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## A DEA/Goal Programming Model for Incineration Plants Performance in the UK

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### Abstract

Incineration plants in UK carry out two important tasks: reduction of waste disposed to landfills, and power/heat production from waste incineration distributed to the grid. However, incinerating waste produces, except for desirable outputs like exported power, harmful emissions, too. In this work, a DEA/Goal Programming model is presented to assess the performance of each incineration plant. Data from 22 incineration plants have been collected regarding capacity (waste and power), power exported, annual availability and levels of harmful emissions. The proposed model provides an allocation of the examined incineration plants, by shutting down a plant if it doesn't meet environmental targets. Additional constraints are considered regarding levels of power exported and annual availability. The model is solved for multiple scenarios regarding the number of incineration plants that will be eventually installed. Results are provided regarding the optimal allocation of each incineration plant and the optimal values of under and over achievement of each environmental target. Additionally, a comparative analysis is conducted on the scores derived from the proposed method and DEA models that handle both desirable and undesirable outputs. No differences between the two rankings are derived by applying statistical analysis.

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### 1. Introduction

A total of 170 million tonnes of waste is produced from households and businesses in England and Wales each year [ *Sambrook L, 2015*]. There are multiple ways of handling and treating municipal waste. These are: a) landfilling, b) mass burn with energy/heat recovery, c) waste recycling and finally d) waste composting [ *Daskalopoulos E, et al, 1998*]. Regarding landfilling, a waste reduction has been reported in years 2005 – 2006. This

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waste reduction in UK over the past years is partly ought to Directive 1999/31/EC [ *Directive C. DIRECTIVE 1999*], which imposed regulations and set targets regarding biodegradable municipal waste. An alternative way of reducing waste is by incinerating it. With municipal waste incineration, there are benefits (power export, heat produced) but also undesirable excipients (harmful gas emissions, bottom ash etc). However from bottom ash, ferrous and non-ferrous products can be derived. Concentrating mostly on the hazardous gas emissions from municipal waste incineration, several studies are presented which provide information regarding concentrations of gas emissions. Thus, it has also been reported that municipal waste incineration can help in greenhouse gas (GHG) emissions reduction, under specific conditions [ *Papageorgiou A et al, 2009*]. Except for the contribution in GHG reduction, during incineration procedure for power/heat production, it has been reported that polychlorinated biphenyls (PCB) and polycyclic aromatic hydrocarbons (PAH) are emitted [ *et al, 2003*]. Life cycle analysis (LCA) is also another tool of assessing environmental impact of wastewater [ *Gallejo A, et al, 2008*] and MSW facilities [ *Zografidou E, et al, 2015*].

## 2. Methodology

The need to assess incineration plants is of great significance as multiple characteristics must be taken into account. One of the methods that handle this multi-dimensional analysis is Data Envelopment Analysis (DEA). However, as DEA is a methodology that provides efficiency scores based on pre-determined inputs and outputs, information regarding the performance and each Decision Making Units (DMUs) is extracted. Integrating Goal Programming (GP) and DEA, decisions regarding not only efficiency assessment, but also regarding the selection (or not) of a DMU is possible, setting goals modeling multiple aspects of the problem (economic, environmental, social etc). [8].

In this paper a hybrid GP/DEA model is presented. The model integrates binary variables in order to demonstrate which incineration plant is selected based on environmental and operational targets. The proposed model is based on an existing GP/DEA model [ *Izadikhah M, et al, 2003*] and extends its features by providing additional information to Decision Maker (DM). The proposed GP/DEA model selects the entities (DMUs) based on a selection of environmental goals and operational constraints. This characteristic allows the model to examine the DMUs (in this case incineration plants) that over or under perform based on the goals.

The data used in this paper are derived from environmental agencies and papers published in the relevant literature and are selected to capture model environmental, capacity and operational characteristics of each incineration plant. The production process that is assumed in this case is that based on waste and power capacity of each incineration plant, exported power and harmful gasses and particles are emitted. Also, annual availability of each plant is taken into account, even if, it is not part of the production process. Decision variables are considered in this model in order to select those incineration plants, and eventually the number of incineration plants that must operate in order to satisfy environmental constraints, and to maintain the power exported to the grid. As these two characteristics (power produced and harmful gas emissions) are linearly dependent (i.e. the more power generated, the more the environmental pollution) and are therefore taken into account in the form of goals (for gas emissions) and of constraints (for power generated). Annual availability is considered also in this model, as based on statistical analysis, the more the annual availability is independent from power exported to the grid. Scenarios regarding the number of possible facilities that will be selected are also conducted.

With the proposed formulation, it is possible to evaluate and eventually select that incineration plant that satisfies a series of data that are defined in advance from the DM. Due to the flexibility of GP modeling, it is possible to incorporate, except for straightforward data for technical, economic and environmental characteristics of each incineration plant, data regarding people's opinions and views towards these plants.

### 2.1 Mathematical Formulation

#### 2.1.1 Nomenclature

Table 1. Indices, variables and parameters of the proposed model

Index	$j = 1, \dots, n$	DMU (Incineration plant)
	$r = 1, \dots, s$	Outputs
	$i = 1, \dots, m$	Inputs
	$\varepsilon = 1, \dots, E$	Scenarios
	$\lambda_j$	Peers of incineration (DMU) $j$
Variables	$\hat{\lambda}_j$	Auxiliary variable for linearization of bilinear term $\lambda_j \cdot \xi_j$
	$\xi_j$	1 if incineration plant is selected, 0 otherwise
	$\eta$	Auxiliary binary variable for disjunction of constraints
	$d_r^-$	Under achievement of target $r$
	$d_r^+$	Over achievement of target $r$
	$y_{r,j}^{und}$	Undesirable output $r$ of DMU $j$
	$x_{i,j}$	Input $i$ of DMU $j$
Parameters	$G_r^{und}$	Goal for undesirable output $r$
	$PE_j$	Power exported of incineration plant (DMU) $j$ (KWh)
	$AV_j$	Annual availability of incineration plant (DMU) $j$ (%)

2.1.2 Model description

Economic systems entail production processes; assuming that  $i=1, \dots, m$  inputs are consumed to produce  $r=1, \dots, s$  outputs, then the following DEA models are used in order to assess the efficiency of the entities that are examined (DMUs). Focusing only on envelopment models, the following Linear Programming (LP) formulations are input or output oriented DEA models [Charnes A, et al, 1984; Charnes A, et al, 2981]:

Table 2. Input (a) and output (b) oriented DEA models

$\min \theta$ $s.t.$ $\sum_{j=1}^n x_{ij} \cdot \lambda_j \leq x_{io} \cdot \theta$ $\sum_{j=1}^n y_{rj} \cdot \lambda_j \geq y_{ro}$ $\lambda_j \geq 0, j = 1, \dots, n$ $\theta \text{ free}$ <p>(a)</p>	$\max \varphi$ $s.t.$ $\sum_{j=1}^n x_{ij} \cdot \lambda_j \leq x_{io}$ $\sum_{j=1}^n y_{rj} \cdot \lambda_j \geq y_{ro} \cdot \varphi$ $\lambda_j \geq 0, j = 1, \dots, n$ $\varphi \text{ free}$ <p>(b)</p>
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Models presented in Table 2 (a) and (b) are the conventional DEA models. The outputs are treated all as desirable. However, every economic entity may produce undesirable outputs by consuming inputs. In the present study, incineration plants produce, among desirable outputs, undesirable outputs as well. During the incineration procedure, municipal waste is consumed producing power but also harmful gas emissions. The DEA model that assesses a production procedure based on desirable and undesirable outputs is presented with the following LP formulation [Sueyoshi T, et al, 2011][Sueyoshi T, et al, 2010][Sueyoshi T, et al, 2011].

$$\begin{aligned}
 & \max \beta \\
 & s.t. \\
 & \sum_{j=1}^n \lambda_j \cdot x_{j,i} \leq x_{o,j}, \quad i = 1, \dots, m \\
 & \sum_{j=1}^n \lambda_j \cdot y_{j,r_1} \geq y_{o,r_1}^{des} \cdot (1 + \beta), \quad r_1 = 1, \dots, s_1 \\
 & \sum_{j=1}^n \lambda_j \cdot y_{j,r_2} \leq y_{o,r_2}^{undes} \cdot (1 - \beta), \quad r_2 = 1, \dots, s_2 \\
 & \lambda_j \geq 0, \quad j = 1, \dots, n \\
 & \beta \text{ free}
 \end{aligned} \tag{1}$$

In DEA model (1), a DMU is considered efficient if  $\beta^* = 0$ . However, DEA models assess the production process based on a number of given datasets for inputs and outputs. Real world problems generally have more than one objective, based on which criteria are optimized (maximized or minimized). Goal Programming (GP) models are generally used in order to such problems. A typical GP model is formulated as follows:

$$\begin{aligned}
 & \min w^1 \cdot \delta^- + w^2 \cdot \delta^+ + w^3 \cdot (\delta^- + \delta^+) \\
 & s.t. \\
 & \sum_{k=1}^K a_k \cdot x_k + \delta^- - \delta^+ = G \\
 & \sum_{k=1}^K a_k \cdot x_k \leq b \\
 & x_k \geq 0, \quad \forall k
 \end{aligned} \tag{2}$$

In formulation (2), slack variables are minimized in the objective function according to the direction of each goal. The flexibility of this formulation provides decisions for variables regarding location – allocation models, based on several criteria (social, economic, environmental etc). The methodology used in this paper, is a hybrid DEA/GP model [9]. With this combination, the inputs and outputs are utilized, while each combination of incineration plants is examined with the GP approach. The resulting model is a MILP mathematical programming model and is presented in formulation (3). In this model, the aim is to minimize the slack variables that over estimate the goals of undesirable outputs; in this case, these undesirable outputs concern harmful gas emissions. The first set of constraints are introduced to model the goals for undesirable outputs; the second set of constraints, concern the inputs. In both cases, a binary variable is introduced in the model, examining only those DMUs, which satisfy constraints third or fourth set of constraints. Binary variable  $\eta$  is introduced to model the disjunction  $\sum_{j=1}^n PE_j \cdot \xi_j \geq PE^U \vee \sum_{j=1}^n AV_j \cdot \xi_j \geq AV^U$  (either power exported or annual availability of each incineration plant should be over a specific threshold). Parameters  $M^1$  and  $M^2$  are sufficient upper bounds of each constraint.

As it can be seen from Figure 1, parameters for power exported and annual availability have very low correlation ( $p = 0.10$ ); thus, DMUs should be selected on one of the two parameters as the more the annual availability does not imply more power exported to the grid.

Fifth constraint guarantees Variable Returns to Scale (VRS) technology for the selected DMUs while sixth and seventh constraints are introduced to bound the number of selected DMUs in the range of  $[\mu, K]$ .

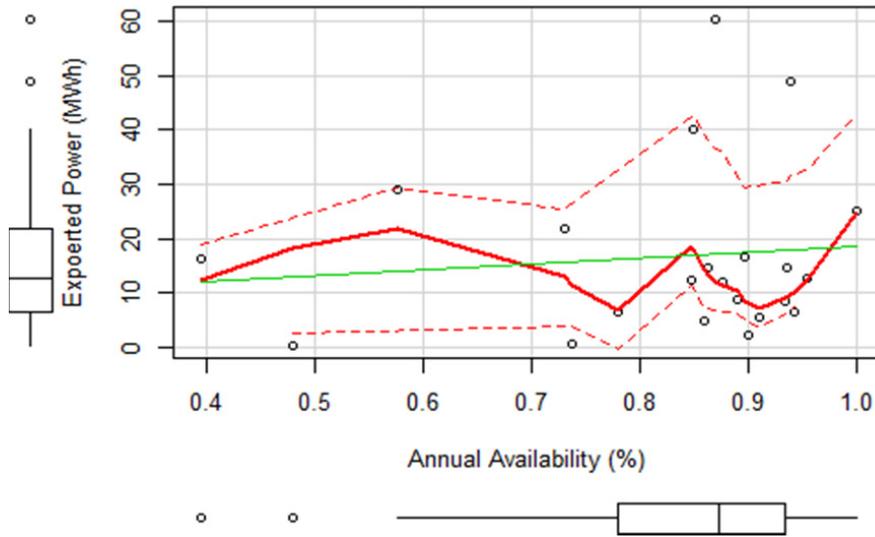


Fig. 1. Scatterplot and boxplot for power exported and annual availability

$$\begin{aligned}
 & \min \sum_{r=1}^s \frac{d_r^+}{G^r} \\
 & \text{s.t.} \\
 & \sum_{j=1}^n \lambda_j \cdot \xi_j \cdot y_{rj}^m + d_r^+ - d_r^- = G^r, \quad r=1, \dots, s & \sum_{j=1}^n \xi_j \leq K \\
 & \sum_{j=1}^n \lambda_j \cdot \xi_j \cdot x_{ij} \leq x_{i0}, \quad i=1, \dots, m & \sum_{j=1}^n \xi_j \geq \mu \\
 & \sum_{j=1}^n PE_j \cdot \xi_j \geq PE^p + M \cdot \eta & \xi_j \in \{0,1\}, \\
 & \sum_{j=1}^n AV_j \cdot \xi_j \geq AV^p + M \cdot (1-\eta) & \eta \in \{0,1\}, \\
 & \sum_{j=1}^n \lambda_j \cdot \xi_j = 1 & \lambda_j \geq 0, \quad j=1, \dots, n
 \end{aligned} \tag{3}$$

Due to the existence of bilinear terms (product of binary and continuous variable), model (3) is a Mixed Integer Non – Linear Programming (MINLP) model. In order to avoid any local optima (due to existence of a non linear model), the following constraints are introduced. Triplets of constraints (4) – (6) are introduced to linearize bilinear term  $(\lambda \cdot \xi)$ .

$$\hat{\lambda}_j - M_j \cdot \xi_j \leq 0, \quad j = 1, \dots, n \tag{4}$$

$$\hat{\lambda}_j - \lambda_j \leq 0, \quad j = 1, \dots, n \tag{5}$$

$$\lambda_j - \hat{\lambda}_j + M_j \cdot \xi_j \leq M_j, \quad j = 1, \dots, n \tag{6}$$

In linearization constraints (4), (6) the upper bound of  $\lambda$  value ( $M_j$ ) is 1. Model (3) is formulated, based on (4) – (6) as follows:

In order to evaluate all possible combinations of DMUs, constraints that bound the number of selected DMUs change, leading to different scenarios. Scenarios for upper bounds are introduced, leading to formulation (8). In this case, 10 scenarios are introduced for the maximum number of selected DMUs; namely  $K^1 = 22, K^2 = 21, \dots, K^9 = 14,$

$K^{10} = 13$ . The minimum number of incineration plants ( $\mu$ ) is defined as 13. Thus, for  $\varepsilon = 10$  constraints  $\sum_{j=1}^n \xi_j \leq K^\varepsilon$  and  $\sum_{j=1}^n \xi_j \geq \mu$  lead to  $\sum_{j=1}^n \xi_j = 13$ .

$$\begin{aligned}
 & \text{for } \varepsilon = 1, \dots, E \\
 & \min \sum_{r=1}^s \frac{d_r^+}{G_r^{und}} \qquad \sum_{j=1}^n \xi_j \geq \mu \\
 & \text{s.t.} \qquad \sum_{j=1}^n \hat{\lambda}_j \cdot y_{r,j}^{und} + d_r^- - d_r^+ = G_r^{und}, \quad r = 1, \dots, s \qquad \hat{\lambda}_j - \xi_j \leq 0, \quad j = 1, \dots, n \\
 & \qquad \sum_{j=1}^n \hat{\lambda}_j \cdot x_{i,j} \leq x_{i,0}, \quad i = 1, \dots, m \qquad \hat{\lambda}_j - \lambda_j \leq 0, \quad j = 1, \dots, n \\
 & \qquad \sum_{j=1}^n PE_j \cdot \xi_j \geq PE^E + M \cdot \eta \qquad \lambda_j - \hat{\lambda}_j + \xi_j \leq 1, \quad j = 1, \dots, n \\
 & \qquad \sum_{j=1}^n AV_j \cdot \xi_j \geq AV^E + M^1 \cdot (1 - \eta) \qquad \xi_j \in \{0, 1\}, \\
 & \qquad \sum_{j=1}^n \hat{\lambda}_j = 1 \qquad \eta \in \{0, 1\}, \\
 & \qquad \sum_{j=1}^n \xi_j \leq K^\varepsilon \qquad \lambda_j \geq 0, \quad j = 1, \dots, n \\
 & \qquad \qquad \qquad \qquad \qquad \hat{\lambda}_j \geq 0, \quad j = 1, \dots, n \\
 & \qquad \qquad \qquad \qquad \qquad \text{and for}
 \end{aligned} \tag{7}$$

### 3. Conclusions

In this work, a hybrid GP/DEA model is presented. The ability of DEA technique to consider inputs and outputs is utilized to provide a performance measurement model of each incineration plant. The presented formulation aims at the reduction of the examined facilities that exceed the environmental targets set but satisfy the constraints of power exported and annual availability.

$$\begin{aligned}
 & \min \sum_{r=1}^s \frac{d_r^+}{G_r^{und}} \\
 & \text{s.t.} \qquad \sum_{j=1}^n \hat{\lambda}_j \cdot y_{r,j}^{und} + d_r^- - d_r^+ = G_r^{und}, \quad r = 1, \dots, s \qquad \sum_{j=1}^n \hat{\lambda}_j = 1 \qquad \hat{\lambda}_j - \lambda_j \leq 0, \quad j = 1, \dots, n \\
 & \qquad \sum_{j=1}^n \hat{\lambda}_j \cdot x_{i,j} \leq x_{i,0}, \quad i = 1, \dots, m \qquad \sum_{j=1}^n \xi_j \leq K \qquad \lambda_j - \hat{\lambda}_j + \xi_j \leq 1, \quad j = 1, \dots, n \\
 & \qquad \sum_{j=1}^n PE_j \cdot \xi_j \geq PE^U + M^1 \cdot \eta \qquad \sum_{j=1}^n \xi_j \geq \mu \qquad \xi_j \in \{0, 1\}, \\
 & \qquad \sum_{j=1}^n AV_j \cdot \xi_j \geq AV^U + M^2 \cdot (1 - \eta) \qquad \lambda_j \geq 0, \quad j = 1, \dots, n \qquad \eta \in \{0, 1\}, \\
 & \qquad \qquad \qquad \qquad \qquad \hat{\lambda}_j - \xi_j \leq 0, \quad j = 1, \dots, n \qquad \hat{\lambda}_j \geq 0, \quad j = 1, \dots, n
 \end{aligned} \tag{8}$$

The basic idea of not setting power exported and annual availability in GP constraints is that the more each incineration plant works and the more power is exported to the grid, the more emissions are generated. The resulting model is a MILP model, with decision variables about which incineration plant satisfies the environmental targets and remains open, and the peers of that plant. As due to MINLP formulation, local optima may arise, linearization constraints are introduced in order to make the model linear and obtain global optimal solution. With this formulation, it is possible to evaluate different combinations of plants and numbers of facilities that will be open by solving the model for various scenarios of the number of facilities that will be eventually remain open. Operational constraints regarding power exported and annual availability of each incineration plant are introduced.

Preprocessing of data indicated that these constraints are “either-or” type of constraints; constraints regarding the number of incineration plants that will be potentially installed are also taken into account. Such model has not been proposed before in the known literature as provides decision levels, not only regarding the “peers” of each incineration plant, but also about whether the incineration plant can be eventually be installed subjected to constraints regarding the power that should be exported and the annual availability. Each incineration plant that does not meet these requirements is not selected and therefore the total overachievement target is minimized.

Treating municipal waste is of major importance as due to consumerism way of life and the vast quantities of waste generated, landfilling is not a viable option. Besides the inefficiency of landfilling for waste municipal, Britain and other countries are subjected to several EU directives for waste reduction. Considering all the aforementioned factors, a turn to incineration of municipal waste has been made in the previous years. During the incineration procedure waste is treated as input; the products of this procedure are both desirable and undesirable. As desirable outputs are considered the power/heat exported to grid, ferrous and non-ferrous metals, while as undesirable outputs harmful gas emissions and fly particulates. However, this production process may not be efficient. In order to measure the efficiency of this production process for each incineration plant, DEA models that take into account both desirable and undesirable outputs, have been employed in the relevant literature. In this paper, a hybrid GP/DEA model is presented for the assessment and selection of those incineration plants satisfies environmental goals and operational constraints. Due to the use of GP, the analysis is possible to capture qualitative characteristics like people’s perceptions and opinions, measuring the social impact. This, type of modeling, besides taking into account the straightforward quantitative factors (mostly economic, environmental) can set goals regarding factors that are often captured with qualitative data. Based on DEA principle, the proposed model can assess the performance of each DMU (incineration plant in this case), compared with the other DMUs.

The novelty of the model is that binary variables for selecting DMUs are introduced in the constraints; only those DMUs that satisfy environmental goals regarding harmful gas emissions and operational constraints are selected. After data preprocessing, it has been concluded that power exported and annual availability are not linked and can be taken into account as a disjunction. Furthermore, constraints for the bounds of the number of DMUs selected are introduced. Scenarios have been also considered for the upper bound of selected DMUs. Results demonstrate that the target for Nitrogen Oxides ( $\text{NO}_x$ ) is not met as in each scenario it is over achieved.

A comparison between the proposed GP/DEA model and a DEA model that takes desirable and undesirable outputs into account is also demonstrated; DMUs are ranked based on the efficiency derived from the two models. Non-parametric statistical analyses applied conclude that the two rankings come from the same distribution and follow the same pattern. The proposed model can be applied to assess the efficiency of any economic entity, providing a flexible modeling framework.

## References

- 1) Charnes A, Cooper WW, Rhodes E. Evaluating program and managerial efficiency: an application of data envelopment analysis to program follow through. *ManagSci* 1981;27:668–97.
- 2) Charnes A, Clark CT, Cooper WW, Golany B.A developmental study of data envelopment analysis in measuring the efficiency of maintenance units in the US air forces. *Ann Oper Res* 1984;2:95–112.
- 3) Cherubini F, Bargigli S, Ulgiati S. Life cycle assessment (LCA) of waste management strategies: Landfilling, sorting plant and incineration. *Energy* 2009;34:2116–23. doi:10.1016/j.energy.2008.08.023.
- 4) Daskalopoulos E, Badr O, Probert SD. An integrated approach to municipal solid waste management. *ResourConservRecycl* 1998;24:33–50.
- 5) Directive C. DIRECTIVE 1999/31/EC of 26 April 1999, L 182/1. *Off J Eur Communities* n.d.
- 6) Dyke PH, Foan C, Fiedler H. PCB and PAH releases from power stations and waste incineration processes in the UK. *Chemosphere* 2003;50:469–80. doi:10.1016/S0045-6535(02)00627-6.
- 7) Gallego A, Hospido A, Moreira MT, Feijoo G. Environmental performance of wastewater treatment plants for small populations. *ResourConservRecycl* 2008;52:931–40. doi:10.1016/j.resconrec.2008.02.001.
- 8) Izadikhah M, Roostae R, HosseinzadehLotfi F. Using goal programming method to solve DEA problems with value judgments. *Yugosl J Oper Res ISSN 0354-0243 EISSN 2334-6043* 2014;24
- 9) Papageorgiou A, Barton JR, Karagiannidis A. Assessment of the greenhouse effect impact of technologies used for energy recovery from municipal waste: A case for England. *J Environ Manage* 2009;90:2999–3012. doi:10.1016/j.jenvman.2009.04.012.
- 10) Sambrook L. How do we regulate landfill sites? U K n.d.<http://apps.environment-agency.gov.uk/wiyby/37825.aspx> (accessed March 26, 2015).

- 11) Sueyoshi T, Goto M. DEA approach for unified efficiency measurement: assessment of Japanese fossil fuel power generation. *Energy Econ* 2011;33:292–303.
- 12) Sueyoshi T, Goto M, Ueno T. Performance analysis of US coal-fired power plants by measuring three DEA efficiencies. *Energy Policy* 2010;38:1675–88.
- 13) Sueyoshi T, Goto M. Measurement of Returns to Scale and Damages to Scale for DEA-based operational and environmental assessment: How to manage desirable (good) and undesirable (bad) outputs? *Eur J Oper Res* 2011;211:76–89.
- 14) Zografidou E, Petridis K, Arabatzis G, Dey PK. Optimal design of the renewable energy map of Greece using weighted goal-programming and data Envelopment analysis. *ComputOper Res* n.d. doi:10.1016/j.cor.2015.03.012