Optimised Orthogonal Matching Pursuit
Approach

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Abstract

An adaptive procedure for signal representation is proposed. The representation is built up through functions (atoms) selected from a redundant family (dictionary).

At each iteration the algorithm gives rise to an approximation of a given signal, which is guaranteed to be the orthogonal projection of a signal onto the subspace generated by the selected atoms, and to minimise the norm of the corresponding residual error.

The approach is termed Optimised Orthogonal Matching Pursuit because it improves upon the earlier proposed Matching Pursuit and Orthogonal Matching Pursuit approaches.

I. Introduction

Traditional methods for signal representation involve the use of orthogonal bases. Nevertheless, recent methodologies for this purpose have been developed outside the basis setting. These new methodologies operate by decomposing an arbitrary signal into a linear expansion of waveforms, called “atoms”, that are selected from a large and, in general, redundant family of functions called a “dictionary”[1]. Most of the new dictionaries are redundant and have been constructed within the frame structure [3], [4], [5], [6], [7], [8], [9]. However, dictionaries of the same redundant nature also arise as the result of merging bases [10]. Among the existing methods for decomposing a signal in terms of dictionary atoms [1], [6], [7], [10], [11], [12], [13], [14] we focus here on the Matching Pursuit (MP) approach [1], since we wish to discuss a natural improvement to that methodology.

MP is a technique to compute adaptive signal representations. The general goal of this technique is to obtain a sparse signal representation by choosing, at each iteration, a dictionary atom that is best adapted to approximate part of the signal. Nonetheless, the MP algorithm in its original form [1] does not provide at each iteration the linear expansion of the selected atoms that approximate the signal at best. A later refinement which does provide such approximation has been termed Orthogonal Matching Pursuit (OMP) [15].

The OMP approach improves upon the MP in the following sense: from the selected atoms through the MP criterion, the OMP approach gives rise, at each iteration, to the set of coefficients yielding the linear expansion that minimises the distance to the signal. However, since it selects the atoms according to the MP prescription, the selection criterion is not optimal in the sense of minimising the residual of the new approximation.

We show here that a natural requirement within the MP philosophy results in a selection criterion amenable to be implemented by the Orthogonal Least Square (OLS) learning approach earlier introduced in the neural networks context [16]. Nevertheless, the mathematical derivation we present in DRAFT
this letter yields an iterative procedure allowing us to simultaneously adapt the signal representation so as to provide, at each iteration, the coefficients of the linear expansion in terms of the selected atoms that represent the signal optimally. Such a procedure, that we term Optimised Orthogonal Matching Pursuit (OOMP), improves upon the OMP one by providing, at each step, the coefficients of the linear expansion that minimises the distance to the signal, and the dictionary atom which is optimal in the same sense.

The letter is organised as follows: In section II the MP procedure is briefly discussed. Section III gives the foundations of the proposed OOMP approach and sketches the algorithm for its implementation. The final remarks are presented in section IV.

II. THE MP APPROACH

Let $\mathcal{H}$ be a Hilbert space, $\Gamma$ a set of indices, and $\mathcal{D} = \{\alpha_n ; n \in \Gamma\}$ a family of functions in $\mathcal{H}$ each of them normalised to unity. Since in practice one deals with a finite number of such functions we shall assume that $\Gamma$ is a finite set of, say, $N$ indices and re-label them as: $n = 1, \ldots, N$. The family $\mathcal{D}$ is referred to as a dictionary and we denote $S$ as the linear span of the dictionary functions, which are called atoms. Thus, a dictionary is complete if for lim$_{N \to \infty}$ the closure of $S$ is equal to $\mathcal{H}$. As already mentioned, a dictionary is in general redundant, which implies that in the finite dimensional case the dimension of $S$ is less than $N$ and in the lim$_{N \to \infty}$ a complete dictionary is actually overcomplete.

Given a signal $f \in \mathcal{H}$ the aim is to represent it as a linear combination of atoms selected from $\mathcal{D}$. The MP approach proposes to make the selection by successive approximations of $f$. At each step the corresponding residue is sub-decomposed by projecting it on the dictionary atom that matches the residue at best. Let $R_k$ be the $k$th order residue and $l_k$ the index $n$ for which the corresponding dictionary atom $\alpha_{l_k}$ yields a maximal value of $|\langle \alpha_n, R_k \rangle|; n = 1, \ldots, N$, where $\langle \cdot, \cdot \rangle$ indicates the inner product in $\mathcal{H}$.

Starting with an initial approximation $f_1 = 0$ and residue $R_1 = f$ the algorithm evolves by sub-decomposing the $k$th order residue into

$$R_k = \langle \alpha_n, R_k \rangle \alpha_n + R_{k+1},$$

which defines the residue at order $k + 1$. Since $R_{k+1}$ given in (1) is orthogonal to $\alpha_n$

$$||R_k||^2 = |\langle \alpha_n, R_k \rangle|^2 + ||R_{k+1}||^2$$

hence, the dictionary atom $\alpha_{l_k}$ yielding a maximal value of $|\langle R_k, \alpha_n \rangle|$ minimises $||R_{k+1}||^2$.

From Eq (1) it follows that at iteration $k$ the MP algorithm results in an intermediate representation of the form:

$$f = f_k + R_{k+1}$$

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with

\[ f_k = \sum_{n=1}^{k} \langle \alpha_{l_n}, R_n \rangle \alpha_{l_n}. \tag{4} \]

Provided that the dictionary is a complete one, in the limit \( k \to \infty \) the sequence \( f_k \) given by (4) has been shown to converge to \( f \) [1]. Nevertheless, if the algorithm is stopped at iteration \( k \) the function \( f_k \) recovers an approximation of \( f \) with an error equal to \( R_{k+1} \). Since the family of atoms \( \alpha_{l_n}; n = 1, \ldots, k \) is in general not orthogonal, the residual \( R_{k+1} \) will not be in general orthogonal to the subspace generated by the selected atoms. Let us denote \( V_k \) to such a subspace and let operator \( \hat{P}_{V_k} \) be the orthogonal projector on \( V_k \). Then \( \hat{P}_{V_k} \) is self-adjoint (i.e., it satisfies the relation \( \langle \hat{P}_{V_k} f, g \rangle = \langle f, \hat{P}_{V_k} g \rangle \), for all \( f \) and \( g \in \mathcal{H} \)) and idempotent (i.e., \( \hat{P}_{V_k} \hat{P}_{V_k} = \hat{P}_{V_k} \)). It is appropriate to recall at this point that the closest function to \( f \) that can be written as a linear expansion of the \( k \)-atoms \( \alpha_{l_n}; n = 1, \ldots, k \) is \( \hat{P}_{V_k} f \). Indeed, let \( g \) be an arbitrary function in \( V_k \) and let us write it as \( g = g - \hat{P}_{V_k} f + \hat{P}_{V_k} f \). If we calculate the distance \( \| f - g \| \), since \( (f - \hat{P}_{V_k} f) \in V_k^\perp \) (the orthogonal complement of \( V_k \) in \( \mathcal{H} \)) we have: \( \| f - g \|^2 = \| f - \hat{P}_{V_k} f + \hat{P}_{V_k} f \|^2 = \| f - \hat{P}_{V_k} f \|^2 + \| \hat{P}_{V_k} f - g \|^2 \), hence the distance is minimised if \( g = \hat{P}_{V_k} f \). Since, unless the family \( \alpha_{l_n}; n = 1, \ldots, k \) is orthogonal, \( f_k \) given in (4) is not the orthogonal projection of \( f \) on \( V_k \), (4) is not the closest approximation of \( f \) that we can obtained in \( V_k \). A refinement to the MP method, which does yield an orthogonal projection approach at each step, has been termed OMP in [15]. The OMP approach improves MP convergence rate and therefore amounts to a better approximation of a signal after a finite number of iterations. However, at each iteration OMP keeps selecting the dictionary atom as prescribed by the MP approach, although such a selection is no longer an optimal one. In the next section we introduce a new method that we term Optimised Orthogonal Matching Pursuit (OOMP) which, in addition to providing the orthogonal projection approximation, at each step selects the dictionary atom that minimises the corresponding residual error.

III. THE OOMP APPROACH

The above mentioned OMP approach provides a decomposition of the signal as given by:

\[ f = \sum_{n=1}^{k} c_n^{(k)} \alpha_n + \hat{R}_k \tag{5} \]

where the coefficients \( c_n^{(k)} \) are computed so as to guarantee that \( \sum_{n=1}^{k} c_n^{(k)} \alpha_n = \hat{P}_{V_k} f \). The superscript of \( c_n^{(k)} \) indicates the dependence of these figures on the approximation step, which entails that at each iteration the coefficients are to be re-calculated. Although the OMP approach gives rise to the best approximation of \( f \) that one can obtain by means of the selected atoms, since the atoms are selected as prescribed by the MP criterion, the choice is not optimum. Indeed, maximisation of \( | \langle \alpha_n, R_k \rangle | \)
minimizes the residue $R_{k+1}$ of the MP decomposition (3) but it does not guarantee that the residue of the OMP decomposition (5) is minimised. In order to achieve optimality in the latter sense we propose a method, that we term OOMP, which, at each step yields the coefficients $c_n^{(k)}$ giving rise to the orthogonal projection approximation of $f$, and selects the dictionary atom that minimizes the corresponding residue.

Let $\alpha_{l_1}$ be a function taken out of the set $\alpha_n$; $n = 1, \ldots, N$ and let us define $V_1 = \alpha_{l_1}$ and $V_{k+1} = V_k \oplus \alpha_{l_{k+1}}$. Thus, by denoting $W_{k+1}$ as the orthogonal complement of $V_k$ in $V_{k+1}$, the orthogonal projector operator onto $V_{k+1}$ can be written as: $\hat{P}_{V_{k+1}} = \hat{P}_{V_k} + \hat{P}_{W_{k+1}}$. Consequently, since by definition $\alpha_{l_{k+1}} \in V_{k+1}$, the orthogonal projection of $\alpha_{l_{k+1}}$ onto $W_{k+1}$ is a function, $\psi_{k+1}$ say, which satisfies

$$
\psi_{k+1} = \hat{P}_{W_{k+1}} \alpha_{l_{k+1}} = \hat{P}_{k+1} \alpha_{l_{k+1}} - \hat{P}_{V_k} \alpha_{l_{k+1}} = \alpha_{l_{k+1}} - \hat{P}_{V_k} \alpha_{l_{k+1}}.
$$

(6)

The subspace $W_{k+1}$ is spanned by the single function $\psi_{k+1}$ so that disregarding those functions $\psi_{k+1}$ of zero norm we can define the normalised to unity functions

$$
\tilde{\psi}_{k+1} = \frac{\psi_{k+1}}{||\psi_{k+1}||}
$$

(7)

each of which provides the representation of the corresponding orthogonal projection operator onto $W_{k+1}$ as given by: $\hat{P}_{W_{k+1}} f = \tilde{\psi}_{k+1} \langle \tilde{\psi}_{k+1}, f \rangle$.

Our aim is to select, at each step, the function $\alpha_{l_{k+1}}$ minimising the residue $||f - \hat{P}_{V_{k+1}} f||^2$. The following proposition is in order.

**Proposition 1:** Let functions $\tilde{\psi}_{k+1}$ and $\psi_{k+1}$ be given by (7) and (6), respectively, with $\psi_1 = \tilde{\psi}_1 = \alpha_{l_1}$.

Let function $\beta_{l_1}^{(1)}$ be equal to $\alpha_{l_1}$ and functions $\beta_n^{(k+1)}$ be defined as:

$$
\beta_n^{(k+1)} = \beta_n^{(k)} - \tilde{\psi}_{k+1} \langle \tilde{\psi}_{k+1}, \beta_n^{(k)} \rangle ; \quad n = 1, \ldots, k
$$

(8)

$$
\beta_{l_{k+1}}^{(k+1)} = \frac{\tilde{\psi}_{k+1}}{||\tilde{\psi}_{k+1}||}
$$

(9)

The above defined functions provide a representation of the orthogonal projection operator onto $V_{k+1}$ as given by:

$$
\hat{P}_{V_{k+1}} f = \sum_{n=1}^{k+1} \langle \beta_n^{(k+1)}, f \rangle \alpha_n.
$$

(10)

The proof is achieved by induction. For $k + 1 = 1 \langle \beta_{l_1}^{(1)}, f \rangle \alpha_{l_1} = \langle \alpha_{l_1}, f \rangle \alpha_{l_1}$ is obviously equal to $\hat{P}_{V_1} f$ because the residue $R_1 = f - \langle \alpha_{l_1}, f \rangle \alpha_{l_1}$ is orthogonal to $\alpha_{l_1}$ as it readily follows, i.e. $\langle \alpha_{l_1}, R_1 \rangle = 0$.

Assuming that (10) holds for label $k$ we shall show it holds for $k + 1$. Indeed, using (9) and (8) we obtain:

$$
\sum_{n=1}^{k+1} \langle \beta_{l_n}^{(k+1)}, f \rangle \alpha_n = \sum_{n=1}^{k} \langle \beta_{l_n}^{(k+1)}, f \rangle \alpha_n + \langle \tilde{\psi}_{k+1} \rangle \alpha_{l_{k+1}}
$$
\[
\sum_{n=1}^{k} \langle \beta^{(k)}_{l_n}, f \rangle \alpha_{l_n} - \sum_{n=1}^{k} \langle \beta^{(k)}_{l_n}, \alpha_{l_{k+1}} \rangle \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle \alpha_{l_n} + \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle \alpha_{l_{k+1}} \tag{11}
\]

By hypothesis \(\sum_{n=1}^{k} \langle \beta^{(k)}_{l_n}, f \rangle \alpha_{l_n} = \hat{P}_{V_k} f\) and \(\sum_{n=1}^{k} \alpha_{l_n} \langle \beta^{(k)}_{l_n}, \alpha_{l_{k+1}} \rangle = \hat{P}_{V_k} \alpha_{l_{k+1}}\) so that from (11) we have:

\[
\begin{align*}
\sum_{n=1}^{k+1} \langle \beta^{(k+1)}_{l_n}, f \rangle \alpha_{l_n} &= \hat{P}_{V_k} f - \hat{P}_{V_k} \alpha_{l_{k+1}} \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle + \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle \alpha_{l_{k+1}} \\
&= \hat{P}_{V_k} f + (\alpha_{l_{k+1}} - \hat{P}_{V_k} \alpha_{l_{k+1}}) \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle = \hat{P}_{V_k} f + \hat{P}_{W_{k+1}} f = \hat{P}_{V_k} f \tag{12}
\end{align*}
\]

**Corollary 1:** The coefficients \(c^{(k+1)}_n\) yielding the orthogonal projection of \(f\) onto \(V_{k+1}\) can be recursively obtained as:

\[
c^{(k+1)}_n = c^{(k)}_n - \langle \beta^{(k)}_{l_n}, \alpha_{l_{k+1}} \rangle \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle ; \quad n = 1, \ldots, k \tag{13}
\]

\[
c^{(k+1)}_{l_{k+1}} = \langle \frac{\psi_{k+1}}{\|\psi_{k+1}\|}, f \rangle , \tag{14}
\]

with \(c^{(k)}_1 = \langle \alpha_t, f \rangle\).

Proof: For coefficients \(c^{(k+1)}_n\) to provide the orthogonal projection of \(f\) onto \(V_{k+1}\) they should satisfy

\[
\hat{P}_{V_{k+1}} f = \sum_{n=1}^{k+1} c^{(k+1)}_n \alpha_{l_n} \tag{15}
\]

Thus (13) and (14) readily follow by using (8) and (9) in (10) and identifying \(c^{(k+1)}_n\) with \(\langle \beta^{(k+1)}_{l_n}, f \rangle\).

We are now ready to settle the OOMP selection criterion.

**Theorem 1:** The dictionary atom \(\alpha_{l_{k+1}}\) that at iteration \(k+1\) minimises the norm of the residue \(\tilde{R}_{k+1}\) is the one yielding a maximal value of the functionals \(e_n; n = 1, \ldots, N\) given by

\[
e_n = \frac{b_n}{d_n} = \frac{\|\alpha_n \tilde{R}_k\|^2}{1 - \langle \alpha_n, \hat{P}_{V_k} \alpha_n \rangle} ; \quad b_n > 0. \tag{16}
\]

Proof: As discussed above, at iteration \(k+1\) the OOMP residue should verify: \(\tilde{R}_{k+1} = f - \hat{P}_{V_{k+1}} f\) so that \(\|\tilde{R}_{k+1}\|^2 = \|f\|^2 - \langle \hat{P}_{V_{k+1}} f, f \rangle\). Thereby, in order to minimise \(\|\tilde{R}_{k+1}\|^2\) we should maximise \(\langle \hat{P}_{V_{k+1}} f, f \rangle\). From Proposition 1 (cf. eq. (12)) we have:

\[
\langle \hat{P}_{V_{k+1}} f, f \rangle = \langle \hat{P}_{V_k} f, f \rangle + \frac{1}{\|\psi_{k+1}\|^2} \|\alpha_{l_{k+1}}\|^2 - \langle \hat{P}_{V_k} \alpha_{l_{k+1}}, f \rangle^2 = \tag{17}
\]

so that, since \(\langle \hat{P}_{V_k} f, f \rangle\) is fixed in the previous iteration, (17) is maximised if

\[
\frac{\|\alpha_{l_{k+1}}\|^2 - \langle \hat{P}_{V_k} \alpha_{l_{k+1}}, f \rangle^2}{\|\psi_{k+1}\|^2} = \frac{\|\alpha_{l_{k+1}}\|^2 - \langle \hat{P}_{V_k} \alpha_{l_{k+1}}, f \rangle^2}{1 - \langle \alpha_{l_{k+1}}, \hat{P}_{V_k} \alpha_{l_{k+1}} \rangle} = \frac{\|\alpha_{l_{k+1}}\|^2}{1 - \langle \alpha_{l_{k+1}}, \hat{P}_{V_k} \alpha_{l_{k+1}} \rangle} \tag{18}
\]

is maximised \(\square\).

From the last equation it is clearly seen that the atoms that at iteration \(k+1\) have a linear dependence with the previously selected atoms yield a value of \(b_n\) (cf. eq. (16)) equal to zero, because they
fulfill the condition $P_{V_k} \alpha_n = \alpha_n$. Hence, through the restriction $b_n > 0$, all the selected atoms are guaranteed to be linearly independent.

Notice that the additional computational task the OOMP condition introduces, with respect to the original MP one, is the evaluation of $d_n$, the denominator of (16). Corollary 2 below shows that this condition admits a fast implementation through the so called OLS technique, earlier introduced in the context of neural network structure optimisation [16].

**Corollary 2** The OOMP selection criterion given in (16) is amenable to be implementing by maximising the expression:

$$\frac{|\langle \phi_n, f \rangle|^2}{\|\phi_n\|^2} : \|\phi_n\| \neq 0$$  \hspace{1cm} (19)

where for each $n$ the corresponding function $\phi_n$ is obtained by orthogonalising $\alpha_n$ through the Gram Schmidt technique, with respect to the previously selected atoms, i.e., $\phi_n = \alpha_n - \sum_{j=1}^{k} \tilde{\phi}_j \langle \tilde{\phi}_j, \alpha_n \rangle$, where $\tilde{\phi}_j = \phi_j / \|\phi_j\|$. \hspace{1cm} (18)

Proof: From the left-hand-side of (18) we have that (16) can be expressed as $\|\phi_n\|^2 = \|\phi_n\|^2 - \|\psi_n\|^2 = \|\psi_n\|^2 - \|\phi_n\|^2$

On the other hand, if we define $\hat{P}_0$ as the zero operator, by successive applications of the relation $\hat{P}_{V_k} = \hat{P}_{V_{k-1}} + \hat{P}_{W_k}$ one arrives at $\hat{P}_{V_k} = \sum_{j=1}^{k} \hat{P}_{W_j}$. By using this in (6) it follows that $\psi_n = \alpha_n - \sum_{j=1}^{k} \tilde{\psi}_j \langle \tilde{\psi}_j, \alpha_n \rangle \equiv \phi_n$. \hspace{1cm} (19)

**Algorithm Sketch**

We sketch here an algorithm for selecting functions by implementing condition (19), and successively adapting the corresponding coefficients yielding the orthogonal projection of the signal onto the selected subspace.

From $\hat{R}_0 = f$ and the inner products $\langle \alpha_n, f \rangle ; n = 1, \ldots, N$ the OOMP algorithm evolves as follows:

i) Initially set:

- $k = 1; \gamma_n = \alpha_n; d_n = 1$ ($n = 1, \ldots, N$) and $l_1$ equal to the index $n$ for which $|\langle \alpha_n, f \rangle|$ takes a maximal value as $n$ ranges from 1 to $N$.
- Assign $\psi_l = \alpha_{l_1} = \beta_1, c_1 = \langle \alpha_{l_1}, f \rangle$ and $\|\hat{R}_1\|^2 = \|\hat{R}_0\|^2 - |c_1|^2$

ii) for $n = 1, \ldots, N$ compute:

- $\gamma_n = \gamma_n - \psi_k \langle \psi_k, \alpha_n \rangle$
- $b_n = \langle \gamma_n, f \rangle$
- $d_n = d_n - |\langle \psi_k, \alpha_n \rangle|^2$ (or $d_n = \|\gamma_n\|^2$)
- if $|b_n| < \epsilon$ (tolerance parameter); $e_n = 0$, otherwise $e_n = |b_n|^2 / d_n$
iii) Increase $k$ to $k+1$ and set $l_k$ equal to the index $n$ for which $e_n$ takes a maximal value as $n$ ranges from 1 to $N$. Assign $\|\hat{R}_k\|^2 = \|\hat{R}_{k-1}\|^2 - e_{l_k}$. Assign $\psi_k = \gamma_k / \sqrt{d_k}$ and $\beta_k = \gamma_k / d_k$.

Compute $c_k = \langle \beta_k, f \rangle$

iv) For $n = 1 \ldots, k - 1$ compute:

$\beta_n = \beta_n - \beta_{n-1} \langle \alpha_{l_{n-1}}, \beta_n \rangle$

$c_n = c_n - \overline{\langle \alpha_{l_{n-1}}, \beta_n \rangle} c_k$ (where $\overline{\langle \alpha_{l_{n-1}}, \beta_n \rangle}$ indicates the complex conjugate of $\langle \alpha_{l_{n-1}}, \beta_n \rangle$)

v) Repeat steps ii), and iii) and iv).

As in the previous MP algorithms, the OOMP algorithm is to be stopped when some criterion is achieved. In the context of signal representation a popular stopping criterion is given by

\[ \|\hat{R}_k\|^2 \leq \delta \] (20)

where $\delta$ is the desired precision.

It should be stressed that in the case of finite dictionaries, or in the case of incomplete dictionaries when $N \rightarrow \infty$, if such dictionaries are inappropriate for representing the signal at hand within the desired precision, all linearly independent atoms will be selected before the stopping criterion is satisfied. If that were the situation, after a number of iterations we would obtain $b_n = 0; n = 1, \ldots, N$. Then the algorithm should be stopped given the warning “convergence failure”.

Let us assume that the given stopping criterion is reached at iteration $K$. At such stage the above algorithm has selected $K$ indexes $l_k; k = 1, \ldots, K$ and has computed the corresponding coefficients $c_{l_k}$.

Then we are now in a position to represent the signal $f$ as a linear combination of the corresponding $K$ dictionary atoms $\alpha_{l_n}; n = 1, \ldots, K$, i.e.

\[ f = \sum_{n=1}^{K} c_{l_n} \alpha_{l_n} + \hat{R}_K. \] (21)

IV. CONCLUSIONS:

A method for representing signals by selecting functions (atoms) from a redundant family (dictionary) has been advanced. The approach, that we termed OOMP, improves upon the previous MP and OMP approaches in that at each step it provides: a) the orthogonal projection of the signal onto the subspace generated by the selected atoms and b) the dictionary atoms minimising the norm of the corresponding residual error. It has been shown that the proposed selection criterion is amenable to be implemented by the OLS technique earlier introduced in the neural network context. Nevertheless, the derivation we give here amounts to a procedure for adapting, at each iteration, the coefficients of the linear expansion approximating the signal.

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The above remarks, along with the fact that the technique allows for the use of atoms of different nature for representing different properties of a given signal, leads us to expect the proposed method will be of assistance in a broad range of signal processing applications.

References


