

A novel inverse DEA model with application to allocate the CO₂ emissions quota to different regions in Chinese manufacturing industries

Ali Emrouznejad^{a1}, Guoliang Yang^b, Gholam R. Amin^c

^a *Operations & Information Management, Aston Business School, Aston University, Birmingham, UK*

^b *Institute of Science and Development, Chinese Academy of Sciences, Beijing 100190, China*

^c *Faculty of Business, University of New Brunswick at Saint John, NB E2L 4L5, Canada*

Abstract

This paper aims to address the problem of allocating the CO₂ emissions quota set by government goal in Chinese manufacturing industries to different Chinese regions. The CO₂ emission reduction is conducted in a three-stage phases. The first stage is to obtain the total amount CO₂ emission reduction from the Chinese government goal as our total CO₂ emission quota to reduce. The second stage is to allocate the reduction quota to different two-digit level manufacturing industries in China. The third stage is to further allocate the reduction quota for each industry into different provinces. A new inverse data envelopment analysis (InvDEA) model is developed to achieve our goal to allocate CO₂ emission quota under several assumptions. At last we obtain the empirical results based on the real data from Chinese manufacturing industries.

Keywords: Data envelopment analysis (DEA); Inverse DEA; CO₂ emissions, Manufacturing Industries

1. Introduction

Since the reform and opening up policy in 1978, China's economy has maintained long-term rapid development and made great achievements. As reported in the China Statistical Yearbook 2016, between 1978 and 2015 the China's nominal Gross Domestic Product (GDP) grew significantly from 367.87 to 68263.51 in billion RMB

¹ Corresponding author: Ali Emrouznejad, Professor and Chair in Business Analytics, Aston Business School, Aston University, Birmingham, UK, Fax: 0121 204 5271, Tel: 0121 204 3092, Email: a.emrouznejad@Aston.ac.uk

Yuan, an increase of about 186 times. Bian et al. (2015) also argued that China's nominal industrial GDP increased by 66.02 times between 1981 and 2009. At the same time, however, the contradiction between the rapid growth of economic development and the environmental problem has been increasingly prominent. The economic development brought about a severe pressure on the natural environment and resources in China, especially in recent several years. China Statistical Yearbook 2016 shows that in year 2015 the Total Waste Water Discharged and Common Industrial Solid Wastes Produced reach 7353.227 and 327.079 million tons, respectively. In particular, the number of Days of Air Quality Equal to or Above Grade II in China's Capital city Beijing is only 186 in the year 2015. Bian *et al.* (2015) also reported the total amount of industrial solid waste produced in 2009 was 5.42 times that of 1981. In 2007 the total consumption of energy in China in 2007 reaches 311, 442 in millions of standard coal equivalent (SCE), and the total consumption of energy in China grew from 57.144 in 1978 to 430.000 in 2015 in million tons of SCE, which is reported clearly in China Statistical Yearbook 2016.

To address the issues of environmental protection, especially reducing CO₂ emissions, China government has been searching the viable solutions to balance the economic growth and CO₂ emissions reduction. At June 30 2015, at the upcoming climate conference in France, Chinese Premier Li Keqiang announced China's latest voluntary reduction commitment: the CO₂ emissions in China will reach the peak at about 2030 and seek to reach it as early as possible.

The Chinese government goal motivates us to investigate the problem of allocating the CO₂ emissions quota in Chinese manufacturing industries to different Chinese regions. In this paper, we use a three-stage way to conduct the CO₂ emission reduction. Firstly, we obtain the total amount CO₂ emission reduction from the Chinese government goal as our total CO₂ emission quota to reduce in the first stage. Secondly, we allocate the reduction quota to different two-digit level manufacturing industries in China. Thirdly, we further allocate the reduction quota for each industry into different provinces. In the CO₂ emissions reduction process, we develop a new

inverse data envelopment analysis (InvDEA) model to achieve our goal to allocate CO₂ emission quota under several assumptions.

The remainder of the paper is organized as follows: Section 2 summarizes the existing literatures on CO₂ emissions and DEA models. Section 3 describes the dataset and input/output indicators of Chinese manufacturing industries in our study. Section 4 gives the detailed information on our proposed InvDEA method and the empirical results of CO₂ emission quota allocation. Section 5 concludes this paper and provides some remarks for future research.

2. Literature review on CO₂ emission and DEA

In this section we provide latest development on measuring CO₂ emission using DEA models.

2.1. Literatures of using DEA for CO₂ emission

Regarding the efficiency analysis with respect to CO₂ emissions, Murty et al. (2007) estimated the technical and environmental efficiency and firm-specific shadow prices of pollutants of some coal-fired thermal power plants in India based on directional output distance function with the given resources and technology. Mukherjee (2010), Riccardi et al. (2012) and Vlontzos et al. (2014) respectively examined the efficiency considering reduction of CO₂ emissions in Indian manufacturing sector, 21 industrialized countries and EU member state countries, using directional distance function or non-radial DEA model allowing for non-proportional adjustments of outputs.

Further, Molinos-Senante et al. (2014) who applied measured the efficiency of wastewater treatment plants and estimated the pure and mixed environmental performance indices for a sample of 60 Spanish wastewater treatment plants using DEA models. Sueyoshi and Goto (2014a) and Sueyoshi and Goto (2014b) applied a radial-based DEA model which is shaped by the Debreu-Farrell and Cui and Li (2015) proposed a new virtual frontier DEA model to measure unified environmental efficiency.

Currently, China has become one of the world's largest contributors of CO₂ emissions, so the environmental efficiency including CO₂ emission in Chinese industries has been a popular research topic. Some of previous studies on Chinese environmental efficiency have been reported Table 1.

Table 1. Previous studies on Chinese environmental efficiency.

| Authors (year) | Research field and data | Methodological approaches | Major issues addressed |
|----------------------------|------------------------------------|---|--|
| Zhang et al. (2015) | Province-level | Output-based CCR model with DDF+ ML index | Total-factor carbon emission performance of the Chinese transportation industry |
| Yang et al. (2015) | Province-level | Input-based CCR and super efficiency CCR | Regional environmental efficiencies in China |
| Wang et al. (2015) | City-level | Output-based BCC model with DDF | Environmental protection mechanisms and economic development of 211 cities in China |
| Fan et al. (2015) | Industrial sub-sectors of Shanghai | Output-based CCR model with DDF+ ML index | Industrial total factor CO ₂ emission performance |
| Bian et al. (2015) | Regional-level data | Two-stage SBM DEA | Chinese regional industrial systems efficiency |
| An et al. (2015) | Plant-level | Enhanced Russell measure DEA | Environmental efficiency evaluation of thermal power enterprises |
| Zhu et al. (2014) | Pesticide-level | Input-based two-stage DEA | Eco-efficiency of Pesticides |
| Zhou et al. (2014) | Plant-level | CCR, BCC, NIRS | Energy efficiency performance of China's transport sector |
| Zhang et al. (2014) | Province-level | CCR model with DDF | Sustainability performance for China |
| Yin et al. (2014) | City-level | Input-based CCR | Eco-efficiency of Chinese cities |
| Wu et al. (2014) | Regional-level | Input-based fixed sum output DEA | Environmental efficiency evaluation of industry in China |
| Mahdiloo et al. (2014) | Regional-level | Output-based network DEA | Environmental quality efficiency |
| Huang et al. (2014) | Regional-level | SBM model with DDF | Regional eco-efficiency in China |
| Hou et al. (2014) | Agricultural systems-level | Input-based CCR with DDF | Sustainable value of degraded soils |
| Du et al. (2014) | Province-level | Output-based CCR with DDF+ML index | Measurement of the sources of economic growth |
| Bi et al. (2014a) | Thermal power sector | SBM model with DDF | Environmental regulation affect energy efficiency in China's thermal power generation |
| Long et al. (2013) | Chinese provinces | | Environmental regulatory cost |
| Wang et al. (2013a) | Province-level | SFA model | Energy and CO ₂ performance |
| He et al. (2013) | Iron and steel firm | Output-based CCR with DDF + ML index | Traditional energy efficiency, productivity, and environmentally sensitive productivity growth |
| Yang and Wang (2013) | Province-level | BCC model with DDF | Environmental efficiency and regulatory cost |
| Yuan et al. (2013) | Prefecture-level | Output-based BCC model with DDF | Environmental efficiency and determinants |
| Zhang and Choi (2013a) | Plant-level | CCR model with DDF + ML index | Total-factor carbon emission change |
| Zhang and Choi (2013b) | Plant-level | CCR model with DDF + ML index | Pure CO ₂ emission change |
| Zhang and Choi (2013c) | Regional-level | SBM model | Environmental energy efficiency of China's regional economies |
| Wu et al. (2012) | Regional industrial sector | Input-based CCR with DDF + ML index | Total-factor energy efficiency change |
| Zhang et al. (2011) | Province-level | Output-based CCR with DDF + ML index | Environmentally sensitive productivity growth and environmental regulatory cost |
| Chang and Hu (2010) | Chinese provinces | CCR with DDF + ML index | Energy productivity growth |
| Kaneko et al. (2010) | Thermal power sector | Output-based CCR with DDF | Shadow price of SO ₂ |
| Watanabe and Tanaka (2007) | Province-level industry | Output-based BCC with DDF | Efficiency with SO ₂ and determinants |
| Kaneko and Managi (2004) | Province-level | Output-based CCR with DDF + ML index | Environmentally sensitive productivity growth |

Note: (1) ML index denotes Malmquist–Luenberger productivity index; (2) SBM denotes slack-based measure

2.2. Inverse DEA

This section briefly reviews the origin and development of the inverse DEA methodology. The origin of inverse DEA is inverse optimization. Unlike normal optimization where the objective is finding an optimal solution, in an inverse optimization a feasible solution, which is not necessarily optimal, is given and the objective is to perturb the original data as less as possible in order to make that solution optimal (Ahuja and Orlin, 2001). Burton and Toint (1992) first studied an inverse problem in network flows specifically for the shortest path problems. Since then inverse optimization has been continuously enriched by new applications and a variety of inverse optimization problems in combinatorial optimization have been studied by researchers in the operations research community (Jiang et al. 2011; Pibernik et al. 2011; Ruiz et al. 2013; Wang et al. 2014). However, there are few articles about inverse continuous optimization like inverse linear programming and inverse DEA. Zhang and Liu (1996) investigated the first inverse linear programming model in the literature. Further research studies on inverse linear programming problems are given in Zhang and Liu (1999) and Huang and Liu (1999). One of the few applications of inverse linear programming in the literature is for predicting more accurate forecasting parameters developed in Amin and Emrouznejad (2007). The first inverse DEA methodology as a special case of the general inverse linear programming suggested in Wei et al. (2000) and further developed in Yan et al. (2002). Unlike the standard DEA whose objective is to find the efficiency score, the InvDEA assumes the efficiency given and aims to find the levels of inputs and outputs that are required to realize the desired efficiency score. Despite the potential applicability of the standard DEA in different contexts, there are few applications of inverse DEA that are reported in the literature such as application in resource allocation suggested in Hadi-Vencheh et al. (2008). Further recent of inverse DEA studies can be found in Jahanshahloo et al. (2015), Ghobadi and Jahangiri (2015), Ghiyasi (2017) and Amin et al. (2017a). In addition, Zhang and Cui (2016) discussed an extension of the inverse DEA model and Lim (2016) addressed the frontier change for setting a new product target using a new inverse DEA method. Gattoufi et al. (2014) extended the concept of inverse DEA to the context of mergers and acquisitions (M&A). The proposed inverse DEA in

Gattoufi et al. (2014) determines the optimal levels of inputs and outputs that are required from merging decision making units (DMUs) in order to allow the merged entity to realize a predefined efficiency target. More recently, Amin and Al-Muharrami (2016) addressed new inverse DEA models for mergers with negative data. Moreover, the potential of the inverse DEA has been used in Amin et al. (2017b) to anticipate whether a given restructuring between a group of DMUs makes a minor or a major consolidation. The successful result of the inverse DEA in M&A shows the potential power of this methodology in other sectors. In this paper we introduce an inverse DEA for allocation of CO₂ emissions reduction goal into different two-digit manufacturing industries and different regions.

3. Dataset and indicators

The country level data of Chinese manufacturing industries in 2012 used in this study is mainly derived from China Statistical Yearbook 2013 and China Energy Statistical Yearbook 2013. The province level data is from 31 statistical yearbooks of each province in 2013 respectively. We select the two-digit manufacturing industries in China as the DMUs. According to the new standard on Industrial Classification for National Economic Activities (GB/T4754-2011) enforced by National Bureau of Statistics of China (NBS) from 2012, the number of two-digit manufacturing industries changed to 31. See the following Table 2. The industry statistics cover all industries above designated size, which is 20 million yuan of annual revenue from primary business.

In this paper, we use three indicators including Labor, Asset and Energy as the inputs and two indicators as the outputs, including Gross Industrial Output Value (GIOV) as the desirable output and CO₂ emissions as the undesirable one.

Table 2. The two-digit manufacturing industries in China.

| No. | Two-digit manufacturing |
|------------|---|
| 1 | Processing of Food from Agricultural Products |
| 2 | Manufacture of Foods |
| 3 | Manufacture of Liquor, Beverages and Refined Tea |
| 4 | Manufacture of Tobacco |
| 5 | Manufacture of Textile |
| 6 | Manufacture of Textile, Wearing Apparel and Accessories |
| 7 | Manufacture of Leather, Fur, Feather and Related Products and Footwear |
| 8 | Processing of Timber, Manufacture of Wood, Bamboo, Rattan, Palm and Straw Products |
| 9 | Manufacture of Furniture |
| 10 | Manufacture of Paper and Paper Products |
| 11 | Printing and Reproduction of Recording Media |
| 12 | Manufacture of Articles for Culture, Education, Arts and Crafts, Sport and Entertainment Activities |
| 13 | Processing of Petroleum, Coking and Processing of Nuclear Fuel |
| 14 | Manufacture of Raw Chemical Materials and Chemical Products |
| 15 | Manufacture of Medicines |
| 16 | Manufacture of Chemical Fibres |
| 17 | Manufacture of Rubber and Plastics Products |
| 18 | Manufacture of Non-metallic Mineral Products |
| 19 | Smelting and Pressing of Ferrous Metals |
| 20 | Smelting and Pressing of Non-ferrous Metals |
| 21 | Manufacture of Metal Products |
| 22 | Manufacture of General Purpose Machinery |
| 23 | Manufacture of Special Purpose Machinery |
| 24 | Manufacture of Automobiles |
| 25 | Manufacture of Railway, Ship, Aerospace and Other Transport Equipment |
| 26 | Manufacture of Electrical Machinery and Apparatus |
| 27 | Manufacture of Computers, Communication and Other Electronic Equipment |
| 28 | Manufacture of Measuring Instruments and Machinery |
| 29 | Other Manufacture |
| 30 | Utilization of Waste Resources |
| 31 | Repair Service of Metal Products, Machinery and Equipment |

The variables used in this study are as follows: (1) Labor refers to the amount of labors in Chinese manufacturing industries. Due to the mobility of Labor, the amount of labor variable is different at different time in one year, so the number of annual average employed persons is taken as the indicator. (2) Asset refers to the amount of total assets. Data on this indicator are obtained by the year-end figures of total assets in the Assets and Liability Table of accounting

records of enterprises. (3) Energy refers to the total consumption of energy of various kinds by the production sectors in the country in a given period of time. (4) In this paper the GIOV is used as a desirable output. This variable has been estimated by dividing Industrial Sales Output Value (ISOV) to Sales Ratio of Products (SRP), as both variables are available for each sub-level manufacturing industry for the year 2013. (5) The CO₂ emission is the undesirable output in our study, which is also estimated based on the consumption of different types of energy. For details on data collection please see Emrouznejad and Yang (2016a, 2016b). The descriptive statistics for the country level dataset can be found also in Emrouznejad and Yang (2016a, 2016b).

4 Methodology and empirical results

Our main idea in this paper is to conduct the CO₂ emission reduction in a three-stage way. The first stage is to obtain the total amount CO₂ emission reduction from the Chinese government goal, denoted by $CO2_{total}$, as our total CO₂ emission quota to reduce. The second stage is to allocate the reduction quota $CO2_{total}$ to different manufacturing industries, denoted by $CO2_i$, where i denotes different two-digit Chinese manufacturing industries, which satisfy $\sum_i CO2_i = CO2_{total}$. The third stage is to further allocate the reduction quota for each industry $CO2_i$ into different provinces, denoted by $CO2_{ij}$, where j denotes different provinces and the following formula holds: $\sum_j CO2_{ij} = CO2_i$.

4.1 Determining the total amount of CO₂ emission in Chinese manufacturing industries

In manufacturing industries, the Gross Industrial Output Value (GIOV) plays the same role as GDP for the country. Chinese State Council released officially the "National Climate Change Plan (2014-2020)" in the September 2014 and announced China's CO₂ emissions to gross domestic product in 2020 would be reduced by 40% to 45% on the basis of 2005. At the world climate conference in France in June 2015, Chinese Premier Li Keqiang announced China's latest voluntary reduction commitment: China government aim to cut its greenhouse gas emissions intensity by 60-65% (per unit of

gross domestic product) from 2005 levels. Based on the above goal, we can propose CO₂ reduction goal as CO₂ emission/GIOV decrease 60% to 65% based on the level of 2005. The CO₂ emission/GIOV in China from 2004 to 2012 is listed in the following Table 3.

Table 3. The CO₂ emission/GIOV in China from 2004 to 2012.

| Year | CO ₂ emission (10 000 tons) | Gross Industrial Output Value (current prices-2010) (100 million yuan) | CO ₂ emission /GIOV | Consumer Price Index (CPI) of China |
|------|---|--|-----------------------------------|---|
| 2004 | 232270.3895 | 193961.0561 | 1.1975 | 81.8313 |
| 2005 | 253527.1366 | 217835.7400 | 1.1638 | 85.0227 |
| 2006 | 275441.6447 | 274571.6700 | 1.0032 | 86.5673 |
| 2007 | 293235.3426 | 353630.8400 | 0.8292 | 87.8369 |
| 2008 | 325151.5258 | 441358.3600 | 0.7367 | 92.0238 |
| 2009 | 341118.8413 | 479199.7200 | 0.7119 | 97.4532 |
| 2010 | 370079.7298 | 609558.5000 | 0.6071 | 96.7834 |
| 2011 | 395088.9957 | 733984.0100 | 0.5383 | 100.0000 |
| 2012 | 413471.1638 | 809255.1324 | 0.5109 | 105.4706 |

*Source: China Statistical Yearbooks 2005 - 2013, China Energy Statistical Yearbook (Note: According to OECD statistics, we set Index 2010=100)

As it is been explained in Emrouznejad and Yang (2016a, 2016b) the value of GIOV transform to constant price in 2010 using the Consumer Price Index (CPI) of China, as shown in the last column of Table 3. This transformation approach is used in many other researches, *e.g.* Oh and Heshmati (2010). The CPI data is derived from OECD (2010).

Therefore in this paper we set the goal to decrease 60% to 65% of the level of CO₂ emission/GIOV in 2012 based on that in 2005. Thus CO₂ emission/GIOV in 2012 should be in the range of [0.4073, 0.4655]. However the real ratio of CO₂ emission/GIOV reaches 0.5109. If Chinese government achieves the goal of the CO₂ emission in 2012, the CO₂ emission in 2012 should be [329646.7686, 376739.1641]. However the real amount of CO₂ emission in manufacturing industries in China is 413471.1638 (10,000 tons). Thus the CO₂ emission reduction gap should be [36731.9997, 83824.3952] in the unit of 10,000 tons. As the CO₂ emission reduction

of Chinese government is an interval, we use the lower bound, which is 36731.9997 (unit: 10 thousand tons), as the minimal CO₂ reduction goal in this paper.

4.2. A new InvDEA model for CO₂ emission quota allocation

Assume that there are n DMUs where the j^{th} DMU use M inputs x_{ij} ($i = 1, \dots, M$) and produces R good outputs y_{rj}^g ($r = 1, \dots, R$) and P undesirable or bad outputs y_{pj}^b ($p = 1, \dots, P$), for each $j = 1, \dots, n$. Let L be the set of selected DMUs for reducing undesirable outputs. Generally in our modeling, we assume that $L \subseteq \{1, \dots, n\}$ and reducing undesirable outputs from all DMUs means that $L = \{1, \dots, n\}$. Assume all the DMUs in L would keep their efficiency scores at least the same as before reducing bad outputs. Moreover, let α_{ik} , β_{rk} , γ_{pk} be the levels of the i^{th} input, r^{th} good output and p^{th} bad output of the k^{th} DMU, respectively, after reducing the bad outputs (for each $i = 1, \dots, M$, $r = 1, \dots, R$, $p = 1, \dots, P$ and every $k \in L$).

First, we propose the following assumptions for the CO₂ emission reduction in our paper:

Assumption 1. *The efficient frontier will remain constant in the process of CO₂ emissions reduction.*

Based on this assumption, we assume F be the set of all efficient DMUs identified by the following model (1).

$$\begin{aligned} \vec{D}_{DDF,v}^G(X_k, Y_k, B_k, g_Y, g_B) = \max \vec{\beta}_k \\ s. t. \begin{cases} \sum_{j=1}^n \lambda_j X_j \leq X_k \\ \sum_{j=1}^n \lambda_j Y_j \geq (1 + \vec{\beta}_k) Y_k \\ \sum_{j=1}^n \lambda_j B_j = (1 - \vec{\beta}_k) B_k \\ \sum_{j=1}^n \lambda_j = 1 \\ \lambda_j \geq 0, j = 1, \dots, n \end{cases} \end{aligned} \quad (1)$$

Based on the results from model (1), we can have the inefficient DMUs as the targets of our CO₂ emission reduction.

The proposed inverse DEA method in this paper is the first attempt in the literature to

determine optimal allocation of CO₂ emissions. The base DEA model for the inverse problem can be any DEA model developed for undesirable output. In this paper, we consider the directional distance DEA model (1) as the base model simply because it is more relevant to the application.

It should be noted that there is enough space for CO₂ emission reduction goal of Chinese government using the inefficient DMUs as the reduction targets in this paper. Therefore, we can assume the *Assumption 1* holds. Otherwise, if we cannot achieve the government goal of reducing CO₂ emissions by inefficient DMUs only, we need to consider to reduce CO₂ emissions from efficient DMUs, which means the efficient frontiers will shift towards the direction of more desirable output(s) and less undesirable output(s). In such case, the problem will be more complex. A possible solution is to assume all the DMUs reduce further the same proportion of CO₂ emissions to achieve this goal, which technically means the frontiers shift in an average way.

Assumption 2: The efficiencies of all DMUs will not decrease in the process of CO₂ emissions reduction.

This assumption indicates the CO₂ emissions reduction will not damage the DMUs' efficiencies including both efficient and inefficient ones. Thus, the efficiency of none of DMUs will be deteriorated after the CO₂ emissions reduction.

Assumption 3: There exist the possible policy thresholds for certain input or output.

In the real scenario of policy making, the policy makers often need to consider some policy thresholds for certain input or output indicators. For example, in China, it is very difficult to fire too much employee in the manufacturing industries. Furthermore, the gross industrial output value (GIOV) cannot be reduced too much, because the Chinese government needs to keep the growth rate of gross domestic product (GDP) at a certain level. Therefore, in our model we consider such types of policy thresholds to make our model more reasonable and flexible.

Based on the above three assumptions, we propose the following InvDEA model for

allocation the given amount of bad outputs reductions to different DMUs.

Remark 1. In certain case, we have to shift the efficient frontier in the process of CO₂ emissions reduction to meet the CO₂ emission reduction targets. We will discuss this issue in the following subsection 4.4.

$$\begin{aligned}
& \min \sum_{k \in L} \sum_{i=1}^m \alpha_{ik} - \sum_{k \in L} \sum_{r=1}^R \beta_{rk} \\
& \text{s.t.} \\
& \sum_{j \in F} \lambda_j^k x_{ij} + \sum_{j \in L} \alpha_{ij} \lambda_{kj} - \alpha_{ik} \leq 0, \quad \forall k \in L, i = 1, \dots, m \\
& \sum_{j \in F} \lambda_j^k y_{rj}^g + \sum_{j \in L} \beta_{rj} \lambda_{kj} - (1 + \hat{\beta}_k) \beta_{rk} \geq 0, \quad \forall k \in L, r = 1, \dots, R \\
& \sum_{j \in F} \lambda_j^k y_{pj}^b + \sum_{j \in L} \gamma_{pj} \lambda_{kj} - (1 - \hat{\beta}_k) \gamma_{pk} = 0, \quad \forall k \in L, p = 1, \dots, P \\
& \sum_{j \in F} \lambda_j^k + \sum_{j \in L} \lambda_{kj} = 1, \quad \forall k \in L \tag{2} \\
& \sum_{j \in L} \gamma_{pj} = a_p, \quad p = 1, \dots, P \\
& 0 \leq \alpha_{ik} \leq x_{ik} \quad \forall k \in L, i = 1, \dots, m \\
& (1 - c_{rk}) y_{rk}^g \leq \beta_{rk}, \quad \forall k \in L, r = 1, \dots, R \\
& 0 \leq \gamma_{pk} \leq y_{pk}^b \quad \forall k \in L, p = 1, \dots, P \\
& \lambda_j^k \geq 0, \quad \forall j \in F_k, k \in L \\
& \lambda_{kj} \geq 0, \quad k, j \in L
\end{aligned}$$

The objective of the InvDEA model (2) is to minimize the sum of the amount of the inputs that should be kept and minimizing the amount of good outputs that should be dropped from each DMU in L in a way that the amount of a_p from the p^{th} ($p = 1, \dots, P$) bad output of DMUs in L should be reduced. There is also limitation on the amount of reduction of good outputs shown by the constraints $(1 - c_{rk}) y_{rk}^g \leq \beta_{rk} \leq y_{rk}^g$ ($\forall k \in L, r = 1, \dots, R$) where c_{rk} is a constant given by decision makers. For instance, a policy of reducing at most 5% of good outputs in order to reduce a given amount of bad outputs, if feasible, can be employed by considering $c_{rk} = 0.05$. Furthermore, $\hat{\beta}_k$ is a parameter that guarantees the efficiency scores of DMUs in L would not be decreased after bad outputs reduction since $0 \leq \hat{\beta}_k \leq \overline{\beta}_k^*$, where $\overline{\beta}_k^*$ is the optimal value of DEA model (1).

It should be noted that the Assumptions 1-3 are given to simplify the implementation of the suggested inverse DEA model (2). In fact, the non-linear model (2) can be simplified to a linear programming problem (3). Assumption 1 guarantees that there would be no frontier change after CO₂ emission reduction and this would simplify the non-linear model to a linear model. The following theorem shows the possibility of this relaxation.

Theorem 1: The NLP InvDEA model (2) can be simplified to the following relaxed LP InvDEA model.

$$\begin{aligned}
& \min \sum_{k \in L} \sum_{i=1}^m \alpha_{ik} - \sum_{k \in L} \sum_{r=1}^R \beta_{rk} \\
& s.t. \\
& \sum_{j \in F} \lambda_j^k x_{ij} - \alpha_{ik} \leq 0, \quad \forall k \in L, i = 1, \dots, m \\
& \sum_{j \in F} \lambda_j^k y_{rj}^g - (1 + \hat{\beta}_k) \beta_{rk} \geq 0, \quad \forall k \in L, r = 1, \dots, R \\
& \sum_{j \in F} \lambda_j^k y_{pj}^b - (1 - \hat{\beta}_k) \gamma_{pk} = 0, \quad \forall k \in L, p = 1, \dots, P \\
& \sum_{j \in F} \lambda_j^k = 1, \quad \forall k \in L \\
& \sum_{j \in L} \gamma_{pj} = a_p, \quad p = 1, \dots, P \\
& 0 \leq \alpha_{ik} \leq x_{ik} \quad \forall k \in L, i = 1, \dots, m \\
& (1 - c_{rk}) y_{rk}^g \leq \beta_{rk}, \quad \forall k \in L, r = 1, \dots, R \\
& 0 \leq \gamma_{pk} \leq y_{pk}^b \quad \forall k \in L, p = 1, \dots, P \\
& \lambda_j^k \geq 0, \quad \forall j \in F, k \in L
\end{aligned} \tag{3}$$

Proof: We first assume that L contains only inefficient DMUs. This means that none of the DMUs in L can be a benchmark for itself and/or other DMUs, implying that $\lambda_j^{k*} = 0$ for all $k, j \in L$ in any optimal solution of the InvDEA model (2). The NLP InvDEA Model (2) can be similarly relaxed to model (3) even if some of the inefficient DMUs in L targeted to be fully efficient after reducing bad outputs, or equivalently $\hat{\beta}_k = 0$ for some $k \in L$. In fact, these new efficient DMUs fall on the efficiency frontier and therefore can be presented in terms of the a convex combination of the existing efficient DMUs.

Now, consider a case when reducing bad outputs from an efficient DMU_k is at concern or equivalently $k \in L$. According to the assumption we have $0 \leq \hat{\beta}_k \leq \overrightarrow{\beta}_k^* = 0$, and so $\hat{\beta}_k = 0$. Therefore, DMU_k is efficient before and after reducing bad outputs and therefore can be presented in terms of DMU_k itself. This concludes that

$$y_{pk}^b - \gamma_{pk}^* = 0, p = 1, \dots, P$$

Or equivalently reducing bad outputs from an efficient DMU would be zero. It worth noting that this would be the case if we wouldn't change the efficiency frontier. This completes the proof. ■

It should be noted that in certain situations, there may be the cases that model (3) will not have feasible solutions because of the setting of policy thresholds. For example, as we mentioned above, there is a limitation on the amount of reduction of good outputs shown by the constraints $(1 - c_{rk})y_{rk}^g \leq \beta_{rk} \leq y_{rk}^g$ ($\forall k \in L, r = 1, \dots, R$) where c_{rk} is a constant given by decision makers. Those policy thresholds may not provide enough space for CO₂ emission reduction. Thus, we suggest to decide the lower bound of those thresholds C_r^* using the following model (4) as the parameters in model (3), which mean the decision makers have to allow to reduce the good outputs at least to the level of $(1 - C_r^*), r = 1, \dots, R$.

$$\begin{aligned}
& \min \sum_{r=1}^R C_r \\
& s. t. \\
& \sum_{j \in F} \lambda_j^k x_{ij} - \alpha_{ik} \leq 0, \quad \forall k \in L, i = 1, \dots, m \\
& \sum_{j \in F} \lambda_j^k y_{rj}^g - (1 + \hat{\beta}_k) \beta_{rk} \geq 0, \quad \forall k \in L, r = 1, \dots, R \\
& \sum_{j \in F} \lambda_j^k y_{pj}^b - (1 - \hat{\beta}_k) \gamma_{pk} = 0, \quad \forall k \in L, p = 1, \dots, P \\
& \sum_{j \in F} \lambda_j^k = 1, \quad \forall k \in L \\
& \sum_{j \in L} \gamma_{pj} = a_p, \quad p = 1, \dots, P \\
& 0 \leq \alpha_{ik} \leq x_{ik} \quad \forall k \in L, i = 1, \dots, m \\
& (1 - c_{rk}) y_{rk}^g \leq \beta_{rk}, \quad \forall k \in L, r = 1, \dots, R \\
& c_{rk} \geq C_r, \quad \forall k \in L, r = 1, \dots, R \\
& 0 \leq \gamma_{pk} \leq y_{pk}^b \quad \forall k \in L, p = 1, \dots, P \\
& \lambda_j^k \geq 0, \quad \forall j \in F, k \in L
\end{aligned} \tag{4}$$

Therefore, we use the following procedure to conduct the allocation the CO₂ emission reduction among designated DMUs.

Procedure 1.

Step 1: Use model (1) to divide all DMUs into two sets of efficient and inefficient DMUs respectively, which are denoted as F and L respectively.

Step 2: Select all inefficient DMUs in the set L as the targets for CO₂ reduction.

Step 3: Set policy thresholds for certain input or output for CO₂ reduction.

Step 4: Use model (3) to allocate CO₂ emission reduction into inefficient DMUs in the set L .

Table 4. The results of model (1) and two sets.

| DMUs | | $\bar{\beta}_k^*$ | Sets |
|-------------------|---|-------------------|----------|
| DMU ₁ | Processing of Food from Agricultural Products | 0.0000 | <i>F</i> |
| DMU ₂ | Manufacture of Foods | 0.4274 | <i>L</i> |
| DMU ₃ | Manufacture of Liquor, Beverages and Refined Tea | 0.4269 | <i>L</i> |
| DMU ₄ | Manufacture of Tobacco | 0.0000 | <i>F</i> |
| DMU ₅ | Manufacture of Textile | 0.3688 | <i>L</i> |
| DMU ₆ | Manufacture of Textile, Wearing Apparel and Accessories | 0.0699 | <i>L</i> |
| DMU ₇ | Manufacture of Leather, Fur, Feather and Related Products and Footwear | 0.0000 | <i>F</i> |
| DMU ₈ | Processing of Timber, Manufacture of Wood, Bamboo, Rattan, Palm and Straw Products | 0.0000 | <i>F</i> |
| DMU ₉ | Manufacture of Furniture | 0.1795 | <i>L</i> |
| DMU ₁₀ | Manufacture of Paper and Paper Products | 0.8209 | <i>L</i> |
| DMU ₁₁ | Printing and Reproduction of Recording Media | 0.3142 | <i>L</i> |
| DMU ₁₂ | Manufacture of Articles for Culture, Education, Arts and Crafts, Sport and Entertainment Activities | 0.0000 | <i>F</i> |
| DMU ₁₃ | Processing of Petroleum, Coking and Processing of Nuclear Fuel | 0.0000 | <i>F</i> |
| DMU ₁₄ | Manufacture of Raw Chemical Materials and Chemical Products | 0.0000 | <i>F</i> |
| DMU ₁₅ | Manufacture of Medicines | 0.4288 | <i>L</i> |
| DMU ₁₆ | Manufacture of Chemical Fibres | 0.3603 | <i>L</i> |
| DMU ₁₇ | Manufacture of Rubber and Plastics Products | 0.2527 | <i>L</i> |
| DMU ₁₈ | Manufacture of Non-metallic Mineral Products | 0.3148 | <i>L</i> |
| DMU ₁₉ | Smelting and Pressing of Ferrous Metals | 0.0000 | <i>F</i> |
| DMU ₂₀ | Smelting and Pressing of Non-ferrous Metals | 0.0000 | <i>F</i> |
| DMU ₂₁ | Manufacture of Metal Products | 0.0771 | <i>L</i> |
| DMU ₂₂ | Manufacture of General Purpose Machinery | 0.0491 | <i>L</i> |
| DMU ₂₃ | Manufacture of Special Purpose Machinery | 0.2183 | <i>L</i> |
| DMU ₂₄ | Manufacture of Automobiles | 0.0000 | <i>F</i> |
| DMU ₂₅ | Manufacture of Railway, Ship, Aerospace and Other Transport Equipment | 0.3776 | <i>L</i> |
| DMU ₂₆ | Manufacture of Electrical Machinery and Apparatus | 0.0000 | <i>F</i> |
| DMU ₂₇ | Manufacture of Computers, Communication and Other Electronic Equipment | 0.0000 | <i>F</i> |
| DMU ₂₈ | Manufacture of Measuring Instruments and Machinery | 0.0664 | <i>L</i> |
| DMU ₂₉ | Other Manufacture | 0.7736 | <i>L</i> |
| DMU ₃₀ | Utilization of Waste Resources | 0.0000 | <i>F</i> |
| DMU ₃₁ | Repair Service of Metal Products, Machinery and Equipment | 0.0000 | <i>F</i> |

4.3 Allocate the CO₂ emission reduction to different two-digit Chinese manufacturing industries

As we discussed in subsection 4.1, we use the lower bound of CO₂ reduction interval, which is 36732(unit: 10 thousand tons), as the minimal CO₂ reduction goal in this paper. We use the above **Procedure 1** to conduct the allocation the CO₂ emission reduction among different two-digit Chinese manufacturing industries.

Step 1: Two sets of efficient and inefficient two-digit Chinese manufacturing industries are as follows (See Table 4):

Step 2: We select all inefficient DMUs in set L as the targets for CO₂ reduction in the following Table 5. Also we assume the parameter $\hat{\beta}_k$ that guarantees the efficiency scores of DMUs in L wouldn't be decreased after CO₂ emission reduction.

Table 5. The inefficient DMUs in set L .

| Inefficient DMUs | | $\overrightarrow{\beta}_k^*$ in model (1) | Sets | $\hat{\beta}_k$ (case 1) | $\hat{\beta}_k$ (case 2) |
|-------------------|---|---|------|--------------------------|--------------------------|
| DMU ₂ | Manufacture of Foods | 0.4274 | L | 0.4274 | 0.3847 |
| DMU ₃ | Manufacture of Liquor, Beverages and Refined Tea | 0.4269 | L | 0.4269 | 0.3842 |
| DMU ₅ | Manufacture of Textile | 0.3688 | L | 0.3688 | 0.3319 |
| DMU ₆ | Manufacture of Textile, Wearing Apparel and Accessories | 0.0699 | L | 0.0699 | 0.0629 |
| DMU ₉ | Manufacture of Furniture | 0.1795 | L | 0.1795 | 0.1616 |
| DMU ₁₀ | Manufacture of Paper and Paper Products | 0.8209 | L | 0.8209 | 0.7388 |
| DMU ₁₁ | Printing and Reproduction of Recording Media | 0.3142 | L | 0.3142 | 0.2828 |
| DMU ₁₅ | Manufacture of Medicines | 0.4288 | L | 0.4288 | 0.3859 |
| DMU ₁₆ | Manufacture of Chemical Fibres | 0.3603 | L | 0.3603 | 0.3243 |
| DMU ₁₇ | Manufacture of Rubber and Plastics Products | 0.2527 | L | 0.2527 | 0.2274 |
| DMU ₁₈ | Manufacture of Non-metallic Mineral Products | 0.3148 | L | 0.3148 | 0.2833 |
| DMU ₂₁ | Manufacture of Metal Products | 0.0771 | L | 0.0771 | 0.0694 |
| DMU ₂₂ | Manufacture of General Purpose Machinery | 0.0491 | L | 0.0491 | 0.0442 |
| DMU ₂₃ | Manufacture of Special Purpose Machinery | 0.2183 | L | 0.2183 | 0.1965 |
| DMU ₂₅ | Manufacture of Railway, Ship, Aerospace and Other Transport Equipment | 0.3776 | L | 0.3776 | 0.3398 |
| DMU ₂₈ | Manufacture of Measuring Instruments and Machinery | 0.0664 | L | 0.0664 | 0.0598 |
| DMU ₂₉ | Other Manufacture | 0.7736 | L | 0.7736 | 0.6962 |

We propose two ways to determine the parameter $\hat{\beta}_k$:

Case 1: The first one is to keep the $\hat{\beta}_k$ as the value of $\overrightarrow{\beta}_k^*$ in model (1), which

means all inefficient DMUs keep their efficiencies in the process of reducing CO₂ emission.

Case 2: The second one is to improve the directional distance $\vec{\beta}_k^*$ by 10%, which means that we define $\hat{\beta}_k = 90\% \times \vec{\beta}_k^*$ for each $k \in L$.

Step 3: We set the policy threshold for at least 95% of the good output GIOV should be kept. Thus we have the following constraints in model (3):

$$(1 - 0.05) y_{rk}^g \leq \beta_{rk}, \forall k \in L, r = 1, \dots, R$$

Step 4: We use model (3) to allocate CO₂ emission reduction into inefficient DMUs in the set L. See Table 6.

Table 6. The CO₂ emission allocation. (unit: 10 thousand tons)

| Inefficient DMUs | | CO ₂ emission allocation | |
|-------------------|---|-------------------------------------|---------------------------------|
| | | $(\hat{\beta}_k\text{-Case 1})$ | $(\hat{\beta}_k\text{-Case 2})$ |
| DMU ₂ | Manufacture of Foods | 0.000 | 175.515 |
| DMU ₃ | Manufacture of Liquor, Beverages and Refined Tea | 0.000 | 0.000 |
| DMU ₅ | Manufacture of Textile | 0.000 | 0.000 |
| DMU ₆ | Manufacture of Textile, Wearing Apparel and Accessories | 0.000 | 0.000 |
| DMU ₉ | Manufacture of Furniture | 3.299 | 4.988 |
| DMU ₁₀ | Manufacture of Paper and Paper Products | 1380.958 | 1109.357 |
| DMU ₁₁ | Printing and Reproduction of Recording Media | 0.000 | 0.000 |
| DMU ₁₅ | Manufacture of Medicines | 0.000 | 0.000 |
| DMU ₁₆ | Manufacture of Chemical Fibres | 758.759 | 0.000 |
| DMU ₁₇ | Manufacture of Rubber and Plastics Products | 0.000 | 0.000 |
| DMU ₁₈ | Manufacture of Non-metallic Mineral Products | 34016.294 | 34740.234 |
| DMU ₂₁ | Manufacture of Metal Products | 0.000 | 0.000 |
| DMU ₂₂ | Manufacture of General Purpose Machinery | 0.000 | 0.000 |
| DMU ₂₃ | Manufacture of Special Purpose Machinery | 0.000 | 0.000 |
| DMU ₂₅ | Manufacture of Railway, Ship, Aerospace and Other Transport Equipment | 0.000 | 0.000 |
| DMU ₂₈ | Manufacture of Measuring Instruments and Machinery | 0.000 | 0.000 |
| DMU ₂₉ | Other Manufacture | 572.689 | 701.904 |
| Total | | 36731.999 | 36731.999 |

4.4 Allocate the CO₂ emission reduction to different regions

Without loss of generality, we assume that we select the Case 2 in subsection 4.3 as the results for the further allocation of the CO₂ emission reduction to different regions.

That means we assume that we aim to improve the directional distance $\overline{\beta}_k^*$ by 10%, i.e., $\hat{\beta}_k = 90\% \times \overline{\beta}_k^*$ for each $k \in L$. Therefore we use the following procedure to conduct the second stage allocation of CO₂ emission reduction.

Step 1. We first select the DMUs for the second stage of allocating the CO₂ emission reduction to different regions in China. Based on the results in the above Table 6, we have the following Table 7 for the further allocation of CO₂ emission reduction.

Table 7. The DMUs to be further allocated. (unit: 10 thousand tons)

| Inefficient DMUs | | CO ₂ emission allocation ($\hat{\beta}_k$ -Case 2) |
|-------------------|--|---|
| DMU ₂ | Manufacture of Foods | 175.515 |
| DMU ₉ | Manufacture of Furniture | 4.988 |
| DMU ₁₀ | Manufacture of Paper and Paper Products | 1109.357 |
| DMU ₁₈ | Manufacture of Non-metallic Mineral Products | 34740.234 |
| DMU ₂₉ | Other Manufacture | 701.904 |

Step 2. We conduct the similar procedure to Procedure 1 in subsection 4.3 where we substitute the Chinese manufacturing in Procedure 1 for the 31 different provinces of China. Furthermore, we also assume that we aim to improve the directional distance, which is obtained from model (1) when applied to the 31 different provinces, by 10%. We repeat this process for DMU₂, DMU₉, DMU₁₀, DMU₁₈, and DMU₂₉. Thus we have the final results as follows (See Table 8):

It should be noted here that for the Manufacture of Foods, Manufacture of Furniture, Manufacture of Paper and Paper Products, and Other Manufacture, the policy thresholds for good output reduction are all 5%, which provides enough space for CO₂ emissions reduction. However for the Manufacture of Non-metallic Mineral Products, model (3) cannot find feasible solution for CO₂ emission reduction with the constraints of the policy thresholds for good output reduction are all 5%. Therefore,

we first use model (4) to find the lower bound of thresholds on GIOV as $C^* = 37.73\%$ using model (4), which mean the decision makers have to allow to reduce the GIOV at least to the level of $1 - C^* = 62.27\%$.

Table 8. The CO₂ emission allocation in the second stage. (unit: 10 thousand tons)

| Regions | DMU₂ | DMU₉ | DMU₁₀ | DMU₁₈ | DMU₂₉ |
|---|------------------------|------------------------|-------------------------|-------------------------|-------------------------|
| Beijing | 0.000 | 0.000 | 0.000 | 309.717 | 0.550 |
| Tianjin | 0.000 | 0.000 | 0.000 | 137.314 | 0.439 |
| Hebei | 0.000 | 0.000 | 0.000 | 2146.731 | 3.054 |
| Shanghai | 0.000 | 0.000 | 83.583 | 774.586 | 8.095 |
| Jiangsu | 0.000 | 0.000 | 34.813 | 3081.870 | 34.393 |
| Zhejiang | 0.000 | 2.151 | 0.000 | 1759.959 | 46.269 |
| Fujian | 0.000 | 0.000 | 0.000 | 1469.718 | 15.602 |
| Shandong | 0.000 | 0.000 | 0.000 | 0.000 | 17.738 |
| Guangdong | 0.000 | 0.000 | 49.630 | 3787.145 | 87.358 |
| Hainan | 0.000 | 0.000 | 0.000 | 0.000 | 17.074 |
| Liaoning | 0.000 | 0.000 | 0.000 | 0.000 | 25.137 |
| Jilin | 0.000 | 0.000 | 31.932 | 0.000 | 1.011 |
| Helongjiang | 0.000 | 0.000 | 0.000 | 0.000 | 0.286 |
| Anhui | 0.000 | 0.000 | 0.000 | 2547.221 | 1.023 |
| Jianxi | 0.000 | 0.000 | 0.000 | 0.000 | 4.067 |
| Henan | 0.000 | 0.000 | 265.378 | 1665.366 | 2.624 |
| Hubei | 41.888 | 0.000 | 0.000 | 1606.724 | 4.764 |
| Hunan | 0.000 | 0.000 | 131.797 | 1654.607 | 409.330 |
| Shanxi | 0.000 | 0.000 | 0.000 | 0.000 | 4.228 |
| Inner Mongolia | 0.000 | 0.000 | 0.000 | 0.000 | 2.672 |
| Guangxi | 0.000 | 2.837 | 97.441 | 2148.201 | 1.579 |
| Chongqing | 0.000 | 0.000 | 71.824 | 0.000 | 4.129 |
| Sichuan | 119.836 | 0.000 | 206.662 | 3572.914 | 1.423 |
| Guizhou | 0.000 | 0.000 | 3.763 | 1636.424 | 0.837 |
| Yunnan | 0.000 | 0.000 | 59.840 | 2100.445 | 2.933 |
| Tibet | 0.000 | 0.000 | 0.000 | 0.000 | 5.291 |
| Shaanxi | 13.790 | 0.000 | 8.627 | 1387.635 | 0.550 |
| Gansu | 0.000 | 0.000 | 0.000 | 991.986 | 0.439 |
| Qinghai | 0.000 | 0.000 | 0.000 | 198.552 | 3.054 |
| Ningxia | 0.000 | 0.000 | 54.753 | 364.846 | 8.095 |
| Xinjiang | 0.000 | 0.000 | 9.315 | 1398.272 | 34.393 |
| Total CO₂ emission reduction allocation | 175.515 | 4.988 | 1109.357 | 34740.234 | 701.904 |

For Other Manufacture, another case happens. We first use model (1) to find the efficient regions and inefficient regions. Here we list the inefficient DMUs in set L as follows (See Table 9):

Table 9. Total CO₂ emission in inefficient regions of Other Manufacture (unit: 10 thousand tons)

| Inefficient regions | Sets | CO ₂ emission | CO ₂ emission of projections of inefficient regions on the frontier of model (1) |
|---------------------|------|--------------------------|---|
| Hebei | L | 3.250 | 2.606 |
| Shanghai | L | 8.394 | 3.965 |
| Guangdong | L | 93.845 | 85.944 |
| Liaoning | L | 17.743 | 8.855 |
| Henan | L | 4.090 | 0.309 |
| Inner Mongolia | L | 4.420 | 2.552 |
| Chongqing | L | 1.704 | 1.664 |
| Sichuan | L | 4.424 | 3.908 |
| Total | | 137.870 | 109.803 |

From the above Table 9, we can see that the total CO₂ emission is 137.870. However the CO₂ emission reduction quota for Other Manufacture is 701.904, which means using only inefficient regions as the reduction targets cannot meet the requirements. Here we have to use the efficient regions as the CO₂ emission reduction targets also. See Column 3 in Table 9. As mentioned in Remark 1, in certain cases we need to replace the Assumption 1-3 as the following Assumption 4:

***Assumption 4.** The efficient frontier can be shift in the process of CO₂ emissions reduction using an average way, which means the existing technology need to be improved by reducing the same proportion of CO₂ emission for each DMU.*

From this assumption, we can see that, for Other Manufacture, we first find the amount of CO₂ emission of projections of inefficient regions on the frontier of model (1). See Column 4 in Table 9. Therefore, we can see that if we fix the efficient frontier in model (1), the maximum amount of CO₂ emission reduction is 28.068. There is still a big gap between our CO₂ emission reduction target 701.904, which is $701.904 - 28.068 = 673.836$. Thus we allocate this 673.836 CO₂ emission to all regions using a proportional way and we can have the final allocation results as shown in the

Column 6 of Table 8. That means there is a strong need for Other Manufacture to improve its technology to meet the CO₂ emission reduction targets.

The conventional and inverse DEA are two different methods in nature. They solve two different type of problems. Being completely two different methods, the results are not really comparable. The conventional DEA focuses on the data and finds the efficiency score while the inverse DEA focuses on the efficiency and finds the data point.

5. Concluding Remarks

In this paper we tried to tackle the problem of allocating the CO₂ emissions quota set by government goal in Chinese manufacturing industries to different Chinese regions. This objective is implemented using a three-stage way based on several assumptions. In the first stage, we obtained the total amount CO₂ emission reduction from the Chinese government goal as our total CO₂ emission quota to allocate to different regions to reduce. Based on this, we further allocate the reduction quota to different two-digit level manufacturing industries in China in the second stage. In the last stage we allocate the CO₂ emissions reduction quota for each industry into different provinces. The empirical results can provide an alternative solution for the allocation of CO₂ emissions reduction in China for policy making.

Acknowledgment

We would like to acknowledge the supports of National Natural Science Foundation of China (71671181) and Newton Fund from Royal Academy of Engineering (NRCP/1415/80).

References

- Ahuja, R.K. and Orlin J.B., (2001). Inverse Optimization. *Operations Research*, 49(5): 771-783.
- Amin, GR., Al-Muharrami, S. (2016). A new inverse data envelopment analysis model for mergers with negative data. *IMA Journal of Management Mathematics*, In press.
- Amin, GR., Emrouznejad, A. (2007). Inverse forecasting: A new approach for predictive modeling. *Computers & Industrial Engineering* 53(3): 491-498.
- Amin, GR., Emrouznejad, A., Gattoufi, S. (2017a). Minor and major consolidations in inverse DEA: Definition and determination. *Computers and Industrial Engineering*, 103(1), 193-200.

- Amin, G.R., Emrouznejad, A., Gattoufi, S. (2017b). Modelling generalized firms' restructuring using inverse DEA. *Journal of Productivity Analysis*, 48(1), 51-61.
- An, Q., Pang, Z.Q., Chen, H.X., Liang, L.(2015). Closest targets in environmental efficiency evaluation based on enhanced Russell measure. *Ecological Indicators* 51: 59-66.
- Arabi, B., Munisamy, S., Emrouznejad, A. (2015). A new slacks-based measure of Malmquist–Luenberger index in the presence of undesirable outputs. *Omega* 51: 29-37.
- Bi, G.-B., Song, W., Zhou, P., Liang, L. (2014a). Does environmental regulation affect energy efficiency in China's thermal power generation? Empirical evidence from a slacks-based DEA model. *Energy Policy* 66: 537-546.
- Bian, Y., Liang, N.N., Xu, H. (2015). Efficiency evaluation of Chinese regional industrial systems with undesirable factors using a two-stage slacks-based measure approach. *Journal of Cleaner Production* 87: 348-356.
- BP (2011). *Statistical Review of World Energy*. Available at: <http://www.bp.com/statisticalreview>.
- Burton, D., Toint, Ph.L. (1992). On an instance of the inverse shortest paths problem. *Mathematical Programming* 53: 45-61.
- Chang, T.-P. and J.-L. Hu (2010). Total-factor energy productivity growth, technical progress, and efficiency change: An empirical study of China. *Applied Energy* 87(10): 3262-3270.
- Chen, S.Y. (2009). Energy Consumption, CO₂ Emissions and Sustainable Development of Chinese Industries. *Economic Research Journal* 4, 41-54. (in Chinese).
- Cui, Q. and Y. Li (2015). An empirical study on the influencing factors of transportation carbon efficiency: Evidences from fifteen countries. *Applied Energy* 141: 209-217.
- Du, M., Wang, B., Wu, Y.R. (2014). Sources of China's Economic Growth: An Empirical Analysis Based on the BML Index with Green Growth Accounting. *Sustainability* 6(9): 5983-6004.
- Emrouznejad, A., Yang, G. (2016a). A framework for measuring global Malmquist-Luenberger productivity index with CO₂ emissions on Chinese manufacturing industries. *Energy* 115, 840-856.
- Emrouznejad, A., Yang, G. (2016b). CO₂ emissions reduction of Chinese light manufacturing industries: A novel RAM-based global Malmquist–Luenberger productivity index. *Energy Policy* 96, 397-410.
- Fan, M., Shao, S., Yang, L.L. (2015). Combining global Malmquist-Luenberger index and generalized method of moments to investigate industrial total factor CO₂ emission performance: A case of Shanghai (China). *Energy Policy* 79: 189-201.
- Gattoufi, S., Amin, G.R., Emrouznejad, A., (2014). A new inverse DEA method for merging banks. *IMA Journal of Management Mathematics*, 25(1), 73-87.
- Ghiyasi, M., (2017). Industrial sector environmental planning and energy efficiency of Iranian provinces. *Journal of Cleaner Production*, 142, 2328-2339.
- Ghobadi, S., Jahangiri, S. (2015). Inverse DEA: Review, Extension and Application. *International Journal of Information Technology and Decision Making* 14(4): 805-824.
- Green, C. (2000). Potential Scale-related problems in estimating the cost of CO₂ mitigation policies. *Climatic Change* 44, 331-349.

- Hadi-Vencheh, A., Foroughi, A.A., Soleimani-damaneh, M. (2008). A DEA model for resource allocation. *Economic Modelling* 25(5): 983-993.
- He, F., Zhang, Q.Z., Lei, J.S., Fu, W.H., Xu, X.N.(2013). Energy efficiency and productivity change of China's iron and steel industry: Accounting for undesirable outputs. *Energy Policy* 54: 204-213.
- Hou, L., Hoag, D., Keske, C.M.H., Lu, C.(2014). Sustainable value of degraded soils in China's Loess Plateau: An updated approach. *Ecological Economics* 97: 20-27.
- Huang, J., Yang, X., Cheng, G., Wang, S.(2014). A comprehensive eco-efficiency model and dynamics of regional eco-efficiency in China. *Journal of Cleaner Production* 67: 228-238.
- Huang, S., Liu, Z. (1999). On the inverse problem of linear programming and its application to minimum weight perfect k-matching. *European Journal of Operational Research* 112, 421-426.
- IPCC (2006). IPCC Guidelines for National Greenhouse Gas Inventories. <http://www.ipcc-nggip.iges.or.jp/public/2006gl/vol2.html>.
- Jahanshahloo, G.R., Soleimani-damaneh, M., Ghobadi, S. (2015). Inverse DEA under inter-temporal dependence using multiple-objective programming. *European Journal of Operational Research* 240(2): 447-456.
- Jiang, Y., Xiao, X., Zhang, L., Zhang, J. (2011). A perturbation approach for a type of inverse linear programming problems. *International Journal of Computers Mathematics* 88(3): 508-516.
- Kaneko, S. and S. Managi (2004). Environmental productivity in China. *Economics bulletin* 17(2): 1-10.
- Kaneko, S., Fujii, H., Sawazu, N., Fujikura, R. (2010). Financial allocation strategy for the regional pollution abatement cost of reducing sulfur dioxide emissions in the thermal power sector in China. *Energy Policy* 38(5): 2131-2141.
- Krautzberger, L. Wetzel, H.(2012). Transport and CO₂: productivity growth and carbon dioxide emissions in the european commercial transport industry. *Environmental and Resource Economics* 53(3): 435-454.
- Lee, M. (2011). Potential cost savings from internal/external CO₂ emissions trading in the Korean electric power industry. *Energy Policy* 39(10): 6162-6167.
- Lim, D.J., (2016). Inverse DEA with frontier changes for new product target setting. *European Journal of Operational Research*, 254(2), 510-516.
- Long, X., Oh, K., Cheng, G. (2013). Are stronger environmental regulations effective in practice? The case of China's accession to the WTO. *Journal of Cleaner Production* 39: 161-167.
- Mandiloo, M., Tavana, M., Saen, R.F., Noorizadeh, A. (2014). A game theoretic approach to modeling undesirable outputs and efficiency decomposition in data envelopment analysis. *Applied Mathematics and Computation* 244: 479-492.
- Matsushita, K. , Yamane, F. (2012). Pollution from the electric power sector in Japan and efficient pollution reduction. *Energy Economics* 34(4): 1124-1130.
- Molinos-Senante, M., Hernandez-Sancho, F., Mocholi-Arce, M., Sala-Garrido, R. (2014). Economic and environmental performance of wastewater treatment plants: Potential reductions in greenhouse

- gases emissions. *Resource and Energy Economics* 38: 125-140.
- Mukherjee, K. (2010). Measuring energy efficiency in the context of an emerging economy: The case of Indian manufacturing. *European Journal of Operational Research* 201(3): 933-941.
- Murty, M. N., Kumar, S., Dhavala, K.K.(2007). Measuring environmental efficiency of industry: a case study of thermal power generation in India. *Environmental and Resource Economics* 38(1): 31-50.
- Nakano, M. Managi, S. (2008). Regulatory reforms and productivity: An empirical analysis of the Japanese electricity industry. *Energy Policy* 36(1): 201-209.
- Oh, D., Heshmati, A. (2010). A sequential Malmquist–Luenberger productivity index: Environmentally sensitive productivity growth considering the progressive nature of technology. *Energy Economics* 32, 1345–1355.
- Pibernik, R. Zhang, Y. Kerschbaum, F. and Schröpfer, A., (2011). Secure collaborative supply chain planning and inverse optimization–The JELS model. *European Journal of Operational Research* 208(1): 75-85.
- Riccardi, R., Oggioni, G., Toninelli, R. (2012). Efficiency analysis of world cement industry in presence of undesirable output: application of data envelopment analysis and directional distance function. *Energy Policy* 44, 140-152.
- Ruiz, C., Conejo, A.J., and Bertsimas, D.J., (2013). Revealing rival marginal offer prices via inverse optimization. *IEEE Transactions on Power Systems* 28(3): 3056- 3064.
- Sueyoshi, T., Goto, M. (2014a). Photovoltaic power stations in Germany and the United States: A comparative study by data envelopment analysis. *Energy Economics* 42: 271-288.
- Sueyoshi, T., Goto, M. (2014b). DEA radial measurement for environmental assessment: A comparative study between Japanese chemical and pharmaceutical firms. *Applied Energy* 115: 502-513.
- Vlontzos, G., Niavis, S., Manos, B. (2014). A DEA approach for estimating the agricultural energy and environmental efficiency of EU countries. *Renewable & Sustainable Energy Reviews* 40: 91-96.
- Wang, K., Wei, Y. M., Zhang, X. (2013b). Energy and emissions efficiency of Chinese regions: a multidirectional efficiency analysis. *Applied Energy* 104, 105–116.
- Wang, M., Xu, F., and Wang, G., (2014). Sparse portfolio rebalancing model based on inverse optimization. *Optimization Methods and Software*, 29(2): 297-309.
- Wang, Q., Zhao, Z., Shen, N., Liu, T. (2015). Have Chinese cities achieved the win-win between environmental protection and economic development? From the perspective of environmental efficiency. *Ecological Indicators* 51: 151-158.
- Wang, Q., Zhou, P., Shen, N., Wang, S. (2013a). Measuring carbon dioxide emission performance in Chinese provinces: a parametric approach. *Renewable and Sustainable Energy Reviews* 21: 324-330.
- Watanabe, M., Tanaka, K. (2007). Efficiency analysis of Chinese industry: a directional distance function approach. *Energy Policy* 35(12): 6323-6331.
- Wei, Q. Zhang, J. and Zhang, X., (2000). An inverse DEA model for inputs/outputs estimate. *European*

- Journal of Operational Research, 121(1): 151-163.
- Wu, F., Fan, L.W., Zhou, P., Zhou, D.Q. (2012). Industrial energy efficiency with CO₂ emissions in China: a nonparametric analysis. *Energy Policy* 49: 164-172.
- Wu, J., An, Q., Yao, X., Wang, B. (2014). Environmental efficiency evaluation of industry in China based on a new fixed sum undesirable output data envelopment analysis. *Journal of Cleaner Production* 74: 96-104.
- Yan, H., Wei, Q., Hao, G. (2002). DEA models for resource reallocation and production input/output estimation. *European Journal of Operational Research* 136(1): 19-31.
- Yang, L., Ouyang, H., Fang, K., Ye, L., Zhang, J. (2015). Evaluation of regional environmental efficiencies in China based on super-efficiency-DEA. *Ecological Indicators* 51: 13-19.
- Yang, L., Wang, K.-L. (2013). Regional differences of environmental efficiency of China's energy utilization and environmental regulation cost based on provincial panel data and DEA method. *Mathematical and Computer Modelling* 58(5): 1074-1083.
- Yin, K., Wang, R., An, Q., Yao, L., Liang, J. (2014). Using eco-efficiency as an indicator for sustainable urban development: A case study of Chinese provincial capital cities. *Ecological Indicators* 36: 665-671.
- Yörük, B. K., Zaim, O. (2005). Productivity growth in OECD countries: A comparison with Malmquist indices. *Journal of Comparative Economics* 33(2): 401-420.
- Yuan, P., Cheng, S., Sun, J., Liang, W. (2013). Measuring the environmental efficiency of the Chinese industrial sector: A directional distance function approach. *Mathematical and Computer Modelling* 58(5-6): 936-947.
- Zhang, C., Liu, H., Bressers, H.T.A., Buchanan, K.S. (2011). Productivity growth and environmental regulations-accounting for undesirable outputs: Analysis of China's thirty provincial regions using the Malmquist-Luenberger index. *Ecological Economics* 70(12): 2369-2379.
- Zhang, J., Liu, Z. (1996). Calculating some inverse linear programming problems. *Journal of Computational and Applied Mathematics* 72: 261-273.
- Zhang, J., Liu, Z. (1999). A further study on inverse linear programming problems. *Journal of Computational and Applied Mathematics* 106: 345-359.
- Zhang, M., Cui, J.C., (2016). The extension and integration of the inverse DEA method. *Journal of the Operational Research Society*, In press.
- Zhang, N., Choi, Y. (2013a). A comparative study of dynamic changes in CO₂ emission performance of fossil fuel power plants in China and Korea. *Energy Policy* 62: 324-332.
- Zhang, N., Choi, Y. (2013b). Environmental energy efficiency of China's regional economies: A non-oriented slacks-based measure analysis. *The Social Science Journal* 50(2): 225-234.
- Zhang, N., Choi, Y. (2013c). Total-factor carbon emission performance of fossil fuel power plants in China: A metafrontier non-radial Malmquist index analysis. *Energy Economics* 40: 549-559.
- Zhang, N., Kong, F., Choi, Y. (2014). Measuring sustainability performance for China: A sequential generalized directional distance function approach. *Economic Modelling* 41: 392-397.

- Zhang, N., Zhou, P., Kung, C.-C. (2015). Total-factor carbon emission performance of the Chinese transportation industry: A bootstrapped non-radial Malmquist index analysis. *Renewable & Sustainable Energy Reviews* 41: 584-593.
- Zhou, G., Chuang, W., Zhang, Y. (2014). Measuring energy efficiency performance of China's transport sector: A data envelopment analysis approach. *Expert Systems with Applications* 41(2): 709-722.
- Zhu, Z., Wang, K., Zhang, B. (2014). Applying a network data envelopment analysis model to quantify the eco-efficiency of products: a case study of pesticides. *Journal of Cleaner Production* 69: 67-73.