Foreign Direct Investment with Tax Holidays and Policy Uncertainty*

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Abstract

We study foreign direct investment agreements that entitle firms to a lower tax rate during a tax holiday period. Our model considers both finite and uncertain tax holiday period settings. We show that the tax holiday duration may have, for small tax rate reductions, a non-monotonic effect on the investment timing. For sufficiently high tax reductions, a longer tax holiday speeds up investment. A higher tax reduction during the tax holiday and a lower uncertainty are shown to have a monotonic effect on the threshold, hastening investment. However, in case of a finite tax holiday, for exceptional high salvage values, a higher uncertainty can speed up investment. We show the usefulness of our model to design an optimal incentives package that prompts investment.

Keywords: FDI, Uncertainty, Real Options, Tax Holidays, Taxation Policy.

JEL codes: F21, G31, H25.

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1 Introduction

Governments use corporate tax incentives to enhance foreign direct investment (FDI). The offer to a foreign firm of a more attractive tax rate is often enough to make an investment profitable, or the relocation of a business to another country optimal. For instance, amongst the EU countries, Ireland is well known for its aggressive corporate tax policy, which attracts FDI.

A FDI agreement can be seen as a contract between a country and a foreign firm through which, over a given time period, the two parties are entitled to a set of financial benefits and obligations. The benefits for the firm are usually given through subsidies, guarantees or lower tax rates, whereas the obligations are normally required through the promotion of new jobs, investment in human capital, establishment of business partnerships with local firms, or, as we will consider, the commitment to remain in the country, not divesting during a given time period.

We develop a real options model which determines the optimal time to undertake a FDI when there is a tax holiday period over which the firm agrees not to divest. This means that, after investing, instead of the (usual) divestment option, the firm holds a forward start option to abandon the investment, which can only be exercised after the expiration date of the FDI agreement. By considering this constrain on the divestment option, we depart from the previous literature. We believe that this is a realistic setting, since it is not plausible that a country offers a tax holiday to a foreign firm without any constrain.

Our model considers two different settings: a finite and a random duration of the tax holiday period. In the former case the firm is offered a tax reduction lasting for a certain period of time, whereas in the latter the tax reduction is offered as permanent, but is perceived as reversible by the firm, as a result of a tax policy change.

Typically, FDI agreements hold during relatively long time periods, over which investments can face very adverse economic conditions, where a change in the tax rate, a size contraction, or an early abandonment may have to be considered.[1] Although the abandonment of FDI projects before the agreed termination date is not very frequent, it often happens, due to political disputes between countries, or the bankruptcy or financial distress of the parent firm.

The main results of this paper can be summarized as follows. For both the finite and random cases, the tax holiday duration may have, for small tax rate reductions, a non-monotonic effect on the investment timing. This is because of the trade-off between the gains from the tax reduction and the loss in divesting flexibility during the tax holiday period. For a sufficiently high tax reduction, a longer tax holiday hastens investment. However, despite of this trade-off, we show that a higher tax reduction has a monotonic effect on the threshold, speeding up investment. We also show that for most cases, the effect of uncertainty is to deter investment. However, in case of a finite tax holiday, a higher uncertainty can speed up investment for exceptional high salvage values.

The effect of taxation policy on investment decisions under uncertainty has been a relevant research topic in accounting and finance. Most of the available theoretical results are based on model settings where market uncertainty is taken into account, the investment cost is irreversible and fixed, the tax and the fiscal depreciation rates are both known (e.g. MacKie-Mason, 1990; Pennings, 2000; Agliardi, 2001; Sureth, 2002; Niemann and Sureth, 2004; Yu et al., 2007; Wong, 2009; Gries et al., 2012; Niemann and Sureth, 2013; Barbosa et al., 2016; Tian, 2018). Nevertheless, these models make the assumption that there is not taxation policy uncertainty.

[1]For instance, the EU countries that were bailed out after the 2008-09 financial crisis were advised to renegotiate some FDI agreements. Specifically, Ireland was pressed by France and Germany, during the negotiations of the 2010 bailout, to rise its very competitive corporate tax rate in return for an aid bailout package, and Portugal renegotiated some Public-Private Partnerships (PPP) after the 2011 bailout, in order to balance its public budget deficit (Burger et al., 2009; Sarmento and Renneboog, 2017).

The theoretical literature which considers both market and taxation policy uncertainty is still very limited. The works of Hassett and Metcalf (1999) and Niemann (2004) are amongst the few exceptions. Specifically, Hassett and Metcalf (1999) study the effect of taxation policy uncertainty (i.e., changes in the investment tax credits can occur in the near future due to a random discrete jump) and show that the gains from delaying the investment is negatively affected by the likelihood of an unfavorable tax switch. Niemann (2004) investigates the effect of the tax rate uncertainty on the timing of the investment and conclude that a rise of the tax rate uncertainty has an inconclusive effect on the timing of the investment.

Alvarez et al. (1998) consider taxation policy uncertainty, but neglects market uncertainty. He examines the effect of the timing and the nature of a corporate tax reform uncertainty on investment decisions, and show that the expectation of a reduction in the corporate tax rate enhances investment, whereas the expectation of a contraction in the tax base (i.e., the fiscal depreciation rate) deters investment.

Very few works study the effect on investment decisions of both market uncertainty and taxation policy together with the divestment option. The few exceptions are Agliardi (2001); Wong (2009); Niemann and Sureth (2013). Agliardi (2001) studies the effect on the timing of the investment of the taxation policy and the uncertainty about both the operating income and the replacement value of the firm's capital, and concludes that fiscal policies can have an ambiguous effect on investment timing. Wong (2009) considers progressive taxation, and concludes that the threshold to abandon the investment decreases with the tax exemption threshold and increases with the tax rate. Niemann and Sureth (2013) investigate the effect of the capital gains tax rate on the entry and exit timing of depreciable investment projects. Their results show that a higher tax rate does not necessarily delay investment if salvage values are relatively high.

Our model departs from these models by considering the divestment flexibility under a tax policy which includes tax holidays incentives. Jou (2000) model has some similarities with ours in the sense that both study the optimal investment timing considering market uncertainty and tax holidays. However, instead of the divestment option constrained by the FDI the agreement, he only considers a temporary unconstrained suspension. In addition, he considers a full tax exemption and neglects tax policy uncertainty, whereas our model accounts for different levels of tax reductions as well as policy uncertainty.

The rest of the paper is organized as follows. Section 2 presents the base model for the finite tax holiday incentive which constrains the divestment flexibility. Section 3 extends the model to the case where there is taxation policy uncertainty. Section 4 concludes.

2 The model

Let us suppose that a country and a foreign firm make an agreement regarding a FDI project, according to which if the firm undertakes the project it will be entitled to a more favorable tax rate over a given time period (T), during which it cannot divest. Before investing, the firm holds the option to invest, whose value can be determined following standard real option backward induction procedures. Thus, we start by the derivation of the value function for the period when the firm is active, and proceed then backwards in order to derive the value function for the period when the firm is inactive.

2.1 The active firm

Let us assume that an all-equity firm is active with a FDI project that generates a pre-tax profit flow $x(t)$ which fluctuates over time according to the following geometric Brownian motion (gBm) process:[2]

$$dx(t) = \alpha x(t)dt + \sigma x(t)dw(t), \qquad x(0) = x \tag{1}$$

where $\alpha < r$, σ, and dw are, respectively, the drift under the risk-neutral measure, the volatility, and the increment of a Wiener process, and r is the constant risk-free interest

[2] In FDI the currency exchange rate is a factor to be considered. However, for the sake of simplicity, we assume that $x(t)$ is the profit flow in the currency of the investor and it incorporates both the profit flow in the foreign currency and the currency exchange rate.

rate.

In addition, assume that τ_h and τ_c are the profit tax rates which hold, respectively, over and after a tax holiday period (T), with $0 \leq \tau_h < \tau_c$. Therefore, the after-tax profit flow over the tax holiday period is $x(1-\tau_h)$, whereas the after-tax profit flow for after the tax holiday period is $x(1-\tau_c)$.

Thus, the firm's value is given by:

$$V(x, \tau_h, \tau_c, T) = \mathbb{E}_0 \left[\int_0^T x(t)(1-\tau_h)e^{-rt}dt + \int_T^\infty x(t)(1-\tau_c)e^{-rt}dt \right] \quad (2)$$

whose solution is:

$$V(x, \tau_h, \tau_c, T) = \frac{x}{r-\alpha}\left(1 - \tau(\tau_h, \tau_c, T)\right) \quad (3)$$

with

$$\tau\left(\tau_h, \tau_c, T\right) = \tau_h + (\tau_c - \tau_h)e^{-(r-\alpha)T} \quad (4)$$

where T is the tax holiday period, and $\tau\left(\tau_h, \tau_c, T\right)$ is a time-weighted average tax rate.[3]

Notice that the FDI contract entitles the firm to a tax holiday benefit $(\tau_c - \tau_h)$, over a given time (tax holiday) period T. In exchange the firm contractually accepts not to abandon the investment during that period. The abandonment option has value for the firm because it provides management flexibility if in the future market conditions deteriorate significantly. Following standard procedures, the value of the option to abandon $(A(x, \tau_c))$ solves the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 A(x, \tau_c)}{\partial x^2} + \alpha x \frac{\partial A(x, \tau_c)}{\partial x} - rA(x, \tau_c) = 0 \quad (5)$$

[3] Note that for $\tau\left(\tau_h, \tau_c, 0\right) = \tau_c$, the firm does not benefit from a more favourable tax rate, whereas for $\tau\left(\tau_h, \tau_c, \infty\right) = \tau_h$ the firm benefits from a more favourable tax rate perpetually. If during a given finite time period (T) the firm benefits from a full tax exemption, the average tax rate over a perpetual time period is $\tau\left(0, \tau_c, T\right) = \tau_c e^{-(r-\alpha)T}$.

and it is given by (McDonald and Siegel, 1985; Dixit and Pindyck, 1994):

$$A(x,\tau_c) = \begin{cases} S - \dfrac{x(1-\tau_c)}{r-\alpha} & \text{for } x < x_A \\ \left(S - \dfrac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\dfrac{x}{x_A}\right)^{\beta_2} & \text{for } x \geqslant x_A \end{cases} \tag{6}$$

where S is the project's salvage value, and x_A is the optimal abandonment threshold value, given by:

$$x_A = \frac{\beta_2}{\beta_2 - 1}\frac{S(r-\alpha)}{1-\tau_c} \tag{7}$$

with β_2 expressed by:

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{8}$$

Notice that the firm cannot abandon the investment while the FDI agreement prevails (i.e. during the tax holiday period, T), but it can do so as soon as it ends. This means that the FDI agreement comprises a forward abandonment option with a starting date on T. Thus, the value of the active firm is given by the sum of the present value of the project's future cash flows plus both the present value of the tax holiday benefits and the value of the forward start abandonment option.

Proposition 1. *The value of an active firm ($F(x,.)$) that is under a FDI agreement is given by:*

$$F(x,\tau_h,\tau_c,T) = V(x,\tau_h,\tau_c,T) + F_A(x,\tau_c,T) \tag{9}$$

where $V(x,.)$ and $F_A(x,.)$ represent, respectively, the firm's value, given by Equation (3)*, and the value of the forward start option to abandon, which is represented by:*

$$\begin{aligned} F_A(x,\tau_c,T) = {} & Se^{-rT}N(-d_2(x)) - \frac{x(1-\tau_c)}{r-\alpha}e^{-(r-\alpha)T}N(-d_1(x)) \\ & + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x}{x_A}\right)^{\beta_2} N(d_3(x)) \end{aligned} \tag{10}$$

where S is the project's salvage value, x_A is the optimal abandonment threshold value, represented by Equation (7), β_2 is given by Equation (8), $N(.)$ is the cumulative normal integral, and

$$d_1(x) = \frac{\ln\left(\frac{x}{x_A}\right) + \left(\alpha + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{11}$$

$$d_2(x) = d_1(x) - \sigma\sqrt{T} \tag{12}$$

$$d_3(x) = d_1(x) + (\beta_2 - 1)\,\sigma\sqrt{T} \tag{13}$$

The economic interpretation for Equation (10) is as follows: on the right-hand side, the first two terms represent the value of the abandonment option at time T, conditional on the threshold value x_A being reached; the third term represents the value of the abandonment option after T if $x(T) > x_A$ (i.e. if when T is reached the value of the profit cash flows is above the abandonment threshold value). The following corollaries also hold:

Corollary 1. *When $T \to 0$, firm's value converges to that of a firm with the abandonment option and that pays a profit tax rate τ_c forever:*

$$\lim_{T\to 0} F(x, \tau_h, \tau_c, T) = \frac{x(1-\tau_c)}{r-\alpha} + A(x, \tau_c) \tag{14}$$

Corollary 2. *When $T \to \infty$, the firm's value converges to that of a firm without the abandonment option that pays a profit tax rate τ_h forever:*

$$\lim_{T\to \infty} F(x, \tau_h, \tau_c, T) = \frac{x(1-\tau_h)}{r-\alpha} \tag{15}$$

2.2 The idle firm

Let us now assume that the firm is currently inactive, waiting for the optimal time to invest. Following a standard real options framework, the value if the idle firm $(O(x,.))$

solves the following ODE:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 O(x,.)}{\partial x^2} + \alpha x \frac{\partial O(x,.)}{\partial x} - rO(x,.) = 0 \tag{16}$$

Using the appropriate boundary conditions the following proposition holds.

Proposition 2. *The value of an idle firm ($x < x_I$) with the above described FDI agreement is given by:*

$$O(x, \tau_h, \tau_c, T) = \Big(V(x_I, \tau_h, \tau_c, T) + F_A(x_I, \tau_c, T) - I \Big) \left(\frac{x}{x_I} \right)^{\beta_1} \tag{17}$$

where I is the investment cost, and x_I is the optimal investment threshold value, which can be determined numerically by solving the following equation:

$$\begin{aligned} &(\beta_1 - \beta_2) \left(S - \frac{x_A(1-\tau_c)}{r-\alpha} \right) \left(\frac{x_I}{x_A} \right)^{\beta_2} N(d_3(x)) \\ &+ (\beta_1 - 1) \left[V(x_I, \tau_h, \tau_c, T) - \frac{x_I(1-\tau_c)}{r-\alpha} e^{-(r-\alpha)T} N(-d_1(x)) \right] \\ &- \beta_1 \left[I - Se^{-rT} N(-d_2(x)) \right] = 0 \end{aligned} \tag{18}$$

where β_1 is given by:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{19}$$

For the limiting cases of the tax holiday period (T), the following corollaries hold:

Corollary 3. *When $T \to 0$, the value of the idle firm is given by:*

$$\lim_{T \to 0} O(x, \tau_h, \tau_c, T) = \left(\frac{x_I^*(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha} \right) \left(\frac{x_I^*}{x_A} \right)^{\beta_2} - I \right) \left(\frac{x}{x_I^*} \right)^{\beta_1} \tag{20}$$

where x_I^* is the optimal investment threshold value, which is a solution of the following *equation:*

$$(\beta_1 - \beta_2)\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_I^*}{x_A}\right)^{\beta_2} + (\beta_1 - 1)\frac{x_I^*(1-\tau_c)}{r-\alpha} - \beta_1 I = 0 \tag{21}$$

Corollary 4. *When $T \to \infty$, the value of the idle firm is given by:*

$$\lim_{T\to\infty} O(x,\tau_h,\tau_c,T) = \left(\frac{x_I^{**}(1-\tau_h)}{r-\alpha} - I\right)\left(\frac{x}{x_I^{**}}\right)^{\beta_1} \tag{22}$$

where x_I^{**} is the optimal investment threshold value:

$$x_I^{**} = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{(1-\tau_h)} I \tag{23}$$

In addition, the following corollaries also hold:

Corollary 5. *The effect on the optimal investment threshold value (x_I) of the tax holiday period (T) is non-monotonic: $\partial x_I/\partial T \gtrless 0$.*

Corollary 6. *The optimal investment threshold value (x_I) increases with the tax holiday rate (τ_h): $\partial x_I/\partial \tau_h > 0$.*

Corollary 7. *The effect on the optimal investment threshold value (x_I) of the market uncertainty (σ) is non-monotonic: $\partial x_I/\partial \sigma \gtrless 0$.*

Corollary 5 is of particular importance since it asserts that there are cases where widening the tax holiday period does not decrease the investment threshold value, speeding up the investment, as we would expect.

Figure 1 illustrates more clearly this important finding: for a relatively low tax holiday benefit (i.e. as τ_h approaches τ_c), widening T does not necessarily accelerates investment. On the contrary, it can delay investment if T is sufficiently high. However, this behavior is because, if the tax holiday period rises, it reduces the tax payment, which enhances investment, but it also constrains the firm's abandonment option for a longer time period, which precludes investment. These two forces have counteracting effects on the investment threshold, and the effect that prevails depends on the length of the tax holiday period. Figure 1 also shows that, when τ_h is relatively close to τ_c (i.e. there is a relatively low tax holiday gain), increasing T from zero accelerates investment but only up until a given T is reached, after which, if T increases, it delays investment.

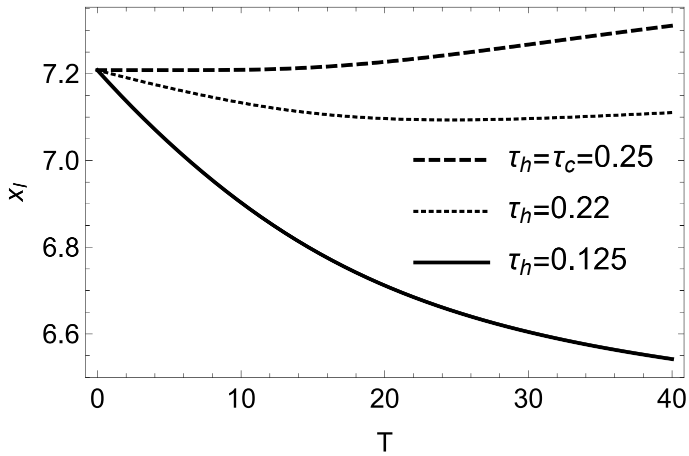

Figure 1: The effect of the tax holiday on the investment threshold (x_I). The model parameters are $\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, and $\tau_c = 0.25$.

Figure 2 shows the effect of uncertainty on the investment threshold value, for different salvage values. As expected, a higher salvage value reduces the threshold, hastening the investment. For salvage values below the investment cost (partial reversibility), the investment threshold increases with uncertainty (σ). Nevertheless, for the exceptional case where the salvage value is higher than the investment cost, increasing σ from zero, delays investment but only up until a given σ is reached, after which, if σ increases, it accelerates investment, illustrating the non-monotonic effect shown in Corollary 7. Thus, the relative value of the salvage value can determine to some extent the effect of the uncertainty on the timing of the investment, in particular if the uncertainty is relatively high. Notice that, although it is not very common to find projects with salvage values higher that the investment costs, there are investments where this can happen, for instance in real estate investments, or other investments which comprise assets which are prone to appreciate significantly. We note that divestment is only possible after the tax holiday period, which makes this situation more plausible.

This result is of some relevance, in particular to those firms operating in industries with both high future fixed assets value and high market uncertainty. For instance, a FDI project which involves the construction of a manufacturing plant that is outsourced by the IT industry inherits both the high uncertainty of the IT sector and the high fixed assets of the manufacturing industry.

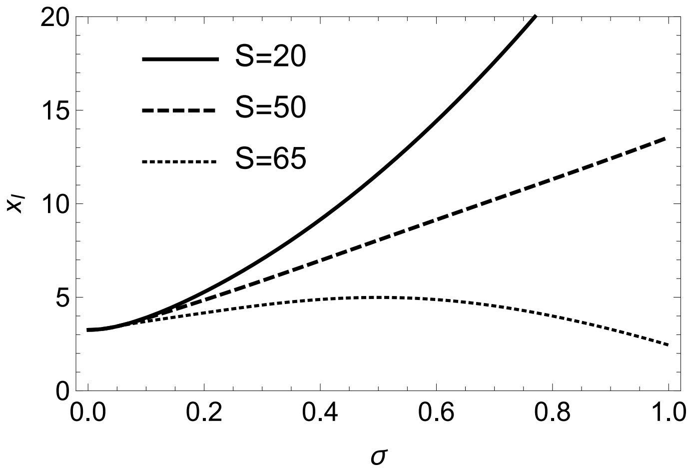

Figure 2: The effect of uncertainty (σ) on the optimal investment threshold (x_I) for different values of S. The model parameters are: $r = 0.05$, $\alpha = 0.02$, $I = 50$, $\tau_h = 0.125$, and $\tau_c = 0.25$.

Figure 3 shows iso-treshold curves for three levels of uncertainty, that represent different (T,τ_h) agreements which trigger investment. A point above the iso-threshold curves represent scenarios where the country offers the firm an unnecessary generous tax holiday incentive, whereas a point below the iso-threshold curves represent scenarios where tax holiday incentive that is offered to the firm is not sufficient to trigger investment.

3 Taxation policy uncertainty

In the previous section, we assume that a firm and a country make a FDI agreement according to which, if the investment is undertaken, the firm will pay a more favorable a tax rate (τ_h) over a given time period (T), during which it cannot abandon investment. Additionally, we assume that τ_h does not change over time. In this section, we consider taxation policy uncertainty. Specifically we consider the case where the Government offers

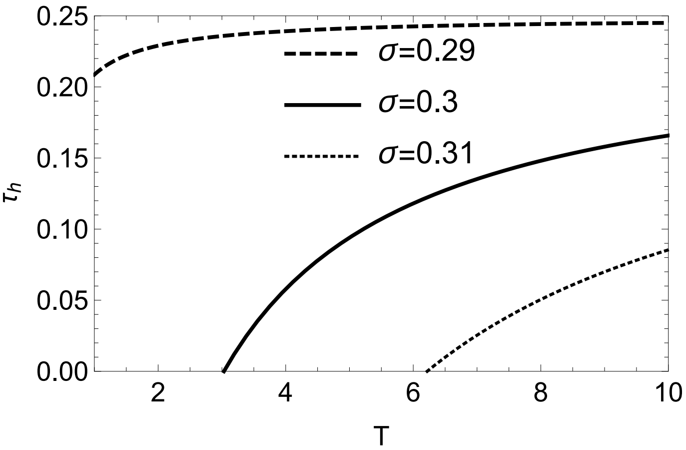

Figure 3: Iso-threshold curves which represent scenarios where investing is optimal. In this simulation we set $x(0) = 7$, with the following model parameters: $\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$.

to a firm a permanent tax rate reduction incentive. Nevertheless, it makes sense from the firm's perspective to assume that this tax reduction may not be permanent, as it can be reversed as a result an unexpected policy change. This tax policy uncertainty can be modeled as a random event whose arrival date follows a Poisson jump process with a rate λ. As before, in exchange for the tax holiday, the firm accepts not to divest.

The case of Ireland is perhaps a good illustrative example of the application of this model setting. Currently, Ireland offers a much more attractive corporate tax rate to some firms (for instance Apple). But, as the recent 2008-09 financial crisis has shown, from the firm's perspective, when evaluating a FDI project that requires a long-term non-abandonment commitment, it may make sense to assume that the taxation policy that is offered today may not hold all over the life-time of the investment.

We start by the derivation of the value function for when the firm is active, and proceed then backwards in order to derive the value function for when the firm is inactive.

3.1 The active firm

Under this setting the active firm benefiting from a reduce tax rate τ_h faces the risk of a sudden tax policy change, where the tax is reversed to the normal tax rate τ_c. This event is modeled to arrive according to a Poisson jump with intensity λ. The value of an active firm ($V_R(x,.)$) must satisfy the following ODE:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_R(x,.)}{\partial x^2} + \alpha x \frac{\partial V_R(x,.)}{\partial x} - rV_R(x,.) + x(1-\tau_h) + \lambda\left[G(x,\tau_c) - V_R(x,.)\right] = 0 \quad (24)$$

where $G(x,\tau_c)$ is the firm's value after a rise of the tax rate from τ_h to τ_c.

$$G(x,\tau_c) = \frac{x(1-\tau_c)}{r-\alpha} + A(x,\tau_c) \quad (25)$$

The last term of the left-hand side of the equation represents the expected value loss due to the possibility of a rise of the tax rate in the next instant.

Proposition 3. *The value of an active firm paying currently a tax rate τ_h which can increase to τ_c at a random future date is given by:*

$$V_R(x,\tau_h,\tau_c,\lambda) = \frac{x(1-\tau_h)}{r-\alpha+\lambda} + \begin{cases} b_1 x^{\eta_1} + \frac{\lambda}{r+\lambda} S & \text{for } x < x_A \\ b_4 x^{\eta_2} + \frac{\lambda}{r-\alpha+\lambda}\frac{x(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x}{x_A}\right)^{\beta_2} & \text{for } x \geqslant x_A \end{cases} \quad (26)$$

where x_A is the optimal abandonment threshold value, provided by Equation (7)*, with:*

$$b_1 = \frac{S}{\eta_1-\eta_2}\left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{\beta_2}{\beta_2-1}(\eta_2-1) - \frac{r}{r+\lambda}\eta_2\right)\left(\frac{1}{x_A}\right)^{\eta_1} \quad (27)$$

$$b_4 = \frac{S}{\eta_1-\eta_2}\left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{\beta_2}{\beta_2-1}(\eta_1-1) - \frac{r}{r+\lambda}\eta_1\right)\left(\frac{1}{x_A}\right)^{\eta_2} \quad (28)$$

$$\eta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} > 1 \quad (29)$$

$$\eta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} < 0 \quad (30)$$

For the limiting cases of λ, the following corollaries hold:

Corollary 8. *When $\lambda \to \infty$ (i.e. a change in the tax rate is certain) the firm's value converges to that of a firm which holds an abandonment option and profits are taxed at*

τ_c:

$$\lim_{\lambda\to\infty} V_R(x,\tau_h,\tau_c,\lambda) = \frac{x(1-\tau_c)}{r-\alpha} + A(x,\tau_c) \tag{31}$$

Corollary 9. *When $\lambda \to 0$ (i.e. a change in the tax rate will not happen) the firm's value converges to that of a firm which does not hold an abandonment option and profits are taxed at τ_h:*

$$\lim_{\lambda\to 0} V_R(x,\tau_h,\tau_c,\lambda) = \frac{x(1-\tau_h)}{r-\alpha} \tag{32}$$

3.2 The idle firm

While waiting to invest, the firm holds an option to undertake the project by paying an investment cost K. The value of this option ($O_R(x,.)$) must satisfy the following ODE:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 O_R(x,.)}{\partial x^2} + \alpha x \frac{\partial O_R(x,.)}{\partial x} - rO_R(x,.) = 0 \tag{33}$$

Using the appropriate boundary conditions, the following proposition holds.

Proposition 4. *The value of the option to invest in a project which benefits from a favorable tax rate τ_h that can reversed to τ_c at a random future date is given by:*

$$O_R(x,\tau_h,\tau_c,\lambda) = (V_R(x_R,\tau_h,\tau_c,\lambda) - I)\left(\frac{x}{x_R}\right)^{\beta_1} \tag{34}$$

where x_R is the investment threshold value, which is the solution of the following equation:

$$(\beta_1-\eta_2)b_4 x_R^{\eta_2} + (\beta_1 - 1)\left(\frac{x_R(1-\tau_h)}{r-\alpha+\lambda} + \frac{\lambda}{r-\alpha+\lambda}\frac{x_R(1-\tau_c)}{r-\alpha}\right) - \beta_1 I = 0 \tag{35}$$

For the limiting cases of λ, the following corollaries hold:

Corollary 10. *When $\lambda \to \infty$ (i.e. a change in the tax rate is certain), the value of the option to invest converges to:*

$$\lim_{\lambda\to\infty} O_R(x,\tau_h,\tau_c,\lambda) = \left(\frac{x_R^*(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_R^*}{x_A}\right)^{\beta_2} - I\right)\left(\frac{x}{x_R^*}\right)^{\beta_1} \tag{36}$$

where x_R^* *is the optimal investment threshold, which is a solution for the following equation:*

$$(\beta_1 - \beta_2)\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_R^*}{x_A}\right)^{\beta_2} + (\beta_1 - 1)\frac{x_R^*(1-\tau_c)}{r-\alpha} - \beta_1 I = 0 \qquad (37)$$

Corollary 11. *When* $\lambda \to 0$ *(i.e. a change in the tax rate will not happen), the value of the option to invest converges to:*

$$\lim_{\lambda \to 0} O_R(x, \tau_h, \tau_c, \lambda) = \left(\frac{x_R^{**}(1-\tau_h)}{r-\alpha} - I\right)\left(\frac{x}{x_R^{**}}\right)^{\beta_1} \qquad (38)$$

where x_R^{**} *is the optimal investment threshold value to invest:*

$$x_R^{**} = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{1-\tau_h}I \qquad (39)$$

Below are some corollaries that summarize our findings regarding the effect of the market uncertainty and the taxation policy uncertainty on the timing of the investment.

Corollary 12. *The effect of the taxation policy uncertainty* (λ) *on the optimal investment threshold* (x_R) *is non-monotonic:* $\partial x_R/\partial\lambda \gtreqless 0$.

Corollary 13. *The investment threshold* (x_R) *increases with the tax holiday rate* (τ_h)*:* $\partial x_R/\partial\tau_h > 0$.

Corollary 14. *The investment threshold* (x_R) *increases with the uncertainty* (σ)*:* $\partial x_R/\partial\sigma > 0$.

Corollary 12 is important, since it asserts that there are market conditions in which a rise of the taxation policy uncertainty does not discourage investment. This is because, for a relatively low tax holiday benefit (i.e. when τ_h is relatively close to τ_c) the losses from a rise of the tax rate are more limited. Thus, if the likelihood of a rise of tax rate increases, the value loss, due to the increase in the expected tax payment, can more easily be offset by the value gain from the elimination of the non-abandonment option constrain. These two forces have counteracting effects on the investment threshold. The force which prevails over time depends on the terms of the FDI agreement and the market conditions.

Figure 4 illustrates more clearly how the above described forces interact with the investment threshold. For a relatively high tax holiday benefits (i.e. when τ_h is significantly lower than τ_c), the investment threshold increases with λ. However, if the tax holiday benefit is very small (i.e. τ_h is very close to τ_c), a rise of the taxation policy uncertainty leads to a decreases of the investment threshold. For intermediate tax holiday benefits, the relationship between λ and x_R tends to be non-monotonic, being the sensitivity of x_R to changes in λ more acute when the taxation policy uncertainty is low and the tax holiday benefit is small.

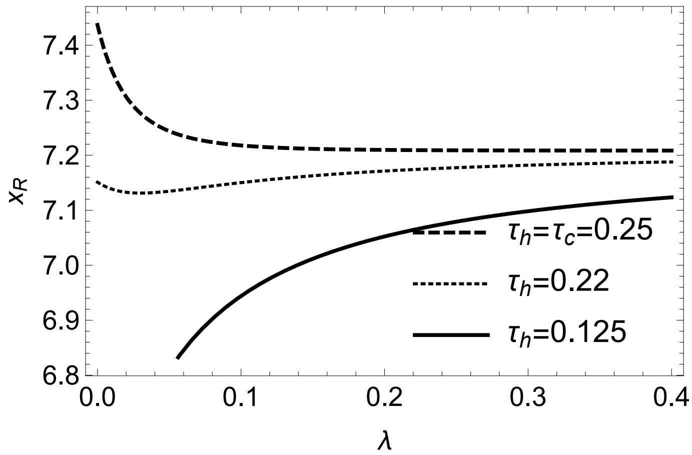

Figure 4: The effect of the taxation policy uncertainty (λ) on the optimal investment threshold (x_R), for different tax holiday rates (τ_h). The model parameters are: $\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$.

Figure 5 shows iso-treshold curves that represent pairs of λ and τ_h, which trigger investment, for different levels of σ. Points above or below these curves represent, respectively, scenarios where the investment incentive is unnecessary generous or not sufficient to trigger investment. For relatively low value of λ and relatively high values σ a rise of λ does not necessarily delays investment.

4 Conclusion

This paper studies optimal FDI tax holiday incentive packages considering market and taxation policy uncertainty. In exchange for the tax benefit, the firm agrees not to divest

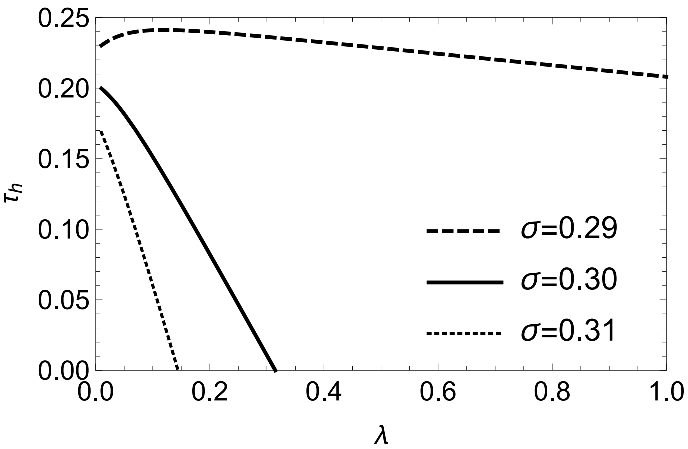

Figure 5: Iso-threshold curves, for different values of uncertainty (σ): (λ, τ_h) pair values that trigger immediate investment. We use $x(0) = 7$ and the following model parameters: $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$.

during the tax holiday period. We derive a real options investment model considering two different settings. One where the tax holiday period is finite and certain, and another where a firm is offered a permanent tax reduction, which is perceived as reversible, due to a tax policy change, resulting in a random tax holiday duration.

We show that, for both the finite and random cases, the tax holiday duration may have, for small tax rate reductions, a non-monotonic effect on the investment timing. In fact, the benefit of a longer tax holiday period may not compensate to forgo the flexibility of divesting when market conditions deteriorate. For sufficient tax reductions the expected effect holds, i.e., a longer tax holiday speeds up investment.

However, despite of this trade-off between the lost flexibility and the tax benefit, a higher tax reduction during the tax holiday is shown to have a monotonic effect on the threshold, hastening investment.

Finally, for the common reversibility situations, the effect of uncertainty on the investment threshold is in line with the previous literature, where a higher uncertainty deters investment. However, in case of a finite tax holiday, we show that for exceptional high salvage values a higher uncertainty can speed up investment.

Our model can be extended in several ways, for instance by considering competition amongst firms and/or amongst FDI host countries, possibly, relying on Smets (1993) framework. The incorporation in our model of assets depreciation and/or a more diverse set of taxation policies which could include tax exemptions, tax credits, or progressive taxation, would also be a interesting research. Our model can also be easily adapted to determine a fair reimbursement amount that is due to the foreign firm, or the FDI host country, when there is a breach of the FDI agreement. Finally, the innovative features of our model lead to interesting results which can be empirical tested in future research.

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Proof of Proposition 1. The value of the forward start option to abandon ($F_A(.)$) is represented by:

$$F_A(x, \tau_c, T) = e^{-rT} E\left[A(x(T), \tau_c)\right] \tag{40}$$

where $A(x, \tau_c)$ is the value of the abandonment option at time T, given by equation (6). In addition, $A(x, \tau_c)$ has two regions, thus:

$$F_A(x, \tau_c, T) = e^{-rT} E\left[\left(S - \frac{x(T)(1-\tau_c)}{r-\alpha}\right) \mathbf{1}_{x(T)<x_A}\right] \tag{41}$$

$$+ e^{-rT} E\left[\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right) \left(\frac{x(T)}{x_A}\right)^{\beta_2} \mathbf{1}_{x(T)\geqslant x_A}\right] \tag{42}$$

where $\mathbf{1}_{\text{condition}}$ is equal to 1 if the condition is met, and is equal to 0 otherwise.

From Shackleton and Wojakowski (2007) we acknowledge that the first term of $F_A(x, \tau_c, T)$ represents the difference between a *cash-or-nothing* put option on S, and an *asset-or-nothing* put option on $x(T)(1-\tau_c)/(r-\alpha)$, both with a maturity T and exercise price x_A:

$$e^{-rT} E\left[A(x(T), \tau_c)\right] \mathbf{1}_{x(T)<x_A} = S e^{-rT} N\left(-d_2(x, T)\right) - \frac{x(1-\tau_c)}{r-\alpha} e^{-(r-\alpha)T} N\left(-d_1(x, T)\right) \tag{43}$$

where:

$$d_1(x, T) = \frac{\ln\left(\frac{x}{x_A}\right) + \left(\alpha + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \tag{44}$$

$$d_2(x, T) = d_1(x, T) - \sigma\sqrt{T} \tag{45}$$

The second term of $F_A(x, \tau_c, T)$ is:

$$e^{-rT} E\left[A(x(T), \tau_c)\right] \mathbf{1}_{x(T)\geqslant x_A} = \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right) \left(\frac{x}{x_A}\right)^{\beta_2} N\left(d_3(x, T)\right) \tag{46}$$

where:

$$d_3(x,T) = \frac{\ln\left(\frac{x}{x_A}\right) + \left(\alpha + \left(\beta_1 - \frac{1}{2}\right)\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$= d_1(x,T) + (\beta_1 - 1)\,\sigma\sqrt{T} \tag{47}$$

□

Proof of Proposition 2. The value function for an idle firm $(O(x,.))$ must satisfy the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 O(x,.)}{\partial x^2} + \alpha x \frac{\partial O(x,.)}{\partial x} - rO(x,.) = 0 \tag{48}$$

whose general solution is given by $O(x,.) = c_1 x^{\beta_1} + c_2 x^{\beta_2}$. In addition, $\lim_{x\to 0} O(x,.) = 0$, thus c_2 must be set equal to 0. Using the following value-matching and smooth-pasting boundary conditions we determine obtain c_1 and x_I:

$$c_1 x_I^{\beta_1} = F(x_I,.) - I \tag{49}$$
$$\beta_1 c_1 x_I^{\beta_1 - 1} = \frac{\partial F(x,.)}{\partial x}|_{x=x_I} \tag{50}$$

Substituting in the equation system above $F(x_I,.)$ by Equation (9) we obtain:

$$\begin{aligned}\beta_1 c_1 x_I^{\beta_1} =& \frac{x}{r-\alpha}\left(1 - \tau(\tau_h,\tau_c,T)\right) + x\frac{\partial}{\partial x}\left[Se^{-rT}N(-d_2(x))\right] \\ & - x\frac{\partial}{\partial x}\left[\frac{x(1-\tau_c)}{r-\alpha}e^{-(r-\alpha)T}N(-d_1(x))\right] \\ & + x\frac{\partial}{\partial x}\left[\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x}{x_A}\right)^{\beta_2} N(d_3(x))\right]\end{aligned} \tag{51}$$

The solutions for the above derivatives are provided by Shackleton and Wojakowski

(2007, section 4). Substituting $c_1 x_{CA}^{\beta_1}$ by Equation (51) we obtain:

$$(\beta_1 - \beta_2)\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_I^*}{x_A}\right)^{\beta_2} + (\beta_1 - 1)\frac{x_I^*(1-\tau_c)}{r-\alpha} - \beta_1 I = 0 \tag{52}$$

where x_I is the numerical solution of this equation. The constant c_1 is determined through Equation (51) and is given by $c_1 = (F(x_I, .) - I)\, x_I^{-\beta_1}$. □

Proof of Proposition 3. The value function of an active firm ($V_R(x,.)$) must satisfy the following non-homogeneous ODE:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_R(x,.)}{\partial x^2} + \alpha x \frac{\partial V_R(x,.)}{\partial x} - rV_R(x,.) + x(1-\tau_h) + \lambda\left[G(x,\tau_c) - V_R(x,.)\right] = 0 \tag{53}$$

where $G(x,\tau_c)$ is the firm's value after a rise of the tax rate from τ_h to τ_c.

$$G(x,\tau_c) = \frac{x(1-\tau_c)}{r-\alpha} + A(x,\tau_c) \tag{54}$$

The last term of the left-hand side of the equation represents the expected value loss due to the possibility of a rise of the tax rate in the next instant. The solution to this ODE corresponds to the sum of the homogeneous solution for each region:[4]

$$V_R(x,.) = \begin{cases} b_1 x^{\eta_1} + b_2 x^{\eta_2} + \dfrac{x(1-\tau_h)}{r-\alpha+\lambda} + \dfrac{\lambda}{r+\lambda} S & \text{for } x < x_A \\[2ex] b_3 x^{\eta_1} + b_4 x^{\eta_2} + \dfrac{x(1-\tau_h)}{r-\alpha+\lambda} + \dfrac{\lambda}{r-\alpha+\lambda}\dfrac{x(1-\tau_c)}{r-\alpha} & \\ \qquad + \left(S - \dfrac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\dfrac{x}{x_A}\right)^{\beta_2} & \text{for } x \geqslant x_A \end{cases} \tag{55}$$

[4]Note that the value function $G(x,\tau_c)$ has two regions depending on x and x_A.

where b_1, b_2, b_3 and b_4 are arbitrary constants which remain to be determined, and

$$\eta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} > 1 \tag{56}$$

$$\eta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} < 0 \tag{57}$$

Given that $\lim_{x\to 0} V_R(x,.) = 0$ and $\lim_{x\to +\infty} V_R(x,.) = +\infty$, so the constants b_2 and b_3 must be set equal to zero. The first condition ensures that the active firm is worthless if cash flows drop to zero. For the remaining arbitrary constants we need two additional conditions. However, the two regions of the value function must met at $x = x_A$, therefore, $V_R(x,.)$ is continuous and differentiable along x, from which we obtain:

$$b_1 x_A^{\eta_1} + b_2 x_A^{\eta_2} + \frac{\lambda}{r+\lambda} S = b_3 x_A^{\eta_1} + b_4 x_A^{\eta_2} + \frac{\lambda}{r-\alpha+\lambda} \frac{x_A(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right) \tag{58}$$

$$\begin{aligned} \eta_1 b_1 x_A^{\eta_1 - 1} + \eta_2 b_2 x_A^{\eta_2 - 1} = {} & \eta_1 b_3 x_A^{\eta_1 - 1} + \eta_2 b_4 x_A^{\eta_2 - 1} + \frac{\lambda}{r-\alpha+\lambda} \frac{1-\tau_c}{r-\alpha} \\ & + \beta_2 \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{1}{x_A}\right) \end{aligned} \tag{59}$$

Solving the above equation system we obtain:

$$b_1 = \frac{S}{\eta_1 - \eta_2}\left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{\beta_2}{\beta_2 - 1}(\eta_2 - 1) - \frac{r}{r+\lambda}\eta_2\right)\left(\frac{1}{x_A}\right)^{\eta_1} \tag{60}$$

$$b_4 = \frac{S}{\eta_1 - \eta_2}\left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{\beta_2}{\beta_2 - 1}(\eta_1 - 1) - \frac{r}{r+\lambda}\eta_1\right)\left(\frac{1}{x_A}\right)^{\eta_2} \tag{61}$$

□

Proof of Proposition 4. The value function of the idle firm $(O_R(x,.))$ must satisfy the following ODE:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 O_R(x,.)}{\partial x^2} + \alpha x \frac{\partial O_R(x,.)}{\partial x} - r O_R(x,.) = 0 \tag{62}$$

whose general solution is given by: $O_R(x,.) = e_1 x^{\beta_1} + e_2 x^{\beta_2}$. In addition, $\lim_{x\to 0} O_R(x,.) = 0$, therefore e_2 must be set equal to 0. Using the following value matching and smooth-pasting boundary conditions, we determine x_I and e_1:

$$e_1 x_R^{\beta_1} = V_R(x_I, .) - I \tag{63}$$

$$\beta_1 e_1 x_R^{\beta_1 - 1} = \frac{\partial V_R(x, .)}{\partial x}|_{x = x_R} \tag{64}$$

Although $V_R(x, .)$ has two branches (see equation (26)), we can show that there is not solution for the first branch, therefore, $x_R \geqslant x_A$. Substituting $F_R(x_I, .)$ by the second branch of Equation (26), it yields:

$$(\beta_1 - \eta_2) b_4 x_R^{\eta_2} + (\beta_1 - 1)\left(\frac{x_R(1-\tau_h)}{r - \alpha + \lambda} + \frac{\lambda}{r - \alpha + \lambda}\frac{x_R(1-\tau_c)}{r-\alpha}\right) - \beta_1 I = 0 \tag{65}$$

□

Proof of Corollary 1. If $x < x_A, \lim_{T\to 0} N\left(d_3(x)\right) = 0$, $\lim_{T\to 0} N\left(-d_1(x)\right) = 1$, and $\lim_{T\to 0} N\left(-d_2(x)\right) = 1$, therefore:

$$\lim_{T\to 0} F_A(x, \tau_c, T) = \left(S - \frac{x}{r-\alpha}(1-\tau_c)\right)$$

where this expression corresponds to the lower branch of Equation (6).

If $x \geqslant x_A, \lim_{T\to 0} N\left(d_3(x)\right) = 1$, $\lim_{T\to 0} N\left(-d_1(x)\right) = 0$, and $\lim_{T\to 0} N\left(-d_2(x)\right) = 0$, therefore:

$$\lim_{T\to 0} F_A(x, \tau_c, T) = \left(S - \frac{x_A}{r-\alpha}(1-\tau_c)\right)\left(\frac{x}{x_A}\right)^{\beta_2} \tag{66}$$

where this expression corresponds to the upper branch of Equation (6).

Also, $\lim_{T\to 0} V(x, \tau_h, \tau_c, T) = \dfrac{x(1-\tau_c)}{r-\alpha}$, thus: $\lim_{T\to 0} F(x, \tau_h, \tau_c, T) = \dfrac{x(1-\tau_c)}{r-\alpha} + A(x, \tau_c)$. □

Proof of Corollary 2. Given that $\lim_{T\to\infty} N\left(d_3(x)\right) = 1$, $\lim_{T\to\infty} e^{-(r-\alpha)T} = 0$, and $\lim_{T\to\infty} e^{-rT} = 0$, thus, $\lim_{T\to\infty} F_A(x, \tau_c, T) = 0$, $\lim_{T\to\infty} V(x, \tau_h, \tau_c, T) = \dfrac{x(1-\tau_h)}{r-\alpha}$, and $\lim_{T\to\infty} F(x, \tau_h, \tau_c, T) =$

$\dfrac{x(1-\tau_h)}{r-\alpha}$. □

Proof of Corollary 3. See Proof of Corollary 1. □

Proof of Corollary 4. See Proof of Corollary 2. □

Proof of Corollary 5. Differentiating Equation (18) with respect to T yields:

$$\begin{aligned}
&\Bigg[\beta_2(\beta_1-\beta_2)\left(S-\frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{1}{x_A}\right)^{\beta_2}x_I^{\beta_2-1}N(d_3(x))\\
&+\frac{\beta_1-1}{r-\alpha}\left[1-\tau_h-(\tau_c-\tau_h)e^{-(r-\alpha)T}-(1-\tau_c)e^{-(r-\alpha)T}N(-d_1(x))\right]\Bigg]\frac{\partial x_I}{\partial T}\\
=&-(\beta_1-\beta_2)\left(S-\frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{1}{x_A}\right)^{\beta_2}x_I^{\beta_2}n(d_3(x))\frac{\partial d_3}{\partial T}\\
&-(\beta_1-1)\left[x_I(\tau_c-\tau_h)e^{-(r-\alpha)T}+x_I(1-\tau_c)e^{-(r-\alpha)T}N(-d_1(x))\right]\\
&-(\beta_1-1)\left[\frac{x_I(1-\tau_c)}{r-\alpha}e^{-(r-\alpha)T}n(-d_1(x))\frac{\partial d_1}{\partial T}\right]\\
&+\beta_1\left[Sre^{-rT}N(-d_2(x))+Se^{-rT}n(-d_2(x))\frac{\partial d_2}{\partial T}\right]
\end{aligned}$$

where $N(.)$ is the normal density function.

We can show that the cross derivatives of $N(.)$ (i.e. $\partial d_i/\partial x_I\times\partial x_I/\partial T, i\in 1,2,3$) cancel each other out. Given that $N(-d1(x)\leqslant 1$, $1-\tau_h-(\tau_c-\tau_h)e^{-(r-\alpha)T}-(1-\tau_c)e^{-(r-\alpha)T}N(-d_1(x))\geqslant(1-\tau_h)(1-e^{-(r-\alpha)T)})\geqslant 0$, therefore, the coefficient of $\partial x_I/\partial T$ is positive.

However the the sign of the right-hand side of the equation above is undetermined, thus: $\partial x_I/\partial T\gtreqless 0$. □

Proof of Corollary 6. Differentiating Equation (18) with respect to τ_h, it yields:

$$\begin{aligned}
&\Bigg[\beta_2(\beta_1-\beta_2)\left(S-\frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{1}{x_A}\right)^{\beta_2}x_I^{\beta_2-1}N(d_3(x))\\
&+\frac{\beta_1-1}{r-\alpha}\left[1-\tau_h-(\tau_c-\tau_h)e^{-(r-\alpha)T}-(1-\tau_c)e^{-(r-\alpha)T}N(-d_1(x))\right]\Bigg]\frac{\partial x_I}{\partial\tau_h}\\
=&(\beta_1-1)\frac{x_I}{r-\alpha}\left(1-e^{-(r-\alpha)T}\right)
\end{aligned}$$

In the previous proof we show that the right hand-side of the equation and the coefficient of $\partial x_I/\partial \tau_h$ are both positive, thus: $\partial x_I/\partial \tau_h > 0$. □

Proof of Corollary 7. Differentiating Equation (18) with respect to σ, it yields:

$$
\begin{aligned}
&\Bigg[\beta_2(\beta_1-\beta_2)Ax_I^{\beta_2-1}N(d_3(x))\\
&+\frac{\beta_1-1}{r-\alpha}\left[1-\tau_h-(\tau_c-\tau_h)e^{-(r-\alpha)T}-(1-\tau_c)e^{-(r-\alpha)T}N(-d_1(x))\right]\Bigg]\frac{\partial x_I}{\partial \sigma}\\
=&-(\beta_1-\beta_2)\left[Ax_I^{\beta_2}n(d_3(x))\frac{\partial d_3}{\partial \sigma}+\frac{\partial A}{\partial \sigma}x_I^{\beta_2}N(d_3(x))\right]-\left(\frac{\partial \beta_1}{\partial \sigma}-\frac{\partial \beta_2}{\partial \sigma}\right)Ax_I^{\beta_2}N(d_3(x))\\
&-(\beta_1-1)\left[\frac{x_I(1-\tau_c)}{r-\alpha}e^{-(r-\alpha)T}n(-d_1(x))\frac{\partial d_1}{\partial \sigma}\right]\\
&-\frac{\partial \beta_1}{\partial \sigma}\frac{x_I}{r-\alpha}\left[1-\tau_h-(\tau_c-\tau_h)e^{-(r-\alpha)T}-(1-\tau_c)e^{-(r-\alpha)T}N(-d_1(x))\right]\\
&+\beta_1\left[Se^{-rT}n(-d_2(x))\frac{\partial d_2}{\partial \sigma}\right]+\frac{\partial \beta_1}{\partial \sigma}\left[I-Se^{-rT}N(-d_2(x))\right]
\end{aligned}
$$

with $A=\left(S-\dfrac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\dfrac{1}{x_A}\right)^{\beta_2}$

From the previous proofs we acknowledge that the coefficient of $\partial x_I/\partial \sigma$ is positive. In addition, we can show that $\partial \beta_1/\partial \sigma < 0$, $\partial \beta_2/\partial \sigma > 0$, $\partial A/\partial \sigma > 0$, $\partial d_1(x)/\partial \sigma \gtreqless 0$, $\partial d_2(x)/\partial \sigma < 0$, $\partial d_3(x)/\partial \sigma \gtreqless 0$. Therefore, the sign of the right-hand side of the equation is undeterminate. □

Proof of Corollary 8. $\lim_{\lambda\to\infty} b_4 = 0$, $\lim_{\lambda\to\infty} b_1 = 0$, $\lim_{\lambda\to\infty} \lambda/(r+\lambda) = 1$, $\lim_{\lambda\to\infty} \lambda/(r-\alpha+\lambda) = 1$. Thus, the two branches simplify to: $x(1-\tau_c)/(r-\alpha)+A(x,\tau_c)$. □

Proof of Corollary 9. $\lim_{\lambda\to 0} \eta_1 = \beta_1$, $\lim_{\lambda\to 0} \eta_2 = \beta_2$, $\lim_{\lambda\to 0} b_1 = 0$, $\lim_{\lambda\to 0} b_4 = S/(\beta_2-1)$. Thus, the two branches simplify to: $x(1-\tau_h)/r-\alpha$. □

Proof of Corollary 10. See Proof of Corollary 8. □

Proof of Corollary 11. See Proof of Corollary 9. □

Proof of Corollary 12. Differentiating Equation (35) with respect to λ, it yeilds:

$$\begin{aligned}
&\left[\eta_2(\beta_1-\eta_2)b_4x_R^{\eta_2-1}+(\beta_1-1)\left(\frac{1-\tau_h}{r-\alpha+\lambda}+\frac{\lambda}{r-\alpha+\lambda}\frac{\lambda(1-\tau_c)}{r-\alpha}\right)\right]\frac{\partial x_R}{\partial\lambda}\\
=&-(\beta_1-\eta_2)\left(\frac{\partial b_4}{\partial\lambda}x_R^{\eta_2}+b_4\log x_R\frac{\partial\eta_2}{\partial\lambda}\right)+\frac{\partial\eta_2}{\partial\lambda}b_4x_R^{\eta_2}\\
&+(\beta_1-1)\left[\frac{x_R}{(r-\alpha+\lambda)^2}(1-\tau_h)-\frac{x_R}{(r-\alpha+\lambda)^2}(1-\tau_c)\right]
\end{aligned}$$

Given that $b_4<0$, $\eta_2<0$, and $\beta_1>0$, so the coefficient of $\partial x_R/\partial\lambda$ is positive. However, as $\partial b_4/\partial\lambda>0$ and $\partial\eta_2/\partial\lambda<0$, the sign of the right-hand side of the equation is undetermined: $\partial x_R/\partial\lambda \gtrless 0$. □

Proof of Corollary 13. Differentiating Equation (35) with respect to τ_h, it yields:

$$\begin{aligned}
&\left[\eta_2(\beta_1-\eta_2)b_4x_R^{\eta_2-1}+(\beta_1-1)\left(\frac{1-\tau_h}{r-\alpha+\lambda}+\frac{\lambda}{r-\alpha+\lambda}\frac{\lambda(1-\tau_c)}{r-\alpha}\right)\right]\frac{\partial x_R}{\partial\tau_h}\\
=&(\beta_1-1)\frac{x_R}{r-\alpha+\lambda}
\end{aligned}$$

From the previous proof we acknowledge that the coefficient of $\partial x_R/\partial\tau_h$ is positive, and the sign of the right-hand side of the above equation is negative. Thus, $\partial x_R/\partial\tau_h>0$. □

Proof of Corollary 14. Differentiating Equation (35) with respect to σ, it yields:

$$\begin{aligned}
&\left[\eta_2(\beta_1-\eta_2)b_4x_R^{\eta_2-1}+(\beta_1-1)\left(\frac{1-\tau_h}{r-\alpha+\lambda}+\frac{\lambda}{r-\alpha+\lambda}\frac{(1-\tau_c)}{r-\alpha}\right)\right]\frac{\partial x_R}{\partial\sigma}\\
=&-(\beta_1-\eta_2)\left(\frac{\partial b_4}{\partial\sigma}x_R^{\eta_2}+b_4x_R^{\eta_2}\frac{\partial\eta_2}{\partial\sigma}\right)+\left(\frac{\partial\beta_1}{\partial\sigma}-\frac{\partial\eta_2}{\partial\sigma}\right)b_4x_R^{\eta_2}\\
&-\frac{\partial\beta_1}{\partial\sigma}\left[\frac{x_R(1-\tau_h)}{r-\alpha+\lambda}+\frac{\lambda}{r-\alpha+\lambda}\frac{x_R(1-\tau_c)}{r-\alpha}\right]
\end{aligned}$$

From the previous proofs, we acknowledge that the coefficient of $\partial x_R/\partial\sigma$ is positive. As $b_4<0$, $\eta_2<0$ and $\beta_1>0$, so $\partial x_R/\partial\lambda$ is positive. We can also show that $\partial\beta_1/\partial\sigma<0$, $\partial\eta_2/\partial\sigma>0$, $\partial b_4/\partial\sigma<0$. Therefore, the sign of the right-hand side of the equation above is positive, and $\partial x_R/\partial\sigma>0$. □