Nonlinear Fourier transform for optical data processing and transmission: advances and perspectives

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Abstract *The nonlinear Fourier transform is a transmission and signal processing technique that makes positive use of the Kerr nonlinearity in optical fibre channels. I will overview recent advances and some of challenges in this field.*

Introduction

Optical fibre channels are principally and significantly different from many communication media studied in the classical communication theory. The major difference is that optical fibre channels are nonlinear, their properties are dependent on signal amplitude and might be varied with changing signal power. This makes their understanding and optimal exploitation a challenging problem (see e.g. [1-7] and references therein). There is no “nonlinear communication theory” adequately governing information transmission in fibre channels.

In the linear channel the increase of the signal power (respectively increasing the signal-to-noise-ratio) always leads to improved performance, while in the optical fibre channel, higher signal power enhances the nonlinear signal-to-signal and signal-to-noise interactions leading to substantial signal distortions. The fibre Kerr nonlinearity is one of the key physical effects limiting the spectral efficiency and transmission reach of modern fibre-optic communications. It should be pointed out that any optical fibre-based parallelisation of data transmission, e.g. spatial-division-multiplexing (SDM) will face at some stage the same challenge from the fibre nonlinearity. Therefore, compensation or mitigation of nonlinear signal distortion is not competing with SDM techniques, but rather is a complimentary side-by-side challenge.

Here I focus on the theoretical aspects of this problem and consider truly nonlinear technique available for a simplified nonlinear fibre channel model. Consider the lossless nonlinear Schrödinger equation (NLSE) (written here in dimensionless form) as a master model of the nonlinear fibre channel, term η accounts for an effective distributed optical noise ((see e.g. [4] and discussion therein for detail of the model and noise properties):

$$i\frac{\partial q}{\partial z}+\frac{1}{2}\frac{\partial^2 q}{\partial t^2}+|q|^2 q=\eta(z,t) \qquad (1)$$

This lossless NLSE model can be derived under certain conditions by averaging over periodic gain and loss variation in EDFA-based systems [11, 4, 24]. Moreover, quasi-lossless fibre spans in which gain/loss variations can be compensated continuously along the fibre can be achieved using ultra-long fibre concept [54, 55].

From the view point of the linear communication theory, nonlinearity imposes constraints on the transmission of information. An alternative and not yet broadly popular viewpoint is not to pre-impose use of methods developed for linear channels to the nonlinear ones, but to work out new techniques that will be appropriate and adequate for nonlinear channels. Since fibre communication channels are inherently and inevitably nonlinear, rather than treating nonlinearity as a completely detrimental effect, it potentially can be contemplated as an essential element in the design of fibre transmission systems.

In this talk I will update on the recent progress in using the so-called nonlinear Fourier transform (NFT) (known as the inverse scattering transform in the mathematical and nonlinear science communities [8-12]), in optical communications. This field is fast growing and it is beyond the scope of this talk to overview here all recent advances (see [13-53] and references therein).

Nonlinear Fourier Transform

The inverse scattering transform, introduced in 1970’ties, is a powerful mathematical framework that allows one to present nonlinear evolution of the signal q(t) governed by Eq. (1) (without noise term) in a special basis - “nonlinear spectrum”. Evolution of such nonlinear spectral components (eigenvalues) along the fibre is trivial and they do not interact with each other, potentially offering a way to remove nonlinear cross talk [12]. In 1993 Hasegawa and Nyu [12] proposed the concept of eigenvalue communications based on exploitation of the invariance of the eigenvalues (nonlinear spectral components) to encode to transmit information avoiding nonlinear distortions. However, only recent progress in coherent detection and digital signal processing made possible implementation of this idea.

In general, nonlinear spectrum related to Eq. 1 with return-to-zero signal consists of the discrete data $\{\lambda_n, C_n\}$ and the continuous nonlinear spectrum $r(\xi)$ that is the nonlinear analogue of the Fourier spectrum, converging (after some rescaling) to the standard FT of q(t) in the low power limit. The evolution with distance z of the nonlinear continuous spectrum $r(\xi)$ is trivial: $r(\xi, z) = r(\xi, 0)e^{2i\xi^2 z}$. The backward NFT is given by the solution of the so-called Gelfand-Levitan-Marchenko equation (see [8, 10, 38] for details).

Note that all the Kerr nonlinearity induced fibre nonlinear effects, such as self-phase modulation, cross-phase modulation and four-wave-mixing, are mitigated using the NFT.

Most of the NFT based optical communication systems studied so far deal with the burst mode operation that substantially reduce achievable spectral efficiency. The burst mode requirement emerges due to the very nature of the commonly used version of the NFT processing method: it can process only rapidly decaying signals (return-to-zero signals), requires zero-padding guard intervals for processing of dispersion-induced channel memory, and does not allow one to control the time-domain occupation well. Some of the limitations and drawbacks imposed by this approach can be rectified by the recently-introduced more mathematically demanding periodic NFT processing tools. However, the studies incorporating the signals with cyclic prefix extension into the NFT transmission framework have so far lacked the efficient digital signal processing method of synthesising an optical signal, the shortcoming that diminishes the approach flexibility.

Conclusions

The application of NFT-decomposition opens fundamentally new possibilities for advanced coding and modulation, which are resistant to nonlinear transmission impairments. By applying the NFT technique, it is possible to develop a new signal processing routine for compensating nonlinear distortions. From a practical standpoint, the fibre nonlinearity greatly increases the difficulty of understanding system behavior. On the other hand, new techniques may be developed that cannot be realized in linear systems.

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