

Financial Distress and Tail Risk

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Abstract

Considering the declining number of bankruptcy filings, and increasing out-of-court negotiations and debt reorganisations, we argue in favour of penalising firms for becoming sufficiently close to bankruptcy that they have questionable going-concern status. Thus, we propose a definition of financial distress contingent upon firms' earnings, financial expenses, market value and operating cash flow. Subsequently, we investigate the role of tail risk measures (Value-at-risk and Expected Shortfall) in aggravating likelihood of financial distress. Our results show that longer horizon (three- and five-year) tail risk measures contributes positively toward firms' likelihood of experiencing financial distress.

Keywords: tail risk; value-at-risk; downside risk; expected shortfall; bankruptcy; financial distress

JEL Classification Codes: G17, G33, C53

1. Introduction

Financial Distress and *Tail Risk* are two apparently diverse topics that are gaining increasing attention in corporate finance literature. The financial crisis of 2007-08 was the alarm bell that augmented global awareness toward tail risk among financial risk managers. Since then, we have witnessed increasing concern among stakeholders toward firms' risk of bankruptcy or financial distress. Although tail risk has been an active area of investigation in the domain concerned with large financial institutions and financial stability, to the best of our knowledge this study is the first academic attempt to address the relationship between firms' extreme negative daily equity returns and their likelihood of experiencing financial distress. We hypothesise that more frequent extreme negative daily equity returns result in higher tail risk, and this subsequently increases firms' likelihood of entering financial distress.

The vast majority of academic literature on bankruptcy prediction gravitates around either the choice of explanatory variables (e.g. Campbell *et al.* 2008, Jones 2017), or modelling methodologies (e.g. Shumway, 2001, Gupta *et al.* 2018) targeted towards optimising models' classification performance. However, a model's performance is significantly dependent on how the distress or bankruptcy event is defined in the first place. Unfortunately, this aspect has not received sufficient attention in bankruptcy literature. Moreover, legal bankruptcy filings in the United States are becoming an increasingly rare phenomenon,¹ and this might provide false signals on firms' bankruptcy likelihood to external stakeholders. Waiting until the bankruptcy event might lead to significant erosion in firms' value (see Glover, 2016), losses to creditors and unfavourable business decisions in the years prior to the bankruptcy filing. Thus, we also argue for the need for an alternative mechanism to identify firms in financial difficulties, and penalise them for getting close to questionable going-concern status. This might be considered

¹ Out of about 7,538 firms in our sample, between 1990 and 2016, only 216 exit the Compustat database due to Chapter 7/11 filings.

analogous to identifying the symptoms of a disease (financial distress) and starting the treatment, rather than waiting until the fully-grown disease (bankruptcy). This is expected to improve/stabilise credit pricing and minimise losses arising due to mispricing of credit risk.

As a consequence, we propose a definition of financial distress contingent upon firms' earnings, financial expenses, market value and operating cash flow. Our proposed new definition of financial distress essentially builds on that offered by Pindado *et al.* (2008), which is considered to be more functional than its predecessors. We define a firm as financially distressed if it satisfies three simple conditions: (i) there is negative growth in the average market value (AMV) of the firm in years $t - 1$ and $t - 2$, where AMV is the geometrically declining weighted average of a firm's monthly market values over the past twelve months; (ii) its earnings before interest tax depreciation and amortisation (EBITDA) is less than its financial expenses (FE) in the years $t - 1$ and $t - 2$; and (iii) its operating cash flow is less than its financial expenses in the years $t - 1$ and $t - 2$.

Subsequently, we explore the information content of tail risk measures, namely Value-at-risk (VaR) and Expected Shortfall (ES), in explaining firms' likelihood of entering financial distress. Downside or tail risk measures focus only on the risk of underperforming a defined benchmark return and ignore the risk of outperforming the same. We consider the relationship between downside risk measures and financial distress for several reasons. First, there exists substantial literature pertaining to safety-first investors, whose objective is to minimise the probability of financial distress/bankruptcy. A safety-first investor's portfolio is intended to maximise expected return subject to downside risk constraint (see, among others, Roy, 1952, Arzac and Bawa, 1977). Second, as part of efficient risk management practices, financial and non-financial firms are increasingly required to quantify the amount of risk their portfolio or assets may incur over specified time horizons. For instance, banks assess their potential losses in order to maintain adequate levels of buffer capital. Similarly, credit rating agencies track the

value of firms' assets in order to provide them with an accurate credit rating. Finally, empirical evidence typically shows a skewed distribution of stock returns, with peaks around the mode and fat tails (see, among others, Jansen and De Vries, 1991, Conrad *et al.* 2013). This implies that occurrence of negative extreme events is more frequent than suggested by the normal distribution. Therefore, traditionally used measures of market risk (i.e. variance or standard deviation) might be insufficient to approximate the likelihood of maximum loss that a firm may witness under highly volatile or normal periods.²

There is a growing literature on downside risk and expected stock returns (see, for example, Bali *et al.* 2009, Huang *et al.* 2012). However, we did not come across any suitable literature that explores the relationship between tail risk measures and financial distress. Thus, we explore this relationship and, in addition to proposing a definition of financial distress, we assess the marginal discriminatory power of tail risk measures, namely VaR and ES, in predicting the financial distress of publicly-traded firms in the United States. The relationship we explore is intertemporal as we use three-month, six-month, one-year, three-year and five-year daily, and sixty-month monthly returns to measure the VaR and ES of all firms in our sample. Following Zangari (1996), Jaschke (2002) and Liang and Park (2010), we add the Cornish and Fisher (1938) expansion into these risk measures. Contrary to the normal assumption of these risk measures, the Cornish and Fisher (1938) expansion brings skewness and kurtosis into the equation, which is more appropriate for non-normal financial returns. We estimate the risk measure of each firm at the year-end to predict firms' financial distress in the following year. In terms of tail risk measures, our study closely relates to Bali *et al.* (2009), who study the intertemporal relationship between tail risk and expected return, and Liang and

² Empirical literature reports that VaR is reasonable in predicting catastrophic financial market events. It also efficiently captures the rate of occurrence of such events, including extreme cases. On the other hand, the traditional measures of market risk, such as standard deviation and conditional variance, provide an inaccurate prediction of extreme events in financial markets (Jorion, 2000, Longin, 2000, Neftci, 2000, Bali, 2003).

Park (2010), who explore the relationship between downside risk measures and hedge funds' failure.

We test our hypothesis using a sample of publicly-traded firms in the United States obtained from the Compustat database, covering an analysis period from 1990 to 2016. To gauge the statistical significance of respective tail risk measures in predicting likelihood of financial distress, we begin the empirical validation by reporting univariate regression estimates for them. Although tail risk measures and their respective average marginal effects (AME) are highly significant in predicting financial distress likelihood, VaR measures estimated at the 5% significance level show highest values of AME for all respective durations. In support of our hypothesis, the univariate regression results establish the positive relationship between firms' distress likelihoods and extreme negative returns. We also test the marginal discriminatory power of tail risk measure in the multivariate setup. First, we develop our one-year financial distress prediction model using panel logistic regression technique and standard financial ratios found significant in recent literature (e.g. Campbell *et al.* 2008, Gupta *et al.* 2018). Next, we supplement respective tail risk measures to this baseline distress model and analyse its significance in the presence of other competing covariates.

Empirical results suggest that VaR estimates over longer duration (five years) are significant in the multivariate setup and are superior to standard deviation as a proxy for risk measure (as AME of standard deviation is much lower than AME of significant VaR measures). ES also exhibit explanatory behaviour similar to VaR in predicting financial distress. Longer duration (three and five years) are significant, but unlike VaR estimates they show lower values of AMEs. Our results are also robust to financial distress definition of varying intensities. Considering the theoretical superiority of ES over VaR (see Righi and Ceretta, 2016) for additional discussion) we suggest using longer duration ES measures as a proxy for risk

measure. Finally, we find strong support for our hypothesis that firms that show more frequent extreme negative daily equity returns are more likely to experience the risk of financial distress.

The remainder of the paper is organised as follows: Section 2 defines and provides the rationale behind our proposed definition of financial distress; Section 3 provides definitions of downside risk measures employed in this study; and Section 4 discusses data and financial covariates. Section 5 presents our empirical results; Section 6 and 7 present additional results on robustness checks; and Section 8 concludes this study.

2. Defining Financial Distress

Traditionally, most academic studies on bankruptcy prediction gravitated around the choice of explanatory variables and modelling methodologies targeted towards improving prediction models' classification performance. However, these performances depend significantly on how the distress or bankruptcy event is defined in the first place. Unfortunately, this aspect has not received sufficient attention in the bankruptcy literature.

The vast majority of studies employ some legal definition of bankruptcy in line with the relevant bankruptcy code (e.g. Chapter 7/11 filings in the United States), or legal bankruptcy filings supplemented with other related events, such as delisting from stock exchanges due to financial reasons (e.g. Shumway, 2001), or receiving below investment grade credit ratings (e.g. Campbell *et al.* 2008). Plausible reasons behind this additional use of proxy bankruptcy events are attributed to the lack of a standard dataset of bankruptcy events, supplemented with the very small frequency of such events. Since these events significantly threaten firms' degree of solvency, these may be used to model firms' default risk. However, these definitions of default may suffer from several noteworthy issues.³

³ The issues discussed in the subsequent paragraph are drawn from studies that employ sample of listed or unlisted firms of the United States, France, Germany, Hungary, United Kingdom, etc. Since, we employ a sample of the

Firstly, by combining all of these different forms of default, we are trying to predict the probability of a heterogeneous outcome variable, which may be inappropriate. Additionally, since events like bankruptcy resolution involve lengthy legal processes, there often exists a significant time gap between real/economic default date and legal default date (see Theodossiou 1993, Balcaen and Ooghe 2006, Tinoco and Wilson 2013).⁴ Moreover, these events are significantly affected by differences in bankruptcy codes between different countries, thus cross comparison of default prediction models between different countries might be inappropriate. Even in cases of uniform codes, instances of judicial discretion cannot be completely ignored where bankruptcy outcomes are often biased toward debtors or creditors (see Gennaioli and Rossi, 2010 for additional discussion). Additionally, differences in creditors' rights also play a significant role in bankruptcy or default outcomes. Davydenko and Franks (2008) report large differences in creditors' rights across France, Germany and the United Kingdom, leading banks to adjust their lending and reorganisation practices in an attempt to avoid costly proceedings of bankruptcy filings. This might lead to false impressions about firms' financial health, which might otherwise have questionable going-concern value. A recent study by Franks and Loranth (2014) confirms that the large majority of bankrupt Hungarian firms in their sample were maintained as going-concerns despite these firms having suffered large operating losses, primarily due to varying controlling rights of creditors as per the Hungarian bankruptcy code.

However, if we define financial default as debtors' inability to honour the terms of debt contract (specifically failure to make payment when it is due), then this usually precedes events

U.S. listed firms in our study, we do not try to imply that all these issues exist or might exist in the U.S. as well. We merely present an exposition in support of financial distress as an appropriate default definition.

⁴ To gain protection from the bankruptcy code, in the U.S., firms usually file for Chapter 11 before they actually default on a significant debt obligation. This gives them the opportunity to renegotiate the outstanding debt obligation or related credit terms with its creditors. However, this may cause significant cost to creditors in the form of administration/recovery cost, subsidised loan, partial loan waiver, etc. Thus, if lenders can identify the timing of economic default or financial distress (which usually precedes legal default), they may undertake pre-emptive measures to minimise subsequent costs arising in the event of any bankruptcy litigation.

such as legal bankruptcy filing, delisting, or rating downgrade. Frequent occurrence of financial default events might lead to a state of financial distress, where a firm's going-concern aspect is seriously threatened. Once a firm is financially distressed, it may opt for formal reorganisation involving the court system, or an informal reorganisation through the market participants (see John *et al.* 2013 for a comparative discussion). Debt restructuring, asset sale and infusion of new capital from external sources are the three most commonly used market-based or private methods of resolving financial distress (Senbet and Wang, 2010). Debt restructuring allows a financially distressed firm to renegotiate the outstanding debt obligation or related credit terms with its creditors, but is critically subject to whether the debt obligation is due to private or public entity.

As an alternative to this, a distressed firm may sell off some of its existing assets to reduce its outstanding liability, or may undertake new profitable investment opportunities, which may eventually help it to overcome its misery. Despite having profitable investment opportunities, a financially distressed firm might not be able to generate additional funding, due to the high risk involved in financing distressed firms and the 'debt overhang' problem as discussed by Myers (1977). As a consequence, infusion of new capital through informal reorganisation involving market participants is rarely observed in the resolution of financial distress.⁵ Thus, we cannot rule out the possibility that a financially distressed firm may not file for Chapter 7 or Chapter 11 protection and, instead, choose a private workout method for resolving financial distress.

⁵ However, in case of a formal reorganisation involving the court system, new debt financing may be made available through the debtor-in-possession (DIP) provision intended to encourage new lending to firms in bankruptcy reorganisation. Once a firm files for bankruptcy, the DIP provision allows the financially distressed firm to avail new debt financing usually on a seniority basis (violating any absolute priority rule by placing the new financing ahead of the firm's existing debts for payment).

Gilson *et al.* (1990) and Gilson (1997) report that firms avoid legal bankruptcy processes by out-of-court negotiation with creditors. However, it should be noted that such out-of-court negotiations are nearly impossible if the firm under formal restructuring has any public debt. Thus, under the binary classification based on legal consequences, a financially distressed firm which has not filed for Chapter 7 or Chapter 11 is not considered to have questionable going-concern value. There is, therefore, a clear need for an *ex-ante* mechanism to identify firms vulnerable to bankruptcy risk in the near future. Waiting until the bankruptcy event might lead to significant erosion in firms' value, losses to creditors and unfavourable business decisions in the years prior to the bankruptcy filing year. It is in the interest of most stakeholders to raise the flag when the symptoms of financial distress are visible, in order to avoid or prepare for the forthcoming crisis. In this regard, lending or credit decisions based on firms' *financial distress* likelihood may be more relevant in addressing respective stakeholders' objectives.

Moreover, from the literature pertaining to bankruptcy costs, we understand that indirect costs of bankruptcy (which are generally in the form of opportunity costs) usually precede direct costs (including legal and liquidation expenses). Thus, leading to declining degrees of solvency much before the actual bankruptcy filing event date. This is in line with the theoretical arguments of Purnanandam (2008), who suggests that financial distress is a state in which firms are unable to meet their debt obligations, ultimately leading them to insolvency as the debt matures. Prominent signs of indirect costs include loss of valuable employees, restrictive terms of trade credit and decline in consumer confidence and investment growth, which, over time, may lead to significant loss in market value even if the firm never files for legal bankruptcy (Chen and Merville, 1999). Thus, a financially distressed firm is left with the challenge of persuading employees, customers, suppliers and trade creditors to engage in business with it. The closer to bankruptcy a firm is, the stronger this challenge becomes. Stakeholders begin abandoning distressed firms, leading to even faster depletion of operating profits and

shareholder wealth. Thus, rather than considering a firm bankrupt when it files for legal bankruptcy, it is appropriate to penalise it for being sufficiently close to bankruptcy/default for their going-concern status to become questionable.

One might argue the economic rationale for imposing this additional cost on firms, which apparently seems to be counterproductive. However, we know that a penalty on rash driving is expected to make drivers more careful, which, in turn, is expected to reduce the number of accidents and, thus, reduce the overall cost associated with such accidents. Similarly, if a firm is penalised for approaching financial distress in the form of higher cost of credit, reduced access to credit or receiving simply a warning bell, the firm may be more mindful towards its deteriorating financial conditions and take proactive measures to avoid potential costly bankruptcy.⁶ Penalising firms for approaching financial distress will add additional marginal cost to these firms, but it is expected to reduce cost to other stakeholders (lenders, judiciary, trade creditors, etc.).

Through this, to an extent, we try to imply that the threat of a penalty would decrease the likelihood of firms entering financial distress and eventually bankruptcy. However, we cannot rule out the possibility of firms filing for bankruptcy to gain strategic advantage. Gupta and Barzotto (2018) report evidence of strategic bankruptcy resolution among large corporations in the United States and argue that firms evaluate their likelihood of emerging from Chapter 11 bankruptcy filings before make filing decisions.

Considering the discussion above, we believe that financial distress is practically more relevant than legal bankruptcy events and, in the following discussion, we propose a new definition of financial distress that builds on suggestions and findings from earlier related

⁶ Through this statement we do not imply that lenders are currently unable to assess credit risk and aren't imposing higher costs of credit to riskier borrowers. We rather suggest that our proposed definition of financial distress will aid them in better assessment and pricing of credit risk.

studies. In this study, we define a firm as financially distressed in the year t if it satisfies the following three conditions in years $t - 1$ and $t - 2$:

Condition 1: There is negative growth in the average market value (AMV) of the firm in years $t - 1$ and $t - 2$. Where, AMV is the geometrically declining weighted average of a firm's monthly market values (MV) over the past twelve months computed as follows:

$$AMV_{t-1,t-12} = \frac{1 - \varphi}{1 - \varphi^{12}} (MV_{t-1} + \varphi MV_{t-2} + \dots \dots \dots \varphi^{11} MV_{t-12}) \quad (1)$$

Here, $\varphi = 2^{-\frac{1}{3}}$, implying that the weight is halved each month. To avoid losing observations, missing values are replaced with AMV computed using the available number of observations at time t .

Although decline in firms' market value in two consecutive time periods has been suggested in past literature (e.g. Pindado *et al.* 2008), we suggest considering the average market value rather than the end of the analysis year market value, as the latter represents a static snapshot of a firm's market capitalisation at the end of a given trading day, which may be significantly influenced by negative or positive developments around that trading day. Thus, instead of market value on a given day, we use AMV, as it is a better representation of the average state of a firm. Further, by imposing geometrically declining weighted average in Equation 1, we impose higher weight to more recent observations similar to Campbell *et al.* (2008) so that our average is not flawed/biased by rapid growths or declines in firms' monthly market values.

Condition 2: the firm's earnings before interest tax depreciation and amortisation (EBITDA) is less than financial expenses (FE) in the years $t - 1$ and $t - 2$.

Traditionally, excessive leverage is believed to be the primary cause of bankruptcy or financial difficulties (e.g. John *et al.* 2013), however significant volumes of literature argue that firms with a higher cost of default choose a lower level of leverage to mitigate potential default likelihood (e.g. George and Hwang, 2010, Glover, 2016), and, thus, we cannot rule out the possibility of financial distress triggered by reasons other than leverage. By including a condition on firms' ability to meet their financial expenses, we are essentially trying to capture those firms that enter into financial difficulties due to their inability to meet financial commitments to the providers of debt capital. Pindado *et al.* (2008) also suggest that financially distressed firms should report financial expenses higher than their earnings in two consecutive years. This effectively implies that the ratio between earnings to financial expenses should be less than one in these two time periods.

Condition 3: the firm's operating cash flow (OCF) is less than its financial expenses in the years $t - 1$ and $t - 2$.

If we define financial default as debtors' inability to make payment when it is due, then cash flow is of prime importance. Since the adoption of an accrual accounting system for recording and reporting business transactions, balance sheets and income statements have emerged as dominant sources of information for business decision-making. However, accruals are the non-cash component of earnings and represent adjustments made to cash flows to generate a profit measure largely unaffected by the timing of receipts and payments of cash (Ball *et al.* 2016). Thus, just having earnings above financial expenses ($EBITDA > IE$ or $EBITDA/IE > 1$) does not guarantee a firm's ability to repay its debt on time. Inefficient management of cash may also result in premature closure of even profitable businesses, as loans, most operating expenses, taxes etc. are required to be paid in cash. However, this important cash flow information is often overlooked in the academic literature. For instance,

Pindado *et al.* (2008) and Keasey *et al.* (2015) did not consider the role of cash flow information while defining financial distress.

Ball *et al.* (2016) argue that cash-based measures of operating profitability (a measure that excludes accruals) outperform measures of profitability that include accruals. Additionally, Grullon *et al.* (2017) report that, for large firms, cash flow remains the primary driver of investment spending and the marginal effect of cash flow on aggregate investment has actually increased over the past 30 years. Recent literature also highlights the importance of financing constraints and free-cash-flow problems in making sound investment decisions (Lewellen and Lewellen, 2016). The relative importance of cash flow in times of financial distress is also realised in recent literature. Lee *et al.* (2016) argue that, in times of financial distress, investors place significantly higher weight on OCF information than on earnings information, as they find that a firm's stock returns show stronger association with its OCF than its earnings when the firm is in financial distress. Considering the discussion above, and to be consistent with the two previous conditions, we include *Condition 3* in our proposed definition of financial distress, with an aim to overcome some of the limitations of firms' failure definitions discussed earlier.

3. Estimating Downside Risk Measures

We use daily three-month, six-month, one-year, three-year, five-year and monthly sixty-month returns to measure VaR and ES of all firms in our sample. We estimate the risk measure of each firm as of the month-end (of the latest available daily/monthly return data) in a given year to predict firms' financial distress in the next year. For instance, if the date on which financial statements were filed is June 2015, we calculate its risk measures as of May 2015 to predict a firm's financial distress likelihood in the next one year.

Value-at-risk: Estimation of VaR requires the definition of two basic parameters, the confidence level ($1 - \alpha$) and the time horizon (τ), along with the estimation model. These are

then used to estimate the worst loss that may occur over the specified time horizon and confidence level.

Traditionally, the risk horizon is measured in the number of trading days,⁷ rather than the number of calendar days for measuring VaR, and it is this period over which we measure the expected loss. In the absence of any defined rules or guidelines, time horizon is usually chosen by considering the liquidity of the risk and the duration of exposure to that risk (Alexander, 2008). The higher the assets' liquidity, the lower the time required to close or fully hedge the exposure. Thus, the time horizon required is shorter for the VaR model. A longer time horizon is suggested when measuring VaR in stressful and volatile market conditions, as markets tend to lose liquidity during such economic conditions. Credit rating agencies assign top ratings to those companies that exhibit extremely low default probability (such as 0.03%) over the next one-year horizon. Hence, companies looking for AA or above credit rating should apply a confidence level of 99.97% to measure their enterprise-wide VaR over the next one-year horizon. Considering the arguments presented above, and the nature of our application, we estimate downside risk measures using five different time horizons to detect the presence of any intertemporal differences. We estimate the risk measure of each firm as of the year-end using the past three-month, six-month, twelve-month, three-years and five-years daily returns, and past five-year monthly returns to predict firms' financial distress the following year.

The choice of confidence level is also highly subjective; it primarily depends on the area of application and users' attitude towards risk. For instance, the Basel II Accord requires banks using internal VaR models to assess their capital requirements to measure VaR at a 99% confidence level, while a credit rating agency may maintain a more stringent confidence level of about 99.97% to assess the probability of firms' insolvency hazard (Jorion, 2000). On the

⁷ In this study, our risk estimates are computed using 252 trading days in a year.

other hand, a risk manager may allow a lower confidence level (95%) and shorter risk horizon (1 day) while setting VaR based trading limits. Thus, we consider two different loss probability levels, namely 99% ($\alpha = 0.01$) and 95% ($\alpha = 0.05$) confidence levels, to estimate the downside risk measures.

Let τ denote the time horizon, $R_{t+\tau}$ denote a firm's return between the time period t and $t + \tau$, and let $F_{R,t}$ represent the cumulative distribution function (CDF) of $R_{t+\tau}$ conditional upon the set of information available at time t . Then, $F_{R,t}^{-1}$ represents the inverse function of $F_{R,t}$. Given this, the VaR of a firm's return as of time t with a time horizon τ and at $(1 - \alpha)$ confidence level can be estimated as follows:

$$VaR_t(\alpha, \tau) = -F_{R,t}^{-1}(\alpha) \quad (2)$$

Unlike normal VaR, the semi-parametric Cornish-Fisher VaR (VaR_{CF}) considers higher moments in the return distribution, thus relaxing the normality assumption. The Cornish and Fisher (1938) approximation incorporates higher moments (skewness and kurtosis) into the estimation process and, thus, accounts for non-normality in the return distribution. Zangari (1996) is the first to introduce this in his study that estimates the VaR of option portfolios. The justification behind the use of this formula is that any distribution can be approximated with these known moments, in terms of any other given distribution (Johnson *et al.* 1994, Jaschke, 2002). Thus, we use VaR_{CF} ⁸ to accommodate non-symmetrical and fat-tailed returns distribution. The fourth order Cornish and Fisher (1938) expansion for α percentile of $(R - \mu)/\sigma$ is shown in equation (2), while equation (3) defines the Cornish-Fisher VaR (VaR_{CF}).

$$\Omega(\alpha) = Z(\alpha) + \frac{1}{6}(Z(\alpha)^2 - 1)S + \frac{1}{24}(Z(\alpha)^3 - 3Z(\alpha))K - \frac{1}{36}(2Z(\alpha)^3 - 5Z(\alpha))S^2 \quad (3)$$

⁸ We use VaR and VaR_{CF} interchangeably in this manuscript.

$$VaR_{CF} = -(\mu + \Omega(\alpha) \times \sigma) \quad (4)$$

where σ is the standard deviation, μ is the average return, S is the measure of skewness, K is the excess kurtosis of past n -month daily returns, $(1 - \alpha)$ is the confidence level, and $Z(\alpha)$ is the critical value obtained from the standardised normal distribution.⁹

Expected Shortfall: Although VaR is the maximum loss that can take place over a defined time horizon and confidence level, there still exists a very small probability that a loss greater than VaR may be experienced. Thus, VaR does not reveal any information about the size of the loss in the event when that level is breached. Expected Shortfall (ES) provides this information. It is the conditional expected loss that is greater than or equal to the VaR. ES expressed in terms of return rather than dollar amount is formulated as follows:

$$\begin{aligned} ES_t(\alpha, \tau) &= -E_t[R_{t+\tau} | R_{t+\tau} \leq -VaR_t(\alpha, \tau)] \\ &= -\frac{\int_{v=-\infty}^{-VaR_t(\alpha, t)} v f_{R,t}(v) dv}{F_{R,t}[-VaR_t(\alpha, \tau)]} \\ &= -\frac{\int_{v=-\infty}^{-VaR_t(\alpha, t)} v f_{R,t}(v) dv}{\alpha} \end{aligned} \quad (5)$$

Here, $R_{t+\tau}$ represents a firm's return in the time period t , and $t + \tau$ and $f_{R,t}$ denotes the conditional probability density function (PDF) of $R_{t+\tau}$. $F_{R,t}$ is the conditional CDF of $R_{t+\tau}$ conditional upon the information set available at time t , $F_{R,t}^{-1}$ represents the inverse function of $F_{R,t}$, and $(1 - \alpha)$ is the confidence level. Artzner *et al.* (1999) provide the theoretical argument that VaR is inferior to ES as a risk measure. They argue that, unlike VaR, ES possesses mathematical properties like continuity and subadditivity which are desirable to a coherent

⁹Note that standard deviation is always positive, while the original VaR and ES are usually negative. To avoid confusion, the original VaR and ES numbers are multiplied by -1 in equations (3) and (4). Therefore, the VaR and ES numbers presented in this paper are usually positive.

measure of risk. We use Cornish-Fisher VaR (VaR_{CF}) calculated using equations (3) and (4) to estimate Expected Shortfall, denoted as ES_{CF} .¹⁰

4. Data and Covariates

This section presents discussion on our sources of data and choice of covariates used in this study to perform required empirical analysis.

4.1 Data

Unlike several others (e.g. Chava and Jarrow, 2004), we do not try to predict the financial distress event over the next month or quarter. We believe that an efficient distress prediction model should work as an Early Warning System (Edison, 2003) and should signal any adverse distress event well in advance. This may provide sufficient time to avert or manage the forthcoming crisis. Thus, to predict the financial distress over the next one-year horizon, our empirical analysis employs annual firm-level accounting and market data of the United States firms from the Compustat database. We consider a relatively long analysis period from 1990 to 2016. We also limit our sample to firms which are publicly-traded either on AMEX, NASDAQ or NYSE stock exchanges only. Transportation, Communications & Public Utilities, Finance, Insurance & Real Estate, and Public Administration firms have been excluded from our analysis (see Table 1).

[Insert Table 1 Here]

4.2 Covariates

Dependent Variable: We use the definition of financial distress proposed in Section 2 as the binary dependent variable for this study.

¹⁰ We use ES and ES_{CF} interchangeably in this manuscript.

Independent Variables: A large volume of empirical literature has evolved over time which efficiently addresses the issues pertaining to prediction of corporate bankruptcy. However, the default prediction models gravitate toward the use of accounting variables (e.g. Altman, 1968, Ohlson, 1980) or market information to predict firms' insolvency hazard (e.g. Bharath and Shumway, 2008). The market-based approach of modelling default risk has become popular since their commercial introduction by KMV, CreditMetrics and others. Hillegeist *et al.* (2004) report that market-based structural models perform better than popular accounting-based models, in particular the Altman Z-score (Altman, 1968) and Ohlson O-score (Ohlson, 1980) models.

In a similar study, Das *et al.* (2009) report that their distress prediction models, built using accounting metrics and market metrics, respectively, exhibit very similar classification performances. Models which employ both accounting and market information substantially outperform both accounting and market-based models, suggesting that accounting and market information are complementary to each other. Findings of other related studies (see, among others, Agarwal and Taffler, 2008; Trujillo-Ponce *et al.* 2014) also support the findings of Das *et al.* (2009). Considering the above arguments, we use accounting and market information to develop our distress prediction models.

In particular, we employ all eight (three accounting-based and five market-based) measures of bankruptcy employed by Campbell *et al.* (2008) to predict the bankruptcy of US firms, as they report that this set of covariates outperforms other standard sets of covariates employed in prior studies. We supplement this with two additional covariates, financial expenses to sales (FES) and tax-to-market valued total assets (TMTA), as they exhibit significant explanatory power and very high average marginal effects in several recent studies on SMEs financial distress (e.g. Gupta *et al.* 2018). Furthermore, we also test the marginal

predictive ability of tail risk measures discussed in Section 3, i.e. VaR and ES. Detailed definitions of financial covariates are as follows (see Table 2 for complete list of covariates along with their respective Compustat data items):

- I. Total liabilities to Market-valued Total Assets (TLMTA), calculated as:

$$TLMTA_{i,t} = \frac{Total\ Liabilities_{i,t}}{(Market\ Value\ of\ Equity_{i,t} + Total\ Liabilities_{i,t})}$$

Here a Market-valued Total Asset (MTA) is the sum of the market value of a firm's equity and its total liabilities. In case of missing values of Total Liabilities, MTA is equal to the firm's equity, and vice-versa.

- II. Cash and short-term assets to Market-valued Total Assets (CMTA), calculated as:

$$CMTA_{i,t} = \frac{Cash\ and\ Short - Term\ Investments_{i,t}}{(Market\ Value\ of\ Equity_{i,t} + Total\ Liabilities_{i,t})}$$

- III. Weighted average of Net Income to Market-valued Total Assets (*NIMTAAVG*) over the previous three-year period (*NIMTAAVG*), calculated as:

$$NIMTAAVG_{t-1,t-3} = \frac{1}{1.75} NIMTA_{t-1} + \frac{0.5}{1.75} NIMTA_{t-2} + \frac{0.25}{1.75} NIMTA_{t-3}$$

where

$$NIMTA_{i,t} = \frac{Net\ Income_{i,t}}{(Market\ Value\ of\ Equity_{i,t} + Total\ Liabilities_{i,t})}$$

Following Chen and Hill (2013), we modify Campbell *et al.*'s (2008) measure of average *NIMTA* to allow for the fact that we employ annual, rather than quarterly, accounting measures. Missing values of *ANIMTA* are replaced with the cross-sectional mean of *NIMTA*.

- IV. Weighted average of monthly log excess return relative to value-weighted S&P 500 return over the previous 12-month period (*EXRETAVG*), calculated as:

$$EXRETAVG_{t-1,t-12} = \frac{1 - \phi}{1 - \phi^{12}} (MEXRET_{t-1} + \dots + \phi^{11} MEXRET_{t-12})$$

Where;

$$MEXRET_{i,t} = \log(1 + Return_{i,t}) - \log(1 + Return_{S\&P\ 500,t})$$

Missing values of AMEXRET are replaced with the cross-sectional mean of MEXRET.

- V. Annualised standard deviation of a firm's daily log returns over the previous three months (SIGMA), estimated as:

$$SIGMA_{i,m-1,m-3} = \left(252 \times \frac{1}{N-1} \sum_{k \in \{m-1, m-2, m-3\}} r_{i,k}^2 \right)^{\frac{1}{2}},$$

where $r_{i,k}^2$ represents the log returns of firm i on the day k , and N represents the number of days in the three-month estimation window. Here, following Campbell *et al.* (2008), we use a proxy centred around zero rather than a three months' rolling mean.

- VI. Firm's Market-to-book ratio (MB)
 VII. Log of price per share winsorized above \$15 (PRICE)
 VIII. Logarithm of each firm's size relative to S&P 500 market capitalisation (RSIZE), calculated as:

$$RSIZE_{i,t} = \log \left(\frac{Firm\ Market\ Equity_{i,t}}{Total\ S\&P\ Market\ Value_t} \right)$$

- IX. Financial Expense to Sale (FES), calculated as:

$$FES_{i,t} = \frac{Financial\ Expense_{i,t}}{(Sale_{i,t})}$$

- X. Tax to Market-Valued Total Assets (TMTA), calculated as:

$$TMTA_{i,t} = \frac{Total\ Income\ Tax_{i,t}}{(Market\ Value\ of\ Equity_{i,t} + Total\ Liabilities_{i,t})}$$

Healthy firms are expected to pay higher taxes due to higher business volume in comparison to distressed firms and, hence, a negative relationship is expected between TMTA and financial distress likelihood. Further, following the suggestion of Cohen *et al.* (2003), we adjust the book value of equity to avoid the influence of outliers. Specifically, we add 10% of the difference between the market value and the book value of equity,¹¹ thus increasing book values that are relatively very small and probably wrongly measured. This ultimately prevents outliers when book equity is used in the denominator when calculating financial ratios. For observations which still exhibit negative book value of equity after this adjustment, we replace negative book values with positive \$1 to ensure that market-to-book value ratios for all observations are positive. Further, we restrict the range of all the covariates within 5th and 95th percentiles to limit the influence of any outliers.

Control Variables: To control the volatility in the macroeconomic environment affecting specific industrial sectors, we calculate a measure of industry risk (*INDRISK*) as the financial distress rate (number of firms experiencing the event of interest in the respective industrial sector in a given year/total number of firms in that industrial sector in that year) in each of the seven industrial sectors in a given year. Higher values indicate a higher risk of financial distress, and vice versa.

[Insert Table 2 Here]

5. Empirical Methods

5.1 Panel Logistic Regression

Although we see a significant rise in the popularity of hazard models in modelling bankruptcy or financial distress events, in line with the findings of Gupta *et al.* (2018) we use panel logistic

¹¹ $Total\ Assets(adjusted)_{i,t} = Total\ Assets_{i,t} + 0.1(Market\ Equity_{i,t} - Book\ Equity_{i,t})$

regression with random effects to establish our empirical validation. Gupta *et al.* (2018) argue that the discrete-time hazard model with logit link is essentially a panel logistic model that controls for firms' age. Thus, we assume that marginal probability of firms' financial distress over the next year follows a logistic distribution that is estimated as follows:

$$P(Y_{it} = 1) = \frac{1}{1 + \exp(-\alpha - \beta X_{i,t-1})} \quad (7)$$

where Y_{it} is an indicator variable that equals one if the firm is in financial distress in year t , and $X_{i,t-1}$ is a vector of explanatory variables known at the end of the previous year. To capture any duration dependency, we use the natural logarithm of firms' annual age¹² (AGE) as a control variable in our multivariate models. Additionally, since our panel regression analyses use lagged covariates ($t - 1$), any potential endogeneity problem is expected to be mitigated, as lagged explanatory variables and dependent variables have a low likelihood of being jointly determined (Berger and Bouwman, 2013).

However, Reed (2015) raise concern on this widespread practice of dealing with potential simultaneity, and argue that associated estimates may still remain inconsistent and hypothesis testing may be invalid. Rather they suggest an alternative, is to use lagged values of the endogenous variable in instrumental variable estimation. However, to the best of our knowledge, programming codes required to perform such statistical estimations are currently available only for linear regression specifications. Since we use non-linear panel logistic specification, we are at a disadvantage to act upon this valuable suggestion. But, to be mindful, we also report regression estimates of our main model using $t - 2$ and $t - 3$ lagged covariates. It is unlikely that the dependent and independent variables are simultaneously determined two or three years ahead in advance. Thus, significance of $t - 2$ and $t - 3$ lagged covariates shall

¹² We proxy a firm's age as the earliest year for which financial information for that firm is available in the Compustat database.

imply absence of any severe endogeneity bias in our regression estimates arising due to reverse causality.

5.2 Summary Statistics and Correlations

To get an initial understanding about the variability of our covariates, we report descriptive statistics for financially distressed and non-distressed/healthy groups of firms. Table 3 provides an overview of mean, median, standard deviation, minimum and maximum of all covariates used in our study. Columns 1 and 5 present the list of covariates, while columns 2 and 6 list names of descriptive measures. The remaining columns report descriptive measures for financially distressed (columns 3 and 7) and non-distressed (columns 4 and 8) groups of firms. Descriptive measures of the vast majority of covariates are as per our expectation and without any extreme variability.

The mean/median of covariates we report are very similar to those reported by Campbell *et al.* (2008) in their study, with some differences due to the data period and different definition of the default event. We expect the mean/median of the covariates that are positively related to the insolvency/distress risk to be higher for the distressed group than for the group without distress (see, for example, variable FES in Table 3). Contrarily, we expect the mean/median of the covariates that are negatively related to the insolvency/distress risk to be lower for the distressed groups than for the group without distress (see, for example, variable NIMTAAVG in Table 3). However, we also see some exceptions. The mean value of TLMTA for the distressed group is lower than for the non-distressed group of firms, implying that financially distressed firms have lower total liabilities. This is contrary to our traditional understanding, where we expect firms in financial distress to have higher liabilities compared to their total assets. Similarly, the mean value of CMTA for the distressed group is higher than for the non-distressed group, implying that financially distressed firms have higher cash reserves. This is

contrary to intuition, where we expect financially distressed firms to have liquidity problems and, hence, lower reserves of cash and short-term investments. Also, the mean market-to-book (MB) ratio, which is expected to be lower for the distressed group of firms than its non-distressed counterpart, is actually higher. This may suggest active demand of distressed stocks, which consequently pushes their stock price higher. These contrary revelations might affect the sign of respective regression coefficients.

[Insert Table 3 Here]

A comparison of the distressed and non-distressed groups also reveals expected differences in Table 3. For example, distressed firms make mean loss of about 18%, compared with non-distressed firms that make mean loss of only about 1.9%, as lower mean of net income relative to market value of total assets shown by NIMTAAVG. The mean volatility of distressed firms is quite high at 80%, relative to only 55% of non-distressed firms, as shown by SIGMA. Distressed firms are relatively small (RSIZE) and have a much lower mean log price per share (PRICE), and lower returns (EXRETAVG) compared to their non-distressed counterpart. Similar to Campbell *et al.* (2008), the mean of market-to-book (MB) ratio of financially distressed firms is slightly higher, because the book value of their equity erodes due to frequent losses. This does not seem to be reflected in the market value of the firms because of the proactive nature of stock markets.

A closer look at extreme measures generated using three-month, six-month, one-year, three-year and five-year daily returns, and sixty-month monthly returns (i.e. VaR and ES) reveal that distressed firms have higher values compared to non-distressed firms. This is to be expected, as distressed firms tend to exhibit extreme movements. VaR is the loss that will not exceed from that level with a certain confidence interval during a certain period of time. The higher magnitude reflects the higher probability of loss. For example, in Table 3 the mean

VAR3M1 of financially distressed firms is 13%. It means that we are 99% confident that the mean loss from financial distressed firms will not exceed 13% during a period of 3 months. Similarly, the mean VAR5Y5 of non-distressed firms is 5.37%. It means that we are 95% confident that the mean loss from the firms that are not financially distressed will not exceed 5.37% during a period of 5 years.

On the other hand, expected shortfall is the expected loss conditional upon that the loss is greater than a certain threshold. For example, in Table 3 the mean ES3M1 of financially distressed firms is 21.03%. It means that the mean loss of financially distressed firms will be 21.03% during a period of 3 months with the assumption that the loss is greater than 99% of the loss distribution. Similarly, the mean ES5Y5 of non-distressed firms is 8.47%. It means that the mean loss of firms that are not financially distressed will be 8.47% during a period of 5 years months with the assumption that the loss is greater than 95% of the loss distribution.

Furthermore, the mean/median of downside measures with the 95% confidence interval are lower compared to the measures with the 99% confidence interval. This is also intuitive as, at the lower confidence level, the cut-off point at which the loss should not exceed should be lower in the case of VaR and, at the higher confidence level, the cut-off point should be higher. Similarly, the mean of the losses larger than VaR (i.e., ES) should be lower at lower confidence intervals and higher at higher confidence intervals. We also find intertemporal differences, especially at the 99% confidence interval. We see the mean/median of extreme measures at shorter rolling windows are less than those at longer rolling windows. However, at the 95% confidence interval, the mean/media of extreme measures is slightly higher.

Panel A of Table 4 reports correlation among main (non-tail risk) covariates. As reported, most covariates show low to moderate correlation among each other. However, PRICE shows strong positive correlation with RSIZE (0.7168) and strong negative correlation with SIGMA

(-0.6121). Thus, we take into account the issues of multicollinearity while developing our multivariate models excluding tail risk measures. Subsequently, Panel B of Table 4 reports correlation of tail risk measures with main variables. As reported, most tail risk measures exhibit moderate to low correlation with main variables, except SIGMA. This is expected, as both of these are competing measures of firms' stock price volatility. However, we address this issue effectively while developing multivariate models (see Section 5.4).

[Insert Table 4 Here]

5.3 Univariate Regression and Average Marginal Effects

Table 5 reports the univariate regression results of respective covariates. This table reports point estimates (β), standard errors (SE), z-statistics (Z) and average marginal effects (AME) in percentages.¹³ All financial ratios are highly significant at the 1% level of significance. MB negatively predicts the financial distress of firms, meaning that the stock market is already anticipating the financial distress, driving down the market value of equity and hence the market-to-book ratio. As explained above in the summary statistics, we find another counterintuitive result as CMTA positively predicts the likelihood of financial distress.

Eight out of twelve covariates show AME of less than 5% with AME of TLMTA, MB, RSIZE and AGE even less than 1%. However, the other four covariates reported are highly significant predictors, with more than 5% AME. Two non-Campbell covariates, FES and TMTA, exhibit AME of around 5% and 30%, respectively. Among extreme risk measures, all are highly significant at the 1% level of significance. This implies that all extreme risk measures

¹³ In non-linear regression analysis, Marginal Effects are a useful way to examine the effect of changes in a given variable on changes in the outcome variable, holding other variables constant. These can be computed as marginal change (it is the partial derivative for continuous predictors) when a variable change by an infinitely small quantity and discrete change (for factor variables) when a variable change by a fixed quantity. The Average Marginal Effects (AME) of a given variable is the average of its marginal effects computed for each observation at its observed values. Alternatively, AME can be interpreted as the change in the outcome (financial distress = 1, in our case) probabilities due to unit change in the value of a given variable, provided other covariates are held constant. See Long and Freese (2014) for additional discussion on this topic.

can predict financial distress, but some of the measures have very low values of AME, which raises scepticism about their predictive power in the multivariate setup. We see intertemporal differences as longer horizon rolling estimates show more statistical and economic significance. For example, three-year and five-year VaR, estimated at the 5% level of significance, have the highest AME of 4.49% and 6.03%, respectively. Although the AME of ES is not as high as for the VaR, sixty-month ES estimated at the 5% level of significance have the highest AME of 1.78%. Among ES estimated at the 1% level of significance, five-year and sixty-month, ES have the AME of 1.39% and 1.66, respectively. This shows that its discriminatory power is sensitive to duration and the significance level of tail risk estimates.

[Insert Table 5 Here]

5.4 Baseline Multivariate Model

In order to test the marginal discriminatory power of downside risk measures, we first estimate the baseline multivariate model with all financial covariates, as they are all significant in the univariate analysis. However, considering that few cases of moderate to high correlation among covariates are reported in Table 4, we follow the method suggested by Gupta *et al.* (2018) while developing our baseline multivariate model. Primarily, this requires ranking of the covariates in univariate regression based on the magnitude of their respective AME and, then, introducing each variable at a time in the multivariate setup, in ascending order of their rank. Gupta *et al.* (2018) argue that the higher the value of AME, the higher the change in the predicted probability due to unit changes in the covariate's value. Thus, to develop a parsimonious model, our priority should be include minimum number of covariates with highest explanatory power/AME. Also, we exclude a covariate from the multivariate model if, when introduced: (i) it reverses the expected sign of any previously added covariate; (ii) it bears the opposite sign

to that expected; (iii) it bears the expected sign, but has a p-value greater than 0.10; and (iv) it makes a previously added covariate insignificant with a p-value greater than 0.10.

This allows us to develop a parsimonious baseline multivariate model that excludes PRICE, RSIZE, TLMTA and MB.¹⁴ We also exclude SIGMA from our baseline model, as the purpose of this study is to report the comparative performance of several tail risk measures in comparison to SIGMA, which we do in the subsequent section. Finally, we include INDRISK and natural logarithm of firms' annual age (AGE) as control variables in the model. As reported in column 2 of Table 6, we develop the final parsimonious baseline model with seven covariates, all of which are highly significant and retain their respective signs, as reported in Table 5. We therefore conclude that, all of these covariates are jointly significant in predicting the financial distress of US firms. Also, as reported in Figure 1, the within-sample and out-of-sample area under ROC curves (AUROC¹⁵) of this model is about 92%, suggesting excellent performance of our baseline model in classifying financially distressed firms.

Columns 3 and 4 of Table 6 report regression estimates for $t - 2$ and $t - 3$ lagged covariates of the main multivariate model reported in column 2 of Table 6. Except for the insignificance of EXRETAVG in $t - 3$, all financial covariates are highly significant in explaining a firm's likelihood of experiencing financial distress in time t . This suggest strong intertemporal explanatory power of our model, with excellent classification performance of around 90% (see Figure 1). This also implies, absence of any severe endogeneity bias in our

¹⁴ PRICE makes EXRETAVG insignificant, hence removed; RSIZE makes EXRETAVG insignificant, hence removed; TLMTA makes EXRETAVG insignificant, hence removed; and the coefficient of MB is positive, thus removed.

¹⁵ The higher the AUROC, the better the prediction model. Please see Gupta *et al.* (2018) for additional discussion on this. For out-of-sample validation, we first estimate models using observations until the year 2011, and, using these estimates, we predict financial distress probabilities for the year 2012; we then include 2012 in the estimation sample and predict distress probabilities for 2013 and so on until the year 2016. We then use these predicted probabilities from the year 2012 through 2016 to estimate out-of-sample AUROC.

regression estimates due to reverse causality, as it is unlikely that the dependent and independent variables are simultaneously determined two or three years ahead in advance.

[Insert Table 6 Here]

[Insert Figure 1 Here]

5.5 Multivariate Models with Risk Measures

In order to test the marginal discriminatory power of tail risk measures in comparison to SIGMA, we re-estimate the baseline multivariate model with $t - 1$ lagged covariates (reported in column 2 of Table 6), supplemented with tail risk measures and report the results in Table 7 and 8. Rows 3 to 7 in Table 7 and Table 8 report regression estimates for respective risk measures as listed in the first row of columns 2 to 14. $LR\ Chi^2$, in the seventh row of column (1), is the chi-square for likelihood ratio test that compares the log likelihood of the baseline model supplemented with risk variables with the log likelihood of the baseline model, and tests whether this difference is statistically significant. To capture any variations that may arise due to the difference in significance levels of tail risk estimates, we estimate separate models employing tail risk measures estimated at 1% and 5% significance levels. Furthermore, we estimate separate multivariate models for daily three-month, six-month, one-year, three-year and five-year, and monthly sixty-month tail risk estimates to account for any intertemporal differences that may exist. Following Liang and Park (2010), we compare these models with our baseline multivariate model, supplemented with SIGMA as the risk measure.

We estimate all multivariate models using our proposed definition of financial distress as defined in Section 2. Table 7 reports our multivariate regression results with VaR measures. The first column gives the names of the covariates. Column (2) reports estimates with SIGMA as the risk measure. Columns (3) to (8) report respective regression estimates employing tail

risk measured at the 99% confidence interval. Columns (9) to (14) report the same, but for tail risk measures estimated using the 95% confidence interval.

Table 7 reveals intertemporal differences, as short-duration rolling estimates of VaR measures are insignificant in predicting firms' likelihood of experiencing financial distress. As we measure the extreme downside risk, it might be that short-duration rolling estimates do not have enough observations (Huang *et al.* 2012);¹⁶ therefore, long-duration rolling estimates perform better in predicting the financial distress of firms. The multivariate models with five-year VaR measured at the 95% confidence interval (VAR5Y5) is the best, as it shows the highest value of AME (8.19%) in comparison to other significant risk measures, and the highest value of *LR Chi*² (31.77). The baseline model with SIGMA as a measure of risk is also statistically significant with *LR Chi*² of 7.50; however, the AME is very low at 0.45%. Other significant downside risk measures are five-year and sixty-month VaR measured at the 99% confidence level and three-year and sixty-month VaR measured at the 95% confidence level.¹⁷

Subsequently, we repeat this multivariate analysis with ES as the risk measure and report the results in Table 8. Similar to multivariate regression results with VaR, we find intertemporal differences in that the longer-duration rolling estimates of ES perform better in predicting the financial distress of firms. However, with ES we find that five-year ES measured at the 99% confidence interval performs better (with the *LR Chi*² of 23.61) than the five-year ES measured at the 95% confidence interval. The only exception is the sixty-month ES measured at the 95% confidence interval that significantly predicts financial distress likelihood of firms. Since ES is the mean of the losses greater than VaR, the mean of the losses is lower at the 95% confidence interval because lower values reduce the mean of the losses. The mean is higher at the 99%

¹⁶ While measuring ES and TR, Bali *et al.* (2009) also use a longer rolling window of 100 days compared to shorter windows for measuring VaR.

¹⁷ Three-month VAR measured at the 99% level of significance and three-month and six-month VAR measured at 95% level of significance also marginally significance with the reverse sign.

confidence interval and, therefore, we find stronger results for five-year ES measured at the 99% confidence interval. However, VaR is simply a cut-off point and the cut-off point at the 99% confidence interval covers the cut-off point at the 95% confidence interval. Therefore, we find stronger results for five-year VaR measured at the 95% confidence interval. Other ES measure that significantly predict the financial distress likelihood of firms is three-year ES measures at the 99% confidence interval. Our baseline model with SIGMA as a measure of risk also significantly explains the financial distress of firms, but shows lower values of AME and $LR\ Chi^2$ than ES5Y1. The AME of ES3Y1 are also lower than the AME of ES5Y1. Overall, as the superiority is ES over VaR is well-documented in the literature, we conclude that five-year and sixty-month ES measured at the 99% confidence interval (ES5Y1) are superior to SIGMA (higher AME than SIGMA) and other competing tail risk measures.

[Insert Tables 7 and 8 Here]

6. Multivariate Models with Thresholds

Conditions 1, 2 and 3 of our definition of financial distress presented in Section 2 emphasise only on the decline in respective values in the past two consecutive years. One might argue that the decline should be significant enough to threaten firms' degree of solvency. Thus, imposing a threshold (a decline of X% or higher) might appear reasonable. Addressing this concern relating to model calibration or stress-testing in our existing framework is relatively straightforward. However, imposing such threshold will alter the distress status of a firm and, thus, needs to be based upon the risk appetite of the decision-maker. Imposing 0% threshold will imply most conservative behaviour or lowest risk appetite, while higher values will imply higher risk appetites. For a given threshold, this will also lead to decline in number of distressed firms in the sample, which will keep decreasing with increasing threshold values.

To investigate the effect of imposing such threshold on our distress definition, and its subsequent impact on the performance of the main distress prediction model (presented in column 2 of Table 6), we impose further restrictions on our proposed definition of financial distress and report the results in Table 9. In particular, we report results obtained from arbitrarily imposing 10%, 20% and 30% threshold levels respectively on conditions 1, 2 and 3. For instance, a 10% threshold implies - *Condition 1*: There is negative growth in the AMV of the firm in years $t - 1$ and $t - 2$, and this decline in AMV is at least 10% or higher in both these respective years; *Condition 2*: the firm's EBITDA is less than financial expenses in the years $t - 1$ and $t - 2$, and this shortfall in earnings is at least 10% or higher in both these respective years; and *Condition 3*: the firm's OCF is less than its financial expenses in the years $t - 1$ and $t - 2$, and this shortfall in OCF is at least 10% or higher in both these respective years. Subsequently we do the same for 20% and 30% threshold levels respectively. Although our choice of threshold levels is arbitrary, one might choose any threshold level subjected to their risk appetite, or any required combination of different thresholds in the same definition of financial distress.

In order to compare the baseline multivariate model with the models with different thresholds, we first report the baseline model of Table 6 in column 2 of Table 9. Columns 3 to 5 of Table 9 report multivariate regression estimates for models with 10%, 20% and 30% threshold levels respectively. As we see in Table 9, all five financial covariates are highly significant and retain their respective expected signs at different threshold levels. This shows consistency of these covariates in identifying different intensities of financial distress. However, the AME of respective covariates is highest for the model without any threshold and declines gradually with increasing threshold levels (see Table 9). This implies that the explanatory power of covariates decreases with the increase in the intensity of financial distress. This appears reasonable as firms experiencing higher intensity of financial distress

makes the distress group of firms appear more homogeneous than their healthy counterparts. This is also supported from the fact that the number of financially distressed firms in our sample decreases with increasing threshold levels. From 1,219, it falls to 696 with the imposition of 10% threshold; 504 with the imposition of 20% threshold; and 350 with the imposition of 30% threshold. However, the R^2 of these models stays around 32%, implying no loss in overall explanatory power of these models with increasing threshold levels. Additionally, we also see in Table 9 that the log-likelihood values of respective models increase with increasing threshold levels. This implies that the sample quality improves with increasing threshold levels.

Overall, we find our proposed definition of financial distress is robust to varying intensities of distress, with positive association between the sample quality and threshold levels.

[Insert Table 9 Here]

7. Multivariate Models with Thresholds and Risk Measures

In order to test whether our finding relating to the statistical significance of tail risk measures in explaining financial distress are robust to financial distress definitions of varying intensities, we re-estimate the multivariate models reported in Table 9 supplementing risk measures listed in Table 2 (see Tables 10 and 11)¹⁸.

Similar to what we see in Table 7, we find that short-duration rolling estimates of VaR measures are insignificant in predicting firms' likelihood of experiencing financial distress, whereas long-duration rolling estimates perform significantly better (see Table 10). Similar to our baseline results in Table 7, we find that multivariate models with five-year VaR estimate at the 95% confidence interval (VAR5Y5) is the best at all respective threshold levels of 10%,

¹⁸ Please note that, to save space we report only risk variables and models' goodness of fit measures in these tables. The statistical significance of unreported covariates remain identical to the one reported in Table 9.

20% and 30%, with the highest value of AME, and the highest value of LR Chi². Models with SIGMA as a measure of risk are also statistically significant at all threshold levels but with lower AMEs. Our results are also robust for three-year (VAR3Y5) and sixty-month (VAR60M5) VaR estimates at the 95% confidence level, and for sixty-month (VAR60M1) VaR estimate at the 99% confidence level. However, five-year VaR estimate at the 99% confidence level (VAR5Y1) is significant only at 10% threshold level. Overall, these results at respective threshold levels are broadly identical to what we report in Table 7, with the only difference being insignificance of VAR5Y1 at 20% and 30% threshold levels. Additionally, as reported in the earlier section, here also we see decline in the AME of risk measure with increasing threshold levels.

We repeat the above multivariate analysis with ES as the risk measure at three selected threshold levels of 10%, 20% and 30% respectively. We report our results in Table 10. At all threshold levels, our results are robust and are very similar to what we report in Table 8 without any threshold; with three-year, five-year and sixty-month ES estimates at the 99% confidence interval being significant predictor of financial distress. Similarly, at 95% confidence level, the only significant predictor is sixty-month ES (ES60M). These results are also similar to multivariate regression results with VaR, that, long-duration rolling estimates of ES perform better in predicting financial distress than its short-duration counterparts. Our baseline model with SIGMA as a measure of risk also significantly explain financial distress of firms at all threshold levels but show lower AME values. Additionally, as reported in the earlier section and similar to VaR, here also we see decline in the AME of respective ES measures with increasing threshold levels.

Overall, our robustness checks substantiate our earlier conclusion, and show that our proposed definition of financial distress is consistent and robust. Because of the theoretical

superiority of ES over VaR, we suggest using five-year or sixty-month ES estimates in predicting financial distress.

[Insert Tables 10 and 11 Here]

8. Conclusion

Appropriate pricing of credit risk is integral to maintaining sustainable and competitive lending practices. This also informs the estimation of capital reserves that lending institutions are required to maintain in proportion to their risk-weighted assets. Overestimation of credit risk will lead to higher capital reserves and, thus, an opportunity loss, whereas underestimation results in lower capital reserves, but may trigger large shortfalls and significant capital losses (see Demircuc-Kunt *et al.* 2013, for brief discussion). We contribute to this strand of literature by documenting the impact of tail risk measures on firms' likelihood of financial distress.

At first, we argue the need for a mechanism to identify firms in financial difficulties rather than waiting until they file for bankruptcy. Waiting until the bankruptcy filing might lead to significant erosion in firms' value, losses to creditors and unfavourable business decisions. Thus, there is a need for an alternative mechanism to identify firms in financial difficulties and it would be appropriate to penalise them for getting close to questionable going-concern status. As a consequence, we propose a definition of financial distress contingent upon firms' earnings, financial expenses, market value and operating cash flow. Specifically, we define a firm as financially distressed if it satisfies three simple conditions: (i) there is negative growth in the average market value (AMV) of the firm in years $t - 1$ and $t - 2$, where AMV is geometrically declining weighted average of a firm's monthly market values over the past twelve months; (ii) its earnings before interest tax depreciation and amortisation is less than its financial expenses in the years $t - 1$ and $t - 2$; and (iii) its operating cash flow is less than its financial expenses in the years $t - 1$ and $t - 2$.

Next, we hypothesise that more frequent extreme negative daily equity returns result in higher tail risk, and this subsequently increases firms' likelihood of entering financial distress. Thus, we assess the marginal discriminatory power of tail risk measures, namely value-at-risk and expected shortfall, in predicting financial distress likelihood of United States publicly-traded firms. Empirical results suggest that longer duration tail risk measures exhibit significant discriminatory powers. Our robustness checks substantiate these findings, and show that our proposed definition of financial distress is consistent and robust. Because of the theoretical superiority of ES over VaR, we suggest using five-year or sixty-month ES estimates in predicting financial distress.

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Tables & Figures

Table 1: Sample Industrial Classification

Industry Code	SIC Code	Industry	Included/Excluded
1	< 1000	Agriculture, Forestry, Fishing	Included
2	1000 to < 1500	Mining	Included
3	1500 to < 1800	Construction	Included
4	2000 to < 4000	Manufacturing	Included
5	5000 to < 5200	Wholesale Trade	Included
6	5200 to < 6000	Retail Trade	Included
7	7000 to < 8900	Services	Included
Excluded	4000 to < 5000	Transportation, Communications & Public Utilities	Excluded
Excluded	6000 to < 6800	Finance, Insurance & Real Estate	Excluded
Excluded	9100 to < 10000	Public Administration	Excluded

Notes: This table reports Standard Industrial Classification (SIC) of the United States firms. SIC Code is a four digit code that represents a given industrial sector. The last column reports the industrial sectors that we included or excluded from our sample.

Table 2: Variable Description

Variable	Definition	Compustat Data Item
TLMTA	Total liabilities to Market-valued Total Assets	LT, PRCC_F, CSHO
CMTA	Cash and short-term assets to Market-valued Total Assets	CHE, PRCC_F, CSHO, LT
NIMTAAVG	Weighted average of Net Income to Market-valued Total Assets over the previous three-year period	NI, PRCC_F, CSHO, LT
EXRETAVG	Weighted average of monthly log excess return relative to value-weighted S&P 500 return over the previous 12-month period	PRCCM, AJEXM, SPTRN
SIGMA	Annualised standard deviation of a firm's daily log returns over the previous three months	PRCCD, AJEXDI
MB	Firm's Market-to-book ratio	PRCC_F, CSHO, SEQ
PRICE	Log of price per share winsorized above \$15	PRCC_F
RSIZE	Logarithm of each firm's size relative to S&P 500 market capitalisation	PRCC_F, CSHO, TOTVAL
FES	Financial Expense to Sale	IE, SALE
TMTA	Tax to Market-valued Total Assets	TXT, PRCC_F, CSHO, LT
AGE	Natural logarithm of firms' annual age	
INDRISK	Number of firms experiencing financial distress in respective industrial sectors in a given year/total number of firms in that industrial sector in that year	
VAR3M1	Value-at-Risk estimated using daily returns over past 3 months at 1% significance level	PRCCD, AJEXDI
VAR6M1	Value-at-Risk estimated using daily returns over past 6 months at 1% significance level	PRCCD, AJEXDI
VAR1Y1	Value-at-Risk estimated using daily returns over past 1 year at 1% significance level	PRCCD, AJEXDI
VAR3Y1	Value-at-Risk estimated using daily returns over past 3 years at 1% significance level	PRCCD, AJEXDI
VAR5Y1	Value-at-Risk estimated using daily returns over past 5 years at 1% significance level	PRCCD, AJEXDI
VAR60M1	Value-at-Risk estimated using monthly returns over past 5 years at 1% significance level	PRCCM, AJEXDI
VAR3M5	Value-at-Risk estimated using daily returns over past 3 months at 5% significance level	PRCCD, AJEXDI
VAR6M5	Value-at-Risk estimated using daily returns over past 6 months at 5% significance level	PRCCD, AJEXDI
VAR1Y5	Value-at-Risk estimated using daily returns over past 1 year at 5% significance level	PRCCD, AJEXDI
VAR3Y5	Value-at-Risk estimated using daily returns over past 3 years at 5% significance level	PRCCD, AJEXDI
VAR5Y5	Value-at-Risk estimated using daily returns over past 5 years at 5% significance level	PRCCD, AJEXDI
VAR60M5	Value-at-Risk estimated using monthly returns over past 5 years at 5% significance level	PRCCM, AJEXDI
ES3M1	Expected Shortfall estimated using daily returns over past 3 months at 1% significance level	PRCCD, AJEXDI
ES6M1	Expected Shortfall estimated using daily returns over past 6 months at 1% significance level	PRCCD, AJEXDI
ES1Y1	Expected Shortfall estimated using daily returns over past 1 year at 1% significance level	PRCCD, AJEXDI
ES3Y1	Expected Shortfall estimated using daily returns over past 3 years at 1% significance level	PRCCD, AJEXDI
ES5Y1	Expected Shortfall estimated using daily returns over past 5 years at 1% significance level	PRCCD, AJEXDI
ES60M1	Expected Shortfall estimated using monthly returns over past 5 years at 1% significance level	PRCCM, AJEXDI
ES3M5	Expected Shortfall estimated using daily returns over past 3 months at 5% significance level	PRCCD, AJEXDI
ES6M5	Expected Shortfall estimated using daily returns over past 6 months at 5% significance level	PRCCD, AJEXDI
ES1Y5	Expected Shortfall estimated using daily returns over past 1 year at 5% significance level	PRCCD, AJEXDI
ES3Y5	Expected Shortfall estimated using daily returns over past 3 years at 5% significance level	PRCCD, AJEXDI
ES5Y5	Expected Shortfall estimated using daily returns over past 5 years at 5% significance level	PRCCD, AJEXDI
ES60M5	Expected Shortfall estimated using monthly returns over past 5 years at 5% significance level	PRCCM, AJEXDI

Notes: This table reports the set of accounting- and market-based covariates that we use in our empirical analysis. The first column lists names of covariates, while the second column provides their respective definitions. Financial information and stock price information are sourced from the Compustat database, covering an analysis period from 1990 to 2016. The third column states the specific Compustat data items that we use to calculate respective covariates. Value-at-Risk (VAR) and Expected Shortfall (ES) are estimated at 1% and 5% significance levels using three-month, six-month, one-year, three-year and five-year daily returns, and five-year monthly returns.

Table 3: Descriptive Statistics

Variable	Descriptive Measures	Financially Distressed	Healthy	Variable	Descriptive Measures	Financially Distressed	Healthy
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
TLMTA				MB			
	Mean	0.2816	0.3440		Mean	3.1660	2.4520
	Median	0.2078	0.2970		Median	2.6513	2.0025
	SD	0.2284	0.2407		SD	2.0469	1.5767
	Minimum	0.0352	0.0352		Minimum	0.6588	0.6588
	Maximum	0.8446	0.8446		Maximum	6.6199	6.6199
CMTA				PRICE			
	Mean	0.2025	0.1129		Mean	1.0365	2.1295
	Median	0.1681	0.0648		Median	0.8755	2.6301
	SD	0.1489	0.1249		SD	0.8451	0.7971
	Minimum	0.0028	0.0027		Minimum	0.1178	0.1178
	Maximum	0.4510	0.4510		Maximum	2.7081	2.7081
NIMTAAVG				RSIZE			
	Mean	-0.1812	-0.0192		Mean	-11.9120	-10.2861
	Median	-0.1780	0.0192		Median	-12.0701	-10.3200
	SD	0.1046	0.0987		SD	1.3269	1.8228
	Minimum	-0.3039	-0.3039		Minimum	-13.4014	-13.4014
	Maximum	0.0750	0.0750		Maximum	-6.8133	-6.8133
EXRETAVG				FES			
	Mean	-0.0071	-0.0079		Mean	0.0445	0.0242
	Median	-0.0056	-0.0051		Median	0.0170	0.0112
	SD	0.0572	0.0430		SD	0.0516	0.0333
	Minimum	-0.0984	-0.0984		Minimum	0.0000	0.0000
	Maximum	0.0712	0.0712		Maximum	0.1290	0.1290
SIGMA				TMTA			
	Mean	0.8002	0.5529		Mean	0.0001	0.0115
	Median	0.7433	0.4734		Median	0.0000	0.0090
	SD	0.3059	0.2993		SD	0.0079	0.0159
	Minimum	0.1962	0.1963		Minimum	-0.0174	-0.0174
	Maximum	1.2851	1.2851		Maximum	0.0494	0.0494
AGE				RISKFD			
	Mean	2.4457	2.3227		Mean	0.0337	0.0146
	Median	2.3979	2.3979		Median	0.025	0.0094
	SD	0.5469	1.0483		SD	0.0227	0.0164
	Minimum	1.3863	0.0000		Minimum	-0.0009	0.0000
	Maximum	4.1897	4.2047		Maximum	0.1667	0.1667
VAR3M1				VAR3M5			
	Mean	0.1300	0.0880		Mean	0.0788	0.0544
	Median	0.0961	0.0672		Median	0.0636	0.0438
	SD	0.1106	0.0779		SD	0.0552	0.0424
	Minimum	1.0000	1.0000		Minimum	0.6277	1.0000
	Maximum	0.0033	0.0001		Maximum	0.0019	4.5e-05
VAR6M1				VAR6M5			
	Mean	0.1550	0.1058		Mean	0.0785	0.0557
	Median	0.1111	0.0798		Median	0.0654	0.0460
	SD	0.1343	0.0942		SD	0.0513	0.0400
	Minimum	1.0000	1.0000		Minimum	0.4815	1.0000
	Maximum	0.0004	8.2e-05		Maximum	0.0005	2.4e-06
VAR1Y1				VAR1Y5			
	Mean	0.1867	0.1243		Mean	0.0761	0.0546
	Median	0.1354	0.0922		Median	0.0648	0.0464
	SD	0.1566	0.1139		SD	0.0464	0.0363
	Minimum	1.0000	1.0000		Minimum	0.4388	1.0000
	Maximum	0.0052	4.2e-05		Maximum	0.0003	1.1e-05
VAR3Y1				VAR3Y5			

Mean	0.2829	0.1689	Mean	0.0808	0.0537
Median	0.2009	0.1202	Median	0.0717	0.0475
SD	0.2240	0.1595	SD	0.0450	0.0323
Minimum	1.0000	1.0000	Minimum	0.3717	1.0000
Maximum	0.0480	0.0023	Maximum	0.0031	2.3e-05
VAR5Y1			VAR5Y5		
Mean	0.3475	0.1945	Mean	0.0760	0.0530
Median	0.2408	-0.1366	Median	0.0708	0.0477
SD	0.2605	0.1832	SD	0.0373	0.0300
Minimum	1.0000	1.0000	Minimum	0.3197	1.0000
Maximum	0.0510	0.0011	Maximum	5.4e-05	2.6e-07
VAR60M1			VAR60M5		
Mean	0.5797	0.3758	Mean	0.3792	0.2416
Median	0.5418	0.3223	Median	0.3542	0.2121
SD	0.2339	0.2137	SD	0.1442	0.1325
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
Maximum	0.0628	0.0004	Maximum	0.0248	0.0006
ES3M1			ES3M5		
Mean	0.2103	0.1287	Mean	0.1297	0.0817
Median	0.1243	0.0841	Median	0.0900	0.0609
SD	0.2277	0.1555	SD	0.1188	0.0761
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
Maximum	0.0248	0.0007	Maximum	0.0048	7.2e-05
ES6M1			ES6M5		
Mean	0.2250	0.1389	Mean	0.1293	0.0870
Median	0.1439	0.0987	Median	0.0914	0.0649
SD	0.2173	0.1405	SD	0.1157	0.0828
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
Maximum	0.0009	0.0001	Maximum	0.0084	2.4e-06
ES1Y1			ES1Y5		
Mean	0.2717	0.1660	Mean	0.1215	0.0861
Median	0.1842	0.1191	Median	0.0886	0.0656
SD	0.2307	0.1572	SD	0.1163	0.0815
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
Maximum	0.0003	4.2e-05	Maximum	0.0034	5.5e-05
ES3Y1			ES3Y5		
Mean	0.3686	0.2044	Mean	0.1154	0.0861
Median	0.2828	0.1497	Median	0.0860	0.0674
SD	0.2487	0.1778	SD	0.1133	0.0878
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
Maximum	0.0034	2.9e-05	Maximum	0.0002	2.3e-05
ES5Y1			ES5Y5		
Mean	0.3968	0.2201	Mean	0.1151	0.0847
Median	0.3028	0.1603	Median	0.0851	0.0666
SD	0.2605	0.1897	SD	0.1273	0.0929
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
ES60M1			ES60M5		
Mean	0.6828	0.4607	Mean	0.5275	0.3375
Median	0.6620	0.4004	Median	0.4783	0.2862
SD	0.2407	0.2481	SD	0.2207	0.2006
Minimum	1.0000	1.0000	Minimum	1.0000	1.0000
Maximum	0.1531	0.0363	Maximum	0.1236	0.0310

Notes: This table provides descriptive statistics for covariates listed in columns (1) and (5). To facilitate comparison, descriptive measures are reported separately for financially distressed (columns 3 and 7) and healthy (columns 4 and 8) groups. SD is standard deviation. Except EXRETAVG, two groups mean comparison test for respective covariates are statistically significant at 1% significance level.

Table 4: Correlation Matrix

Variable	1	2	3	4	5	6	7	8	9	10
Panel A: Correlation among Main Variables										
TLMTA (1)	1.0000									
CMTA (2)	-0.2351	1.0000								
ANIMTA (3)	-0.0311	-0.3074	1.0000							
AMEXRET (4)	-0.2141	-0.1351	0.1993	1.0000						
SIGMA (5)	0.0758	0.1975	-0.4696	-0.2595	1.0000					
MB (6)	-0.3104	-0.1883	-0.1360	0.2007	0.0156	1.0000				
PRICE (7)	-0.2216	-0.2598	0.5217	0.3033	-0.6121	0.1109	1.0000			
RSIZE (8)	-0.1516	-0.2596	0.3490	0.2140	-0.5189	0.2372	0.7168	1.0000		
FES (9)	0.3265	-0.0744	-0.2872	-0.0836	0.1448	0.1631	-0.1706	-0.0569	1.0000	
TMTA (10)	-0.0907	-0.1179	0.4489	0.0757	-0.2525	-0.1162	0.2874	0.1751	-0.2394	1.0000
Panel B: Correlation among Main and Tail Risk Variables										
VAR3M1	-0.0860	-0.1368	0.3489	0.2263	-0.7607	-0.0195	0.4636	0.3784	-0.1142	0.1768
VAR6M1	-0.0743	-0.1441	0.3521	0.2774	-0.6806	-0.0341	0.4320	0.3654	-0.1042	0.1799
VAR1Y1	-0.0490	-0.1538	0.3482	0.2566	-0.5946	-0.0485	0.4035	0.3484	-0.1042	0.1797
VAR3Y1	-0.0166	-0.1604	0.3474	0.1556	-0.4454	-0.0835	0.3493	0.3092	-0.1253	0.2015
VAR5Y1	-0.0096	-0.1620	0.3427	0.1250	-0.3836	-0.0943	0.3341	0.2976	-0.1300	0.1997
VAR6M1	-0.0050	-0.2147	0.4741	0.1698	-0.5438	-0.0934	0.3840	0.3304	-0.1650	0.2747
VAR3M5	-0.1004	-0.1573	0.4021	0.2570	-0.8464	-0.0058	0.5315	0.4274	-0.1309	0.2031
VAR6M5	-0.1012	-0.1661	0.4210	0.3059	-0.8335	-0.0072	0.5475	0.4454	-0.1286	0.2134
VARIY5	-0.0858	-0.1746	0.4390	0.2813	-0.8154	-0.0179	0.5688	0.4677	-0.1306	0.2260
VAR3Y5	-0.0419	-0.1740	0.4525	0.1842	-0.7576	-0.0453	0.5664	0.4866	-0.1378	0.2545
VAR5Y5	-0.0049	-0.1803	0.4413	0.1511	-0.7144	-0.0579	0.5598	0.4961	-0.1275	0.2538
VAR6M5	0.0118	-0.2385	0.5485	0.1994	-0.6432	-0.1059	0.4541	0.3896	-0.1764	0.3065
ES3M1	-0.0500	-0.1115	0.2676	0.1478	-0.5639	-0.0311	0.3263	0.2806	-0.0924	0.1423
ES6M1	-0.0667	-0.1336	0.3214	0.2064	-0.5859	-0.0330	0.3920	0.3392	-0.0994	0.1646
ES1Y1	-0.0543	-0.1514	0.3463	0.1939	-0.5485	-0.0371	0.4193	0.3713	-0.1116	0.1820
ES3Y1	-0.0185	-0.1747	0.4060	0.1491	-0.5130	-0.0851	0.4375	0.3889	-0.1460	0.2257
ESSY1	-0.0042	-0.1760	0.3908	0.1331	-0.4524	-0.0981	0.4085	0.3632	-0.1499	0.2231
ES6M1	-0.0070	-0.2074	0.4590	0.1627	-0.5459	-0.0915	0.4087	0.3651	-0.1633	0.2727
ES3M5	-0.0749	-0.1414	0.3582	0.2088	-0.7499	-0.0251	0.4491	0.3687	-0.1142	0.1777
ES6M5	-0.0718	-0.1337	0.3369	0.2546	-0.6519	-0.0254	0.4096	0.3444	-0.0935	0.1678
ES1Y5	-0.0503	-0.1346	0.3129	0.2390	-0.5811	-0.0280	0.3782	0.3143	-0.0823	0.1580
ES3Y5	-0.0083	-0.1067	0.2509	0.1199	-0.4184	-0.0413	0.2733	0.2345	-0.0713	0.1434
ESSY5	0.0072	-0.0931	0.2046	0.0757	-0.3091	-0.0432	0.2137	0.1907	-0.0582	0.1177
ES6M5	-0.0009	-0.2105	0.4795	0.1747	-0.5582	-0.0986	0.4074	0.3497	-0.1637	0.2756

Notes: This table reports correlation among the set of covariates. Panel A displays correlations among main variables suggested by Campbell *et al.* (2008) to which we supplemented FES and TMTA. Panel B provides correlations among main variables and the respective tail risk measures (VaR and ES).

Table 5: Univariate Regression

Variables	Coefficient	Standard Error	z-statistics	AME %	Rank
(1)	(2)	(3)	(4)	(5)	(6)
TLMTA	1.2407 ^a	0.1443	8.60	0.41 ^a	8
CMTA	5.0517 ^a	0.2107	23.98	2.78 ^a	5
NIMTAAVG	-9.9555 ^a	0.2553	-38.99	-7.20 ^a	2
EXRETAVG	-11.7030 ^a	0.6643	-17.62	-6.71 ^a	3
SIGMA	1.7099 ^a	0.0952	17.97	1.05 ^a	----
MB	-0.0822 ^a	0.0208	-3.95	-0.03 ^a	9
PRICE	-1.8438 ^a	0.0843	-38.21	-1.47 ^a	6
RSIZE	-1.0774 ^a	0.0376	-28.69	-0.62 ^a	7
FES	10.6394 ^a	0.7990	13.32	4.80 ^a	4
TMTA	-53.7919 ^a	2.7701	-19.42	-30.07 ^a	1
AGE	0.5636 ^a	0.0465	12.12	0.14 ^a	----
INDRISK	22.6630 ^a	1.3730	16.51	8.72 ^a	----
VAR3M1	2.1574 ^a	0.2666	8.09	1.05 ^a	----
VAR6M1	1.7075 ^a	0.2256	7.57	0.84 ^a	----
VAR1Y1	1.4859 ^a	0.1900	7.82	0.72 ^a	----
VAR3Y1	1.7141 ^a	0.1404	12.21	0.91 ^a	----
VAR5Y1	2.1107 ^a	0.1274	16.57	1.20 ^a	----
VAR60M1	2.8442 ^a	0.1303	21.83	1.87 ^a	----
VAR3M5	4.1758 ^a	0.4938	8.46	2.04 ^a	----
VAR6M5	4.5172 ^a	0.5386	8.39	2.26 ^a	----
VAR1Y5	5.4346 ^a	0.6092	8.92	2.70 ^a	----
VAR3Y5	9.1312 ^a	0.7207	12.67	4.49 ^a	----
VAR5Y5	12.2205 ^a	0.8433	14.49	6.03 ^a	----
VAR60M5	4.4384 ^a	0.1906	23.29	2.99 ^a	----
ES3M1	1.2239 ^a	0.1327	9.22	0.59 ^a	----
ES6M1	1.5158 ^a	0.1447	10.48	0.76 ^a	----
ES1Y1	1.5925 ^a	0.1355	11.74	0.81 ^a	----
ES3Y1	2.2144 ^a	0.1239	17.87	1.13 ^a	----
ES5Y1	2.3195 ^a	0.1236	18.77	1.39 ^a	----
ES60M1	2.5213 ^a	0.1231	20.47	1.66 ^a	----
ES3M5	2.5530 ^a	0.2506	10.19	1.25 ^a	----
ES6M5	1.9224 ^a	0.2429	7.91	0.91 ^a	----
ES1Y5	1.5578 ^a	0.2548	6.11	0.72 ^a	----
ES3Y5	0.8846 ^a	0.2630	3.36	0.39 ^a	----
ES5Y5	1.2673 ^a	0.2349	5.39	0.56 ^a	----
ES60M5	2.7446 ^a	0.1319	20.81	1.78 ^a	----

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). This table reports univariate regression estimates of respective covariates using our proposed definition of financial distress as the dependent variable. AME is average marginal effects in percentages. Column (6) reports the ranking of the main variables based on the magnitude of their average marginal effects (AME).

Table 6: Baseline Multivariate Regression Models

Variables	(T - 1)	(T - 2)	(T - 3)
(1)	(2)	(3)	(4)
TLMTA			
β	----	----	----
SE	----	----	----
CMTA			
β	3.9430 ^a	3.3276 ^a	1.8415 ^a
SE	0.2480	0.2598	0.2618
AME	5.1450 ^a	4.4193 ^a	2.7795 ^a
NIMTAAVG			
B	-9.6073 ^a	-11.7358 ^a	-9.3950 ^a
SE	0.3362	0.3711 ^a	0.3333
AME	-12.5361 ^a	-15.5853	-14.1800 ^a
EXRETAVG			
B	-3.6941 ^a	-12.6168 ^a	-0.5815
SE	0.7194	0.7968	-0.8200
AME	-4.8202 ^a	-16.7559 ^a	-0.8776
SIGMA			
B	----	----	----
SE	----	----	----
MB			
B	----	----	----
SE	----	----	----
PRICE			
B	----	----	----
SE	----	----	----
RSIZE			
B	----	----	----
SE	----	----	----
FES			
B	7.0887 ^a	3.8007 ^a	3.2327 ^a
SE	0.7688	0.8400	0.8155
AME	9.2497 ^a	5.0475 ^a	4.8792 ^a
TMTA			
B	-26.3068 ^a	-22.0542 ^a	-23.7398 ^a
SE	2.9939	2.9722	2.9429
AME	-34.3264 ^a	-29.2894 ^a	-35.8306 ^a
AGE			
B	0.3017 ^a	0.1799 ^a	-0.2586 ^a
SE	0.0558	0.0522	0.0459
AME	0.3937 ^a	0.2389 ^a	-0.3903 ^a
INDRISK			
B	18.7770 ^a	-2.5321 ^a	-5.6689 ^a
SE	1.6898	2.2349	2.0998
AME	24.5011 ^a	-3.3628 ^a	-8.5562 ^a
Model's goodness of fit and prediction performance measures			
Chi2	2236.11 ^a	2242.69 ^a	1817.70 ^a
Log likelihood	-4150.7302	-3741.3455	-4182.0543
R ²	0.3917	0.3536	0.2508
AUROC-W	0.9240	0.9431	0.9026
AUROC-H	0.9193	0.9428	0.8995
N = 0+1	59,602	53,003	48,010
N = 1	1,160	1,225	1,168

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). This table reports multivariate regression estimates that employ only main variable discussed in Section 4.2 for T-1 (column 2), T-2 (column 3) and T-3 (column 4) lagged covariates. The baseline multivariate model in column 2 is developed following the method suggested by Gupta *et al.* (2018) (see section 5). TLMTA, MB, RSIZE, PRICE and SIGMA are excluded from the model for the reasons mentioned in section 5 (related to the expected coefficient sign and the p-value, or the

strong correlation with other covariates). $N = 0 + 1$ represent the total number of firm-year observations for the two groups of firms between 1990 and 2016. $N = 1$ is the number of observations for the financially distressed firms. This table also reports the chi-square, the likelihood ratio and the coefficient of determination (McKelvey & Zavoina's R^2) to measure the model's goodness of fit. AWROC-W is within-sample area under ROC curve and AWROC-H is for hold-out sample.

Figure 1: Area under ROC Curves

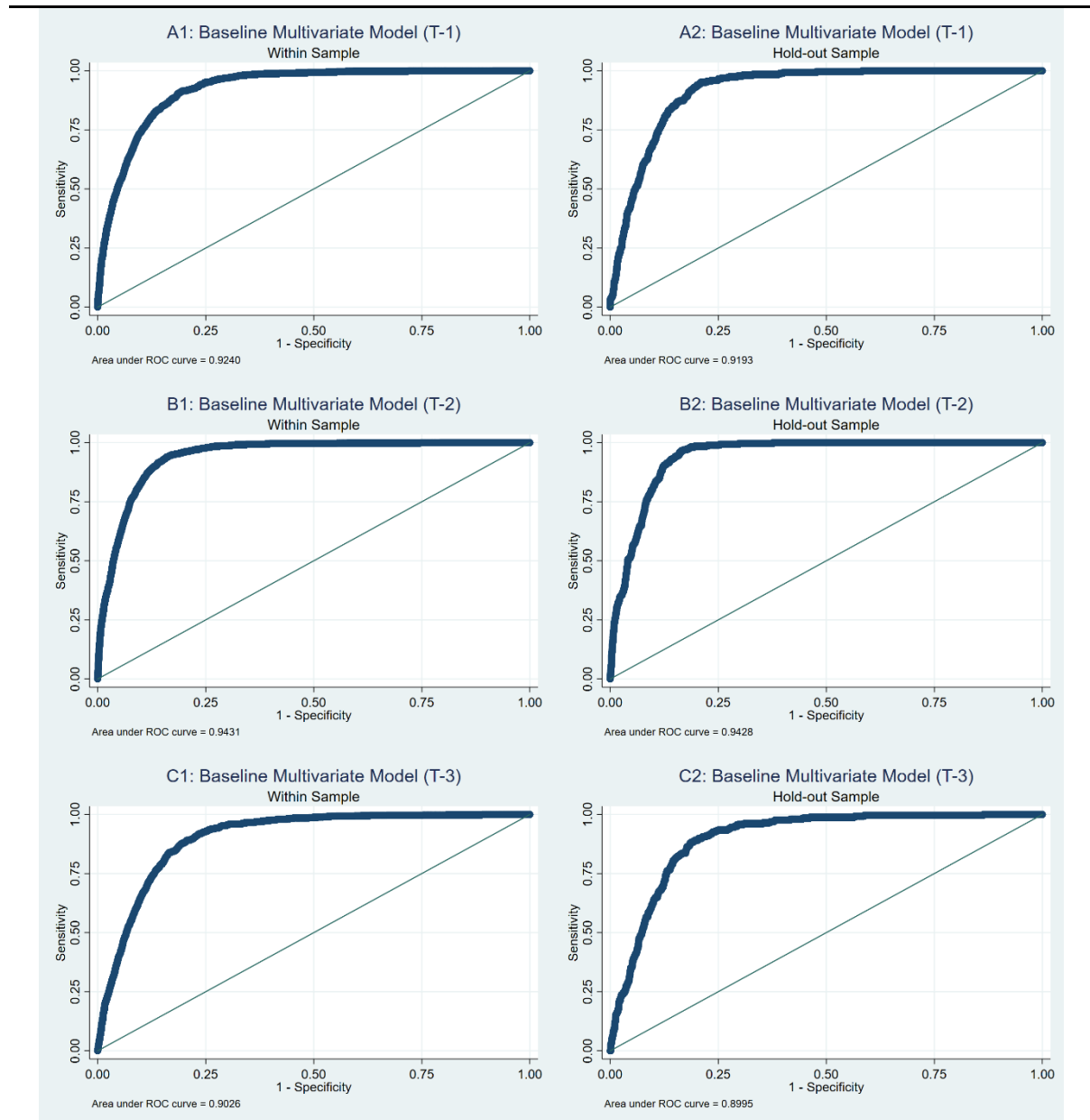


Table 7: Multivariate Regression Models with VaR

Variables	SIGMA	VAR3M1	VAR6M1	VARIY1	VAR3Y1	VAR5Y1	VAR60M1	VAR3M5	VAR6M5	VARIY5	VAR3Y5	VAR5Y5	VAR60M5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Risk Measure													
β	0.3446 ^a	-0.6634 ^c	-0.4875	-0.2922	0.2044	0.5732 ^a	0.6907 ^a	-1.3422 ^c	-1.7765 ^b	-1.3476	2.8789 ^a	6.5853 ^a	1.3093 ^a
SE	0.1251	0.3682	0.3045	0.2512	0.1726	0.1503	0.1667	0.7093	0.7879	0.9011	1.0149	1.1449	0.2819
AME	0.4495 ^a	-0.8697 ^c	-0.2649	-0.3773	0.2662	0.7437 ^a	0.8826 ^a	-1.7443 ^c	-2.2969 ^b	-1.7376	3.6191 ^a	8.1862 ^a	1.6723 ^a
LR Chi ²	7.5000 ^a	3.4400 ^c	2.6500	1.3800	1.3800	14.100 ^a	16.9500 ^a	3.7300 ^c	5.3000 ^b	2.3100	7.7200 ^a	31.7700 ^a	21.2300 ^a
CMTA													
β	4.0596 ^a	3.8820 ^a	3.9273 ^a	3.9347 ^a	3.9766 ^a	3.9924 ^a	3.9923 ^a	3.8381 ^a	3.8347 ^a	3.9314 ^a	4.2236 ^a	4.2015 ^a	4.01309 ^a
SE	0.2504	0.2522	0.2511	0.2512	0.2492	0.2511	0.2511	0.2524	0.2547	0.2555	0.2591	0.2610	0.2819
NIMTAAVG													
β	-9.1909 ^a	-9.8214 ^a	-9.8028 ^a	-9.7502 ^a	-9.5584 ^a	-9.1629 ^a	-9.1629 ^a	-9.8917 ^a	-9.9514 ^a	-9.8193 ^a	-9.4009 ^a	-9.1925 ^a	-8.9686 ^a
SE	0.3721	0.3551	0.3542	0.3536	0.3463	0.3609	0.3608	0.3599	0.3655	0.3672	0.3638	0.3620	0.3706
EXRETAVG													
β	-3.4113 ^a	-3.8365 ^a	-3.8214 ^a	-3.7963 ^a	-3.5371 ^a	-3.4006 ^a	-3.4005 ^a	-3.7818 ^a	-3.8942 ^a	-3.9179 ^a	-3.3195 ^a	-3.4141 ^a	-3.2654 ^a
SE	0.7217	0.7318	0.7280	0.7283	0.7233	0.7234	0.7234	0.7293	0.7351	0.7375	0.7486	0.7464	0.7238
FES													
β	7.1077 ^a	7.0961 ^a	7.1864 ^a	7.0929 ^a	7.0809 ^a	6.9784 ^a	6.9783 ^a	7.0919 ^a	7.1285 ^a	7.1382 ^a	7.4160 ^a	7.4645 ^a	6.9472 ^a
SE	0.7692	0.7760	0.7733	0.7769	0.7735	0.7786	0.7786	0.7754	0.7789	0.7844	0.8043	0.8126	0.7784
TMTA													
β	-25.7929 ^a	-26.0233 ^a	-25.9140 ^a	-25.9666 ^a	-26.1026 ^a	-25.5957 ^a	-25.5957 ^a	-26.0049 ^a	-25.4098 ^a	-25.8002 ^a	-24.5883 ^a	-23.8689 ^a	-25.4876 ^a
SE	3.0013	3.0130	3.0053	3.0144	3.0070	3.0193	3.0192	3.0107	3.0224	3.0310	3.0441	3.0585 ^a	3.0212 ^a
AGE													
β	0.3235 ^a	0.2862 ^a	0.2985 ^a	0.2920 ^a	0.3038 ^a	0.3375 ^a	0.3375 ^a	0.2854	0.2831 ^a	0.2815 ^a	0.3275 ^a	0.3673 ^a	0.3610 ^a
SE	0.0568	0.0565	0.0563	0.0565	0.0563	0.0576	0.0576	0.0565	0.0568	0.0574	0.0591	0.0608	0.05858
RISK													
β	19.5882 ^a	18.8046 ^a	18.8477 ^a	18.8847 ^a	18.7999 ^a	19.4803 ^a	19.4802 ^a	18.8948 ^a	18.8983 ^a	19.0909 ^a	20.2097 ^a	21.0505 ^a	19.8380 ^a
SE	1.7130	1.7131	1.7045	1.7078	1.6952	1.7022	1.7022	1.7141	1.7249	1.7286	1.7444	1.7411	1.7033
Model's goodness of fit and prediction performance measures													
chi2	2232.06 ^a	2209.66 ^a	2234.30 ^a	2208.21 ^a	2226.58 ^a	2184.84 ^a	2184.84 ^a	2212.63 ^a	2198.73 ^a	2186.41 ^a	2126.69 ^a	2083.52 ^a	2185.96 ^a
Log likelihood	-4128.5475	-4032.4688	-4063.9586	-4066.6788	-4120.0273	-4084.6001	-4084.6001	-4053.8316	-4010.7991	-3976.7497	-3878.275	-3821.1612	-4089.0856
R ²	0.3299	0.3267	0.3280	0.3257	0.3270	0.3305	0.3305	0.3252	0.3291	0.3257	0.3264	0.3301	0.3311
N = 0+1	59,601	58,659	57,698	57,869	58,175	58,185	58,185	57,415	58,507	57,019	56,332	55,968	58,258
N = 1	1,215	1,183	1,196	1,194	1,213	1,205	1,205	1,176	1,118	1,167	1,140	1,124	1,206

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). This table reports multivariate regression estimates obtained by supplementing the baseline model reported in Table 6 with SIGMA and VaR measures. Estimates of the risk measures as listed in columns (2) to (14) are reported in rows (4) to (7). Column (2) reports regression estimates with SIGMA being the risk measure. Columns (3) to (8) report respective multivariate regression estimates supplementing VaR as the risk measure (three-month, six-month, one-year, three-year and five-year daily returns and sixty-month monthly return) at the 95% confidence interval, and columns (9) to (12) report the same at 99%

confidence interval. The chi-square for likelihood ratio test (LR Chi^2) reported in row (7) of column (1) tests the existence of any significant difference between the log likelihood of the baseline model supplemented with risk variable, and log likelihood of baseline model. The last five rows of this table provide goodness of fit measures. Chi-square (Chi2), likelihood ratio, and the coefficient of determination (McKelvey & Zavoina's R^2) of respective multivariate models with respective risk variables are reported in the first three of the last five rows. $N = 0 + 1$ represents the total number of firm-year observations for the two groups of firms between 1990 and 2016. $N = 1$ is the number of firm-year observations for financially distressed group of firms.

Table 8: Multivariate Regression Models with ES

Variables	SIGMA	ES3M1	ES6M1	ES1Y1	ES3Y1	ES5Y1	ES60M1	ES3M5	ES6M5	ES1Y5	ES3Y5	ES5Y5	ES60M5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Risk Measure													
β	0.3446 ^a	0.1632	0.0947	0.2682	0.6783 ^a	0.7360 ^a	0.6769 ^a	0.0173	-0.2254	-0.2083	-0.4769	0.2433	0.7264 ^a
SE	0.1251	0.1636	0.1840	0.1667	0.1536	0.1489	0.1558	0.3286	0.3265	0.3348	0.3512	0.2876	0.1724
AME	0.4495 ^a	0.2124	0.1238	0.3472	0.8813 ^a	0.9523 ^a	0.8715 ^a	0.0225	-0.2698	-0.4941	-0.6016	0.3077	0.9334 ^a
LR Chi^2	7.500 ^a	0.9800	0.2600	2.5400	18.7800 ^a	23.6100 ^a	18.7700 ^a	0.0000	0.4800	0.3900	1.9400	0.7000	17.4300 ^a
CMTA													
β	4.0596 ^a	3.9649 ^a	3.9807 ^a	3.9924 ^a	3.9973 ^a	3.9547 ^a	3.9882 ^a	3.9634 ^a	3.9270 ^a	4.0290 ^a	4.0523 ^a	4.0146 ^a	3.9931 ^a
SE	0.2504	0.2492	0.2493	0.2503	0.2491	0.2496	0.2498	0.2496	0.2504	0.2510	0.2548	0.2547	0.2501
NIMTAAVG													
β	-9.1909 ^a	-9.5841 ^a	-9.5781 ^a	-9.4613 ^a	-9.2437 ^a	-9.2368 ^a	-9.0949 ^a	-9.6406 ^a	-9.7323 ^a	-9.7013 ^a	-9.7663 ^a	-9.6136 ^a	-9.1244 ^a
SE	0.3721	0.3433	0.3486	0.3499	0.3490	0.3476	0.3592	0.3487	0.3498	0.3478	0.3483	0.3461	0.3589
EXRETAVG													
β	-3.4113 ^a	-3.5959 ^a	-3.7082 ^a	-3.095 ^a	-3.3643 ^a	-3.4783 ^a	-3.4067 ^a	-3.6033 ^a	-3.5967 ^a	-3.5680 ^a	-3.5561 ^a	-3.6227 ^a	-3.4280 ^a
SE	0.7217	0.7226	0.7230	0.7255	0.7199	0.7185	0.7192	0.7226	0.7242	0.7260	0.7415	0.7398	0.7197
FES													
β	7.1077 ^a	7.1047 ^a	7.1388 ^a	7.0795 ^a	6.8732 ^a	6.7409 ^a	6.9813 ^a	7.0960 ^a	7.1997 ^a	7.1340 ^a	7.4299 ^a	7.3272 ^a	6.9324 ^a
SE	0.7692	0.7708	0.7701	0.7742	0.7734	0.7768	0.7737	0.7710	0.7729	0.7769	0.7924	0.7929	0.7752
TMTA													
β	-25.7929 ^a	-26.0431 ^a	-26.1313 ^a	-26.1694 ^a	-25.9099 ^a	-25.6348 ^a	-25.3468 ^a	-26.1866 ^a	-25.9843 ^a	-26.0025 ^a	-25.1342 ^a	-25.0697 ^a	-25.5604 ^a
SE	3.0013	3.0031	3.0021	3.0124	3.0168	3.0206	3.0065	3.0024	3.0063	3.0078	3.0406	3.0360	3.0091
AGE													
β	0.3235 ^a	0.3026 ^a	0.3023 ^a	0.3027 ^a	0.3176 ^a	0.3196 ^a	0.3427 ^a	0.2993 ^a	0.3047 ^a	0.3027 ^a	0.2874 ^a	0.2970 ^a	0.3420 ^a
SE	0.0568	0.0562	0.0563	0.05630	0.0565	0.0566	0.0574	0.0562	0.0563	0.0565	0.0623	0.0576	0.0574
RISK													
β	19.5882 ^a	18.9429 ^a	19.1011 ^a	19.1011 ^a	18.9332 ^a	18.9319 ^a	19.3062 ^a	18.8269 ^a	18.8518 ^a	18.7541 ^a	16.2860 ^a	19.3562 ^a	19.3286 ^a
SE	1.7130	1.6969	1.6992	1.6992	1.6965	1.6971	1.7005	1.6975	1.6991	1.7042	2.3852	1.7211	1.6997
Model's goodness of fit and prediction performance measures													
chi2	2232.06 ^a	2229.36 ^a	2238.87 ^a	2211.94 ^a	2223.00 ^a	2211.40 ^a	2196.33 ^a	2227.25 ^a	2216.58 ^a	2214.75 ^a	1938.18 ^a	2142.31 ^a	2198.83 ^a

Log likelihood	-4128.5475	-34122.0715	-4110.1135	-4096.2486	-4114.9963	-4115.2167	-4110.5026	-4126.6347	-4104.8703	-4077.1546	-3784.0133	-3955.3607	-4111.3689
R ²	0.3299	0.3267	0.3282	0.3269	0.3290	0.3287	0.3313	0.3263	0.3256	0.3259	0.3259	0.3218	0.3297
N = 0+1	58,386	58,236	58,131	58,138	58,311	58,331	58,343	58,261	58,148	57,743	58,201	56,891	58,348
N = 1	1,215	1,213	1,210	1,202	1,213	1,214	1,212	1,214	1,206	1,201	1,108	1,156	1,212

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). This table reports multivariate regression estimates obtained by supplementing the baseline model reported in Table 6 with SIGMA and ES measures. Estimates of the risk measures as listed in columns (2) to (14) are reported in rows (4) to (7). Column (2) reports regression estimates with SIGMA being the risk measure. Columns (3) to (8) report respective multivariate regression estimates supplementing ES as the risk measure at the 95% confidence interval, and columns (9) to (14) report the same at 99% confidence interval. The chi-square for likelihood ratio test (LR Chi^2) reported in row (7) of column (1) tests the existence of any significant difference between the log likelihood of the baseline model supplemented with risk variable, and log likelihood of baseline model. The last five rows of this table provide goodness of fit measures. Chi-square (Chi^2), likelihood ratio and the coefficient of determination (McKelvey & Zavoina's R^2) of respective multivariate models with respective risk variables are reported in the first three of the last five rows. N = 0 + 1 represents the total number of firm-year observations for the two groups of firms between 1990 and 2016. N = 1 is the number of firm-year observations for financially distressed group of firms.

Table 9: Baseline Multivariate Model at different threshold levels

Variables	Main (0% Threshold)	10% Threshold	20% Threshold	30% Threshold
(1)	(2)	(3)	(4)	(5)
CMTA				
β	3.9430 ^a	2.908 ^a	2.9629 ^a	3.1287 ^a
<i>SE</i>	0.2480	0.3063	0.3432	0.4006
<i>AME</i>	5.1450 ^a	2.4574 ^a	1.9692 ^a	1.4829 ^a
NIMTAAVG				
β	-9.6073 ^a	-9.1548 ^a	-9.5856 ^a	-10.3909 ^a
<i>SE</i>	0.3362	0.4180	0.4787	0.5766
<i>AME</i>	-12.5361 ^a	-7.7360 ^a	-6.3709 ^a	-4.9247 ^a
EXRETAVG				
<i>B</i>	-3.6941 ^a	-4.5370 ^a	-4.3899 ^a	-4.1111 ^a
<i>SE</i>	0.7194	0.9034	1.0367	1.2236
<i>AME</i>	-4.8202 ^a	-3.8334 ^a	-2.9177 ^a	-1.9484 ^a
FES				
<i>B</i>	7.0887 ^a	11.0839 ^a	9.5835 ^a	8.3311 ^a
<i>SE</i>	0.7688	0.8878	0.9944	1.1687
<i>AME</i>	9.2497 ^a	9.3660 ^a	6.3695 ^a	3.9485 ^a
TMTA				
<i>B</i>	-26.3068 ^a	-22.8561 ^a	-19.1492 ^a	-13.6018 ^a
<i>SE</i>	2.9939	3.8382	4.4013	5.1473
<i>AME</i>	-34.3264 ^a	-19.3139 ^a	-12.7271 ^a	-6.4465 ^a
AGE				
<i>B</i>	0.3017 ^a	0.1957 ^a	0.1071	0.0378
<i>SE</i>	0.0558	0.0682	0.0779	0.0934
<i>AME</i>	0.3937 ^a	0.1653 ^a	0.0712	0.0179
INDRISK				
<i>B</i>	18.7770 ^a	192664 ^a	19.4829 ^a	20.9762 ^a
<i>SE</i>	1.6898	2.0835	2.3834	2.7946
<i>AME</i>	24.5011 ^a	16.2805 ^a	12.9489 ^a	9.9415 ^a
Model's goodness of fit and prediction performance measures				
Chi2	2236.11 ^a	1506.61 ^a	1196.95 ^a	903.11 ^a
Log likelihood	-4150.7302	-2730.9877	-2101.0535	-1532.2126
R ²	0.3263	0.3211	0.3262	0.3340
N = 0+1	59,691	58,995	59,187	59,341
N = 1	1,219	696	504	350

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). Column 2 reports the baseline multivariate regression estimates for $t - 1$ lagged covariates, similar to the model reported in column 2 of Table 6. Columns 3, 4 and 5 report results obtained from arbitrarily imposing 10%, 20% and 30% threshold levels respectively on conditions 1, 2 and 3 of our proposed financial distress definition (see section 6 for details). All regression estimates are reported for $t - 1$ lagged covariates. N = 0 + 1 represent the total number of firm-year observations for the two groups of firms between 1990 and 2016. N = 1 is the number of firm-year observations for financially distressed group of firms. This table also reports the chi-square, the likelihood ratio and the coefficient of determination (McKelvey & Zavoina's R²) to measure the model's goodness of fit.

Table 10: Multivariate Regression Models at different thresholds with VaR

Panel A: 10 % Threshold Estimates													
Risk Measures	SIGMA	VAR3M1	VAR6M1	VARIY1	VAR3Y1	VAR5Y1	VAR60M1	VAR3M5	VAR6M5	VARIY5	VAR3Y5	VAR5Y5	VAR60M5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
β	0.3724 ^a	-0.4628	-0.3947	-0.3527	0.2804	0.6081 ^a	1.1308 ^a	-1.0255	-1.0302	0.1388	5.01948 ^a	8.9486 ^a	2.0060 ^a
<i>SE</i>	0.1550	0.4366	0.3639	0.3054	0.2045	0.1774	0.2010	0.8398	0.9206	1.0121	1.1076	1.2545	0.3270
<i>AME</i>	0.3158 ^a	-0.3932	-0.3330	-0.2940	0.2363	0.5113 ^a	0.9312 ^a	-0.8713	-0.8612	0.1161	4.0686 ^a	7.3382 ^a	1.6607 ^a
<i>LR Chi</i> ²	5.7000 ^b	1.1600	1.2100	1.3800	1.8400	11.3200 ^a	31.2100 ^a	1.5500	1.2900	0.0200	18.7400 ^a	47.2600	36.4100 ^a
Model's goodness of fit and prediction performance measures													
Chi2	1508.75 ^a	1493.39 ^a	1491.64 ^a	1479.09 ^a	1500.05 ^a	1491.80 ^a	1464.23 ^a	1498.72 ^a	1472.06 ^a	1467.99 ^a	1438.60 ^a	1432.81 ^a	1475.48 ^a
Log likelihood	-2716.0946	-2669.8853	-2677.1974	-2671.3670	-2710.5895	-2709.0528	-2680.1741	-2680.6081	-2658.3682	-2619.9340	-2527.0812	-2490.9328	-2684.0485
R ²	0.3249	0.3208	0.3215	0.3189	0.3212	0.3224	0.3266	0.3221	0.3192	0.3190	0.3241	0.3327	0.3268
N = 0+1	58,907	57,979	58,211	58,384	58,695	58,724	58,702	58,185	57,914	57,521	56,826	56,452	58,775
N = 1	694	680	683	679	693	694	688	684	677	665	646	640	689
Panel B: 20 % Threshold Estimates													
β	0.4900 ^a	-0.4411	-0.6696	-0.5887 ^c	-0.0557	0.2963	1.0120 ^a	-0.8340	-1.1595	0.1232	4.3923 ^a	8.7093 ^a	1.8207 ^a
<i>SE</i>	0.1761	0.4899	0.4258	0.3555	0.2378	0.2031	0.2280	0.9344	1.0463	1.1439	1.2448	1.4112	0.3652
<i>AME</i>	0.3263	-0.2924	-0.4399	-0.3806 ^c	-0.0369	0.1965	0.6509 ^a	-0.5564	-0.7582	0.0794	2.7035 ^a	5.3361 ^a	1.1833 ^a
<i>LR Chi</i> ²	7.6600 ^a	0.8400	2.6300	2.9100 ^c	0.0600	2.0800	19.4700 ^a	0.8200	1.2700	0.0100	11.3000 ^a	35.3800 ^a	24.0000 ^a
Model's goodness of fit and prediction performance measures													
Chi2	1196.56 ^a	1177.35 ^a	1177.57 ^a	1166.62 ^a	1192.22 ^a	1189.53 ^a	1153.79 ^a	1186.60 ^a	1167.53 ^a	1151.52 ^a	1118.10	1099.30 ^a	1168.44 ^a
Log likelihood	-2084.8657	-2045.4045	-2045.3282	-2053.0042	-2086.1928	-2084.6411	-2057.9970	-2054.0931	-2031.8816	-2004.9210	-1946.1826	-1921.0403	-2062.0946
R ²	0.3322	0.3254	0.3261	0.3241	0.3266	0.3270	0.3312	0.3272	0.3242	0.3240	0.3288	0.3346	0.3316
N = 0+1	59,099	58,169	58,403	58,571	58,886	58,916	58,893	58,376	58,104	57,707	57,001	56,625	58,966
N = 1	502	490	491	492	502	502	497	493	487	479	471	467	498
Panel C: 30 % Threshold Estimates													
β	0.3990 ^c	-0.4631	-0.9538 ^c	-0.5987	-0.1221	0.3454	1.2543 ^a	-0.9422	-1.5617	0.0318	4.2145 ^a	8.9213 ^a	2.1655 ^a
<i>SE</i>	0.2089	0.5694	0.5175	0.4129	0.2782	0.2345	0.2680	1.0952	1.2463	1.3375	1.4047	1.5886	0.4192
<i>AME</i>	0.1891 ^c	-0.2167	-0.4466 ^c	0.2788	-0.0577	0.1628	0.5748 ^a	-0.4430	0.7290	0.0147	0.1879 ^a	3.9878 ^a	0.9998 ^a
<i>LR Chi</i> ²	3.6200 ^c	0.6900	3.7300 ^c	2.2400	0.2000	2.1100	21.7700 ^a	0.7700	1.6500	0.0000	8.0800 ^a	29.1900 ^a	25.6100 ^a
Model's goodness of fit and prediction performance measures													
Chi2	901.62 ^a	884.89 ^a	888.47 ^a	889.10 ^a	899.74 ^a	895.38 ^a	877.62 ^a	888.14 ^a	883.17 ^a	869.39 ^a	857.57 ^a	849.02 ^a	888.23 ^a
Log likelihood	-1517.1064	-1493.5122	-1492.1717	-1499.9776	-1517.2735	-1515.9201	-1500.6078	-1498.4049	-1481.1960	-1463.0975	-1434.2777	-1408.3170	-1499.4390
R ²	0.3398	0.3327	0.3331	0.3334	0.3349	0.3353	0.3416	0.3337	0.3316	0.3322	0.3362	0.3450	0.3419
N = 0+1	59,253	58,317	58,552	58,719	59,040	59,070	59,043	58,526	58,252	57,852	57,140	56,763	59,117
N = 1	348	342	342	344	348	348	347	343	339	334	332	329	347

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). This table reports multivariate regression estimates obtained by supplementing models estimated at different threshold levels of conditions 1, 2 and 3 of our proposed financial distress definition (see section 6 for details) with SIGMA and VaR measures. Estimates of the risk measures as listed in columns (2) to (14) are provided in rows (4) to (7). Column (2) reports regression estimates with SIGMA being the risk measure. Columns (3) to (8) report

respective multivariate regression estimates supplementing VaR as the risk measure (three-month, six-month, one-year, three-year and five-year daily returns and sixty-month monthly returns) at the 95% confidence interval and columns (9) to (14) report the same at 99% confidence interval. The chi-square for likelihood ratio test ($LR\ Chi^2$) reported in row (7) of column (1) tests the existence of any significant difference between the log likelihood of the baseline model supplemented with risk variable, and log likelihood of baseline model. The last 5 rows of respective panels provide goodness of fit measures. Chi-square (Chi2), log likelihood and the coefficient of determination (McKelvey & Zavoina's R^2) of respective multivariate model with respective risk variables are reported in first three of last five rows. All regression estimates are reported for $t - 1$ lagged covariates. To save space, we do not report the baseline covariates of Table 9 in this table. $N = 0 + 1$ represent the total number of firm-year observations for the two groups of firms between 1990 and 2016. $N = 1$ is the number of firm-year observations for financially distressed group of firms.

Table 11: Multivariate Regression Models at different thresholds with ES

10 % Threshold Estimates													
Risk Measures	SIGMA	ES3M1	ES6M1	ES1Y1	ES3Y1	ES5Y1	ES60M1	ES3M5	ES6M5	ES1Y5	ES3Y5	ES5Y5	ES60M5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
β	0.3724 ^a	-0.0509	-0.1867	0.05307	0.7653 ^a	0.81118 ^a	1.0569 ^a	-0.1283	-0.1199	0.1434	-0.0045	0.4964	1.1631 ^a
<i>SE</i>	0.1550	0.2069	0.2311	0.2600	0.1829	0.1759	0.1938	0.4012	0.3876	0.3861	0.3872	0.3170	0.2038
<i>AME</i>	0.3158 ^a	-0.04230	-0.1576	0.0443	0.7653 ^a	0.6807 ^a	0.8826 ^a	-1.0819	-1.0061	0.1204	-0.0036	0.4137	0.9674 ^a
<i>LR Chi²</i>	5.7000 ^b	1.0600	0.6600	0.0700	16.6900 ^a	20.3900 ^a	29.6200 ^a	0.1000	0.1000	0.1400	0.0000	2.3100	31.5600 ^a
Model's goodness of fit and prediction performance measures													
Chi2	1508.75 ^a	1503.03 ^a	1501.64 ^a	1483.50 ^a	1497.98 ^a	1488.17 ^a	1478.02 ^a	1503.58 ^a	1494.30 ^a	1492.68 ^a	1461.82 ^a	1462.74 ^a	1481.95 ^a
Log likelihood	-2716.0946	-2717.0580	-2712.8445	-2697.4317	-2705.2087	-2707.2534	-2694.5402	-2716.9205	-2706.4733	-2689.3026	-2591.0134	-2595.1436	-2693.6768
R^2	0.3249	0.3208	0.3213	0.3198	0.3228	0.3232	0.3311	0.3210	0.3192	0.3192	0.3193	0.3204	0.3275
$N = 0+1$	58,907	58,755	58,648	58,654	58,831	58,851	58,863	58,781	57,914	58,257	57,627	57,388	58,868
$N = 1$	694	694	693	686	693	694	692	694	677	687	660	659	692
20 % Threshold Estimates													
β	0.4900 ^a	-0.0444	-0.1753	0.2059	0.6863 ^a	0.6492 ^a	0.9652 ^a	-0.1209	0.1899	0.2395	-0.5234	0.2915	1.0660 ^a
<i>SE</i>	0.1761	0.2322	0.2583	0.2285	0.2052	0.1981	0.2233	0.4476	0.4156	0.4257	0.4795	0.3682	0.2298
<i>AME</i>	0.3263 ^a	-0.0294	-0.1157	0.1347	0.4544 ^a	0.4291 ^a	0.6295 ^a	-0.0800	0.1250	0.1577	-0.3280	0.1840	0.6960 ^a
<i>LR Chi²</i>	7.6600 ^a	0.0400	0.4700	0.8000	10.6700 ^a	10.3300 ^a	18.7000 ^a	0.0700	0.2100	0.3100	1.2800	0.6000	20.8600 ^a
Model's goodness of fit and prediction performance measures													
Chi2	1196.56 ^a	1193.69 ^a	1189.37 ^a	1179.15 ^a	1189.81 ^a	1183.88 ^a	1166.31 ^a	1194.27 ^a	1184.98 ^a	1182.21 ^a	1146.37 ^a	1135.97 ^a	1173.36 ^a
Log likelihood	-2084.8657	-2087.3259	-2083.2399	-2072.9820	-2082.3212	-2082.4929	-2069.5539	-2087.1803	-2076.3988	-2068.3888	-1985.8420	-1990.4561	-2068.5422
R^2	0.3322	0.3260	0.3260	0.3253	0.3280	0.3277	0.3353	0.3262	0.3253	0.3241	0.3248	0.3231	0.3321
$N = 0+1$	59,099	58,947	58,840	58,843	59,022	59,043	59,055	58,973	58,856	58,447	57,808	57,570	59,060
$N = 1$	502	502	501	497	502	502	500	502	498	497	479	477	500

30 % Threshold Estimates													
β	0.3990 ^c	-0.3343	-0.3030	0.1054	0.6213 ^a	0.7005 ^a	1.0475 ^a	-0.5214	-0.3611	0.0797	-0.4601	-0.1651	1.1884 ^a
<i>SE</i>	0.2089	0.2865	0.3051	0.2681	0.2397	0.2298	0.2677	0.5445	0.5262	0.5092	0.5499	0.4703	0.2689
<i>AME</i>	0.189 ^c	-0.1570	0.1428	0.0497	0.2930 ^a	0.3286 ^a	0.4855 ^a	-0.2452	-0.1676	0.0372	-0.2053	-0.0754	0.5512 ^a
<i>LR Chi</i> ²	3.6200 ^c	1.4300	1.0200	0.1500	6.4300 ^b	8.9300 ^a	15.4300 ^a	0.9600	0.4900	0.0200	0.7500	0.1300	18.9900 ^a
Model's goodness of fit and prediction performance measures													
Chi2	901.62 ^a	897.95 ^a	898.80 ^a	896.52 ^a	899.25 ^a	891.50 ^a	878.39 ^a	899.22 ^a	888.47 ^a	888.66 ^a	866.12 ^a	870.76 ^a	883.40 ^a
Log likelihood	-1517.1064	-1517.2659	-1516.1788	-1509.3053	-1515.0614	-1513.8117	-1506.6564	-1517.3928	-1509.0097	-1505.0721	-1454.9139	-1452.9582	-1504.9120
R ²	0.3398	0.3340	0.3347	0.3354	0.3360	0.3357	0.3450	0.3342	0.3325	0.3318	0.3313	0.3313	0.3413
N = 0+1	59,253	59,101	58,993	58,994	59,176	59,197	59,208	59,127	59,009	58,599	57,952	57,714	59,213
N = 1	348	348	348	346	348	348	347	348	345	345	335	333	347

Notes: a (b) [c] significant at the 1% (5%) [10%] level (two-sided test). This table reports multivariate regression estimates obtained by supplementing models estimated at different threshold levels of conditions 1, 2 and 3 of our proposed financial distress definition (see section 6 for details) with SIGMA and ES measures. Estimates of the risk measures as listed in columns (2) to (14) are provided in rows (4) to (7). Column (2) reports regression estimates with SIGMA being the risk measure. Columns (3) to (8) report respective multivariate regression estimates supplementing ES as the risk measure (three-month, six-month, one-year, three-year and five-year daily returns and sixty-month monthly returns) at the 95% confidence interval and columns (9) to (14) report the same at 99% confidence interval. The chi-square for likelihood ratio test (*LR Chi*²) reported in row (7) of column (1) tests the existence of any significant difference between the log likelihood of the baseline model supplemented with risk variable, and log likelihood of baseline model. The last 5 rows of respective panels provide goodness of fit measures. Chi-square (Chi2), log likelihood and the coefficient of determination (McKelvey & Zavoina's R²) of respective multivariate model with respective risk variables are reported in first three of last five rows. All regression estimates are reported for $t - 1$ lagged covariates. To save space, we do not report the baseline covariates of Table 9 in this table. N = 0 + 1 represent the total number of firm-year observations for the two groups of firms between 1990 and 2016. N = 1 is the number of firm-year observations for financially distressed group of firms.