Persistence and the Random Bond Ising Model in Two Dimensions

S. Jain

Information Engineering

The Neural Computing Research Group

School of Engineering and Applied Science

Aston University

Birmingham B4 7ET

U.K.

H. Flynn

School of Mathematics and Computing

University of Derby

Kedleston Road

Derby DE22 1GB

U.K.

Abstract

We study the zero-temperature persistence phenomenon in the random bond $\pm J$ Ising model on a square lattice via extensive numerical simulations. We find strong evidence for 'blocking' regardless of the amount disorder present in the system. The fraction of spins which *never* flips displays interesting non-monotonic, double-humped behaviour as the concentration of ferromagnetic bonds p is varied from zero to one. The peak is identified with the onset of the zero-temperature spin glass transition in the model. The residual persistence is found to decay algebraically and the persistence exponent $\theta(p) \approx 0.9$ over the range $0.1 \leq p \leq 0.9$. Our results are completely consistent with the result of Gandolfi, Newman and Stein for infinite systems that this model has 'mixed' behaviour, namely positive fractions of spins that flip finitely and infinitely often, respectively. [Gandolfi, Newman and Stein, Commun. Math. Phys. **214** 373, (2000).]

PACS numbers: 05.20-y, 05.50+q, 05.70.Ln, 64.60.Cn, 75.10.Hk, 75.40.Mg

I. INTRODUCTION

In recent years there has been considerable interest in the 'persistence' problem [1-11] and it has been studied theoretically in a wide range of systems. Generically, this problem is concerned with the fraction of space which persists in its initial ($t = 0$) state up to some later time.

Thus, when studying the non-equilibrium dynamical behaviour of spin systems at zero-temperature we are interested in the fraction of spins, $P(t)$, that persists for $t > 0$ in the same state as at $t = 0$.

It has now been established for quite sometime that for the pure ferromagnetic two-dimensional Ising model, $P(t)$ decays algebraically [1-4]

$$P(t) \sim t^{-\theta}, \tag{1}$$

where $\theta = 0.209 \pm 0.002$ [5]. Similar algebraic decay has been found in numerous other systems displaying persistence [10, 11].

However, computer simulations of the Ising model in high dimensions [3], $d > 4$, the $q-$state Potts [12] ($q > 4$) have suggested the presence of a non-vanishing persistence probability as $t \to \infty$; this feature is sometimes referred to as 'blocking' and has also been found to be present in some models containing disorder [5-6, 13-15]. Clearly, if $P(\infty) > 0$, the problem can be reformulated by restricting attention only to those spins that eventually do flip. Therefore, we can study the behaviour of the residual persistence

$$r(t) = P(t) - P(\infty). \tag{2}$$

Most of the initial effort was restricted to studying pure systems and, it' s only fairly recently that the persistence behaviour of systems containing disorder has been studied [5-6, 13-15]. Very recently [16], the local persistence exponent for the axial next-nearest neighbour Ising model has been estimated to be $\theta = 0.69 \pm 0.01$; a value considerably different to that found for the ferromagnetic Ising model.

Numerical simulations of the bond diluted Ising model [5,6] indicate that the long time behaviour of the system depends on the amount of disorder present. For the weakly diluted system [5], there is evidence of non-algebraic decay prior to 'blocking'. For the strongly diluted model, on the other hand, the residual persistence probability decreases exponentially for large times [6].

Although the presence of a 'blocked' state has also been suggested [13-15] for the random bond Ising model in $2d$, the long time behaviour of the residual persistence has not been investigated to-date and is still an open question.

In this work we attempt to fill the gap by presenting new results of extensive computer simulations of the $2d$ random bond Ising model on a square lattice for a wide range of bond concentrations. Our main objective is to investigate the persistence behaviour as a function of the ferromagentic bond concentration. As we shall see, we find strong evidence for 'blocking' regardless of the amount of disorder present in the system. Furthermore, unlike the bond-diluted case [5,6], here the qualitative behaviour of the model does not appear to depend on the concentration of the disorder.

In the next section we introduce the model and give brief details about the method used to perform the simulations. In section III we discuss the results and finish with some concluding remarks.

II. THE MODEL

The Hamiltonian for our model is given by

$$H = -\sum_{<ij>} J_{ij} S_i S_j \tag{3}$$

where $S_i = \pm 1$ are Ising spins situated on every site of a square lattice with periodic boundary conditions and the quenched ferromagnetic exchange interactions are selected from a binary distribution given by

$$P(J_{ij}) = (1-p)\delta(J_{ij} + J) + p\delta(J_{ij} - J) \tag{4}$$

where p is the concentration of ferromagnetic bonds and we set $J = 1$; the summation in Eqn. (3) runs over all nearest-neighbour pairs only. Note that for $p = 1/2$ and $p = 1$ we obtain the Ising spin glass and the pure ferromagnetic Ising models, respectively.

Initial runs for a range of ferromagnetic bond concentrations were performed for lattices of linear dimensions ranging from $L = 250$ to $L = 1000$. No appreciable finite-size effects were evident for the range of values considered. As a result, the data presented in this work were obtained for a lattice with dimensions 500×500 $(= N)$.

Each simulation run begins at $t = 0$ with a random starting configuration of the spins and then we update the lattice via single spin flip zero-temperature Glauber dynamics [5]. The

updating rule we use is: always flip if the energy change is negative, never flip if the energy change is positive and flip at random if the energy change is zero.
The number, $n(t)$, of spins which have never flipped until time t is then counted.
The persistence probability is defined by [1]

$$P(t) = [< n(t) >]/N \tag{5}$$

where $< \ldots >$ indicates an average over different initial conditions and $[\ldots]$ denotes an average over samples. Averages over at least 100 different initial conditions and samples were performed for each run undertaken.

III. RESULTS

We now discuss our results. The behaviour of the persistence probability is displayed in the log-log plot shown in Figure 1 ($0.1 \leq p \leq 0.5$). The problem is symmetric about the spin-glass ($p = 0.5$) case and, as a check on the numerics, we confirmed that similar plots were obtained for ($0.5 \leq p \leq 0.9$). Note that the error-bars are smaller than the size of the data points. It's clear from the figure that $P(t)$ is finite in the long time limit. Hence, the system is 'blocked'. The 'blocking' probability depends on p, the concentration of ferromagnetic bonds present. However, it would appear that 'blocking' occurs for all of the values of p considered.

To explore the 'blocking' feature further, we plot in Figure 2 the data over a narrow range very close to the pure case, namely $0.95 \leq p \leq 0.999$. For reference purposes, the straight line in Figure 2 has a slope of -0.21 and corresponds to the well established persistence exponent for the pure case $p = 1.0$. It's clear from Figure 2 that we have deviations from the pure case even when $p = 0.999$. For all values of ferromagnetic bond concentrations $p \neq 1$ we have a finite fraction of spins which *never* flips. Furthermore, the blocking probability, $P(\infty)$, also appears to be highly sensitive to the value of p.

In order to examine the behaviour of the 'blocking' probability, we plot in Figure 3 the values extracted for $P(\infty)$ from Figures 1 and 2 against the bond concentration. Note that Figure 3 shows the values of $P(\infty)$ for a wide range of $p : 0 \leq p \leq 1$, including some values which have not been displayed in the earlier figures for clarity. Once again, the symmetry of the plot about $p = 0.5$ acts as a consistency check on the numerics. The plot itself appears

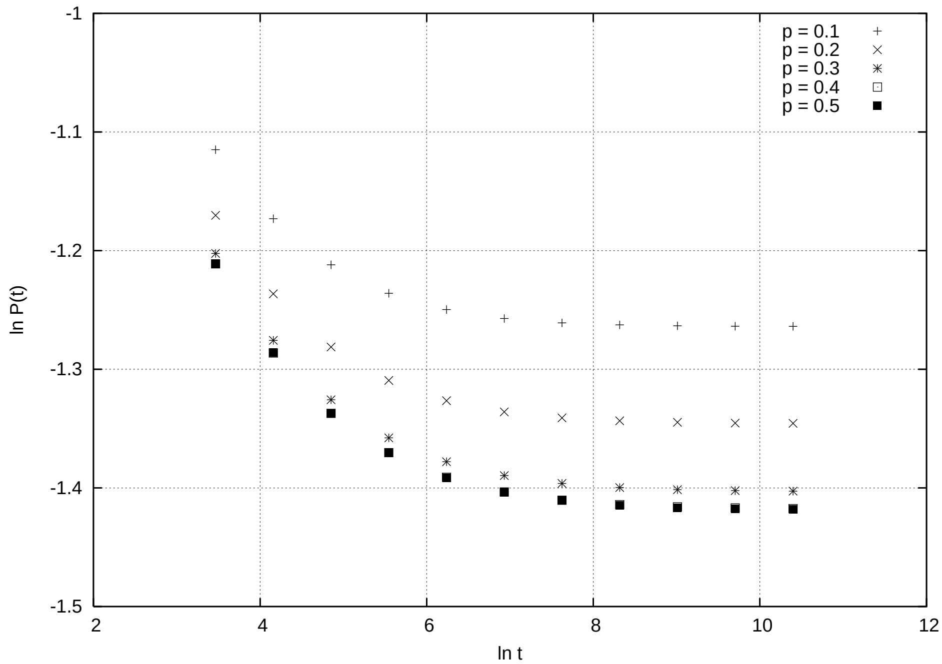

FIG. 1: A log-log plot of the persistence against time for a range of bond concentrations, $0.1 \leq p \leq 0.5$. Note that the data for $p = 0.5$ are superimposed over those for $p = 0.4$.

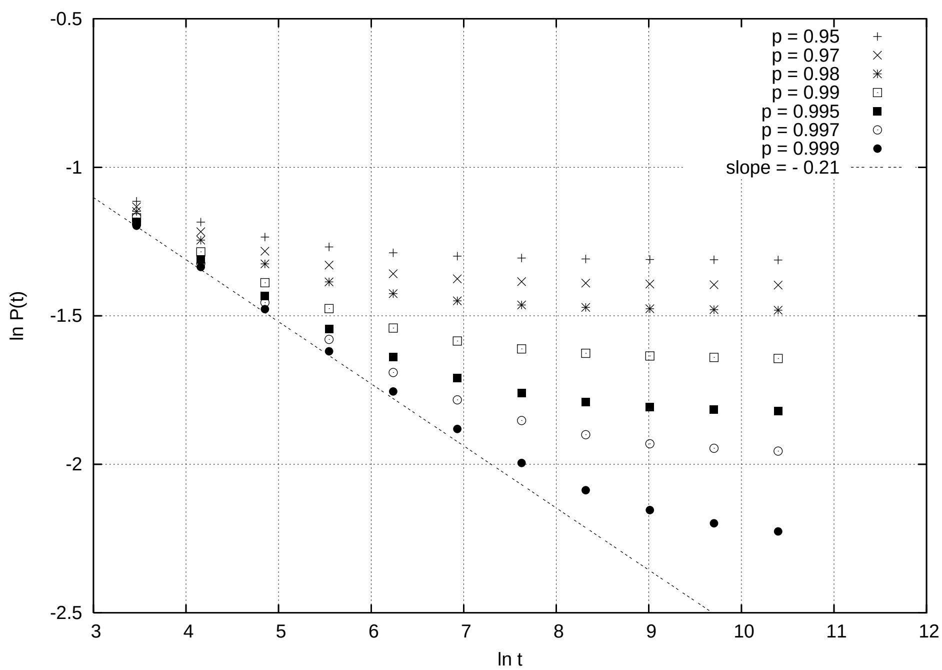

FIG. 2: A plot of $\ln P(t)$ against $\ln t$ for $0.95 \leq p \leq 0.999$. The straight line, corresponding to the behaviour for the pure ($p = 1.0$) case, has gradient -0.21.

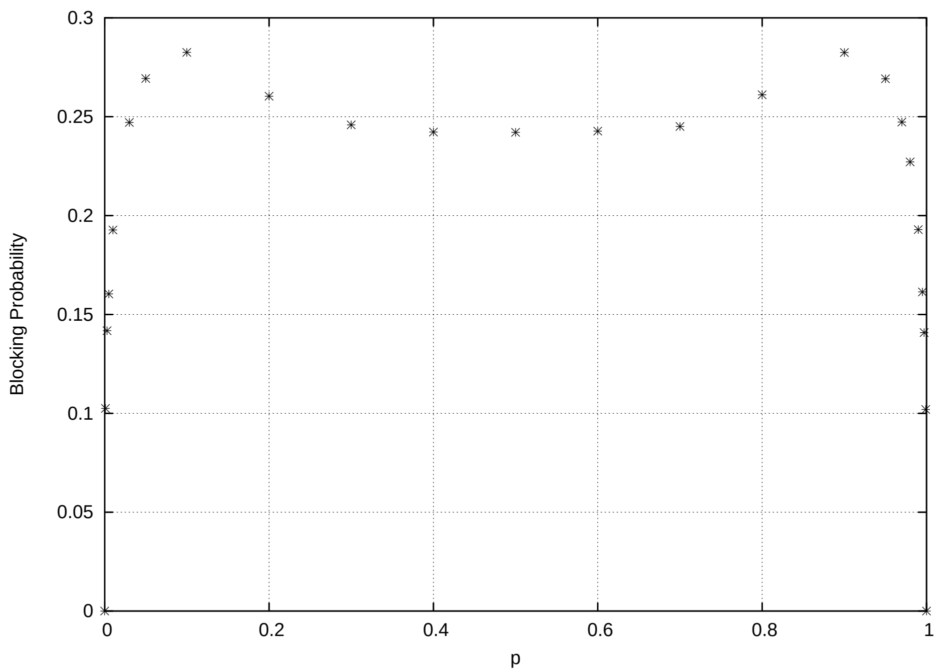

FIG. 3: A plot of the 'Blocking Probability' , $P(\infty)$, against the bond concentration, p.

to have an interesting non-monotonic, double-humped feature. In our model the average fraction of frustrated plaquettes, $Plaq_f$, is given by [17]

$$Plaq_f = 4p(1-p)(p^2 + (1-p)^2) \tag{6}$$

and there is a zero-temperature spin glass transition at $p_c \approx 0.11$ [18]. We see from Figure 3 that the peak in the blocking probability coincides with this value of p_c. Furthermore, $Plaq_f(p_c \approx 0.11) \approx 0.31 < 1/2$, the maximum value of $Plaq_f$.

As explained earlier, for a blocked system, we can study the residual persistence $r(t) = P(t) - P(\infty)$. After having extracted the blocking probability for each p, we calculate $r(t)$. However, there is an error involved in estimating $P(\infty)$. As a consequence, the error in $r(t)$ is much greater than that in the original persistence probability. In Figure 4 we show log-log plots of $r(t)$ against t for three selected values of $p = 0.1, 0.5$ and 0.9. In each case, we see that the decay of the residual persistence is algebraic over the time-interval concerned. However, because of the uncertainty in the blocking probability, there are not inconsiderable error bars attached to the resulting (residual) persistence exponents.

Our estimates for the persistence exponents, $\theta(p)$, are plotted against the bond concentrations in Figure 5. It would appear that $0.8 \leq \theta(p) \leq 1.1$ when $0.1 \leq p \leq 0.9$. For

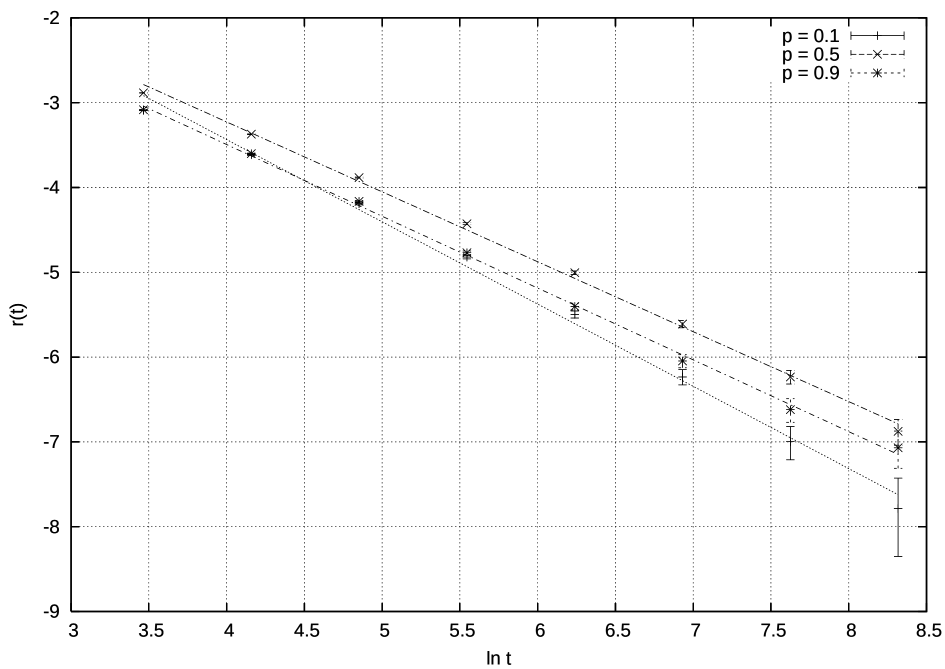

FIG. 4: Here we show a log-log plot of the residual persistence against time for selected bond concentrations. The straight lines are guides to the eye and indicate that $\theta = 0.97(8), 0.83(3), 0.85(4)$ for $p = 0.1, 0.5, 0.9$, respectively.

reference, the exponent for the pure case is indicated by the arrow. Note that although our data are not influenced by finite-size effects, it's nevertheless a non-trivial matter to extract the residual persistence exponent for $0 < p < 0.1$ because of the sensitivity to the estimate of $P(\infty)$. As can be deduced from Figure 2, the closer we are to the pure case, the more difficult it is to estimate the blocking probability.

IV. CONCLUSION

To conclude, we have presented new data for the random bond Ising models on a square lattice. Our results confirm the existence of 'blocking' in the system regardless of the amount of disorder present. The results are consistent with the presence of positive fractions of spins which flip finitely and infinitely often, respectively. We have also investigated the 'blocking' probability and find interesting non-monotonic behaviour as a function of the ferromagnetic bond concentration. The persistence exponent has been extracted and found to be ≈ 0.9, independent of the bond concentration over the range $0.1 \leq p \leq 0.9$. Although we know that

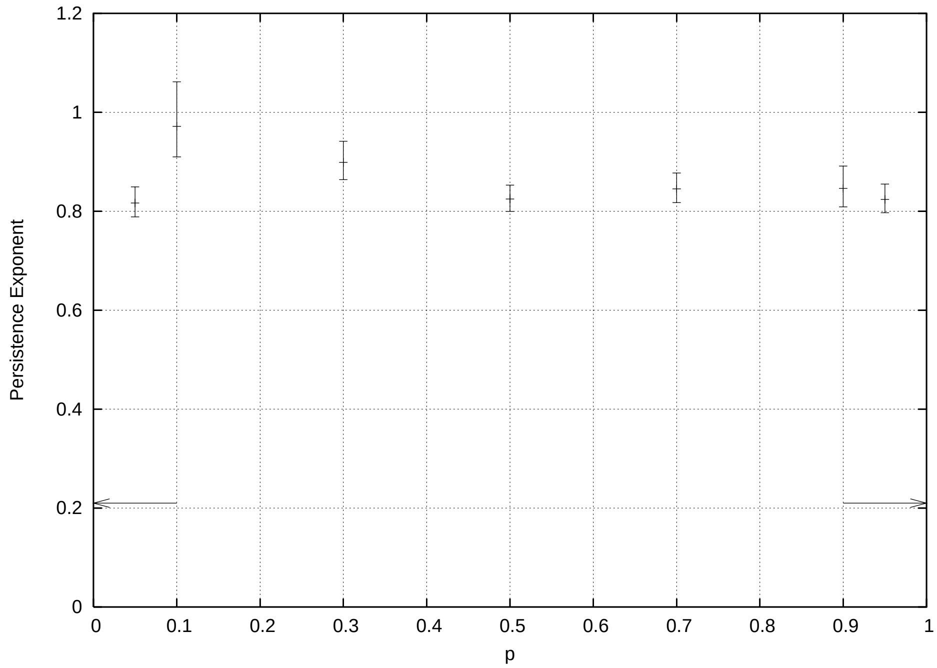

FIG. 5: A plot of $\theta(p)$ against p. For reference, the arrows indicate where the pure, ferromagnetic ($p = 1.0$) and anti-ferromagnetic ($p = 0$), values appear on the plot.

$\theta(p = 1.0) = 0.209 \pm 0.002$ [5], an accurate extraction of the residual persistence exponent for p close to 1 is highly sensitive to the (assumed) value for $P(\infty)$. As a consequence, the complete nature of $\theta(p)$ over the entire range of bond concentrations remains to be established.

V. ACKNOWLEDGEMENT

HF would like to thank the University of Derby for a Research Studentship while this work was carried out.

References

[1] B. Derrida, A. J. Bray and C. Godreche, J.Phys. A **27**, L357 (1994).

[2] A.J. Bray, B. Derrida and C. Godreche, Europhys. Lett. **27**, 177 (1994).

[3] D. Stauffer J.Phys.A **27**, 5029 (1994).

[4] B. Derrida, V. Hakim and V. Pasquier, Phys. Rev. Lett. **75**, 751 (1995); J. Stat. Phys. **85**, 763 (1996).

[5] S. Jain, Phys. Rev. E**59**, R2493 (1999).

[6] S. Jain, Phys. Rev. E**60**, R2445 (1999).

[7] S.N. Majumdar, C. Sire, A.J. Bray and S.J. Cornell, Phys. Rev. Lett. **77**, 2867 (1996).

[8] B. Derrida, V. Hakim and R. Zeitak, Phys. Rev. Lett. **77** 2971 (1996).

[9] S.N. Majumdar and A.J. Bray, Phys. Rev. Lett. **81** 2626 (1998).

[10] S.N. Majumdar, *Curr. Sci.* **77** 370 (1999).

[11] P. Ray, Phase Transitions **77** (Nos. 5-7), 563 (2004).

[12] B. Derrida, P.M.C. de Oliveira, D. Stauffer, Physica A**224**, 604 (1996).

[13] C.M. Newman and D.L. Stein, Phys. Rev. Lett. **82**, 3944 (1999).

[14] C.M. Newman and D.L. Stein, Physica A**279**, 159 (2000).

[15] A. Gandolfi, C.M. Newman and D.L. Stein, Communication Math. Phys. **214**, 373 (2000).

[16] P. Sen and S Dasgupta, J. Phys. A **37**, 11949 (2004).

[17] S. Kirkpatrick, Phys. Rev. B**16**, 4630 (1977).

[18] N. Kawashima and H. Rieger, Europhys. Lett. **39**, 85 (1997).