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Highlights:

– A continuous strategy public goods game is proposed which supports emergence of cooperation: Compulsory Persistent Cooperation

– Persistent Cooperation is an alternative model for evolution of cooperation, similar but different to punishment mechanisms

– This continuous version of persistent cooperation is more realistic as agents can choose to invest into the game in different levels of commitment

– A multi-group version of the game outperforms the single-group version

Compulsory Persistent Cooperation in Continuous Public Goods Games

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Abstract

The public goods game (PGG), where players either contribute an amount to the common pool or do nothing, is a paradigm for exploring cooperative behaviors in biological systems, economic communities and other social systems. In many situations, including climate game and charity donations, any contribution, however large or small, should be welcome. Consequently, the conventional PGG is extended to a PGG with continuous strategy space, which still can't escape the tragedy of commons without any enforcing mechanisms. Here we propose the persistent cooperation investment mechanisms based on continuous PGG, including single-group games, multi-group games with even investment, non-even investment and non-even investment with preference. We aim to reveal how these investment strategies promote the average cooperation level in the absence of any other enforcing mechanisms. Simulations indicate that the multi-group game outperforms the single-group game. Among the multi-group game, non-even investment is superior to even investment, but inferior to non-even investment with preference. Our results may provide an explanation to the emergence of cooperative actions in continuous phenotypic traits based on inner competition and self-management without extrinsic enforcing mechanisms.

Keywords: Evolutionary Game Theory, Public Goods Game, Evolution of Cooperation
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1. Introduction

The public goods game (PGG) [1, 2, 3] is a widely used paradigm for discovering the mechanism of cooperative behaviors, which are abundant both in human and animal societies, and can also be viewed as a basic model of economic interactions [4, 5]. In a conventional PGG, cooperators benefit the population at personal cost c while defectors do nothing. The resulting public resource is shared equally among all participants irrespective of their individual contributions. Players will acquire more profit if they invest nothing into the common pool, which leads to the tragedy of the commons [6]. Many mechanisms including social diversity [7, 8], voluntary participation [9] and persistent cooperation [10, 11] were put forward to overcome these social dilemmas. Other mechanisms such as reward and punishment [12, 13] can also promote cooperation, but both of them may induce the second free riders [14, 15, 16, 17, 18, 19].

In a typical PGG, players select a strategy randomly from the discrete set S={C (cooperation), D (defection)}, either cooperation or defection. However, in many real systems, each individual engaging in a game has heterogeneity on investing owing to his or her own financial strength. Without losing generality, we let the minimum and maximum investment of each individual be 0 and 1 in the PGG. At this point we generalize towards continuous contributions: An individual may invest any value on the interval [0, 1] into

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the public pool. The strategies are no longer pure cooperation or pure defection, all the strategies may be viewed as cooperative actions with various extents of cooperation. Hereby a PGG on a continuous strategy space S=[0, 1] is constructed. The conventional PGG where S=1 (cooperation) or S=0 (defection) is a special case of the continuous PGG.

Many researchers deemed it more reasonable to consider the evolutionary game behavior on a continuous strategy set than a discrete one [20, 21]. For example, in the low-carbon game, due to the heterogeneity of each enterprise in scale, finance, technique and products, etc, all the enterprises may have different degrees of cooperation. Actually, any amount of contribution would be grateful in the public benefit, including environment protection and charity donations. Another common example in biological system is the body size of animals [22], e.g., Anolis lizards [23] and Geospiza finches [24]. Such phenotypes are obviously continuous variables on some certain intervals. Consequently, continuous strategies have been investigated in various evolutionary games. The Prisoner's Dilemma was discussed on continuous space to study the evolution of cooperation [25, 26, 27]. It was concluded that cooperation can evolve easily and remain at relatively high levels [28]. Later, some researchers studied the repeated game and spatial game as well as public goods games based on continuous investment [29, 30, 31, 32]. For the continuous PGG, the temptation to adopt antisocial behaviors wins over taking prosocial actions without any reinforcing mechanisms [33, 34].

Here we present the continuous persistent cooperation model. In the continuous PGG, behaviors of players are considered in a quantitative trait rather than a qualitative one, hence there is no distinct boundary between cooperation and defection.

The remainder of the paper is organized as follows. In section 2, the game model in well-mixed populations is introduced, and its superiority is also illustrated in contrast to the conventional continuous PGG. In section 3, the model is discussed in structured populations, and several mechanisms are presented to enhance the degree of cooperation of the population. Simulations on lattices are performed to show the efficiency of the mechanisms. In section 4, a summary is made based on the analysis and computer simulations in the preceding sections.

2. The Evolution of Continuous Persistent Cooperation in well-mixed populations

2.1. Model

Suppose that in a well-mixed population of size n ($n \geq 2$), each individual plays the PGG with heterogeneous decision ability. They contribute an amount to the common pool. Without losing generality, we let $x_i \in$ [0,1] be the investment of agent i, i=1,$\cdots$,n. The strategies of all the individuals are denoted by a random vector (x_1,$\cdots$,x_n). Thus the continuous PGG on S=$[0, 1]^n$ is constructed. The resulting profit is the sum of all the investments multiplied by a synergic factor r, which reflects the effect of cooperation. Firstly, only a fraction s of the total benefit, i.e., $sr\sum_{i=1}^{n} x_i$, is shared equally among all the players. The remaining fraction (1$-s$) will be used as the public profit, and redistributed to each player according to his or her contribution at an additional personal cost $x_i d$, which we denote as the second cost. Thus, the total payoff of individual i is given by

$$
\begin{aligned}
\pi_i(x_1, \cdots, x_i, \cdots, x_n) &= \frac{sr}{n}\sum_{j=1}^{n} x_j - x_i + (1-s)r\sum_{j=1}^{n} x_j \frac{x_i}{\sum\limits_{j=1}^{n} x_j} - x_i d \\
&= (r - 1 - \frac{n-1}{n}sr - d)x_i + \frac{sr}{n}\sum_{j\neq i} x_j \\
&= \pi_i(x_i; x_{-i}).
\end{aligned}
\tag{1}
$$

Here $d \in [0, 1]$ is the coefficient of the cost in the second distribution. The second cost is proportional to the contribution for each player to gain the additional $(1-s)$ of the total benefit. For convenience, we let

$\pi_i(x_i;x_{-i})$ be a substitution of the payoff function $\pi_i(x_1,\cdots,x_i,\cdots,x_n)$ of agent i, i=1,$\cdots,n$. If s=1 and d=0, it is in accordance with the conventional continuous PGG mentioned in ref.[33], wherein the payoff of individual i is $\pi_i(x_i;x_{-i}) = (\frac{r}{n}-1)x_i + \frac{r}{n}\sum\limits_{j\neq i} x_j$. If $r < n$, then $r/n - 1 < 0$. That is to say, a player will obtain less with the increase of his or her contribution. The only Nash equilibrium is $(0,\cdots,0)$, i.e. none of the players contributes to the common pool. The continuous PGG would be trapped in the tragedy of the commons [6].

2.2. Replicator Dynamics

At time t, the strategy or investment of each player denoted by $(x_1,\cdots,x_i,\cdots,x_n)$ can be viewed as a random vector, which describes the state of the population. Each population state during the evolution can be described by a Borel probability measure Q_t^n defined on S=$[0,1]^n$. In the game, players determine to invest simultaneously and anonymously, they are just aware of their own investments, so $x_1,\cdots,x_i,\cdots,x_n$ are independent of each other. The statistic rule of $x_i(i=1,\cdots,n)$ is described by a Borel probability measure Q. Then the average payoff of player i in state Q is

$$\begin{aligned} E(\delta_{x_i}) &= \int_{[0,1]^{n-1}} \pi_i(x_i;x_{-i})Q(\mathrm{d}x_1)\cdots Q(\mathrm{d}x_{i-1})\cdots Q(\mathrm{d}x_n) \\ &= (r-1-\frac{n-1}{n}sr-d)x_i + \frac{sr}{n}\sum_{j\neq i}\int_0^1 x_j Q(\mathrm{d}x_j) \\ &= (r-1-\frac{n-1}{n}sr-d)x_i + \frac{n-1}{n}sr\bar{x}, \end{aligned} \tag{2}$$

herein $\delta_{x_i}(i=1,\cdots,n)$ is the Dirac delta measure. It means that the total probability mass is concentrated on the single point x_i. Further

$$\bar{x} = \int_0^1 x_j Q(\mathrm{d}x_j) \tag{3}$$

is the average investment of an agent in state Q. Let A be a Borel subset of S, then $P(A)$ is the proportion of the individuals whose strategies are selected in set A. Suppose that the population is large enough to remove the effect of finite size. Finally, when the support of Q is not empty, the replicator dynamics is

$$\frac{\mathrm{d}P}{\mathrm{d}t}(A) = \int_A [E(\delta_x, Q) - E(Q,Q)]Q(\mathrm{d}x), \tag{4}$$

where

$$E(Q;Q) = \int_0^1 \pi_i(x_i;Q)Q(\mathrm{d}x_i) \tag{5}$$

is the average investment of the whole population in state Q. Since the strategy set is compact, and the payoff function is a continuous function with boundary, the equation (4) has a unique solution $Q_t(t \geq 0)$ for all the states for any initial state Q_0 [35].

Now we consider the evolutionary dynamics of the average investment $\bar{x}$

$$\begin{aligned} \frac{\mathrm{d}\bar{x}}{\mathrm{d}t} &= \int_0^1 x_i \frac{\mathrm{d}Q}{\mathrm{d}t}\mathrm{d}x_i \\ &= \int_0^1 x_i[E(\delta_{x_i},Q) - E(Q,Q)]Q(\mathrm{d}x) \\ &= (r-1-\frac{n-1}{n}sr-d)\int_0^1 (x_i-\bar{x})^2 Q(\mathrm{d}x_i). \end{aligned} \tag{6}$$

In equation (6), let $z = r-1-\frac{n-1}{n}sr-d$. If $d < r-1-\frac{n-1}{n}sr$, for fixed synergic factor r and population size n, there exist s and d such that $z > 0$, therefore, $\frac{\mathrm{d}\bar{x}}{\mathrm{d}t}$ is positive in these cases. $\frac{\mathrm{d}\bar{x}}{\mathrm{d}t} = 0$ if and only if $Q_t = \delta_{x_i}$. Due to the support Q_t is invariant of the dynamics equation (4), $\bar{x}$ will be convergent to the maximum value x^* in the support of Q_t. Hence the solution to the equation (4) converges to δ_{x^*} in weak topology [36]. In other words, if there are unselfish individuals who invest their maximum possession 1 in the support of Q_0, then the proportion of players whose investments are $1-\epsilon$ will decrease, no matter how small ϵ is, Q_t will be convergent to δ_1 in weak topology, that is, all the players will contribute their maximum property 1 into the common pool.

3. The Evolution of Continuous Persistent Cooperation in structured populations

The above result was discussed in well-mixed populations, which ignore the effect of spatial structures to the evolution. However in practical systems, limited by region, resource, information and so on, agents have little chance to interact with anyone else randomly, they can only be involved in games in a certain range, so, the populations are often structural. A number of researchers have studied continuous evolutionary games in spatially structured populations [31, 37]. In this section, we are to discuss the continuous persistent cooperation mechanism in structured populations.

3.1. Continuous Persistent Cooperation in a Single-Group Game

3.1.1. Model

Suppose that all the individuals are assigned to the nodes of a network of size n ($n \geq 2$). Edges denote the interactions between pairs of the players. All the players invest into the common pool simultaneously and independently. Each player can only be engaged in one group, here we call it a single-group game. Agent i sitting on node i plays the game with his or her k_i neighbors connected with edges and he or she is the focal one of the group. k_i is the degree of node $i, i = 1, \cdots, n$. In this situation, the total benefit is the sum of the investment from each individual multiplied by a synergic factor r. Firstly, only a fraction s of the total profit is shared equally among all the members in the same group regardless of their contributions. The remaining benefit is distributed again according to the investment of each player at an additional cost $x_i d$ in the second stage, where $d\in$[0,1] is the coefficient of the second cost. The second cost is proportional to the investment. Then the income of player i is given by

$$\begin{aligned}
&\pi_i(x_1, \cdots, x_i, \cdots, x_{k_i+1}) \\
&= \frac{sr}{k_i+1}\sum_{j=1}^{k_i+1} x_j - x_i + (1-s)r\sum_{j\in\sigma} x_j \frac{x_i}{\sum_{j=1}^{k_i+1} x_j} - x_i d \\
&= (r - 1 - \frac{k_i}{k_i+1} sr - d)x_i + \frac{sr}{k_i+1}\sum_{j\neq i} x_j \\
&= \pi_i(x_i; x_{-i}).
\end{aligned} \tag{7}$$

Similarly, the payoff of player j is denoted by $\pi_j(x_j; x_{-j})$.

Obviously, expression (1) is a special case of expression (7) if the network is homogeneous. The evolution is a dynamically learning process. Each individual may learn from one of his or her neighbors. We perform a random sequential update where each Monte Carlo step (MCS) is defined as follows:

(i) A randomly selected agent i plays the game with his or her k_i neighbors and obtains his or her payoff $\pi_i(x_i; x_{-i})$;

(ii) A randomly selected neighbor of agent i , say j, gains his or her payoff $\pi_j(x_j; x_{-j})$ in the game;

(iii) In each MCS, player i either learns from player j with probability

$$P_{i\to j} = 1/(1 + \exp\{[\pi_i(x_i; x_{-i}) - \pi_j(x_j; x_{-j})]/T\} \tag{8}$$

or keeps his or her own strategy with probability $1-P_{i\to j}$.

In this pairwise updating rule [38], T is the strength of noise. $1/T$ serves as the selection intensity in the population dynamics as well as the stochastic errors in the replacement process [39, 40]. High values of $1/T$ correspond to very strong selection, whereas for $1/T \to 0$, selection becomes so weak that evolution proceeds by random drift. If $1/T \to \infty$, we arrive at $Q \to 1$, or $Q \to 0$, depending solely on the sign of the payoff difference, hence the pairwise comparison updating rule becomes deterministic, indicating that an individual always adopts his neighbor with higher income and refuses to imitate one with lower payoff.

Here, we define $\rho = \frac{1}{n}\sum_{i=1}^{n} x_i$ as the average cooperation level in the stable state to measure the effect of the continuous persistent cooperation.

3.1.2. Simulation Results

Now we investigate the evolution dynamics of the model on a square lattice with periodic boundary conditions. During a MCS, initially, all the strategies on [0,1] are assigned randomly to the nodes of a square lattice. Then an individual is selected at random to play the game with k_i=4 (i=1,$\cdots$,n) neighbors. Each player is the focal one of his or her group. It is known that the system size can influence the dynamics to a large degree, so we have confirmed our model on two different linear system sizes, 100×100 and 400×400, and up to 10^5 full Monte Carlo steps before determining the average cooperation level ρ. The phase diagrams in equilibrium have similar distributions in the phase planes. These simulations indicate that our results are robust in even larger systems.

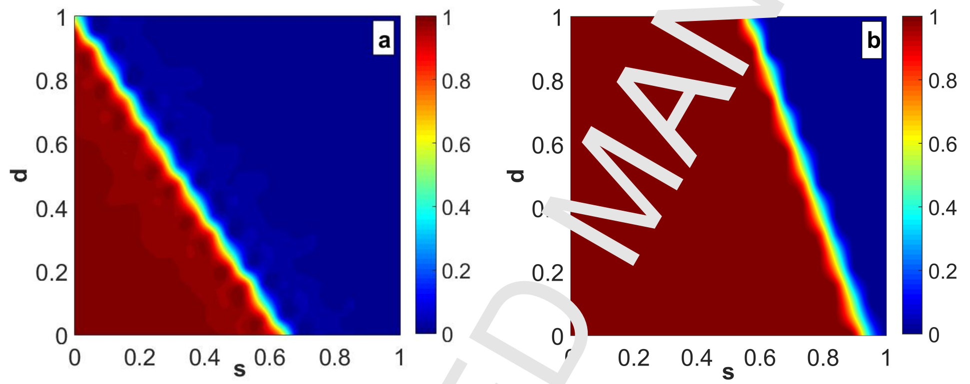

Figure 1: (Color online) The average cooperation level depending on s and d on a 400×400 lattice in the stable state. (a) r=2; (b) r=3.5. Without losing generality, we set $T = 0.1$, implying that better performing individuals are readily to be imitated. The color blue represents a low average cooperation level (upper right region), the darker the blue, the lower the contribution. The color red represents a high average cooperation level (lower left region), the darker the red, the higher the contribution.

Fig.1 is the contour plot of the average cooperative level in the equilibrium state on the $s-d$ plane. The darker in red, the higher the average cooperative level, the darker in blue, the opposite. When r=2, the red area occupys nearly one third of the plane, however, when r=3.5, the red area takes over more than two-thirds. If s is larger than the threshold (in Fig.1a, s≈0.63; in Fig.1b, s≈0.91), the persistent cooperation mechanism fails to spur the cooperative action of all agents even without any second cost, because the value of $(r-1-0.8sr-d)$ is negative. Players will get less benefit if they contribute more into the game, so, they prefer to play the game in a manner of “contributing less, getting more” without any surprise. That leads to the dilemma of the continuous PGG. For any fixed value of s within the certain interval (in Fig.1a, s∈(0, 0.63); in Fig.1b, s∈(0.53, 0.91)), which covers the transition region, the average cooperative level will decrease sharply towards a very low level with the increase of d. If d is large enough such that $(r-1-0.8sr-d)$ is less than zero, an individual will obtain less than ever. In other words, the temptation of drawing back the $(1-s)$ of the profit dwindles down due to the high second cost.

Fig.1 also shows that the synergic factor r plays an important role in the evolutionary dynamics. For the same s and d, a higher synergic factor r results in a higher average cooperation level. As is seen in

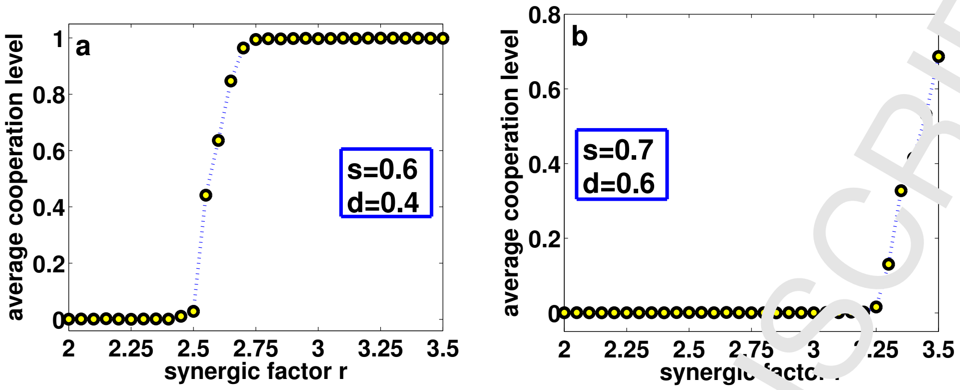

Figure 2: (Color online) Variation of the average cooperation level with r. (a) s=0.6, d=0.4; (b)s=0.7, d=0.6.

Fig.2a, we set s=0.4, d=0.6, when r=2, no agents would like to contribute any property to the public pool. With the increase of r, the average cooperation level will go up rapidly until reaching its maximum value 1 at r=2.75. Similarly in Fig.2b, we let the two parameters be larger than that in Fig.2a, s=0.7, d=0.6, the enthusiasm of all players is frustrated, however, when r increases to 3.25, cooperation begins to emerge.

Fig.3 exhibits the evolutionary dynamics snapshots of the individuals strategies for r=3.5, s=0.6 and d=0.4 in different stages. In the initial state, all the individuals invest into the common pool at random. It is found that even though the overwhelming majority of players are not so active in cooperating at the very beginning (Fig.3a is covered largely with blue), but with the enforcement of continuous persistent cooperation, agents switch their strategies in a short time period (in Fig.3b, blue areas become less and red prevails in Fig.3c). Owing to spatial reciprocity [41, 42], players who have higher contributions form clusters (red clusters) to defend the invasion of neighbors with lower contributions, until all the individuals are very positive in contributing into the common pool (Fig.3d is almost covered with red).

3.2. Continuous Persistent Cooperation in a Multi-group Game

Since the population is composed of rational and inhomogenous agents, they have various decision abilities, learning abilites, social intercourse and investing preferences, etc. These inhomogeneities play different roles in the evolutionary dynamics of cooperation. In this subsection, we take them into consideration to discuss how the average cooperation level varies with these factors. The synergic factor r reflects the conflicts between the social interest and the individual benefit. Larger the synergic factor yields less social dilemma. However, in the PGG, the synergic factor is not very large. Now the question is how to promote the average cooperation level even if the synergic factor is small? Still the agents are arranged on the nodes of a network of size $n(n > 2)$ with periodic boundary conditions. They decide upon their investments into the common pool independently. For the case of agent i, he or she is involved in $k_i + 1$ groups of games, where k_i is the degree of node i. Among these $k_i + 1$ groups, one group is focal at player i, the other k_i groups are focal at the k_i neighbors of player i, respectively. We call it a multi-group game in the context. The influence of agent i is to be reflected in its degree k_i, $(i = 1, \cdots, n)$.

3.2.1. Even investment in a multi-group game

An agent involved in a multi-group game may invest the same amount into each group, so his or her total investment is proportional to his or her degree. On an irregular network, the degree of each node may be different, thereby social diversity is introduced. But if an agent has limited resource, it is hard to invest so generously, then the agent can invest evenly into all the groups he or she is involved in so as to avoid

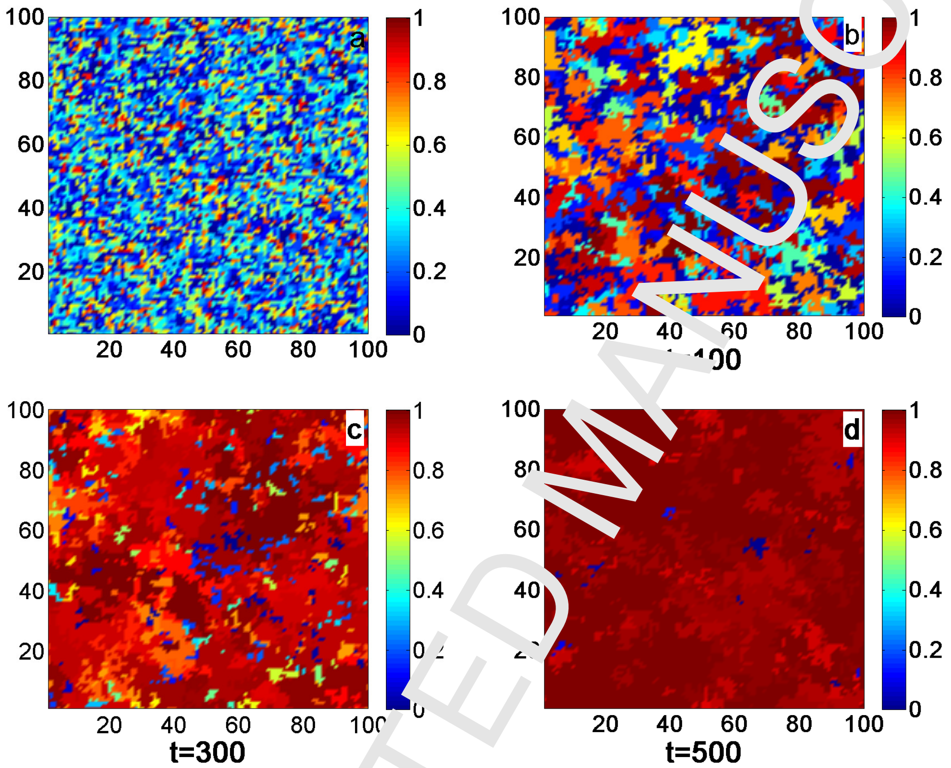

Figure 3: (Color online) Snapshots of the evolutionary dynamics of the agents strategies in a single-group game for different time steps on a 100×100 lattice, where r=3.5, s=0.[illegible], d=0.[illegible] The color blue represents a low average cooperation level, the darker the blue, the lower the contribution. The color red represents a high average cooperation level, the darker the red, the higher the contribution.

risks. Santos et al. studied the promotion of cooperation by social diversity in conventional PGG on discrete strategy space [7].

In our model, agent i who has k_i neighbors, attends k_i+1 groups of games, and invests an amount x_i into the common pool totally. According to the even-investment assumption, he or she shall invest $x_i/(k_i+1)$ to each group, wherein the profit is the sum of the contributions from each member multiplied by a synergic factor r. Only a fraction s of the total profit is shared among all the players, the remaining $(1-s)$ profit is relocated proportionally to their contribution at a second cost. Hence the payoff of agent i in group g is given by

$$\begin{aligned}
&\pi_i^g(x_1,\cdots,x_i,\cdots,x_{N(\sigma_i^g)})\\
&=\frac{1}{N(\sigma_i^g)}sr\sum_{j\in\sigma_i^g}\frac{x_j}{k_j+1}+\frac{1}{k_i+1}[-x_i+(1-s)rx_i-x_id]\\
&=\frac{1}{k_i+1}[(r-1-\frac{k_i}{k_i+1}sr-d)]x_i+\frac{sr}{N(\sigma_i^g)}\sum_{j\neq i}\frac{x_j}{k_j+1}\\
&=\pi_i^g(x_i;x_{-i}).
\end{aligned}\tag{9}$$

Note that σ_i^g is the set of all the co-players of agent i involved in the game of group g, and $N(\sigma_i^g)$ in the number of co-players of agent i in group g. $N(\sigma_i^g)$ is k_i+1 in the group which is local at agent i. The overall payoff of agent i from all the groups of games is obtained by $\pi_i(x_i;x_{-i})=\sum_g\pi_i^g(x_i;x_{-i})$.

3.2.2. Non-even investment in a multi-group game

When the agents play the continuous PGG game, they do expect to obtain more without contributing more than planned. Hence, we take more heterogeneities into consideration such as inner competition. The ecological and economical benefit of the k_i+1 groups are the critical points at present. Those groups who have higher ecological and economical benefit will magnetize more investments and can make more profit than others. Thus agents will fare better in these groups. To make the idea come true, we need to define a new evaluating indicator. Firstly, we denote the average cooperation level of group g at time t by

$$\rho_i^g(t)=\frac{\sum\limits_{j\in\sigma_i^g}x_j}{N(\sigma_i^g)},g=1,\cdots,k_i+1;i=1,\cdots,n.\tag{10}$$

Then, we take sum of all the average cooperation level of the k_i+1 groups which agent i attends, $\sum\limits_{g=1}^{k_i+1}\rho_i^g(t)$. Finally, we compute the ratio of the average cooperation level of group g over the sum of all

$$m_i^g(t)=\frac{\rho_i^g(t)}{\sum\limits_{g=1}^{k_i+1}\rho_i^g(t)},g=1,\cdots,k_i+1;i=1,\cdots,n.\tag{11}$$

Obviously, $m_i^g(t)$ is monotonously increasing with $\rho_i^g(t)$. A group with a higher evaluating indicator can magnetize more investment. Assuming that each player can get the average cooperation levels of all the groups at last time step. By evaluating this indicator, to gain more profit, agents have to adjust their investments to different groups, not $x_i/(k_i+1)$ again but $m_i^g(t-1)x_i$. The payoff of agent i in group g is

$$
\begin{aligned}
&\pi_i^g(x_1, \cdots, x_i, \cdots, x_{N(\sigma_i^g)}) \\
&= \frac{1}{N(\sigma_i^g)} sr \sum_{j \in \sigma_i^g} m_j^g(t-1)x_j + m_i^g(t-1)[-x_i + (1-s)rx_i - x_i d] \\
&= m_i^g(t-1)[(r - 1 - \frac{k_i}{k_i+1} sr - d)x_i] + \frac{sr}{N(\sigma_i^g)} \sum_{j \neq i} m_j^g(t-1)x_j] \\
&= \pi_i^g(x_i; x_{-i}).
\end{aligned} \tag{12}
$$

The overall benefit of agent i from the game is $\pi_i(x_i; x_{-i}) = \sum_g \pi_i^g(x_i; x_{-i})$.

3.2.3. Non-even investment with preference in a multi-group game

In the study of human behavior, it is believed that various individual preferences lead to various individual behaviors and motivations. So we now modify the evaluating indicator $m_i^g(t)$ with a preference coefficient α to reflect preference of agents on investing into the groups with a higher average cooperation level. Thus $m_i^g(t)$ changes into a new one

$$
u_i^g(t) = \frac{(\rho_i^g(t))^\alpha}{\sum\limits_{g=1}^{k_i+1} (\rho_i^g(t))^\alpha}, g = 1, \cdots, k_i + 1; i = 1, \cdots, N. \tag{13}
$$

The investment that agent i contributed into group g is $u_i^g(t-1)x_i$. Now his or her payoff in group g is expressed by

$$
\begin{aligned}
&\pi_i^g(x_1, \cdots, x_i, \cdots, x_{N(\sigma_i^g)}) \\
&= \frac{1}{N(\sigma_i^g)} sr \sum_{j \in \sigma_i^g} u_j^g(t-1)x_j + u_i^g(t-1)[-x_i + (1-s)rx_i - x_i d] \\
&= u_i^g(t-1)[(r - 1 - \frac{k_i}{k_i+1} sr - d)x_i] + \frac{sr}{N(\sigma_i^g)} \sum_{j \neq i} u_j^g(t-1)x_j] \\
&= \pi_i^g(x_i; x_{-i}).
\end{aligned} \tag{14}
$$

The overall benefit of agent i from the game is $\pi_i(x_i; x_{-i}) = \sum_g \pi_i^g(x_i; x_{-i})$.

$u_i^g(t)$ is increasing with α. Parameter α reflects the degree of preference. A larger α results in a higher preference. When α=0, the non-even investment game with preference returns to an even investment game, and if α=1, it is non-even investment in a multi-group game.

3.2.4. Simulation results

In this subsection, we perform numerical simulations for the mechanisms introduced above. Santos et al. showed that cooperation performs better on scale-free networks than on regular networks with social diversity [7], which implys that a structured population usually facilitates the evolution of cooperation in most cases. Therefore, our simulations are performed on square lattice to examine the efficiency of these mechanisms.

Fig.4 displays the variation tendency of average cooperation level with synergic factor r for fixed s and d. Obviously, compared to a single-group game (as illustrated in Fig.3), all these types of investment can promote the positivity in cooperative action for the whole population effectively even if the synergic factor is low. As for the three different types of investment in a multi-group game, the non-even investment performs better than the even investment but not so effective as the non-even investment with preference for some synergic factors. With the increase of coefficient $u_i^g(t)$, agents invest more into the groups which have a high average cooperation level or reputation, contribute less into the other groups. To gain more investment,

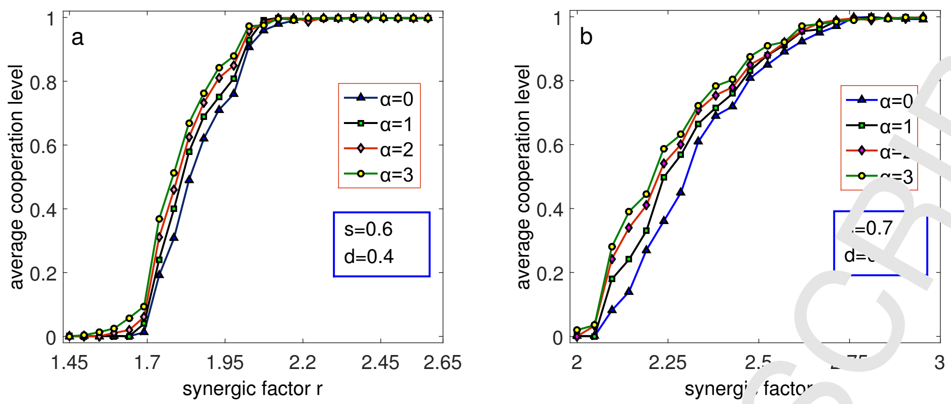

Figure 4: (Color online) The variation of average cooperation level with synergic factor r for fixed s and d. (a) s=0.6, d=0.4; (b)s=0.7, d=0.6.

the individuals in the same group have to contribute as much as they can to enhance the average cooperation level. With this mechanism, the average cooperation level is improved in each group, so is that of the whole population. After r approaches a threshold (r=2.2 for s=0.6, d=0.4 and r=2.8 for s=0.7, d=0.6), there is no obvious difference in enhancing the average cooperation level among the three mechanisms. From Fig.4, we also find that a larger value of α makes cooperation begin to emerge in a shorter period with a smaller synergic factor, e.g., when α=3, cooperation begins to emerge for r=1.75 (Fig.4a) and r=2 (Fig.4b). All in all, the mechanism of non-even investment with preference promotes the activity in cooperating of the players and mollify the social conflicts by self-management.

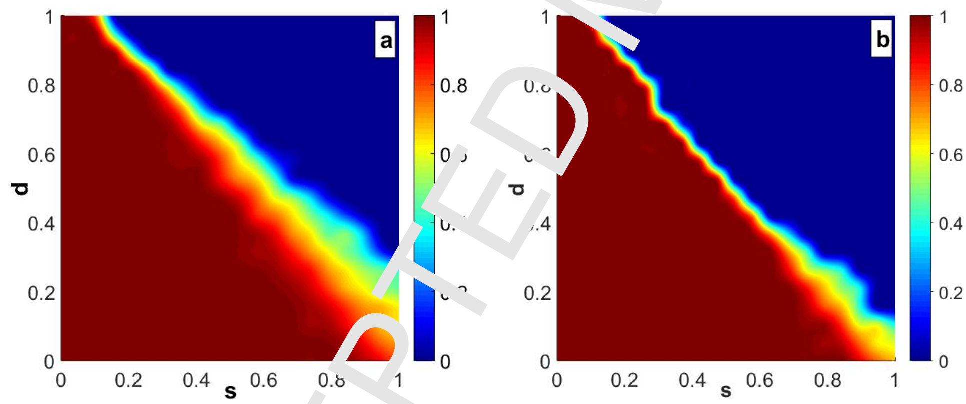

Figure 5: (Color online.) The average cooperation level depending on s and d on a 400×400 lattice in the stable state for r=2, α=3. (a)T=0.1; (b)T=2 The color blue represents a low average cooperation level (upper right region), the darker the blue, the lower the contribution. The color red represents a high average cooperation level (lower left region), the darker the red, the higher the contribution.

Fig.5 illustrates the influence of s and d on the average cooperation level with r=2, α=3. Compared to Fig.1a, the red area covers larger part both in Fig.5a and Fig.5b, which again indicates the efficiency of the investment with preference. Notably, Fig.5(a) shows some qualitative difference from Fig.1(a) on the $s-d$ plane. On one hand, since group size plays a decisive role in the evolution of cooperation in the public goods game on the square lattice, thus the increase in the group size from $N(\sigma_i^g)$=5(single-group) to $N(\sigma_i^g)$=9(multi-group) changes the interaction topology effectively in that the joint membership in the larger groups indirectly links the no-linking players. On the other hand, the noise level T also plays an important role in the difference if $T \ll 1$ in spatial PGG. We set T=0.1 and T=2 in Fig5(a) and Fig5(b), respectively. They have similar distributions on the $s-d$ plane. Topology-independent impact of noise remains valid multi-group systems for larger T values. The simulation results agree with the conclusions as reported in [43]

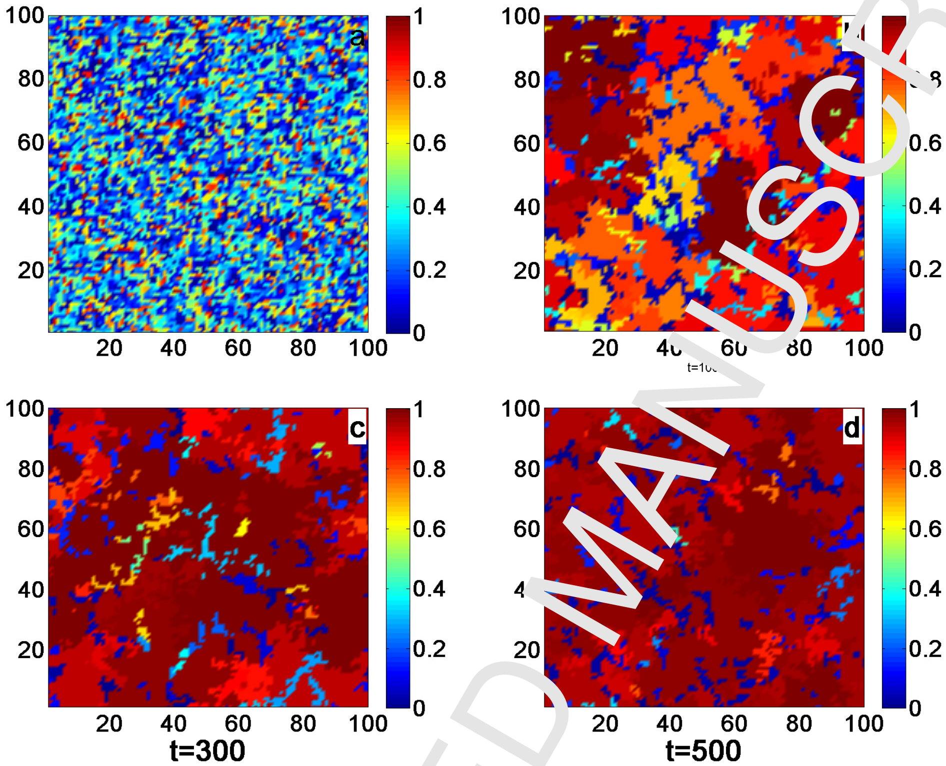

Figure 6: (Color online.) Snapshots of the evolutionary dynamics of agents' strategies in a non-even investment multi-group game with preference on a 100×100 lattice for different time steps, where r=2, α=3, s=0.6, d=0.4. The color blue represents a low average cooperation level,the darker the blue, the lower the contribution. The color red represents a high average cooperation level,the darker the red, the higher the contribution.

Fig.6 presents the snapshots of evolutionary dynamics contour plot of strategies of all the players on a square lattice. The parameters are set as r=2, α=3, s=0.6, d=0.4. In the initial state, each agent is endowed with a strategy randomly, most players are not active in cooperating, or they contribute only a little into the common pool, so the color blue covers almost all the snapshot in Fig.6a, which results in a low cooperation level. To gain more profit, players adjust their investments with preference. The groups which get more investments raise their average cooperation level, players involved in these groups fare well. This encourage other players to contribute more into the game. The legend of "contributing more, getting more" spreads among whole population. Individuals with higher investments also construct clusters in order to defend their opponents with lower contributions. Selfish contributors also form clusters. With the promotion of non-even investment with preference, individuals turn to invest more than ever. The blue clusters begin to shrink, until the plane is covered with more and more red area (Fig.6d). Due to the spatial reciprocity, agents with similar strategies form clusters more easily in a multi-group game than in a single-group game, thus the boundaries among agents in Fig.6b are much clearer than that in Fig.3b.

4. Conclusion and Discussion

In a conventional PGG, some supporting mechanisms such as reward and punishment are often applied to promote the emergence and evolution of cooperation. However, these mechanisms need cooperators who perform rewarding or punishing to pay an additional personal cost, which leads to the second free riders. In this continuous PGG, there are no distinct boundaries between cooperators and defectors. No agents need to punish or reward others to sustain the cooperative actions, they just do their own best in the game. In order to encourage the agents to "contribute more, get more back", several investing mechanisms are presented here. It is found that all the mechanisms can promote the positivity of the agents in investing (cooperative actions) in appropriate range of distribution fraction and second cost coefficient even the synergic factor is low. To make comparisons among the proposed mechanisms, numerical simulations are performed on regular networks. Simulations indicate that players involved in multi-group games are more active than in single-group games. Multi-group games are supposed to disperse risk. An agent should try not to put all his or her eggs in one basket. As for the three different investing styles in the continuous persistent cooperation mechanism, non-even investment is superior to even investment, but inferior to non-even investment with preference. Those groups with higher average cooperation levels or reputation appeal more investments and gain more confidence from the agents. This inner competition and self-management play important roles in promoting the average cooperation level of the body system. In summary, the proposed mechanisms enhance cooperative actions, and a continuous variant of persistent cooperation should be considered where the level of cooperative actions (and hence strategies) is reflected in a continuous phenotypic trait.

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