

# Dissipation Induced Modulation Instability: New Applications for Frequency Combs and Pulses Generation

A. M. Perego<sup>(1)</sup>, S. K. Turitsyn<sup>(1), (2)</sup>, K. Staliunas<sup>(3),(4)</sup>

<sup>(1)</sup>AIPT, Aston University, Aston Triangle B4 7ET, Birmingham, UK, ([peregoa@aston.ac.uk](mailto:peregoa@aston.ac.uk))

<sup>(2)</sup>Novosibirsk State University, Novosibirsk 630090, Russia,

<sup>(3)</sup>Institució Catalana de Recerca i Estudis Avançats, Pg. Lluís Companys 23, 08010, Barcelona, Spain,

<sup>(4)</sup>Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Rambla Sant Nebridi 22, 08222 Terrassa, Barcelona, Spain

**Abstract** *We present a new technique based on a dissipation induced modulation instability, caused by unbalanced spectral dissipation for signal and idler waves, with applications in amplifiers and parametric oscillators but especially for frequency combs and pulses generation in fiber resonators.*

## Introduction

It is in general believed that in order to achieve modulation instability of continuous wave solutions, or parametric amplification in fiber optics phase-matching conditions should be satisfied. In fiber optics, phase-matching conditions are obtained in anomalous group velocity dispersion regime. It was shown however by Tanemura and co-authors that even when phase-matching conditions can not be satisfied amplification of spectral sidebands can be achieved nevertheless: unbalanced spectral losses for signal and idler frequency can lead to modulation instability of continuous wave solution in the normal dispersion regime<sup>1</sup>. In Tanemura and co-authors experiment, losses were induced on the idler wave using a counterpropagating probe inducing Brillouin scattering; amplification of the signal wave at frequency symmetrically located with respect to the pump frequency mode. More generally speaking it is possible to show that even damping selectively the signal wave in the nonlinear Schrödinger equation leads to energy transfer from the pump to both signal and idler frequencies<sup>2</sup>. Such kind of dissipation induced modulation instabilities relies upon a true gain-through-losses dynamics, where very counterintuitively, lossy modes suffer net amplification exactly thanks to the dissipation acting on them.

## Results

Dissipation induced modulation instability caused by spectrally unbalanced losses for signal and idler wave in equations of the nonlinear Schrödinger type have an efficiency proportional to the input pump power, Kerr nonlinearity and losses strength, but inversely proportional to the

group velocity dispersion.

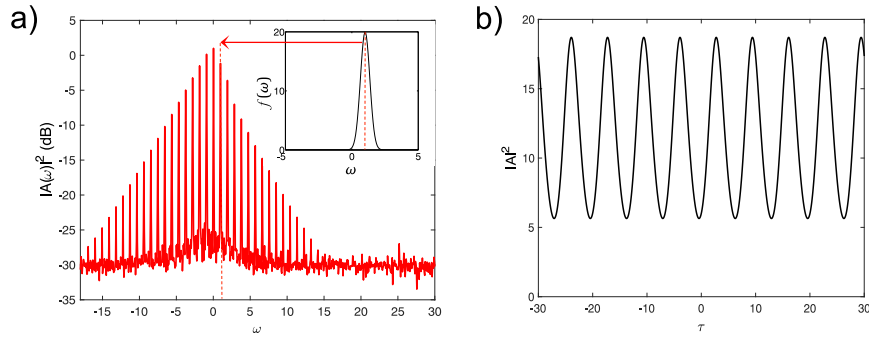
A very interesting application of such losses-enabled signal amplification can be found in the field of frequency comb generation in externally driven ring passive fiber resonators, if a suitable intracavity spectral filter is used. Starting from the Ikeda map approach we have derived a mean field generalized Lugiato-Lefever equation for the normalized field envelope  $A$  defined in the local time reference frame  $\tau$  and evolving in the slow time  $T$ :

$$\frac{\partial A}{\partial T} = -i \frac{\partial^2 A}{\partial \tau^2} + i|A|^2 A + i\Delta A - A + S + F \star A. \quad (1)$$

The second derivative with respect to  $\tau$  describes dispersion, the cubic term accounts for Kerr nonlinearity,  $\Delta$  denotes the cavity detuning, losses and injection are described by the terms  $-A$  and  $S$  respectively. The filter action is modeled in time domain by a convolution (denoted by the  $\star$  symbol) between  $f$  and  $A$ ,  $F$  is the inverse Fourier transform of the filter reflectivity spectral function

$$f(\omega) = \mu e^{-\frac{\omega - \omega_f}{\sigma^2}} \quad (2)$$

where  $\mu$  describes the filter strength,  $\omega_f$  is the filter frequency position and  $\sigma$  its width. In frequency domain the action of the filter corresponds to a simple multiplication of  $f(\omega)$  with the Fourier transform of the field envelope. The filter central frequency  $\omega_f$  is chosen to be detuned with respect to the injection frequency. An analytical and numerical study of the generalized Lugiato-Lefever equation reveals instability dynamics and associated temporal pattern formation excited by a process different from the well-known Benjamin-Feir or Turing instability. Results



**Fig. 1:** The comb-like spectrum induced by the dissipative instability in the stationary state is depicted in a) in the inset the loss profile (filter reflectivity profile is shown). The filter is centered at  $\omega = \omega_f$  and hence the first excited mode in the comb is separated from the pump mode ( $\omega = 0$ ) by a shift equal to  $\omega_f$ . Higher harmonics are excited too. In b) the corresponding temporal trace is plotted: in the stationary state we observe a stable train of pulses with a repetition rate determined by  $\omega_f$ . Parameters used are  $S=40$ ,  $\Delta = 0$ ,  $\omega_f = 1$ ,  $\sigma_f = 0.5$  and  $\mu = 20$ .

of numerical simulations are plotted in Fig. 1. It is observed that in presence of filter, the otherwise stable continuous wave solution of the generalized Lugiato-Lefever equation is destabilized and spectral sidebands located at frequencies  $\pm\omega_f$  and at their integer multiples are amplified. In the stationary state a stable pulse train on the finite field background is observed. The spectrum consists in a frequency comb with triangular shape, in semilogarithmic scale, where the modes spacing is given by  $\omega_f$  i.e. by the detuning between the filter and the pump frequency. Very counter-intuitively dissipation applied to certain spectral modes initiates and preserves a substantial energy flow from the pump frequency to the lossy modes and indirectly to its harmonics and symmetric idler waves.

## Conclusions

Dissipation induced modulation instability caused by the asymmetric losses for signal and idler waves can hence lead to the generation of frequency combs whose mode spacing could be tuned depending on the filter detuning, whose time domain counterparts are ultra-high repetition rate light pulse trains. Furthermore the same gain-through-losses dynamics can be used for selective signal amplification in fiber amplifiers and optical parametric oscillators operating in regimes where classical phase-matching conditions are difficult to achieve, allowing a tunable design of the gain spectrum that assumes the shape of the dissipation spectral profile. Our findings suggest that dissipation instability induced by specific spectral losses landscapes, already successfully applied to achieve high-repetition rate mode-locking in lasers<sup>3,4</sup>, may find a wealth of new applications in nonlinear optics.

## References

- [1] T. Tanemura, Y. Ozeki, and K. Kikuchi, "Modulational Instability and Parametric Amplification Induced by Loss Dispersion in Optical Fibers", *Phys. Rev. Lett.* **93**, 163902 (2004).
- [2] A. M. Perego, S. K. Turitsyn and K. Staliunas, "Instabilities and gain through losses in nonlinear optics", Submitted for publication, (2018).
- [3] A. M. Perego, N. Tarasov, D. V. Churkin, S. K. Turitsyn and K. Staliunas, "Pattern generation by dissipative parametric instability", *Phys. Rev. Lett.* **116**, 28701 (2016).
- [4] N. Tarasov, A. M. Perego, D. V. Churkin, K. Staliunas and S. K. Turitsyn, "Mode-locking via dissipative Faraday instability", *Nature communications* **7**, 12441 (2016).