



# Dynamics of soliton explosions in ultrafast fiber lasers at normal-dispersion

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**Abstract:** We found two kinds of soliton explosions based on the complex Ginzburg-Landau equation without nonlinearity saturation and high-order effects, demonstrating the soliton explosions as an intrinsic property of the dissipative systems. The two kinds of soliton explosions are caused by the dual-pulsing instability and soliton erupting, respectively. The transformation and relationship between the two kinds of soliton explosions are discussed. The parameter space for the soliton explosion in a mode-locked laser cavity is found numerically. Our results can help one to obtain or avoid the soliton explosions in mode-locked fiber lasers and understand the nonlinear dynamics of the dissipative systems.

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**OCIS codes:** (060.5530) Pulse propagation and temporal solitons; (140.4050) Mode-locked lasers; (140.3510) Lasers, fiber.

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## 1. Introduction

Ultrafast fiber lasers have become important tools in scientific researches and industry [1–3]. They are also important test beds to research the nonlinear dynamics of dissipative systems [4–6]. A better understanding of the nonlinear dynamics can help one to make a better design of a ultrafast fiber laser. The complex cubic-quintic Ginzburg-Landau equation (CQGLE) and its coupled version are widely used to describe the ultrafast dynamics of the mode-locked fiber lasers [7–10]. The soliton explosion was numerically found for the first time in [11], where the soliton explosion manifests itself as a chaotic and quasi-periodic process when the dissipative system is in a meta-stable state. The soliton in the meta-stable dissipative system erupts into pieces in temporal domain abruptly and gradually recover its original state after the eruption, which is similar to exploding behavior and thus regarded as the so-called soliton explosion [11]. The soliton explosion was experimentally demonstrated in [12] in a solid-state Kerr-lens mode-locked laser. After the first theoretical and experimental finding of soliton explosions, several theoretical works on this phenomenon have been done based on the CQGLE [13–19]. Linear stability analysis was proposed to explain the origin of the soliton explosion [13, 14]. High-order effects (H.O.Es) such as the third-order dispersion (TOD), self-steepening (SST) and soliton self-frequency shifting (SSFS) were demonstrated to control to dynamics of the soliton explosion [15–17]. Noise-induced soliton explosion was demonstrated in [18]. For experimental findings, the dissipative soliton explosions accompanied with Raman emission were found in a nonlinear amplifying loop mirror (NALM) mode-locked fiber laser [20, 21]. The explosions in [20, 21] were linked to the multi-pulsing instability, which is different from the typical soliton explosion caused by the soliton eruption. The typical soliton explosion and the following rogue waves in a carbon-nanotube (CNT) mode-locked fiber laser were found in [22, 23], where the explosions manifested themselves as abrupt collapse of the soliton spectrum without Raman emission. Very recently, the polarization dynamics of the soliton explosion was experimentally researched in [6].

All the theoretical works on the soliton explosions were based on the distributed CQGLE model [13–19] and sometimes H.O.E effects were added in the model [15–17]. Though the distributed model helps one to understand the dynamics of the soliton explosions very well, it is not straightforward for one to obtain or avoid the soliton explosions in a mode-locked laser in the high dimension parameter space of the CQGLE. For an actual fiber laser system, it is easy for one to adjust the net-dispersion, pump strength or the modulation depth of a saturable absorber (SA) to obtain different mode-locking states. The birefringence of the laser cavity can also be easily adjusted by the polarization controllers (PCs) used in the cavity, however, the polarization properties of the solitons are not in the scope of this paper. To obtain the qualitative and quantitative relationship between the soliton explosion and actual laser parameters is not only important to understand the dynamics of the dissipative systems but also important to the design of mode-locked lasers. The simulation of soliton explosions based on a lumped model in [20, 21] reproduced the experimental results well, however, one wants to know whether the soliton explosion can happen and how they evolve in the lumped model without H. O. E effects and nonlinearity saturation.

In this paper, we report our numerical simulations of the soliton explosions based on the complex Ginzburg-Landau equation (CGLE) in a lumped cavity model at normal-dispersion without H. O. Es and nonlinearity saturation. The cavity length in our simulations ranges from 2m-6.35m in order to demonstrate the universality of the soliton explosion with different laser cavity length at normal-dispersion. We find that two kinds of explosions exist in the laser system, one is the dual-pulsing instability (DPI) and the other is the typical erupting soliton explosion. Pump strength, lumped loss and spectral filtering have strong effects on the dynamics of the soliton explosions. The DPI explosion can be transformed into the erupting soliton explosion by reducing the lumped loss. Both DPI and erupting soliton explosion can be transformed into the stable multi-pulse state through spectral filtering. No soliton explosion happens under small net-dispersion (below 0.064ps<sup>2</sup> in our case) with 70% modulation depth of the SA. The detailed behaviors of the two kinds of soliton explosions are presented and discussed. Our results enrich the dynamics of the dissipative soliton explosions and are useful for the design of ultrafast laser.

## 2. Model and parameters in simulations

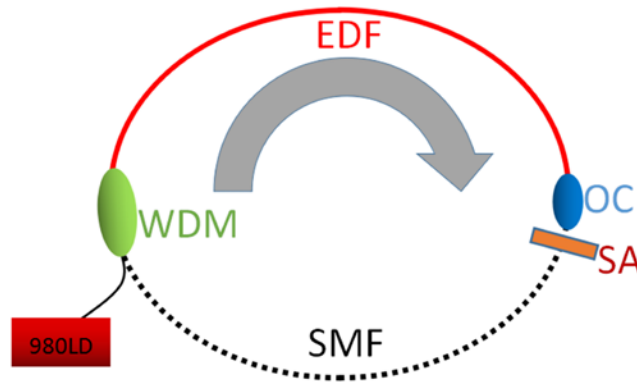


Fig. 1. Schematic of the fiber laser in our simulation. LD:laser diode, WDM:wavelength division multiplexer, OC: output coupler, SA: saturable absorber, EDF: Er-doped fiber, SMF: single mode fiber.

The lumped cavity model that we use in our simulation is schematically shown in Fig. 1. The main parts of the cavity include 2m Er-doped fiber (EDF), a segment of single-mode fiber (SMF), a lumped SA and an output coupler (OC). Our simulation runs from the EDF to the SMF in each round as shown in Fig. 1. We use the CGLE to describe the pulse propagating in the laser cavity:

$$\frac{\partial u}{\partial z} = -\frac{1}{2}i\beta_2 \frac{\partial^2 u}{\partial T^2} + i\gamma|u|^2 u + \frac{1}{2}g \left( 1 + \frac{1}{\Omega^2} \frac{\partial^2}{\partial T^2} \right) u \quad (1)$$

where  $u$  represents the slow varying envelop of the optical field and  $T$  is the retarded time.  $\beta_2$  is the second-order dispersion, which is 50ps<sup>2</sup>/km in EDF and -23ps<sup>2</sup>/km in SMF.  $\gamma$  is the nonlinearity of the fiber, which is 4.7 W<sup>-1</sup>km<sup>-1</sup> and 1.3 W<sup>-1</sup>km<sup>-1</sup> in EDF and SMF, respectively.  $\Omega$  represents the gain bandwidth of the EDF and the full width of half maximum (FWHM) of the gain bandwidth is 20nm in our simulation.  $g$  is the gain of the EDF, which is represented by:

$$g = g_0 \exp(-\int |u|^2 dt / Es) \quad (2)$$

where  $E_s$  is the saturable energy of the EDF, which also represents the pump strength.  $g_0$  is the small signal gain of the EDF, which is  $2.5\text{m}^{-1}$  in our simulation. The transmission function of the amplitude of the lumped SA is represented by:

$$T = \sqrt{1 - \alpha / (1 + P / P_0)} \quad (3)$$

where  $\alpha$  is the modulation depth of the SA, which is varied in our simulations.  $P_0$  is the saturation power of the SA, which is 100W in our simulations. The parameters are chosen to make the simulation converge quickly. The parameters of the SA can be flexibly controlled by the artificial SA of the nonlinear polarization rotation (NPR) [23]. For simplicity, we use the monotonic model and the parameters of the SA have no obvious effects on the conclusions of this paper. We have checked that the H. O. E effects do not have obvious effects on the pulse properties under the pulse parameters in this paper. In fact, our paper here is to demonstrate that the soliton explosion is an intrinsic property of the dissipative systems, which is independent of the H. O. E effects. We vary the length of SMF to adjust the net-dispersion of the laser cavity in the simulations. We use the symmetric split-step Fourier method to implement our simulations. We run the programs for 50000 rounds for each case in our simulations.

### 3. Results

#### 3.1 Dual-pulsing instability induced soliton explosions

In our simulations, DPI happens when the net-dispersion is larger than  $0.064\text{ps}^2$ . For a certain value of the net-dispersion, the DPI exists in a certain range of pump strength. Weaker or stronger pump strength results in the single or double pulse mode-locking. The parameter space will be summarized in section 3.3, and we present the typical results in sections 3.1 and 3.2. We set the net-dispersion, modulation depth of the SA and the output ratio of the OC to be  $0.091\text{ps}^2$ , 0.7 and 10:90, respectively. By increasing the pump strength to 1020pJ, DPI explosion happens and its dynamics is shown in Fig. 2. Figures 2(a) and 2(b) show the round-to-round evolution of the dissipative soliton in temporal and spectral domains. We can get from Fig. 2(a) that the soliton jumps chaotically in the temporal domain. The new pulse originates from the background noise in temporal domain. One can get from Fig. 2(b) that there are obvious spectral perturbations before the DPI soliton exploding. When the explosion comes into being, the soliton spectrum becomes narrow as well as modulated because of the dual-pulsing state in temporal domain. The pulse energy and its spectral width evolutions are shown in Fig. 2(c) and 2(d), respectively. It is obvious in Fig. 2(c) that the soliton experiences a transformation from the stationary state to the relaxation oscillating state before the abrupt energy increasing. The pulse energy at the stationary and exploding states are 1.2nJ and 2.1nJ, respectively. After the abrupt energy increasing, the pulse energy gradually recovers its stationary state. The period of the DPI is ~995-1050 round trips. The spectrum of the ultrashort pulse experiences an abrupt narrowing during the DPI explosion, which is shown in Figs. 2(b) and 2(d). Figures 2(e) and 2(f) show four pulse states in one period of DPI explosion. One can see from Fig. 2(e) that the new born pulse emerges at the background during the explosion and the old pulse still exists at its original location. Due to the gain competition between the old and new pulses, the old pulse gradually annihilates and new soliton comes into being at the new location. We can get from Fig. 2(f) that the spectrum is modulated due to the double-pulsing state and much narrower than the single soliton state. We should note that the temporal jumping in this case is irregular, for example, the temporal jumping in the first explosion is ~26ps while the value for the third explosion is ~50ps, which means there is no specific position of the new pulse relative to the old pulse. Such DPI explosion is similar to the multi-pulsing instability explosion in [21], however, the explosion in [21] is accompanied with Raman emission and an additional-noise-like pulse, which is different from the two smooth pulses in our DPI explosion. Considering the abrupt changing

(energy increasing, spectrum narrowing) as well as the quasi-periodic evolution of the soliton state [11–19], we think the DPI is a new state of the soliton explosion.

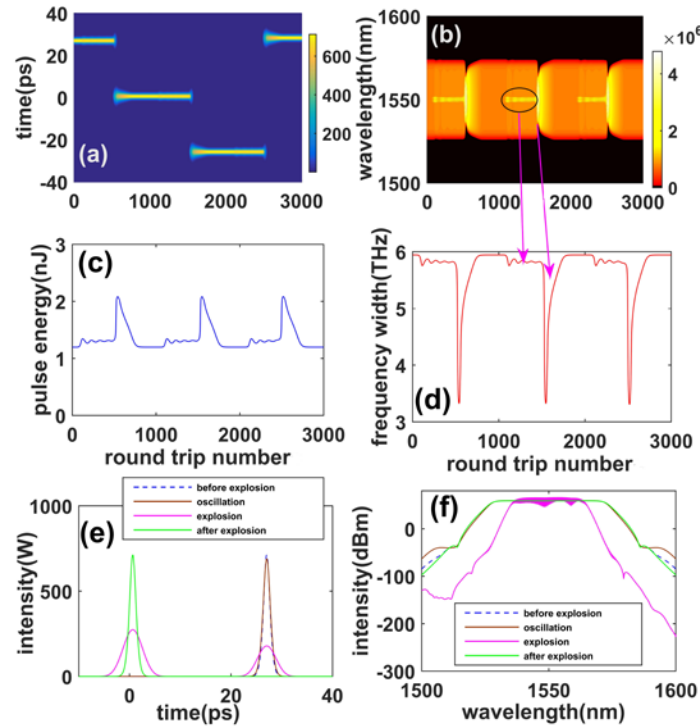


Fig. 2. Pulse characteristics with 1020pJ pump strength, 70% modulation depth and 0.091ps<sup>2</sup> net-dispersion: (a) pulse evolution in temporal domain, (b) pulse evolution in spectral domain, (c) pulse energy evolution, (d) evolution of frequency width of the soliton, (e) pulse intensity profiles at different stages, (f) pulse spectra at different stages.

With the pump strength increasing while keeping the other parameters fixed, the irregular jumping of the soliton in temporal domain can be regular, limited at two fixed temporal locations. Figure 3 shows the pulse characteristics under the pump strength of 1180pJ. We can obtain from Fig. 3(a) that the soliton jumps back and forth at two fixed temporal locations with  $\sim 22.7$ ps separation. Spectral perturbations exist on the spectrum of the soliton before its exploding as one can see from Fig. 3(b). The pulse energy evolution in Fig. 3(c) has a shorter period of  $\sim 75$ -77 round trips than that under the pump strength of 1020pJ ( $\sim 995$ -1050 round trips). We should note that the DPI explosion is not strictly periodic but chaotic, however, we can still observe that the periods for the DPI explosion decrease as the pump strength increases. Spectral narrowing appears when the soliton explodes, which is similar to the case in Fig. 2. There is no relaxation oscillation in Figs. 3(c) and 3(d) and the recovering stage dominates during the explosion, which is accompanied by the spectrum broadening, soliton narrowing and energy decreasing. Figures 3(e) and 3(f) show the pulse characteristics before, during and after the DPI exploding with the insets showing their local enlargement portions. We can see from Fig. 3(e) that the new pulse emerges at one side of the old pulse and the old pulse annihilates after the exploding. From the inset in Fig. 3(e), we can note that a very weak pulse emerges with a quite weak intensity of  $\sim 0.008$ W before the exploding. Such weak pulse originates from the background noise and quickly develops into a new pulse during the explosion. We can obtain from Fig. 3(f) as well as its inset that there is a small spectral perturbation before the soliton exploding, manifesting itself as a localized and modulated part in the center of the spectrum. This spectral perturbation corresponds to the weak new pulse



before exploding in Fig. 3(e). The coherent supposition of the weak new and old strong pulses in temporal domain results in the localized and modulated spectral perturbation on the spectrum. As the weak pulse gradually developing into a strong pulse, the small localized spectral perturbation gradually spreads to the whole spectrum as shown in Fig. 3(f). The spectrum is narrowed and modulated during the DPI exploding, which is shown in Figs. 3(d) and 3(f). From the above simulations in Figs. 2 and Figs. 3, we can get that the DPI explosion is accompanied with pulse energy increasing and spectrum narrowing, which is in agreement with the erupting explosion [11] even there are some differences between the DPI and the erupting explosion. The DPI is an intermediate state between the single pulse and dual-pulses, while the results in [20, 21] locate in transition zone between the single pulse and noise-like pulse (NLP). Visualization 1 in the supplementary shows the explosion process in Fig. 3. In our simulations, we found that the pulse spacing between the new and old pulses in the DPI decreases with the pump strength increasing. We estimate that the temporal location where the total perturbation function of the old pulse has its maximum value depends on the pump strength of the dissipative system, however, this should be strictly checked by the linearity analysis [13].

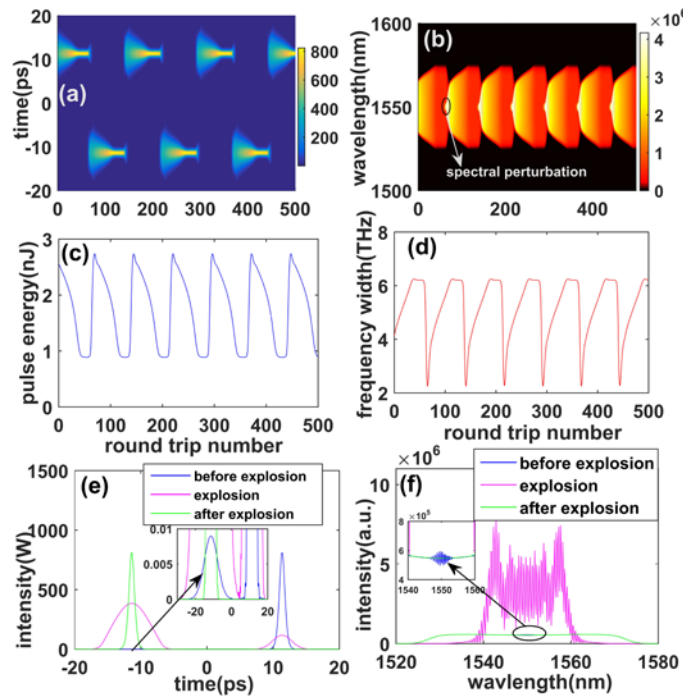


Fig. 3. Pulse characteristics with 1180pJ pump strength, 70% modulation depth and 0.091ps<sup>2</sup> net-dispersion: (a) pulse evolution in temporal domain (see Visualization 1), (b) pulse evolution in spectral domain (see Visualization 1), (c) pulse energy evolution, (d) evolution of frequency width of the soliton, (e) pulse intensity profiles at different stages, (f) pulse spectra at different stages.

### 3.2 Pulse erupting induced soliton explosions

In our simulations, DPI happens when the lumped loss is large enough (in our case, an OC with output ratio from 5:95-10:90) while the typical erupting soliton explosion happens when the lumped loss is reduced (typically, 0:100-4:96 output ratio) in the cavity. We removed the OC in the cavity model and set the net-dispersion and the modulation depth of the SA to be 0.1ps<sup>2</sup> and 0.7, respectively, and gradually increase the pump strength, the soliton explosion emerged when the pump strength was 1610pJ and its exploding period is ~657-680 round

trips. In order to show more periods of explosions in several hundreds of round trips, we set the pump strength to be 1800pJ. The pulse characteristics are shown in Fig. 4. We can see from Fig. 4(a) that the soliton erupts abruptly within  $\sim 25$  round trips as shown in Fig. 4(a) and its inset. The perturbations in temporal domain develops quickly into a broad pulse with many dips in it, which is also regarded as the process of the soliton erupting into multi-pieces in [13–19]. We can get from Fig. 4(b) that the spectrum of the soliton transforms from the flat-top shape into the collapsing state during the soliton exploding. The pulse energy and spectral width evolutions of the soliton in Figs. 4(c) and 4(d) are quasi-periodic as well as chaotic, which also shows the characteristics of energy increasing and spectrum narrowing during the soliton exploding. One can get from Fig. 4(e) that the soliton recovers from the erupting state to its original state after the soliton explosion. The exploding soliton in Fig. 4(e) is cut into pieces with many dips on it. The spectra of the soliton before and after explosion in Fig. 4(f) have flat tops and near-rectangular shapes, which are characteristics of the dissipative soliton at normal-dispersion. The spectrum of the exploding soliton has many peaks and stronger intensity compared to those before and after explosion, the simulated spectrum in Fig. 4(f) is similar to the experimentally measured shape in [22]. The results shown in Fig. 4 are typical characteristics of the soliton explosion in [11], whose dynamic evolutions can be regarded as the homoclinic while chaotic trajectories in the infinite phase space [13]. We can note that the soliton and its exploding state are strictly symmetric in both temporal and spectra domains, which is due to the symmetric perturbations. [Visualization 2](#) in supplementary shows the pulse explosion in Fig. 4.

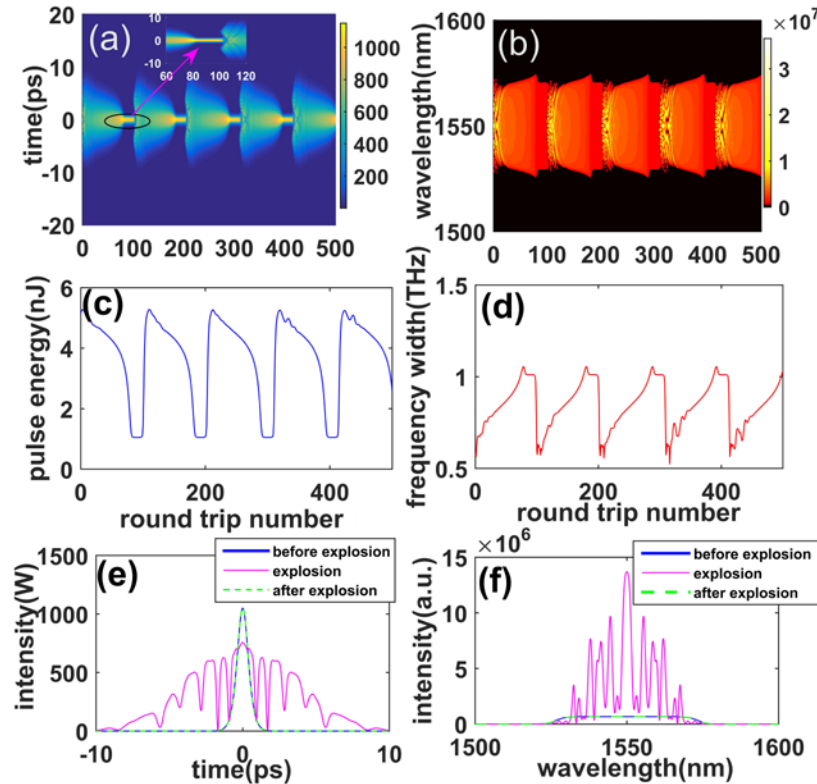


Fig. 4. Pulse characteristics with 1800pJ pump strength, 70% modulation depth and  $0.1\text{ps}^2$  net-dispersion: (a) pulse evolution in temporal domain (see [Visualization 2](#)), (b) pulse evolution in spectral domain (see [Visualization 2](#)), (c) pulse energy evolution, (d) evolution of frequency width of the soliton, (e) pulse intensity profiles at different stages, (f) pulse spectra at different stages.

With the pump strength increased to 2400pJ, a kind of asymmetric soliton explosion happened, manifesting itself as a chaotic process accompanied with irregular temporal jumping and soliton erupting. The results are shown in Fig. 5. One can see from Fig. 5(a) that the explosion is asymmetric in temporal domain, which is different from the symmetric state in Fig. 4. Such asymmetric explosion can be qualitatively explained by the linear stability analysis in [13, 14], which is caused by the composition of different symmetric or asymmetric eigen perturbation functions of the systems. Different from the cases of DPI in Figs. 2 and Figs. 3, the background noise mixes with the old pulse, developing into a collapsed soliton with many pieces during the explosion rather than the dual-pulse state. The spectrum evolution in Fig. 5(b) shows that the exploding state dominates the soliton evolution in one period of explosion and the spectra in Fig. 5(b) is no longer symmetric as the case in Fig. 4(b). One can obtain from Fig. 5(c) that the exploding soliton is cut into an asymmetric state with multi-pieces. We can get from the inset of Fig. 5(c) that the perturbation before explosion is asymmetric in temporal domain, which results in the asymmetric soliton explosion and the temporal jumping of the solitons. The perturbations in spectral domain are also asymmetric as one can get from Fig. 5(d). We can get from Fig. 5(e) that the explosion process dominates the soliton evolution. The pulse energy increases from  $\sim 0.97\text{nJ}$  to  $\sim 7\text{nJ}$  during the soliton explosion. [Visualization 3](#) in the supplementary shows the pulse explosion in Fig. 5.

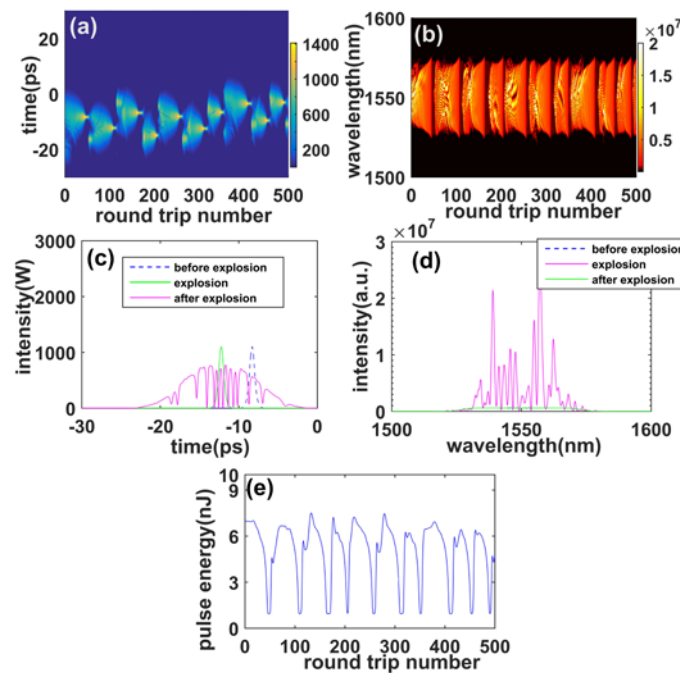


Fig. 5. Pulse characteristics with 2400pJ pump strength, 70% modulation depth and  $0.1\text{ps}^2$  net-dispersion: (a) pulse evolution in temporal domain (see [Visualization 3](#)), (b) pulse evolution in spectral domain (see [Visualization 3](#)), (c) pulse intensity profiles at different stages, (d) pulse spectra at different stages, (e) pulse energy evolution.

The soliton explosion became more chaotic and successive when the pump strength was increased to 10000pJ. We can get from Fig. 6(a) that the soliton evolution is quite chaotic while localized in the temporal domain. The chaotic wave packet occasionally shrinks into a short soliton during evolution, which can be regarded as a successive soliton explosion. The soliton evolution in the spectral domain shown in Fig. 6(b) has similar behavior to Fig. 5(b), which is dominated by the exploding states and recovers to the single pulse state



intermittently. One can get from Figs. 6(c) and 6(d) that the soliton is a smooth pulse with strongly asymmetric perturbations before and after the explosion while it is a wave packet with multi-pieces during the explosion. The pulse evolution in Fig. 6(e) shows that the soliton explosion is successive as there are no pauses for the local minimum energy states. Further increasing the pump strength results in NLP generation.

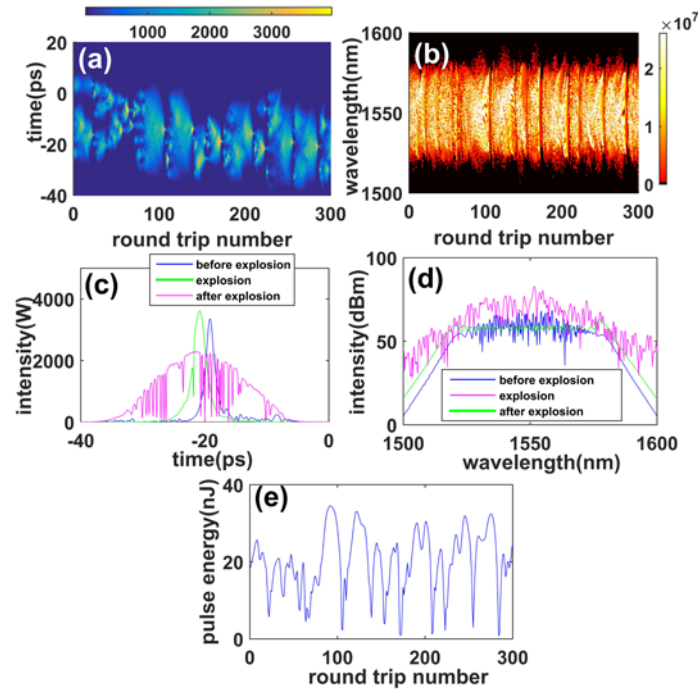


Fig. 6. Pulse characteristics with 10000pJ pump strength, 70% modulation depth and 0.1ps<sup>2</sup> net-dispersion: (a) pulse evolution in temporal domain, (b) pulse evolution in spectral domain, (c) pulse intensity profiles at different stages, (d) pulse spectra at different stages, (e) pulse energy evolution.

### 3.3 Parameters space for the soliton explosion

The soliton explosion in [20–22] are regarded as an intermediate state between the stable single soliton and NLP. In our case of the DPI, the soliton explosion is an intermediate state between the single soliton and soliton molecule. Figures 7(a)-7(c) show the soliton evolutions with 0.091ps<sup>2</sup> dispersion under pump strength of 1180pJ, 1190pJ and 1200pJ, respectively. We can get from Fig. 7(a) the soliton works at the DPI state with the pump strength of 1180pJ. With slightly increasing the pump strength to 1190pJ, the soliton transforms into an asymmetric soliton pair, pulsating during its evolution. With the pump strength of 1200pJ, the soliton transforms into the stable soliton molecule, which is shown in Fig. 7(c). For a specific net-dispersion, the DPI explosion happens in a limited range of pump strength between the single soliton and soliton molecule. The parameters space for the DPI explosion with 0.7 modulation depth of SA is shown in Fig. 8. We can get from Fig. 8 that for a specific net-dispersion, the DPI happens in a limited region of pump strength and the region of pump strength increases with the net-dispersion increasing. No DPI happens if the net-dispersion is less than 0.064ps<sup>2</sup> as we can obtain from Fig. 8. We also research the effect modulation depth of the SA on the soliton explosion with a fixed net-dispersion of 0.091ps<sup>2</sup>. We find that no explosion happens when the modulation depth is below 20%.

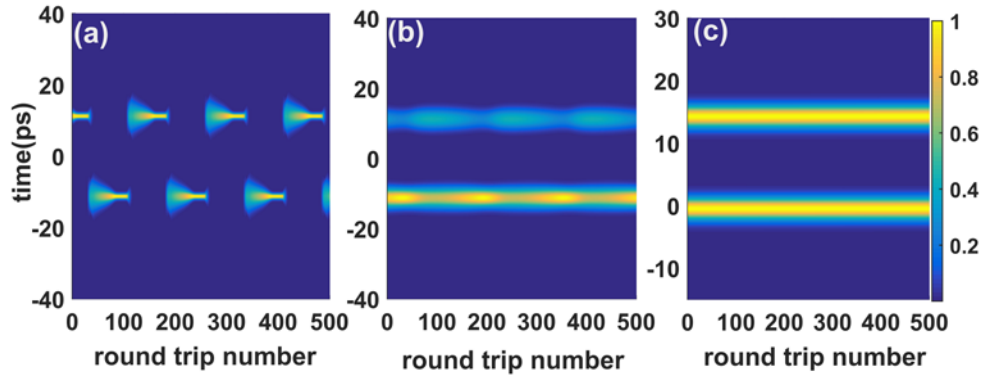


Fig. 7. Pulse evolutions (normalized) with 70% modulation depth and  $0.091\text{ps}^2$  net-dispersion under different pump strength: (a) 1180pJ, (b) 1190pJ, (c) 1200pJ.

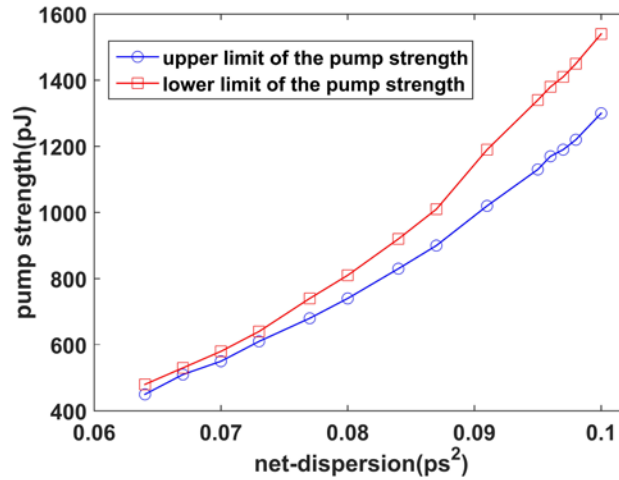


Fig. 8. Upper and lower limits of the pump strength for the soliton explosion happening with 70% modulation depth under different net-dispersion.

We find that the spectral filtering can transform the erupting explosion as well as the DPI into multi-pulse states. If we add a band-pass filter (BPF) at the end of the cavity with a net-dispersion of  $0.1\text{ps}^2$  and pump strength of 1700pJ, the soliton will transform its explosive state to the stable multi-pulse state, which is shown in Fig. 9. We can get from Figs. 9(b) and 9(c) that stronger filtering results in more pulses, which means the spectral filtering has strong effects on the soliton explosion dynamics. The pulse energy evolutions in Figs. 9(d)-9(f) show that the multi-pulse states are quite stable compared to the soliton explosion. We think it is easy for us explain this phenomenon: the explosion originates from the temporal perturbation on the background of the soliton while a BPF can filter the perturbation to avoid the explosion. As for the multi-pulse state, it has been explained in [24] that extra loss by the spectral filtering results in the splitting of the DS. We think the spectral filtering is an alternative method to manipulate the soliton explosion in an ultrafast laser. As has been mentioned in section 3.2, the DPI explosion can be transformed into the typical erupting soliton explosion by decreasing the lumped loss. The reason for such transformation remains unclear, however, it can advise us that soliton erupting tends to be in the higher energy state compared to the DPI.

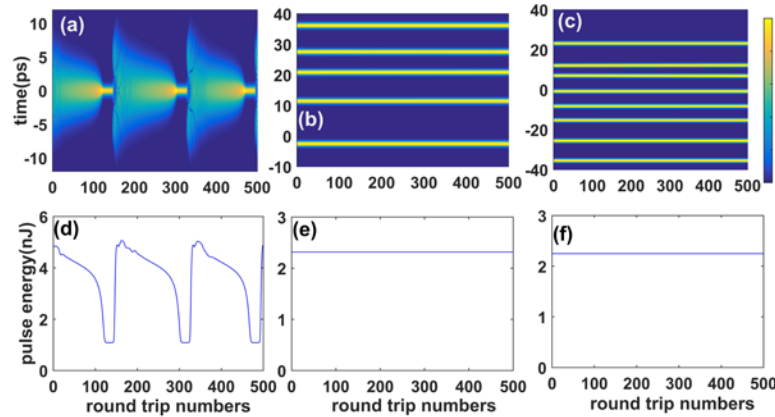


Fig. 9. Pulse evolutions with 70% modulation depth,  $0.1\text{ps}^2$  net-dispersion, 1700pJ pump strength under different spectral filtering: (a) and (d) without BPF, (b) and (e) 10nm(FWHM) BPF, (c) and (f) 5nm BPF.

#### 4. Discussion

The lumped model in our case is different from the distributed model in [13–19]. The model of CQGLE in [13–19] includes two quintic items, the saturation of the nonlinear gain (SNG) and the saturation of the nonlinear index (SNI). It is easy for us to understand that Eqs. (2) and 3 correspond to the SNG in the distributed CQGLE model, which are important for the pulse generation and stabilization. However, there are no items in Eq. (1) corresponding to the SNI in the distributed CQGLE model. As has been proposed in [19], a complex system should have a minimum nonlinear complexity to obtain the robust pulsating solutions, together with the results in our case, we think the SNG is complex enough for a dissipative system to obtain the pulsating solutions such as the soliton explosions while SNI is not necessary. Even the SNI as well as H. O. E are not necessary for the soliton explosion generation, they can affect the behavior of the soliton explosions strongly [11, 16, 20].

The DPI and erupting soliton explosions in our paper seems to be different physical phenomena, however, we can obtain clearly from [Visualization 1](#), [Visualization 2](#) and [Visualization 3](#) that their recovering processes have the very similar behaviors, which are the pulse narrowing, spectrum broadening and energy decreasing. We think they both belong to the soliton explosion proposed in [11], but with different states. It has been demonstrated in experiments that the soliton explosions can have different states, such as Raman accompanied explosion [20], successive explosion [22], blue shifted explosion [12] and vector soliton explosion [6], and we believe more states of soliton explosions and their dynamics are remained to be observed both experimentally and theoretically.

#### 5. Conclusion

In conclusion, we have given a thorough investigation of the soliton explosion by simulations. We find a new kind of soliton explosion, the dual-pulsing instability. The DPI is an intermediate state between the single pulse and soliton molecule while the erupting soliton explosion is an intermediate state between the single pulse and NLPs. The DPI can be transformed into the typical erupting soliton explosion by decreasing the lumped loss under certain parameters. Both the erupting explosion and DPI can be transformed into the multi-pulse state by spectral filtering. For a normal-dispersion mode-locked laser, the DPI happens in a certain range of pump strength when the net-dispersion is large than a threshold ( $0.064\text{ps}^2$  in our case). Both kinds of soliton explosions are sensitive to the dissipative processes such as the nonlinear gain, lump loss and spectral filtering, so we think they can be regarded as the recently proposed item, the incoherent dissipative soliton [6], considering their localized but

incoherent structures. For the applications of the laser systems, the soliton explosion might be undesirable because it can result in time jittering, intensity floating and spectrum eruption, our simulation may give some qualitative instructions for the ultrafast laser designs. Also, our results can give an insight into the nonlinear dynamics of the ultrafast dissipative systems.

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