduli of concrete in compression and tension are equal and remain constant throughout the entire loading range. In fact the tension modulus of concrete is believed by some authorities to be less than the corresponding value in compression and the non-linearity of the stress-strain characteristics in both compression and tension, even over a limited range has been demonstrated many times. Also, it is assumed that redistribution of stresses due to microcracking and time dependant effects such as creep and shrinkage do not occur. Owing to the fact that as yet no analytical technique has been found which is powerful enough to deal with all these deviations from the ideal elastic homogeneous material it has proved necessary to make these rather sweeping assumptions if any attempt is to be made to obtain a theoretical examination of the stress state of a prism under concentrated loads.

2.1.3 Formulation of the Problem

It was thought that the best approach to the analysis was to consider the problem as that of a concentrated load acting on a small area on the free end surface of a prism made of an elastic material, in other words, to consider the problem of bearing in isolation from other effects that were likely to be associated with the plate being cast into the concrete. The general problem of high stress concentrations under forces acting on small areas arises in a number of different aspects of civil engineering construction, e.g. in the case of bridge piers under narrow bearings, in footings and foundations and also in the anchorage blocks of post tensioned prestressed concrete beams. Much of the work that has been carried out has been connected with the last of these examples and several methods have been investigated with varying degrees of accuracy. It is not proposed to analyse here all the existing methods as this has been done adequately elsewhere (8,9) and it is not claimed that the theoretical work which was carried out for these investigations and which is described in the following chapters provides comprehensive solutions to the analysis but in order that this work may be put in its historical perspective, some mention of the more important theoretical attempts at solving the stresses in blocks loaded by concentrated forces is apposite.

2.2 STRENGTH OF MATERIALS APPROACH

The first attempts which were made to calculate the stress in blocks subjected to concentrated forces made use of what is essentially a strength of materials approach when the block was considered as a deep beam and bending moments and sheers were computed using transverse sections as for ordinary beams. Representative of this approach are the theories of Mörsch,[10] Bortsch[11] and Magnel[12].

Mörsch in 1924 used as a basis the results of some early tests that had been performed on masonary blocks. He assumed, as others have done since, that the stresses produced by a concentrated force acting on one face of a prism are distributed uniformly at a distance equal to the width of the prism and also that the curvature of the stress trajectories caused tensile stresses to be developed at right angles to the direction of loading which were distributed according to a second order parabolic law. This latter assumption was based on the rather sparse strain readings of the earlier tests and cannot be said to be very accurate in view of the results of later work. However, he was able to estimate an approximate value of the force which caused splitting and hence of the maximum tensile stress.

Bortsch in 1935 developed a more sophisticated approach in which transverse, longitudinal and bearing stresses were obtained from Fourier series and the load was assumed to be distributed parabolically over part of the loaded surface of the prism. However, it was left to Magnel in 1949 to apply this approach to the particular problem of the anchorages of prestressed concrete beams and he put forward this theory which was based on the assumption that the transverse stress diagram at any plane parallel to the direction of loading took the form of a cubic parabola and from the expression for the transverse stress which he evolved he was able to deduce an expression for the sheer stresses in the block. In view of the assumptions that were made with all this work and the considerable areas of disagreement that exist it was not surprising that a different approach was made and the most well known of these was reported by Guyon (13) in 1951.

2.3. CONTINUUM APPROACH

Guyon's paper on the stresses in prismatic bodies loaded on their surface is of considerable importance in this field of stress analysis and as the results have been used for compartive purposes in a later chapter and have been used with some success in the design of end-blocks of prestressed concrete beams, a somewhat detailed descriptive account is included in this survey.

The first part of his paper (13) deals with a semi-infinite prism of unit thickness loaded by forces of any magnitude and inclination to the top surface (the longitudinal axis being vertical).

Guyon considers the method of obtaining Fourier series used by among other Timoshenko (14) to describe the stress system required to keep the prism in equilibrium under the action of the applied loads. However, the method of applying stresses equal in magnitude and of opposite direction to those which exist on the sides of the prism considered by Timoshenko did not, according to Guyon, yield the correct solution to the problem as some of the boundary conditions were not satisfied. However, it is explained how by using this first method, an approximate solution is obtained by developing a Fourier series, the stresses on the sides of the prism being eliminated by an equal and opposite stress system obtained by the usual methods of strength of materials. In this case an Airy stress function is used. This approximate solution is then expressed in finite terms after using the usual method of finding the values of the coefficients from the boundary conditions. A more exact method is then developed for the case of a single normal force applied on the top face of the prism by superimposing a symmetrical force system, whose resultant equals the value of the applied force, on to an antisymmetrical system whose resultant is zero. A similar method is developed for tangential forces acting on the top surface. Rather complicated expressions for the resulting direct and shoaring stresses are then found which yield, by a process of numerical reiteration the more accurate solution to the problem according to Guyon. The results are presented in six tables which give the axial, transverse and shoaring stresses at selected locations in the prism caused either by pure axial or pure tangential loading at a point on the top surface.

The general shape of the transverse stress distribution on the prisms centre line which is produced by normal symmetrical loading is shown in Fig. 2.1. The value of the maximum tensile stress, its distance from the top of the prism and the point of zero stress all vary with the loading distribution. However, for equilibrium to be maintained, the areas under the compressive and tensile portions of the curves must be equal for all loading distributions. It is of course the presence of these tensile stresses which cause a concrete prism to fail by splitting when it is loaded by a relatively narrow strip load. The high compression stresses which occur just beneath the top of the prism when is is loaded by a relatively narrow strip load do not lead to failure by crushing as might be expected because the material is in a three dimensional state of stress which corresponds closeley to triaxial compression. Under these conditions the load to cause failure by crushing would be far higher than that required to cause splitting lower down the prism. The problem of this compression zone is considered in greater detail in Chapter 13.

The set of stress profiles for the central vertical plane reproduced in Fig. 2.1 summarises the results obtained from Guyon's tables for different values of the "concentration factor" b/D. b is the width of the loading strip and D the total width of the prism. Only the tensile portions of the curves have been drawn and loading is normal to the upper surface and symmetrical about the vertical centre line. For this analysis the total height of the portion of the prism under consideration is equal to D, the width. It will be noticed that the profile for the case for which b/D = 0 has no positive portion which seems to contradict the statement made earlier concerning equilibrium. In fact, the compressive stress is supplied by a compressive force acting on an infinitesmal area for the theoretical case of a knife edge load. This force balances the tensile stresses. In practice however compressive stresses are always produced by forces which do act on a finite area.

The second part of Guyon's paper deals with the problem of a finite prism of rectangular cross-section, e.g. cube. The case of normal symmetrical loading only is developed. An approximate solution is obtained by using double Fourier

TRANSVERSE STRESS PROFILES ON THE CENTRE LINE OF A CENTRALLY LOADED PRISM ACCORDING TO GUYON.

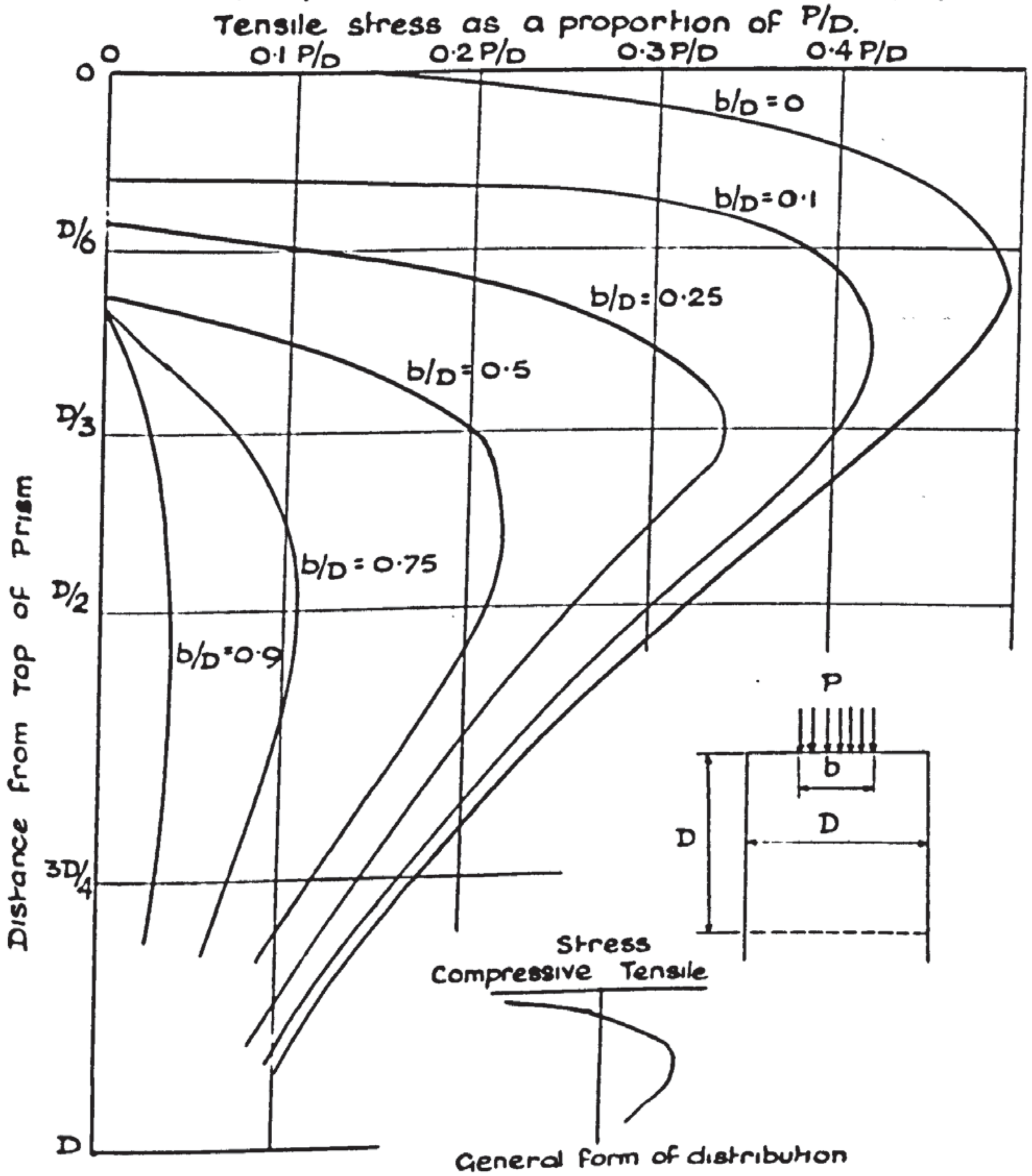


Figure 2.1

series and a method of numerical calculation is given for the case of a single normal point load on the central axis. However, the method is too restricted and it is felt that the results obtained from the first part of the paper are generally more applicable to the subject of these investigations even when the analysis of a cube loaded by a narrow strip load is being considered.

It is worth noting that Guyon, in his paper gave no information concerning any tests that he had performed but merely referred to some early photo-elastic tests that were carried out by Tesar whose results were in close agreement with his own.

In their paper on the stress distribution in the anchorage zone of post-tensioned prestressed concrete members, Rowe and Zielinski[(8)] criticise the theory of Guyon by comparing it with the results of later photo-elastic tests performed by Christodoulides[(15,16)] and with the results of their own tests on concrete prisms. They show that Guyon's theory considerably underestimates the maximum tensile stress developed in the block by, in some cases, as much as fifty per cent. It must be remembered however that Guyon's theory was developed primarily for the two dimensional case of plane stress and was later applied to that of the three dimensional case of small loaded areas on prisms by modifying the coefficients given in the tables. However, the system of loading used for the tests described in these investigations conforms far more closely to the two dimensional case that Guyon considered than it does to the three dimensional end block problem and so it might be expected that Guyon's theory would have some applicability.

The solutions which have been given by Bleich (17) modified by Sievers (18) for application to three-dimensional end blocks of prestressed beams are based on Guyon's initial approximation and have been considered by many authorities to give less reliable results than Guyon's theory. The most recent and probably the best solutions to the two and three-dimensional case are due to Iyengar (9) and it is shown in his paper how closely his results agree with the main ones of Guyon, at least in the case of the transverse stress distributions of the two-dimensional case.

A fourth method of approaching this problem has also been considered, that of a two-dimensional finite element method and this is described in the next Chapter. A comparison between the results of Guyon's analysis and the finite element approach is given in Chapter 4 and comparisons between theoretical and experimental results may be found in Chapter 6.

CHAPTER 3

CHAPTER 3

PLANE FRAMEWORK ANALYSIS

3.1 INTRODUCTION

One of the main advantages of using a finite element approach to stress problems is that a variety of problems with different boundary conditions can be dealt with automatically by means of the electronic digital computer once a suitable analysis programme has been written. The purpose of the analytical method is to obtain the stresses and the strains at the centre of elements of a two-dimensional grid which is made to represent the elastic stress plane.

It was decided to use the method of equivalent plane frameworks for the analysis as it was of some interest to see whether this method could be used successfully to analyse stress systems of the type described in the last chapter.

The method of solving plane stress problems by means of equivalent plane framework models has been investigated by many workers, including Hrennikoff (19), McHenry (20), Grinter (21), McCormick (22) and recently by Yettram and Hussain (23) and the method has been used in these investigations to obtain the stresses on the vertical centre line of a plate loaded on its top edge by a symmetrically placed uniformly distributed load whose width relative to the width of the plate was altered.

3.2 THEORY

The method has been well described by Yettram and Hussein [23] in their paper from which the following extracts have been taken.

" A plane framework model is derived to represent a rectangular element of a plate in extension. The model consists of four side beams having both axial and in-plane flexural rigidities, and two diagonal beams having axial rigidity only. The Poisson's ratio effect is considered automatically.

A plate is first divided into rectangular elements that are connected rigidly at their corners, as shown in Fig.3.1. Each element is then replaced by an equivalent framework model.

Hrenikoff has specified four criteria for the deformability of a rectangular pin-connected framework. Hence, only four cross-sectional properties can be defined for such a model. However, with the introduction of the rigid-body rotation in a rectangular plate element under shear stresses as an independant mode of deformation, five criteria for the deformability of a rectangular rigidly-connected plane framework will be obtained. As a consequence, a model with five cross-sectional properties will uniquely define a rectangular element of a plate. The five properties are chosen to be the axial and the in-plane flexural rigidities of the side beams and the axial rigidity of the diagonal beams. These cross-sectional

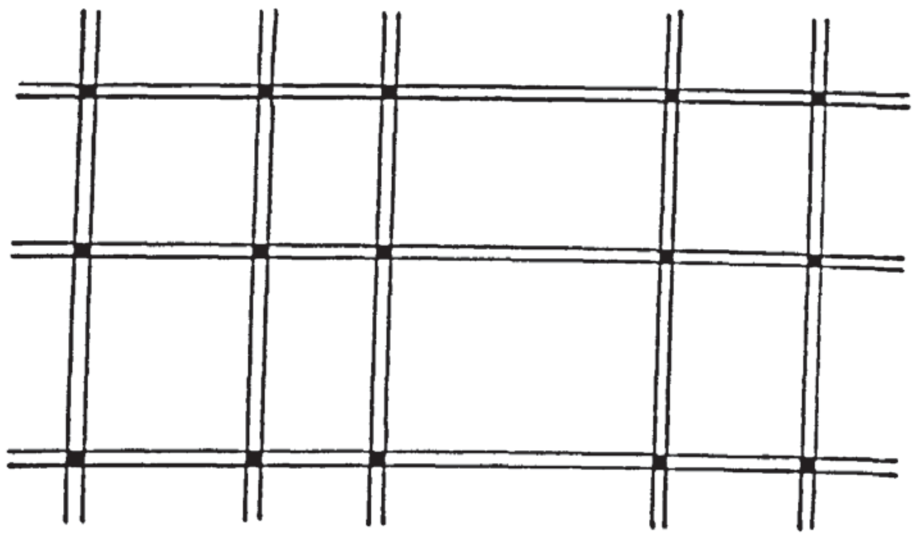


PLATE ELEMENTS CONNECTED AT CORNERS

Figure 3.1

properties are evaluated by equating the deformations of the nodes of the framework model with these of the plate element, when both are acted on by statically equivalent loads. The plate element and its model are shown in Figs. 3.2 (a) and (b).

Extension of a Rectangular Element of a Plate

When a plate element is subjected in turn to systems of stress flows T_1, T_2 and H as shown in Fig. 3.2 (c), (e) and (g) the associated displacements are

$$\delta_1 = \frac{k\lambda T_1}{Eh}$$

$$\delta_2 = \frac{\nu\lambda T_1}{Eh}$$

$$\delta_3 = \frac{\lambda T_2}{Eh}$$

$$\delta_4 = \frac{\nu k\lambda T_2}{Eh}$$

and $$\delta_5 = \frac{(1 + \nu)k\lambda H}{Eh}$$

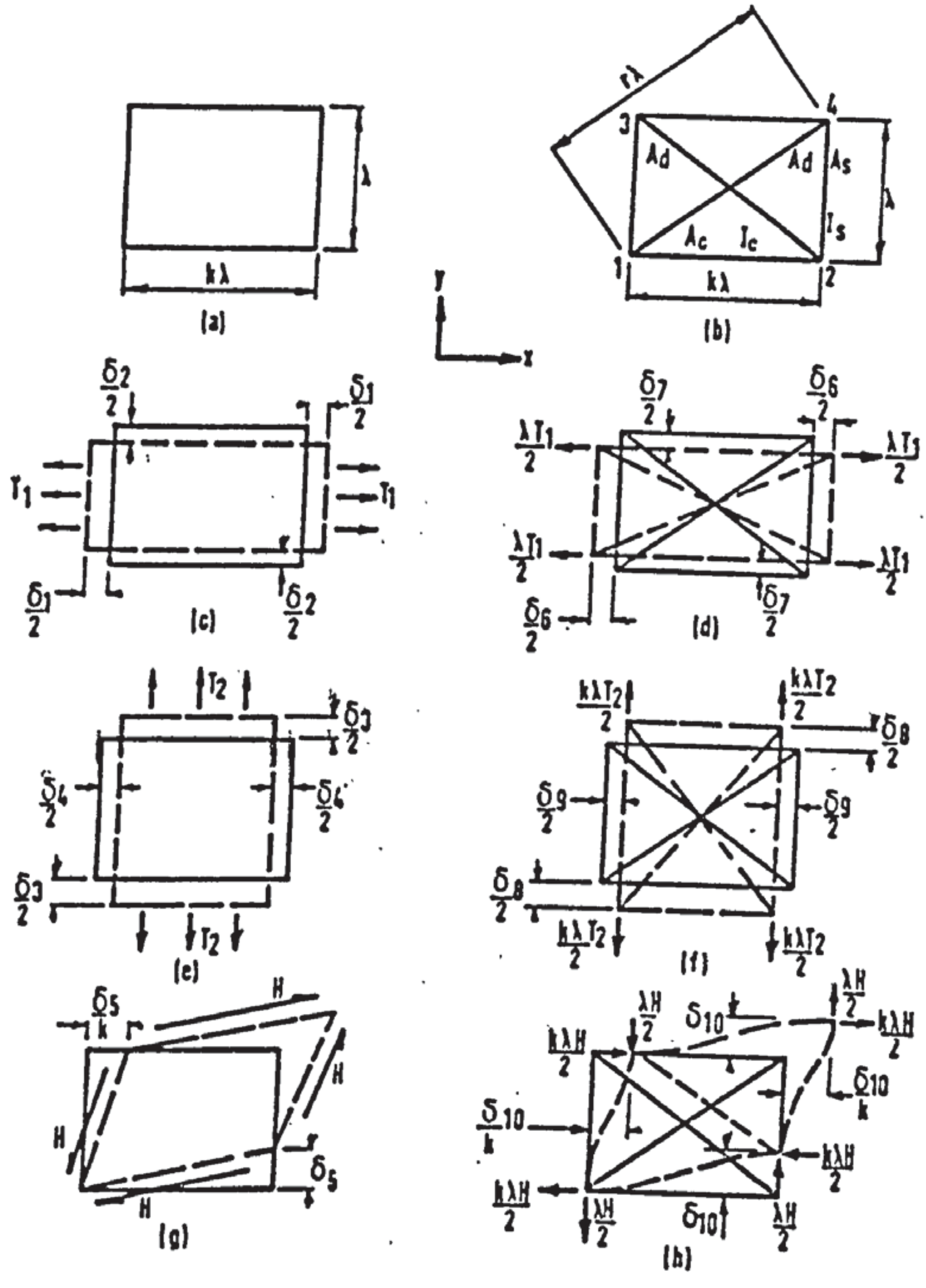


FIG.12.—PLATE ELEMENT AND EQUIVALENT FRAMEWORK MODEL (DIAGONALS INCLUDED)

in which λ and $k\lambda$ are the side lengths of the element, h is its thickness and E and ν are the elastic modulus and Poisson's ratio of its material, respectively. The deformation of the element in Fig. 3.2 (g) is adjusted so that the rigid-body rotation equals zero.

Extension of the Plane Framework Model

Consider the arrangement of six beams as shown in Fig. 3. 2 (b). The side beams of length λ have equal cross-sectional areas A_s and equal second moments of area, I_s; the side beams of length $k\lambda$ have equal cross-sectional areas A_c and equal second moments of area I_c. The diagonals of length $r\lambda$ have equal cross-sectional areas A_d and no flexural rigidity. All the beams are rigidly connected at the nodes.

.... by establishing the governing stiffness matrix equation for the model and considering the systems of forces shown in Fig. 3.2 (d), (f) and (h), the associated displacements are

$$\delta_6 = \frac{k\lambda^2 T_1}{2E} \cdot \frac{r^3 A_s + A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d}$$

$$\delta_7 = \frac{k^2\lambda^2 T_1}{2E} \cdot \frac{A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d}$$

$$\delta_8 = \frac{k\lambda^2 T_2}{2E} \cdot \frac{A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d}$$

$$\delta_9 = \frac{k^3\lambda^2 T_2}{2E} \cdot \frac{A_d}{r^3 A_c A_s + A_c A_d + k^3 A_s A_d}$$

and

$$\delta_{10} = \frac{r^3 k^2 \lambda^4}{4E} \cdot \frac{1}{6r^3 I_s + k^2\lambda^2 A_d}$$

<u>Evaluation Of The Cross-Sectional Properties Of The Members</u>

For the plane framework model to represent the plate element, corresponding displacements must be equal for the two systems; therefore

$$A_s = \frac{(k^2 - \nu)\lambda}{2k(1 - \nu^2)} h$$

$$A_c = \frac{(1 - \nu k^2)\lambda}{2(1 - \nu^2)} h$$

$$A_d = \frac{\nu r^3 \lambda}{2k(1 - \nu^2)} h$$

$$I_s = \frac{(1 - 3\nu)k}{2(1 - \nu^2)} \cdot \frac{h\lambda^3}{12}$$

and

$$I_c = \frac{(1 - 3\nu)k^2}{2(1 - \nu^2)} \cdot \frac{h\lambda^3}{12}$$

For a square element k=1 then [the above] equations reduce to those obtained by McCormick. Also, when Poisson's ratio equals 1/3 the framework model reduces to one consisting of beams with axial rigidity only, as was derived by Hrennikoff. Where Poisson's ratio $\nu=0$ then $A = k\lambda h/2$, $A_o = \lambda h/2$, $I_s = k\lambda^3 h/24$, $I_o = k^2\lambda^3 h/24$ and $A_d = 0$, and thus the model would reduce to one with side beams only."

3.3 PROGRAMME

Using the above equations it is possible to replace a plate by an equivalent framework which can be solved by a standard frame analysis computer programme (24). The resulting nodal displacements will be those for the plate and from these can be obtained the stresses and the strains at the centre of each element. The programme that was used in this case was basically a standard one that was suitable for running on the English Electric KDF 9 and it had been modified so that the data preparation could be simplified.

3.4 INPUT DATA

In order that the simplified method of data preparation could be used it was necessary to use frameworks containing one size of element only. It was therefore necessary to use a considerable degree of judgement to choose an element size which allowed an adequate degree of accuracy without too great a loss of economy as the time required to run these large problems was mainly a function of the number of the elements. It is of interest to record that in the case of the problems that were run, a framework consisting of 162 elements was run in approximately 5 minutes and a framework consisting of 243 elements took approximately $8\frac{1}{2}$ minutes. In view of the effectiveness and economy with which data preparation was carried out for this type of framework analysis where very large numbers of equations required solving, the method of data preparation together with a simple example is given in Appendix A.

3.5 OUTPUT

The line printer facility was used to output the results which consisted of the following data. For each node: the displacements in the x - and y - directions, the rotational displacement, the resultant dorces in the positive x - and y - directions and the moment. For the centre of each element (numbered in the order in which they were submitted in the input), the direct stresses in the x - and y - directions, the shear stress, the two principal stresses, the maximum shear stress and the angle of maximum shear.

CHAPTER 4

CHAPTER 4

RESULTS AND COMPARISON OF PLANE FRAMEWORK ANALYSIS

4.1 COMPARISON USING TWO VALUES OF POISSON'S RATIO

4.1.1 Introduction

Before any data could be written it was necessary to choose a suitable value of Poisson's ratio because, although the calculated stresses should ideally be unaffected by different values of this ratio over most of the region the strains would be affected and it would clearly be of some interest to compare theoretical and experimental values of strain. The value of the modulus of elasticity was also critical in the case of strains but as strain is inversly proportioned to the modulus of elasticity it was only necessary to supply an arbitrary value in the analysis as correction could easily be made to the strain values, if necessary after the results were obtained. Many authorities[25,26,27] give an average value of 0.15 for Poisson's ratio for concrete and this was the value that has been used for all but one of the theoretical problems investigated. However, it was decided to check the programme by determining how the strains were affected and to see whether stresses were affected when different values of Poisson's ratio was used.

4.1.2 Method

For this purpose it was decided to analyse a cube, loaded by a vertical central line load on its upper surface using Poisson's ratio values of zero and 0.15. A plate of dimensions 6 x 6in. and of thickness 6in. was used to similate the cube. It was only necessary to analyse half the plate as loading was symmetrical about the vertical centre line os the nodes on the centre line were given horizontal and rotational fixity. Square elements of side 0.333in. and thickness 6in. were used and the whole mesh was 6in. high and 3in. wide giving 3 x 3 x (6x3) = 162 elements in all. The plate was regarded as being fixed at its base so the nodes along the base were given three degrees of fixity to satisfy the condition that the base of the cube was plane and that friction acted along the lower platen/concrete interface. The load applied at the position corresponding to the centre of the upper face of the cube was 20 tons (10 tons on the actual framework).

4.1.3 Results

The results are shown in Figs. 4.1 and 4.2. Fig.4.1 shows the stress profiles on the vertical centre line. of the cube. The sketch shows the mesh that was used. For convenience the transverse stress is denoted by σ_x in lb/sq.in. positive for tensile stress and the transverse strain by ε_x, positive for tensile strain. D' in inches is the distance from the upper face of the cube.

CHAPTER 4

COMPARISON OF σ_x PROFILES ON CENTRE LINE OF CUBE USING DIFFERENT VALUES OF ν WITH FIXED BASE CONDITION.

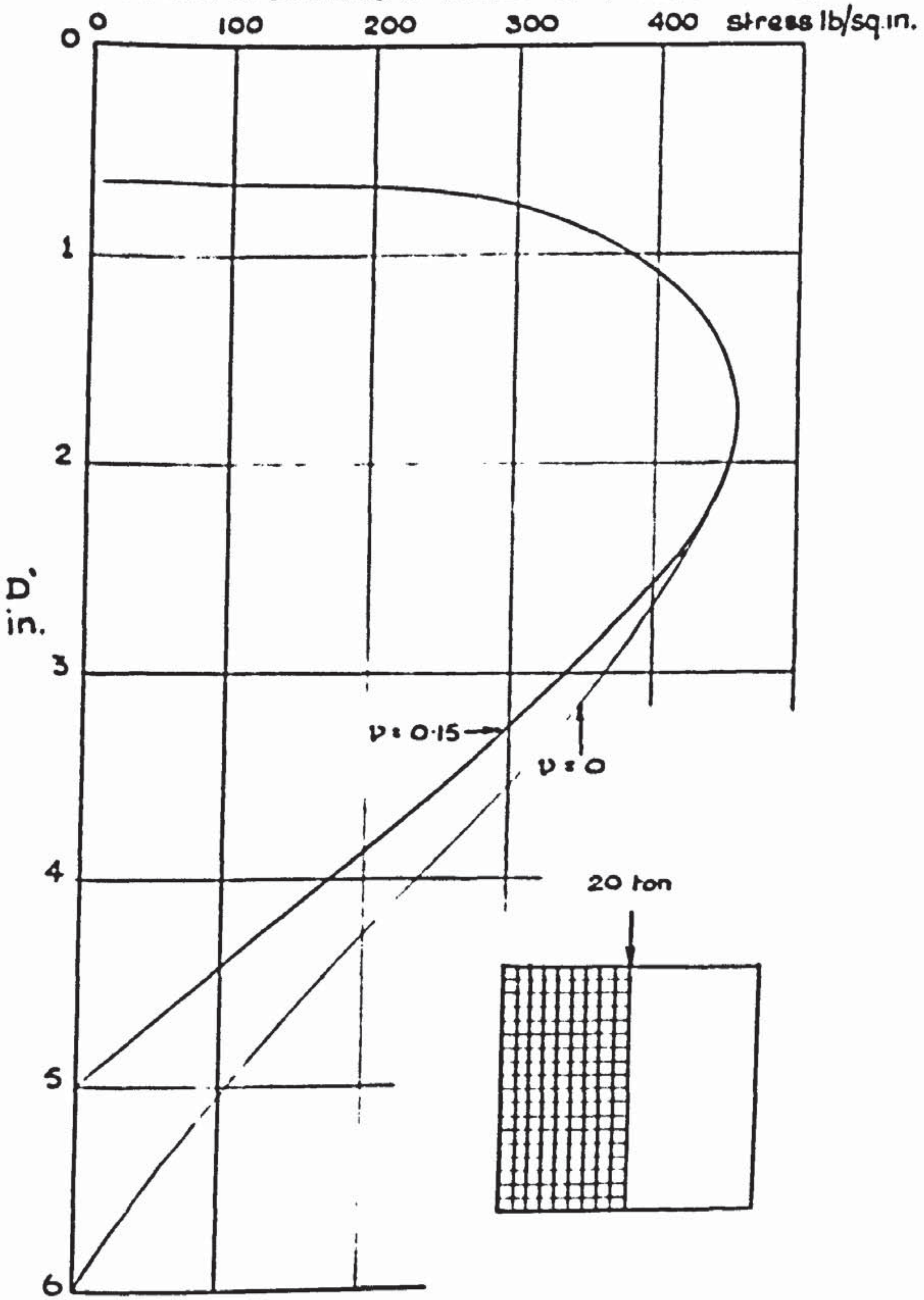


Figure 4.1

COMPARISON OF εx PROFILES ON CENTRE LINE OF CUBE USING DIFFERENT VALUES OF ν WITH FREE BASE CONDITION.

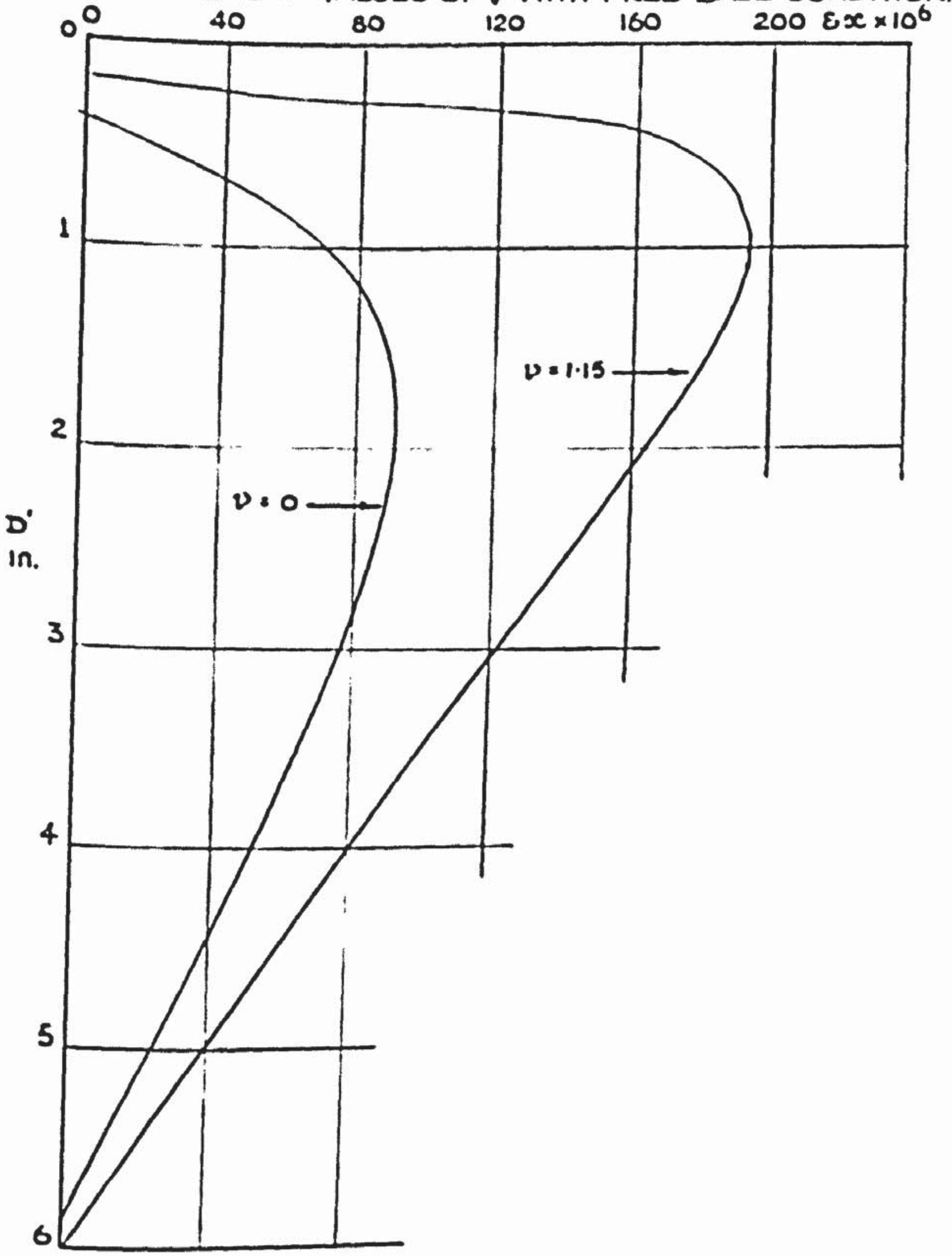


Figure 4.2

It will be seen that the form of the distributions follow the general pattern of Fig. 2.1; while there is good agreement between the values of the peak stress and the positions of the peaks for $\nu = 0$ and $\nu = 0.15$ it is clear that there is some discrepancy in the values of the stresses towards the base. Indeed, compressive stresses are present near the base when $\nu = 0.15$ is used in the calculation. This is not surprising and follows from the fact that the transverse strain at the base of the cube has been set equal to zero. If the elastic equation for plane stress is considered,

$$\sigma_x = \frac{E}{1 - \nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

where ε_y is the strain in the vetical direction, assuming that loading is vertically downwards, it is easy to see that when $\varepsilon_x = 0$,

$$\sigma_x = \frac{E}{1 - \nu^2} \times \nu\varepsilon_y$$

As the longitudinal strain on the centre line is compressive it follows that the transverse stress will be also compressive. However, it can be seen from Fig. 4.2 that the magnitude of the transverse (tensile) strain when $\nu = 0.15$ is considerably larger than it is for $\nu = 0$ over most of the region except at the base when the strain for both values of ν is zero. The values of the peak strains are

$$197 \times 10^{-6} \text{ for } \nu = 0.15$$

and

$$91 \times 10^{-6} \text{ for } \nu = 0$$

4.2 COMPARISON OF PRISMS OF DIFFERENT LENGTHS

4.2.1 Introduction

It was stated in the introduction to Chapter 2 that it was desireable to determine theoretically whether or not a column loaded through a vertical cast in plate could be simulated by a cube under similar loading conditions. For this purpose a framework of members of length 0.333in. was used to simulate the column of width 6in. and length 9in. whose cross-section was square; again it was only necessary to to analyse half the column as loading was symmetrical. It was also of some interest to investigate what differences in the transverse stress and strain distributions would be caused when a free base condition was used in the analysis as well as a fixed one. This case corresponds to the condition that the base is smooth and hence no frictional stresses exist between the lower platen/concrete interface. A matter of more general interest was a comparison of the results from the plane framework method with the continuum solution and for this purpose the analysis due to Guyon was used.

4.2.2 Results

The results of these analyses are compared in Figs. 4.3 to 4.7. Fig. 4.3 shows the comparison of the two restraint conditions for the cube. It is apparent that while the peak stresses are unaffected, the magnitude of the tensile stress towards the base of the cube is larger when the base is free to expand than in the case when the base is fixed. A similar difference can be seen in the case of the transverse stress profiles computed for the two cases, as shown in Fig. 4.4.

A comparison of the stress profiles for the cube and the column is shown in Fig. 4. 5 for the free base condition. The transverse stress distribution due to Guyon (shown by the discontinuous line) is also given. It can be seen from this figure that whereas the cube and the column results are very close and are, for the most part identical, the magnitude of the peak stress given by Guyon's analysis is considerably greater than that from the framework analysis. This point will be considered later. A similar comparison is given in Fig. 4.6 for the fixed base condition. It can be seen from a comparison of Fig. 4.5 and 4.6 that in the case of the column, the stress distributions are very similar for the two base conditions, at least for the top 6in. of its height.

Fig.4.7 shows a comparison between the transverse stress distribution for $\frac{b}{D} = \frac{1}{3}$ due to Guyon and the framework method

COMPARISON OF σx PROFILES ON CENTRE LINE OF CUBE FOR FIXED AND FREE BASE CONDITIONS - $\nu = 0.15$

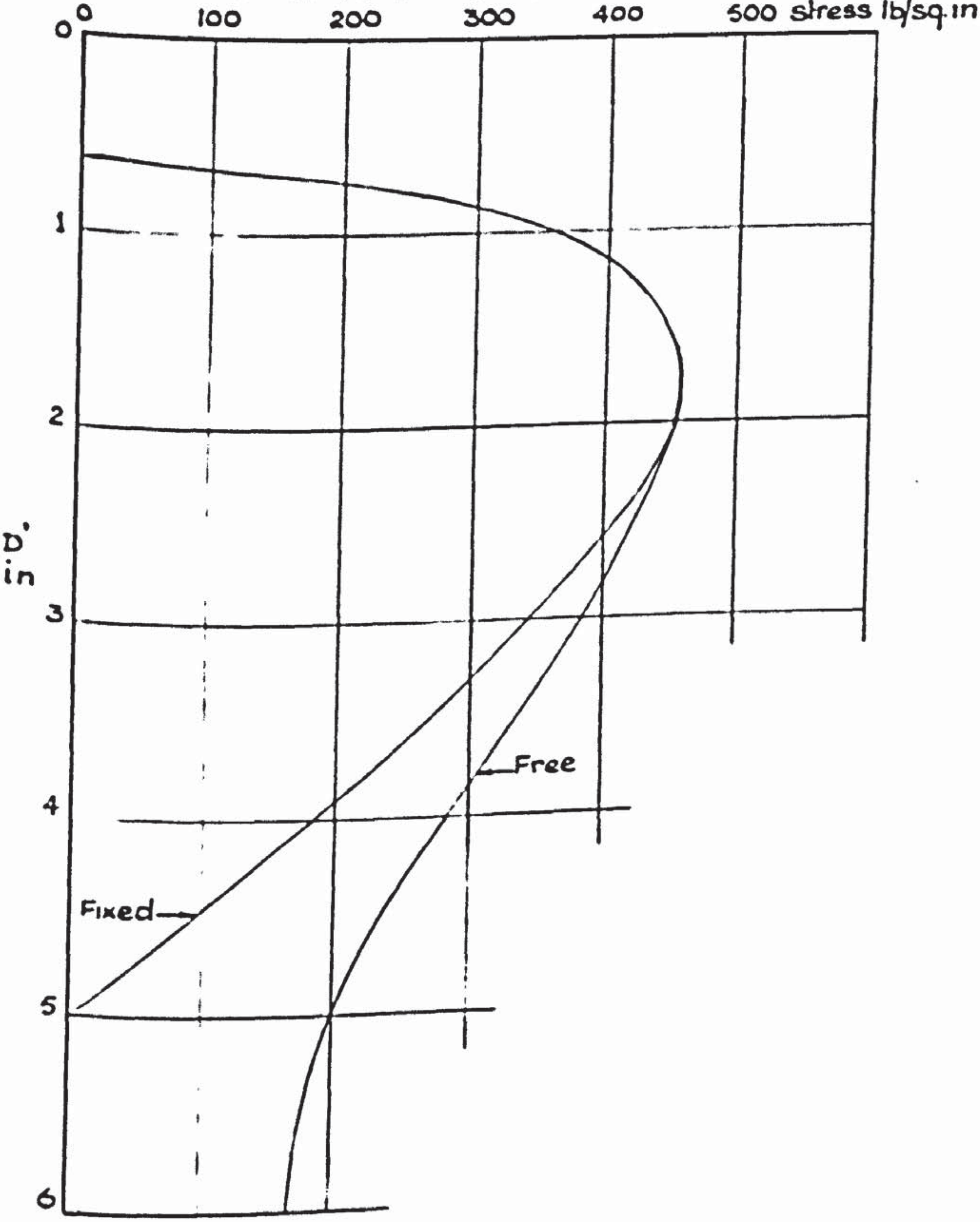


Figure 4.3

COMPARISON OF ϵ_x PROFILES ON CENTRE LINE OF CUBE FOR FIXED AND FREE BASE CONDITIONS - $\nu = 0.15$

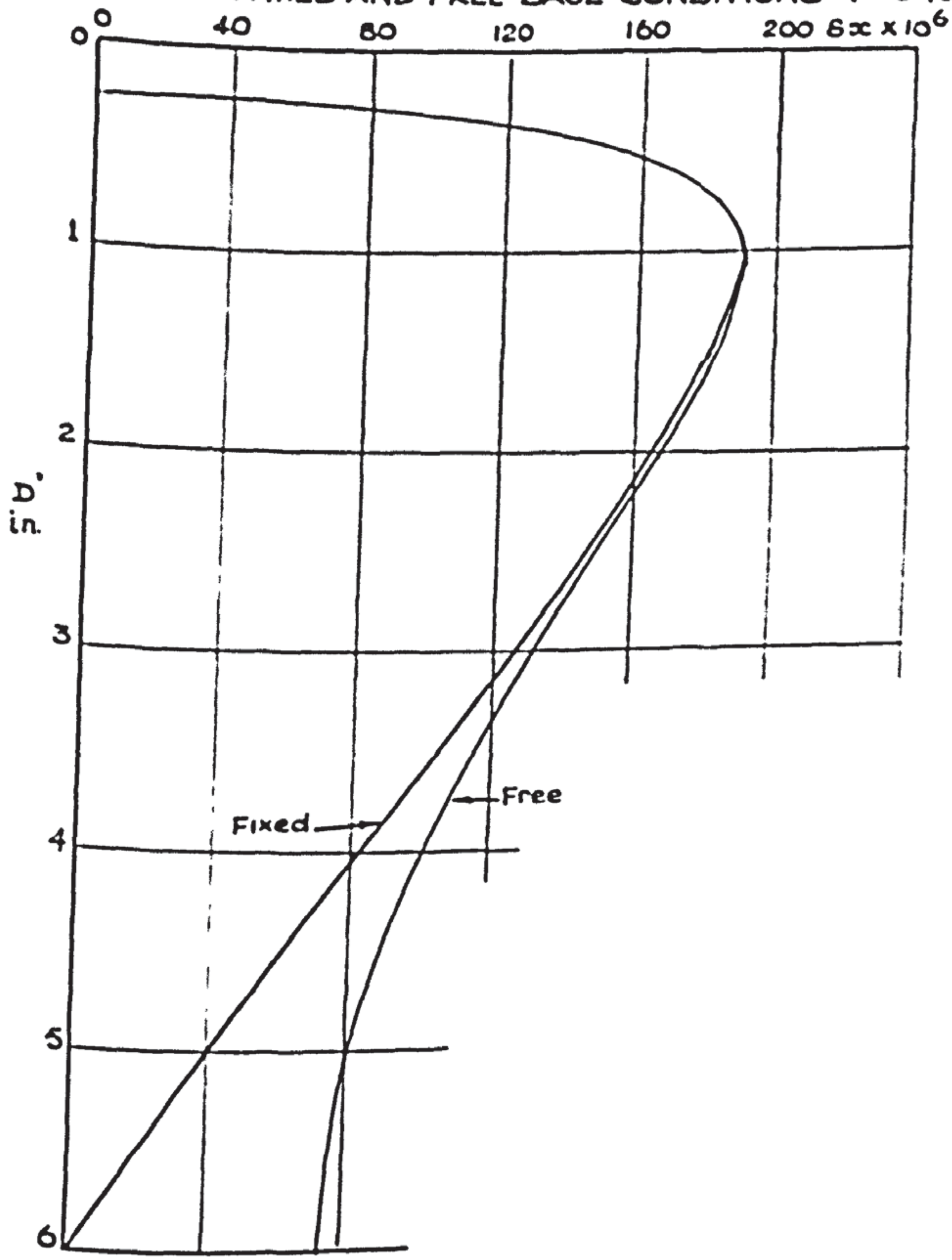


Figure 4.4

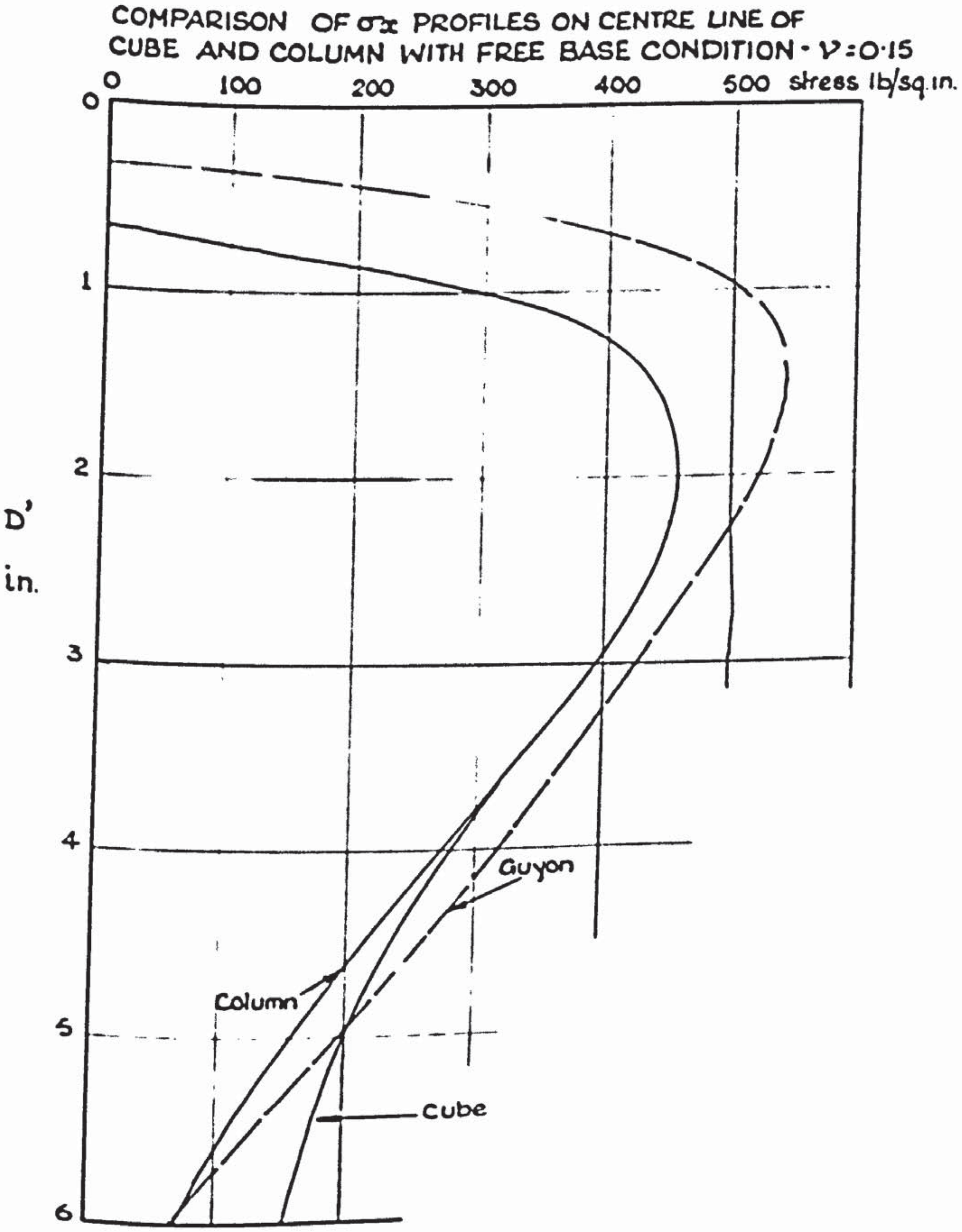

COMPARISON OF σx PROFILES ON CENTRE LINE OF
CUBE AND COLUMN WITH FREE BASE CONDITION • ν=0·15
0
100
200
300
400
500
stress lb/sq.in.
0
1
2
3
4
5
6
D'
in.
Guyon
Column
Cube

Figure 4.5

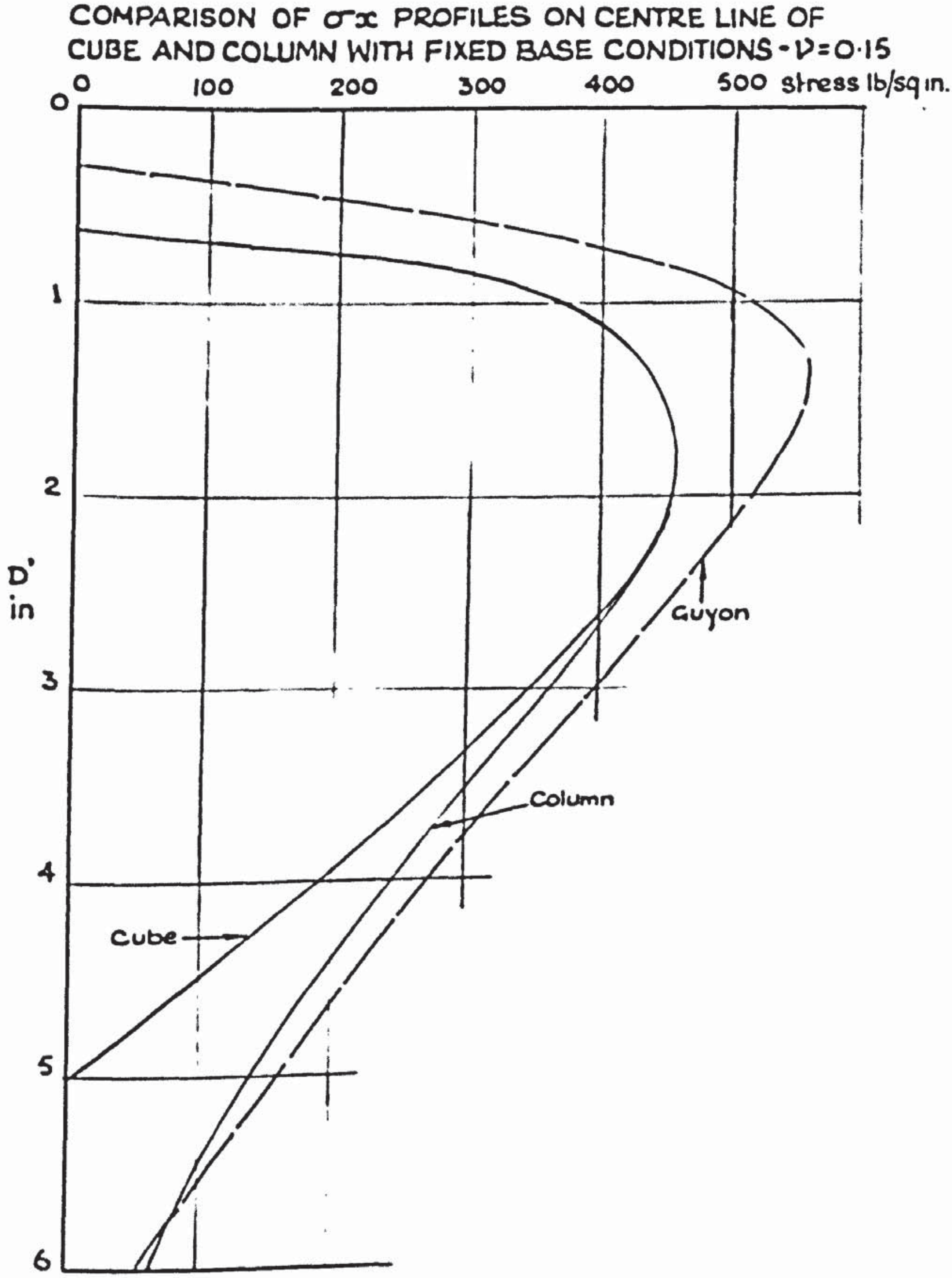

COMPARISON OF σx PROFILES ON CENTRE LINE OF CUBE AND COLUMN WITH FIXED BASE CONDITIONS - ν=0·15
0
100
200
300
400
500 stress lb/sq in.
0
1
2
3
4
5
6
D' in
Guyon
Column
Cube

Figure 4.6

COMPARISON OF σx PROFILES ON CENTRE LINE OF CUBE FOR $b/D = 0{\cdot}333$. $\nu = 0{\cdot}15$.

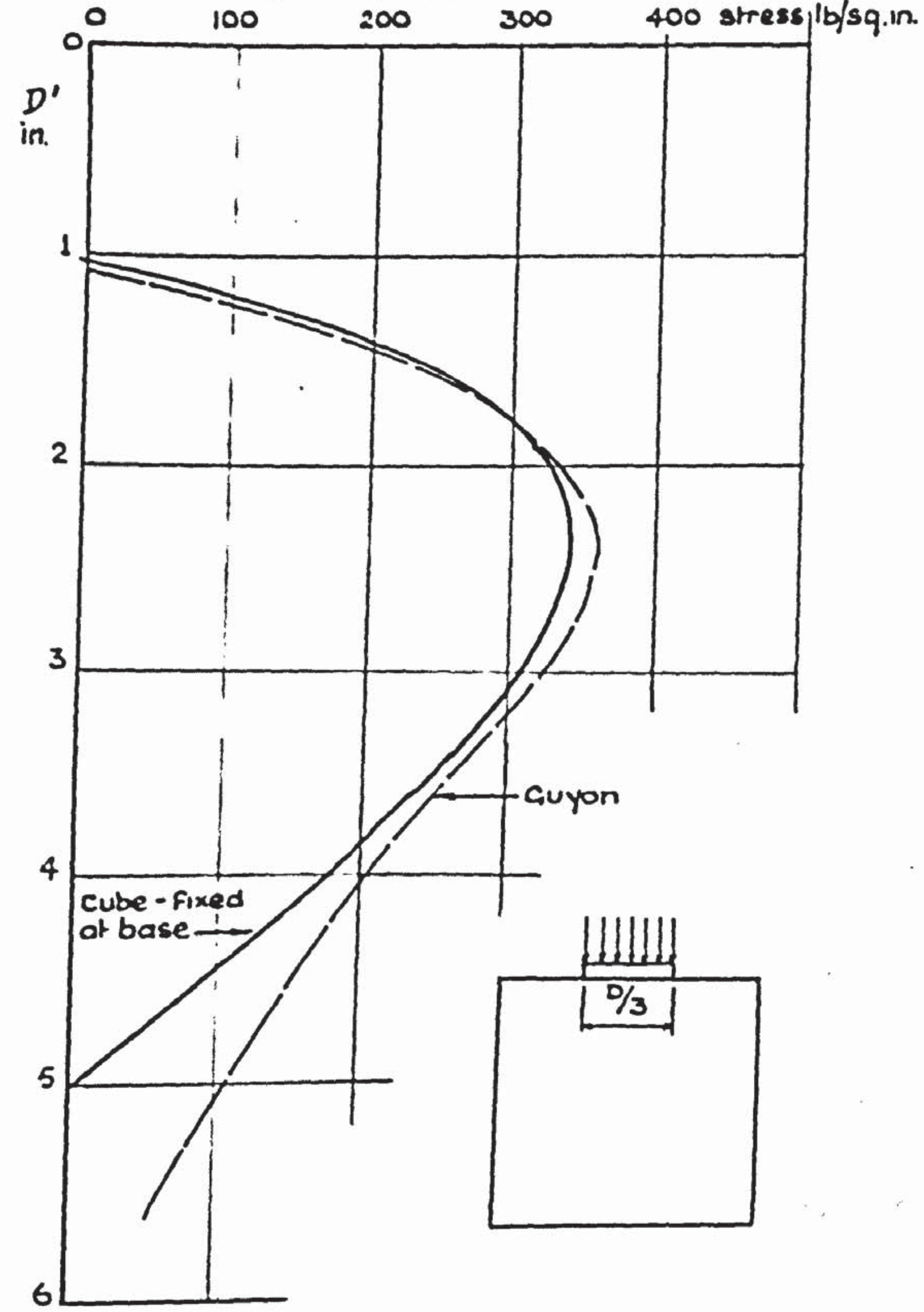


Figure 4.7

(base fixed condition). Good agreement is obtained between the peak stress values and their positions and it can be deduced that, had the cube been analysed using the base free condition, agreement between the two transverse stress distributions would have been very good indeed.

4.3 CONCLUSIONS

It is apparent from the foregoing discussion that the method of analysis using an equivalent plane framework to simulate a solid body under a variety of different loading and boundary conditions can be usefully applied to the type of problem which forms the subject of these investigations. It is clear however that when a prism is loaded by a centrally placed, uniform line load, the accuracy of the analysis was limited in comparison with the continuum analysis. The reason for this undoubtedly lies in the fineness of the mesh that was used. The problem of the mesh size and the resulting accuracy has been adequately dealt with elsewhere, particularly by Hooley and Hibbert[28] who came to the conclusion that it was not only necessary to design a framework model to satisfy the laws of elasticity and equilibrium but also to consider the general deformed shape. They said that the arbitrary choosing of members would generally lead to a violation of at least some of the deformation conditions which would lead in turn to a less accurate model.

However, the results shown in Fig.4.7 indicate that when the relative loading width is greater than zero the accuracy increases considerably. It was not the purpose of this investigation to find a definitive mesh size for all loading widths but clearly one could be obtained although undoubtedly at the expense of economic computer utilization. Nevertheless it has been shown that even when the spread of the load is small a cube can be used as a good approximation for a column when the free base condition is used.

Comparisons of theoretical strains and experimental ones will be found in Chapter 6.

CHAPTER 5

CHAPTER 5

STANDARD APPARATUS AND TESTING PROCEDURES

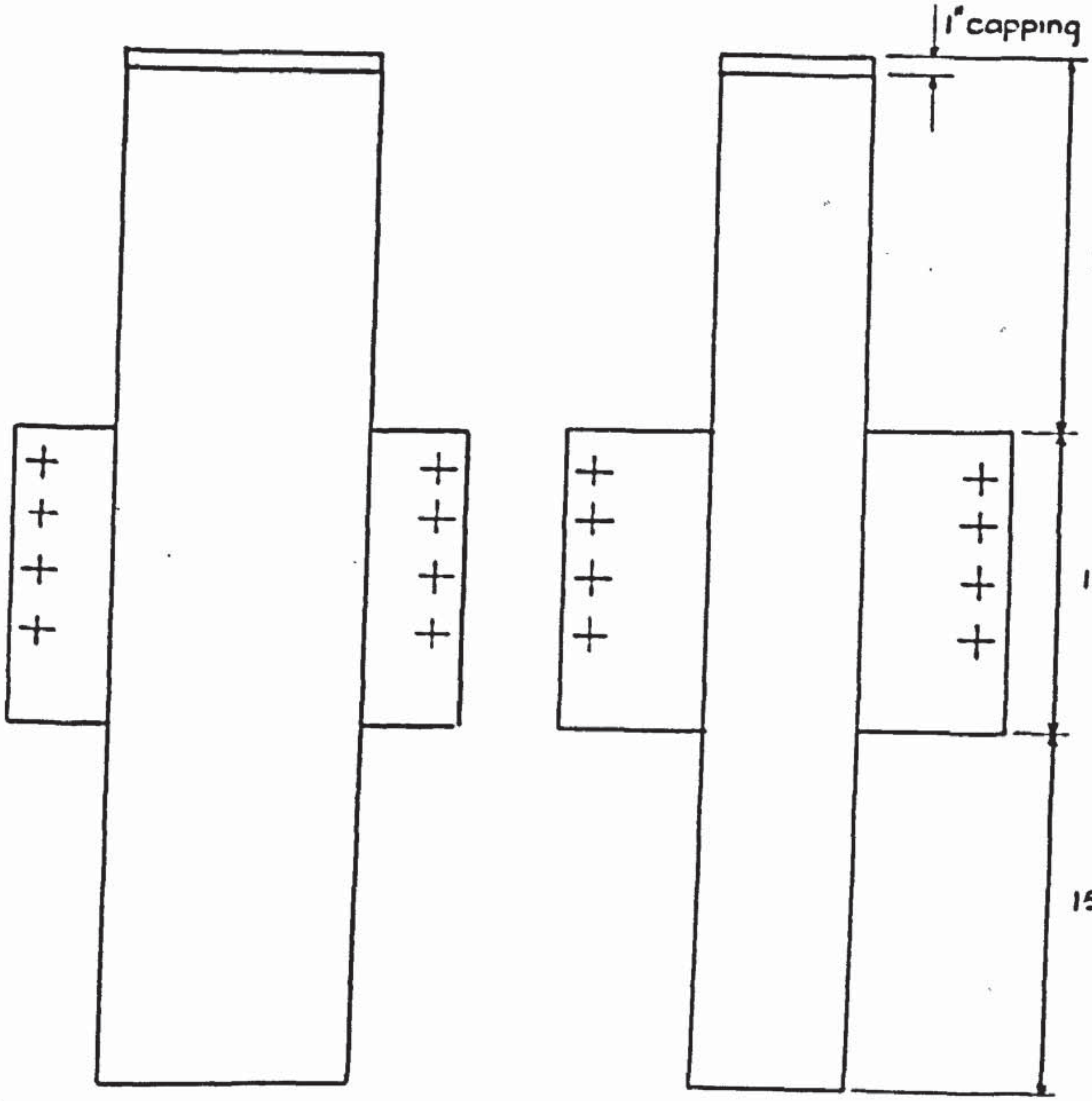
5.1 INTRODUCTION

It was decided to confine the scope of the experimental work in these investigations to a study of the factors involved when a column is loaded through a vertical cast in plate. It was considered that although there were problems concerning the action of a vertical plate cast into the end of a beam, and its connexion to the reinforcement, these were probably better understood and easier to overcome than those associated with the transferring of load through a vertical plate cast into a column. In order to evaluate with any degree of thoroughness the major factors involved, it was found necessary to perform a considerable number of tests on both full size stub columns and on cubes which were used to simplify testing procedure. The standard testing and manufacturing procedures are described in this chapter and variations of these procedures are dealt with in succeeding chapters and also in Appendix B.

5.2 COLUMNS

5.2.1 Dimensions

The standard dimensions of the columns used in the tests are shown in Figs. 5.1 and 5.2. The cross-sections of all the columns tested in these investigations were either 6in. square or 10in. square and in both cases the standard columns were



(b) long 10" square column

(a) long 6" square column

LONG COLUMNS

Figure 5.2

29in. high, the lower edge of the cast in plate being 15in. above the column base. In the light of the experiments reported by Holmes and Bond (4) on a similar beam to column connexion, and also in the light of the theoretical results reported in Chapter 4 this distance of 15in. was considered adequate for the size of columns tested. In almost all of the tests 5/8in. thick plates were cast into the 10in. square columns and 1/2in. thick plates were used with the 6in.square ones. For some of the experimental work columns longer than the standard were cast as shown in Fig.5.2 and these were 42in. high, the top inch being a capping of mortar. Again the lower edge of the plates were 15in. above the base of the column. The vertical plate itself was always set symmetrically on the columns vertical centre line.

5.2.2 Manufacture

The columns were cast vertically in timber moulds consisting of two wall sections and a base section which could be easily taken apart, when required, for stripping. The interior of the earlier moulds used had no special surfacing but were periodically treated with linseed oil. These moulds tended to lose their shape rather quickly and the surface soon deteriorated. The interior of the walls of the later moulds were surfaced with formica sheets which helped to considerably increase the useful life of the moulds and rendered a smoother surface to the concrete. The two halves of the earlier moulds met on the centre lines of the faces of the column through which the plate protruded and a small raised "seam" of concrete resulted at the joint. In order that electrical resistance strain gauges could be located satisfactorily on the centre lines of those faces it was necessary to remove the seams by rubbing with a carborundum stone and sanding. This proved to be a time consuming and laborious process so the design of the later moulds was improved by relocating the joint 2in. from the centre line of the faces, thus allowing the gauges to be fixed to an undisturbed surface.

When the mould had been prepared it was clamped securely to the surface of a 1000 lb wt. capacity vibrating table and filled slowly with concrete. The cylinder and cube moulds required for the casting of control test specimens were also vibrated on a vibrating table but were left unclamped. In order to ensure a high and reasonably constant level of compaction concrete was placed in the moulds in small quantities and no more was added until the "fat" or slurry had risen to the surface and air bubbles were appearing only infrequently. In spite of the care taken, it is unlikely that the relative densities of columns, cubes and cylinders were always constant and this would undoubtedly have caused some of the scatter of experimental results.

5.2.3 Curing

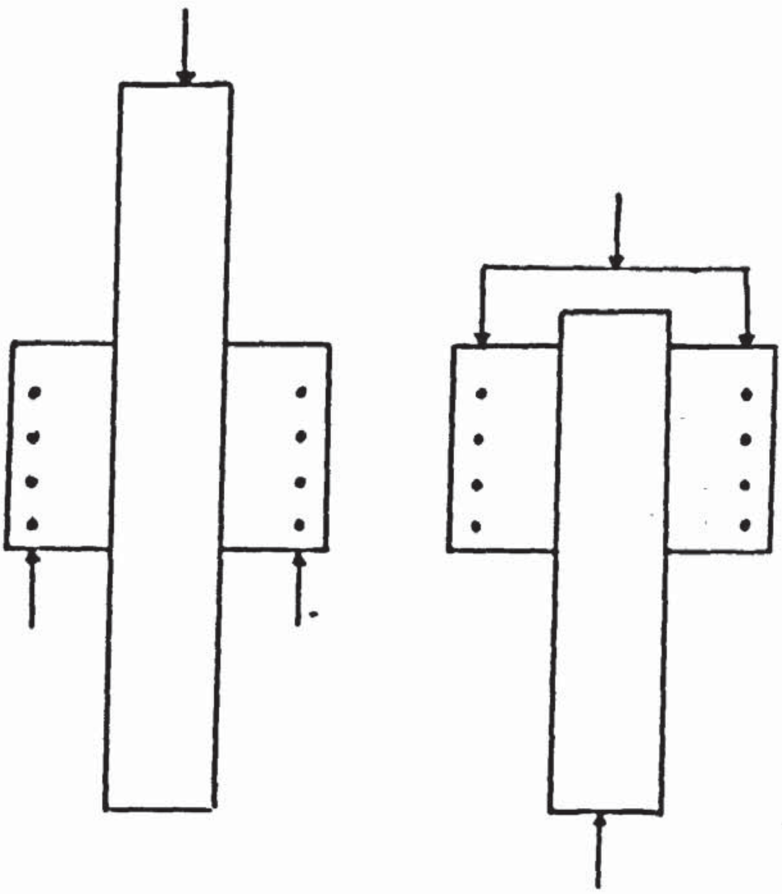
In all but a few cases the curing of the concrete cubes and cylinders followed a standard procedure. As soon as casting had been completed the specimens were covered with polythene sheeting which was held down at the edges so that the relative humidity of the enclosed air was maintained at a high level. After a period of approximately 12 hours when the concrete had set hard the moulds were stripped from the specimens and the latter were transferred to a curing room for a period of 7 days where they were stored on wooden racks to ensure that air could circulate freely over all surfaces. The relative humidity of the air in the curing room was maintained as closely as possible at 100% and the temperature was maintained at between 60° and $65^{\circ}F$. At the end of the 7 day period the specimens were returned to the laboratory and stored in the open, again ensuring that free air circulation was possible, for a further period of three weeks. The specimens were then tested at an age of 28 days. The variations in this procedure are noted in Appendix B and also were necessary in the chapters in which the tests are described.

5.3 LOADING OF COLUMNS

The majority of the tests were performed on columns under symmetrical loading conditions. Two different methods of applying the load were employed, one for the standard columns, and one for the long columns. Diagrams of the two methods are shown in Fig. 5.3

5.3.1 Loading Of Standard Columns

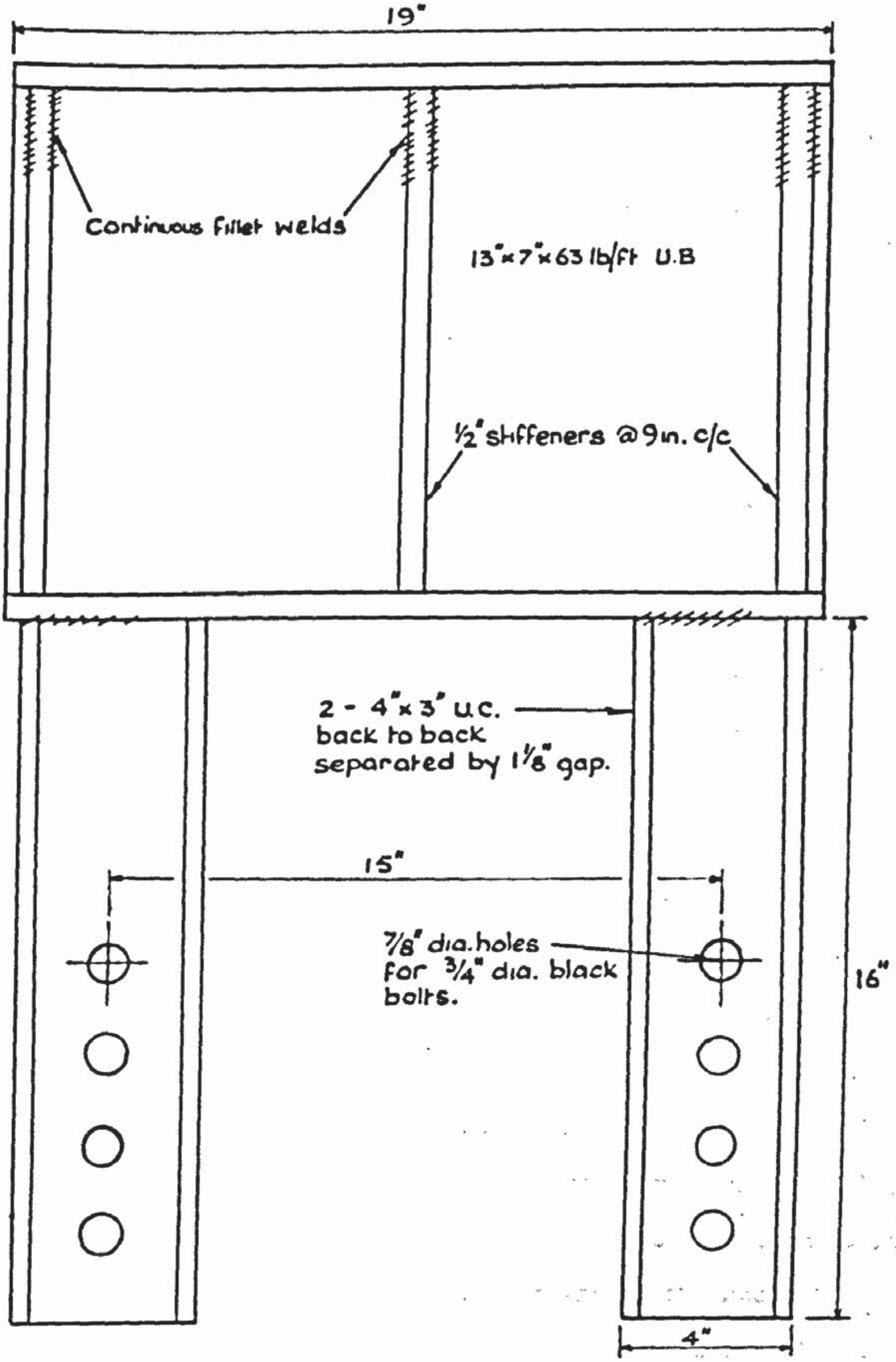
In the case of the standard columns, load was applied through a yoke fabricated from mild steel sections, see Fig. 5.4. The cross piece was made from a piece of universal beam of I-section, 19in.long. Stiffeners of ½ in. thick plate were welded to both sides of the web at the centre and close to the ends of the beam. Two universal 4 x 2 in. channel sections were welded to each end of one of the flanges and perpendicularly to it so that a 1 1/8 in. gap was left between the adjacent webs of each pair of channels. The webs of the channels were parallel to the web of the beam. Prior to welding, four 7/8in. diameter holes at 2 3/16in. centres were drilled to receive 3/4in. diameter black bolts. The yoke could then be bolted to the protruding ends of the cast in plate in which similar holes had been drilled.. Before the bolts were finally tightened, the gaps between the webs of the channel sections and the column plate were packed with steel wedges to ensure that the load was transferred vertically by the bolts to the plate. The whole assembly was then placed in a compression testing machine on a pad of still workable plaster of Paris to allow the column assembly to be levelled; a ball seating assembly was located at the centre of the upper flange of the horizontal



(b) Loading arrangement for long column.

(a) Loading arrangement for standard column.

SYMMETRICAL LOADING ARRANGEMENTS
Figure 5.3



LOADING YOKE
Figure 5.4

member of the yoke. The upper plattern of the machine was brought into contact with the upper seating and loading could be commenced. The arrangement is illustrated in Plate 1 which shows a 10in. square column before testing.

5.3.2 Loading of Long Columns

Two 30 ton capacity hydraulic jacks were used in parallel to apply load to the column plate. The jacks, which were manufactured by Applied Power Industries Inc. (U.K.) Ltd. had a 6in. stroke, with a $2\frac{3}{4}$ in. diameter plunger, an outside diameter of $3\frac{1}{2}$ in. and a collapsed height of $12\frac{3}{4}$ in. They were connected through a T - junction to a pump, manufactured by Tangyes Limited, which was fitted with a high pressure plunger which enabled a pressure of 10 000 p.s.i. to be applied. The pressure gauge attatched to the pump was manufactured by Budenberg Gauge Company Limited. The reaction of the jacks was taken by the lower plattern of the compression testing machine and the reaction of the column was taken by the upper plattern, as shown in Fig. 5.3 (b) and illustrated in Plate 2. In order that it could be levolled satisfactorily, the column was placed on a plaster of Paris pad and more plaster of Paris was placed on the top of the column before it was brought into contact with the imovable upper plattern of the machine.

The load supplied by the jacks, which were positioned on the lower plattern of the machine, was transferred to the

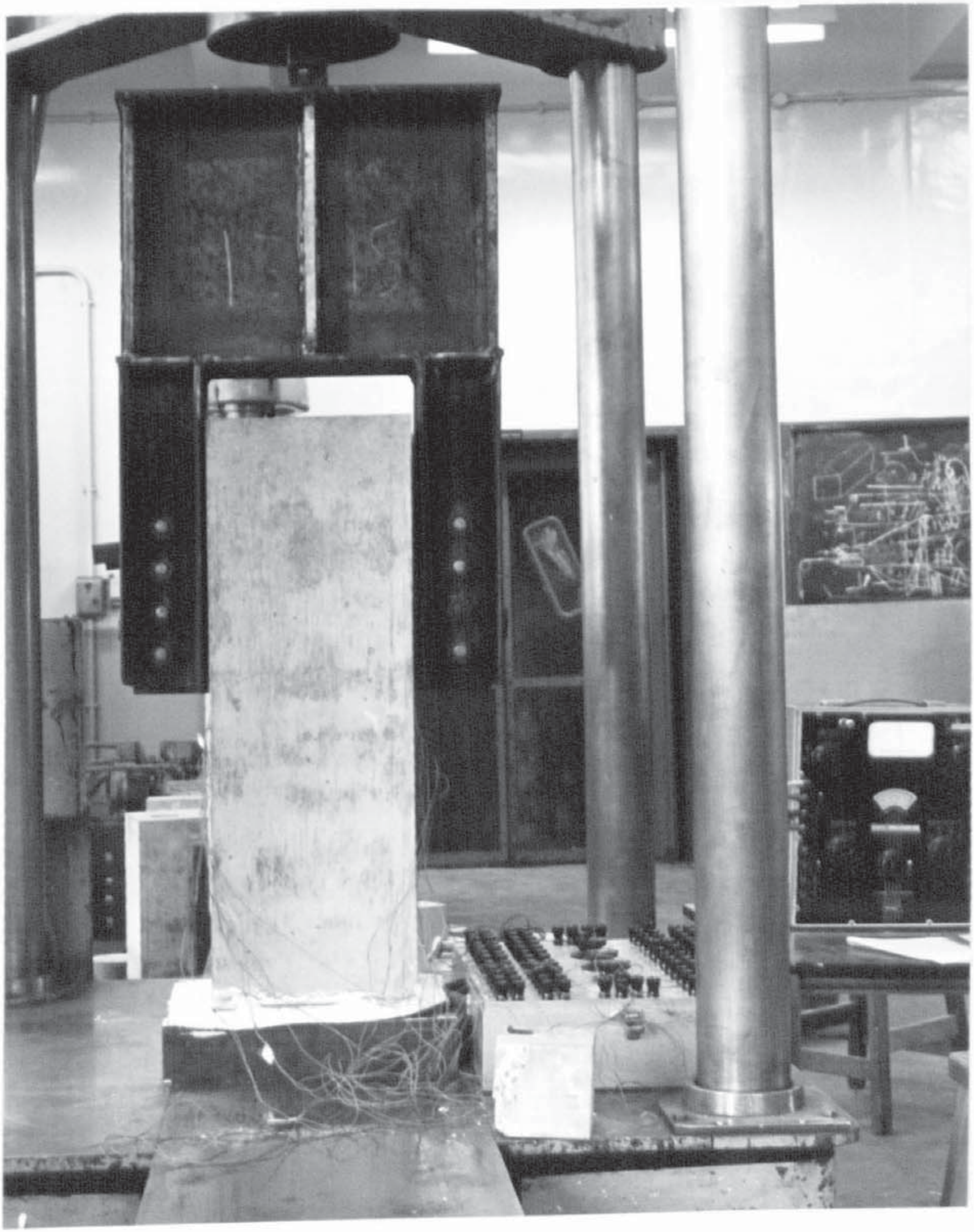


Plate 1

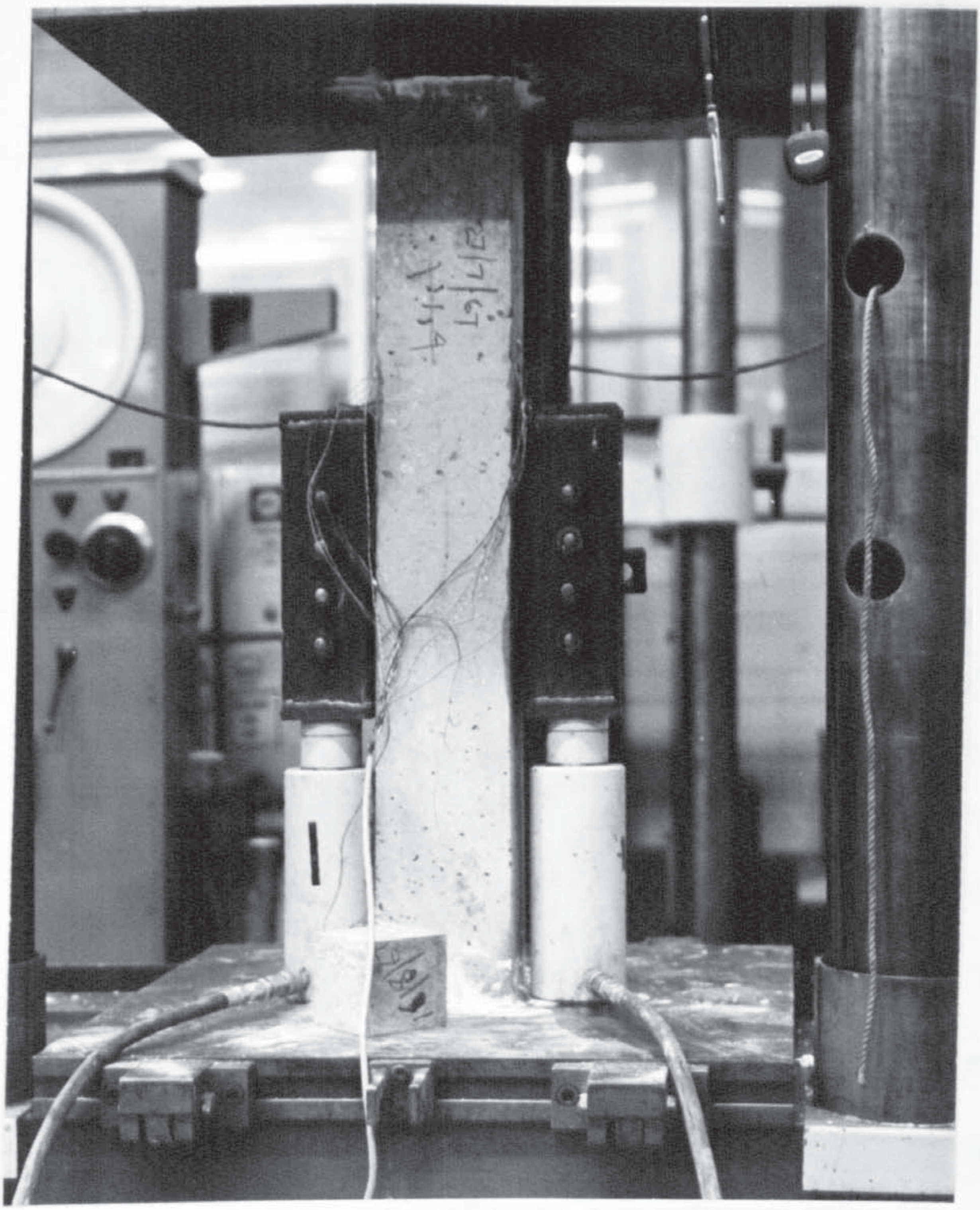
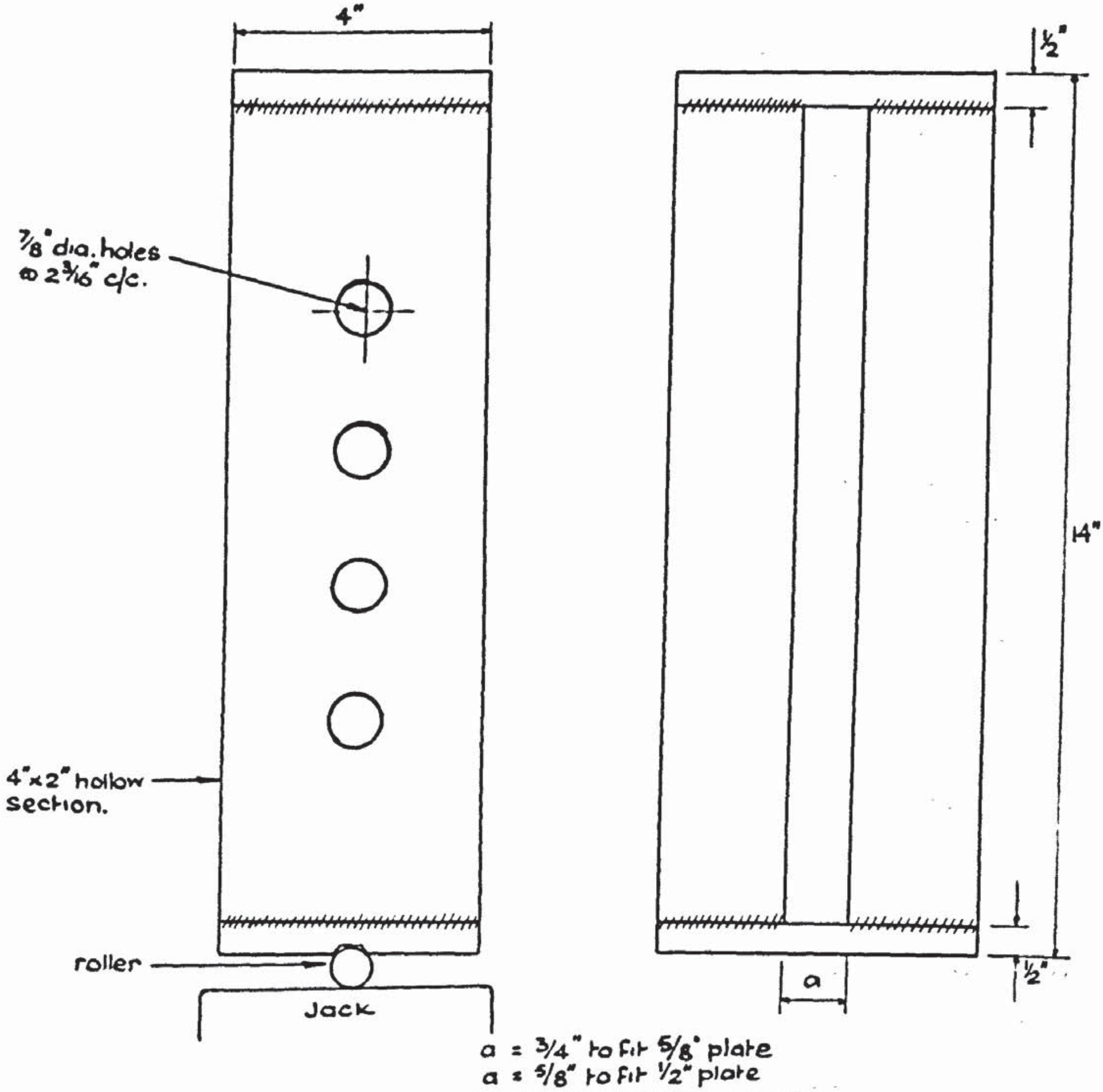


Plate 2

ends of the column plate through special loading blocks illustrated in Fig. 5.5. They each consisted of two 4 x 2in. mild steel hollow sections 13in. long welded to a ½in. thick plate at each end. A gap of sufficient width to allow the block to slip over the column plate was left between the two hollow sections through which four 7/8in. diameter holes at 2 3/16in. centres had been drilled to match those in the column plate. The blocks were secured by 3/4in. diameter bolts. A groove was cut in the centre of the underside of the lower ½in. thick plate so that a ½in. diameter roller could be accommodated between the plate and the plattern of the jack to allow free rotation of the column plate when required. The position of the roller is indicated in Fig.5.5. This was, however, only necessary when the jacks were being used either singly or as a pair to apply a moment to the column plate as described in Chapter 11. Under standard loading conditions the rollers were omitted : any rotation that might have been produced could be easily taken up by the tolerance on the bolts. Plate 2 illustrates the arrangement by which a 6in. long column was being loaded in the standard manner.

5.4 CUBE TESTS

A considerable number of tests were performed using 6in. cubes instead of columns. The mode of failure and other characteristics are fully explained in ensuing chapters and



LOADING PIECE USED WITH JACKS

Figure 5.5

it is only necessary to state here that the cubes were made in standard 6in. steel moulds.

5.5 STRAIN GAUGE MEASUREMENTS

Two different methods of obtaining strain measurements were used. The first one that was used was the Demec strain measuring technique. Gauge points consisting of small aluminium disks about 5 mm in diameter and 1 mm thick were cemented to the concrete with quick setting cement(F 88 Adhesive, manufactured by Tridox Products, U.S.A.) one at each end of the gauge length and the extensions between the disks were measured with an extensometer. Each disk had a circular depression at its centre to enable the rigid and moveable points of the extensometer to be accurately located. On the 2in. gauge length that was used, it was only possible to measure strains to 24.5 microstrains (24.5×10^{-6}). Due to the fact that most of the more important measurements were of tensile strain it was considered that a more sensitive method of measurement was required and so the majority of the tests were carried out with electrical resistance strain gauges which enabled strain measurement down to 5 microstrains. Plate 3 shows the Demec gauge points in position on the lower portion of a 10in. square column which had been tested to failure.

The electrical resistance strain gauges that were used

Plate 3

in the majority of the tests were manufactured by H. Tinsley and Company Limited and consisted of a strip of paper on to which was mounted the wires which formed the active element of the gauge. The wires were protected by a strip of felt. The gauges were conveniently fixed to the concrete by the same cement used to fix the Demec points described above. This method was found to be quick and ensured that, provided adequate care was taken, the gauge was well and firmly bedded, even on to a surface that was not completely smooth. The gauges used were type 7A whose overall dimensions were 69 x 9 mm (gauge length 48 mm) and type 6E whose overall dimensions were 27 x 9 mm (gauge length 17 mm). Two type 7A gauges were used on each of the cylinders which were used to obtain the value of the modulus of elasticity (described in section 5.6) and also on a small number of columns in particular locations, as, for example, those shown in Plate 5. The type 6E gauge was used in the vast majority of cases in which strain gauge measurements were taken in tests performed on columns and cubes. In this connection it should be observed that a gauge length of 17 mm was nearly twice as long as the largest nominal dimension of the aggregate used in all the concrete mixes (3/8in. or 9.5mm) so little innacuracy in the gauge reading was likely to result from this source. Generally, the performance of these gauges was very satisfactory.

he gauges in position on 10in. square columns are shown in Plates 4 and 5. When all the gauges had been fixed to the specimen a wire lead was soldered to each of the tags, two per gauge, and connected to the terminals of a static strain extension box (type 48U or type 23U) which was connected to the static strain recorder (type B103 U). The equipment was manufactured by Peekel Electronica N.V. The two units are seen in Plate 1. Only one dummy gauge was required for each type of gauge as it could be switched into each gauge circuit in turn. The dummy gauge was fixed to a cube of concrete, seen in Plate 1, and placed near the test specimen.

It is of interest to note that one of the first comparitive tests in which electrical resistance strain gauges were used was reported by van der Veen[29] in 1953. In these tests in which mechanical, capacitative and electrical resistance gauges were used on airport landing strips, it was found that the more accurate and reliable results were obtained from the two electrical types of gauges and this has been confirmed many times since then by many workers including Kaplan[30] who used bonded wire electrical resistance strain gauges whose gauge length was 1in. He found that they were suitable for tests of short duration.

5.6 CONTROL TESTS

Control specimens were cast from each mix from which test

Plate 4

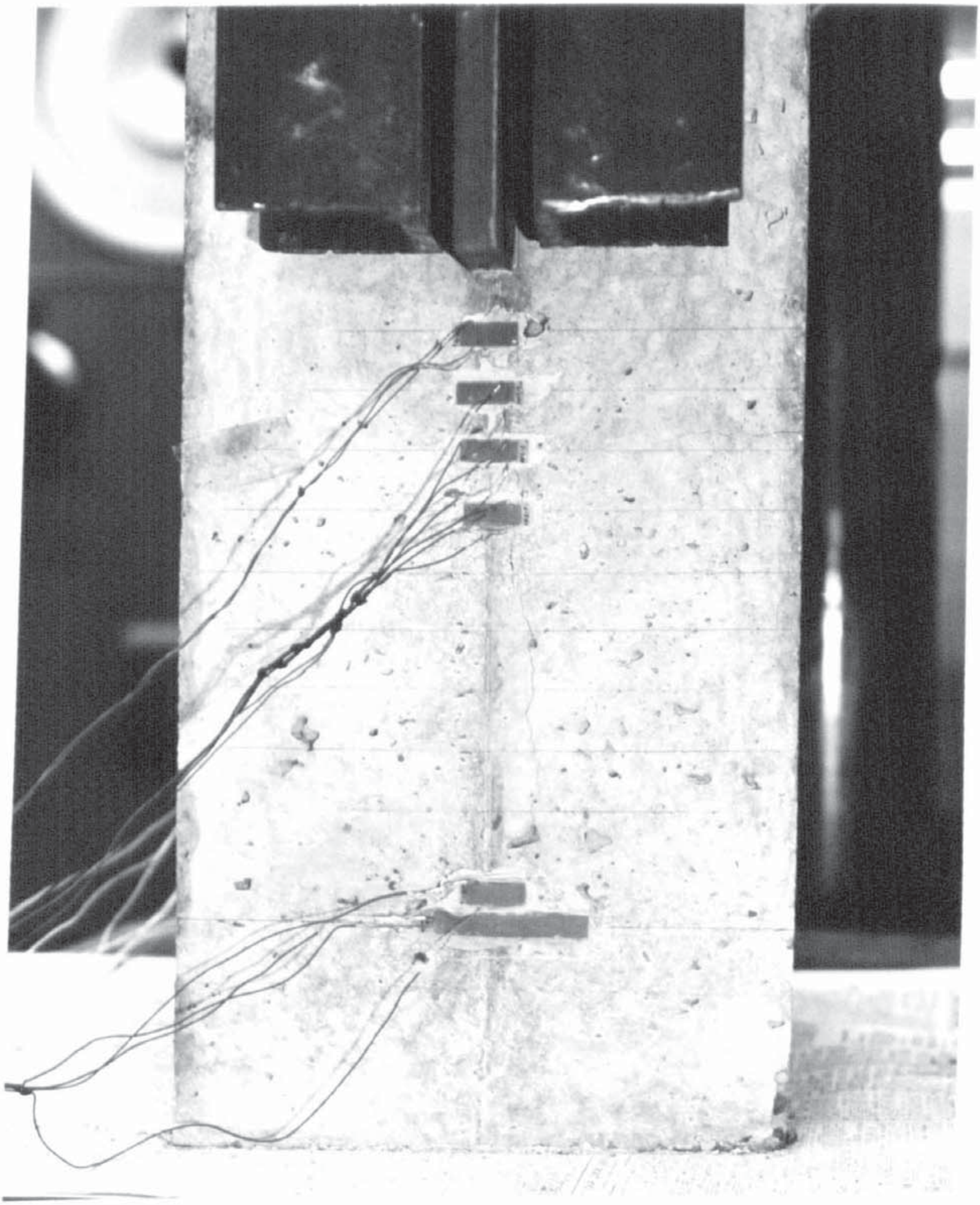


Plate 5

specimens were cast, cured under the same conditions and tested in accordance with the British Standard (31) for the testing of concrete. The cube strength was calculated from the average result of crushing six 4in. cubes.

The tensile strength was calculated from the average splitting load of three 12in. long by 6in. diameter cylinders. The cylinders were tested in the standard manner. Each was placed with the axis set horizontally between the platen of a compression testing machine and loaded across the diameter. A slat of plywood approximately 1/8in. thick, 3/8in. wide and 12in. long was placed between each platen and the surface of the concrete, parallel to the axis of the cylinder to ensure an even distribution of load. The cylinder was then slowly loaded to failure which occurred by sudden splitting on the vertical diameter when the cylinder fell into two halves. The tensile stress was calculated from the average splitting load P_b in pounds from the formula :

$$f_t = \frac{2P_b}{\pi D_1 D_2}$$

Where f_t is the tensile stress in lb/sq.in., D_1 is the diameter and D_2 the length in inches.

The value of the modulus of elasticity was obtained from the result of a test in which a 12 x 6in. cylinder was tested with its axis set vertically in the compression testing machine in the standard manner. As the cylinder required capping the mould in which the cylinder was cast was filled to within $\frac{1}{2}$in. of the top and after the concrete had started to set the remaining space was made up with a rich cement mortar and carefully levelled off. In this way it was possible to ensure that the concrete was loaded on two reasonably parallel and true faces. Even so, a disk of thin cardboard of the same diameter as the cylinder was placed between each platen and the face of the concrete. Before loading two type 7A electrical resistance strain gauges were fixed to the surface at opposite ends of a diameter and parallel to the axis of the cylinder at mid-height. The standard loading procedure was carried out and, after it had been ascertained that the difference in the readings from each gauge were within the prescribed limits, the readings at each value of the applied load were averaged and the modulus of elasticity for the concrete in uniform compression was calculated.

CHAPTER 6

CHAPTER 6

EXPERIMENTAL ANALYSIS OF STRAINS

6.1 INTRODUCTION

Although it was considered that the mode of failure and the loads which caused the failure of test columns were likely to be among the most important factors which were required to establish a rational design method, it was thought necessary to obtain as complete an understanding as possible of the behaviour of columns loaded through a vertical cast in plate before failure was reached. If the mechanism by which load was transferred to the column could be properly understood, safe and economical limits could be set on the value of an estimated load factor.

Whilst strain gauge measurements were made on columns, in order that as much information as possible could be obtained it was decided to carry out an extensive experimental study of the behaviour of cubes loaded in a similar manner to the columns. Although the theoretical work of Chapter 4 showed that the analysis of a cube and a prism whose length was longer than its width were very similar, it was to be expected that the differences that there were, might prove to be significant in practice. Nevertheless, it was considered that the general conclusions which could be drawn from the results of the tests on cubes would be of some considerable interest and these are dealt with in the first part of this chapter. A comparison of the results from the tests on cubes and columns is given in the second part.

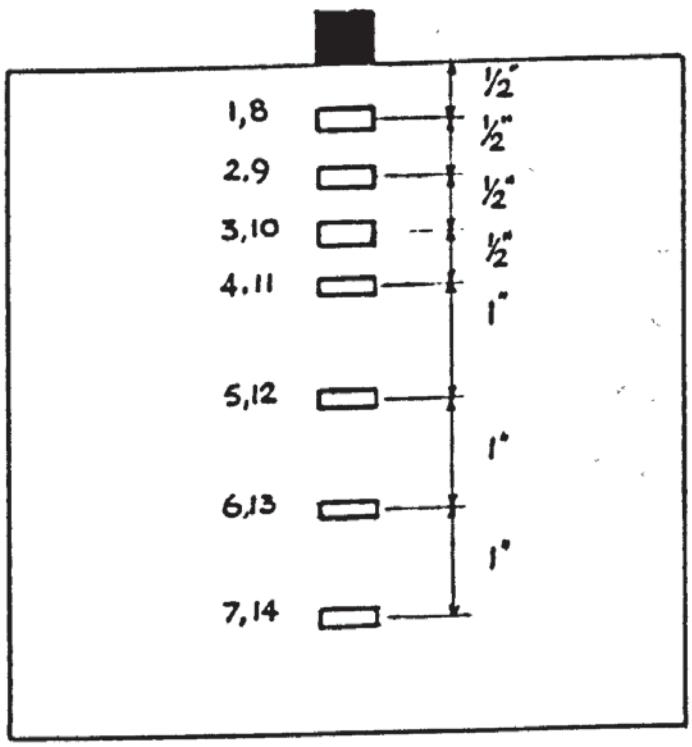
6.2 TESTS ON CUBES

Each 6in. cube was subjected to a load on its upper face applied through a rectangular strip of bright, machined steel, the length being 6in. but the width of the strip was different in different tests. There were a number of advantages in this method of testing. A suitable quantity of cubes from one batch of concrete and of uniform quality could be produced quickly and easily from standard steel moulds. The cubes were easier to handle than columns and could be produced to a high standard of accuracy and surface finish, Moreover, the effects of the plate being cast into the concrete in the case of columns could be disregarded so that the number of unknown factors which would have to be left out of account if any realistic comparison was to be made between a theoretical and an experimental strain analysis was thus considerably reduced.

6.3 TESTING PROCEDURE

6.3.1 Gauging

Electrical resistance gauges were fixed to the vertical centre line of the two vertical faces which were at right angles to the longitudinal axis of the loading strip. The disposition of the gauges is shown in Fig. 6.1, seven being



LOCATIONS OF GAUGES ON 6 IN. CUBES
Figure 6.1

fixed to each face.

In order to measure stress at a point on the surface of a material whose value of Poisson's ratio is not equal to zero it is normally required that three strain measurements are taken in three different directions, unless of course the principal directions are known in which case only two measurements are made, one in each of the two principal directions. In the case of the vertical centre line of the faces of the cubes on which strains were in fact measured, it was known that the principal directions were vertical and horizontal because of the symmetry of the test arrangement. Consequently only two gauges set at right angles at each point would have been required in order that the stress at the point might be determined. However, in the tests that were carried out for these investigations only transverse (horizontal) strains were measured. There were two reasons for this procedure. Firstly, it was felt that because the degree of accuracy of the measurements could not be readily assessed owing to the fact that the elastic properties of the material changed as load was increased, the use of two gauge measurements in a calculation would considerably reduce the overall accuracy of the calculated stress. Secondly, owing to the fact that the theoretical analysis had indicated the peaked nature of the transverse stress distribution, it was desireable to space gauges as closely as possible in the region in which peaking was expected. Owing to the small scale of the test specimens and of the small width of loading ($\frac{1}{2}$in. in many of the tests on the cubes) it was not possible to fix more than one gauge horizontally at any one measuring point if the results were to be at all meaningful.

6.3.2 Loading

The reading of each gauge was taken before loading was commenced and then at each increment of load until the stage was reached when it became impossible to read one or more of the gauges because straining was continuing while the load was being held constant. No fast scanning strain recording apparatus was available and consequently, as it took approximately two minutes to read the fourteen gauges which were normally used in each test it was decided to stop recording at the stage described above. The strain recorder was then disconnected from the extension box and loading was continued until the cube failed by splitting. The average of the readings from corresponding gauges on opposite faces were taken to plot the load v. strain graphs which are shown in this chapter.

6.3.3 Effect of Unsymmetrical Loading

It is considered to be of interest to examine the method by which the cubes were loaded in view of the importance which has been attached to the general conclusions which have been drawn from the results of the tests reported here and in Chapter 12.

It was found that however carefully loading was applied, the readings from corresponding gauges on opposite faces were never exactly the same and were often considerably different in magnitude. It was therefore necessary to take the mean reading of corresponding gauges. This disagreement, caused by the fact that straining on opposite faces of the cube was taking place at different rates under the action of the load, may have been due both to non-uniformity of the concrete itself and also due to certain characteristics of the machine which was used for testing.

It is possible that if the cubes had been loaded so that the axis of the loading piece had been parallel to the direction in which the cubes had been cast, the effect of variation of the density of the concrete might have been significant. This could arise from the segregation of the mix which can occur when fresh concrete is vibrated, the heavier particles tending to settle in the softer cement paste. However, it was ensured that the cubes were loaded at right-angles to the direction in which they were cast so it is reasonably safe to assume that density in the loaded plane was sensibly constant.

The conclusions reached by Sigvaldason[32] from his work on the influence of testing machine characteristics have some relevance to this discussion although his work was concerned with the testing of cubes and cylinders in direct compression. In the usual type of testing machine and the one used for these investigations the lower plattern is fixed to the ram and is thus effectively fixed but the upper platen is located in a ball seating and can tilt in any direction. However its ability to do so under load depends on the roughness and size of the spherical surfaces, the type and degree of lubrication of the surfaces and the magnitude of the load. If good lubrication is maintained, the upper surface of the specimen can be regarded as pinned in which case the specimen will have, effectively, the upper end pinned and the lower end fixed. If there is no lubrication at the spherical interface, the upper plattern is effectively fixed after loading is commenced and the specimen is deformed uniformly as the centroid of the action of the machine is always colinear with the centroid of resistance of the specimen provided the loaded surfaces are both normal to the axis of the specimen. (Of course if the loaded surfaces of the specimen were not normal to the machine's centre line, then the specimen would not be loaded uniformly and the values of the measured strains on different vertical faces would be different). On the other hand, if, owing to the presence of a suitable lubricant, the upper platen could move freely under load, and unless the specimen was completely isotropic and centred accurately in the machine, the specimen would have a tendency to bend. In reality, when no special precautions concerning lubrication are taken, the situation is probably intermediate between the two conditions described above. At low loads, movement can take place and as the load is increased the upper platen becomes fixed as the normal reaction at the interface of the spherical surfaces and hence the frictional resistance is increased. Thus once tilting of the upper platen has taken place it becomes fixed as load is increased and the difference between strain rates at corresponding places on opposite vertical faces of a cuboidal specimen becomes constant.

It should be noted that the results from Sigvaldason's tests show that the measured value of the ultimate compressive strength of cubes loaded under the one end pinned condition were extremely sensitive to misalignment of the specimen in the machine and it is this which is thought to be a major factor contributing to the considerable differences of strains and to the variability of the failure loads which occurred in the tests performed for these investigations, both on cubes and columns. Moreover in the case of the tests that were performed on cubes considered here, when cubes were loaded through a $\frac{1}{2}$in. square loading piece the alignment was probably even more critical than in those reported by Sigvaldason because of the effect of concentration of the load. It was in fact no easy matter in practice to ensure that the geometrical centre lines of the machine, the loading piece and the specimen were colinear in spite of the considerable care that was taken. In order to ensure that full contact was made between the plattern and the loading piece at the start of each test the plattern was left unlocked.

6.4 RESULTS OF TESTS ON CUBES

It can be seen from Fig. 6.2 that the load v.strain characteristics for the upper half of the cube under the action of the load applied through a $\frac{1}{2}$in. wide centrally placed loading piece (b/D = 1/12) on its upper surface are linear only over the very early part of the loading range but as the load was increased the value of the strain recorded on the upper gauges increased at a greater rate than the applied load. For the lowest gauges however, the characteristics are linear up to a load of at least 27 ton (in this particular test there was a fault in gauge number 4 so the characteristic for the mean of the readings for numbers 4 and 11 were omitted).

From Fig. 6.3, which shows the strain profiles on the vertical centre line for different values of load, it can be seen that the load v. strain characteristics which exhibited

LOAD v STRAIN GRAPHS FOR CUBE

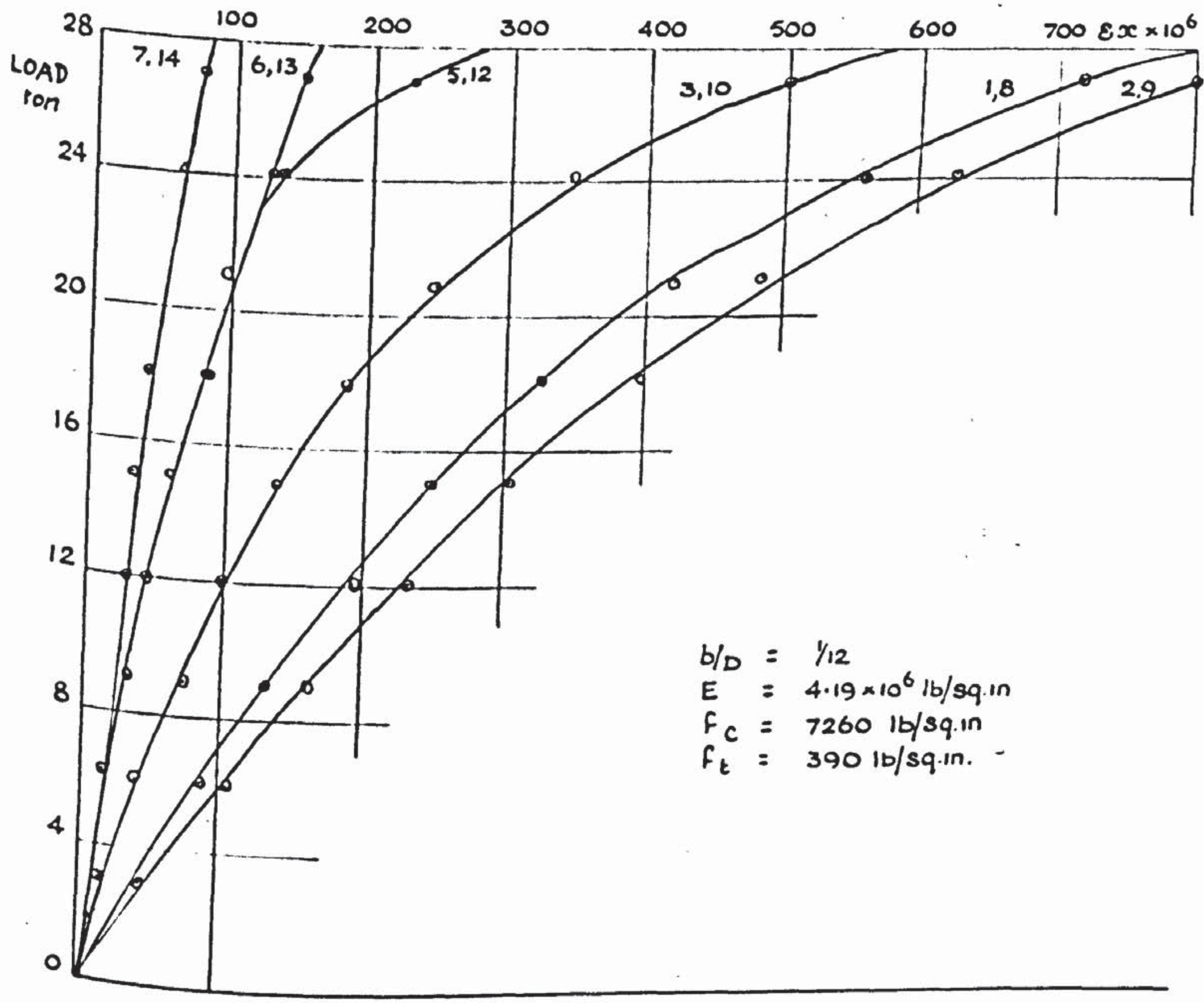


Figure 6.2

STRAIN PROFILES AT DIFFERENT LOADS FOR CUBE OF FIGURE. 6.2.

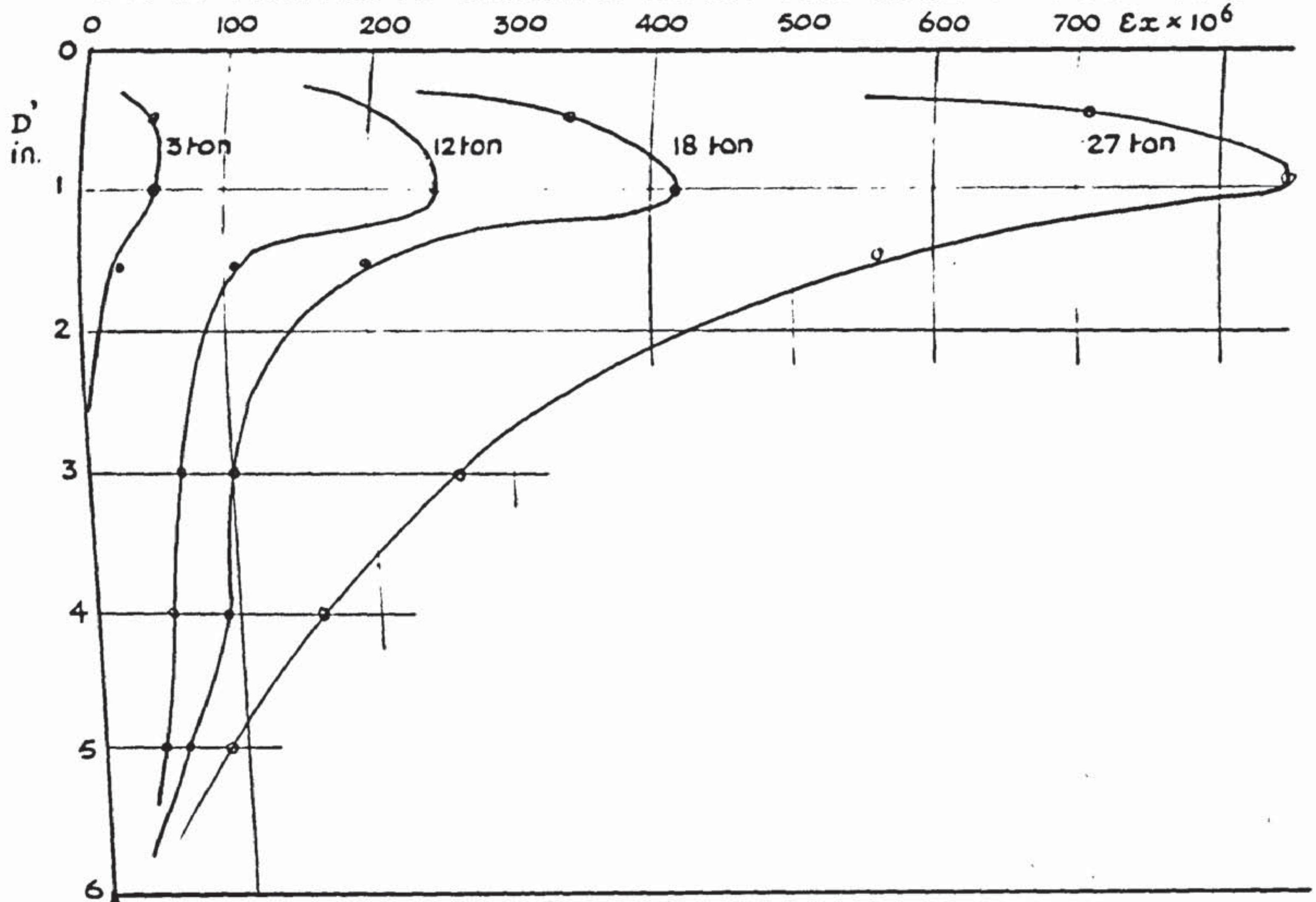


Figure 6.3

greatest cervature were those for the gauges which were in the region where there was peaking of the profile. Load v.strain characteristics for tests on cubes loaded under plates for values of b/D of 1/9, 1/4 and 1/2 are given in Figs.6.5, 6.7 and 6.8 respectively and it can be seen that the trend noted above is repeated for all the values of b/D. Representative profiles for each of the four values of b/D are given in Fig. 6.6 and 6.9.

Fig. 6.3 shows strain profiles drawn for the test for which b/D = 1/12 for different values of load and it is apparent that as the load was increased the shape of the profiles changed somewhat in relation to each other. This can be seen more clearly in Fig. 6.4. Here the values of the peak strain at the different load values have been made equal to the value of the peak strain at 3 ton and the values of the other strains at the different loads have been reduced in proportion. Thus each value of strain has been multiplied by the peak strain at 3 ton and divided by the peak strain at the value of load for which the particular value of strain was obtained. It can be seen from the figure that as the load was increased some redistribution of strain from the more highly stressed regions to the adjacent regions was taking place.

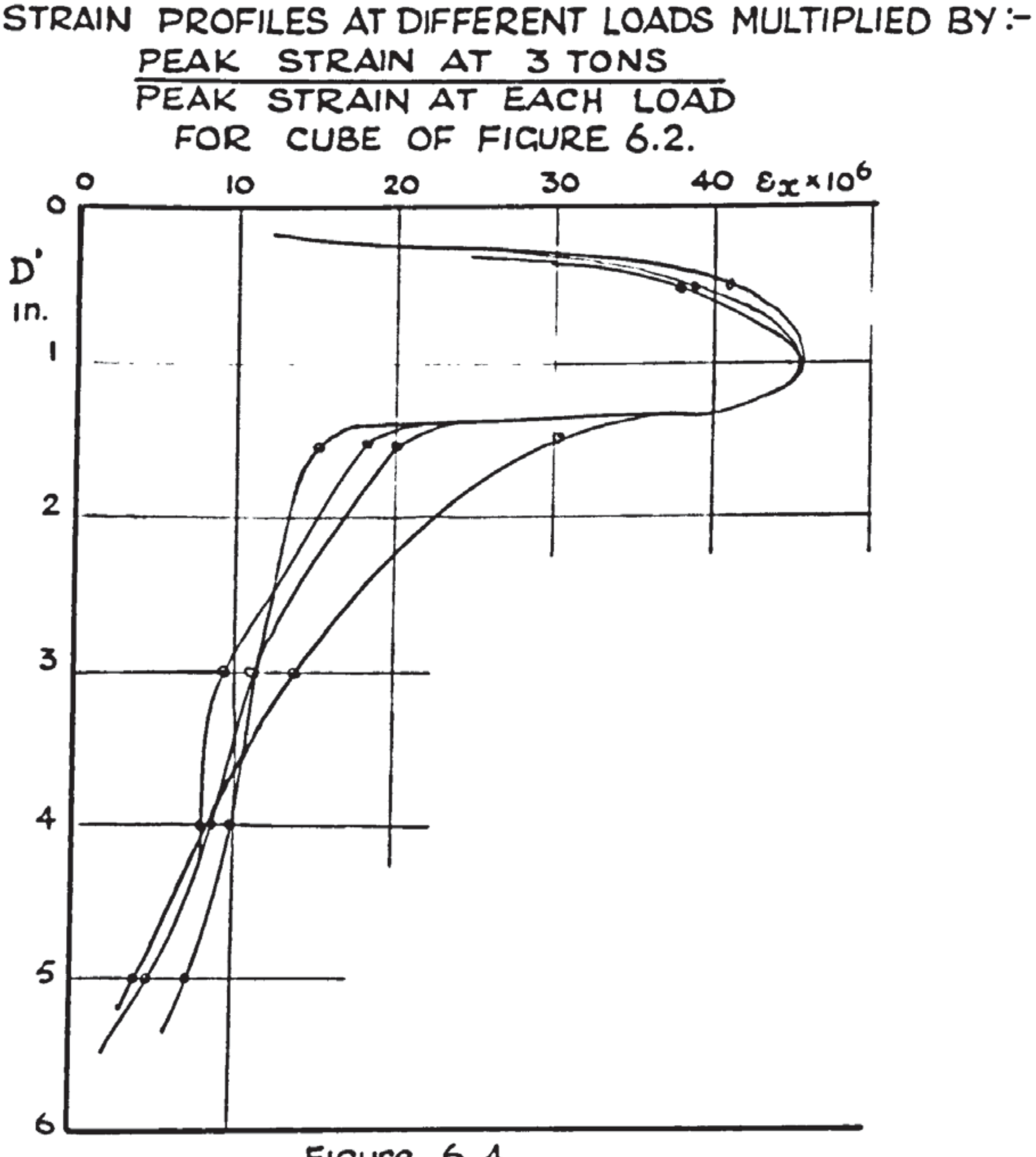

STRAIN PROFILES AT DIFFERENT LOADS MULTIPLIED BY :-
PEAK STRAIN AT 3 TONS
PEAK STRAIN AT EACH LOAD
FOR CUBE OF FIGURE 6.2.
0
10
20
30
40
εx × 10^6
D'
in.
0
1
2
3
4
5
6

Figure 6.4

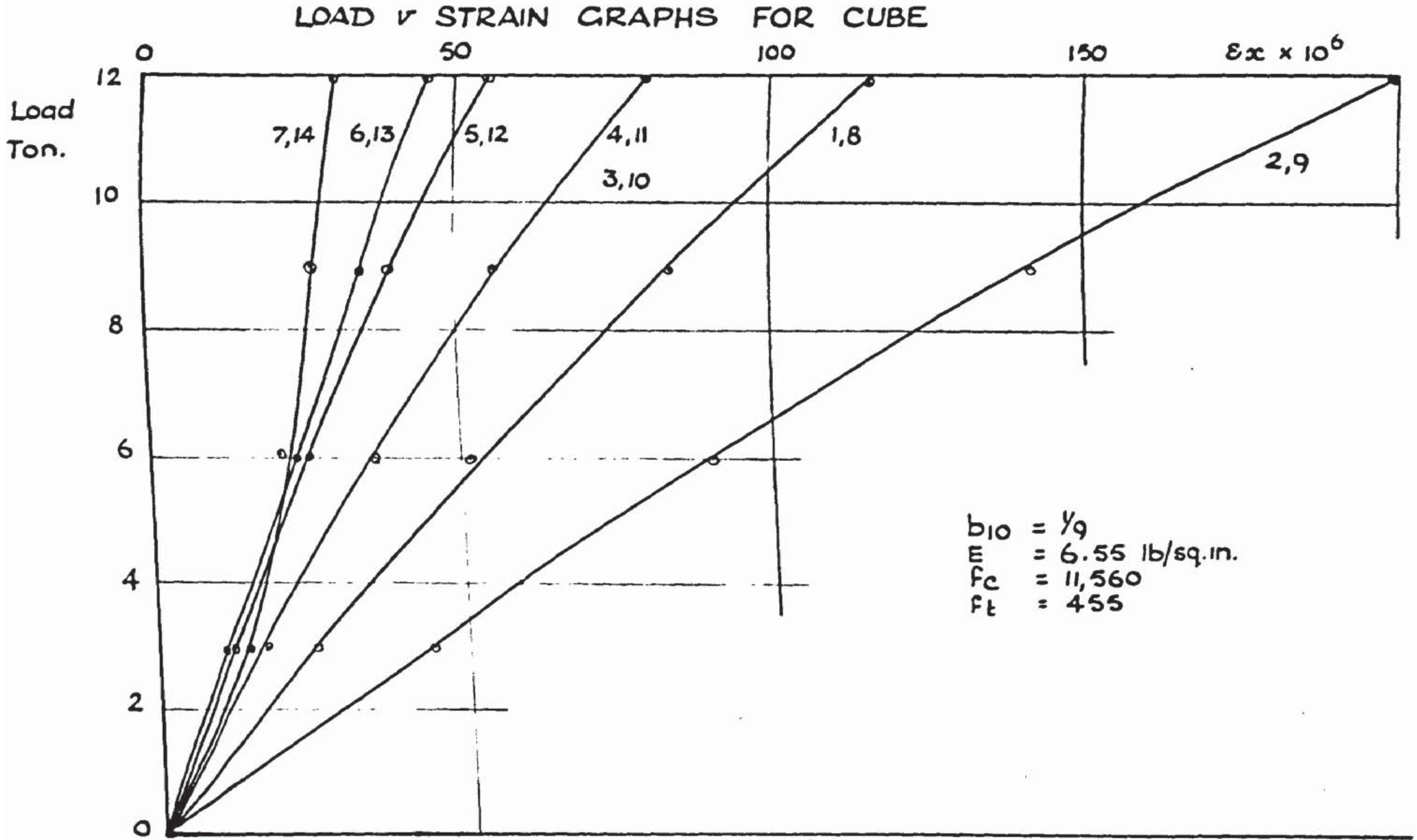

LOAD v STRAIN GRAPHS FOR CUBE
0
50
100
150
εx × 10^6
12
10
8
6
4
2
0
Load
Ton.
7,14
6,13
5,12
4,11
3,10
1,8
2,9
b10 = 1/9
E = 6.55 lb/sq.in.
fc = 11,560
ft = 455

Figure 6.5

COMPARISON OF THEORETICAL AND EXPERIMENTAL ~~STRESS~~ STRAIN PROFILES

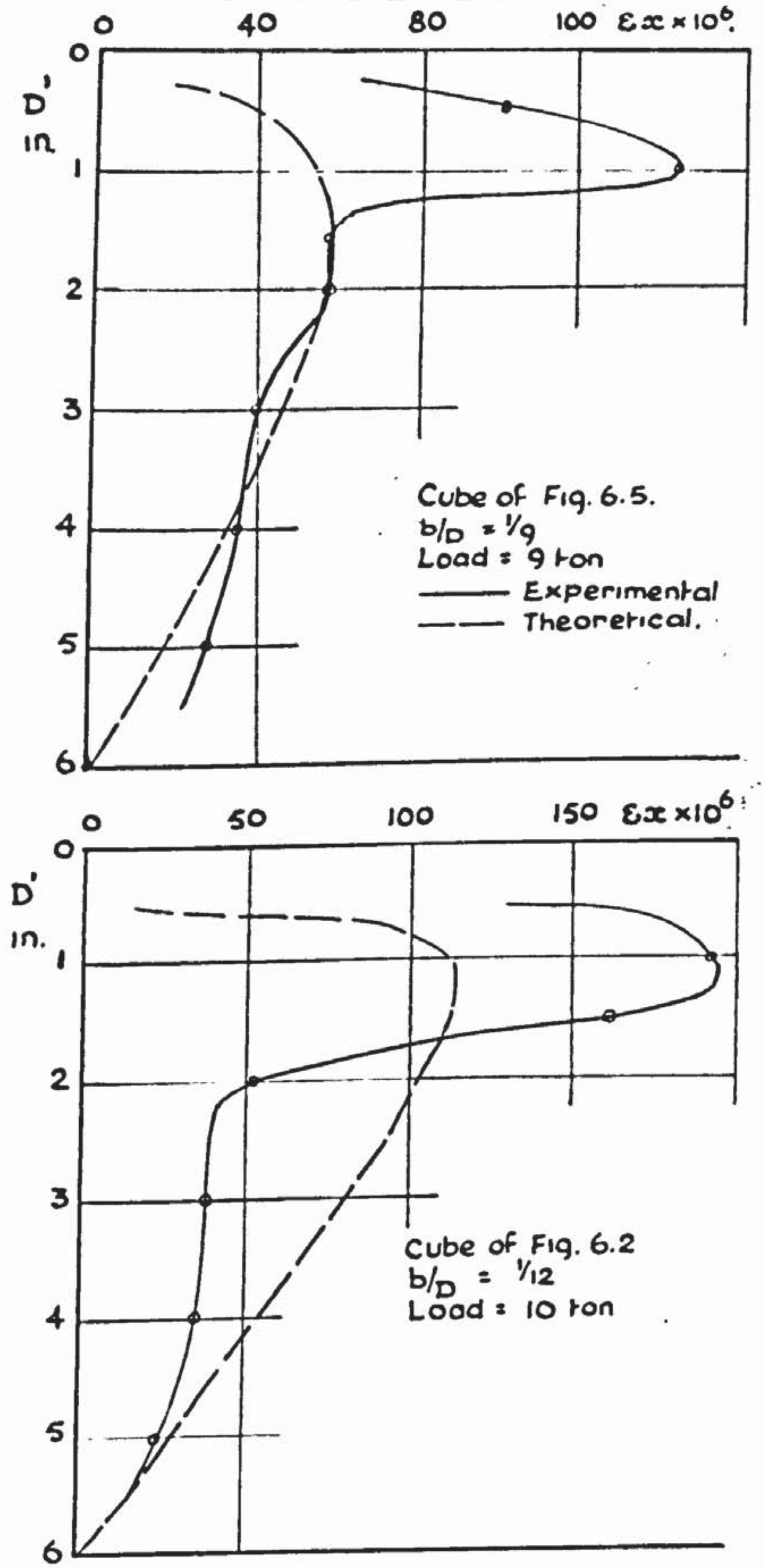


Figure 6.6

LOAD v STRAIN GRAPHS FOR CUBE.

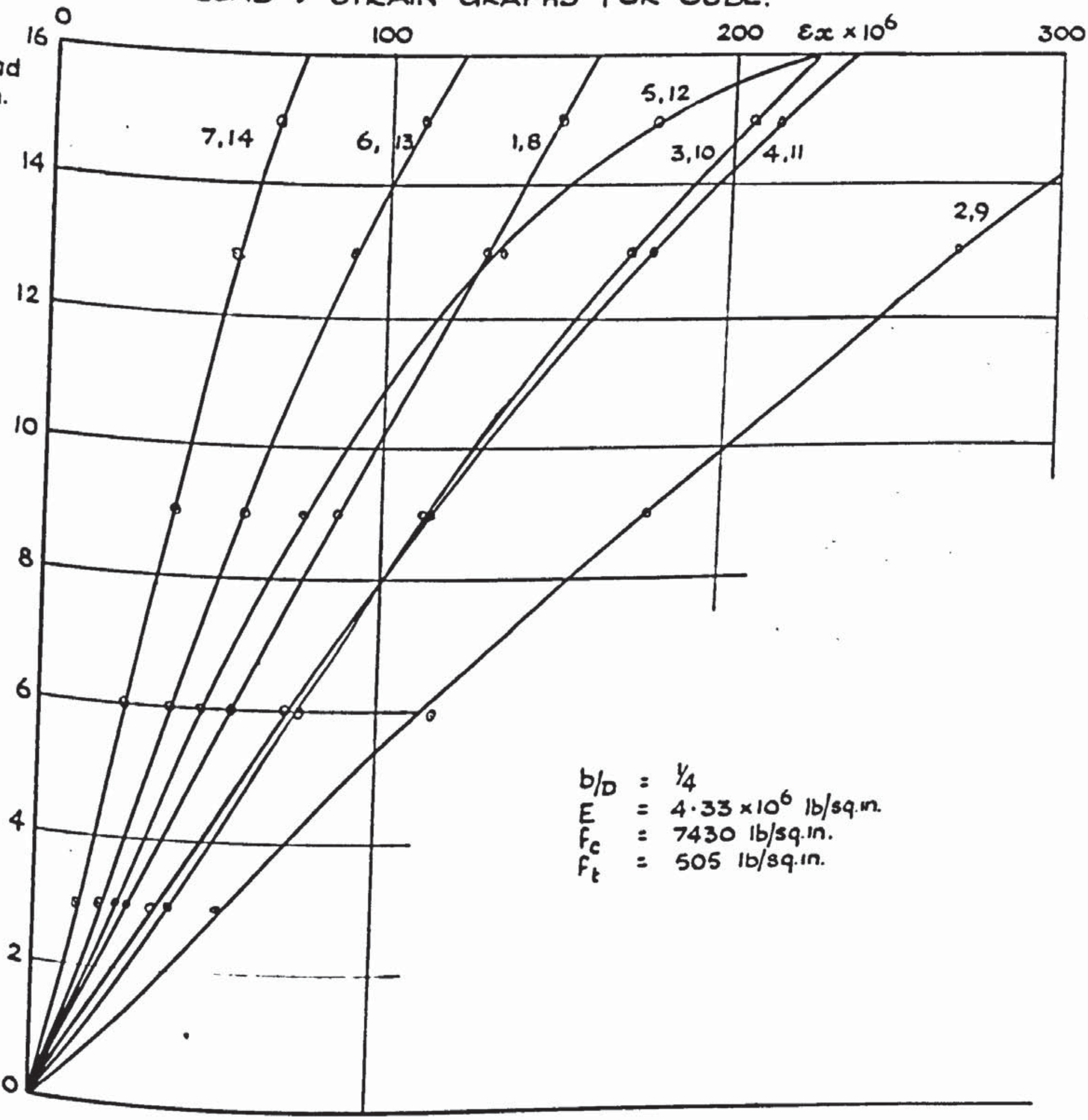


Figure 6.7

LOAD V STRAIN GRAPHS FOR CUBE

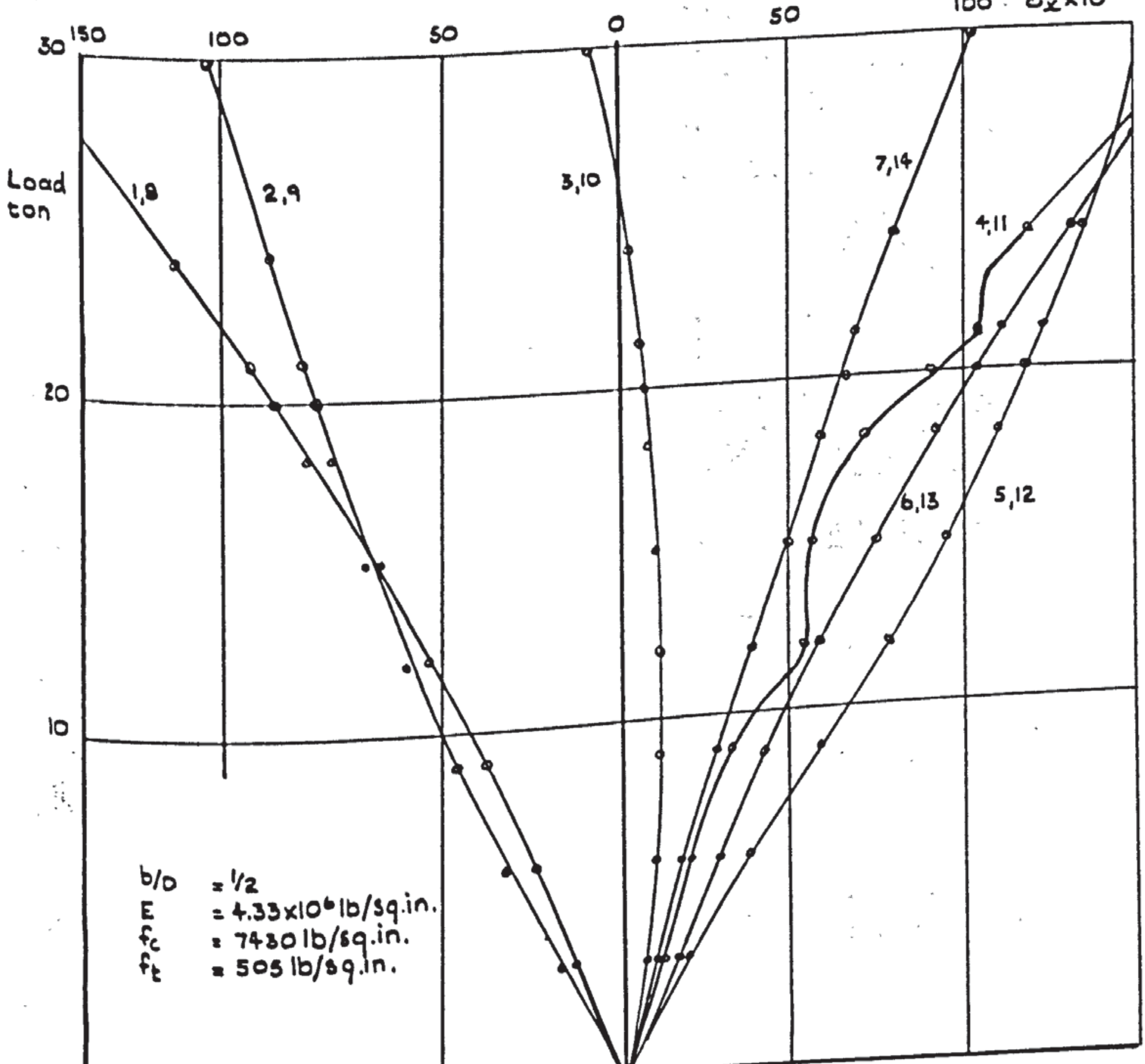
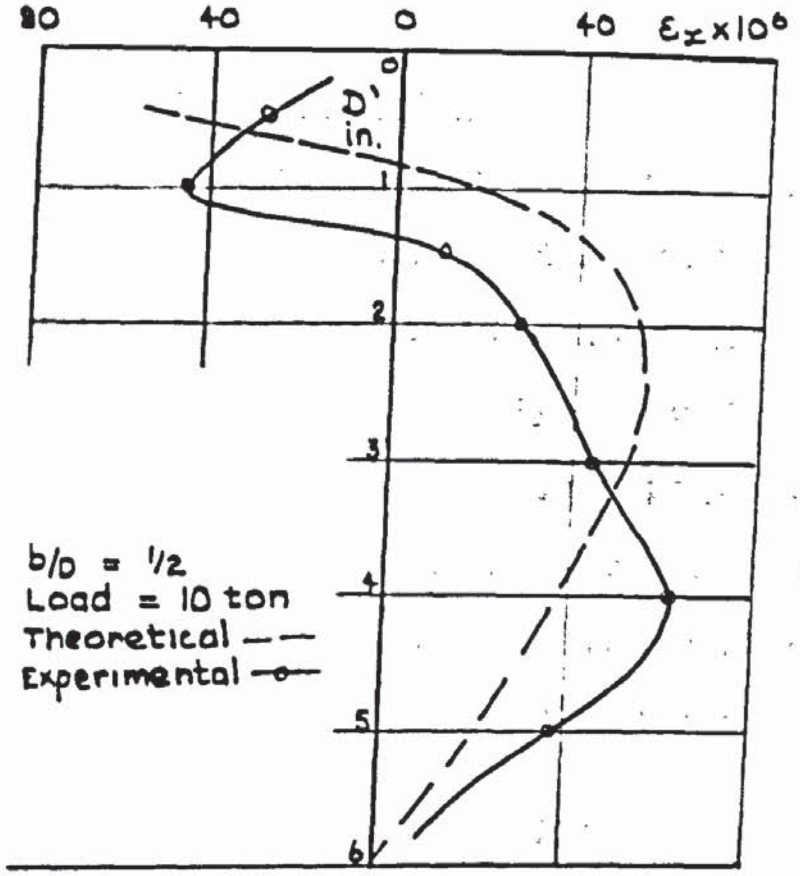


Figure 6.8

COMPARISON OF THEORETICAL AND EXPERIMENTAL STRAIN PROFILES



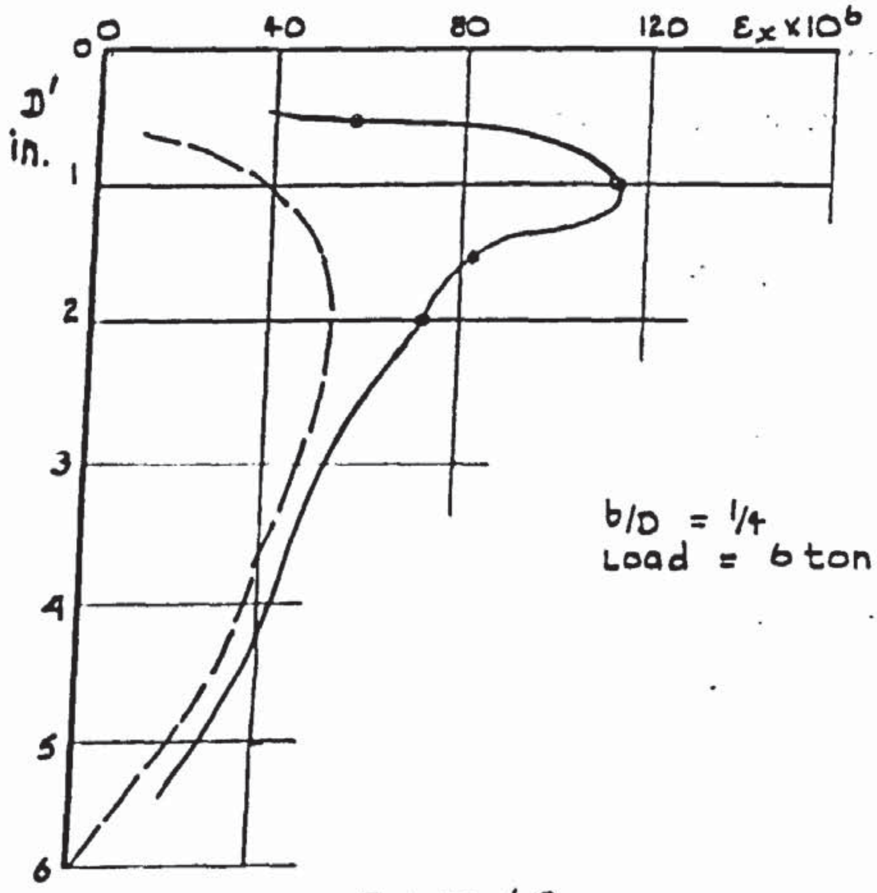


Figure 6.9

6.5 DISCUSSION

6.5.1 Magnitude of Experimental Strains

It was of some considerable interest to consider the results quantitatively in view of the importance that has been placed on the limiting strain (the strain when cracking has been deemed to have occurred) in concrete by many workers in the field over the past two decades. It is not proposed to give here a complete resume of all the work that has been carried out on the subject but it is of some interest to consider how the results of some previous work is relevant to those presented here.

It has been said by many, e.g. Neville (27) that the strength of concrete is determined not so much by the limiting stress it can sustain as by the more fundamental concept of a limiting (tensile) strain which Neville put at between 100 and 200 x 10^{-6} no matter how the concrete is tested. Of course the value of the limiting strain is not a constant and is to some extent dependant on the intrinsic strength of the elements which comprise the complete matrix, on the volume and the shape of the coarse aggregate in the mix and other properties. Many of the other workers in the field have produced results which tend to confirm those of Neville, e.g. Todd (33) who tested beams in flexure and Sturman, Shah and Winter (34) who reported the results of some extensive research into the straining of concrete. They tested short specimens in compression and tension under eccentric loading using electrical resistance strain gauges 5/8in. (16 mm) long and found that first cracking occurred at a strain of approximately 100×10^{-6} for concrete of 3790 lb/sq.in. (6in. dia. by 12in. long cylinders). Kaplan[35] tested cylinders 6in.dia. by 12in. long in the manner of the indirect tensile test for concrete (described in Chapter 5) and found that the first cracking strain was of the order of 150×10^{-6} using 1in. (25.4 mm) gauges. However in some tests reported by Kajfasz and Rowe[36] on the composite action of prestressed floor beams which were finished with a slab of concrete cast in situ it was found that the in situ concrete was capable of being strained in tension to approximately 500×10^{-6} without showing signs of disintegration and it was concluded that it was the restraining action of the prestressed concrete which was responsible for the occurrence of these relatively high strain values.

A similar argument can be used in the case of the straining in the tests on cubes described here. It is considered that the presence of the relatively high strains can at least partly be accounted for by the fact that the friction between the base of the cube and the steel of the lower plattern of the machine does not allow the cube to expand freely and consequently straining can take place without disintegration of the cube. It must be added however that although there did not appear to be any visible cracking until a load of about 90 per cent of the ultimate, this does not mean that micro-cracking did not occur. Sturman, Shah and Winter[34] showed experimentally that micro-cracking caused non-linearity of stress v. strain curves and that these could occur at between 70 and 90 per cent of the ultimate load. Hansen[37] said that these could occur at loads as low as 60 per cent of the ultimate load. It is felt however that the shape of the transverse strain distribution itself is the main cause of the high recorded strain values due to the fact that it is characterised by a peak which is adjacent to less highly strained regions and this inhibits cracking on the macro-scale. It can be added that Zielinski and Rowe[38] also found very large tensile strain magnitudes when they tested short prisms under concentrated loads.

6.5.2 Comparison of Theoretical and Experimental Strains

The presence of the micro-cracks referred to in the preceding paragraph are thought to account for considerable differences in magnitude between the experimental and the theoretical strain profiles (the latter shown by the discontinuous lines) shown in Figs. 6.6 and 6.9. The theoretical strains were obtained from the finite element analysis of Chapter 3. The fixed base condition was used for the theoretical analysis and the experimental results - tend to confirm the validity of this assumption. In Fig.6.9 the experimental compressive strain close to the upper loaded surface of the cube for which b/D = 1/2 appears to be reduced which the theoretical strains are shown to increase as the upper surface is approached. This was thought to be due to the restraining action of the loading plate which seems to suggest that there were frictional stresses between it and the surface of the concrete. These were not accounted for in this analysis. However it is interesting to note that in this example the peak value of the experimental and theoretical tensile strains are in close agreement but are displaced in position.

6.6 RESULTS OF TESTS ON COLUMNS

The arrangement of the gauges which was used on the majority of the columns is shown in Fig. 6.10 and the arrangement used for the tests, the results of which are given in Fig. 6.19, is shown in Fig. 6.11. All the columns were loaded symmetrically.

Fig. 6.11 shows the load v. strain characteristics for a 10in. square column containing a standard 5/8in. thick plate 12in. deep for which transverse gauge measurements were taken on the centre line at a distance of 12in. below the level of the lower edge of the plate. Fig. 6.12 shows the strain

STANDARD STRAIN GAUGE POSITIONS OF COLUMNS

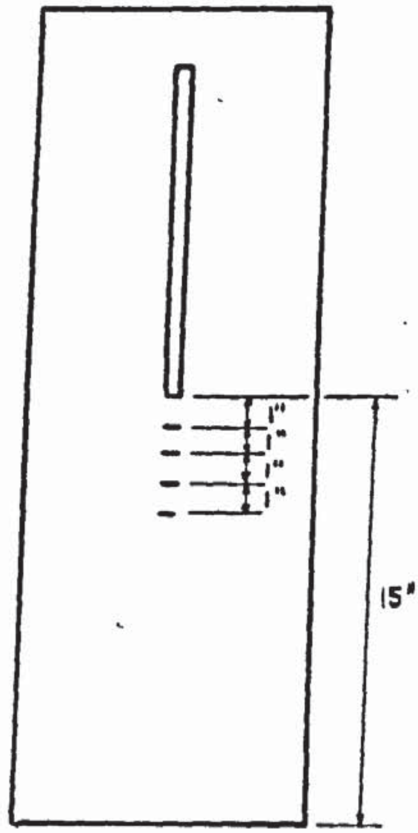


Figure 6.10

LOAD v STRAIN GRAPHS OF 10in. SQUARE COLUMNS

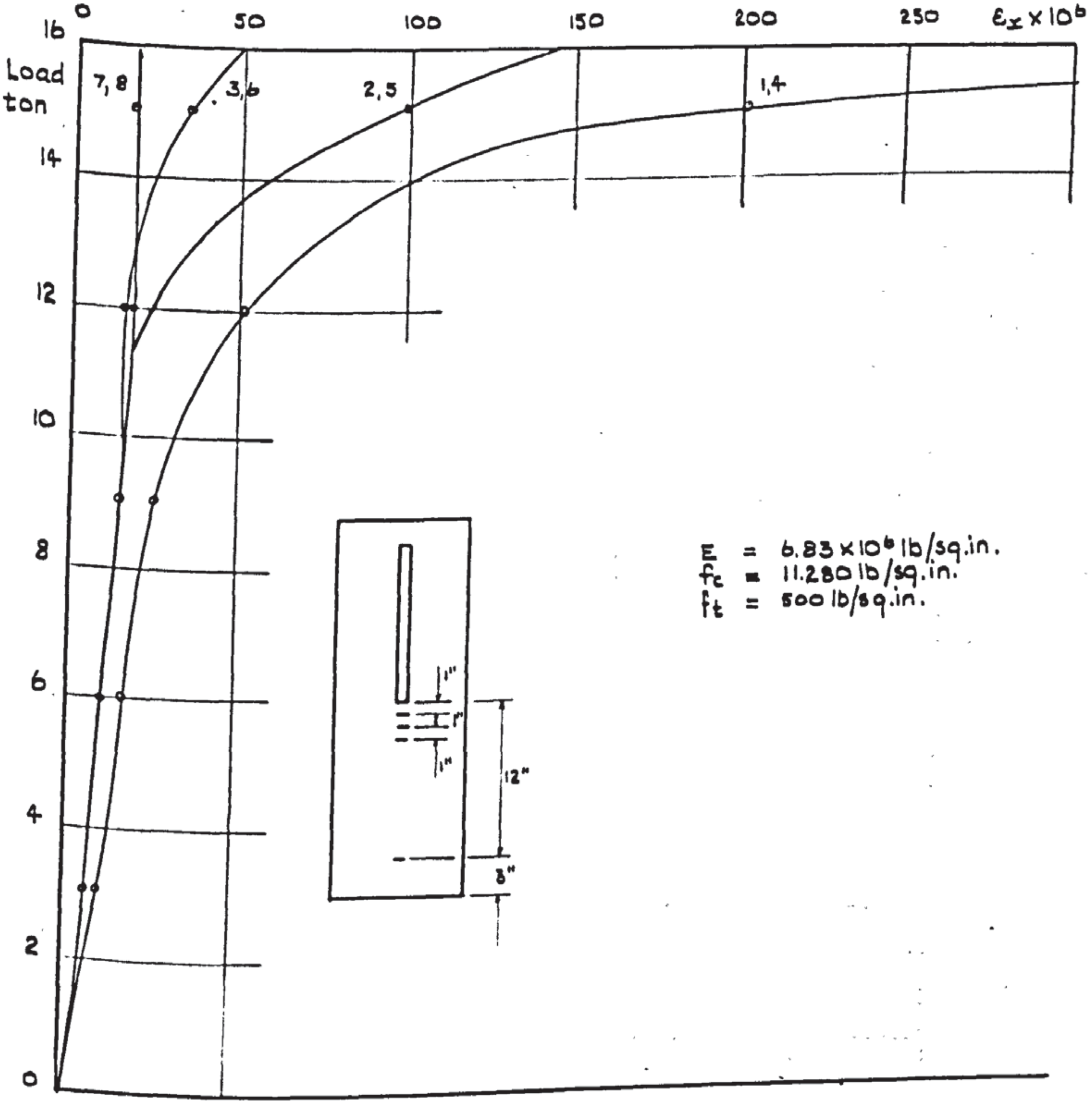


Figure 6.11

EXPERIMENTAL STRAIN PROFILE ON CENTRE LINE OF COLUMN OF FIGURE 6.11.

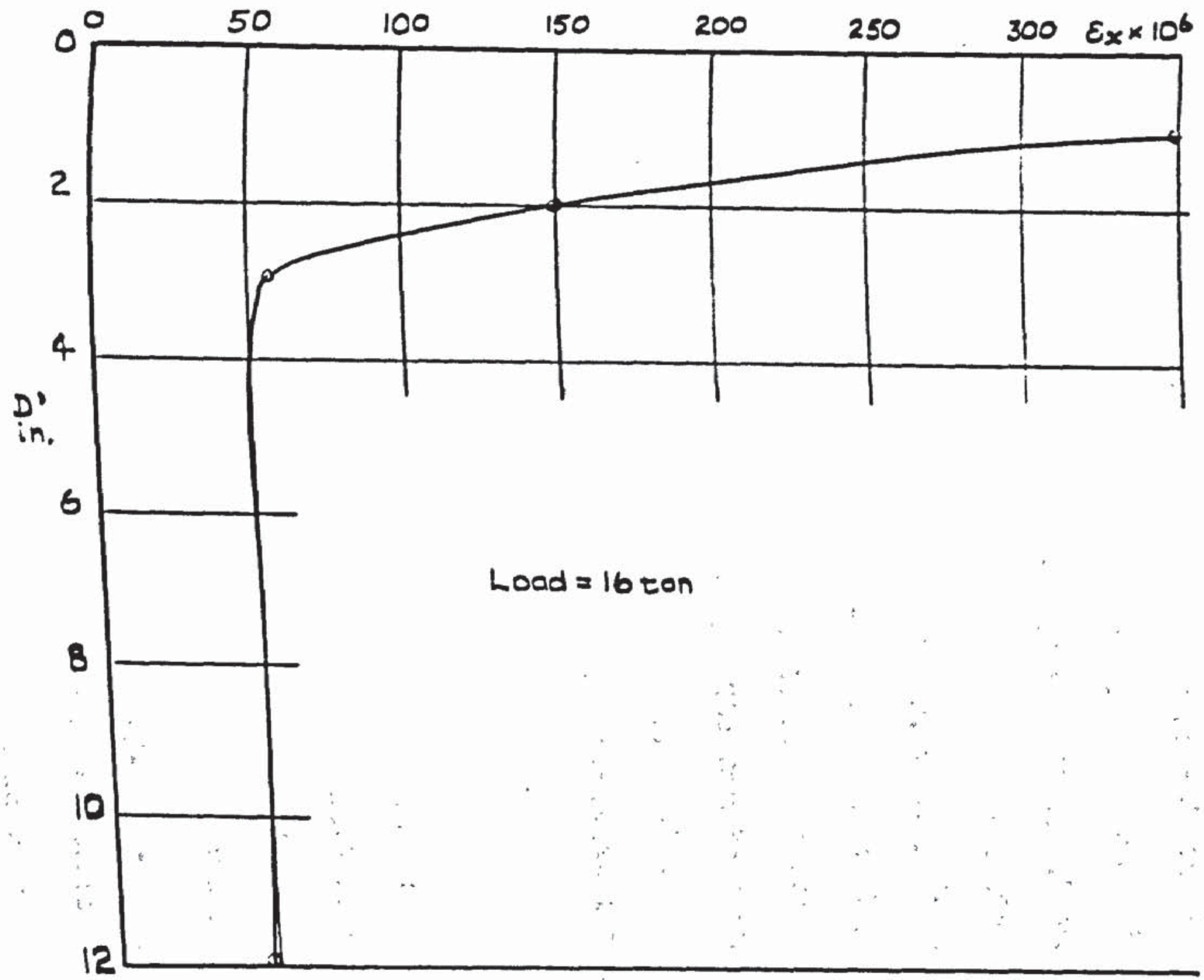


Figure 6.12

profile which was produced. The spacing of the gauges was such that the whole of the peak was not picked up but the extent of the transverse tensile strain is shown by the inclusion of the readings for the gauges at a distance of 12in. below the plate. However the general picture is similar to that of the cube for which b/D = 1/12 (see Figs. 6.2 and 6.3).

Fig. 6.13 shows the load v. strain characteristics for a 6in. square column loaded through a 1/2in. thick plate and a typical profile has been drawn from these curves in Fig. 6.14. For this example the theoretical profile has been drawn for the 9 x 6 x 6in. prism of Chapter 4. The free base condition has been used for the analysis and it is clear that the experimental strains are smaller than the theoretical ones which is in contrast to the results from the tests on cubes described earlier.

Figs. 6.15 and 6.16 show the load v. strain curves for the tests in which columns containing a 1in. thick plate were tested. The depth of the plates was 12in. and the standard method of loading using the yoke (see Chapter 5) was employed. The gauge positions were those shown in Fig. 6.10. Typical profiles can be seen in Fig. 6.17 for these two tests and it is clear that there is no agreement between theory and experiment, the magnitudes of the experimental strains being far larger than those of the theoretical strains. These

LOAD v STRAIN GRAPHS FOR 6in. SQUARE COLUMN WITH 6in. PLATE

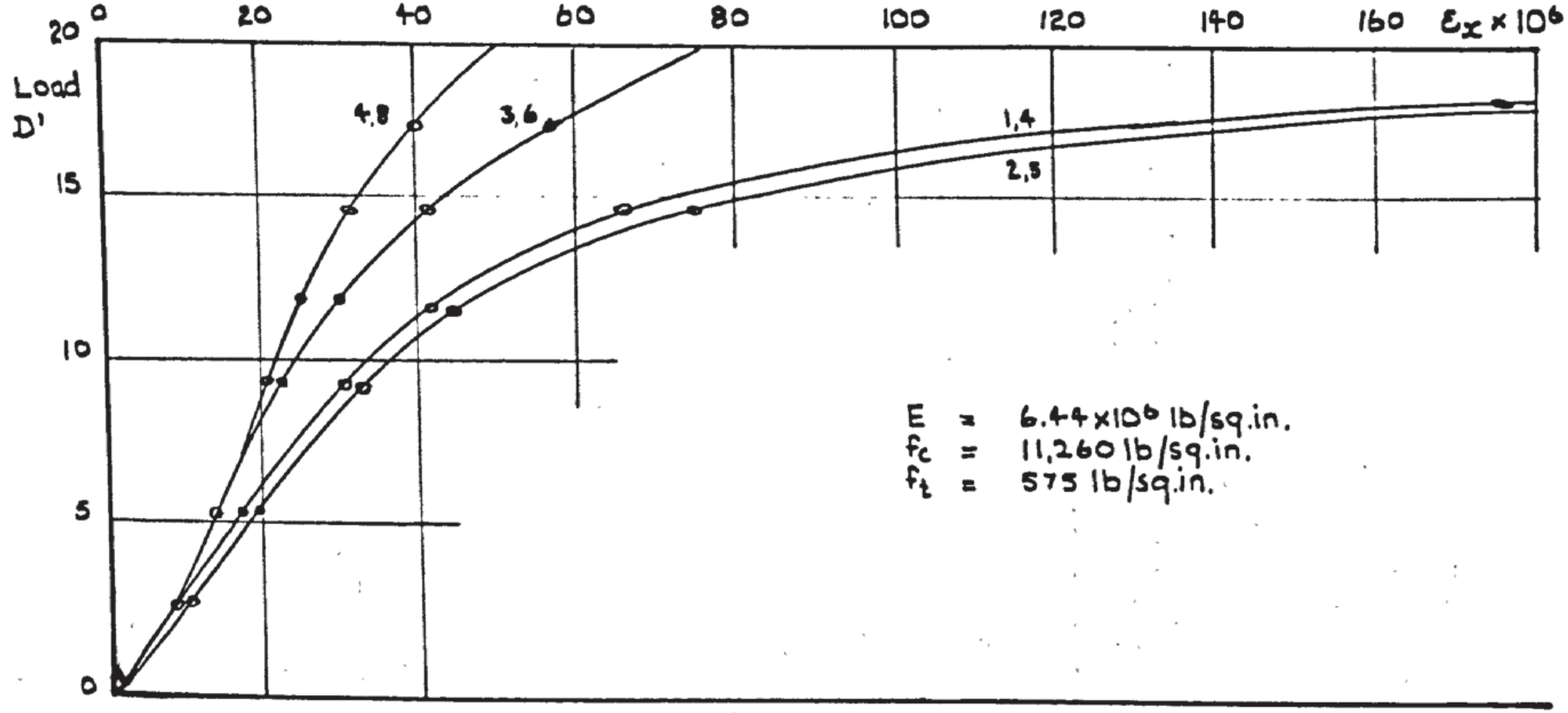


Figure 6.13

COMPARISON OF EXPERIMENTAL AND THEORETICAL STRAIN PROFILES FOR THE 6in. SQUARE COLUMN OF FIGURE 6.13

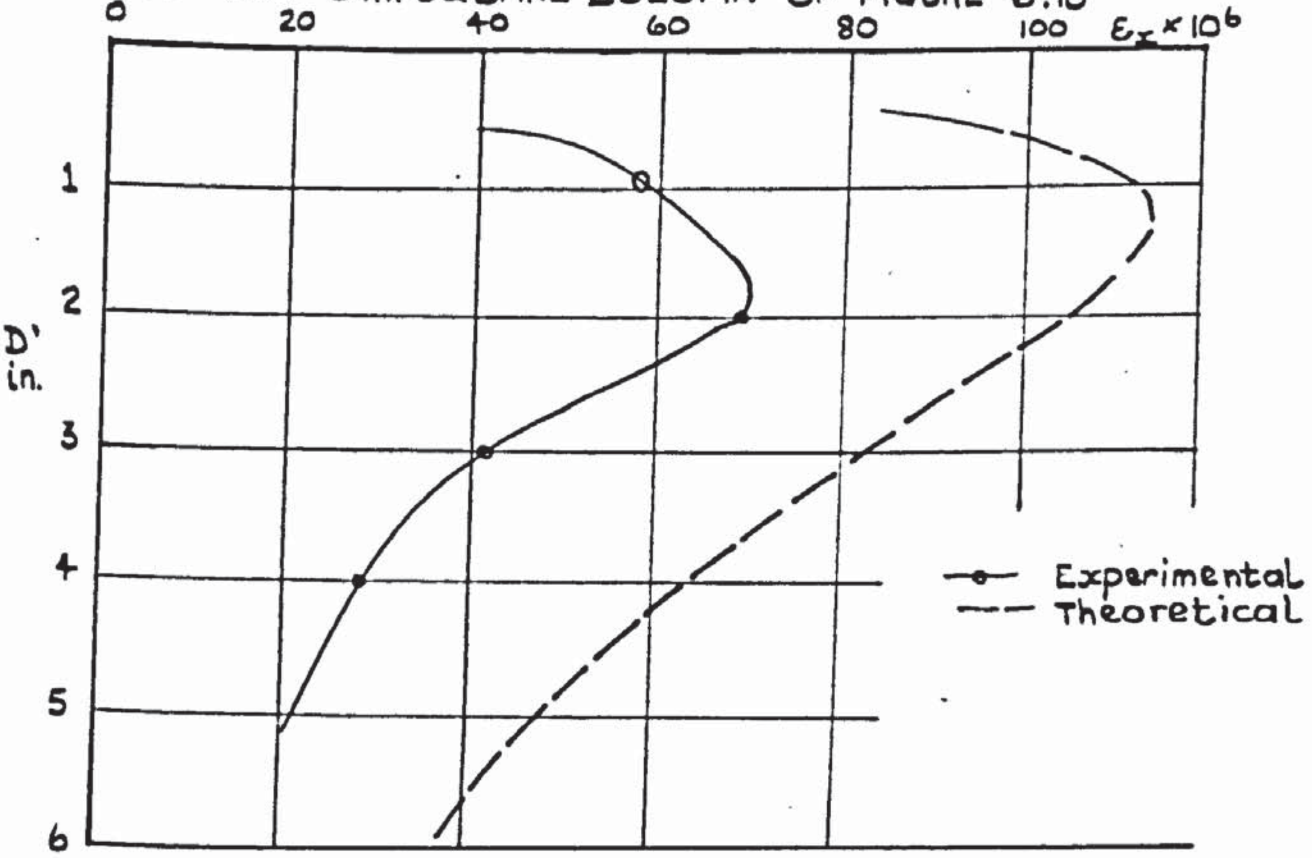


Figure 6.14

LOAD v STRAIN GRAPHS FOR 10 in. SQUARE COLUMN WITH 1 in. PLATE

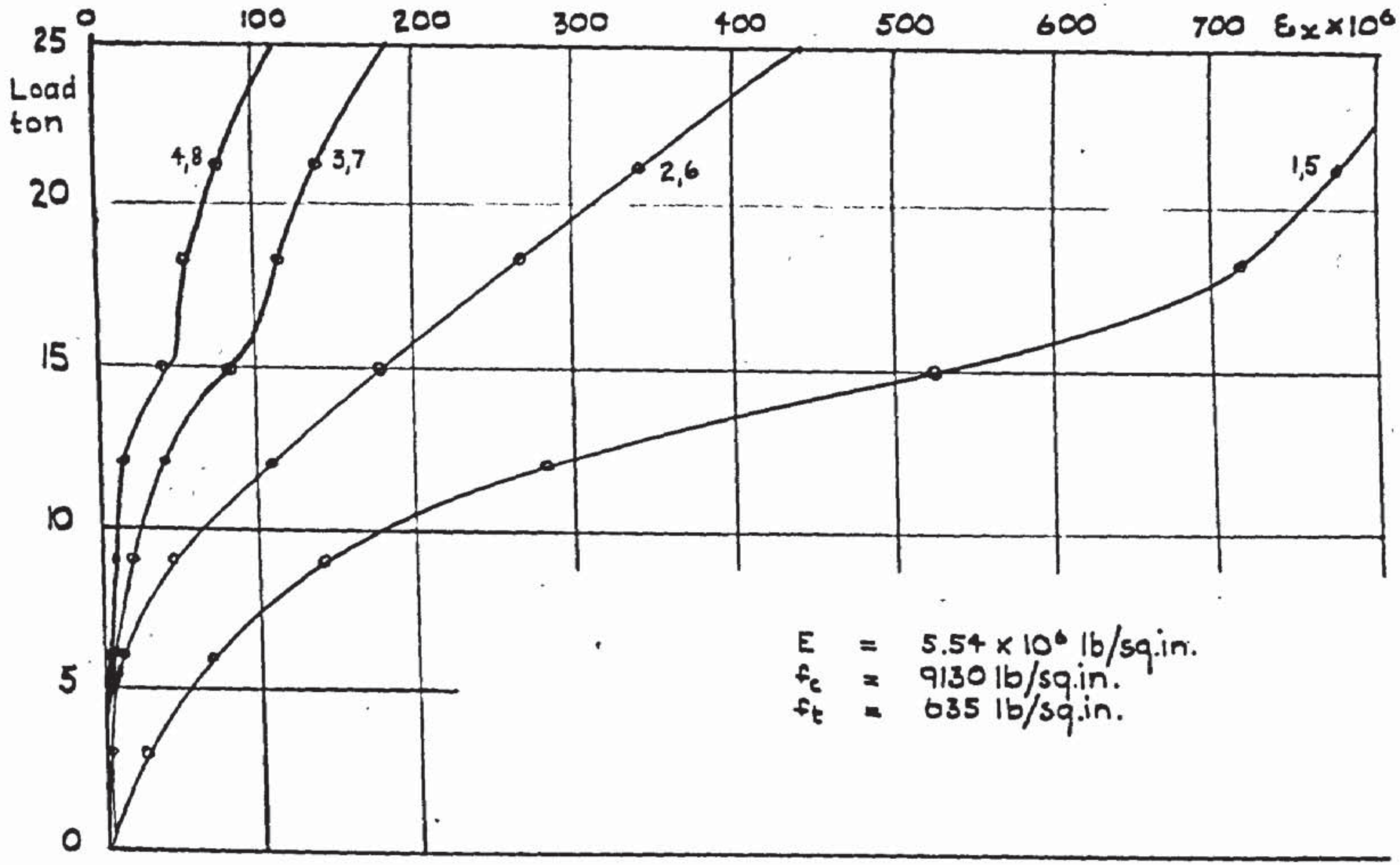


Figure 6.15

LOAD v STRAIN GRAPHS FOR 6in. SQUARE COLUMN WITH 1in. PLATE

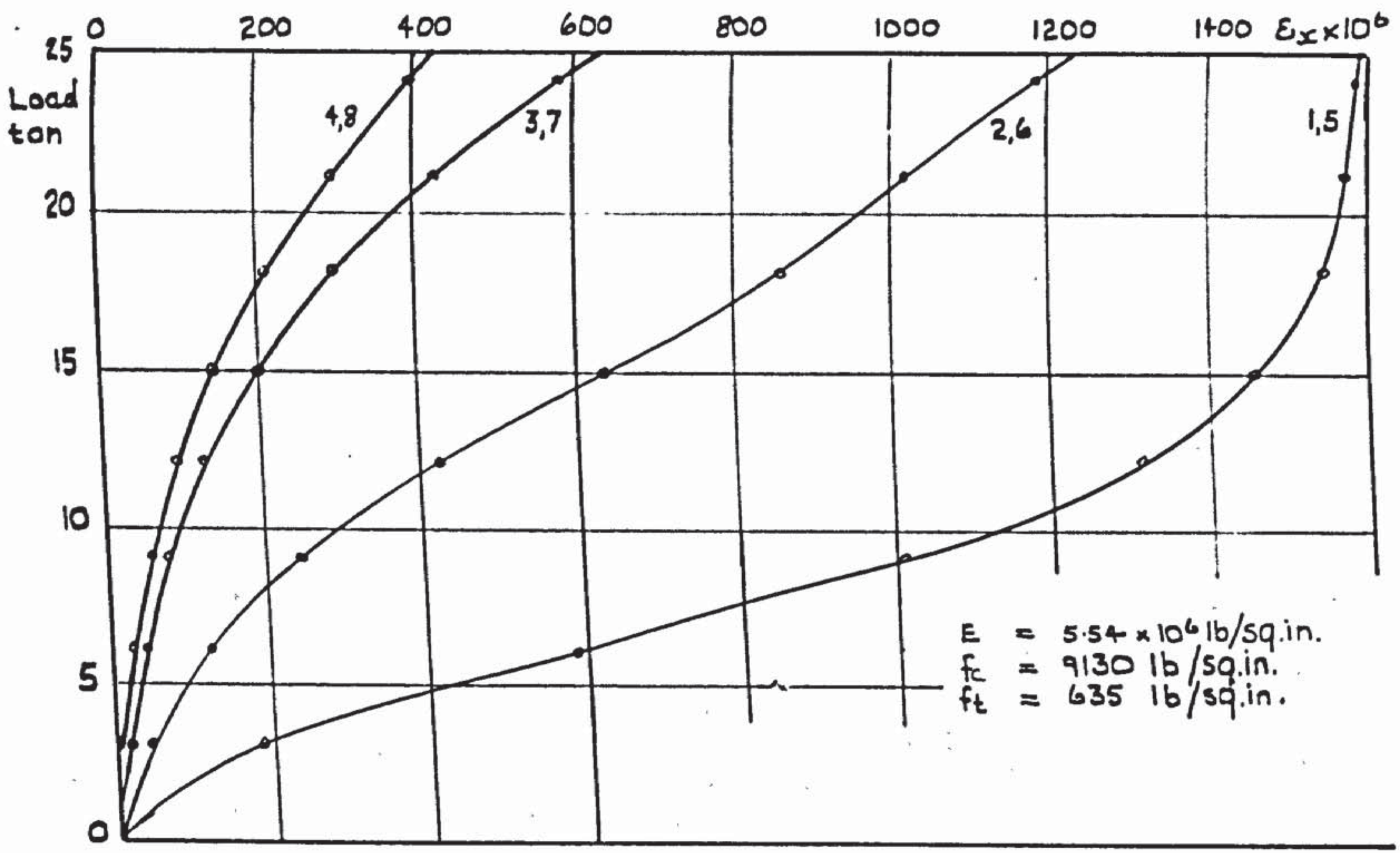
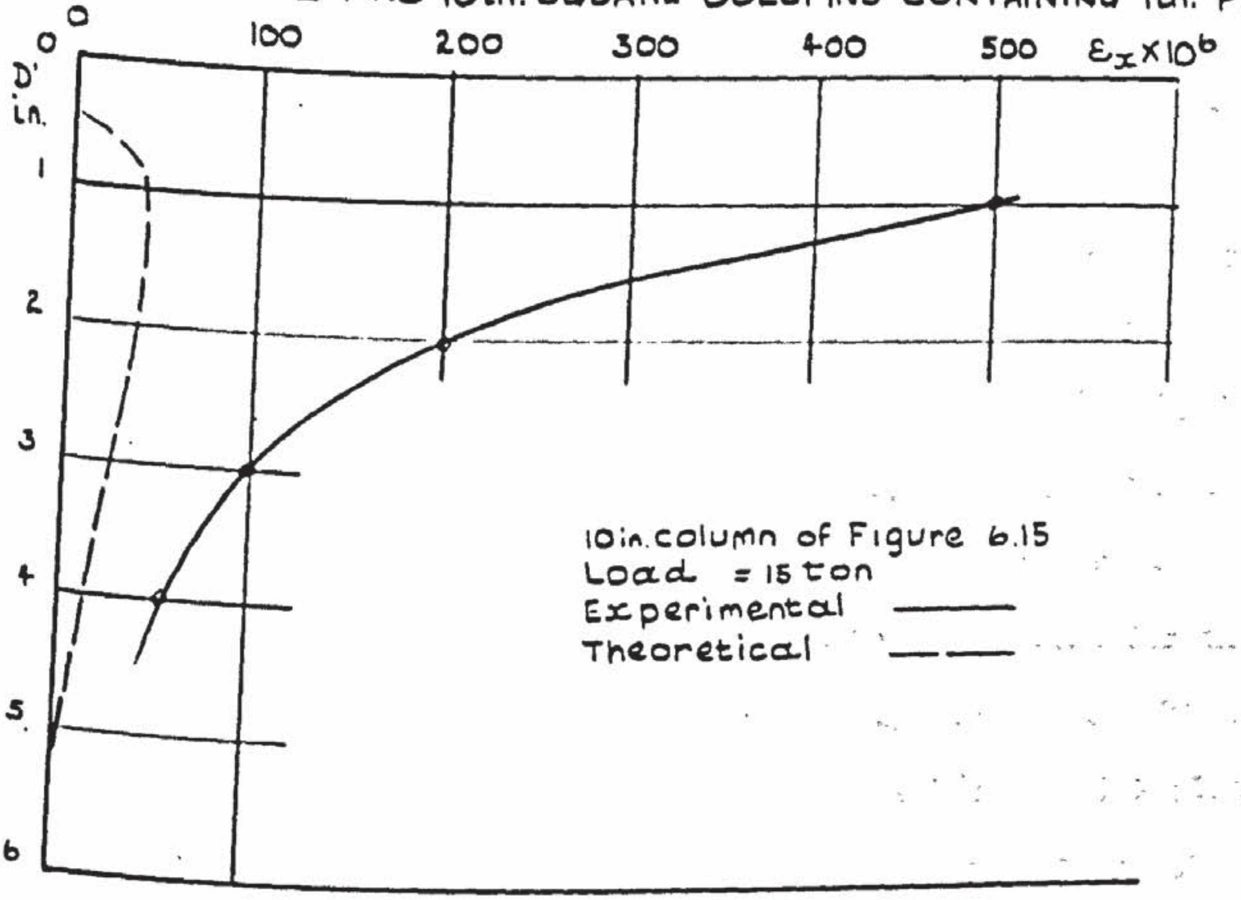


Figure 6.1b

THEORETICAL AND EXPERIMENTAL STRAIN PROFILES FOR 6in. SQUARE AND 10in. SQUARE COLUMNS CONTAINING 1in. PLATES



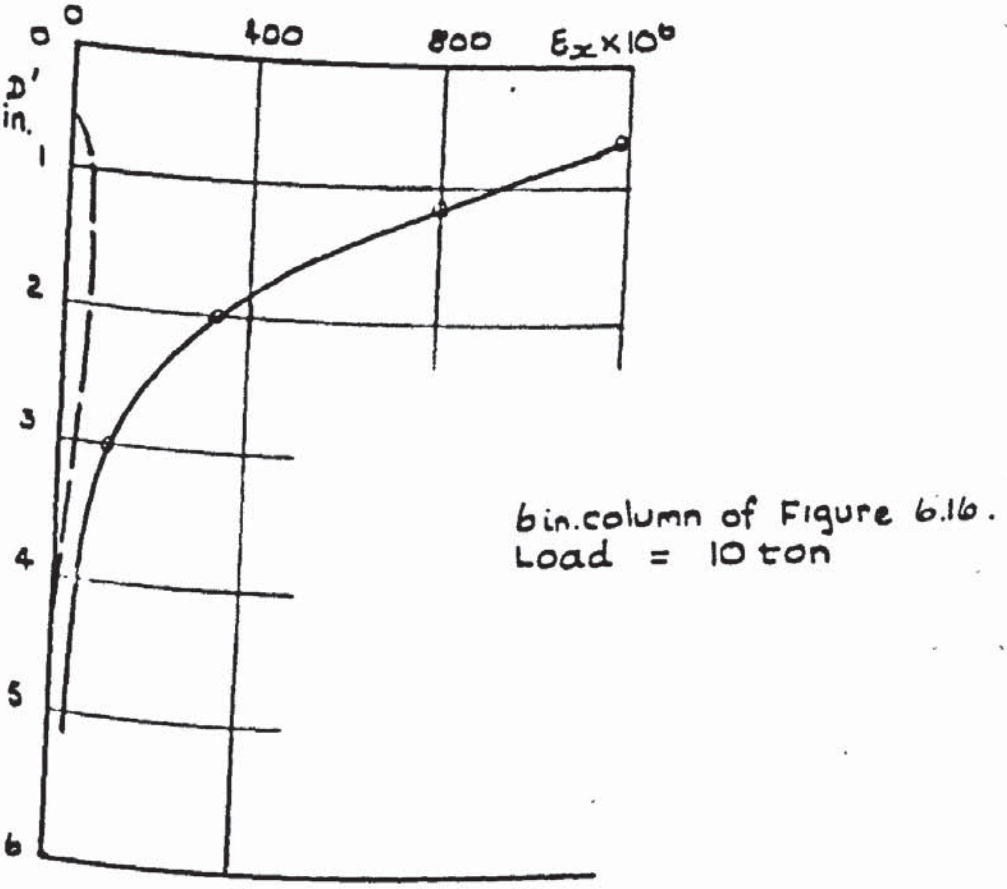


Figure 6.17

results appear to conflict with those of Fig. 6.14 which shows theoretical strains greater than experimental ones but an important difference between the two sets of tests was the age at which testing took place. In the case of the column of Fig. 6.14, the age at testing was 28 days and in the case of the columns of Fig. 6.17 the age at testing was 70 days. A detailed discussion of the implications of the difference in the testing conditions will be left to Chapter 9.

Fig. 6.18 shows a comparison of the profiles for the cube loaded through a 1/2in. wide strip (see Fig. 6.2) and the 6in. square column loaded through a 1/2in. thick plate (see Fig. 6.13). It appears that at the same load, strains for the column at the same distance from the lower edge of the plate as in the case of the cube, were smaller which seems to indicate that the effect of the column plate being cast in was to reduce the load which was sustained in direct bearing by the material below the lower edge of the plate. In order that a direct comparison could be made the tensile strains for the cube were multiplied by the modulus of elasticity of the cube concrete and divided by that for the column concrete. The age at testing was the same for both specimens, 28 days.

6.7 CONCLUSIONS

The tests showed that generally there was poor agreement between the magnitudes of experimental and theoretical transverse

COMPARISON OF EXPERIMENTAL STRAIN PROFILES FOR A 6in. CUBE, $E = 4.19 \times 10^6$ lb/sq.in. AND A 6in. COLUMN, $E = 6.44 \times 10^6$ lb/sq.in. FOR b/D = 1/12

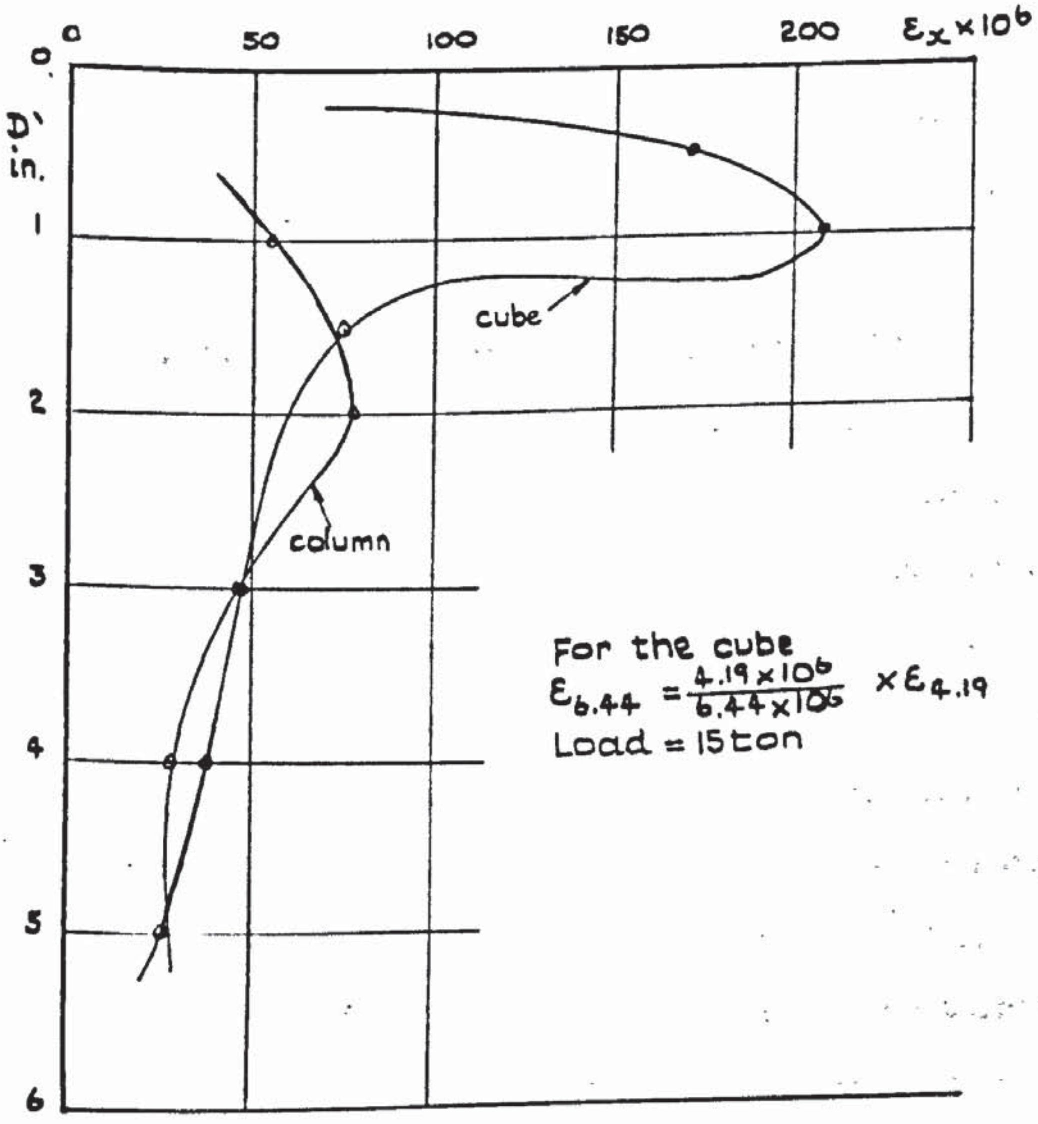


Figure 6.18

strain measurements for cubes loaded through centrally placed loading strips and for columns loaded through cast in plates but there was great similarity between the shapes of the distributions. It would seem that the measured strains were very sensitive to changes in the properties of the material which occurred at quite low levels of loading. This was partly due to the peculiar shape of the transverse strain profile on the centre lines of specimens which caused micro-cracking and yielding of the concrete to occur at well below the load at which failure became imminent, e.g. the cube for which the load v. strain curves were drawn in Fig. 6.2 failed by splitting at a load of 42.8 ton although yielding of the concrete beneath some of the gauges close to the upper (loaded) surface started to occur at a load of only 27 tons). The considerable disagreement was also affected by the innacuracies in the values of the modulus of elasticity that were used for each test : it is felt that the determination of the modulus of elasticity by compressing a cylinder was not a very good way of assessing this value obtaining within a cube or a column tested in the manner described in this chapter and it is this factor which probably accounts for much of the discrepancy between theoretical and experimental values. It should perhaps be added that while the method of determining the modulus of elasticity did not provide a very accurate value for these tests;

it was the simplest and most reliable way of obtaining a value that provided a means of comparing experimental and theoretical strain values which were in a few cases in fairly close agreement.

Fig. 6.18 shows that the strains in a cube are larger at the same load than corresponding ones in a column but this does not necessarily indicate that failure by splitting will occur at a lower load on the cube than on the column, as will be seen from the work described later. This may be due to the fact that whilst surface strains only were measured in all the tests there is no doubt that there is some three dimensional effect which allows stress to be redistributed within the depth of the specimen.

There is no doubt also that the gauge length itself had some influence on the results. In some comparative tests which are not reported here in detail it was found that transverse strains measured over a 2in. (50.8mm) gauge length were somewhat smaller than those measured over the 17mm (which was used in almost all of the tests performed for these investigations) when measurements were made on the same portion of stressed material. This is due to the fact that strain gauges measure the average strain over the gauge length and if there is considerable variation of strain over this length, as there is on the vertical central zone of the specimens.

described in these tests, the measured average strain will not be the same as the maximum which occurs ideally only on the centre line itself. The gauges of shorter length therefore measured the central transverse strains to a greater degree of accuracy although even higher accuracy would have been obtained with gauges of still shorter length. However there were practical drawbacks in using very much shorter gauge lengths. Firstly, it was possible that the most highly strained zone was displaced slightly with respect to the centre line in some cases due to innacuracies of alignment of the loading piece. Secondly, if a very short and consequently very sensitive gauge had been used, it was possible that irregularities of the concrete beneath the gauge could have influenced the readings. It was considered that a gauge length of 17mm was close to the optimum giving a suitable balance between the competing demands of accuracy and sensitivity.

CHAPTER 7

CHAPTER 7

VARIATION OF PLATE DEPTH IN COLUMNS

7.1 INTRODUCTION

One of the main assumptions on which the theoretical determination of stress and strain in the portion of a column loaded through a vertical cast in plate was based, was that the load was distributed in an effectively uniform manner under the plate, assuming that the direction of loading was downwards. Only if it is possible to establish that this was indeed the case can a two dimensional stress analysis be considered. Clearly then, it was important to determine the required minimum stiffness of such a plate that would behave in an effectively rigid manner. It was equally important to determine how the ultimate failing loads of columns were affected when plates of different stiffnesses were used and in order that this might be investigated experimentally, a series of columns of standard length was tested in the manner described in Chapter 5. Columns of two different cross-sectional areas were tested; those of the same cross-section each incorporated a plate of a standard thickness but of different depth.

It was thought possible that, if a column plate became bent in its own plane due to its relatively low stiffness when load was applied to both ends, high stress concentrations would arise in the concrete close to the lower edge of the plate near the faces of the column through which the plate protruded. These would in turn cause splitting of the column at a relatively low load compared with the load at which failure would occur when a relatively stiff plate of the same thickness was used. It will be shown in Chapter 9 how the existence of shrinkage stresses in the concrete above and below the plate acting perpendicularly to its plane could cause the concrete in the vicinity of the edge of the plate to be in an initial state of tensile stress. The magnitude of these stresses would, of course, depend on the surface area of the plate and it follows that this magnitude would be smaller in the case of a column in which a plate of only 6in. depth had been incorporated than one in which the depth of the plate was 12in. This would, all other things being equal, give the concrete an increased potential bearing capacity, consequently, if the effect of reducing the depth of the plate was to make it flexible enough to locally over stress the concrete and so reduce the potential bearing capacity compared with that of a column incorporating a deeper plate, the effect of the reduced area of the more flexible plate might be expected to keep the failing loads at very much the same level. It must be emphasised that this argument is based on the assumptions that the columns are all identical in shape and that the composition of the concrete mix and other properties are all similar.

By the above reasoning, it is possible to hypothesise that, for practical sizes of columns and plates (the minimum size of plates in a 6in. square column being say ½in. thick by 6in. deep) it was unlikely that the failure loads of similar columns incorporating plates of different depth would vary a great deal, if at all. Furthermore, it was also possible that the effects of localised concentrations of compressive stresses in the concrete might be mitigated by some redistribution of stress leading to a more uniform distribution of load in the concrete adjacent to the lower edge of the plate. Thus it was possible that the stiffness and hence the strength of the steel plate, not the relative strength of the concrete, was the critical factor governing the strength of the connexion.

In order that some of these effects might be investigated experimentally, several test methods were used. Columns of different sizes were tested as mentioned above. Cubes were tested by applying load through vertical plates of different depths and the effect of altering the distance between the two loading points was also compared.

7.2 SPECIMENS

Four sets of standard length columns, each incorporating a plain vertical black steel plate were cast. Each set consisted of a 6in. square column incorporating a ½in. thick plate and a 10in. square column incorporating a 5/8in. thick plate. The two columns of each set were cast from the same batch of concrete making a total of eight columns in all. The details of the mix and the curing procedure are given in Appendix B. With each set of columns a suitable number of control specimens were cast from which could be obtained the cube crushing strength, the tensile splitting strength and the value of the modulus of elasticity (see Chapter 5). A different depth of plate was used in each set, either 12, 10, 8 or 6in.; the depth of the plate was the same in each of the two columns comprising a set. The horizontal distance between the centre lines of the bolt holes in all plates was 15in. and the distance from the underside of each plate to the base of the column was 15in. for all columns.

7.3 LOADING

The columns were loaded through the yoke described in Chapter 5 which permitted equal loads to be applied to each end of the column plate; strain gauge readings using the Demec method, which are not reported here, were taken at intervals of 2 ton until vertical cracks which formed beneath the plate were observed. However, prior to the observation of these cracks, spalling of the concrete was noticed in the localised area in the vicinity of the lower edge of the plate on the faces of the column through which the plate protruded. It was noticed that although this spalling occurred early in the loading sequence the affected area never extended for more than 2in. below the lower edge of the plate. The spalled zone can be seen clearly in Plate 3.. As loading was continued small cracks at right angles to the plate close to the lower edge appeared as did small cracks at the upper edges of the plate as shown in Fig. 7.1(a) and these were soon followed by the appearance of a crack running from one of the lower edges of the plate towards the base of the column. This crack gradually extended with the increase in the load until the column was unable to sustain further increments of load. The column was then deemed to have failed and the load at which failure occurred, P_o, was taken as the maximum load that the column could sustain. If the testing machine was kept running for only a few seconds after the maximum load had been reached, the plate was pushed further down into the concrete and the two portions into which the column had been split by the formation of the vertical crack moved apart, in some cases with explosive force accompanied by a resounding report. The general crack pattern at failure is illustrated in Fig.7.1(b) and a 10in. square column which was tested to failure is illustrated in Plate 3. As failure occurred, one or two of the small cracks which had earlier formed at right angles to the plate near its lower edge extended outwards away from the plate towards the sides of the column as shown in Fig.7.1(b).

CRACK PATTERNS IN COLUMNS
BEFORE AND AFTER FAILURE

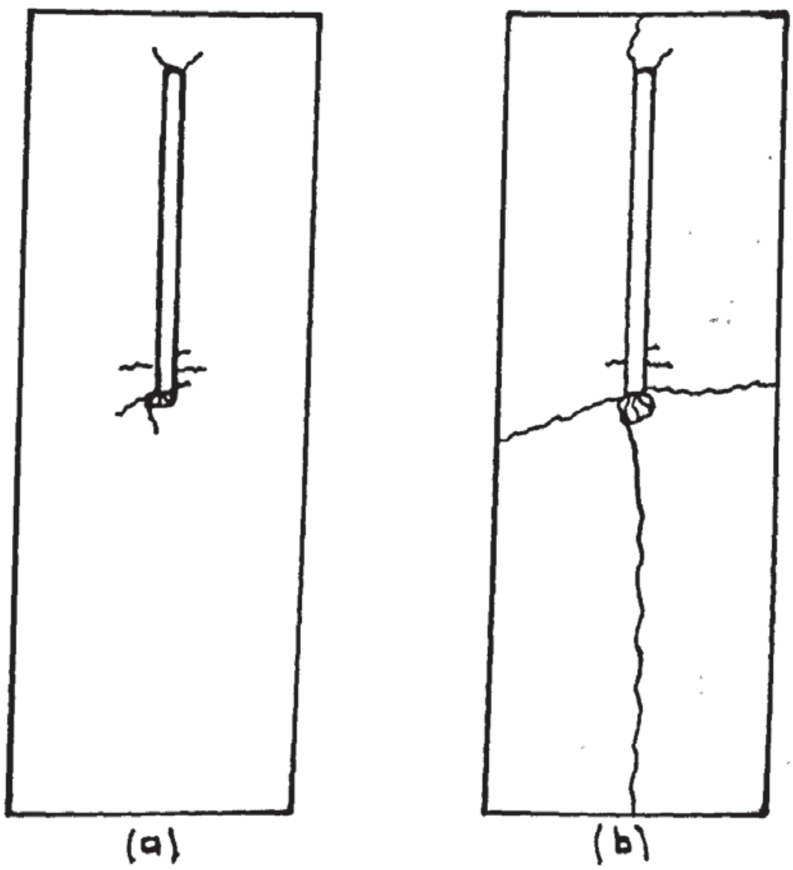


Figure 7.1

One of these cracks only appeared in the column of Plate 3. The formation of this horizontal crack was, however, considered to be a secondary failure mode and was of little significance. A general view of a 10in. square column after failure is shown in Plate 6 and a close up is shown in Plate 7 in which can be seen the wedge which was formed below the plate as failure occurred.

7.4 RESULTS AND DISCUSSION

7.4.1 Spalling Loads

In the tests on the columns of this series, Series I, the load at which spalling was first noticed was usually less than half the ultimate load, as shown in Table 7.1. It can be seen that while there is no significant trend in the results of the loads causing ultimate failure, the percentage of the ultimate load at which spalling was first observed tended to fall with decreasing depth of plate. This leads to the conclusion that whilst the surface of the concrete is damaged by being loaded through a plate of relatively low stiffness the load at which ultimate failure would occur is largely unaffected by the change of plate depth.

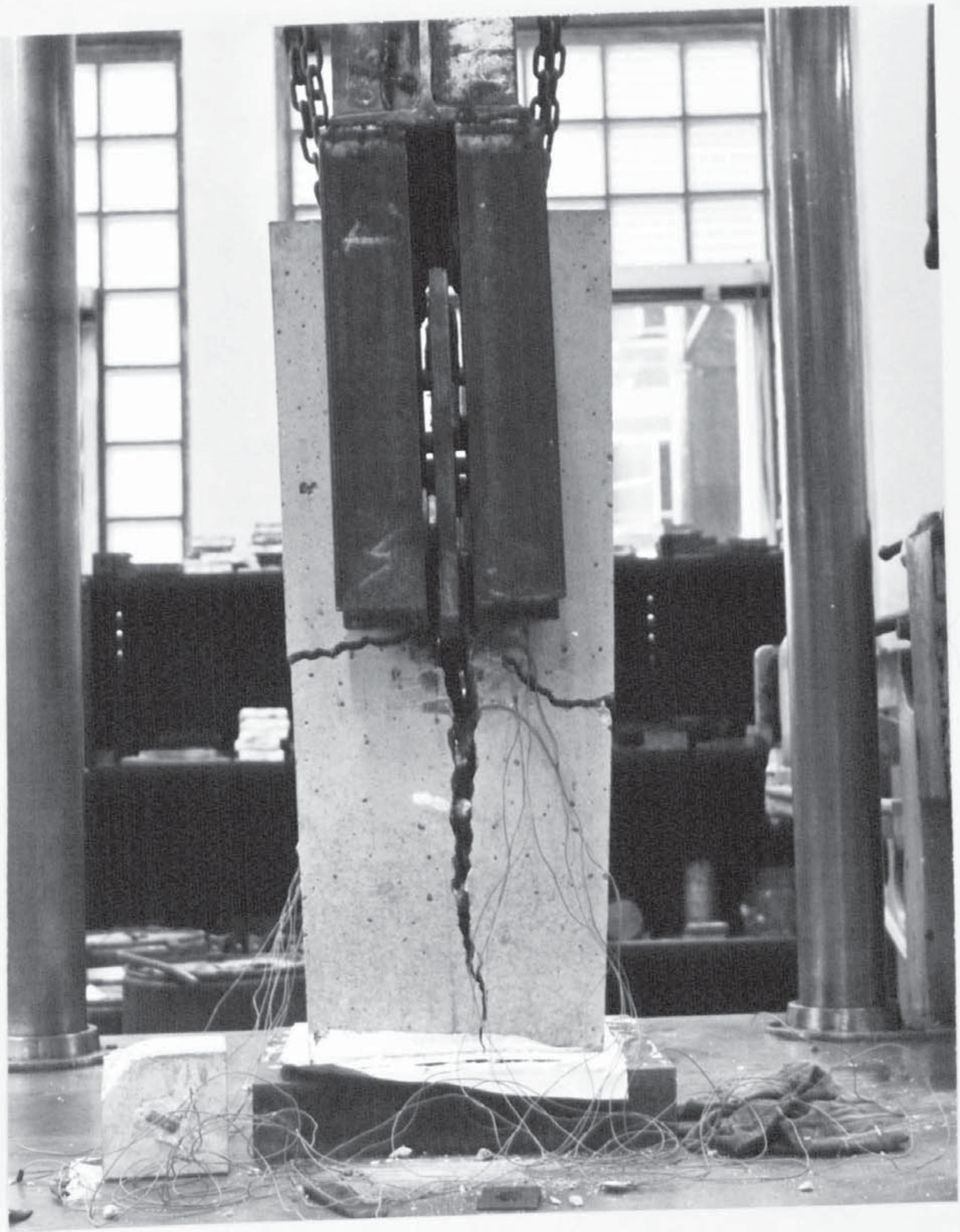


Plate 6

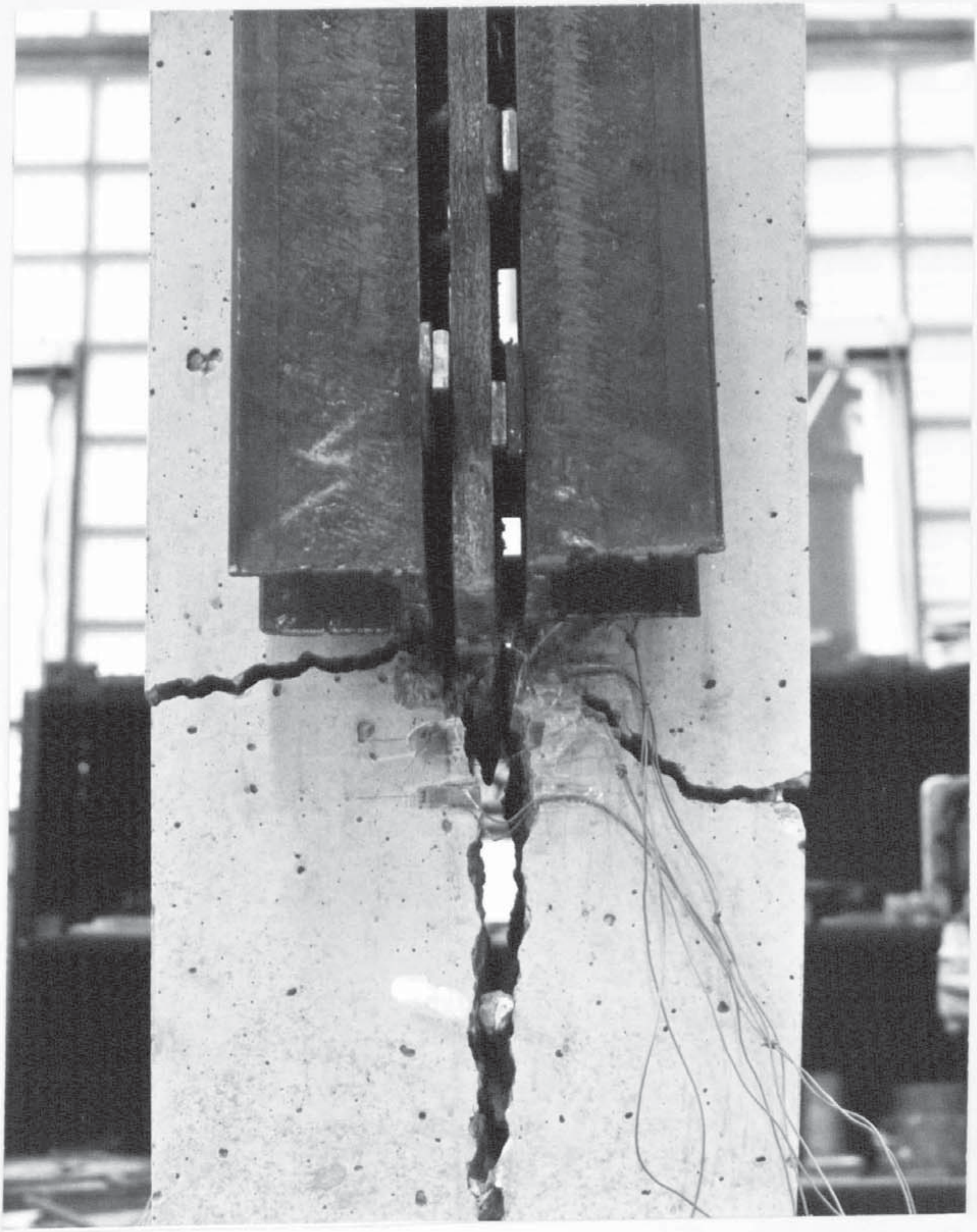


Plate 7

TABLE 7.1

Test No.	d in.	P_o ton	P_s ton	P_s as % P_o
A1	12	18.0	10.0	56
A2	10	22.7	10.0	44
A3	8	18.5	5.0	27
A4	6	14.9	5.6	38
B1	12	9.7	5.0	51
B2	10	9.9	3.0	30
B3	8	7.5	3.0	40
B4	6	8.8	2.0	23

In the table A indicates the 10in. square columns and B the 6in. square columns; d is the plate depth and P_s is the load at which spalling was first observed.

7.4.2 Comparison With Cube Strength

In order that the loads causing failure of columns of different strengths might be compared, the cube compressive strength of the column has been used as an indicator of concrete strength throughout the work reported in this thesis. This also applies in the case of the tests on cubes loaded through strips. It might be thought the value of the tensile splitting strength of the concrete obtained from tests on cylinders might be a more logical choice for a measure of strength in view of the similarity between the modes of failure of the column loaded through a vertical cast in plate and the cylinder loaded on diametrically opposite generators but this was in fact considered not to be the case because of the larger degree of scatter of results that was generally obtained from the splitting tests as compared with that from the results of tests on 4in. cubes. In other words, the results obtained from testing cubes in compression were considered to yield a more reliable measure of strength.

This larger relative variability which can be measured statistically by the coefficient of variation (the ratio of the standard deviation to the mean of a set of results usually expressed as a percentage) has been noted in many papers on the testing of concrete and in the results obtained in these investigations and is due to the presence of flaws which exist in all concrete, no matter how well compacted. When a cube is compressed in a testing machine the flaws, such as air bubbles, or shrinkage cracks play no major part in the failure of the concrete because, owing to the shape of the stress v. strain curve for concrete in uniaxial compression and to the method of testing, the high stresses which could form in the region of the flaws can be redistributed to the surrounding mass and this process can continue until failure is imminent. Thus the strength of the cube obtained in this test represents an average valuefor the concrete of which the cube is made. It might be added that the cube crushing strength obtained from tests on 4in. or 6in. cubes is not a very good approximation to the true compressive strength of the concrete in uniaxial compression because of the proximity of the platens which give rise to a rather complex stress system resulting in a rather high estimate of the true compressive strength but the cube strength is nevertheless a reasonably reliable method for comparing the strengths of different concretes.

However in the case of the cylinder tested in the standard splitting test, the tensile stress developed on the loaded diameter is uniform over most of the length of the diameter; moreover, the length of the stress v. strain characteristic of concrete tested in this way is short with stress increasing with strain until failure occurs by splitting. Thus the concrete exhibits a more brittle behaviour when tested in this manner than when tested in uniaxial compression. Consequently the high stresses associated with flaws are not readily redistributed and once the stress at a flaw has reached the limiting value for the concrete, failure of the whole specimen is precipitated. Thus concrete specimens of the same intrinsic tensile strength and of the same size will fail at different loads if the number of flaws per unit volume (the specific number) is not the same. Moreover larger specimens would be expected to fail at lower loads than smaller ones of the same intrinsic strength when tested in tension because, if the specific number of flaws is the same in the two concretes, the likelihood of a flaw being present increases with the volume of the concrete. This size effect is observed with cubes tested in compression but the cause is associated with the change of the stress system produced in the specimens rather than the increase in the absolute number of flaws.

7.4.3 Main Results

The main results of the tests on the columns of series I are summarised in Table 7.2. C.V. denotes the coefficient of variation (the standard deviation divided by the mean of a set of data) of the results in the first column in each block of results and is expressed as a percentage. P_o is the maximum load sustained by the individual column in tons and f_o is the maximum load in pounds divided by the cross-sectional area of the column which has been called the mean failure stress; f_o/f_c is the quotient of the mean failure stress and the cube strength of a particular column.

In view of the fact that the range of the standard cube crushing test results for the whole of series I was small, the coefficient of variation being 8.3%,it seemed not unreasonable in the first instance to take the ultimate loads for the columns of one size to be independant of the cube strength of individual columns. Thus, it can be argued, any marked trend in the ultimate loads for series IA and IB would be due to the differences in the plate depth. The results of series IA indicate a positive correlation between the ultimate load and the depth of plate provided the result for column I A1 is left out of account. However, there seemed no reason why that result was any less valid than the others. If the results from series IB are considered it is clearly seen that there is no apparent correlation and the ultimate load sustained by the column seems in no way related to the depth of the plate through which the column was loaded. These results are shown in Fig.7.2 which is a plot of f_o v.d, the plate depth.

Another way of considering the results is to use the non-dimensional paramater f_o/f_c and to plot it against the plate depth d. This should ensure that if the differences in the strength of individual columns, as measured by their cube strength, is a factor, then provided f_o/f_c is not itself dependant on f_c the individual value of f_c will be accounted for. The use of the parameter f_o/f_c in this context implies that for columns of the same size and under identical loading conditions f_o/f_c is constant : then if f_o/f_c did vary when plotted against d, any trend would be significant. It will be shown in Chapter 12 that to a very good approximation, f_o/f_c was constant over a wide range of cube strengths, for cubes

TABLE 7.2

Test No.	d in.	f_c p.s.i.	Av. f_c p.s.i.	C.V. %	f_t p.s.i.	Av. f_t p.s.i.	C.V. %	f_o p.s.i.	Av. f_o p.s.i.	C.V. %	$\frac{f_o}{f_c}$	Av. $\frac{f_o}{f_c}$	C.V. %
IA1	12	5180	5225	8.3	385	378	9.3	403	415	15.0	0.0778	0.0791	9.3
IA2	10	5950			335			508			0.0854		
IA3	8	4830			400			414			0.0857		
IA4	6	4940			395			333			0.0675		
IB1	12	5180	5225	8.3	385	378	9.3	603	558	10.5	0.1165	0.1068	7.6
IB2	10	5950			335			615			0.1035		
IB3	8	4830			400			467			0.0966		
IB4	6	4940			395			547			0.1108		

PLOTS OF f_o v d (PLATE DEPTH)

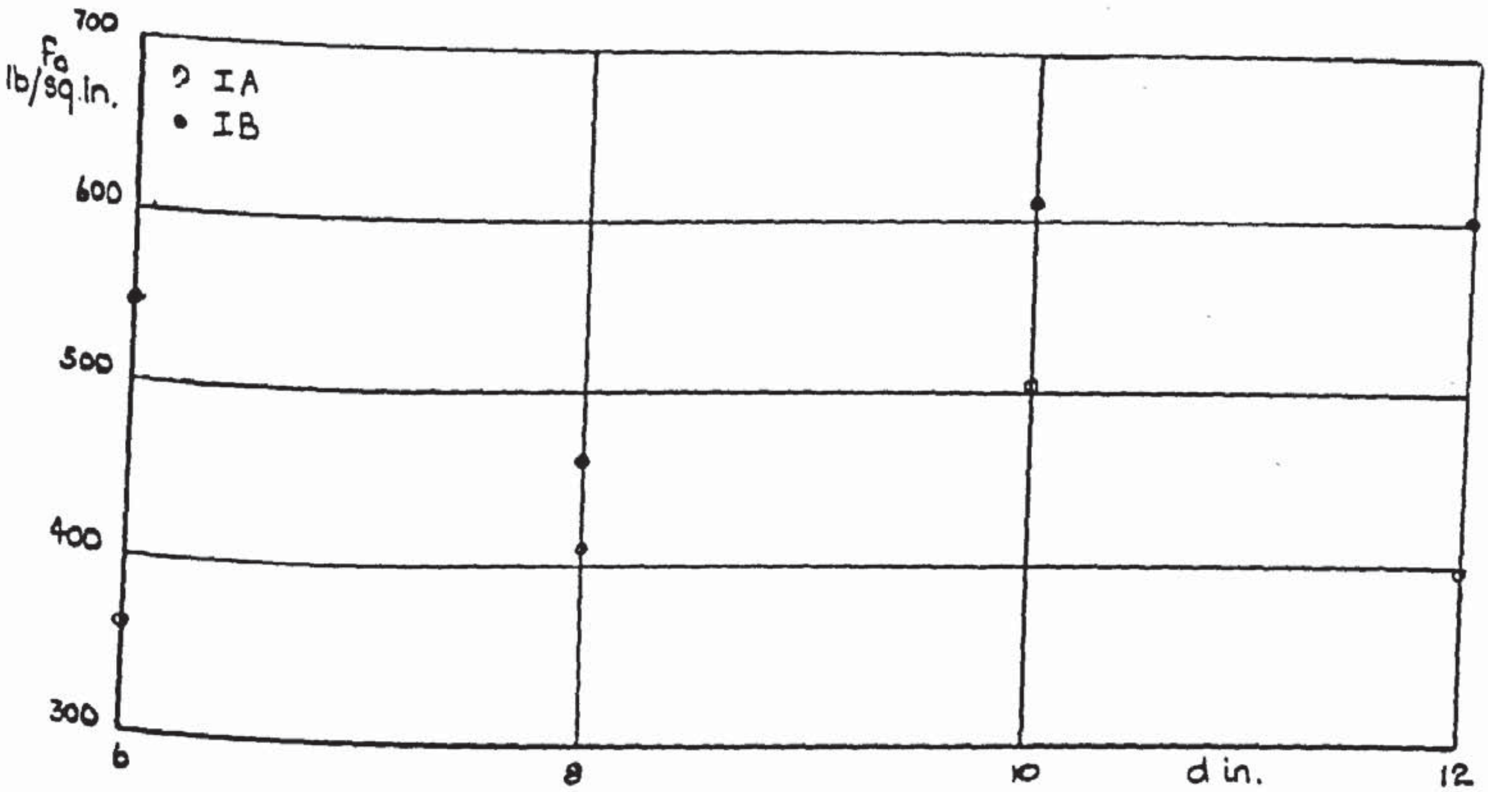


Figure 7.2

tested under centrally placed strip loads, whose b/D ratio was ½in. Although this independance cannot be shown to obtain for all the columns tested in the standard manner throughout the whole of these investigations, it was considered reasonable to assume that for the tests of series I f_o/f_c was independant of f_c because the mix proportions and the type of aggregates used were the same, and also because the range of cube strength was small. Thus f_o/f_c has been plotted in Fig.7.3 and it is again apparent that there is no significant correlation between the variables plotted, either for the 10in. square columns or for the 6in. square columns. Moreover, in spite of the small number of tests performed, the coefficients of variation of the results for each size of column when calculated for f_o/f_c were small for this type of work and adds weight to the assertion that for the sizes of columns and plates used in these investigations, the depth of the plate had no effect on bearing capacity.

7.5 TESTS ON CUBES

7.5.1 Specimens

In order that a greater understanding could be gained of the factors involved when the plate through which the load is transferred to the column is bent in its own plane and to investigate further the hypothesis that a relatively flexible plate would cause premature failure of the column some tests

PLOTS OF f_o/f_c V d (PLATE DEPTH)

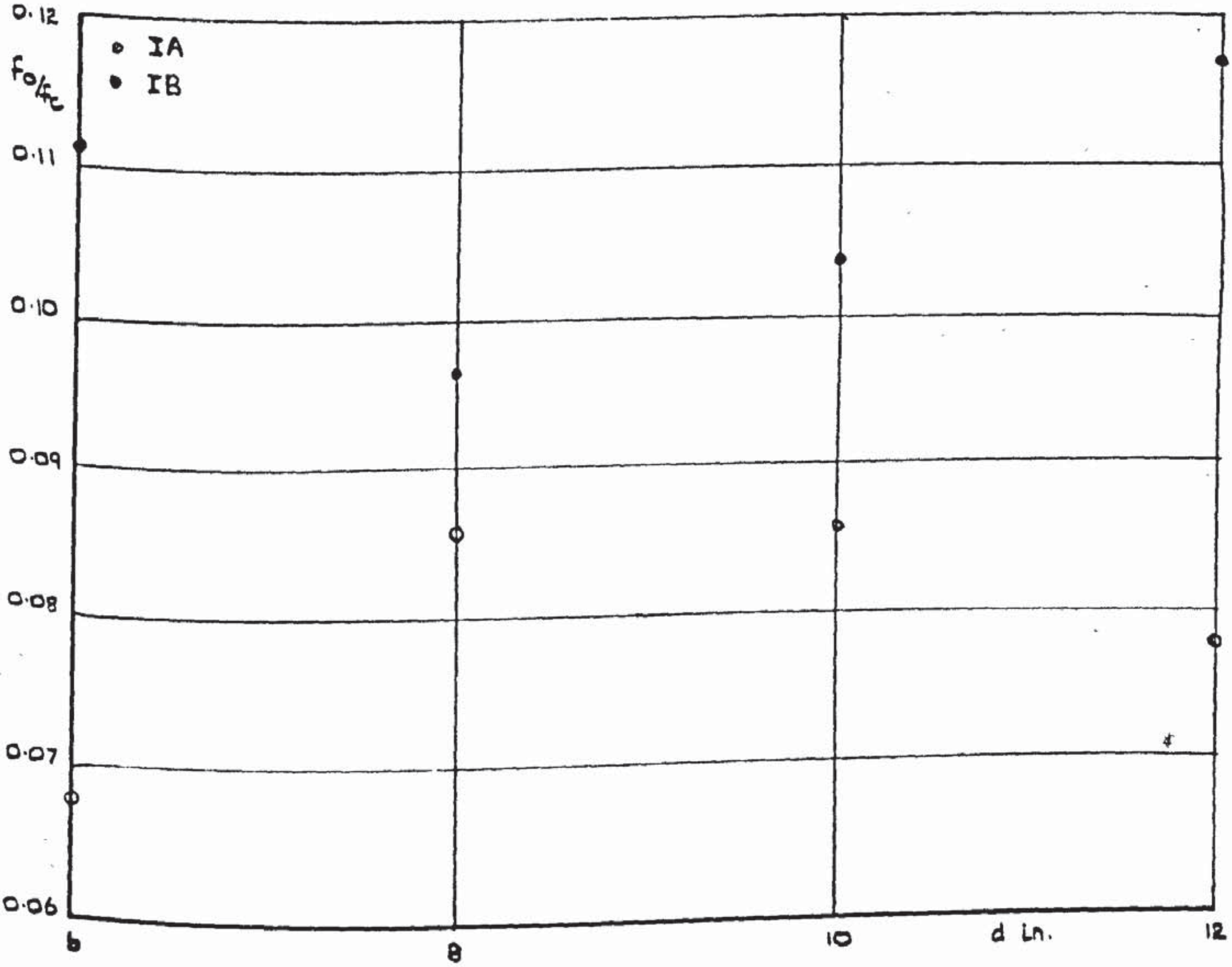


Figure 7.3

were performed on cubes loaded through steel plates of different depths. The general arrangement is shown in Fig. 7.4. A bright steel plate 1in. thick, 12in. long and approximately 6in. deep was shaped so that a tongue of metal 0.46in. deep and 0.6in. wide projected from one of the longer edges. The plate could then be located in a vertical position on a centre line of the upper face of a cube so that the tongue was in contact with the concrete. Two sets of symmetrically placed ball seatings were cut on the upper 1in. thick edge to receive two $\frac{3}{4}$in. diameter steel balls through which load could be applied to the plate, the distance L between the two seatings of each set being 9in. and $11\frac{1}{2}$in. respectively. In this way it was possible to simulate the loading of a 6in. column in which a vertical plate was incorporated, equal loads being applied to each of the projecting ends of the plate at a distance of either $1\frac{1}{2}$in. or $2\frac{3}{4}$in. from the column faces.

Three sets of five cubes each were cast from two batches of concrete, sets CIa and CIb being cast from one mix and set CIc being cast from the other. The distance L used for sets CIa and CIc was 9in. and for CIb was $11\frac{1}{2}$in. The depth d of the steel plate was altered for each cube of a set, five values of d being used in all. From each batch of concrete was cast six 4in. cubes and three 6in. dia. by 12in. long cylinders to obtain the cube and tensile splitting strengths. All the

LOADING ARRANGEMENTS FOR CUBES IN PLATE BENDING TESTS

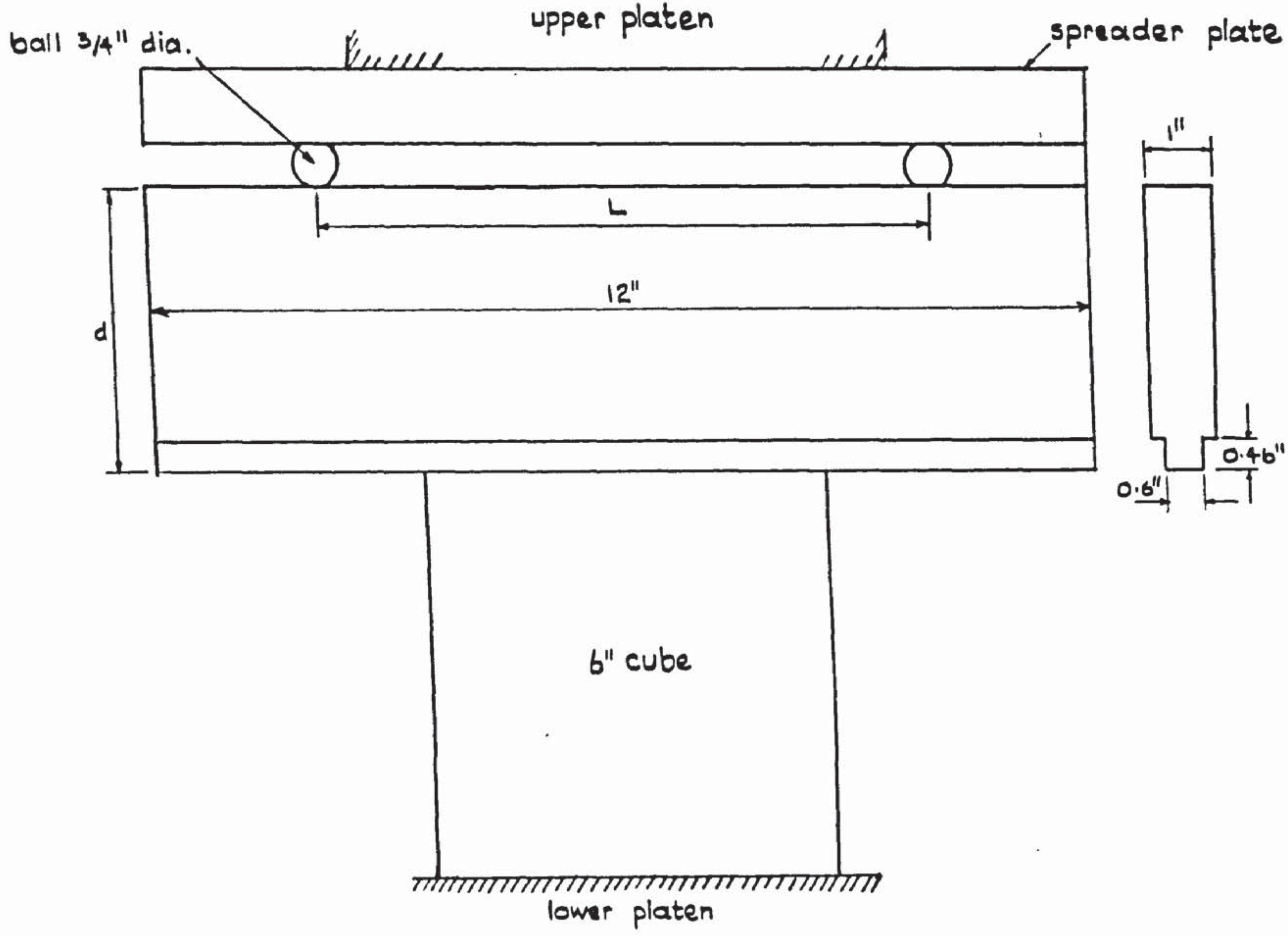


Figure 7.4

cubes were loaded to failure and the results are summarised in Table 7.3.

7.5.2 Results and Discussion

By using different values of L, the relative stiffness of the plate in bending could be altered; thus the plate for which the value of L was 11½in. was effectively more flexible than that for which L was 9in. However it is apparent from a comparison of the results for CIa and CIb that the difference between the mean failing loads of the two sets was very small and it can be shown that statistically the difference was insignificant. It is also clear from the results summarised in Table 7.3 that altering the depth of such a plate did not affect the load at which a cube failed. It was possible that if a more flexible plate had been used in the tests a different result may have occurred but it was observed that when the plate was used with the lowest value of d it started to bend visibly in its own plane as load was increased and after the failure of the last cube of CIc, the deformation of the plate had become permanent. Excessive crushing of the concrete was observed in the cube in the region adjacent to the lower edge of the plate and the opposite vertical faces of the cube normal to the plane of the plate. Thus, as the results show, although the plate was being bent and therefore must have been distributing the load unevenly

TABLE 7.3

Test No.	L in.	d in.	f_c p.s.i.	f_t p.s.i.	P_o ton	Av.P_o ton	C.V. %
CIa1	9.0	5.470	8060	530	16.30	17.70	6.3
2		5.095			18.04		
3		4.720			16.71		
4		4.345			19.20		
5		3.970			18.27		
CIb1	11.5	5.470	8060	530	16.35	18.12	5.7
2		5.095			19.02		
3		4.720			17.94		
4		4.345			18.38		
5		3.970			18.93		
CIc1	9.0	5.470	9920	490	17.00	19.61	12.3
2		5.095			19.80		
3		4.720			19.72		
4		4.345			17.76		
5		3.970			23.80		

to the cube, the load at which the cube actually failed was not less than if it had been loaded through a relatively stiff plate and may in fact have been higher if the last result in CIc is taken at its face value.

7.5.3 Plate Flexibility

An interesting comparison can be made between the relative flexibilities of the plates used in the 6in. square columns of series I and the plates of series CI. It is possible to obtain an approximate measure of the flexibility of a plate, loaded in its own plane in the manner of the tests, by determining the deflection at the loading points relative to the centre caused by a unit load applied at each of those points. For this purpose the following assumptions were made. The plate did not move as a rigid body; the loads applied at each end of the plate were distributed uniformly over the full depth of the plate at its ends; the reaction to the unit loads was distributed uniformly over the full contact length, i.e. 6in. in all the cases considered and finally, in the case of the plate which was cast into the column, the concrete above the level of the lower edge of the plate did not prevent the latter from deforming freely. Whilst the validity of these assumptions is open to question, there is no doubt that, at the lower end of the range of loading, the errors incurred by making them would be small.

The deflections were calculated for the plates of tests IB4, CIa5 and CIb5 whose leading dimensions together with the distance L between loading points were as given in Table 7.4.

TABLE 7.4

Test No.	L in.	t in.	d in.
IB4	15.0	0.5	6.00
CIa5*	9.0	1.0	3.96
CIb5*	11.5	1.0	3.96

* The cross-sectional shape of these plates is shown in Fig.7.4.

These plates were chosen as they had the smallest flexibilities of all the plates used in the tests. The deflection due to shear, calculated by the energy method given by Ryder[(39)], and the deflection due to bending calculated by the engineers theory of bending were added to give the total deflection in each of the three cases and the relative flexibilities were found to be in the ratio of 20.40 : 7.79 : 16.83 showing that the plate used in the 6in. square column of IB4 was the most flexible.

This result tends to confirm the conclusions that were reached using the results of the tests on the columns of series I. That is to say, even with the most flexible plate likely to be used in practice the capacity of a column is not reduced and the only disadvantage of using such a plate is that the considerable deformations that are likely to arise under heavy loads could cause some spalling of the concrete close to the lower edge of the plate.

CHAPTER 8

CHAPTER 8

CONSIDERATION OF THE ULTIMATE LOAD OF COLUMNS

8.1 INTRODUCTION

It was mentioned in Chapter 7 that Holmes and Bond[4] had made use of an analogy between the mode of failure of a cylinder loaded across a diameter as in the standard indirect tensile test for concrete, and the mode of failure of the columns that they had tested in order to determine an estimate of the ultimate load that the columns would sustain when loaded through a cast in plate set vertically on an axial plane of a column. As this is the first reported attempt to make a rational estimate of the bearing capacity of columns loaded in this manner, it is worthwhile considering the results that they obtained.

8.2 CYLINDER ANALOGY

A cylinder whose axis was parallel to the plane of the cast in plate was imagined to be inscribed in the column in such a way that the lower edge of the plate (direction of loading assumed vertically downward) and the uppermost generator of the cylindrical surface were coincident. The diameter of the cylinder was made equal to the width D of the column faces through which the plate protrudod and the length of the cylinder D_1 was made equal to the embedded length of the plate as shown in Fig.8.1. Holmes and Bond asserted

POSITION OF INSCRIBED CYLINDER

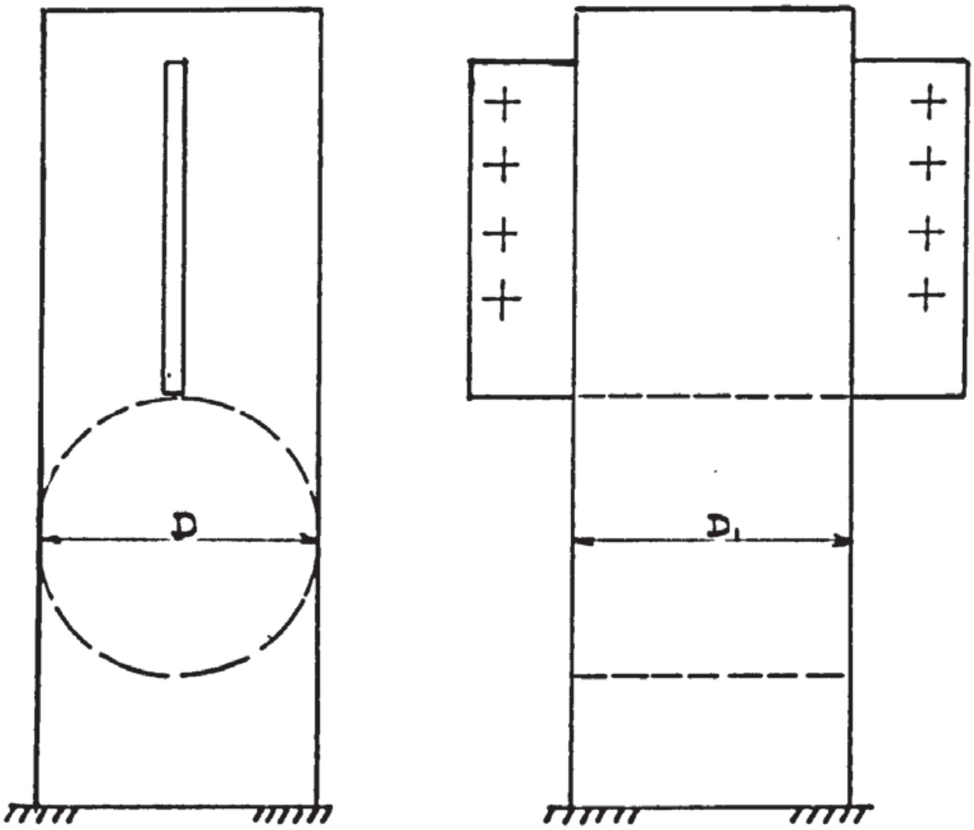


Figure 8.1

that the total ultimate load P_o of the column was compounded from two separate components, P_b and P_f. P_b was derived from the standard formula for the indirect tensile test using cylinders as given in the equation

$$f_t = \frac{2P_b}{\pi D D_1} \qquad (8.1)$$

where f_t was found from tests on standard 6in. dia. by 12in. long cylinders. P_f was the component of the ultimate load which could be accounted for by friction and bond forces acting on the sides of the plates which were in contact with the concrete. These forces were computed from the results of tests similar to those described below. Thus, in their calculation, the ultimate load could be expressed in the form

$$P_o = P_b + P_f \qquad (8.2)$$

In view of the fact that good agreement was obtained between this calculated value of P_o and the experimental value obtained from tests on columns (they tested 8in. square columns using ½in. thick plates) it was decided to test this hypothesis before making further investigations.

8.3 DETERMINATION OF P_f

8.3.1 Test Arrangement

A series of four 10in. square columns of standard length were cast in two batches, a set of 4in. cubes being cast with each batch. Under each 5/8in. thick 12in. deep plate was left a horizontal 1in. square hole so that only frictional stresses and bond stresses could act on the vertical concrete/steel interfaces when the plate was loaded in the standard manner through the yoke, as described in Chapter 5. A dial gauge to measure deflection was positioned under each end of the plate so that the stem was vertical and the anvil was in contact with the lower edge of the plate.

8.3.2 <u>Loading and Results</u>

Loading was applied as slowly as was practicable until the dial gauges indicated that the plate had been pushed bodily downwards a distance of a few thousandths of an inch. This sudden movement on the dial gauge occurred when the load had reached a maximum value and this reading of the load was recorded. The load tended to fall off when the plate had moved and so load was reapplied to the plate until the plate was again caused to move bodily downwards at a load which was slightly lower than the first recorded. This second value of the load which caused movement of the plate in the column was again noted. The difference between these two recorded readings was taken as that component of the load which was caused solely by bond stresses which existed between the concrete and the plate, and the second of the loads recorded was taken to be the load which was caused by frictional stresses. These results are summarised in Table 8.1.

TABLE 8.1

Test No.	f_c p.s.i.	1st load ton	2nd load ton	diff. ton
IIA1	5380	8.06	7.26	0.80
IIA2	5380	7.40	7.32	0.08
IIA3	6060	8.77	7.27	1.50
IIA4	6060	8.29	8.01	0.28

It can be seen from the table that the forces attributed to bond were small compared with those attributed to friction and consequently the first load has been taken to be the value of P_f. This was done because it was observed that once the first load had been reached cracks were formed in most of the columns which extended from the lower corner of the hole beneath the plate making an angle of approximately 45° with the vertical centre lines of the faces through which the plate protruded as indicated in Fig.8.2. Although it was thought that these cracks did not extend into the concrete to any great depth there was some doubt as to whether the second reading was an accurate measure of the load caused by frictional stresses. From the results of these tests the mean of the first load values was found to be 8.13 ton which represented a frictional stress of 76lb/sq.in. calculated by dividing the mean load in pounds by the contact area of the plate, 2 x (10 x 12) sq. in.

COLUMN WITH 1in. SQUARE HOLE BENEATH PLATE
SHOWING LOCATION OF CRACKS AFTER 1st. LOADING

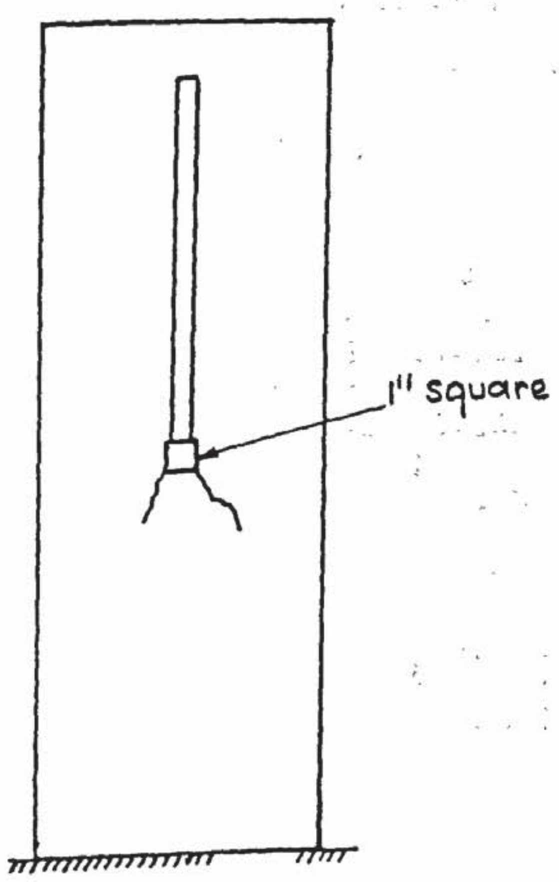


Figure 8.2

Using this result directly with the results of the previous chapter in which columns incorporating plates of different depths were loaded to failure, the calculated failing load using the formula of Holmes and Bond can be compared with the experimental values which were obtained from the tests of series I, as shown in Table 8.2. It will be recalled that there was no measureable difference in the loads which caused failure whatever the depth of the plate.

TABLE 8.2

Test No.	f_t p.s.i.	P_b tons	P_f tons	P_o calc.	P_o test.
IA1	385	26.99	8.13	35.12	18.00
IA2	335	23.58	6.78	30.36	22.70
IA3	400	28.04	5.43	33.47	18.50
IA4	395	23.69	4.07	31.76	14.90
IB1	385	9.90	8.13	18.03	9.70
IB2	335	8.44	6.78	15.22	9.90
IB3	400	10.08	5.43	15.51	7.50
IB4	395	9.95	4.07	14.02	8.80

The penultimate column of the table gives the sum of $P_b + P_f$ ton where P_b was calculated from the particular value of f_t that was obtained from each test and the last column gives the ultimate load (ton) of the columns in the tests of series I. As the cube strengths of the specimens of series II differed little from those of series I, the mean value of 76lb/sq.in. for the frictional stress was used for all the calculated values of P_f.

8.4 OBSERVATIONS

It can be seen from Table 8.2 that the calculated values of the ultimate load using the method of Holmes and Bond are considerably less than the actual failing loads, the ~~latter~~ former being approximately 50 per cent of the former, even allowing for the fact that the value of the frictional stress found in tests on 10in. square columns might not have been the same for the 6in. square columns.

In view of the fact that the actual method of loading of a column incorporating a cast in vertical plate bears little similarity to the method of loading a cylinder in the indirect tensile test for concrete, it is not surprising that the experimental and the calculated values of the ultimate load were so different. Whereas in the case of the cylinder the transverse stress distribution is uniform over most of the loaded diameter as shown in Wright's[40] paper on the indirect tensile test using concrete cylinders, the transverse stress distribution in columns is characterised by a peak close to the lower edge of the plate as shown in Chapter 6. This difference must clearly affect the relationship between the loads causing failure in the two cases.

It is also worth comparing the splitting loads of the 6in. square columns of series IB with the splitting load of 6in. cubes of a cimilar cube strength when loaded through a ½in. centrally placed strip which are referred to in Chapter 12. The mean splitting load of the columns was 9.0 ton (mean fc = 5225 lb/sq.in.) and the mean splitting loads of the cubes was 20.3 ton (mean fc = 5380 lb/sq.in.). It is clear therefore that although some difference between the two loads was perhaps to be expected because in the case of the cube there was possibly a constraining effect due to the proximity of the lower platen to the upper loaded surface which would tend to raisethe ultimate load, there is no doubt that the plate being cast into the concrete had a deleterious effect on the potential bearing capacity of the column.

It is also significant that there is a marked difference between the mean values of f_o/f_c for the two sizes of columns of series I and indicates that the size of the column might have some influence on its potential bearing capacity.

8.5 CONCLUSIONS

These observations led to the important conclusion that properties of the specimens other than the strength of the concrete as measured by the cube compression test or the indirect tensile test would have to be taken into account if the capacity of a column incorporating a cast in plate was to be estimated to an acceptable degree of accuracy. In view of the considerations discussed in this chapter it seemed that the most likely cause of the discrepancies between the results of the tests on cubes and columns and between the results of the tests performed for these investigations and those of Holmes and Bond were due to phenomena associated with shrinkage of the concrete during the curing process. An examination of this hypothesis will form the subject of the next chapter.

CHAPTER 9

CHAPTER 9

EFFECT OF SHRINKAGE

9.1 INTRODUCTION

Before considering the effects of shrinkage on the behaviour and bearing capacities of the columns tested in these investigations it is worth considering the subject more generally in order that its relevance in this context can be more readily appreciated.

The phenomenon of shrinkage in concrete is fundamental to its use as a modern structural material which can be used in members subjected to tensile as well as compressive forces. It is commonly held by the majority of authcrities that it is the irreversible shrinkage of the concrete caused by the hardening process resulting from the chemical reactions which occur between the cement and the water which enables the high gripping, or bond, stresses to be set up between the steel and the concrete in reinforced concrete. Indeed, Glover[41] reports bond stresses as high as 400 lb/sq.in on the surface of reinforcing bars in contact with the concrete. In this way, stress is transferred from the concrete to the steel.

This property is made use of in other ways too. For instance, in the tensile zone of a reinforced concrete member, cracks in the concrete in the tensile zones usually develop as it becomes strained beyond a limiting value of strain but because of the bond between the two materials, these cracks become more evenly distributed and consequently the size of individual cracks can be controlled. This also happens in unstressed concrete which is used for slabs and walls when fine steel mesh is used to limit the size of cracks which develop when shrinkage develops. Cracking on a much smaller scale also occurs in unstressed concrete whether or not steel is present. As concrete hardens and shrinkage of the cement paste commences, tensile stresses develop within the paste itself and between the aggregate particles and the paste. Some of these stresses are relieved by two processes which occur simultaneously; yielding of the weaker material, i.e. the paste, leading to microcracking which occurs at the most highly stressed points and creep of the matrix which is caused by compression of the gel (formed in the hydration of the cement) due to the localised stresses. While the concrete is very young, creep is probably the dominant factor but, as the strength, and also the rigidity of the paste increases with time, cracks have been shown to form in the paste and also at the interfaces of the paste and the particles of aggregate, particularly when the latter has sharp angular surfaces. Of course, as the tensile strength of the matrix develops, the amount of cracking is controlled and may stop altogether when internal equilibrium has been reached.

Tensile stresses can also be developed in a mass of plain concrete when external loading is absent by virtue of the fact that while the concrete is drying out, which can take place over some years, a humidity gradient usually exists between the centre of the mass and the surface. Because the concrete nearer the surface dries out more quickly than that nearer the centre, shrinkage proceeds more rapidly in the outer regions which causes the concrete near the centre to be put into a state of compression. This causes what are known as differential shrinkage stresses which may in turn cause cracking in or near the surface of the concrete.

The type of shrinkage described in the foregoing paragraphs is irreversible and most of this occurs in the first few months after placing of the concrete as the following figures given by Neville(27) indicate:

14 to 34 per cent of 20-year shrinkage occurs in 2 weeks;

40 to 80 per cent of 20-year shrinkage occurs in 3 months; and

66 to 85 per cent of 20-year shrinkage occurs in one year.

This general picture is confirmed by Rao(42) amongst others. It is worth noticing in passing, however that concrete can change volumetrically in other ways which are caused by, for example, temperature changes; changes in the relative humidity of the surrounding atmosphere (lower relative humidities lead to increased rates of shrinkage); changes due to elastic deformation under load. The volumetric changes due to these factors are, however, all reversible and are not of direct importance to the subject of this discussion.

9.2. FACTORS AFFECTING SHRINKAGE

The complicated subject of shrinkage in concrete, particularly with reference to the factors affecting its magnitude and methods of control, are dealt with fully by many authorities including Neville, Rao[42] and Billig[43] and consequently will not be pursued at great length here but it is worth considering some aspects of the subject in view of the fact that it is thought to have an important influence on the strength and the behaviour of columns containing cast in steel plates.

9.2.1 Mix Content

One of the factors judged by many authorities to have the greatest influence on the shrinkage of concrete is the volume of the aggregates used in the mix. The particles of rock (crushed stone or gravel) which form the largest constituent part of any mix act as restraints to the overall shrinkage of the matrix, particularly in young concrete when the strength of the cement paste is low. It follows that the greater the proportion of these rock particles in the mix, the greater will be the restraint and hence the less will be the shrinkage. It is of interest to note that an attempt was made to quantify this by L'Hermite[44] who compared the shrinkage of neat cement paste with concrete containing different proportions of coarse aggregate but he found that the effects of creep of the cement paste tended to give rise to somewhat inaccurate results when the relationship he had derived was used to predict amounts of shrinkage.

The strength of the aggregate particles is not itself of much importance with reference to shrinkage but the modulus of elasticity of these particles, which is often a function of the strength, is an important factor. Soft particles of aggregate will provide less restraint than stronger ones because the rigidity of the former is smaller. Thus the shrinkage of a concrete made from aggregate of granite, quartz or hard limestone will be less by up to two times the shrinkage of a mix made from sandstone or expanded shale.

The size and grading of aggregates do not apparently influence the amount of shrinkage directly but, by using larger sizes of aggregate a leaner mix can be used which shrinks less than a richer one, owing to the smaller amount of cement used in the leaner mix. Also, concrete containing smooth surfaced non-absorbent aggregates will have a reduced tendency to shrink because the amount of cement paste required will be reduced in comparison to the amount required for a mix of the same strength which contains angular, rough and water absorbent aggregates.

To sum up, for mixes of the same strength containing aggregates of the same size but in different amounts, the mix of lower workability will shrink less and this mix will contain the greater proportion of aggregate. Moreover the water content is usually less in a mix of low workability and as this will further increase the relative aggregate content, shrinkage will be reduced. It is held by Neville however that the water content per se is not an important factor although Hillig goes so far as to say that about 80 per cent of shrinkage is influenced by water content.

9.2.2 Effect of Curing

There is still a considerable degree of controversy surrounding the influence that curing exerts on the shrinkage of concrete which has not yet been entirely resolved. There is no doubt however that prolonged curing in conditions of high relative humidity delays the advent of shrinkage although it does not apparently affect the amount of shrinkage to any great extent. Of course, curing concrete in moist conditions allows a greater proportion of the cement to become hydrated which leads to an increase in strength compared with concrete that has not been cured properly. This in turn leads to a reduction in the amount and size of cracks although it has been found that the rate of shrinkage can actually be increased after prolonged curing. However, because a lower capacity for creep is associated with higher strength and an increased rate of shrinkage, increased cracking may be caused by prolonged curing. This conclusion, reached by Neville[45] is disputed by some workers who argue that, generally speaking, curing helps to reduce the effects of shrinkage particularly with regard to cracking.

With these considerations in mind it is now proposed to consider the effects of shrinkage on the column in which a vertical steel plate is cast.

9.3 NORMAL FORCES ON PLATE

It was shown in the last chapter how the frictional stresses which acted on a vertical plate cast into a column could be estimated by loading the plate downwards in a column in which a horizontal hole had been left beneath the lower edge of the plate so that bearing stresses were eliminated and it was shown that a value of 76 lb/sq.in. was obtained for the columns tested. The existence of a frictional stress implies, of course, a normal stress acting on the face of the plate and it is a simple calculation to estimate the value of this stress if a suitable value of the coefficient of friction, μ, is used. A value for the coefficient of friction given in CP 2007 (1960)[46] to be used for prestressing wire in contact with smooth concrete is 0.55 and although the conditions are somewhat different in the case of the contact between concrete and the cast in plate this value is not perhaps too unreasonable to use in an approximate calculation. If f_f is the frictional stress on the plate then the normal stress is f_f/μ. For the values of f_f and μ given above the mean normal stress on the plates was $76/0.55 = 138$ lb/sq. in. and this is clearly not a negligible quantity.

9.4 SHRINKAGE STRESS HYPOTHESIS

The following hypothesis has been put forward to describe the distribution of stresses in the concrete before downward loading of the plate commences. The normal stresses on the plate caused by the contraction of the concrete on to it must be caused by compressive stresses in the concrete and these in turn must be balanced by tensile stresses elsewhere in the concrete. Tensile stresses would be expected to occur just above the upper edge and just below the lower edge of the plate where the maxima of their distributions would occur, as indicated in Fig.9.1. This figure shows an idealization of the stresses which are considered to act in the concrete normal to the vertical axial plane of the column in which the plate lies. Obviously, for the equilibrium condition to be satisfied, the areas of compressive and tensile stress must be equal. Also the sudden change from compressive to tensile

DIAGRAM OF HYPOTHETICAL STRESSES
ACTING NORMALLY TO THE AXIAL PLANE

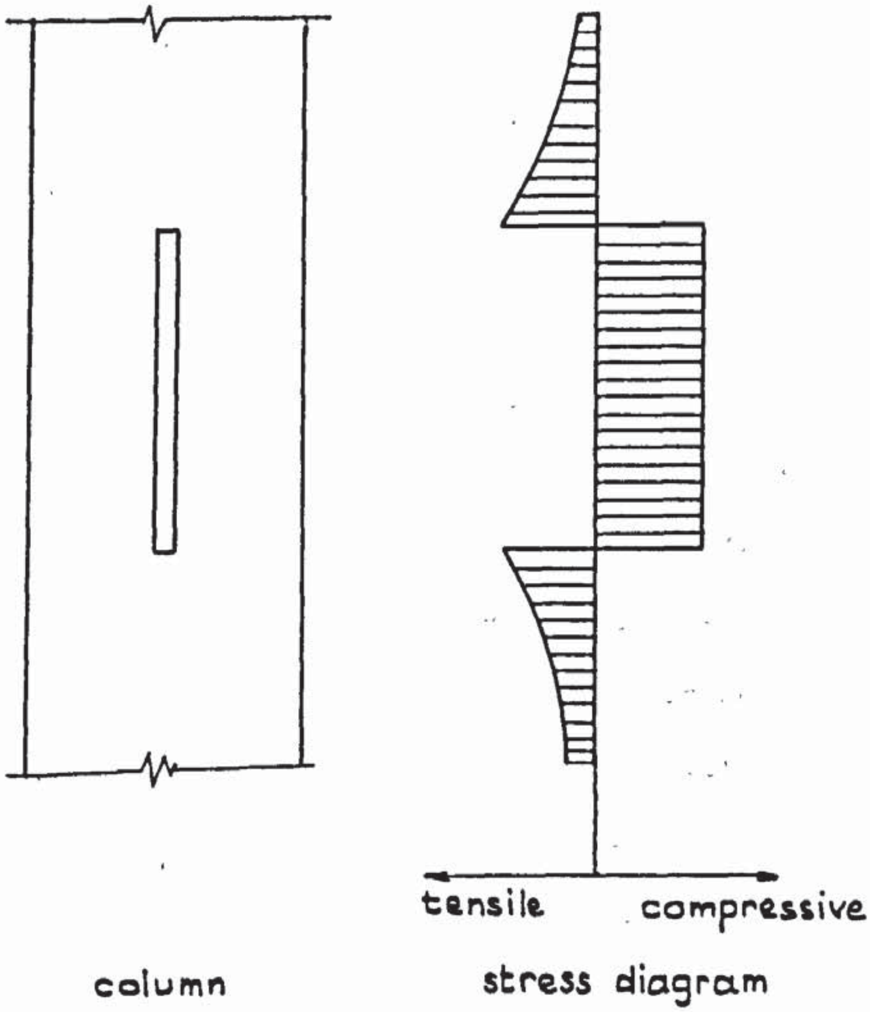


Figure 9.1

stress at the levels of the upper and lower edges of the plate would be more gradual although it is felt that the maximum tensile stresses would occur very close indeed to the edges of the plate.

It is considered that it was primarily the existence of these tensile stresses which caused the considerable discrepancies between the absolute values of the theoretical and the experimental transverse strain distributions in the columns which were described in Chapter 6 and also, and more importantly, the main factor which accounts for the differences between the failure loads of 6 in.square columns and 6 in. cubes when different cube strengths and mix characteristics were used. This is because in the columns, the tensile stresses which were caused by bearing of the lower edge of the plate on to the concrete (loading assumed downwards) were augmented by the stresses which already existed in the concrete due to shrinkage. This would result in a lower bearing capacity of a column compared with that of a cube under similar conditions of loading, strength of concrete and curing conditions. It follows that if the shrinkage stresses, and hence the tensile stresses in the concrete above and below the plate could be controlled, or even eliminated, the bearing capacity of a column could be enhanced. It was decided to test this hypothesis experimentally by using the testing procedure described below.

9.5 TESTS AND RESULTS

In order to allow shrinkage of the concrete surrounding the cast in plate of a column to take place without inducing the concomitant tensile stresses above and below the plate, it was necessary to separate the plate from the surrounding concrete. A set of three 10 in. square columns of standard length each containing one 12 in. deep plate, $\frac{5}{8}$ in. thick, the upper edge of which was 2 in. below the top of the column, as shown in Fig.51 was prepared in the following manner. A rectangular sheet of expanded polystyrene sheeting, 10 x 12 in. approximately $\frac{1}{8}$ in. thick of the type used for the thermal insulation of walls and roofs, was attached by adhesive tape to both sides of the central portion of each plate. Thus the concrete could be placed round the plate without actually coming into contact with its surfaces. Contraction of the concrete would cause the expanded polystyrene sheeting to be compressed between the plate and the concrete but because of its negligible rigidity, compressive stresses in the concrete due to reaction of the plate would be eliminated. In this way the tensile stresses due to this cause would, it was thought, also be eliminated. The columns were loaded in the usual manner through the loading yoke and the results of these tests IIIB1, 2 and 3 are given in Table 9.1. Three similar unmodified columns were also tested and the results from these tests IIIA1, 2 and 3 are also given in the table. The concrete used for all the columns was from mix 3 (see AppendixB) and the moist curing time of 7 days followed by a drying out period of 3 weeks was standard for each column.

TABLE 9.1

Test No.	f_c p.s.i	f_t p.s.i	P_o ton	f_o p.s.i	$\frac{f_o}{f_c}$	Av. $\frac{f_o}{f_c}$
IIIB1	6910	430	37.0	829	0.1199	0.1188
IIIB2	7070	450	36.8	824	0.1165	
IIIB3	6950	360	37.3	835	0.1202	
IIIA1	7160	440	27.8	622	0.0869	0.0885
IIIA2	5870	345	24.6	551	0.0928	
IIIA3	5950	350	22.8	511	0.8858	

9.6 CONCLUSIONS

It is shown in Appendix C that the mean values of f_o/f_c for IIIA and IIIB were significantly different. If f_o/f_c is taken as a constant for columns loaded under similar conditions and is regarded as being independent of f_c, as in Chapter 7, then it is clear that the failure loads obtained for the columns in which the polystyrene sheeting was incorporated were significantly higher than those obtained from the tests on the unmodified ones. It may be added that the stage at which the central vertical crack became visible to the naked eye was delayed in the case of the modified columns which confirmed the assertion that the magnitude of the initial tensile stresses due to shrinkage of the concrete was reduced when the polystyrene sheeting was used.

* * * *

More practical ways on increasing the strength of the joint is given in the following two chapters, together with an account of the way in which the effect of loads from upper storeys was investigated.

CHAPTER 10

CHAPTER 10

FURTHER TESTS ON COLUMNS

10.1 INTRODUCTION

This chapter describes the remainder of the work that was carried out on 10 in. square columns containing a plain $\frac{5}{8}$ in. thick plate and 6 in. square columns containing a $\frac{1}{2}$ in. plate when loading was symmetrical. Reinforced columns, columns with pre-compression and plain columns were tested and the results are plotted on one set of graphs in order that the differences between them can be appreciated. It is proposed to deal with each topic separately before making a general analysis of the results.

10.2 PLAIN COLUMNS

In order that a general picture could be obtained of the variations of the ultimate loads with cube strength of 10 in. square and 6 in. square columns loaded symmetrically through plain plates it was desirable to test a considerable number of columns of both sizes over a wide range of cube strengths. In order that consistency could be maintained it was necessary to ensure that the curing period and conditions of each column were the same and for this purpose the practice outlined in Chapter 5 was adhered to for the columns tested in all of the tests described in this and the following chapters. The results given in Table 10.1 are for the columns tested in

TABLE 10.1

Test No.	f_c p.s.i.	f_t p.s.i.	P_o ton	f_o p.s.i.
VA1	8420	465	48.5	1086
VA2	7790.	440	30.0	672
VA3	11280	500	39.0	874
VA4	9530	450	39.5	885
VA5	12050	505	59.8	1339
VB1	9760	605	22.4	1393
VB2	9760	605	24.2	1505
VB3	8380	510	22.8	1418
VB4	8380	510	25.8	1605
VB5	9240	560	28.2	1755
VB6	9240	560	23.2	1443
VB7	6450	530	15.4	1145
VB8	6450	530	19.6	1220
VB9	7070	485	24.6	1530
VB10	7070	485	23.0	1431
VB11	5750	530	24.0	1493
VB12	5750	530	21.0	1307
VB13	4580	330	19.6	1220
VB14	4580	330	15.2	946
VB15	11260	575	25.2	1568
VB16	11260	575	27.8	1730

series V. Four 10 in. square columns (VA1 to VA4) of standard length and one long one (VA5) and eight pairs of 6 in. square long columns (total height, 42 in.) (VB1 to VB16 were tested. The results are plotted graphically in terms of f_o v. f_c in Figs. 10.2 and 10.3.

10.3 COLUMN PRE-COMPRESSION

10.3.1 Introduction

It has not, it seems, been the usual practice when design of connexions between precast concrete beams and columns has been considered to take into account the effect of column loads from storeys above the level of a particular connexion, the argument being that the stresses due to the loads transferred at the connexions are localised and consequently have no direct influence on the stresses in other parts of the column. This is clearly correct when the local stresses are mainly of a compressive nature but it was felt that some further investigation was necessary in the context of the present work because it had been demonstrated that tensile stresses are developed in the concrete below the plate due to the relatively narrow bearing area of the plate. It was conceivable that the expansion of the column caused by the Poisson's ratio effect when the column was supporting loads from upper storeys could weaken the connexion at, say, the first floor level.

10.3.2 Loading Procedures

In order to test the above hypothesis some 6 in. and 10 in. square long unreinforced columns were pre-loaded in compression and then load was transferred to a single cast in plate, 12 in. in depth, through jacks mounted on the lower platen of the testing machine. The loading arrangement is shown in Fig.10.1. The 6 in. square columns each contained a $\frac{5}{8}$ in. thick plate and the 10 in. square ones a $\frac{5}{8}$ in. square plate. The column was first pre-loaded in compression to a certain load P_c in tons. When the pump to which the jacks were connected in parallel through a T - junction was activated a total load of P (ton) was applied through the two loading blocks, described in Chapter 5, which were bolted to each end of the column plate. The maximum load which was applied by the two jacks is denoted by P_o and P_o in pounds divided by the cross-sectional area of the column in square inches by f_o. The reaction to the jacks was supplied by the force on the lower platen of the testing machine and in order that the load in the portion of the specimen below the level of the plate could be kept at a constant value, the total force being applied by the jacks was compensated for by an equal increment of load in the compression testing machine. Consequently, as the jacks were loaded, the portion of the column above the upper edge of the plate supported a total load of $P + P_c$ and the

LOADING ARRANGEMENT OF COLUMNS
IN PRE-COMPRESSION

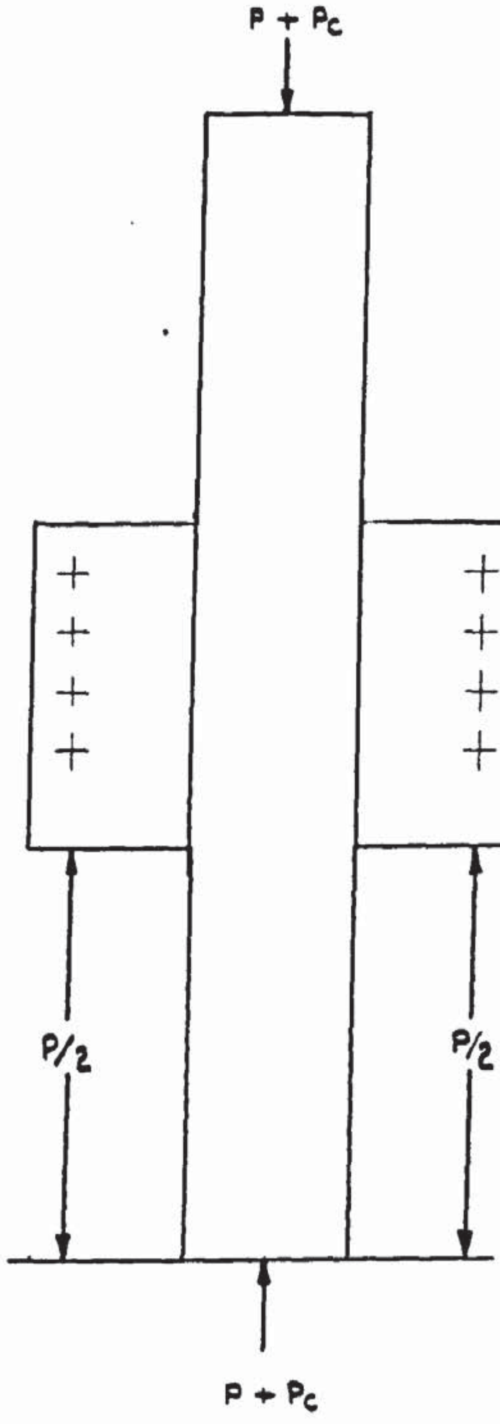


Figure 10.1

portion below the level of the plate supported a load of $P + P_c - P = P_c$. The arrangement is actually inverted with respect to the situation in an actual building but the same principles must apply.

10.3.3 Results and Conclusions

The results from these tests are given in Table 10.2. It will be noticed that VIB1 and VIB2 were tested in compression only, no load being applied to the column plate. The results of this section and the previous one, together with the results of series IIIA are plotted in Figs.10.2 and 10.3. The results for the tests on the 10 in. square columns are shown in Fig.10.2 and for the tests on 6 in. square columns in Fig.10.3. In each case the regression lines for f_o upon f_c have been drawn together with the 95 per cent confidence limits for the regressions. The equations of the regressions are, for the 10 in. square columns,

$$f_o = 0.1043\, f_c - 38 \qquad (10.1)$$

and for the 6 in. square columns,

$$f_o = 0.0749\, f_c + 834 \qquad (10.2)$$

The relevant calculations together with tests of significance of the respective correlation coefficients for the two plots are given in Appendix C.3. It is worth remarking here that the relatively large spread of the confidence limits of the regression shown in Fig.10.2 reflects the relatively small number of samples that were tested.

TABLE 10.2

Test No.	f_c p.s.i.	f_t p.s.i.	P_c ton	P_o ton	f_o p.s.i.
VIA1	10250	515	174.4	44.6	999
VIA2	10810	515	173.0	48.0	1075
VIA3	8770	635	78.0	52.0	1164
VIB1	10800	520	94.5	0	0
VIB2	10800	520	81.0	0	0
VIB3	11300	575	49.8	28.2	1754
VIB4	11300	575	48.8	28.2	1754
VIB5	10340	570	26.2	25.8	1605
VIB6	10340	570	22.2	25.8	1605
VIB7	11520	540	13.1	23.4	1456
VIB8	11520	540	12.5	29.0	1804

From the two graphs it seems very likely that the results of series VI are from the same populations as the remainder of the points plotted for each column cross-section. Thus is seems reasonable to conclude that pre-compression of the columns does not affect the bearing capacity of the joint.

PLOTS OF f_o V f_c FOR 10 in. SQUARE COLUMNS

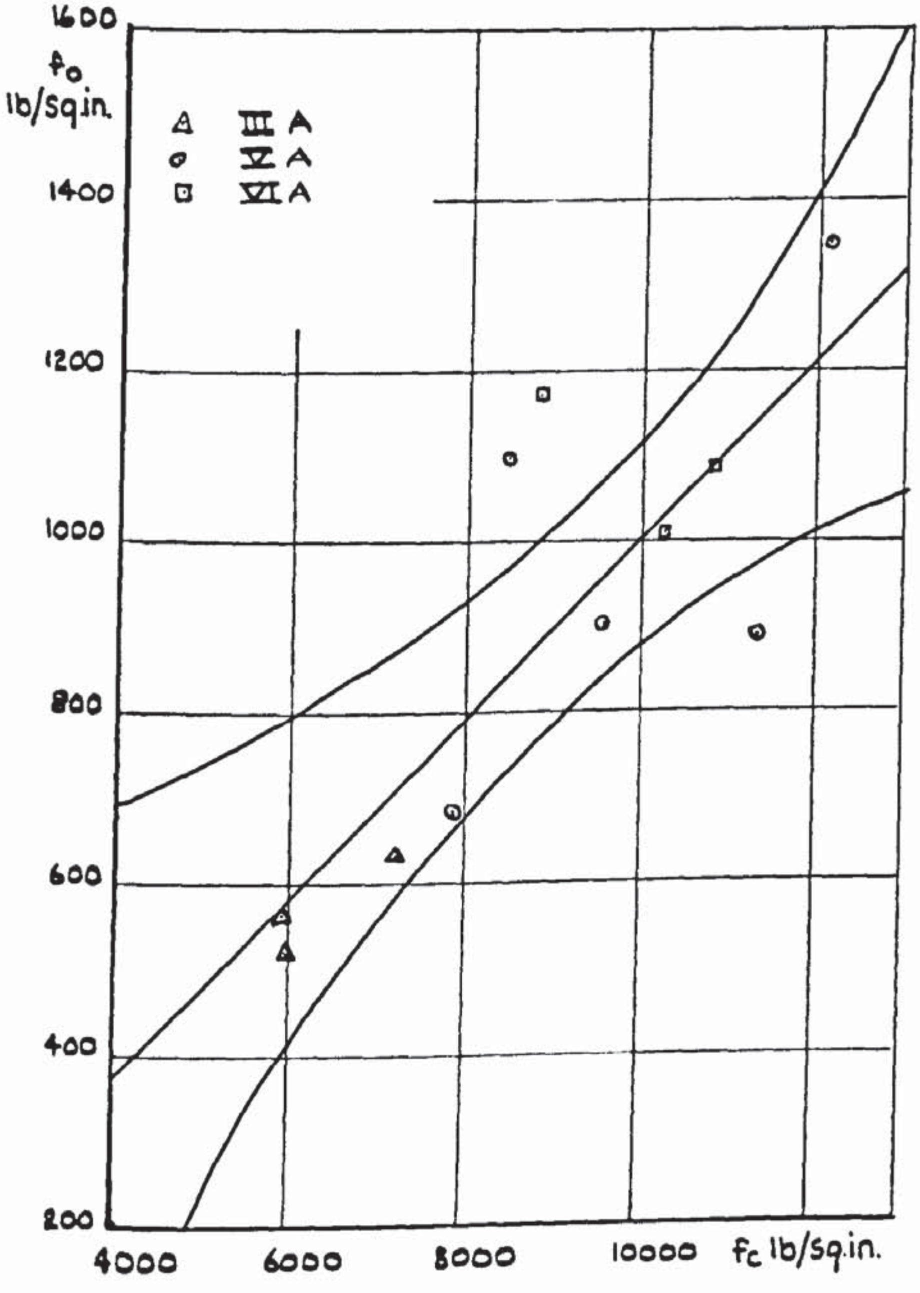


Figure 10.2

PLOTS OF f_o V f_c FOR 6in. SQUARE COLUMNS

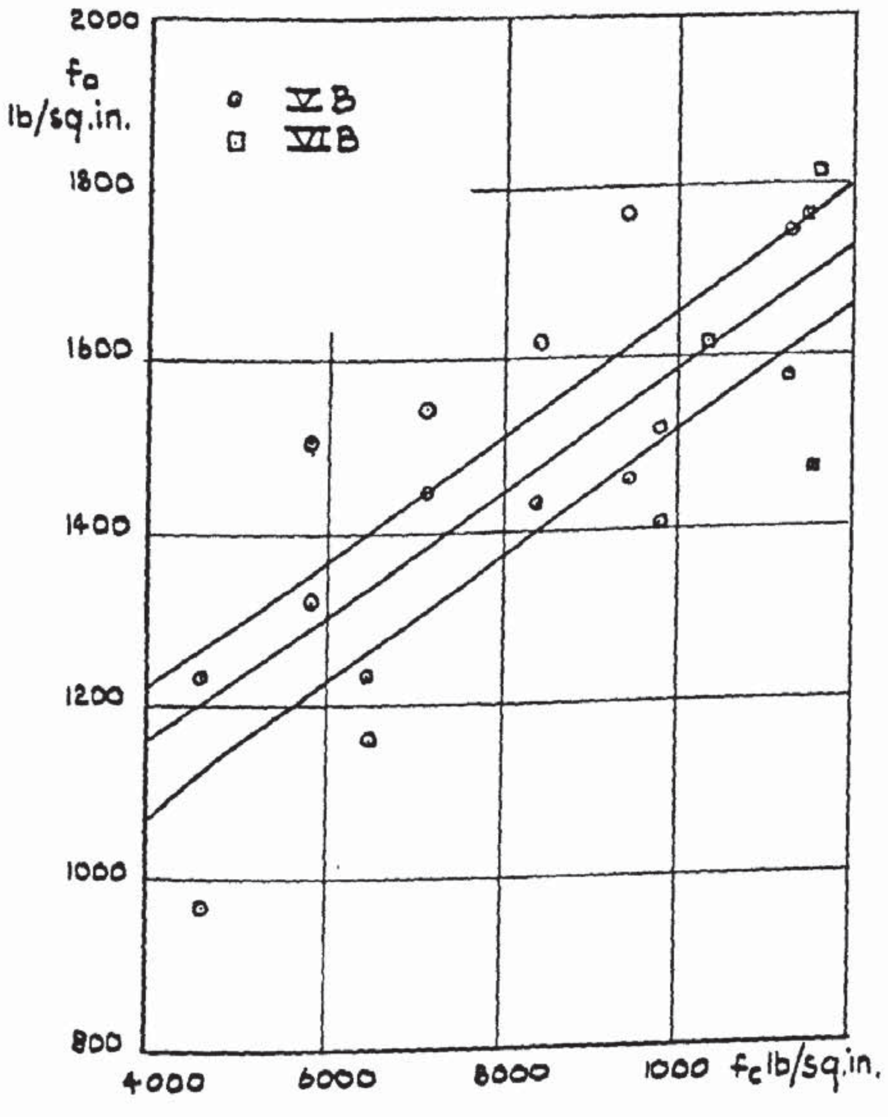


Figure 10.3

10.4. REINFORCED COLUMNS

10.4.1 Introduction

It is clearly of some importance to ascertain the effect on the bearing capacity of a column of reinforcements particularly of link or transverse reinforcement. As all columns used in actual structures have to have a certain amount of transverse reinforcement any increase in the bearing capacity achieved by this means would be provided at little or no extra cost as long as the arrangement was simple. Two different arrangements were investigated, one in which a link was actually in contact with the plate and one in which a gap separated the link from the plate (this gap was in all cases nominally 2 in.).

10.4.2 Specimens

Three 10 in. square columns of standard length were reinforced according to the scheme of Fig.10.4. Ordinary round mild steel was used throughout the tests, consisting of, for the main steel, $\frac{1}{2}$ in. diameter and for the links $\frac{1}{4}$ in. diameter. The mechanical properties of the steel used are given in Appendix B.2. These specimens were used in series VIIA. Three similar columns were also tested in the same manner using the centrally loaded yoke to apply symmetrical loading to the plate but the link S was omitted. These comprised series VIIB. A further four columns of series VIIC were reinforced in the manner shown in Fig.10.5. In

REINFORCEMENT OF 10in. SQUARE COLUMN

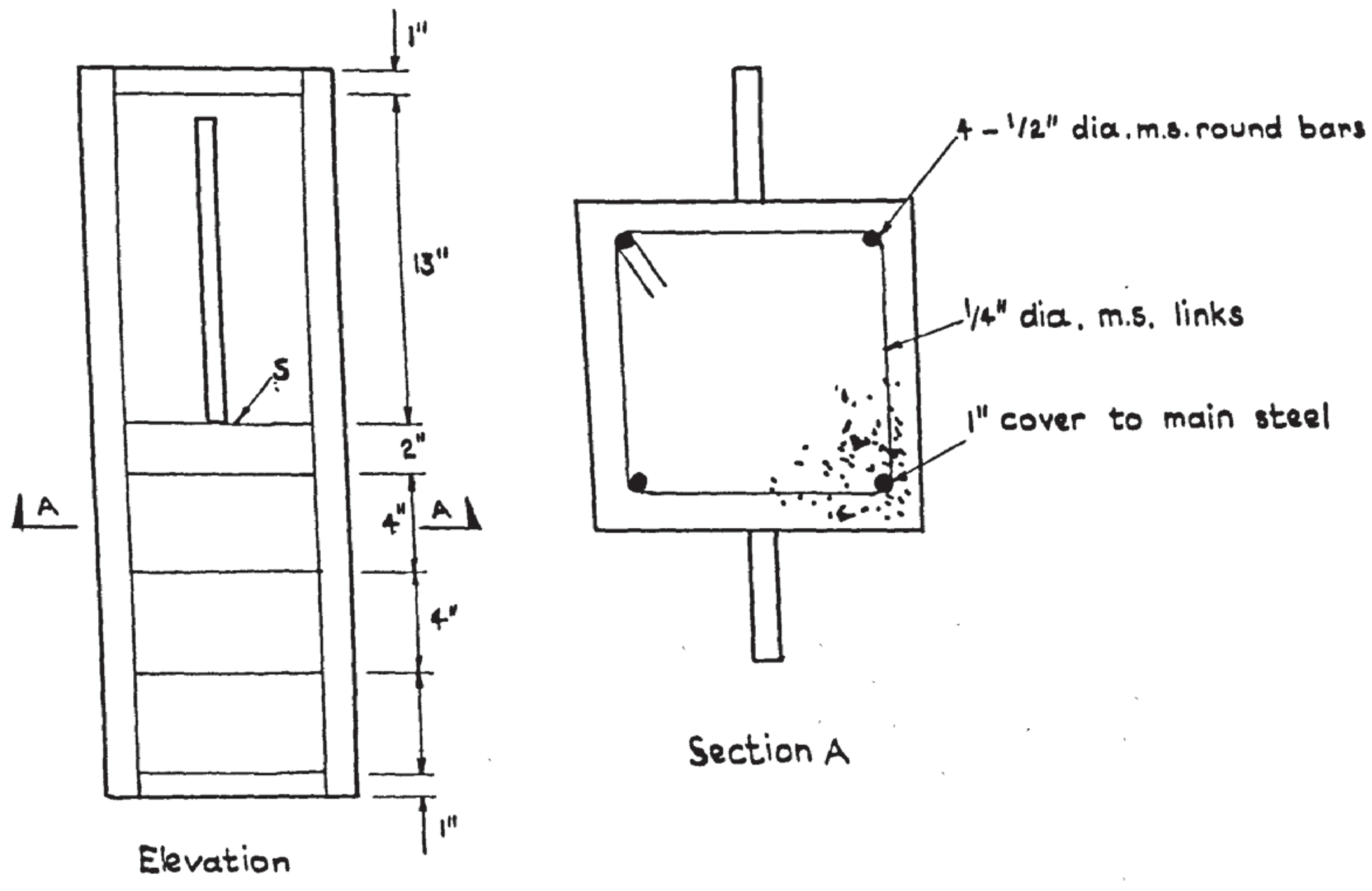


Figure 10.4

REINFORCEMENT OF 6in. SQUARE LONG COLUMN

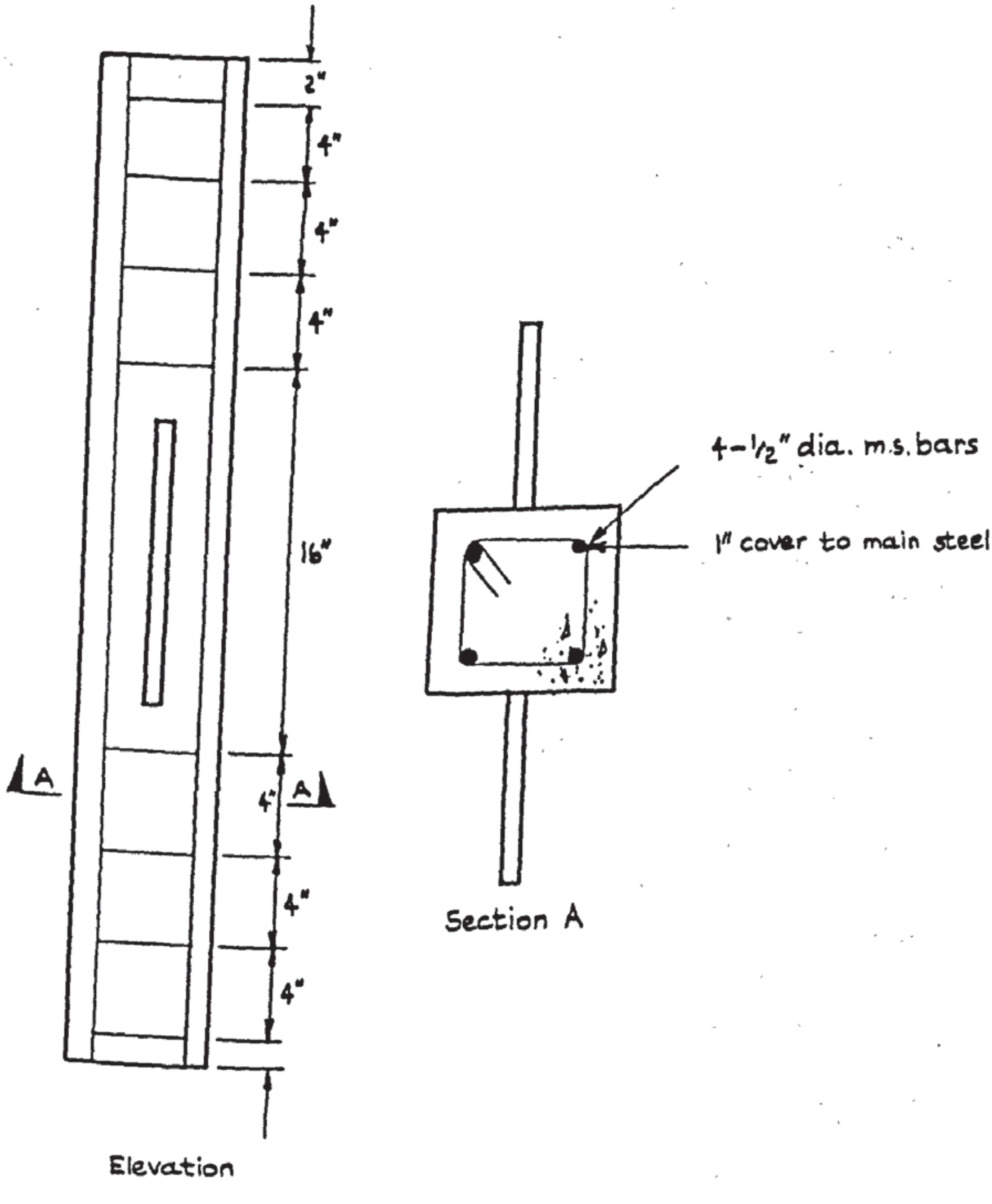


Figure 10.5

Plate 8

In case of these columns, the direction of loading was upwards and load was applied through the two jacks connected in parallel to the pump, as described in section 10.3.2.

10.4.3 Mode of failure

A view of a column in which link S was incorporated is shown in Plate 8. It will be observed that the extent of the spalled area beneath the plate was larger than was the case for the unreinforced columns (compare with Plate 4) and this was found to be the case for the three columns of series VIIA. It can also be seen from Plate 9 how the link adjacent to the lower edge of the plate was bent and this was clearly what caused the large amount of spalling. However, this bending did not occur until the column had cracked considerably and the plate was being pushed downwards, for the most part after the maximum load had been reached.

The mode of failure of the specimens in which the link S had been omitted was similar to that of unreinforced columns except that the portion of the column below the plate in series IIIB and above the plate in series IIIC did not split completely apart when the ultimate load had been reached due to the presence of the transverse reinforcement. The area of spalling was also similar in extent to that in unreinforced columns.

10.4.4 Results and Conclusions

The results of the tests on reinforced columns are given in Table 10.3.

TABLE 10.3

Test No.	f_c p.s.i.	f_t p.s.i.	P_o ton	f_o p.s.i.	$\frac{f_o}{f_c}$
VIIA1	10150	515	75.0	1680	0.1655
VIIA2	8570	360	59.5	1332	0.1554
VIIA3	10340	595	76.0	1762	0.1646
VIIB1	12750	505	67.0	1500	0.1177
VIIB2	12410	575	64.5	1444	0.1164
VIIB3	12410	575	68.5	1534	0.1232
VIIC1	9920	450	27.6	1717	0.1731
VIIC2	9920	450	27.6	1842	0.1857
VIIC3	10080	560	27.8	1729	0.1715
VIIC4	10080	560	29.6	1841	0.1826

The best way of examining these results is to compare them with those from the tests on unreinforced columns which were plotted graphically in Figs.10.2 and 10.3. For clarity only the regressions and the 95 per cent confidence limits have been drawn in Figs.10.6 and 10.7 which show the plots of results of Series VIIA and VIIB, and Series VIIC respectively. If the values of the failing loads for Series VII are compared with the expected

PLOTS OF f_o V f_c FOR SERIES VIIA AND VIIB

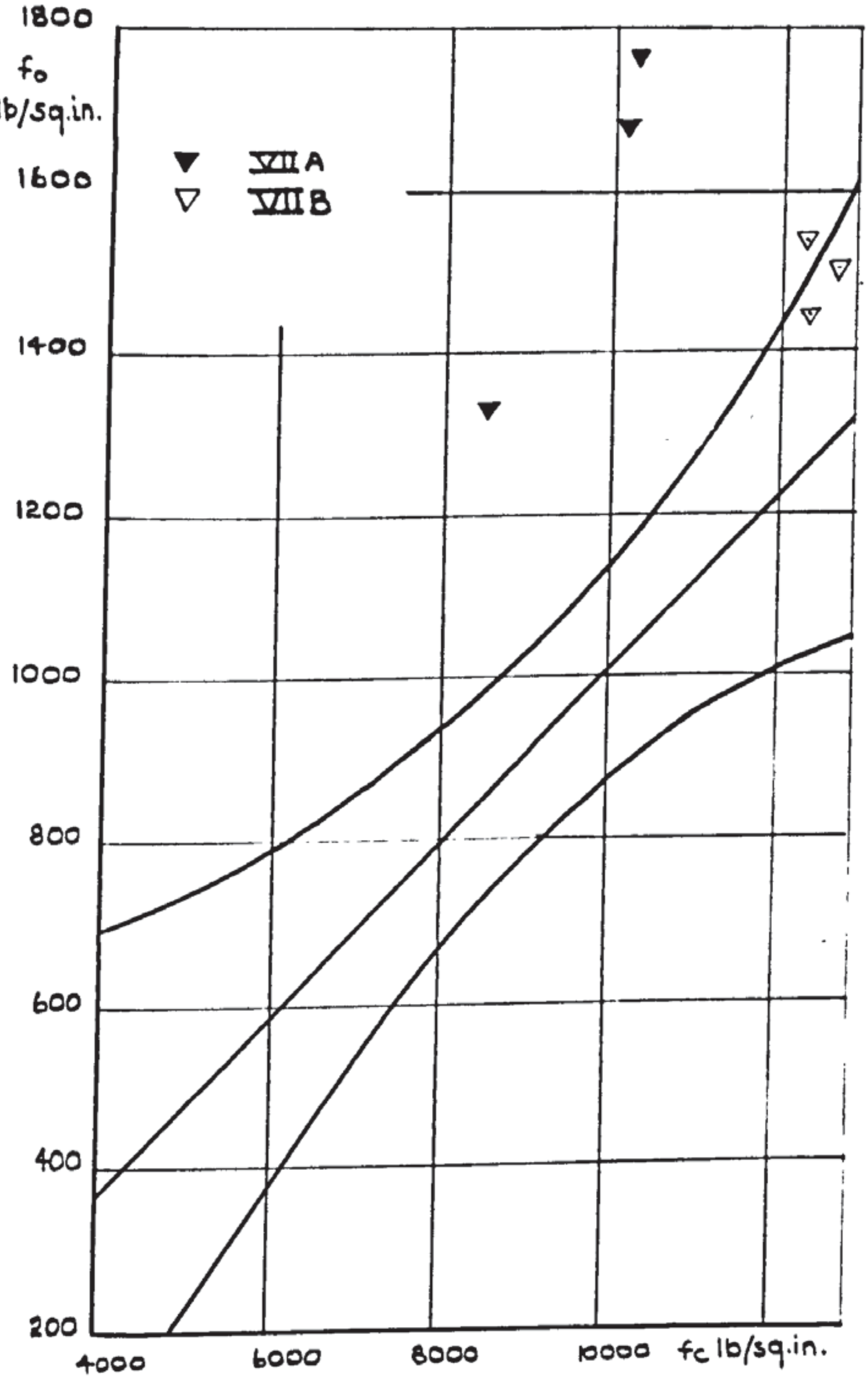


Figure 10.6

PLOTS OF f_o V f_c FOR SERIES VII C

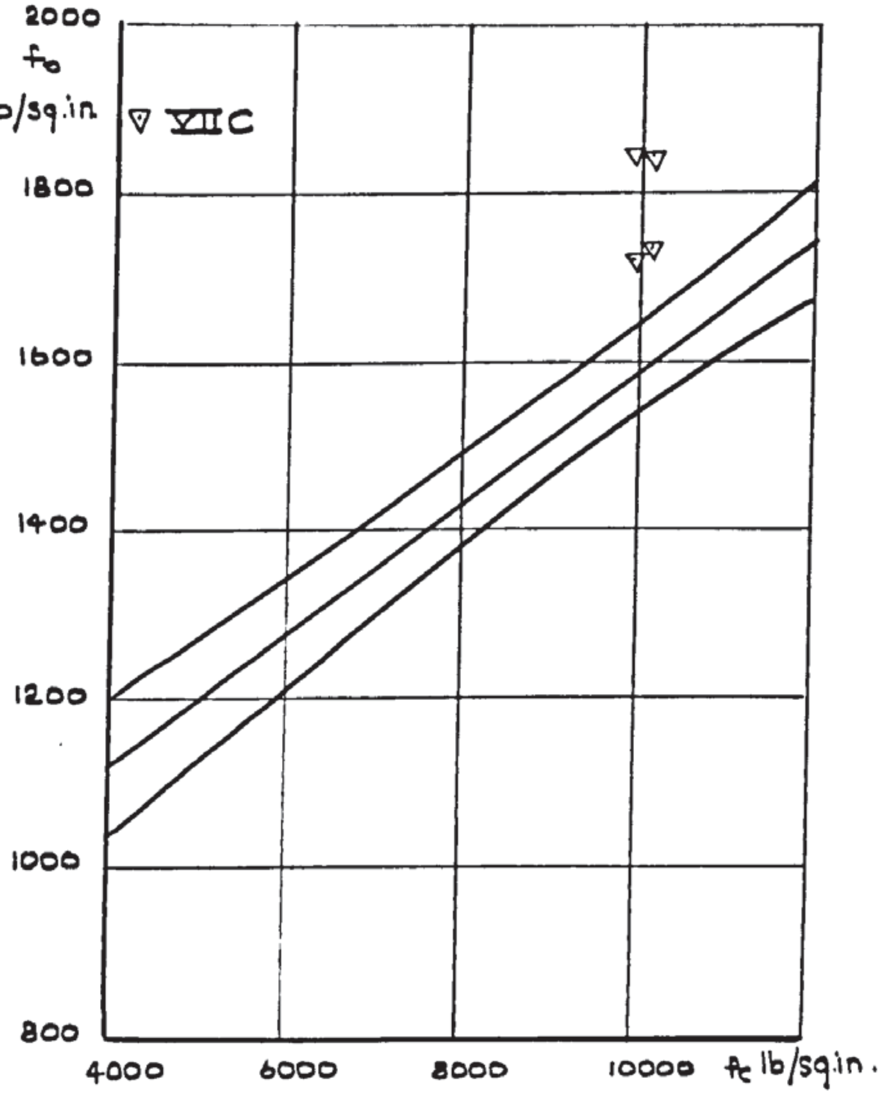


Figure 10.7

failing loads calculated from the regressions it is found that, for series VIIA, f_o was 62.5 per cent greater on average than for a plain column; for series VIIB f_o was 17.7 per cent greater and for series VIIC, 12.6 per cent greater. The fact that the last two percentages are relatively small shows that the incorporation of a ¼ in. diameter mild steel link 2 in. from the lower edge of the column plate (assuming that the direction of loading is downwards) ensures that a column will fail at a load slightly above that predicted for an unreinforced one of the same concrete strength. For the columns in which a link was actually in contact with the plate however, a considerably greater load could be supported. These results indicate that in normal columns which are adequately reinforced, the regression line of Figs. 10.2 and 10.3 can be regarded as the lower limit of the ultimate bearing capacities of 10 in. square and 6 in. square columns.

10.5 RECTANGULAR COLUMNS

Although all the tests reported in this thesis were performed on columns of square cross-section it is a simple matter to compute the ultimate load of a rectangular column when loaded in a similar manner. It has been shown how the ultimate load of a square column when loaded through a relatively thin cast in plate set vertically on a central longitudinal plane can be expressed as

$$f_o = \frac{P_o}{D_1 \times D_1} \quad \text{or} \quad P_o = D_1^2 f_o$$

where P_o is the ultimate load and D_1 is the dimension of each side. Consequently the reaction per unit length of plate is $D_1 f_o$ where D_1 in this case is the dimension of the side through which the plate protrudes. If a column has dimensions D_1 and γD_1 in which the side parallel to the plane of the plate is of length D_1 then the reaction per unit length of plate is still $D_1 f_o$ if the plate thickness is the same in both cases and the ultimate load P_γ is given by

$$f_o = \frac{P_\gamma}{D_1 \times \gamma D_1}$$

$$\text{or } P_\gamma = \gamma D_1^2 f_o = \gamma P_o$$

10.5.1 Example

Find the ultimate load of 6 x 9 in. column whose longer dimension is parallel to the plane of a ½ in. thick cast in plate; the cube strength is 6000 lb/sq.in.

Solution From equation (10.2)

$$f_o = 0.0749 \times 6000 + 834$$

$$= 1283 \text{ lb./sq. in.}$$

$$\gamma = 1.5$$

therefore $P_\gamma = 1.5 \times 6 \times 6 \times 1283$

$$= 30.9 \text{ ton}$$

CHAPTER 11

CHAPTER 11

UNSYMMETRICAL LOADING OF COLUMNS

11.1 INTRODUCTION

All the work reported so far was carried out on columns in which the cast in plate was symmetrically loaded by applying equal loads through bolts located close to the ends of the plate which were at equal distances from the faces of the column. It was clearly of some interest to be able to predict the bearing capacity of a column when unequal loads were applied as this situation could arise in practice, particularly at a corner column. For this purpose a limited testing programme was initiated to enable the affects of applying an out of balance moment to a column plate to be examined.

11.2 TESTING PROCEDURE

All but two of the 6in. square columns that were tested were reinforced according to the scheme shown in Fig. 10.5. One pair of unreinforced columns was also tested for purposes of comparison. Long columns only were used, for convenience, and load was applied through the jacks which were connected in parallel to the pump for the majority of the tests. Thus equal loads could be applied to each end of the column plate (thickness ½ in.) and in order to apply moment the distances between the jacks and the faces of the columns were varied in different tests. The loading arrangement which was used for the tests is shown schematically in Fig.11.1. The distance of the points at which load was applied from the faces of the column is denoted by e_1 and e_2 in inches. In order that standard plates could be used in the columns an extension piece was made which consisted of two ½in. thick portions of plate, 12in. deep which were lap welded together so that no out of plane moments would be induced in the column when it was bolted to one end of a standard plate and loaded through a loading block (described in Chapter 5) which was bolted to it. The loading block could be located in one of three positions as can be seen from Plate 9 which shows the general arrangement before testing. The rollers can be seen in position between the jacks and the loading blocks; these were used so that rotation of the plate could proceed unhindered as load was increased. Plate 10 shows a column after being loaded on one side only. The two horizontal cracks which formed parallel to the upper and lower edges of the plate as the ultimate load was approached can be seen in this photograph. The loading block wedged between the jack and the left hand face of column provided, together with the horizontal hollow steel beam section seen in the top right hand corner, restraint against overturning of the column. Added to this a small pre-compression of the column was applied which was of the order of 10 ton. Some gauge readings were also taken but they added little to the general understanding of the behaviour of the

LOADING ARRANGEMENTS FOR COLUMNS LOADED UNSYMETRICALLY

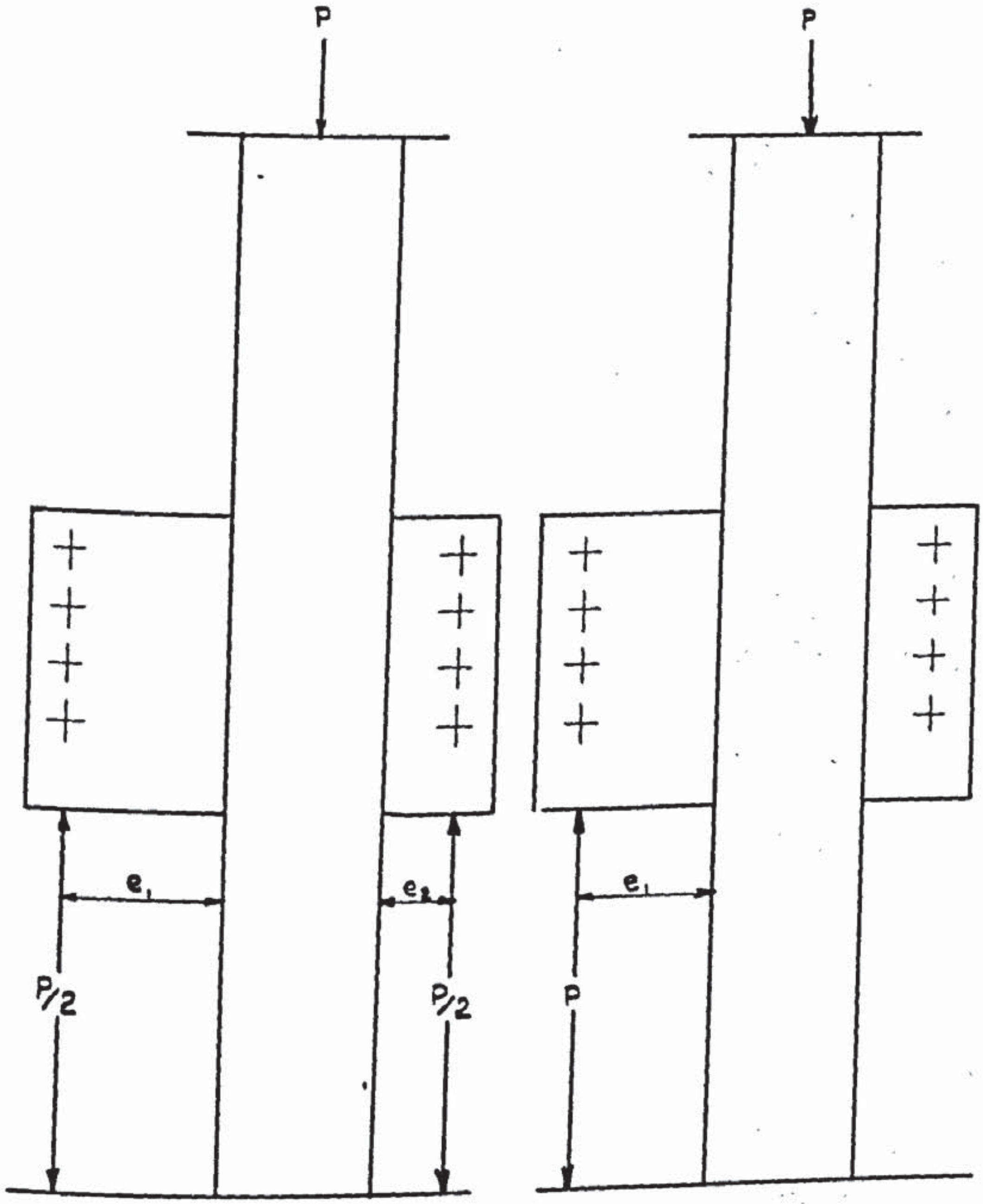


Figure 11.1

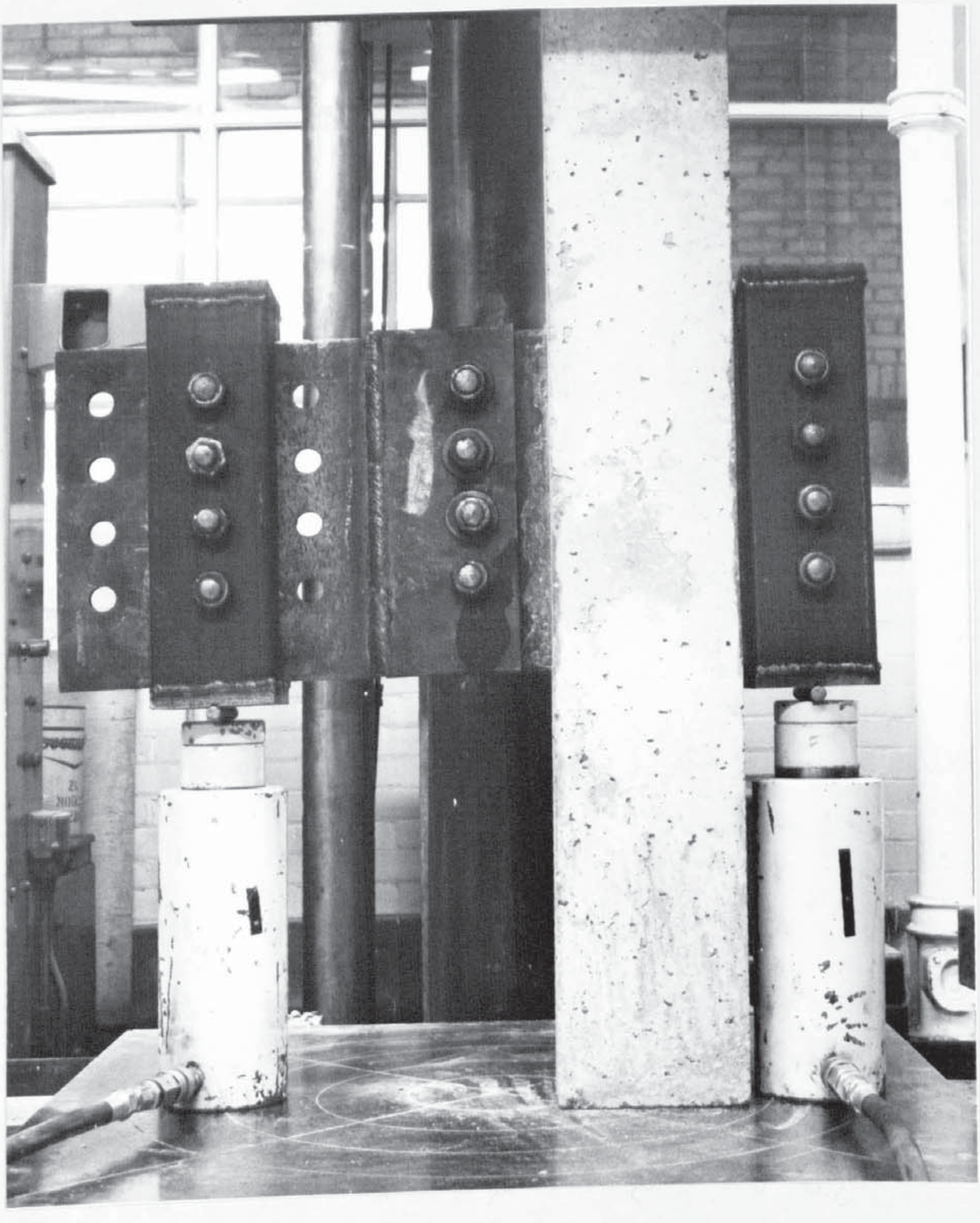


Plate 9

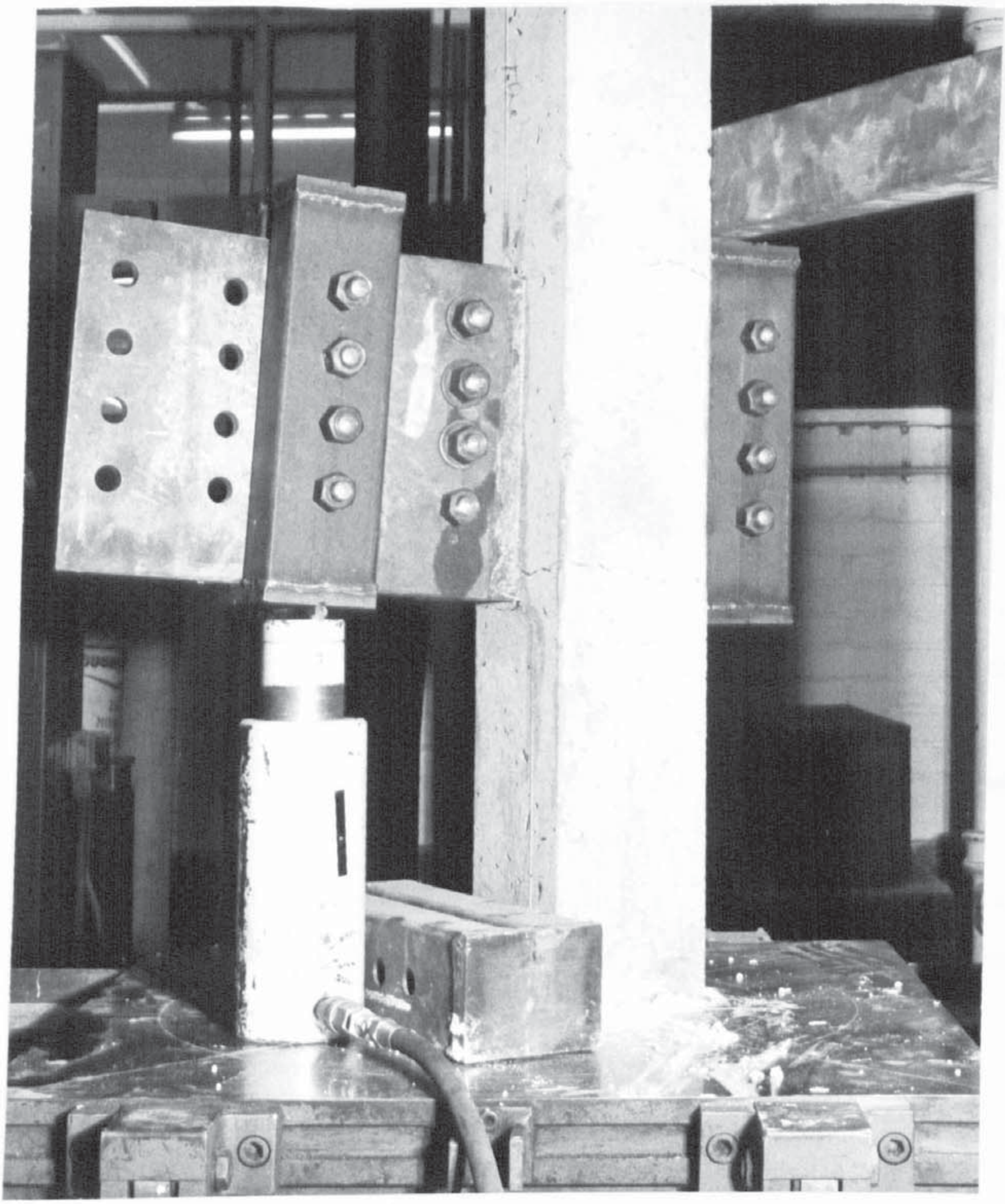


Plate 10

connexion and are therefore not reported here.

11.3 MODE OF FAILURE

The mode of failure of all the columns was dominated by the formation of vertical cracks above and below the plate on the centre lines of the column faces through which the plate protruded. In the case of the columns loaded through both jacks it was observed that at failure the longest crack and the most extensive spalling always occurred above the upper edge of the plate on the side of the column which was furthest from a jack. A shorter crack also occurred on the opposite face above the plate but little cracking was observed below the plate at failure. In the case of the columns loaded on one side only the pattern was similar except that there was only a small crack above the plate on the unloaded side but a quite considerable one beneath the plate on that side, which was almost as long as the one which formed above the plate on the loaded side. As failure was approached the two horizontal cracks referred to in the previous section developed but were limited somewhat by the presence of the reinforcement.

Moreover, if the pre-compression had been greater this horizontal cracking would have been further limited but nevertheless its existence was not considered to have appreciably influenced the values of the loads which caused the ultimate vertical splitting of the columns.

11.4 RESULTS

The results of all the tests are given in Table 11.1. Tests VII B1 and VII B2 were on unreinforced specimens. P_m is the load in tons at which a particular column failed and is the combined maximum load supplied by the two jacks when used together or the maximum load supplied by the one jack when used alone. P_o is the maximum load in tons at which an unreinforced column of the same cube strength and cross-sectional dimensions as a reinforced one would be expected to sustain, and in the case of the columns used for the tests described here was calculated from equation (10.2). M is the moment about the column centre line in in.-ton. These results are plotted in Fig.11. 2 which is a plot of P_m/P_o against M/P_oD which are non-dimensional parameters. The D in this case is the length of the embedded portion of the plate and was for the tests described here, always 6in. The straight line which has been drawn below the plotted points is a theoretical one which is arrived at by induction in the following analysis. The equation of this line is -

$$\frac{P_m}{P_o} = 1.1 - \frac{2.0M}{P_oD} \qquad (11.1)$$

TABLE 11.1

Test No.	f_c p.s.i.	f_t p.s.i.	P_m ton	P_o ton	e_1 in.	e_2 in.	$e_1 - e_2$ in.	$\frac{P_m}{P_o}$	$\frac{M}{P_o D}$
VIIA1	11410	495	22.4	27.12	8.0	2.5	5.5	0.8259	0.4045
VIIA2	10950	605	14.0	26.58	15.0	3.0	12.0	0.5267	0.5267
VIIA3	10950	605	13.6	26.58	15.0	3.0	12.0	0.5116	0.5116
VIIA4	10740	485	16.6	26.32	11.0	2.5	8.5	0.6307	0.4467
VIIA5	10740	485	17.2	26.32	11.0	2.5	8.5	0.6535	0.4629
VIIA6	10300	540	29.0	25.79	5.0	2.5	2.5	1.1244	0.2342
VIIA7	10300	540	29.6	25.79	5.0	2.5	2.5	1.1477	0.2391
VIIA8	9270	485	10.8	24.55	8.0	---	8.0	0.4399	0.5865
VIIA9	9270	485	11.8	24.55	8.0	---	8.0	0.4806	0.5865
VIIB1	10970	560	14.9	26.59	2.5	---	2.5	0.5603	0.5136
VIIB2	10970	560	10.8	26.59	2.5	---	2.5	0.4061	0.3273

11.5 ANALYSIS

The applied system of forces acting on a column with a vertical cast in plate just before failure occurs can be taken as equivalent to a moment N about the column centre line and a force P_m which acts on the centre line. A possible distribution of the reactions acting on the plate caused by this equivalent system is shown in Fig.11.3. For equilibrium to be maintained the resultants of the reactions must be equal and opposite to the applied force, i.e.

PLOTS OF P_m/P_0 V M/P_0D

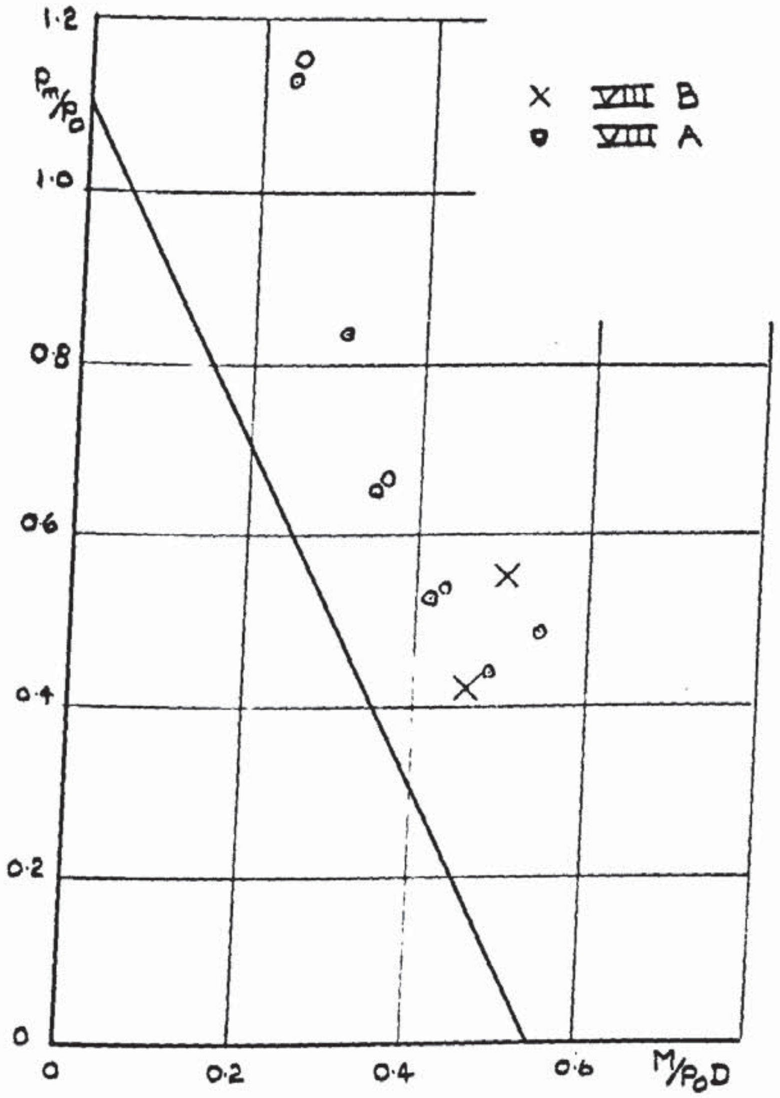


Figure 11.2

REACTION DIAGRAM

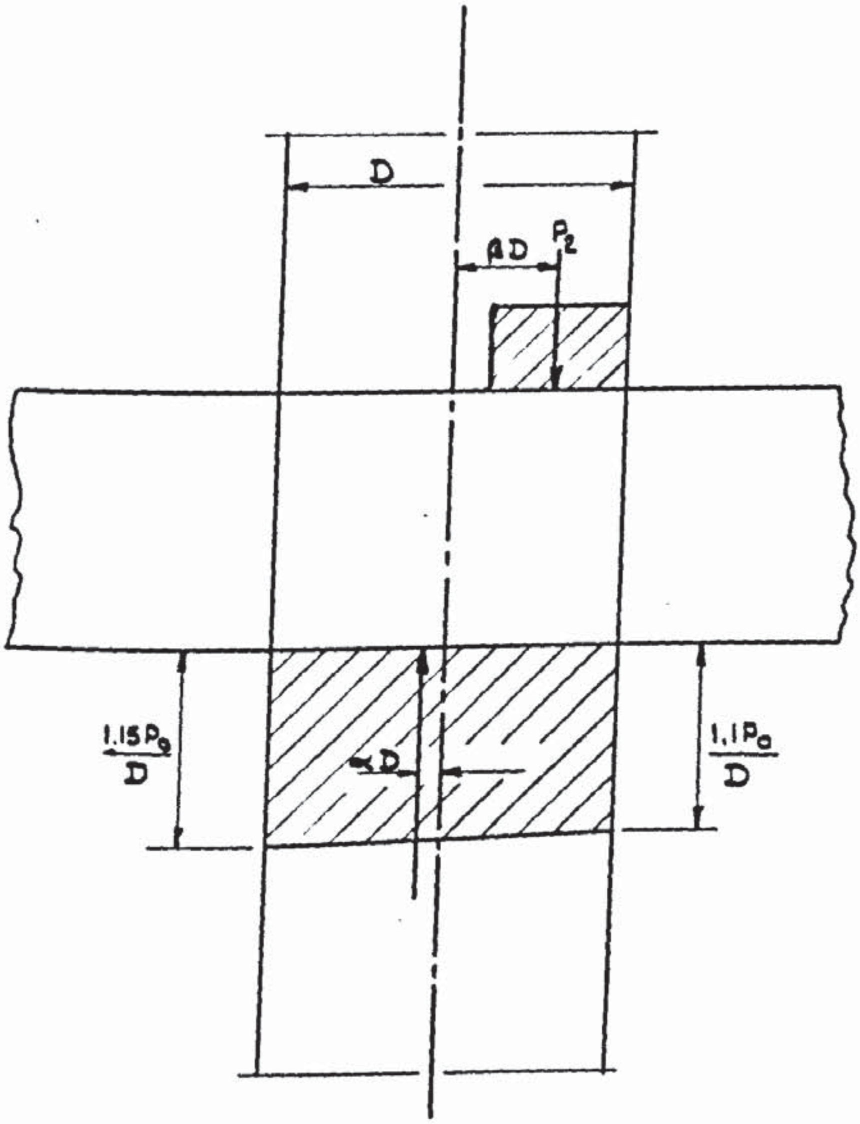


Figure 11.3

$$P_m = P_1 - P_2 \qquad (11.2)$$

and the sum of the moments must be zero so

$$M = P_1 D + P_2 D \qquad (11.3)$$

P_o/D is the reaction per unit length of the cast in portion of the plate which would arise if the plate was being loaded by a symmetrical load in an unreinforced column just before failure. The results of the last chapter showed that the maximum load per unit length of the cast in portion of the plate in a reinforced column was slightly greater than $1.1P_o/D$ and consequently this value has been taken as one of the ordinates of the reaction diagram. The assumption made is that this can be increased by a small amount, say 5 per cent of P_o/D in the local region close to the face of the column which is most highly stressed. This is not unreasonable if it is recalled that the limited region is in the vicinity of a link which allows a certain degree of ductility of the combined material without complete disintegration of the structure. The distribution above the plate is a assumed to vary linearly. The shape of the P_2 distribution is for convenience taken as rectangular but there does not appear to be any required shape to satisfy the equilibrium and moment conditions. The only necessary condition applies to the maximum ordinate of this distribution which should not be greater than $1.1P_o/D$.

Taking moments of area about the centre line for P_1 only to obtain α

$$(1.15 - 1.1)\frac{P_o}{D} \times \frac{D}{2} \times \frac{D}{6} + 1.1\frac{P_o}{D} \times D \times 0$$

$$= \frac{(1.15 + 1.1)}{2} \times \frac{P_o}{D} \times D \times \alpha D \quad (11.4)$$

hence

$$= \frac{0.0500 \times D}{12 \times 1.1250}$$

$$= 0.0037$$

Now equation (11.3) can be re-written as

$$\frac{M}{D} = P_1\alpha + P_2\beta \qquad (11.3a)$$

Substituting equation (11.3a) in (11.1)

$$P_m = 1.1P_o - 2(P_1\alpha + P_2\beta) \qquad (11.1a)$$

substituting equation (11.2) in equation (11.1a) and rearranging,

$$P_1(1 + 2\alpha) + P_2(2\beta - 1) = 1.1P_o \qquad (11.1b)$$

but from Fig.11.3,

$$P_1 = 1.125P_o$$

hence

$$P_2 = P_o\frac{0.0333}{1 - 2\beta} \qquad (11.5)$$

For $P_2 > 0$ $\quad P_o\left(\frac{0.0333}{1 - 2\beta}\right) > 0$

or

$$\beta < \frac{1 - 0.0333}{2} < 0.5$$

which satisfies the condition for $\beta < 0.5$ when $P_2 > 0$

A possible rectangular distribution for P_2 could havd a base length equal to D/3. Then

$$\beta D = \frac{D}{2} - \frac{1}{2}\frac{D}{3} = \frac{D}{3}$$

From equation (11.5)

$$P_2 = P_o \frac{0.0333}{1 - 2x\frac{1}{3}}$$

$$= 0.0998P_o .$$

The height of the rectangle is then

$$P_2 \div \frac{D}{3} = 0.2994\frac{P_o}{D}$$

This satisfies the condition that the maximum ordinate is less than $1.1\frac{P_o}{D}$.

A possible triangular distribution for P_2 could have a base length equal to D/3. Then

$$\beta D = \frac{D}{2} - \frac{1}{3}\frac{D}{3} = \frac{7}{18}D$$

From equation (11.5)

$$P_2 = P_o \frac{0.0333}{1 - 2x\frac{7}{18}}$$

$$= 0.1497P_o$$

The height of the triangle is then

$$P_2 \div \frac{1}{2} x \frac{D}{3} = 0.8982\frac{P_o}{D}$$

This satisfies the condition that the maximum ordinate is less than $1.1\frac{P_o}{D}$.

11.5.1 Example

Find the cube strength required for a 10in. square column to support a 5/8in. thick plate loaded on one side by a load of 20 ton and on the other side by a load of 15 ton, the distances between the lines of action of the loads and the centre line of the column being 7.5in.

Solution

$$M = 7.5 \times (20 - 15) = 32.5\text{in. - ton}$$

$$P_m = 20 + 15 = 35 \text{ ton}$$

Using eqution (11.1)

$$P_o = \frac{1}{1.1} \times \left(35 + 2.0 \times \frac{32.5}{10}\right) = 41.5 \text{ ton}$$

hence $$f_o = \frac{41.5 \times 2240}{10 \times 10} = 929.6 \text{ lb/sq. in.}$$

Using equation (10.1) to obtain f_c for a 10in. square column,

$$f_c = 9260 \text{ lb/sq. in.}$$

This would be the required cube strength based on a somewhat conservative ultimate load analysis.

CHAPTER 12

CHAPTER 12

LOADING THROUGH T - SECTION

12.1 INTRODUCTION

It has already been shown that a connexion consisting only of a relatively thin plate cast into a column and loaded symmetrically can sustain substantial loads. However it was clearly of interest to examine possible ways of increasing the potential bearing capacity of the steel plate type of connexion and some methods have been considered in Chapters 9 and 10. Perhaps the most obvious way of increasing the bearing capacity is simply by increasing the bearing area because by doing so some advantage can be obtained by making use directly of the compressive strength of the concrete. There are several different ways in which this can be achieved; by increasing the width of the simple column plate; by adding a rigid horizontal ledge to the lower edge of the thin column plate which was used in the majority of the tests which have already been described and by adding lugs of various shapes to the vertical surfaces of the plate and so effectively increasing its width. It was described in Chapter 1 how Gifford used an inverted T - section in one type of beam to column connexion and this and the first two of the above mentioned methods have been considered from an experimental point of view in the work which is described in the following pages. Some of the methods have been used with some success in industry but again it must be stated that the extent of the knowledge about rational design criteria appears to be very limited. In the light of the results which were obtained, it has been thought reasonable to use some simplified testing methods to enable conclusions to be drawn for all the different variations suggested above. These are described in the following sections.

In view of the large number of test results which were likely to be required it was decided to test 6in. cubes loaded through steel loading pieces consisting of rectangular plates of various thicknesses and widths rather than to make tests on full scale column specimens which were not only time consuming but to some extent limiting as the results from tests of the latter type had been shown to be dependant on many variables associated with properties of the concrete used in each test. The advantages of using cubes have already been clearly stated in Chapter 6 and whilst the results from such tests may not always be directly applicable to the case of a column loaded through a vertical cast in plate, for instance, it was thought possible to use the results of tests on cubes to provide a basis of a method for estimating column bearing capacities when different plate configurations were used. Loading was applied through both rigid and flexible loading pieces as described below.

12.2 LOADING AND MODES OF FAILURE

12.2.1 Rigid Loading Pieces

A large number of cubes were tested in the manner of Fig.12.1 (a). For this purpose a bar of bright steel 6in. long and cross-section $\frac{1}{2}$in. square was positioned on one of the centre lines of the upper surface of a cube and the upper platen was brought to bear on to it. The cube was then loaded at a constant rate until failure occurred by splitting of the cube, as indicated in Fig.12.1 (a). In most cases a wedge of concrete was formed beneath the loading piece whose base width was the same as the width of loading and whose height was approximately twice this. The wedge clearly formed as a result of the compressive stress field which existed immediately under the load and tensile stresses which existed in the concrete in the regions close to the lower corners of the loading piece (see Chapter 2). Frocht showed by means of photo-elasticity that high shear stresses occurred in these regions when a semi-infinite laminar was loaded along its edge by a short rigid loading piece and it appears that when wider loading pieces were used as in the tests described here these stresses dominated the overall stress field and ultimately the mode of failure of the specimen. In the central zone of the cube between the apex of the wedge and the base of the cube, tensile stresses which acted horizontally

MODES OF FAILURE FOR CUBES UNDER DIFFERENT CONDITIONS OF LOADING

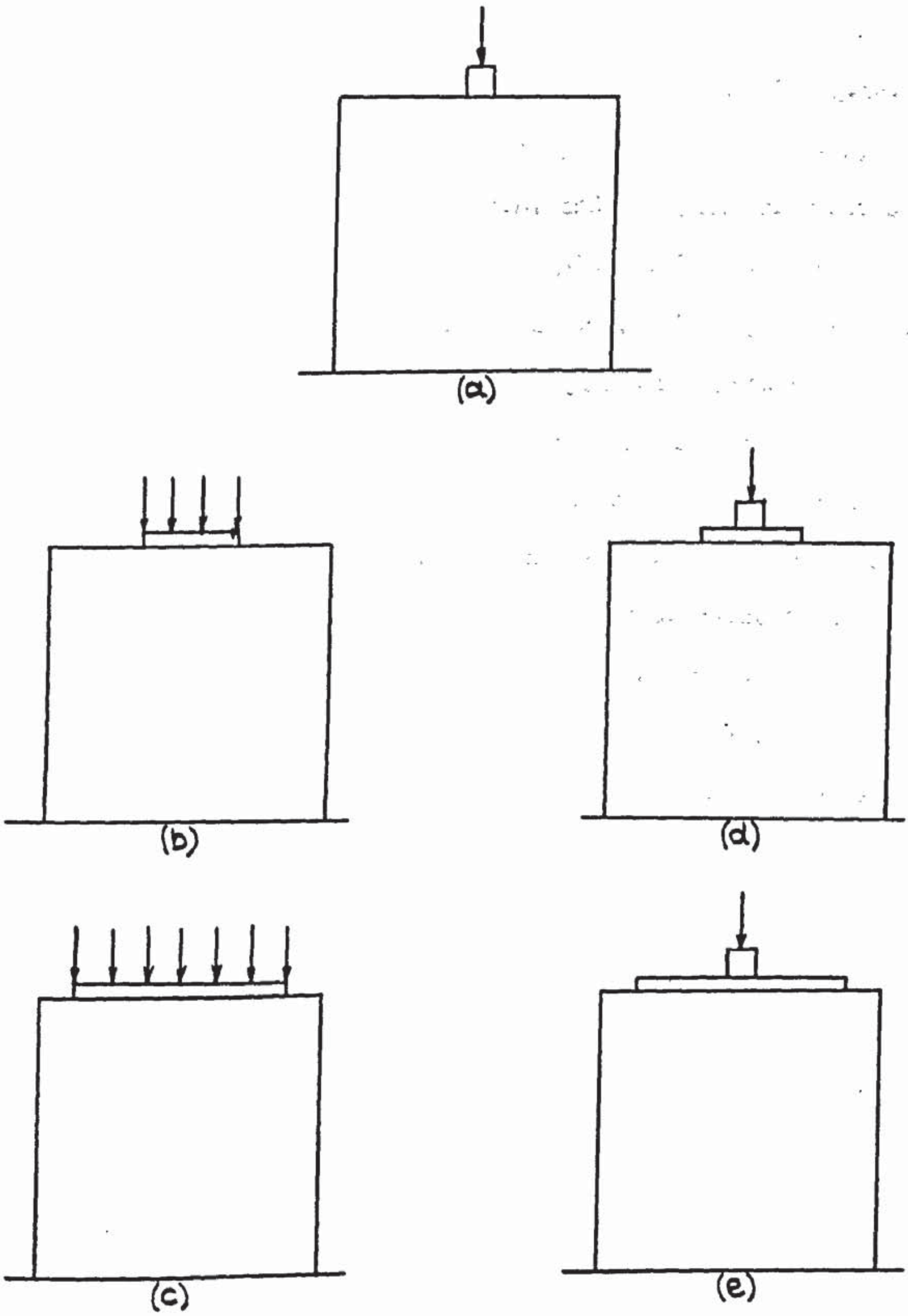


Figure 12.1

became dominant and cracking occurred when they reached their limiting values.

Some cubes were also tested in the manner of Fig.12.1 (b) and (c). The $\frac{1}{2}$in. thick loading piece was replaced by rectangular plates of length 6in. and of different widths so that the effect of using vertical plain column plates of different widths could be investigated. The upper platen was brought to bear directly on to the plate and loading was continued until failure. As the failure load was approached cracks started undet the edges of the plate and travelled downwards and outwards until total failure of the cube had occurred. It was noticed that the load to cause failure of the cube increased more or less uniformly with the width of the plates until the maximum width of plate was used, equal to the side length of the cube, when the test becomes the conventional cube compression test.

12.2.2 Flexible Loading Pieces

Tests were also carried out on cubes loaded in the manner of Fig.12.1 (d) and (e). A steel rectangular plate of length 6in. was placed symmetrically on the upper face of a cube and the $\frac{1}{2}$in. square loading piece was placed on top of the plate so that the longitudinal axes of both the plate and the loading piece were colinear with an axis of the cube. The upper platen of the testing machine was again brought into contact with the

$\frac{1}{2}$in. square loading piece and load was transmitted through the plate to the concrete. In this way it was possible to simulate a column loaded through an inverted T - section. Nine series of tests using this loading arrangement were performed, a different width of plate being used for each test. Four series were carried out using $\frac{1}{4}$in. thick plates and four in which the plate thickness was 5/8in.

When the width b of the plate was small compared to the width D of the cube the cracks in the concrete formed at failure close to the edges of the plate, as shown in Fig.12.1 (d) and as the width of the plate was increased the distance between the top of the two cracks increased to a maximum value (for a particular thickness of plate) as shown in Fig.12.1 (e). Once the maximum distance had been attained the load to cause failure became constant whatever the width of the plate. This horizontal distance varied with plate thickness being larger for plates of relatively large thickness. When failure had occurred it was observed that the wider plates had been bent into a shallow U - shape under the action of the narrow loading piece (the $\frac{1}{2}$in. square bar) and that the central portion of the surface of the concrete between the cracks had yielded to follow the shape of the plate. It was therefore not surprising that there was no increase of failure load after a certain width of plate had been reached because they were transferring load over a limited central area due to bending of the plates.

12.3 RESULTS

The results of series CII, the tests in which the cubes were loaded through the ½in. square bar only are given in Table 12.1 and the results are shown graphically in Fig.12.2 which is a plot of f_o v. f_c. The values of f_o given in the table are the mean values for all the tests in each batch.

TABLE 12.1

Test No.	f_c p.s.i.	f_t p.s.i.	f_o p.s.i.	No. of specs.	$\frac{f_o}{f_c}$
CII1	11280	535	2373	6	0.2103
CII2	11160	600	2370	2	0.2123
CII3	10540	530	2187	3	0.2074
CII4	10020	515	2140	3	0.2135
CII5	7740	505	1670	6	0.2159
CII6	7430	505	1543	3	0.2076
CII7	5630	415	1187	6	0.2107
CII8	5380	340	1264	6	0.2347

Fig.12.2 shows that there is very nearly a functional relationship between f_o and the cube strength f_c and this is born out by a regression analysis given in Appendix C.3 which shows that the correlation coefficient is very close to unity.

PLOTS OF f_o V f_c FOR 6 in. CUBES FOR $b/D = 1/12$

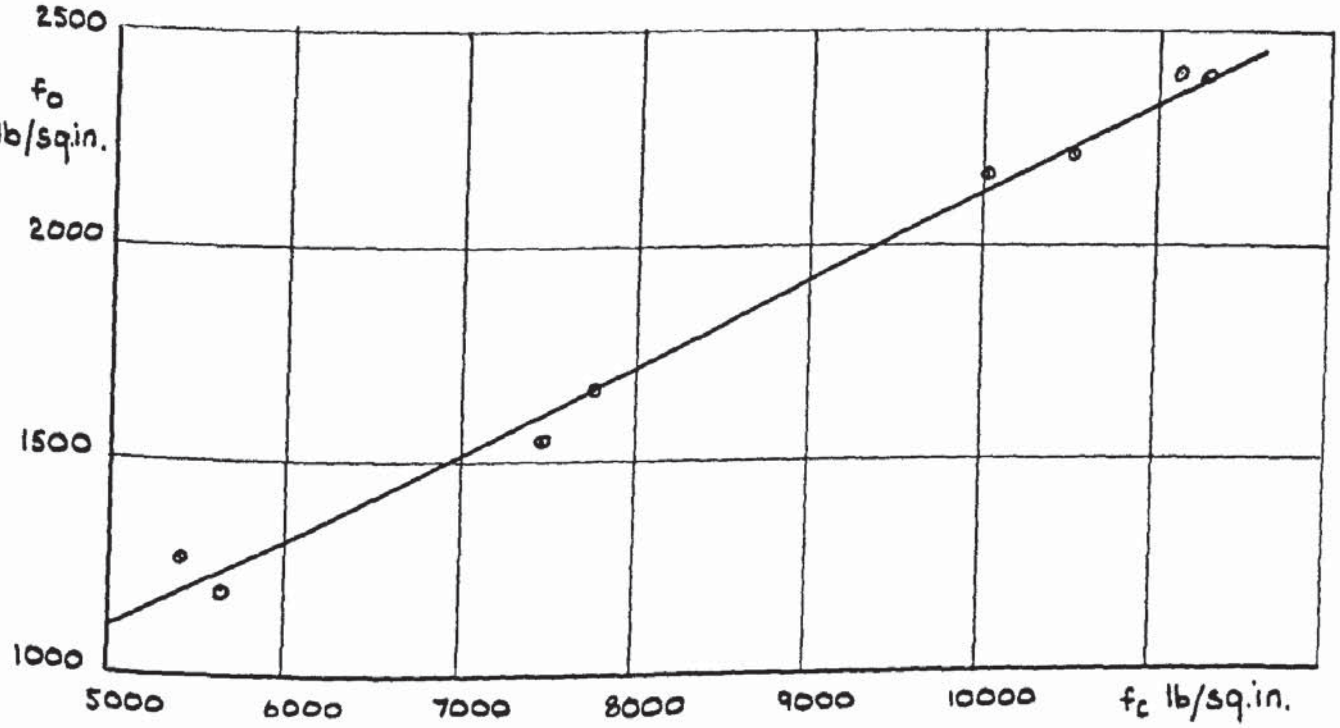


Figure 12.2

The equation of the regression is

$$f_o = 0.1996\ f_c + 117.5 \qquad (12.1)$$

for the range of cube strength 5380 to 11 280lb/sq.in. This indicates that in the case of cubes tested under narrow strip loads of constant width that the non-dimensional factor f_o/f_c is very neary a constant which is equal to 0.2140 ± 0.0043 (35 results), the second figure being the standard error of the mean.

Table 12.2 gives the results of the tests in which the flexible plate system was used, test series CIII. The thickness of the horizontal plate was T inches. The load at failure, i.e. the maximum load was for each cube denoted by P_{ldg} in tons. f_{ldg} is that load in pounds divided by the cross-sectional area of the cube in inches and the results of these tests and of series CII are plotted in Fig.12.3 in terms of f_{ldg}/f_c as a function of b/D. The results of series CII plot as one point as the b/D ratio was constant at 0.0833. When b/D = 1 clearly f_{ldg}/f_c must also equal unity if the ledge is rigid as this corresponds to the standard crushing test for concrete and a straight line has been drawn between the two points

$$\left(\frac{b}{D} = 1,\ \frac{f_{ldg}}{f_c} = 1\right) \text{ and } \left(\frac{b}{D} = 0.0833,\ \frac{f_{ldg}}{f_c} = 0.2140\right)$$

For the sake of clarity points lying along this line, for which $T = \infty$ have not been plotted. The three horizontal lines are

TABLE 12.2

Test No.	T in.	$\frac{b}{D}$	f_c p.s.i.	f_t p.s.i.	P_{ldg} ton	$\frac{f_{ldg}}{f_c}$
CIII1	5/8	0.167	11 160	600	66.7	0.372
		0.333			88.0	0.490
		0.500			86.4	0.481
		0.833			91.3	0.509
		1.000			87.1	0.485
CIII2		0.333	10 020	515	65.2	0.405
		0.500			76.0	0.472
		0.667			80.0	0.496
		1.000			77.0	0.478
CIII3		0.333	8 400	435	49.7	0.368
		0.667			62.6	0.464
		0.883			66.7	0.494
		1.000			64.1	0.475
CIII4		0.333	11 560	455	79.0	0.425
		0.500			87.6	0.471
		0.667			87.6	0.471
		0.833			93.1	0.501
		1.000			85.8	0.462

TABLE 12.2 - Cont'd.

Test No.	T in.	$\frac{b}{D}$	f_c p.s.i.	f_t p.s.i.	P_{ldg} ton	$\frac{f_{ldg}}{f_c}$
CIII5	1/2	0.333	10 180	485	58.0	0.354
		0.500			65.3	0.399
		0.667			66.3	0.405
		0.833			64.6	0.395
		1.000			63.0	0.385
CIII6	1/4	0.167	11 160	600	60.2	0.335
		0.333			63.0	0.351
		0.500			65.1	0.363
		0.833			65.4	0.364
		1.000			58.0	0.323
CIII7		0.333	10 020	515	53.6	0.333
		0.500			51.4	0.319
		0.667			53.0	0.329
		1.000			49.2	0.305
CIII8		0.333	8 400	435	43.1	0.319
		0.667			44.5	0.329
		0.833			44.5	0.329
		1.000			46.6	0.345
CIII9		0.333	10 130	530	45.6	0.280
		0.500			59.6	0.366
		0.667			53.8	0.330
		0.833			50.4	0.310
		1.000			58.6	0.360

PLOTS OF f_{ldg}/f_c V b/D FOR DIFFERENT LEDGE THICKNESSES

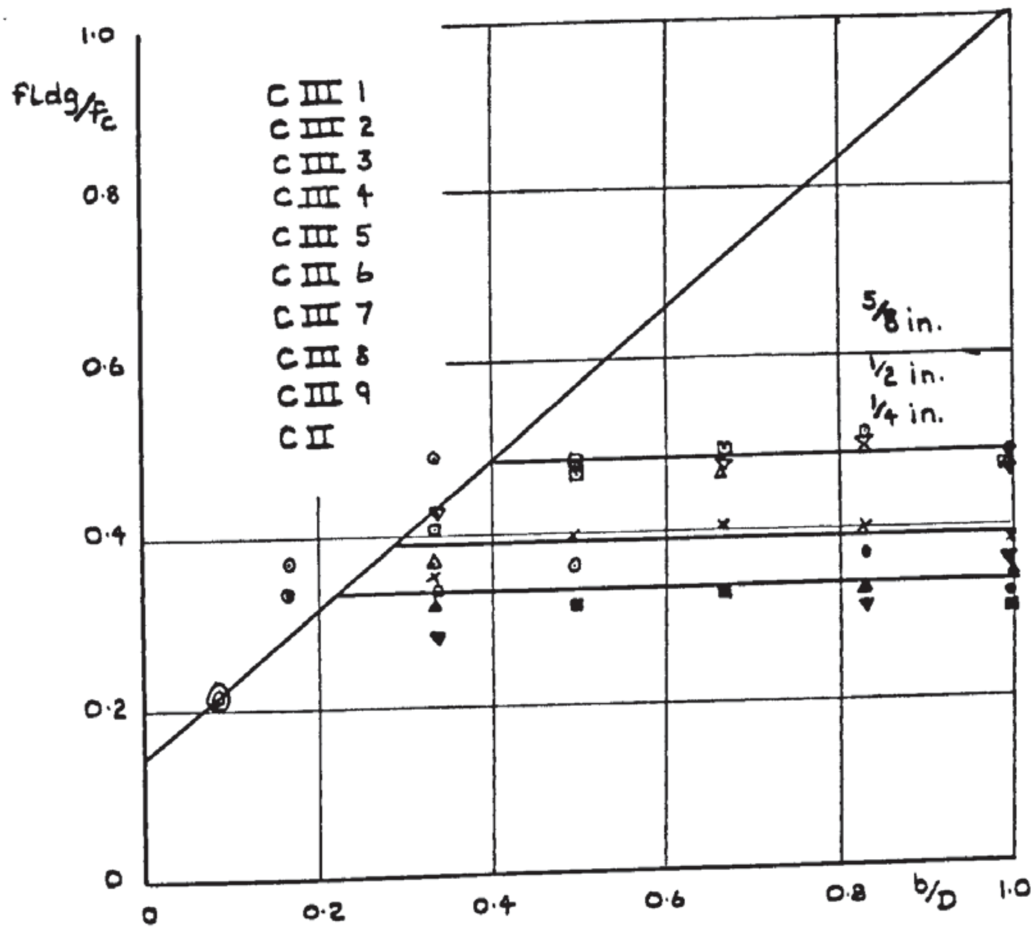


Figure 12.3

drawn for the mean values of the three sets of points which fall to the right of the sloping line corresponding to the three different values of T which were used in the tests. These mean values of f_{ldg}/f_c are given in Table 12.3 and the relevant calculations are given in Appendix C.4.

TABLE 12.3

T in.	Av. $\frac{f_{ldg}}{f_c}$	C.V. %	No. of specs.
5/8	0.4814	2.9	15
1/2	0.3876	4.5	5
1/4	0.3325	6.7	17

In view of the fact that the number of results used to calculate the mean value of f_{ldg}/f_c for the case T = 1/2in. was not greater than a third of those used for each of the other two calculations it was felt that this value must be regarded with some degree of doubt. This is because the difference between 5/8 and 1/2in. is only half of the difference between 1/2 and 1/4in. and consequently it might have been expected that the value of f_{ldg}/f_c for T = 1/2in. would have been, had more tests been carried out using this value of T, closer to the upper value than in fact it is.

The ordinates at which the horizontal lines intersect the sloping line gives the value of b/D for which the ledges ceased to act in an effectively rigid manner and the values of b/D at which this occurred are given in Table 12.4 which also gives the values of the non-dimensional factor b^2/TD which has also been calculated for each value of T.

TABLE 12.4

T in.	$\frac{b}{D}$	$\frac{b^2}{TD}$
5/8	0.3978	1.5191
1/2	0.2889	1.0008
1/4	0.2250	1.2150

If the value of b^2/TD for T = 1/2in. is ignored for the reason stated above it is clear that the mean value of b^2/TD lies between 1.5191 and 1.2150 but for purposes of design it would be necessary to take the lower value. Thus it can be concluded that for the ledge to behave in an effectively rigid manner it would need to have a b^2/TD value less than 1.2150 for a given value of T.

12.4 DESIGN METHOD

It is considered that the results described in the previous section provide an adequate basis for the design of connexions which consist of either a vertical column plate with a ledge welded to its lower edge or of an inverted T - section cast into the column and loaded symmetrically. For this purpose it is necessary to use the graph shown in Fig.12.4 which is a plot of f_{ldg}/f_o v.b/D for the cubes of series CII and CIII. f_{ldg}/f_o

PLOTS OF $fldg/f_o$ V- b/D

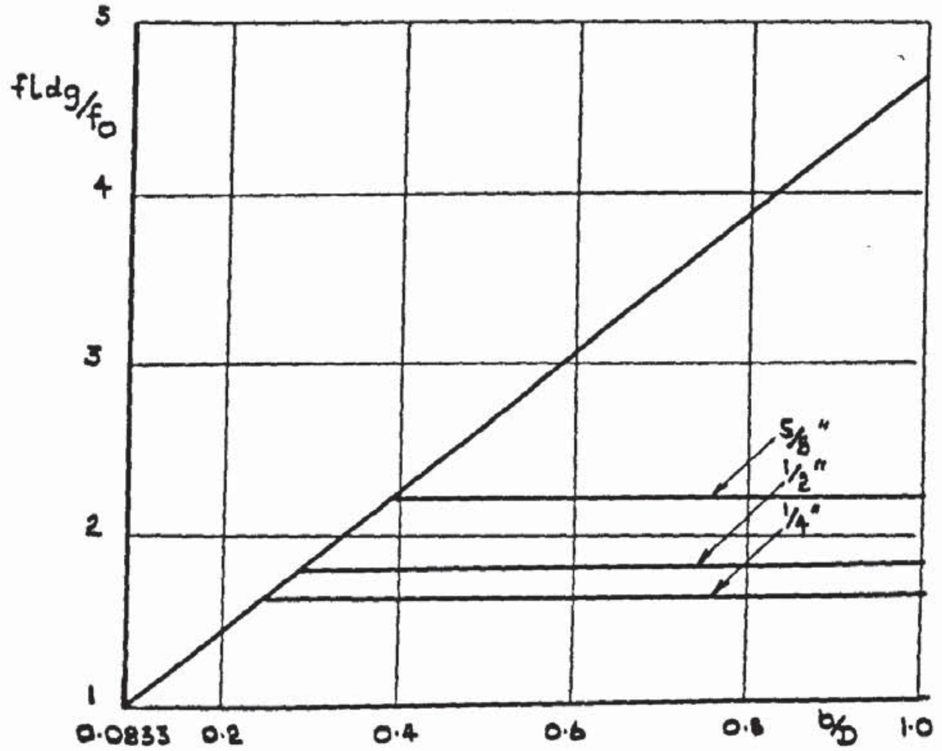


Figure 12.4

was calculated by dividing f_{ldg}/f_c by f_o/f_c taking for f_o/f_c the value obtained in series CII, i.e. 0.2140. In other words, Fig.12.4 is a more general form of Fig.12.3. The equation of this line for which the stiffness of the ledge is effectively infinite is easily shown to be given by

$$\frac{f_{ldg}}{f_o} = 4.0005\,\frac{b}{D} + 0.6663 \qquad (12.2)$$

The proposed design method follows automatically and will be illustrated by example.

12.4.1 Example

Find the ledge dimensions required for a 6in. square column, whose cube strength is 7000lb/sq.in. to support a load of 45 ton applied symmetrically to a vertical column plate.

Solution

From equation (10.2)

$$f_o = 0.0749 \times 7000 + 834$$

$$= 1358 \text{ lb/sq.in.}$$

Therefore $$\frac{f_{ldg}}{f_o} = \frac{45 \times 2240}{36} \times \frac{1}{1358}$$

$$= 2.0617$$

Now, for greatest efficiency it is assumed that the ledge behaves perfectly rigidly so equation (12.2) is used to give the relevant value of b/D i.e.

$$2.0617 = 4.0005\frac{b}{D} + 0.6663$$

hence $$\frac{b}{D} = 0.3488$$

hence $$b = 6 \times 0.3488$$

$$= 2.0938 \text{ or } 2\ 1/8 \text{ in.}$$

For rigid behaviour the condition on T is that

$$\frac{b^2}{TD} < 1.1250$$

hence $$T > \frac{(2.1250)^2}{6 \times 1.2150}$$

or $$T > 0.6194, \text{ say } 5/8 \text{ in.}$$

Thus a ledge of thickness 5/8 in. and breadth 2 1/8 in. would be suitable.

CHAPTER 13

CHAPTER 13

DIFFERENT PLATE CONFIGURATIONS

13.1 PLATES AT RIGHT ANGLES

13.1.1 Introduction

Although the majority of the tests performed for these investigations were on columns (or cubes) in which the load was transferred through a single vertical plate (or loading strip) it was appreciated that in a building, many internal columns supported at the same level not two beams but four. The column would therefore be loaded on both transverse axes requiring two plates at right angles to each other to transfer load from the beam plates to the column. The simplest way of achieving this configuration is to cast into the column a cruciform shape consisting of steel plates set vertically and welded together; in the case of a square column the cruciform has arms of equal length. In many cases that are likely to be met with in practice, the loading in one direction may not be the same as that in the other but once erection is completed the loading due to dead weight, at least, would be symmetrical in each of the two principal vertical planes. It was therefore necessary to establish the relationship between the failure load of a column loaded through a cruciform and that of a column loaded through a single plate of the same thickness and also to determine whether initial loading in one plane affected the load bearing capacity in the other plane. The problem then reduces to a consideration of how effectively the cruciform acts as a stiff spreader plate.

13.1.2 Tests

Tests were carried out using 6in. cubes to compare (i) the failure load produced by loading a cruciform uniformly with that produced by a single strip load alone and (ii) the failure loads produced when a cruciform was loaded uniformly on both axes with that produced when the cruciform was loaded uniformly on one axis only. For comparison (i) the results of series CII3 are compared with those of series CIV1, the loading scheme for which is shown in Fig.13.1. For comparison (ii) the results of series CIV2, the loading scheme for which is the same as for CIV1 are compared with series CV1, the loading scheme for which is shown in Fig.13.2.

The cruciform consisted of a square plate of bright steel of side 6in. and thickness $\frac{1}{2}$in. on to which were welded two plates of depth 6in, length $2\frac{3}{4}$in. and thickness $\frac{1}{2}$in. to form a symmetrical cross whose height and width were 6in. The upper and lower edges were accurately machined true and parallel to each other in order to facilitate symmetrical loading of the cube when pressure was applied uniformly on the upper edges.

LOADING FOR SERIES C IV

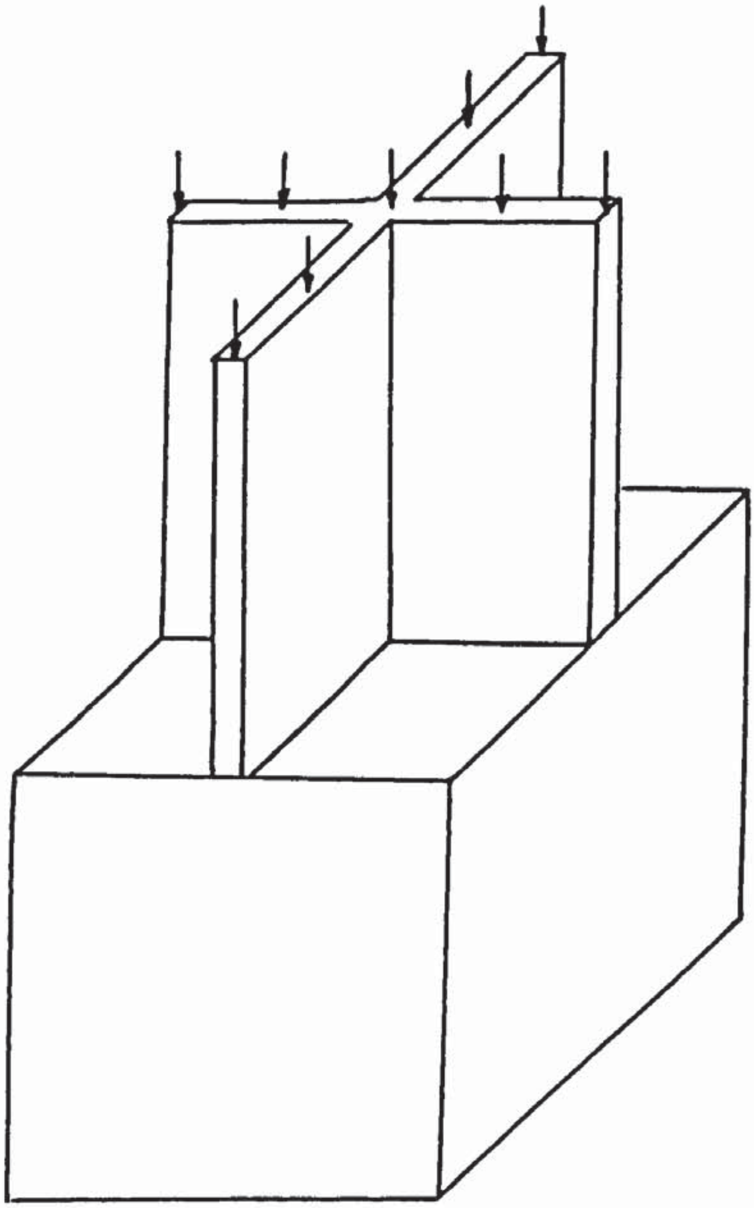


Figure 13.1

LOADING FOR SERIES CV

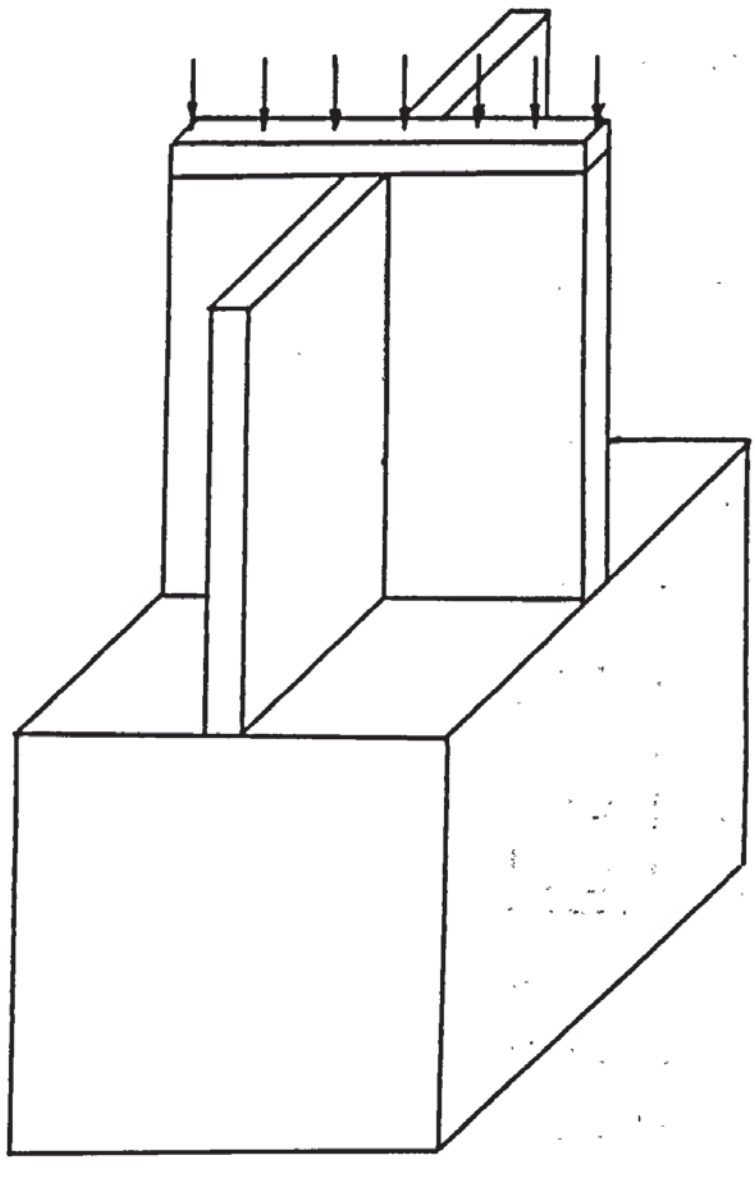


Figure 13.2

The single strip load was applied over a width of $\frac{1}{2}$in. as described in Chapter 12. Again, it was ensured that the cubes were placed in the testing machine with two moulded faces set horizontally. Fig.13.3 shows the locations of electrical resistance gauges (type 6E) cemented to a cube tested in the manner of CII.

13.1.3 Results

The results of the comparative tests are given in Table 13.1 in which P is the ultimate load.

TABLE 13.1.

Test No.	f_c p.s.i.	f_t p.s.i.	P ton	C.V. %
CII3	10 540	530	35.2	3.9
CIV1	10 540	530	72.9	2.1
CV1	8 960	540	62.6	2.2
CIV2	8 960	540	61.8	5.9

The number of specimens for each test was three. It is clearly seen that for comparison (i) the mean cube failing load produced by the loading system shown in Fig.13.1 is twice that produced by loading through a single strip and for comparison (ii) the results show that the cruciform of the dimensions used in these tests acts in a rigid manner and transfers the applied load equally in both planes.

GAUGE LOCATIONS ON 6in. CUBE FOR LOADING TYPE (a)

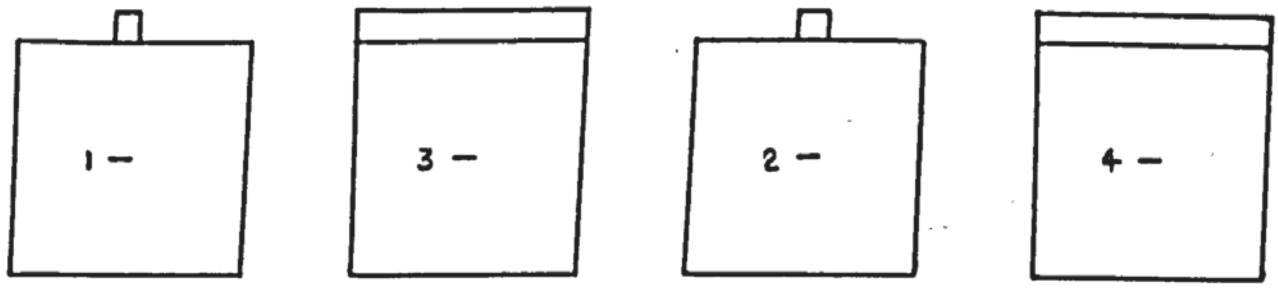


Figure 13.3

LOAD/STRAIN GRAPHS FOR TEST OF TYPE (a) LOADING

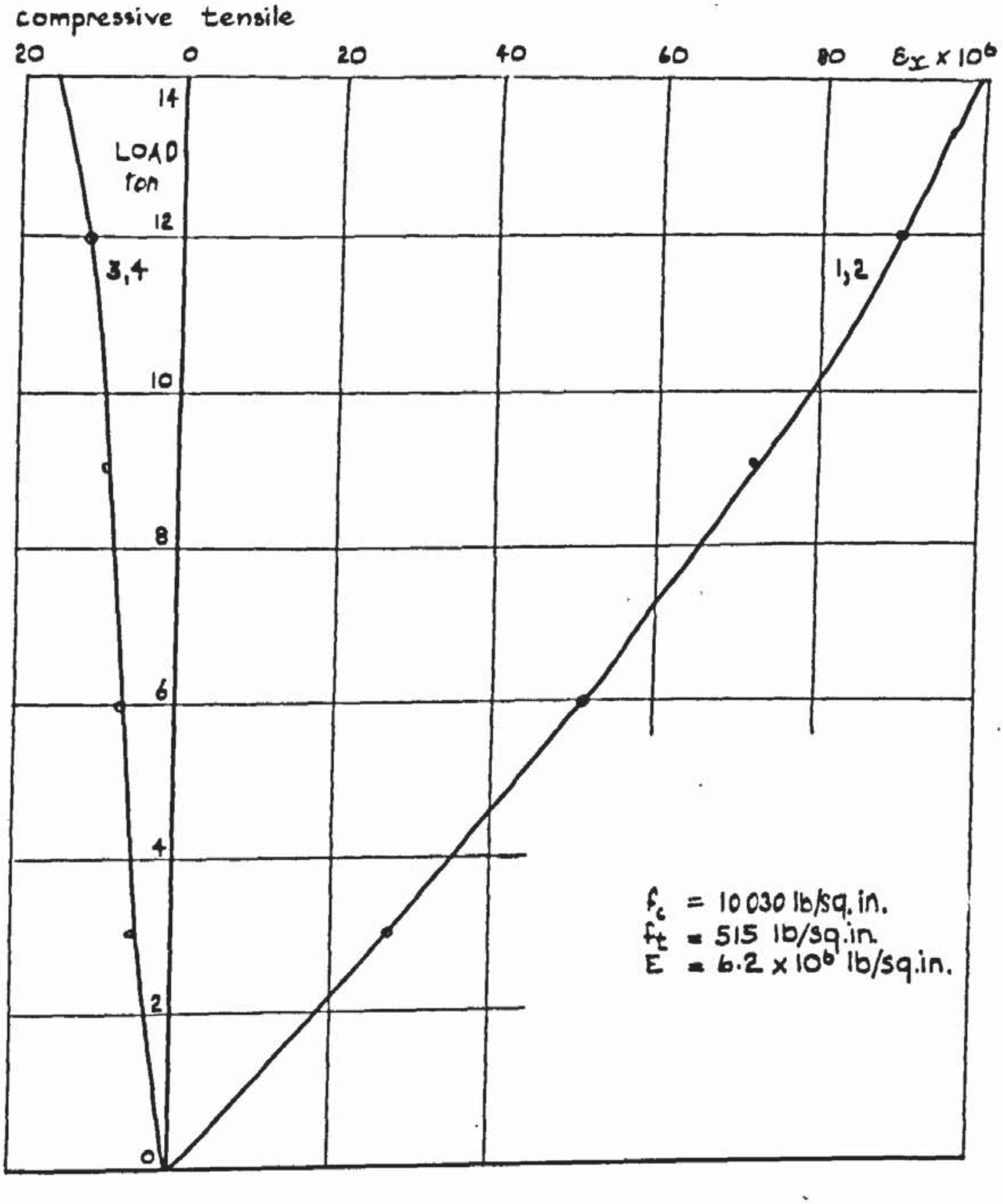


Figure 13.4

Fig.13.4 shows load v. strain characteristics for a cube loaded through a narrow strip and it is apparent that there are no tensile strains caused by the type of loading on the faces of the cube which are parallel to the loading strip. This is taken as an indication of the fact that when load is applied through a cruciform uniformly loaded in both planes the resulting stresses produced in the cube under the load in one direction do not interact with those in the other. This confirms the results of comparison (i).

13.1.4 Conclusions

From the work described above it can be concluded that when a square or rectangular column is loaded in two orthogonal directions the loading in each direction can be considered quite separately for the purpose of calculating the failure loads, at least for relatively narrow loading widths. Although a b/D ration of 1/12 was used in the tests, this assertion is likely to be valid for larger values of b/D and even for b/D values which are different in each direction. Moreover, the maximum loading may be applied in one direction initially without affecting the load carrying capacity in the other. Thus a cruciform could be used to induce the load carrying capacity of a connexion which was used to support any number of beams. At a corner column where only two beams at right angles are supported or when three beams are carried to a side column it may be necessary to calculate the effect of forces produced by moment acting in the vicinity of the embedded plates due to the unsymmetrical loading. This is likely to be of only marginal importance, however, provided reasonably deep plates are provided to distribute the loading. Some examples illustrating the above points are given in Fig.13.5.

13.2 INFLUENCE OF EDGE SHAPE

13.2.1 Introduction

It was appreciated at an early stage in these investigations that the shape of the loaded edge of the embedded plate might have some influence on the load at which failure occurred.

It was observed by Timoshenko (14) that at the point of application of a point load on the boundary of a semi-infinite laminar the stresses were theoretically of infinite magnitude because a finite force was acting on an infinitesmal area. This was realised by Guyon (see Chapter 2) and the stresses in this region given by plane framework method of Chapter 3 are also very large. In practice, Timoshenko argued there is always some yielding of the material at the point of application and the load will become distributed over a finite area. He therefore ignored the region very close to the point of application by imagining that the portion of the material which suffers plastic flow is cut out from the remainder by a cylindrical surface of small radius, and then applied the

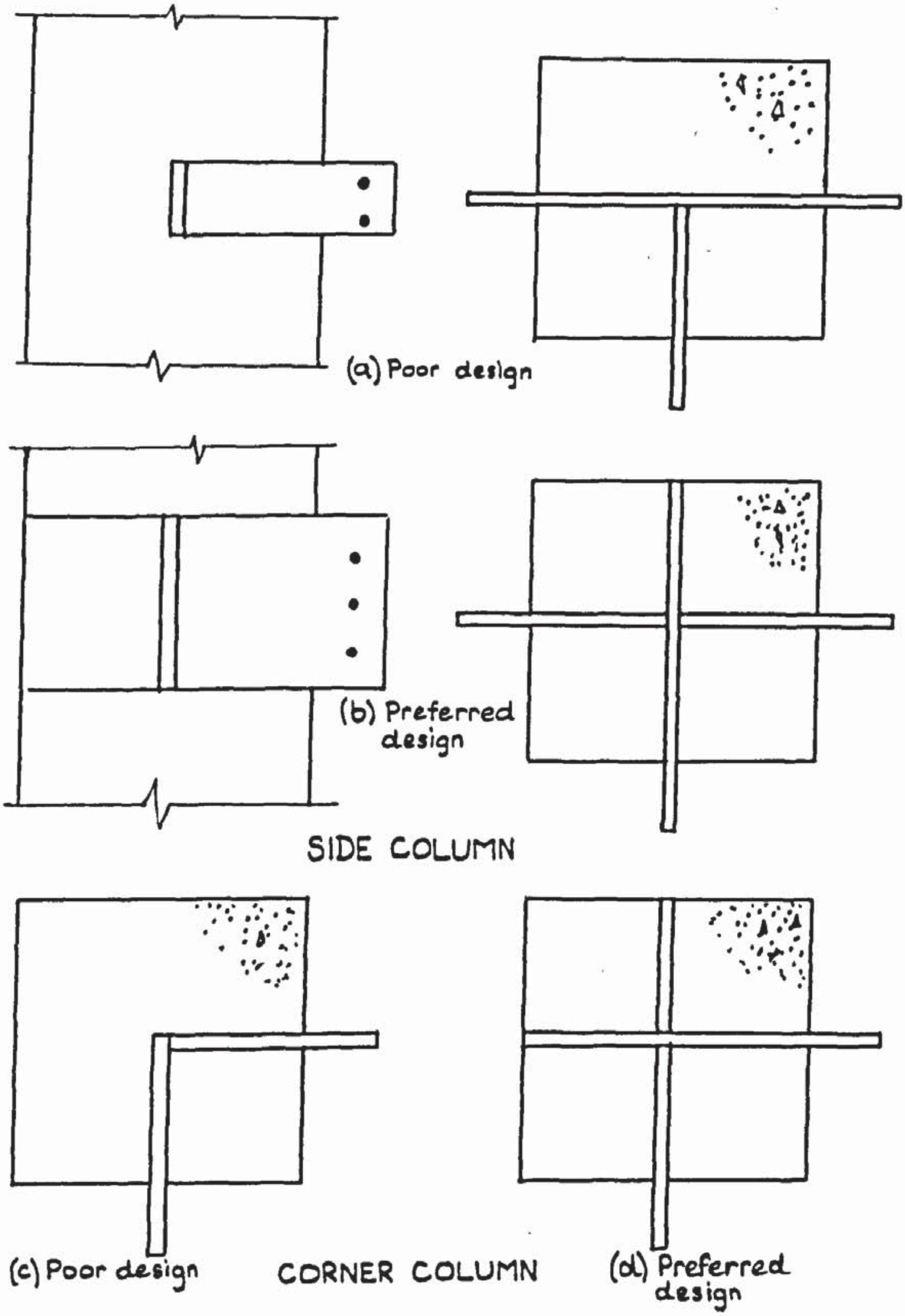
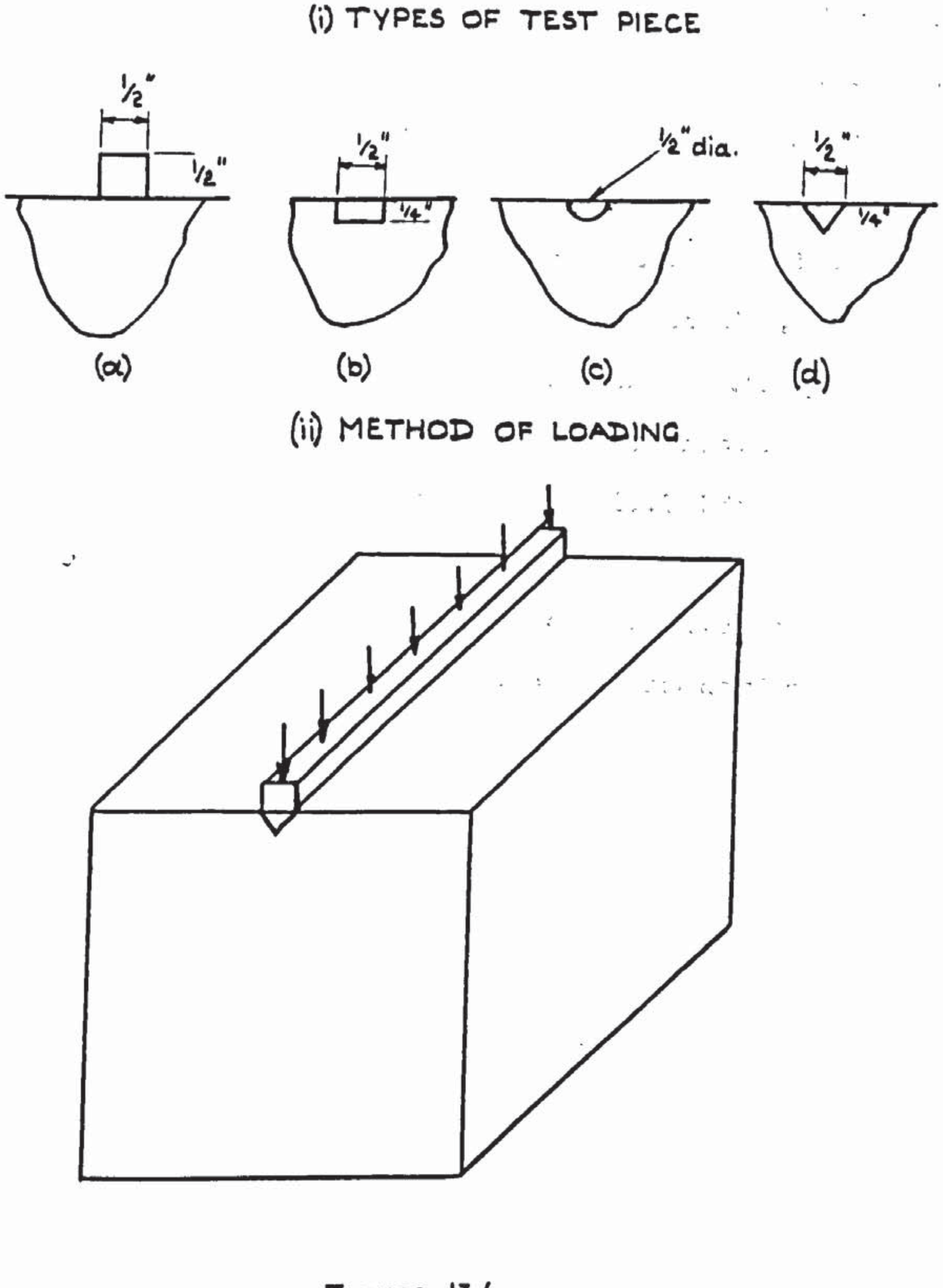

(a) Poor design
(b) Preferred design
SIDE COLUMN
(c) Poor design
CORNER COLUMN
(d) Preferred design

Figure 13.5

equations of elasticity to the rest of the material. It was reasonable to infer therefore that at least from the theoretical point of view, the shape of the very narrow vertical column plate would not have any influence on the stresses set up in the concrete at some distance from the point of application of the load. This follows of course, from the principle of St. Venant. In practice however, whilst the strains at some distance from the point of application would be largely unaffected by the shape of the loaded edge during the early part of the loading sequence, it has been observed that the load to cause failure was in fact affected by the shape. Some simple tests were devised to examine the effect of different shapes.

13.2.2 Tests

A batch of twelve 6in. cubes was cast in four sets of three. One set was cast in the usual manner but the remaining nine cubes each incorporated one of three types of test piece made of bright steel bar. The three types of cross-section were rectangular, semicircular and triangular and are illustrated in Fig.13.6 (i). Their length was 6in., the maximum width of each was $\frac{1}{2}$in. and the maximum depth $\frac{1}{4}$in. One test piece was fixed in a vertical position along the centre line of one wall of each cube mould by a thin smear of a contact ashesive (Evostik), after ensuring that the surfaces to be contacted



(i) TYPES OF TEST PIECE
1/2"
1/2"
1/2"
1/4"
1/2" dia.
1/2"
1/4"
(a)
(b)
(c)
(d)
(ii) METHOD OF LOADING

Figure 13.6

had been cleaned of oil, so that an impression of the test piece could be made in one moulded face of each cube. For testing, the test piece used in each casting was positioned in its groove on the face of the cube and a ½in. square loading piece 6in. long was placed on top. The lower face of the cube was also a moulded one to facilitate uniform loading. The plane set of cubes was tested by loading each cube through a standard ½in. wide loading strip placed centrally on the upper moulded face. The method of loading is exemplified in Fig.13.6(ii).

13.2.3 Results

The results of the four sets of tests are given in Table 13.2. The concrete mix was the same as that used for test CII1.

TABLE 13.2

Test piece	P ton	C.V. %
(a)	38.3	6.1
(b)	38.0	4.4
(c)	29.2	6.6
(d)	20.0	16.3

P is the mean failing load for three tests.

It can be seen that there was no significant difference of failing load when cubes were loaded through a rectangular test piece whether embedded or not. However, when a test piece of semicircular cross-section was used the failure load was reduced by approximately 25 per cent and when a triangular cross-section was used the load was reduced still further to approximately 50 per cent of the load when rectangular test pieces were used.

13.2.4 Conclusions

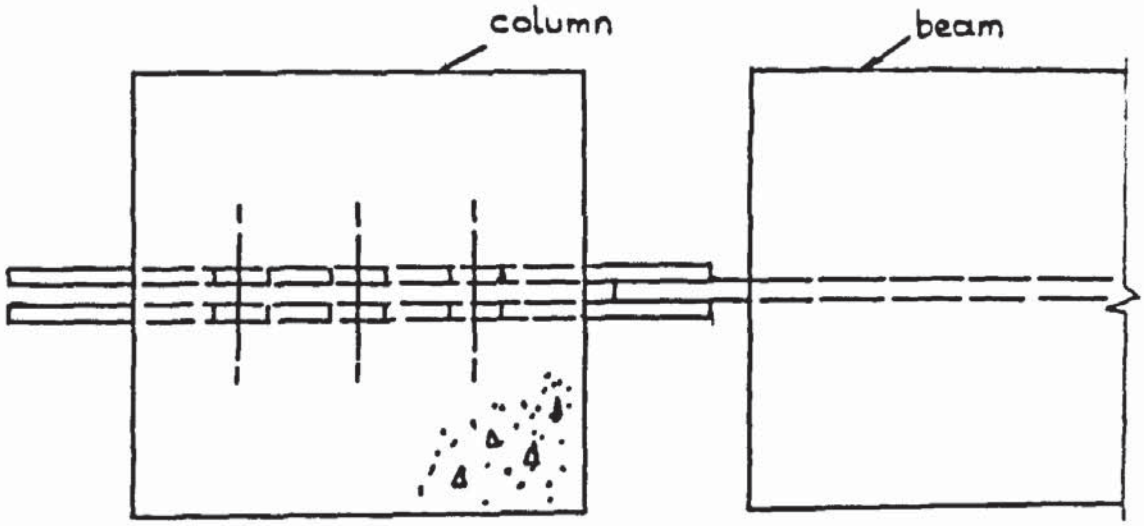
It appears from a consideration of the above results that the shape of the lower edge of a plate which is cast into a column to form a connexion is extremely critical. It would seem that the theoretical device of Timoshenko that was invoked in section 13.2.1 for dealing with stresses in the immediate vicinity of a knife-edge load is not applicable to the case of narrow strip loads. This is probably due to the fact that concrete does not behave like a ductile material like steel but more like a brittle one, particularly in tension when its ability to yield is limited. The results of these tests show that it is necessary to ensure that for maximum efficiency, loading is transferred through plates whose edges are flat and horizontal.

13.3 USE OF PLATES IN PAIRS

13.3.1 Introduction

It was found that when thin column plates were used to transfer large loads, as in the case of 10in. square columns when 5/8in. thick plates were used, the bolts which were employed to transfer the loads from the jacks or the yoke, to the plate had a tendency to yield by double shearing when high loads were applied. Of course, in real structures this problem could be overcome by either increasing the diameter of the bolts, or by increasing their number but these may not be practical alternatives in all instances as the areas of the protruding portions of the beam and column plates may be limited. In practice, the problem has been overcome either by increasing the thickness of the protruding portions of the plate in both the beams and the column, or by using plates in pairs which are separated by a small gap (see Fig.13.7). These are cast into the column and a single beam plate is then located between the two to provide an increased bearing area for the bolts. As a rule, the plates are joined by a small number of dowls of large diameter, say 1½in., to allow the pair to act as a rigid body. Of course, the space between the two plates inside the column is filled with concrete and it is the presence of this infilling material which may allow the pair of plates to behave as one thick plate whose overall thickness is equal to that of the gap plus the sum of the plate thicknesses. This action could of course be obtained by welding a plate to the bottom of the pair but this solution may not be ideal. It was decided therefore, to make an exploratory test.

DOUBLE COLUMN PLATE ASSEMBLY



Horizontal Section

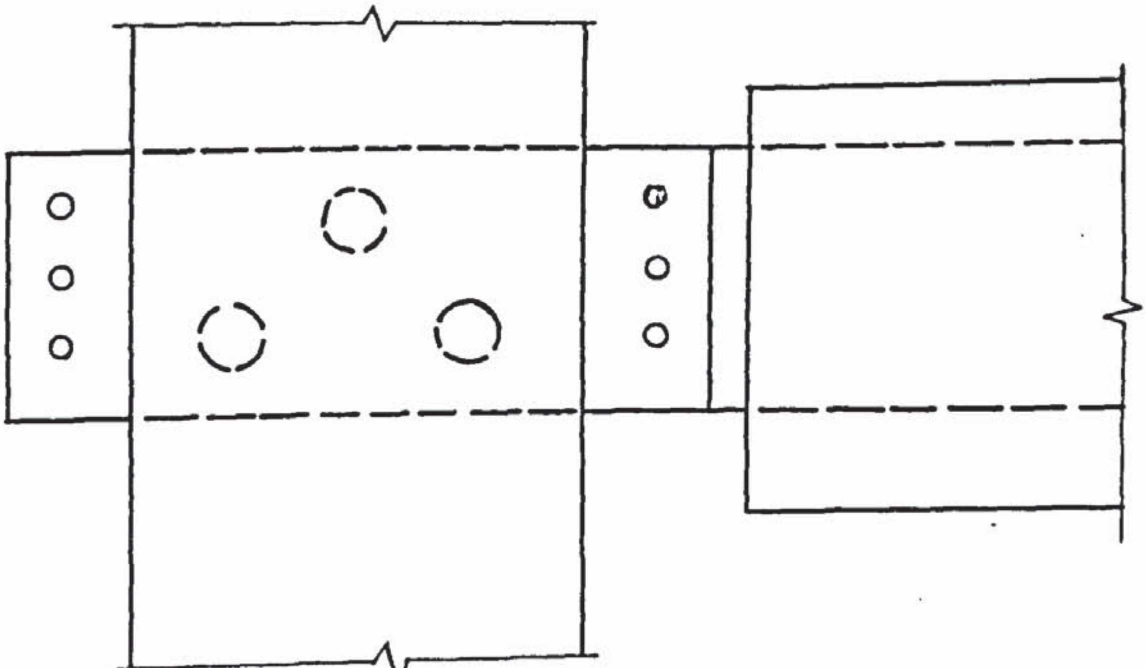


Figure 13.7

13.3.2 Tests

Two sets of three 6in. cubes made from the same batch of concrete were tested. For the first test, CVI1, a $1\frac{1}{4}$in. strip of bright steel of length 6in. was placed centrally on the top face of a cube and the platen was then brought down to bear on the strip and loading was continued to failure. For the second test, CVI2, two 1/2in. square bright steel bars of length 6in. were placed parallel to each other symmetrically on either side of the centre line of the top face of a cube so that they were separated by a gap of $\frac{1}{4}$in. The platen was brought down and loading continued to failure.

13.3.3 Results

The results are given in Table 13.3 where P is the mean failing load for three results.

TABLE 13.3

Test No.	P ton	C.V. %
CVI1	49.1	1.4
CVI2	46.5	4.6

13.3.4 Conclusions

Although these results are too few from which to draw any general conclusions, it can be stated with a fair degree of certainty that for a ratio of gap to total width of the combined plates of less than 1/5 a column loaded through a pair of plates would fail at approximately the same load as one in which a single thick plate of the same overall thickness was used. The presence of the concrete infilling that would exist in the case of a column would reinforce this conclusion. However, more experimental results would have to be obtained from tests on columns using other plate thicknesses and gap dimensions before a complete analysis could be made.

CHAPTER 14

CHAPTER 14

CONCLUSIONS

14.1 GENERAL CONCLUSIONS

The following is a summary of the main conclusions which were reached as a result of the work that was carried out for these investigations.

1. It was found that generally an approach to the problem of loaded vertical plates in columns was not amenable to a theoretical treatment based on an indealised stress analysis and that it was necessary to rely on experimental data obtained from a large programme of tests.

2. The effects associated with shrinkage of the concrete surrounding a plate are of considerable importance and that if these effects could be mitigated, either by using a stronger mix, or one of a lower inherent shrinkage or by some other means, a safer connexion would be obtained. It should be added that the age of the concrete is a factor in this connexion as a large proportion of the total shrinkage takes place in the first month after casting, consequently adequate curing is essential.

3. The effect of reinforcement was found to delay somewhat the first appearance of cracking and to marginally increase the ultimate load that could be carried by the connexion, except when a link was placed in contact with the lower edge of the

plate when a considerable improvement in the bearing capacity was obtained.

4. The fact that the column is loaded in compression in advance of load being applied to a column plate does not prejudice the ability of the connexion to take load.

5. The effects produced by loading a rectangular column in both axial planes can be regarded as being completely independent.

6. A method of design suitable for connexions in which a moment is applied to the plate has been found which in the light of test results is somewhat conservative.

7. An experimental method for estimating the strength of a symmetrically loaded connexion consisting of a cast in T - section has been determined and also for estimating the dimensions of the table of the T - section required for optimum design.

8. The depth of a column plate is not a critical factor and does not affect the ability of the concrete to take load. Provided it is not smaller than the dimensions given in Chapter 7 it may be regarded as being rigid for all practical purposes.

9. It is necessary to ensure that for maximum efficiency the lower edge of a plain column plate is horizontal and flat.

10. It has been shown that the relative load bearing capacity of square columns is not constant for columns whose cross-

sectional areas are different being larger for a 6in. square column than for a 10in. square one. Moreover the bearing capacities of columns of the same cross-sectional area vary as a function of the cube strength of the concrete from which the columns are made although this function is not the same for columns of 10in. and 6in. square cross-sections, as equations (10.1) and (10.2) indicate.

The fact that the smaller columns can sustain relatively higher loads is considered to be at least partly due to the "weakest link" theory which is based on the fact that there are statistically a smaller number of flaws in a small mass of concrete than in a larger mass whatever the intrinsic strength of the mix might be, other things being equal. This theory was considered in greater detail in Chapter 7.

14.2 SUGGESTIONS OF FURTHER WORK

It was clearly not possible in an investigation such as this to provide definitive answers to all the problems associated with the type of beam to column connexion which has formed the subject of this thesis but it was at least possible to ascertain what were the major variables. Some of these were examined in greater detail than others and it would therefore be of some use to suggest further work that might be carried out to complete these investigations.

1. Whilst it is considered that the range of column sizes

that were tested in the course of these investigations was suitable for an initial test programme such as this was, it is considered that the failing loads for different sizes of columns, particularly greater than 10in. square, would have to be investigated before the entire question of the effect of size could be completely resolved. As far as sizes of columns between 6in. square and 10in. square are concerned however the problem should be dealt with by interpolating between the two regressions given by equations (10.1) and (10.2). This is considered to cover the majority of sizes of columns in which this type of connexion would normally be used.

2. Although a method was found for estimating the ultimate loads of columns containing thicker plates than the ones tested, and inverted T - sections, it would clearly be desireable to obtain some results from tests using actual columns, if only to provide confirmation of the method which has been established.

3. The question of a load factor against ultimate collapse has not been taken up because it is felt that this is a matter that must be left largely to the discretion of the designer. It is felt that for purposes of design however, the results presented in the main body of this work provide enough guidance to enable a rational estimate to be made.

4. It would be of considerable interest to investigate more fully the moment carrying capacity of connexions consisting

of plane plates, of inverted T - sections and I - sections. It is true that a small number of tests using inverted T - sections were performed by Gifford (3) as was stated in Chapter 1 but a far more extensive test programme than this would be required to allow anything like a general treatment for purposes of design.

5. The design of the beam plate has not been considered in these investigations and although it is doubtful that it would present serious problems, a limited investigation of an experimental nature would be worthwhile in order that more economical methods than have hitherto been used could be established to obtain a minimum cost design, particularly with regard to the area and thickness of plate and the amount of welding.

6. The effect of using rapid hardening Portland cement in relation to its effect on shrinkage should be examined, particularly as this material is often used in the manufacture of precast units. Besides having an increased strength at an early age compared with ordinary Portland cement the effects of shrinkage, which have been shown to be mainly deleterious to the strength of the connexion, may well be reduced although this is not certain.

APPENDICES

APPENDIX A

DATA PREPARATION

The input data for the plane framework problems consisted of of the following entries:

job number (integer);

number of element (integer);

number of nodes (integer);

number of loads applied (integer);

number of constraints and applied displacements (integer)

exist control (real);

Young's modulus (real);

thickness (real);

Poisson's ratio (real);

x - dimension of element (real);

y - dimension of element (real);

grid references of each element with the elements in any convenient order e.g.

SW; SE; NW; NE;

" " " "

position of constraints (integer);

(these are calculated as 3 times a node number

minus 2 for constraints in x-direction;

minus 1 for constraints in y-direction;

minus 0 for constraints about z-axis;)

constraints (real); (these are in same order as positions)

load position (integer); (calculated like constraints)

load value (real);

next load position (integer);

load value (real);

etc

Example

The analysis of a rectangular plate, 4 in. high by 8 in. long by $\frac{1}{2}$ in. thick loaded by a uniform load of 100 lb/in. run, which rests on a plane, rough surface which effectively constrains the base will be used to illustrate the method of data preparation. A 1 in. square element will be used. As the loading and the plate is symmetrical, only half the plate need be analysed. (see Fig. A.1.)

Poisson's ratio is 0.15 Young's modulus is 5 x 10 lb/sq.in. (in the actual input, the number 5 only would be used for convenience). The spacing of the numbers is immaterial but will be grouped as convenient.

DIAGRAM OF PLATE

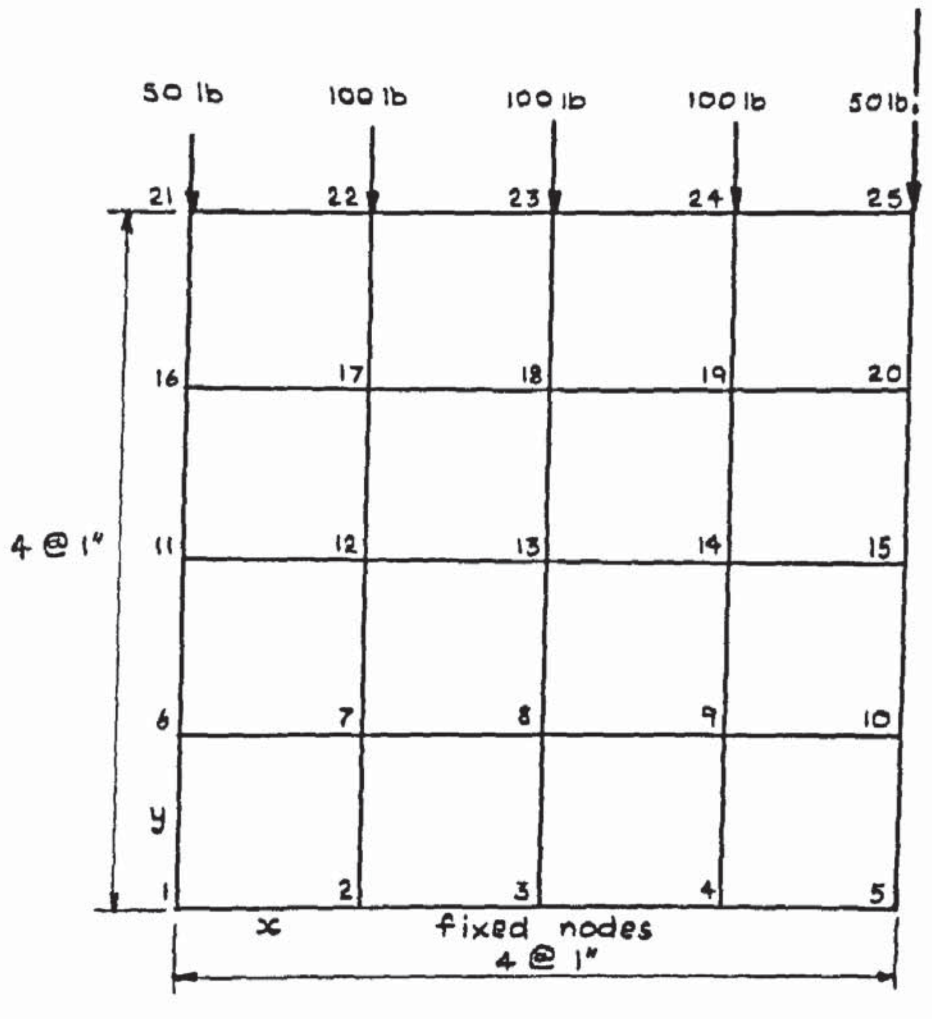


Figure A.1

1; 16; 25; 5; 23; 1.0 -12; 5; 0.5; 0.15; 1; 1;

1; 2; 6; 7;
6; 7; 11; 12;
11; 12; 16; 17;
16; 17; 21; 22;

2; 3; 7; 8;
7; 8; 12; 13;
12; 13; 17; 18;
17; 18; 22; 23;

etc.

1; 4; 7; 10; 13; 28; 43; 58; 73;

2; 3; 8; 11; 14;

3; 6; 9; 12; 15; 30; 45; 60; 75;

0; 0; 0; 0; 0; 0; 0; 0; 0;

0; 0; 0; 0; 0;

0; 0; 0; 0; 0; 0; 0; 0; 0;

62; -50; 65; -100; 68; -100; 71; -100; 74; -100;

APPENDIX B

MATERIALS AND PROPERTIES

B.1 CONCRETE

Mix 1. 1:2:4 w/c = 0.6

Coarse aggregates: crushed stone, 3/8in. down.

Fine aggregates: zone 2 sand.

Source: North Notts. Sand and Gravel Co. Ltd.

Age at test: 21 days comprising 19 days humid curing plus 2 days drying out in the laboratory.

Mix 2. 1:1:2 w/c = 0.45

Other details as for mix I.

Mix 3a. 1:1½:3 w/c = 0.475

Coarse aggregates:
Fine aggregates: } as for mix 1.
Source:

Age at test: 28 days comprising 21 days humid curing plus 7 days drying out in the laboratory.

Mix 3b. 1:1½:3 w/c = 0.475

Coarse aggregates; crushed stone, 3/8in. down, sieved free of dust.

Fine aggregates: zone 3 sand.

Source: Midland Gravel Co. Ltd.

Age at test; as for mix 3b.

Mix 4. 1:1:4 w/c = 0.35

Coarse aggregates: rounded gravel, 80% @ 3/8in.
20% @ 3/16in.

Fine aggregates:
Source: } as for mix 3a.
Age at test:

Mix 5. 1:1½:5 w/c = 0.55
Mix 6a. 1:2:4 w/c = 0.50
Mix 6b. 1:2:4 w/c = 0.55 } as for mix 3b.
Mix 6c. 1:2:4 w/c = 0.60

The mixes used in each test are indicated in Table B.1.

TABLE B.1

Mix No.	Test No.
1	I, II, CIa, CIb
2	CIc
3a	III
3b	VB1-6, CIII5
4	IV, VA1-5, VI, VII, VIII, CII1-4, CIII1,2,4,6,7,9, CIV, CV, CVI
5	VB7,8
6a	CII5, CIII8
6b	VB9-16, CII6,8
6c	CII7

B.2 STEEL

The diameter of each steel specimen was measured at four locations along the length, two measurements being taken at right angles at each location. Four specimens of each of the two sizes used were measured and tested to failure in tension.

$\frac{1}{4}$in. dia.: average dia. = 0.252in.,
average value of E = 30.85×10^6 lb/sq. in.,
average maximum stress = 73 000 lb/sq. in.

$\frac{1}{2}$in. dia.: average dia. = 0.508in.,
average value of E = 30.10×10^6 lb/sq. in.,
average maximum stress = 66 300 lb/sq. in.

APPENDIX C

STATISTICAL CALCULATIONS

C.1 TEST FOR SIGNIFICANCE OF THE DIFFERENCE OF MEANS OF SERIES IIIA AND IIIB

In order to find whether the difference between the mean values of f_σ/f_c obtained from the two sets of samples, series IIIA and IIIB given in Table 9.1 is significant it is necessary to use Bessel's correction because of the small size of the samples.

The pooled estimate of the variance is found, on the Null Hypothesis that the two samples are drawn from populations identical as to mean and variance.

s_A^2 and s_B^2 are the sample variances of series IIIA and IIIB respectively. As there were three results for each sample, the corrected population variance is given by

$$\hat{\sigma} = \frac{3s_A^2 + 3s_B^2}{3 + 3 - 2} = \frac{3 \times 1866 + 3 \times 944}{3 + 3 - 2}$$

$$= 2107.5000$$

where each value of f_σ/f_c has been multiplied by 10^4 for convenience.

Hence $\hat{\sigma} = \sqrt{2107.5000} = 45.9075$

The best estimate of the standard error for the difference of the means of the two samples is given by

$$\hat{\sigma}_W = \hat{\sigma}\sqrt{\frac{1}{3} + \frac{1}{3}}$$

$$= 37.4835$$

Now, the observed difference between the means is, from Table 9.1, 1188 - 885 = 303

hence Student's $t = \frac{303.0000}{37.4835} = 8.0835$

Entering the t tables at 3 +3 -2 degrees of freedom, the observed difference could have arisen by chance in less than 1 per cent of trials, hence the significance is shown to exist.

C.2 <u>10in. SQUARE COLUMN RESULTS</u>

The regression (10.1) was calculated from the results given in Table C.1.

It is convenient to put $f_c = x$ and $f_o = y$,

then $\Sigma x = 97\ 880$

$\Sigma x/N = 8\ 898$

$(\Sigma x/N)^2 = 79\ 177\ 639$

$(\Sigma x^2)/N = 83\ 163\ 527$

$s_x^2 = 3\ 985\ 888$

$s_x = 1\ 996$

and $\Sigma y = 9\ 776$

$\Sigma y/N = 889$

$(\Sigma y/N)^2 = 789\ 836$

$(\Sigma y^2)/N = 857\ 123$

$$s_y^2 = 67\ 287$$

$$s_y = 259$$

also $\Sigma xy/N = 8\ 326\ 055$

$$\frac{\Sigma x}{N} \times \frac{\Sigma y}{N} = 7\ 910\ 322$$

Now, the covariance for the x and y results is given by

$$\text{covxy} = \frac{\Sigma xy}{N} - \frac{\Sigma x}{N} \times \frac{\Sigma y}{N} = 415\ 733$$

and the correlation coefficient r is given by

$$r = \frac{\text{covxy}}{s_x s_y} = 0.8033$$

Test for significance of r·

$$\text{Student's } t = \frac{0.8033\sqrt{11 - 2}}{\sqrt{(1 - (0.8033)^2)}} = 4.0461$$

Entering the t tables at 11 - 2 = 9 degrees of freedom, the value of t at the 1 per cent probability level is 3.25 so the value of r given above is significant.

Hence,

$$y = r \times \frac{s_y}{s_x} (x - 8898) + 889$$
$$= 0.1043x - 38 \qquad (10.1)$$

The 95 per cent confidence limits on y are given by, for $y = Y$

$$Y \pm t\left(\left(\frac{s_y^2 - b^2 s_x^2}{N - 2}\right)\left(1 + \frac{(X - \bar{x})^2}{s_x^2}\right)\right)^{1/2}$$

where t is found at the 5 per cent level for N - 2 degrees of freedom, and b is the regresion coefficient given by

$$b = r \times \frac{s_y}{s_x}$$

Now t = 2.26

and $\frac{s_Y^2 - b^2 s_X^2}{N - 2}$ = 2686

For X = 8898,

the 95 per cent confidence limits on Y are

$$\pm 2.26\sqrt{((2686)(1 + 0))}$$

or $\pm$ 117 lb/sq. in.

For X = 12 000 and 5796,

thr 95 per cont confidence limits on Y are

$$\pm 2.26\sqrt{((2686)(1 + 2.4142))}$$

or $\pm$ 226.3 lb/sq. in.

TABLE C.1

Test No.	f_c p.s.i.	f_o p.s.i.
IIIA1	7160	622
IIIA2	5870	551
IIIA3	5950	510
VA1	8420	1086
VA2	7790	672
VA3	11280	873
VA4	9530	885
VA5	12050	1339
VIA1	10250	999
VIA2	110810	1075
VIA3	8770	1164

C.3 6in. SQUARE COLUMN RESULTS

The regression (10.2) was calculated from the results given in Table C.2.

It is convenient to put $f_c = x$ and $f_o = y$,

then $\Sigma x = 191\ 290$

$\Sigma x/N = 8\ 695$

$(\Sigma x/N)^2 = 75\ 610\ 928$

$(\Sigma x^2)/N = 80\ 962\ 681$

$s_x^2 = 5\ 349\ 969$

$s_x = 2\ 313$

and $\Sigma y = 32\ 670$

$\Sigma y/N = 1\ 485$

$(\Sigma y/N)^2 = 2\ 207\ 250$

$(\Sigma y^2)/N = 2\ 254\ 035$

$s_y^2 \equiv 46\ 656$

$s_y = 216$

also $\Sigma xy/N = 13\ 312\ 866$

$\frac{\Sigma x}{N} \times \frac{\Sigma y}{N} = 12\ 912\ 075$

and covxy $= 415\ 733$

$r = 0.8022$

Test for significance of r:

Student's $t = \dfrac{0.8022\sqrt{22 - 2}}{\sqrt{(1 - (0.8022)^2)}} = 6.0092$

Entering the t tables at 22 - 2 = 20 degrees of freedom, the value of t at the 1 per cent probability level is 2.84

so the value of r given above is significant.

Hence,

$$y = r \times \frac{s_y}{s_x}(x - 8695) + 1485$$

$$= 0.0749x + 834 \qquad (10.2)$$

The 95 per cent confidence limits on y are given by, for y = Y, the same expression that was used in section C.2.

Now t = 2.09

and $\frac{s_y^2 - b^2 s_x^2}{N - 2} = 832$

For X = 8 695

the 95 per cent confidence limits on Y are

$$\pm 2.09\sqrt{((832)(1 + 0))}$$

or ± 60.3lb/sq. in.

For X = 12 000 and 5 390,

the 95 per cent confidence limits on Y are

$$\pm 2.09\sqrt{((832)(1 + 0.2042))}$$

or ± 66.19lb/sq. in.

TABLE C.2

Test No.	f_c p.s.i.	f_o p.s.i.
VB1	9760	1394
VB2	9760	1505
VB3	8380	1419
VB4	8380	1605
VB5	9240	1754
VB6	9240	1443
VB7	6450	1145
VB8	6450	1219
VB9	7070	1530
VB10	7070	1431
VB11	5750	1493
VB12	5750	1306
VB13	4580	1219
VB14	4580	946
VB15	11260	1568
VB16	11260	1730
VIB3	11300	1754
VIB4	11300	1754
VIB5	10340	1605
VIB6	10340	1605
VIB7	11520	1456
VIB8	11520	1804

C.4 RESULTS OF SERIES CII

The regression (12.1) was calculated from the results given in Table 12.1.

Putting $f_c = x$ and $f_o = y$,

then $\Sigma x = 286\ 470$

$\Sigma x/N = 8\ 184$ where $N = 35$

$(\Sigma x/N)^2 = 66\ 977\ 856$

$(\Sigma x^2)/N = 72\ 454\ 448$

$s_x^2 = 5\ 476\ 592$

$s_x = 2\ 340$

and $\Sigma y = 61\ 314$

$\Sigma y/N = 1\ 751$

$(\Sigma y/N)^2 = 3\ 066\ 001$

$(\Sigma y^2)/N = 3\ 286\ 406$

$s_y^2 = 220\ 405$

$s_y = 469$

also $\Sigma xy/N = 15\ 423\ 755$

$\frac{\Sigma x}{N} \times \frac{\Sigma y}{N} = 14\ 330\ 184$

hence $\mathrm{cov}xy = 1\ 093\ 571$

and $r = 0.9955$

hence,

$$y = r \times \frac{s_y}{s_x}(x - 8184) + 1751$$

$$= 0.1996 + 117.5 \qquad (12.1)$$

C.5 RESULTS OF SERIES III

The results given in Table 12.2 were averaged in the manner given below.

It is convenient to put $f_{1dg}/f_c = x$.

CIII1 to CIII4

$\Sigma x = 6.2590$

$\Sigma x/N = 0.4814$ where $N = 15$,

$(\Sigma x/N)^2 = 0.2317$

$(\Sigma x^2)/N = 0.2319$

$$\text{hence C.V.} = \frac{\sqrt{(0.2319 - 0.2317)}}{0.4814} \times 100 = 2.9\%$$

CIII5

$\Sigma x = 1.9380$

$\Sigma x/N = 0.3876$ where $N = 5$,

$(\Sigma x/N)^2 = 0.1502$

$(\Sigma x^2)/N = 0.1505$

$$\text{hence C.V.} = \frac{\sqrt{(0.1505 - 0.1502)}}{0.3876} \times 100 = 4.5\%$$

CIII6 to CIII9

$\Sigma x = 5.6540$

$\Sigma x/N = 0.3325$ where $N = 17$

$(\Sigma x/N)^2 = 0.1105$

$(\Sigma x^2)/N = 0.1111$

$$\text{hence C.V.} = \frac{\sqrt{(0.1111 - 0.1105)}}{0.3325} \times 100 = 6.7\%$$

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